Quantum splitting of electron peaks in ultra-strong fields

Zhang et al , Matter Radiat. Extremes 8, 054003 (2023)

Notebook: Óscar Amaro, September 2023 @ GoLP-EPP

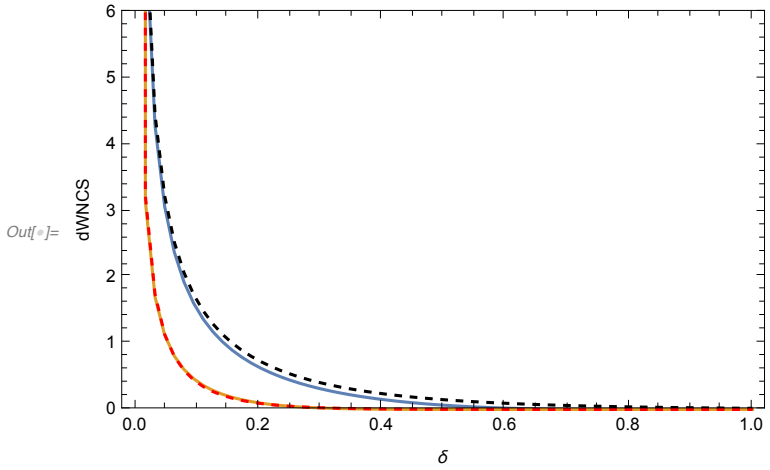
Introduction

In this notebook we reproduce some results from the paper.

Figure 1

In[•]:=
```
Clear[δ, χ, dWNCS, dWQSR, y, m, α, fin, γ]
m = p0 = γ = 1;
α = 1 / 137;
(* NCS *)
dWNCS[χ_, δ_] := α/(π Sqrt[3]) m^2/p0 ((1 - δ + 1/(1 - δ)) BesselK[2/3, 2δ/(3χ(1 - δ))] -
    NIntegrate[BesselK[1/3, y], {y, 2δ/(3χ(1 - δ)), ∞}])
A = 300;
fin[χ_, δ_] := 0.5 α m^2/p0 (χ/δ)^(2/3) Exp[-δ/χ];
Plot[{A dWNCS[0.3, δ], A dWNCS[0.1, δ], A fin[0.3, δ], A fin[0.1, δ]}, {δ, 0, 1},
 PlotPoints → 2, Frame → True, FrameLabel → {"δ", "dWNCS"}, PlotRange → {0, 6},
 PlotStyle → {Default, Default, {Dashed, Black}, {Red, Dashed}}]
```

Out[•]=



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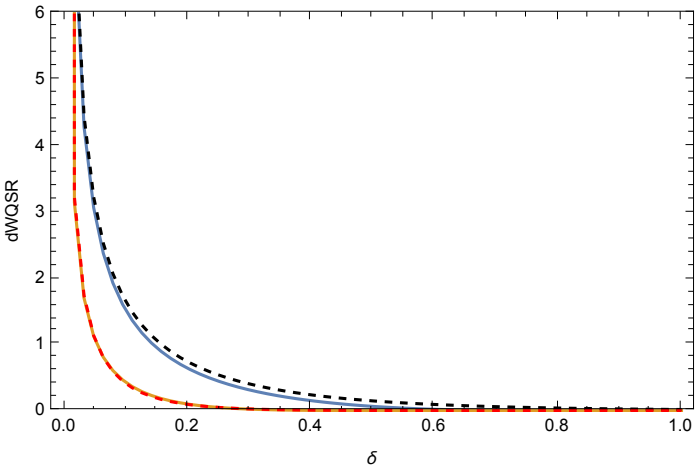
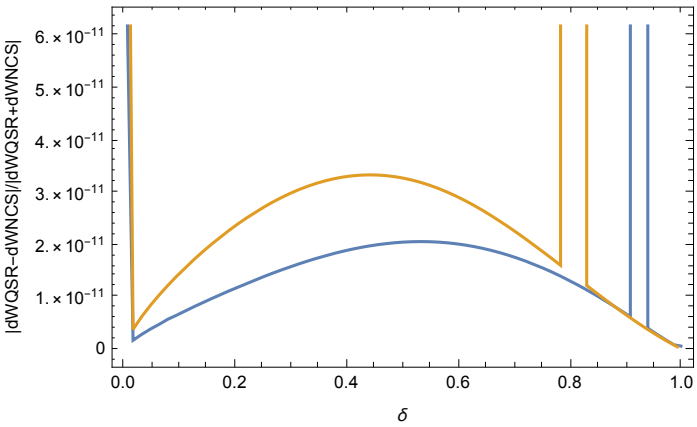
In[•]:=
```
(* QSR *)

dWQSR[χ_, δ_] :=
 Sqrt[3] α / (2 π) m χ / γ (1 - δ) / δ ((2 δ) / (3 χ (1 - δ)) NIntegrate[BesselK[5 / 3, x], {x, (2 δ) / (3 χ (1 - δ)), ∞}] +
   ((2 δ) / (3 χ (1 - δ)))^3 (3 χ / 2)^2 × (1 - δ) BesselK[2 / 3, (2 δ) / (3 χ (1 - δ))])
A = 300;
fin[χ_, δ_] := 0.5 α m^2 / p0 (χ / δ)^(2 / 3) Exp[-δ / χ];
Plot[{A dWQSR[0.3, δ], A dWQSR[0.1, δ], A fin[0.3, δ], A fin[0.1, δ]}, {δ, 0, 1},
 PlotPoints → 2, Frame → True, FrameLabel → {"δ", "dWQSR"}, PlotRange → {0, 6},
 PlotStyle → {Default, Default, {Dashed, Black}, {Red, Dashed}}]
```

