pbdDMAT

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pbdDMAT-package

Distributed Matrix Methods

Description

A package for dense distributed matrix computations. Includes the use of PBLAS and ScaLAPACK libraries via pbdSLAP, communicating over MPI via the BLACS library and pbdMPI.

Details

Package: pbdDMAT
Type: Package
License: GPL
LazyLoad: yes

This package requires an MPI library (OpenMPI, MPICH2, or LAM/MPI).

Author(s)

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References

Programming with Big Data in R Website: http://r-pbd.org/

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ddmatrix-class

Class ddmatrix

Description

Distributed matrix class.

Creating Objects

```
new('ddmatrix', Data = ..., dim = ..., ldim = ..., bldim = ..., ICTXT = ...)
```

Slots

Data: LOCAL: Object of class matrix dim: GLOBAL: Object of class numeric ldim: LOCAL: Object of class numeric bldim: GLOBAL: Object of class numeric ICTXT: GLOBAL: Object of class numeric

Prototype

```
matrix Data matrix(0.0)
numeric dim c(1,1)
numeric ldim c(1,1)
numeric bldim c(1,1)
numeric ICTXT 0
```

Details

ddmatrix is the container for ScaLAPACK-friendly parallel block-cyclically distributed matrices. The class object is instantiated in SPMD fashion, whereby each process owns a piece of the "whole" matrix (which no single R process need ever own in its entirety), and each process stores its piece of the whole into a container with a name common to all processes.

The Data slot contains the data (submatrix) belonging to that process. Accessible via submatrix(). Values in the Data slot will vary from process to process.

The dim slot contains the global dimension; the dimension of the full matrix. Accessible via dim(). The dim slot is global, i.e. each process stores the same information in this slot.

The ldim slot contains the local dimension; here, all(ldim == dim(Data)). Accessible via ldim(). Values in the Data slot will vary from process to process.

The bldim slot contains the blocking factor for the block-cyclic distribution of the data. It consists of two numbers, namely the row and column blocking, respectively. Accessible via bldim(). The bldim slot is global, i.e. each process stores the same information in this slot.

The ICTXT slot contains the BLACS context onto which the matrix information is stored. This is mostly for internal bookkeeping, though advanced users might be able to effectively leverage

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differing BLACS contexts for performance improvements. Accessible via ictxt(). The ICTXT slot is global, i.e. each process stores the same information in this slot.

A very important piece of information is that every process must own something in the Data slot. This is essentially a ScaLAPACK "problem", but one that is not particularly hard to avoid so long as you are aware that it exists. A submatrix of matrix(0, nrow=1, ncol=1) is used if the matrix should not actually, technically, own part of the whole global matrix. You can easily still see if the stored submatrix is indeed part of the global matrix or just a placeholder with the ownany() function, which is just a wrapper on numroc() with argument fixme=FALSE.

See Also

SlotAccessors

pbdDMAT Control

Some default parameters for pbdDMAT.

Description

This set of controls is used to provide default values in this package.

Format

Objects contain several parameters for communicators and methods.

Details

The default blocking .BLDIM is c(4,4), which results in a 4 by 4 blocking dimension for distributed matrices. Any time a function takes the bldim= argument, it will default to this value unless the user specifies an alternative.

The default ICTXT is 0. This is the full 2-dimensional processor grid.

SlotAccessors

Accessor Functions for Distributed Matrix Slots

Description

Functions to get dimension information, local storage, or current BLACS context from a distributed matrix.

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Usage

```
## S4 method for signature 'ddmatrix'
nrow(x)
  ## S4 method for signature 'ddmatrix'
ncol(x)
  ## S4 method for signature 'ddmatrix'
NROW(x)
  ## S4 method for signature 'ddmatrix'
NCOL(x)
  ## S4 method for signature 'ddmatrix'
length(x)
  ## S4 method for signature 'ddmatrix'
dim(x)
  ## S4 method for signature 'ddmatrix'
submatrix(x)
  ## S4 method for signature 'ddmatrix'
ldim(x)
  ## S4 method for signature 'ddmatrix'
bldim(x)
  ## S4 method for signature 'ddmatrix'
ICTXT(x)
  ## S4 method for signature 'ddmatrix'
ownany(x, ...)
  ## S4 method for signature 'missing'
ownany(dim, bldim=.BLDIM, ICTXT=.ICTXT, x)
```

Arguments

X	numeric distributed matrix
dim	global dimension.
bldim	blocking dimension.
ICTXT	BLACS context.
	Extra arguments.

Details

The functions nrow(), ncol(), length() and dim() are the natural extensions of their ordinary matrix counterparts.

ldim() will give the dimension of the matrix stored locally on the process which runs the function. This is a local value, so its return is process-dependent. For example, if the 3x3 global matrix x is distributed as the ddmatrix dx across two processors with process 0 owning the first two rows and process 1 owning the third, then ldim(dx) will return 2 3 on process 0 and 1 3 on process 1.

bldim() will give the blocking dimension that was used to block-cyclically distribute the distributed matrix.

submatrix() will give the local storage for the requested object.

ICTXT() will give the current BLACS context (slot ICTXT) for the requested object.

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ownany() is intended mostly for developers. It answers the question "do I own any of the data?". The user can either pass a distributed matrix object or the dim, bldim, and ICTXT of one.

Value

```
Each of dim(), ldim(), bldim() return a length 2 vector.

Each of nrow(), ncol(), and length() return a length 1 vector. Likewise, so does ICTXT().

submatrix() returns a matrix; namely, submatrix(x) returns a matrix of dimensions ldim(x).
```

Methods

```
signature(x = "ddmatrix")
```

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

x <- ddmatrix(1:9, 3, 3)
x <- as.ddmatrix(x)

y <- list(dim=dim(x), ldim=ldim(x), bldim=bldim(x))
comm.print(y)

finalize()

## End(Not run)</pre>
```

DistributedMatrixCreation

Distributed Matrix Creation

Description

Methods for simple construction of distributed matrices.

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```
ddmatrix(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
  ## S4 method for signature 'missing'
ddmatrix(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
  ## S4 method for signature 'vector'
ddmatrix(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
  ## S4 method for signature 'character'
ddmatrix.local(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., min = 0, max = 1, mean = 0, sd = 1,
                                 rate = 1, shape, scale = 1,
                                  bldim = .BLDIM, ICTXT = .ICTXT)
 ## S4 method for signature 'matrix'
ddmatrix.local(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
  ## S4 method for signature 'missing'
ddmatrix.local(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
  ## S4 method for signature 'vector'
ddmatrix.local(data, nrow = 1, ncol = 1, byrow = FALSE,
                                 ..., bldim = .BLDIM, ICTXT = .ICTXT)
```

Arguments

data	optional data vector.
nrow	number of rows. Global rows for ddmatrix(). Local rows for ddmatrix.local(). See details below.
ncol	number of columns. Global columns for $ddmatrix()$. Local columns for $ddmatrix.local()$. See details below.
byrow	logical. If FALSE then the distributed matrix will be filled by column major storage, otherwise row-major.
	Extra arguments
min, max	Min and max values for random uniform generation.
mean,sd	Mean and standard deviation for random normal generation.
rate	Rate for random exponential generation.
shape,scale	Shape and scale parameters for random weibull generation.
bldim	blocking dimension.
ICTXT	BLACS context number.

Details

These methods are simplified methods of creating distributed matrices, including random ones. These methods involve only local computations, i.e., no communication is performed in the construction of a ddmatrix using these methods (in contrast to using as.ddmatrix() et al).

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For non-character inputs, the methods attempt to mimic R as closely as possible. So ddmatrix(1:3, 5, 7) produces the distributed analogue of matrix(1:3, 5, 7).

For character inputs, you may also specify additional parametric family information.

The functions predicated with .local generate data with a fixed local dimension, i.e., each processor gets an identical amount of data. Likewise, the remaining functions generate a fixed global amount of data, and each processor may or may not have an identical amount of local data.

To ensure good random number generation, you should only consider using the character methods with the comm.set.seed() function from pbdMPI which uses the method of L'Ecuyer via the rlecuyer package.

Value

Returns a distributed matrix.

Methods

```
signature(data = "character")
signature(data = "matrix")
signature(data = "missing")
signature(data = "vector")
```

See Also

```
as.ddmatrix
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

dx <- ddmatrix(data="rnorm", nrow=5, ncol=6, mean=10, sd=100)
dy <- ddmatrix(data=1:4, nrow=7, ncol=5)

print(dx)
print(dy)

finalize()

## End(Not run)</pre>
```

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Diag Distributed Matrix Diagonals	
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Description

Get the diagonal of a distributed matrix, or construct a distributed matrix which is diagonal.

Usage

Arguments

x	distributed matrix or a vector.
nrow,ncol	in the case that x is a vector, these specify the global dimension of the diagonal distributed matrix to be created.
type	character. Options are 'matrix' or 'ddmatrix', with partial matching. This specifies the return type.
	Extra arguments
min, max	Min and max values for random uniform generation.
mean,sd	Mean and standard deviation for random normal generation.
rate	Rate for random exponential generation.
shape,scale	Shape and scale parameters for random weibull generation.
bldim	blocking dimension.
ICTXT	BLACS context number.

Details

Gets the diagonal of a distributed matrix and stores it as a global R vector owned by all processes.

Value

If a distributed matrix is passed to diag() then it returns a global R vector.

If a vector (numeric or character) is passed to diag() and type='ddmatrix', then the return is a diagonal distributed matrix.

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Methods

```
signature(x = "ddmatrix")
signature(x = "vector")
signature(x = "character")
```

See Also

Extract

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:16, 4)
x <- as.ddmatrix(x)

y <- diag(x)
comm.print(y)

finalize()

## End(Not run)</pre>
```

as.ddmatrix

Simplified Syntax to Distribute Matrix Across Process Grid

Description

A simplified interface to the distribute() and redistribute() functions.

```
## S4 method for signature 'NULL'
as.ddmatrix(x, bldim = .BLDIM, ICTXT = .ICTXT)
## S4 method for signature 'vector'
as.ddmatrix(x, bldim = .BLDIM, ICTXT = .ICTXT)
## S4 method for signature 'matrix'
as.ddmatrix(x, bldim = .BLDIM, ICTXT = .ICTXT)
```

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Arguments

x a numeric matrix

bldim the blocking dimension for block-cyclically distributing the matrix across the

process grid.

ICTXT BLACS context number for return.

Details

A simplified wrapper for the distribute() function, especially in the case that the matrix x is global (which you really should not ever let happen outside of testing, but I won't stop you).

The function will only work if x is stored on all processes, or x is stored on a single process (does not matter which) and every other process has NULL stored for x.

If several processes own pieces of the matrix x, then you can not use this function. You will have to create an appropriate ddmatrix on all processes and redistribute the data with the redistribute() function.

As usual, the ICTXT number is the BLACS context corresponding to the process grid onto which the output distributed matrix will be distributed.

Value

Returns a distributed matrix.

See Also

Distribute

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

if (comm.rank()==0){
    x <- matrix(1:16, ncol=4)
} else {
    x <- NULL
}

dx <- as.ddmatrix(x, bldim=c(4,4))
print(dx)

finalize()

## End(Not run)</pre>
```

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Distributed-to-non-distributed Matrix Converters

Description

Converts objects of class ddmatrix to the requested non-distributed type.