Out[•]=



In[•]:=
```
(* difference between NCS and QSR *)
Plot[{Abs[(dWNCS[0.3, δ] - dWQSR[0.3, δ]) / (dWNCS[0.3, δ] + dWQSR[0.3, δ])], Abs[(dWNCS[0.1, δ] - dWQSR[0.1, δ]) / (dWNCS[0.1, δ] + dWQSR[0.1, δ])]},
 {δ, 0, 1}, PlotPoints → 2, Frame → True,
 FrameLabel → {"δ", "|dWQSR-dWNCS|/|dWQSR+dWNCS|"}]
```

Out[•]=



Equation 13

```
In[•]:= Clear[R, χ, δ, f1, f2]
        R[δ_] := χ^(-1 / 3) δ^(-2 / 3) Exp[-δ / χ] / Gamma[1 / 3]
        f1 = R[1 - δ] (* written with argument δ *)
        f2 = Integrate[f1 R[δ - η], {δ, η, 1}] // Normal (* written with argument η *)
        f3 = Integrate[f2 R[η - δ], {η, δ, 1}] // Normal (* written with argument δ *)
```

$$\text{Out[•]}= \frac{e^{\frac{-1+\delta}{\chi}}}{(1-\delta)^{2/3}\,\chi^{1/3}\,\text{Gamma}\left[\frac{1}{3}\right]}$$

$$\text{Out[•]}= \frac{e^{\frac{-1+\eta}{\chi}}\,\text{Gamma}\left[\frac{1}{6}\right]}{2^{2/3}\,\sqrt{\pi}\,\,(1-\eta)^{1/3}\,\chi^{2/3}\,\text{Gamma}\left[\frac{1}{3}\right]}$$

$$\text{Out[•]}= \frac{e^{\frac{-1+\delta}{\chi}}}{\chi}$$

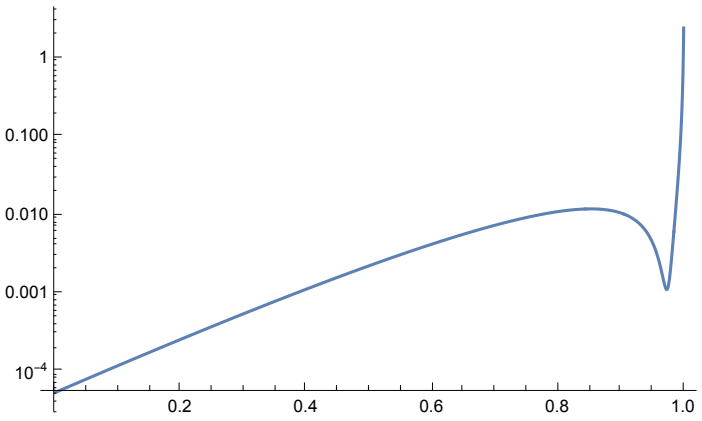
```
In[•]:= (* check for i=2,3 *)
        fi[i_, χ_, δ_] := χ^(-i / 3) / Gamma[i / 3] (1 - δ)^(i / 3 - 1) Exp[-(1 - δ) / χ]
        f2 - fi[2, χ, η] // FullSimplify
        f3 - fi[3, χ, δ] // FullSimplify
```

```
Out[•]= 0

Out[•]= 0
```