Usage

```
## S4 method for signature 'ddmatrix'
as.vector(x, mode = 'any', proc.dest = 'all')
    ## S4 method for signature 'ddmatrix'
as.matrix(x, proc.dest = 'all', attributes = TRUE)
```

Arguments

Х	numeric distributed matrix
mode	A character string giving an atomic mode or "list", or (except for 'vector') "any".
proc.dest	destination process for storing the matrix
attributes	logical specifies whether or not the current attributes should be preserved

Details

Converts a distributed matrix into a non-distributed vector or matrix.

The proc.dest= argument accepts either the BLACS grid position or the MPI rank if the user desires a single process to own the matrix. Alternatively, passing the default value of 'all' will result in all processes owning the matrix. If only a single process owns the undistributed matrix, then all other processes store NULL for that object.

Value

Returns an ordinary R matrix.

Methods

```
signature(x = "ddmatrix")
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()
# don't do this in production code
```

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```
x <- matrix(1:16, ncol=4)
dx <- as.ddmatrix(x)

y <- as.matrix(dx, proc.dest=0)

finalize()
## End(Not run)</pre>
```

Distribute

Distribute/Redistribute matrices across the process grid

Description

Takes either an R matrix and distributes it as a distributed matrix, or takes a distributed matrix and redistributes it across a (possibly) new BLACS context, using a (possibly) new blocking dimension.

Usage

```
distribute(x, bldim = .BLDIM, xCTXT = 0, ICTXT = .ICTXT)
redistribute(dx, bldim = dx@bldim, ICTXT = .ICTXT)
```

Arguments

X	a numeric matrix
dx	numeric distributed matrix
bldim	the blocking dimension for block-cyclically distributing the matrix across the process grid.
xCTXT	the BLACS context number for initial distribution of the matrix x.
ICTXT	BLACS context number for return.

Details

distribute() takes an R matrix x stored on the processes in some fashion and distributes it across the process grid belonging to ICTXT. If a process is to call distribute() and does not yet have any ownership of the matrix x, then that process should store NULL for x.

How one might typically use this is to read in a non-distributed matrix on the first process, store that result as the R matrix x, and then have the other processes store NULL for x. Then calling distribute() returns the distributed matrix which was distributed according to the options bldim and ICTXT.

Using an ICTXT value other than zero is not recommended unless you have a good reason to. Use of other such contexts should only be considered for advanced users, preferably those with knowledge of ScaLAPACK.

redistribute() takes a distributed matrix and redistributes it to the (possibly) new process grid with BLACS context ICTXT and with the (possibly) new blocking dimension bldim. The original BLACS context is dx@CTXT and the original blocking dimension is dx@bldim.

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These two functions are essentially simple wrappers for the ScaLAPACK function PDGEMR2D, with the above described behavior. Of note, for distribute(), dx@CTXT and ICTXT must share at least one process in common. Likewise for redistribute() with xCTXT and ICTXT.

Very general redistributions can be done with redistribute(), but thinking in these terms is an acquired skill. For this reason, several simple interfaces to this function have been written. See SimpleRedistributions for details.

Value

Returns a distributed matrix.

See Also

```
as.ddmatrix
```

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

if (comm.rank()==0){
    x <- matrix(1:16, ncol=4)
} else {
    x <- NULL
}

dx <- distribute(x, bldim=c(4,4))
print(dx)

dx <- redistribute(dx, bldim=c(3,3))
print(dx)

finalize()

## End(Not run)</pre>
```

SimpleRedistributions Distribute/Redistribute matrices across the process grid

Description

Takes either an R matrix and distributes it as a distributed matrix, or takes a distributed matrix and redistributes it across a (possibly) new BLACS context, using a (possibly) new blocking dimension.

Usage

```
as.block(dx, square.bldim = TRUE)
as.rowblock(dx)
as.colblock(dx)
as.rowcyclic(dx, bldim = .BLDIM)
as.colcyclic(dx, bldim = .BLDIM)
as.blockcyclic(dx, bldim = .BLDIM)
```

Arguments

dx numeric distributed matrix

square.bldim logical. Determines whether or not the blocking factor for the resulting redis-

tributed matrix will be square or not.

bldim the blocking dimension for block-cyclically distributing the matrix across the

process grid.

Details

These functions are simple wrappers of the very general redistribute() funciton (see Distribute). Different distributed matrix distributions of note can be classified into three categories: block, cyclic, and block-cyclic.

as.block() will convert ddmatrix into one which is merely "block" distributed, i.e., the blocking factor is chosen in such a way that there will be no cycling. By default, this new blocking factor will be square. This can result in some raggedness (some processors owning less than others — or nothing) if the matrix is far from square itself. However, the methods of factoring ddmatrix objects, and therefore anything that relies on (distributed) matrix factorizations such as computing an inverse, least squares solution, etc., require that blocking factors be square. The matrix will not change BLACS contexts.

as.rowblock() will convert a distributed matrix into one which is distributed by row into a block distributed matrix. That is, the rows are stored contiguously, and different processors will own different rows, but with no cycling. In other words, it block redistributes the data across context 2.

as.colblock() is the column-wise analogue of as.rowblock(). In other words, it block redistributes the data across context 1.

as.rowcyclic() is a slightly more general version of as.rowblock(), in that the data will be distributed row-wise, but with the possibility of cycling, as determined by the blocking factor. In other words it block-cyclically redistributes the data across context 2.

as.colcyclic() is a the column-wise analogue of as.rowcyclic(). In other words, it block-cyclically redistributes the data across context 1.

as.blockcyclic() moves the distributed matrix into a general block-cyclic distribution across a 2-dimensional process grid. In other words, it block-cyclically redistributes the data across context 0.

Value

Returns a distributed matrix.

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See Also

```
as.ddmatrix, Distribute
```

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

dx <- ddmatrix(1:30, nrow=10)

x <- as.block(dx)
x

x <- as.rowblock(dx)
x

x <- as.colblock(dx)
x

x <- as.rowcyclic(dx)
x

x <- as.colcyclic(dx)
x

x <- as.blockcyclic(dx)
x

finalize()

## End(Not run)</pre>
```

Print

Printing a Distributed Matrix

Description

Print method for a distributed matrices.

```
## S4 method for signature 'ddmatrix'
print(x, ..., all = FALSE, name = "x")
```

Print Print

Arguments

X	numeric distributed matrix
	additional arguments
all	control for whether the entire distributed matrix should be printed to standard output
name	character string that will be printed to standard output along with the matrix elements

Details

Print method for class ddmatrix.

If argument all=TRUE, then a modified version of the ScaLAPACK TOOLS routine PDLAPRNT is used to print the entire distributed matrix. The matrix will be printed in column-major fashion, with one element of the matrix per line. If all=FALSE then the name= argument is ignored.

Value

The function silently returns 0.

Methods

```
signature(x = "ddmatrix")
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:16, ncol=4)
dx <- as.ddmatrix(x)

print(dx)

print(dx, all=T)

finalize()

## End(Not run)</pre>
```

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Summary

Distributed Matrix Summary

Description

Summarize a distributed matrix. Gives min, max, mean, etc. by column.

Usage

```
## S4 method for signature 'ddmatrix'
summary(object)
```

Arguments

object

numeric distributed matrix

Details

The return is on process 0 only.

Value

A table on processor 0, NULL on all other processors.

Methods

```
signature(x = "ddmatrix")
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:16, ncol=4)
dx <- as.ddmatrix(x)

summary(dx)

finalize()

## End(Not run)</pre>
```

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Extract

Extract or Replace Parts of a Distributed Matrix

Description

Operators to extract or replace parts of a distributed matrix.

Usage

```
x[i, j, ..., ICTXT]

## S3 method for class 'ddmatrix'
head(x, n = 6L, ...)

## S3 method for class 'ddmatrix'
tail(x, n = 6L, ...)
```

Arguments

X	numeric distributed matrix.
i, j	indices specifying elements to extract or replace. Indices can be numeric, character, empty, or NULL.
n	a single integer. If positive, size for the resulting object: number of elements for a vector (including lists), rows for a matrix or data frame or lines for a function. If negative, all but the n last/first number of elements of x.
•••	additional arguments.
ICTXT	optional BLACS context number for output

Details

[can be used to extract/replace for a distributed matrix exactly as you would with an ordinary matrix

The functions rely on reblocking across different BLACS contexts. If i is not empty, then the input distributed matrix will be redistributed along context 1, where extracting/deleting rows does not destroy block-cyclicality. Likewise, if j is not empty, then the input distributed matrix will be redistributed along context 2. When extraction is complete, the matrix will be redistributed across its input context.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

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Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- x[, -1]
y <- head(y, 2)
print(y)

finalize()

## End(Not run)</pre>
```

Insert

Directly Insert Into Distributed Matrix Submatrix Slot

Description

Allows you to directly replace the submatrix of a distributed matrix.

Usage

```
x[i, j] <- value
submatrix(x) <- value</pre>
```

Arguments

x numeric distributed matrix.

i, j global integer indices.

value replacement value. Can be a global vector or a ddmatrix.

Details

[<- allows the user to insert values into a distributed matrix in exactly the same way one would with an ordinary matrix. The indices here are global, meaning that x[i, j] refers to the (i, j)'th element of the "full", global matrix, and not necessarily the (i, j)'th element of the local submatrix

On the other hand, submatrix<- is different. It is basically syntactic sugar for:

```
x@Data <- newMatrix
```

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It does not alter the distributed matrix x's dim or bldim. It *does* adjust the ldim automatically. However, using this can be dangerous. It is merely provided to give consistent behavior with the submatrix() function.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

x[1, ] <- 0
comm.print(submatrix(x), all.rank=T)

finalize()

## End(Not run)</pre>
```

Comparators

Logical Comparisons

Description

Logical comparisons.

```
x == y
x < y
x > y
x >= y
x <= y
x != y
x & y
x | y
```

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```
## S4 method for signature 'ddmatrix'
any(x, na.rm=FALSE)
  ## S4 method for signature 'ddmatrix'
all(x, na.rm=FALSE)
```

Arguments

x, y distributed matrix or numeric vector

na.rm logical, indicating whether or not NA's should first be removed. If not and an NA

is present, NA is returned.

Details

Performs the indicated logical comparison.

If na.rm is TRUE and only NA's are present, then TRUE is returned.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

See Also

Type

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(sample(0, 1, 9, replace=T), 3)
comm.print(x)

x <- as.ddmatrix(x, bldim=2)

y <- any(x)
comm.print(y)

finalize()

## End(Not run)</pre>
```

Type Type

Type

Type Checks, Including NA, NaN, etc.

Description

Functions to check for various types.

Usage

```
is.ddmatrix(x)
  ## S4 method for signature 'ddmatrix'
is.numeric(x)
  ## S4 method for signature 'ddmatrix'
is.na(x)
  ## S4 method for signature 'ddmatrix'
is.nan(x)
  ## S4 method for signature 'ddmatrix'
is.infinite(x)
```

Arguments

Х

numeric distributed matrix

Details

Performs the appropriate type check.

Value

Returns boolean in the case of is.numeric() and is.ddmatrix(), otherwise a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

See Also

NAs

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NAs

Handle Missing Values in Distributed Matrices

Description

Dealing with NA's and NaN's.

Usage

```
## S4 method for signature 'ddmatrix'
na.exclude(object, ..., ICTXT)
```

Arguments

object numeric distributed matrix

... extra arguments

ICTXT optional BLACS context number for output

Details

Removes rows containing NA's and NaN's.

The function relies on reblocking across different BLACS contexts. The input distributed matrix will be redistributed along context 1, where extracting/deleting rows does not destroy block-cyclicality.

Only advanced users should supply an ICTXT value. Most should simply leave this argument blank.

The context of the return is dependent on the function arguments. If the ICTXT= argument is missing, then the return will be redistributed across its input context object@CTXT. Otherwise, the return will be redistributed across the supplied ICTXT.

Methods

```
signature(object = "ddmatrix")
```

See Also

Type

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()
# don't do this in production code
```

24 Apply

```
x <- matrix(1:9, 3)
x[1, 1] <- NA
x <- as.ddmatrix(x)

y <- na.exclude(x)
comm.print(y)

finalize()

## End(Not run)</pre>
```

Apply

Apply Family of Functions

Description

Apply a function to the margins of a distributed matrix.