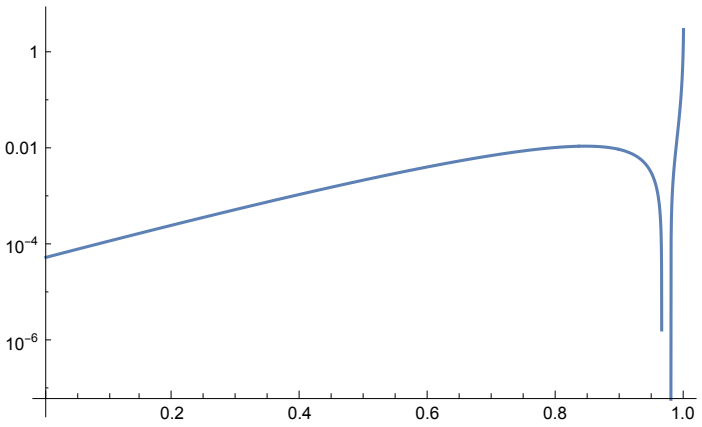
Root of dFdη

In[•]:=
```
(* Showing graphically that the threshold is r~5.35 *)
Clear[dFdη, χ, r, η, ξ]
χ = 0.1;

r = 5.3;
dFdη = -χ Exp[-r] Exp[-ξ]
       Sum[r^i ((i / 3 - 1) ξ^(i / 3 - 2) - ξ^(i / 3 - 1)) / (i ! Gamma[i / 3]), {i, 1, 20}] /. {ξ → (1 - η) / χ} // N;
LogPlot[dFdη, {η, 0, 1}]

r = 5.4;
dFdη = -χ Exp[-r] Exp[-ξ]
       Sum[r^i ((i / 3 - 1) ξ^(i / 3 - 2) - ξ^(i / 3 - 1)) / (i ! Gamma[i / 3]), {i, 1, 20}] /. {ξ → (1 - η) / χ} // N;
LogPlot[
 dFdη,
 {η,
  0,
  1}]
```

Out[•]=

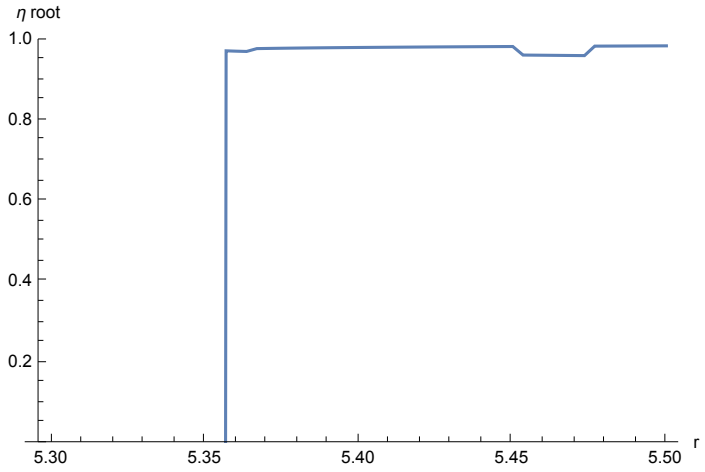


Out[•]=



In[•]:=
```
(* more precise proof that the threshold is r~5.35 *)
Clear[dFdη, χ, r, η, ξ]
χ = 0.1;
dFdη[r_] := -χ Exp[-r] Exp[-ξ]
      Sum[r^i ((i/3-1) ξ^(i/3-2) - ξ^(i/3-1)) / (i! Gamma[i/3]), {i, 1, 40}] /. {ξ → (1-η)/χ} // N;
ListPlot[Table[{r, FindRoot[{dFdη[r] == 0}, {η, 0.9}][[1, 2]]},
   {r, 5.3, 5.5, 0.01/3}], Joined → True,
  PlotRange → {0, 1}, AxesLabel → {"r", "η root"}] // Quiet

Clear[dFdη, χ, r, η, ξ]
χ = 0.5;
dFdη[r_] := -χ Exp[-r] Exp[-ξ]
      Sum[r^i ((i/3-1) ξ^(i/3-2) - ξ^(i/3-1)) / (i! Gamma[i/3]), {i, 1, 40}] /. {ξ → (1-η)/χ} // N;
ListPlot[Table[{r, FindRoot[{dFdη[r] == 0}, {η, 0.9}][[1, 2]]},
   {r, 5.3, 5.5, 0.01/3}], Joined → True,
  PlotRange → {0, 1}, AxesLabel → {"r", "η root"}] // Quiet
```

Out[•]=



Out[•]=