Usage

```
## S4 method for signature 'ddmatrix'
apply(X, MARGIN, FUN, ..., reduce = FALSE,
    proc.dest="all")
```

Arguments

X	distributed matrix
MARGIN	subscript over which the function will be applied
FUN	the function to be applied
	additional arguments to FUN
reduce	logical or string. See details
proc.dest	Destination process (or 'all') if a reduction occurs

Details

If reduce==TRUE then a global matrix or vector (whichever is more appropriate) will be returned. The argument proc.dest= behaves exactly as in the as.vector() and as.matrix() functions of **pbdDMAT**. If reduce=FALSE then a distributed matrix is returned. Other acceptable arguments are reduce="matrix" and reduce="vector" which demand global matrix or vector return, respectively. This should generally be slightly more efficient than running apply and then calling as.vector() or as.matrix().

Value

Returns a distributed matrix unless a reduction is requested, then a global matrix/vector is returned.

Binders 25

Methods

```
signature(x = "ddmatrix")
```

Author(s)

Drew Schmidt <schmidt AT math.utk.edu>, Wei-Chen Chen, George Ostrouchov, and Pragneshkumar Patel.

See Also

prcomp

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- head(x[, -1], 2)
print(y)

finalize()

## End(Not run)</pre>
```

Binders

Row and Column binds for Distributed Matrices

Description

Row and column binds.

```
## S4 method for signature '...'
rbind(..., ICTXT = .ICTXT, deparse.level = 1)
  ## S4 method for signature '...'
cbind(..., ICTXT = .ICTXT, deparse.level = 1)
```

26 Binders

Arguments

... vectors, matrices, or distributed matrices.

ICTXT BLACS communicator number for return object.

deparse.level integer controlling the construction of labels in the case of non-matrix-like ar-

guments. Does nothing for distributed matrices.

Details

The ... list of arguments can be vectors, matrices, or distributed matrices so long as non-distributed objects are not used with distributed objects. This kind of mixing-and-matching will lead to chaos. Currently no check is performed to prevent the user from this mixing-and-matching for performance reasons (it is slow enough already).

Value

Returns a vector, matrix, or distributed matrix, depending on input.

Methods

```
signature(... = "ANY") an R object.
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:16, ncol=4)
dx <- as.ddmatrix(x)

y <- rbind(dx, dx)

print(y)

finalize()

## End(Not run)</pre>
```

Reductions 27

Reductions

Arithmetic Reductions: Sums, Means, and Prods

Description

Arithmetic reductions for distributed matrices.

Usage

```
## S4 method for signature 'ddmatrix'
sum(x, ..., na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
mean(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
median(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
prod(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
rowSums(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
colSums(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
rowMeans(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
colMeans(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
min(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
max(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
rowMin(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
colMin(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
rowMax(x, na.rm = FALSE)
  ## S4 method for signature 'ddmatrix'
colMax(x, na.rm = FALSE)
```

Arguments

X	numeric distributed matrix
na.rm	logical. Should missing (including NaN) be removed?
	additional arguments

28 Arithmetic

Details

Performs the reduction operation on a distributed matrix.

There are four legitimately new operations, namely rowMin(), rowMax(), colMin(), and colMax(). These implementations are not really necessary in R because one can easily (and reasonably efficiently) do something like

```
apply(X=x, MARGIN=1L, FUN=min, na.rm=TRUE)
But apply() on a ddmatrix is very costly, and should be used sparingly.
```

Value

Returns a global numeric vector.

Methods

```
signature(x = "ddmatrix")
```

See Also

Arithmetic

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- sum(colMeans(x))
comm.print(y)

finalize()

## End(Not run)</pre>
```

Arithmetic

Arithmetic Operators

Description

Binary operations for distributed matrix/distributed matrix and distributed matrix/vector operations.

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Usage

```
x + y
x - y
-y
x * y
x / y
x ^ y
x % y
x %/% y
```

Arguments

x, y numeric distributed matrices or numeric vectors

Details

If x and y are distributed matrices, then they must be conformable, on the same BLACS context, and have the same blocking dimension.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix", y = "ddmatrix")
signature(x = "numeric", y = "ddmatrix")
signature(x = "ddmatrix", y = "numeric")
```

See Also

```
Arithmetic, LinAlg, MatMult
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r
library(pbdDMAT, quiet = TRUE)
init.grid()
# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- (2*x) - x^(.5)
print(y)
finalize()</pre>
```

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```
## End(Not run)
```

MatMult

Matrix Multiplication

Description

Multiplies two distributed matrices, if they are conformable.

Usage

```
x %*% y
  ## S4 method for signature 'ddmatrix,ANY'
crossprod(x, y = NULL)
  ## S4 method for signature 'ddmatrix,ANY'
tcrossprod(x, y = NULL)
```

Arguments

х, у

numeric distributed matrices

Details

x and y must be conformable, on the same BLACS context, but they need not be blocked with the same blocking dimension. The return will default to the blocking dimension of x.

If you need to use x and y with differing blocking dimensions and you want the return to have blocking different from that of x, then use the function base.rpdgemm().

The crossprod() and tcrossprod() functions behave exactly as their R counterparts.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix", y = "ddmatrix")
signature(x = "ddmatrix", y = "ANY")
```

See Also

```
Arithmetic, LinAlg, MatMult
```

MiscMath 31

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- x %*% x
print(y)

finalize()

## End(Not run)</pre>
```

MiscMath

Miscellaneous Mathematical Functions

Description

Binary operations for distributed matrix/distributed matrix and distributed matrix/vector operations.

```
## S4 method for signature 'ddmatrix'
abs(x)
  ## S4 method for signature 'ddmatrix'
sqrt(x)
  ## S4 method for signature 'ddmatrix'
exp(x)
  ## S4 method for signature 'ddmatrix'
log(x, base = exp(1))
  ## S4 method for signature 'ddmatrix'
log2(x)
  ## S4 method for signature 'ddmatrix'
log10(x)
  ## S4 method for signature 'ddmatrix'
log1p(x)
  ## S4 method for signature 'ddmatrix'
sin(x)
  ## S4 method for signature 'ddmatrix'
cos(x)
  ## S4 method for signature 'ddmatrix'
tan(x)
```

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```
## S4 method for signature 'ddmatrix'
asin(x)
    ## S4 method for signature 'ddmatrix'
acos(x)
    ## S4 method for signature 'ddmatrix'
atan(x)
    ## S4 method for signature 'ddmatrix'
sinh(x)
    ## S4 method for signature 'ddmatrix'
cosh(x)
    ## S4 method for signature 'ddmatrix'
tanh(x)
```

Arguments

x numeric distributed matrix

base a positive number: the base with respect to which logarithms are computed.

Defaults to e='exp(1)'.

Details

Performs the miscellaneous mathematical calculation on a distributed matrix.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

See Also

```
Arithmetic, Reductions
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- sqrt(abs(log(x/10)))
comm.print(y)</pre>
```

Round 33

```
finalize()
## End(Not run)
```

Round

Rounding of Numbers

Description

Extensions of R rounding functions for distributed matrices.

Usage

```
## S4 method for signature 'ddmatrix'
ceiling(x)
  ## S4 method for signature 'ddmatrix'
floor(x)

## S4 method for signature 'ddmatrix'
round(x, digits = 0)
```

Arguments

x numeric distributed matrix

digits integer indicating the number of decimal places (round()) or significant digits (signif()) to be used. Negative values are allowed (see 'Details').

Details

Rounding to a negative number of digits means rounding to a power of ten, so for example round(x, digits = -2) rounds to the nearest hundred.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
```

See Also

```
MiscMath, NAs
```

LinAlg

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- ceiling(x/3)
print(y)

finalize()

## End(Not run)</pre>
```

LinAlg

Linear Algebra Functions

Description

Linear alegbra functions for distributed matrices with R-like syntax, with calculations performed by the PBLAS and ScaLAPACK libraries.

```
## S4 method for signature 'ddmatrix'
isSymmetric(object, tol = 100 * .Machine$double.eps, ...)
  ## S4 method for signature 'ddmatrix'
t(x)
  ## S4 method for signature 'ddmatrix,ddmatrix'
solve(a, b)
  ## S4 method for signature 'ddmatrix, ANY'
solve(a)
  ## S4 method for signature 'ddmatrix'
La.svd(x, nu, nv)
  ## S4 method for signature 'ddmatrix'
svd(x, nu, nv)
  ## S4 method for signature 'ddmatrix'
eigen(x, symmetric, only.values = FALSE)
  ## S4 method for signature 'ddmatrix'
  ## S4 method for signature 'ddmatrix'
lu(x)
```

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Arguments

object, x, a, l	
	numeric distributed matrices. If applicable, a and b must be on the same BLACS context and have the same blocking dimension.
tol	precision tolerance.
	additional arguments.
nu	number of left singular vectors to return when calculating singular values.
nv	number of right singular vectors to return when calculating singular values.
symmetric	logical, if TRUE then the matrix is assumed to be symmetric and only the lower triangle is used. Otherwise x is inspected for symmetry.
only.values	logical, if TRUE then only the eigenvalues are returned. Otherwise both eigenvalues and eigenvectors are returned.

Details

Extensions of R linear algebra functions.

Value

t() returns the transposed matrix.

solve() solves systems and performs matrix inversion when argument b= is missing.

La.svd() performs singular value decomposition, and returns the transpose of right singular vectors if any are requested. Singular values are stored as a global R vector. Left and right singular vectors are unique up to sign. Sometimes core R (via LAPACK) and ScaLAPACK will disagree as to what the left/right singular vectors are, but the disagreement is always only up to sign.

svd() performs singular value decomposition. Differs from La.svd() in that the right singular vectors, if requested, are returned non-transposed. Singular values are stored as a global R vector. Sometimes core R (via LAPACK) and ScaLAPACK will disagree as to what the left/right singular vectors are, but the disagreement is always only up to sign.

eigen() computes the eigenvalues, and eigenvectors if requested. As with svd(), eigenvalues are stored in a global R vector.

chol() performs Cholesky factorization.

lu() performs LU factorization.

Methods

```
signature(x = "ddmatrix")
signature(a = "ddmatrix")
signature(b = "ddmatrix")
```

See Also

```
Arithmetic, Reductions, MatMult, MiscMath
```

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Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

y <- solve(t(A) %*% A)
print(y)

finalize()

## End(Not run)</pre>
```

QR Decomposition

QR Decomposition Methods

Description

```
qr() takes the QR decomposition.
qr.Q() recovers Q from the output of qr().
qr.R() recovers R from the output of qr().
qr.qy() multiplies y by Q.
qr.qty() multiplies y by the transpose of Q.
```

```
## S4 method for signature 'ddmatrix'
qr(x, tol = 1e-07)
## S4 method for signature 'ANY'
qr.Q(x, complete = FALSE, Dvec = rep.int(if (cmplx) 1 +
   (0+0i) else 1, if (complete) dqr[1] else min(dqr)))
## S4 method for signature 'ANY'
qr.R(x, complete = FALSE)
## S4 method for signature 'ANY'
qr.qy(x, y)
## S4 method for signature 'ANY'
qr.qty(x, y)
```

QR Decomposition 37

Arguments

x, y	numeric distributed matrices for qr(). Otherwise, x is a list, namely the return

from qr().

tol logical value, determines whether or not columns are zero centered.

complete logical expression of length 1. Indicates whether an arbitrary orthogonal com-

pletion of the Q or X matrices is to be made, or whether the R matrix is to be

completed by binding zero-value rows beneath the square upper triangle.

Dvec Not implemented for objects of class ddmatrix. vector (not matrix) of diagonal

values. Each column of the returned Q will be multiplied by the corresponding

diagonal value. Defaults to all 1's.

Details

Functions for forming a QR decomposition and for using the outputs of these numerical QR routines.

Value

```
qr() returns a list consisting of: qr - rank - calculated numerical rank, tau - pivot - "class" - attribute "qr".
```

Methods

```
signature(x = "ddmatrix")
signature(x = "ANY")
```

See Also

```
lm.fit
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(1:9, 3)
x <- as.ddmatrix(x)

Q <- qr.Q(qr(x))
print(Q)

finalize()

## End(Not run)</pre>
```

38 chol2inv

chol2inv

Inverse from Choleski (or QR) Decomposition

Description

```
qr() takes the QR decomposition.
```

Usage

```
## S4 method for signature 'ddmatrix'
chol2inv(x, size = NCOL(x))
```

Arguments

x numeric distributed matrices for size number of columns of x containing the Choleski factorization.

Details

The function returns the inverse of a choleski factored matrix, or the inverse of crossprod(x) if qr.R(qr(x)) is passed.

Value

A numeric distributed matrix.

Methods

```
signature(x = "ddmatrix")
signature(x = "ANY")
```

See Also

```
lm.fit
```

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

comm.set.seed(diff=T)
x <- ddmatrix("rnorm", 3, 3)

R <- qr.R(qr(x))</pre>
```

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```
xtx.inv <- chol2inv(R)
id <- xtx.inv
print(id)
finalize()
## End(Not run)</pre>
```

kappa

Compute or estimate the Condition Number of a Distributed Matrix

Description

Computes or estimates the condition number.

Usage

Arguments

x,z	numeric distributed matrices.
exact	logical. Determines whether exact condition number or approximation should be computed.
norm	character. Determines which matrix norm is to be used.
method	character. Determines the method use in computing condition number.
triangular	logical. If true, only the lower triangle is used.
	Extra arguments.

Value

Returns a number.

Methods

```
signature(x = "ddmatrix")
signature(z = "ddmatrix")
```

40 Norm

See Also

Norm

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

comm.set.seed(diff=T)
x <- ddmatrix("rnorm", 10, 10)

cnm <- rcond(x)

comm.print(cnm)

finalize()
## End(Not run)</pre>
```

Norm

Compute the Norm of a Distributed Matrix

Description

Computes the norm.

Usage

```
## S4 method for signature 'ddmatrix'
norm(x, type = c("0", "I", "F", "M", "2"))
```

Arguments

x numeric distributed matrices.

type character. Determines which matrix norm is to be used.

Value

Returns a number.

Methods

```
signature(x = "ddmatrix")
```

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See Also

ConditionNumbers

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r
library(pbdDMAT, quiet = TRUE)
init.grid()

comm.set.seed(diff=T)
x <- ddmatrix("rnorm", 10, 10)

nrm <- norm(x)

comm.print(nrm)

finalize()
## End(Not run)</pre>
```

scale

Scale

Description

Centers and/or scales the columns of a distributed matrix.

Usage

```
## S4 method for signature 'ddmatrix,ANY,ANY'
scale(x, center = TRUE, scale = TRUE)
```

Arguments

x numeric distributed matrix.

center logical value, determines whether or not columns are zero centered

scale logical value, determines whether or not columns are rescaled to unit variance

Details

Centers and/or scales the columns of a distributed matrix.

Value

Returns a distributed matrix.

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Methods

```
signature(x = "ddmatrix", center="ANY", scale="ANY")
```

Author(s)

R Core Team, Drew Schmidt <schmidt AT math.utk.edu>, Wei-Chen Chen, George Ostrouchov, and Pragneshkumar Patel.

See Also

prcomp

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

comm.set.seed(diff=T)

x <- ddmatrix("rnorm", 10, 10)
y <- scale(x)

print(y)

finalize()
## End(Not run)</pre>
```

sweep

Sweep

Description

Sweep vector or ddmatrix from a distributed matrix.

Usage

```
## S4 method for signature 'ddmatrix,ANY,vector'
sweep(x, MARGIN, STATS, FUN = "-")
## S4 method for signature 'ddmatrix,ANY,ddmatrix'
sweep(x, MARGIN, STATS, FUN = "-")
```

Variance/Covariance 43

Arguments

x numeric distributed matrix.

MARGIN subscript over which the function will be applied

STATS array to be swept out.

FUN function used in the sweep. Only +, -, *, and / are accepted. For more general

operations, use apply().

Details

Sweep vector or ddmatrix from a distributed matrix.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix", MARGIN = "ANY", STATS = "vector")
signature(x = "ddmatrix", MARGIN = "ANY", STATS = "ddmatrix")
```

Variance/Covariance

Variance, Covariance, and Correlation

Description

sd() forms the vector of column standard deviations. cov() and var() form the variance-covariance matrix. cor() forms the correlation matrix. cov2cor() scales a covariance matrix into a correlation matrix.

Usage

```
## S4 method for signature 'ddmatrix'
sd(x, na.rm = FALSE, reduce = FALSE, proc.dest="all")
## S4 method for signature 'ANY'
sd(x, na.rm = FALSE)
## S4 method for signature 'ddmatrix'
var(x, y = NULL, na.rm = FALSE, use)
## S4 method for signature 'ddmatrix'
cov(x, y = NULL, use = "everything", method =
   "pearson")
## S4 method for signature 'ddmatrix'
cov2cor(V)
```

44 Variance/Covariance

Arguments

x, y, V numeric distributed matrices.

na.rm logical, determines whether or not NA's should be dealth with.

reduce logical or string. See details

proc.dest Destination process (or 'all') if a reduction occurs

use character indicating how missing values should be treated. Acceptable values

are the same as R's, namely "everything", "all.obs", "complete.obs", "na.or.complete",

or "pairwise.complete.obs".

method character argument indicating which method should be used to calculate covari-

ances. Currently only "spearman" is available for ddmatrix.

Details

sd() will compute the standard deviations of the columns, equivalent to calling apply(x, MARGIN=2, FUN=sd) (which will work for distributed matrices, by the way). However, this should be much faster and use less memory than apply(). If reduce=FALSE then the return is a distributed matrix consisting of one (global) row; otherwise, an R vector is returned, with ownership of this vector determined by proc.dest.

cov() forms the variance-covariance matrix. Only method="pearson" is implemented at this time.

var() is a shallow wrapper for cov() in the case of a distributed matrix.

cov2cor() scales a covariance matrix into a correlation matrix.

Value

Returns a distributed matrix.

Methods

```
signature(x = "ddmatrix")
signature(V = "ddmatrix")
```

Author(s)

R Core Team, Drew Schmidt <schmidt AT math.utk.edu>, Wei-Chen Chen, George Ostrouchov, and Pragneshkumar Patel.

See Also

prcomp

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r
library(pbdDMAT, quiet = TRUE)
```

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```
init.grid()
x <- ddmatrix("rnorm", nrow=3, ncol=3)
cv <- cov(x)
print(cv)
finalize()
## End(Not run)</pre>
```

PCA

Principal Components Analysis

Description

Performs the principal components analysis.

Usage

```
## S4 method for signature 'ddmatrix'
prcomp(x, retx = TRUE, center = TRUE,
    scale. = FALSE, tol = NULL)
```

Arguments

X	numeric distributed matrix.
center	logical value, determines whether or not columns are zero centered
scale.	logical value, determines whether or not columns are rescaled to unit variance
retx	logical, indicates whether the rotated variables should be returned
tol	a value indicating the magnitude below which components should be omitted. (Components are omitted if their standard deviations are less than or equal to tol times the standard deviation of the first component.) With the default null setting, no components are omitted. Other settings for tol could be tol = 0 or tol = $sqrt(.Machine$double.eps)$, which would omit essentially constant components

Details

prcomp() performs the principal components analysis on the data matrix by taking the SVD. Sometimes core R and pbdDMAT will disagree slightly in what the rotated variables are because of how the SVD is caluclated. See the details section of La.svd() under LinAlg for details. more details.

Value

Returns a list.

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Methods

```
signature(x = "ddmatrix")
```

Author(s)

R Core Team, Drew Schmidt <schmidt AT math.utk.edu>, Wei-Chen Chen, George Ostrouchov, and Pragneshkumar Patel.

Examples

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

comm.set.seed(diff=T)

x <- ddmatrix("rnorm", 10, 10)

y <- prcomp(x)
comm.print(y)

finalize()

## End(Not run)</pre>
```

lm.fit

Fitter for Linear Models

Description

Fits a real linear model via QR with a "limited pivoting strategy", as in R's DQRDC2 (fortran).

Usage

```
## S4 method for signature 'ddmatrix,ddmatrix'
lm.fit(x, y, tol = 1e-07, singular.ok = TRUE)
```

Arguments

x, y numeric distributed matrices
 tol tolerance for numerical rank estimation in QR decomposition.
 singular.ok logical. If FALSE then a singular model (rank-deficient x) produces an error.

Im.fit

Details

Solves the linear least squares problem, which is to find an x (possibly non-uniquely) such that $\|Ax - b\|^2$ is minimized, where A is a given n-by-p model matrix, b is a "right hand side" n-by-1 vector (multiple right hand sides can be solved at once, but the solutions are independent, i.e. not simultaneous), and "||" is the 12 norm.

Uses level 3 PBLAS and ScaLAPACK routines (modified PDGELS) to get a linear least squares solution, using the 'limited pivoting strategy' from R's DQRDC2 (unsed in DQRLS) routine as a way of dealing with (possibly) rank deficient model matrices.

A model matrix with many dependent columns will likely experience poor performance, especially at scale, due to all the data swapping that must occur to handle rank deficiency.

Value

Returns a list of values similar to R's lm.fit(). Namely, the list contains: coefficients - distributed matrix, residuals - distributed matrix, effects - distributed matrix, rank - global numeric, fitted.values - distributed matrix, assign - NULL if lm.fit() is called directly, qr - list, same as return from qr(), df.residual - global numeric.

The return values are, respectively: (1) a solution x to the linear least squares problem, (2) the difference in the numerical fit A %*% x and the observed b, (3) t(Q) %*% b, where Q is the orthogonal matrix from a QR-decomposition of A, (4) the numerical column rank of A, (5) the numerical fit A %*% x, (6) NULL if lm.fit() is directly called, (7) a list containing the return of QR decomposition performed by a modified PDGEQPF, (8) degrees of freedom of residuals, i.e. n minus the column rank of A.

Methods

```
signature(x = "ddmatrix", y = "ddmatrix")
```

See Also

QR

```
## Not run:
# Save code in a file "demo.r" and run with 2 processors by
# > mpiexec -np 2 Rscript demo.r

library(pbdDMAT, quiet = TRUE)
init.grid()

# don't do this in production code
x <- matrix(rnorm(9), 3)
y <- matrix(rnorm(3))

dx <- as.ddmatrix(x)
dy <- as.ddmatrix(y)

fit <- lm.fit(x=dx, y=dy)</pre>
```

lm.fit

```
print(fit)
finalize()
## End(Not run)
```

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