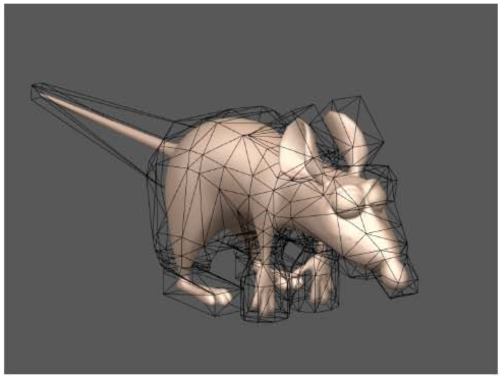
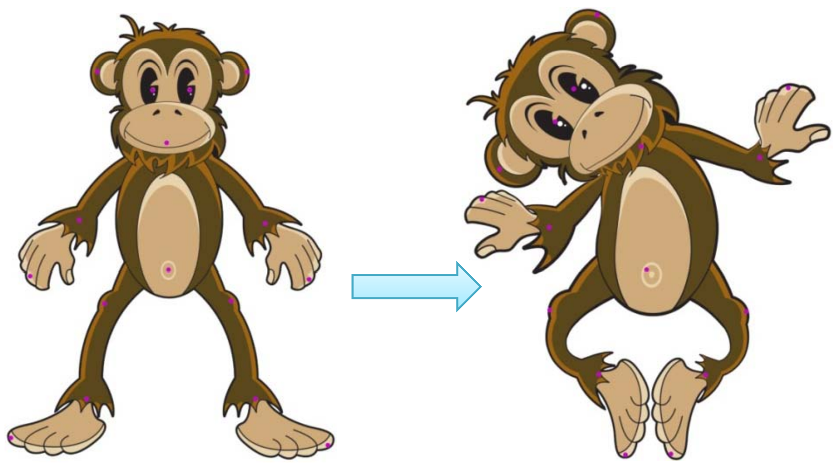
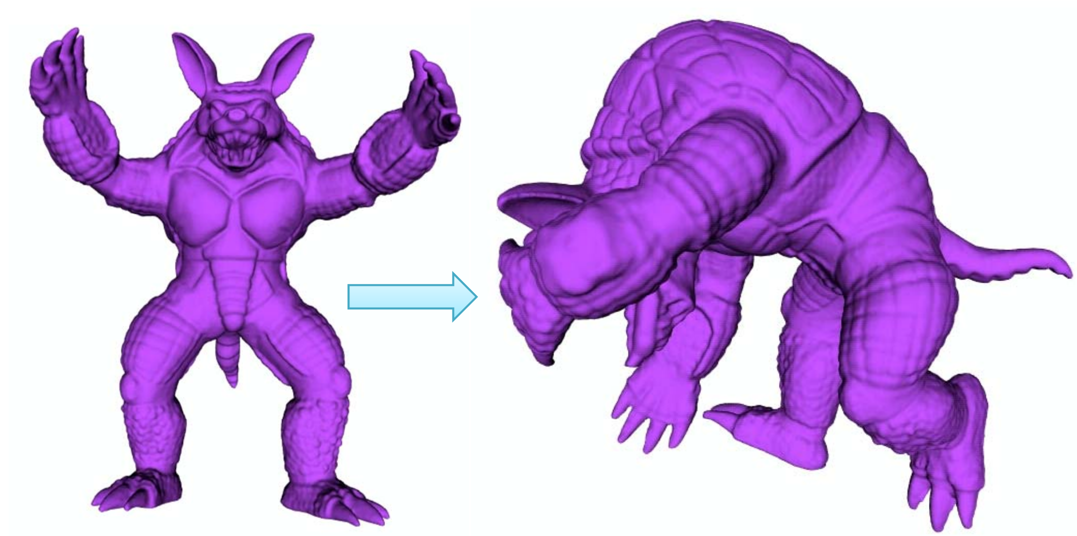
Deformation



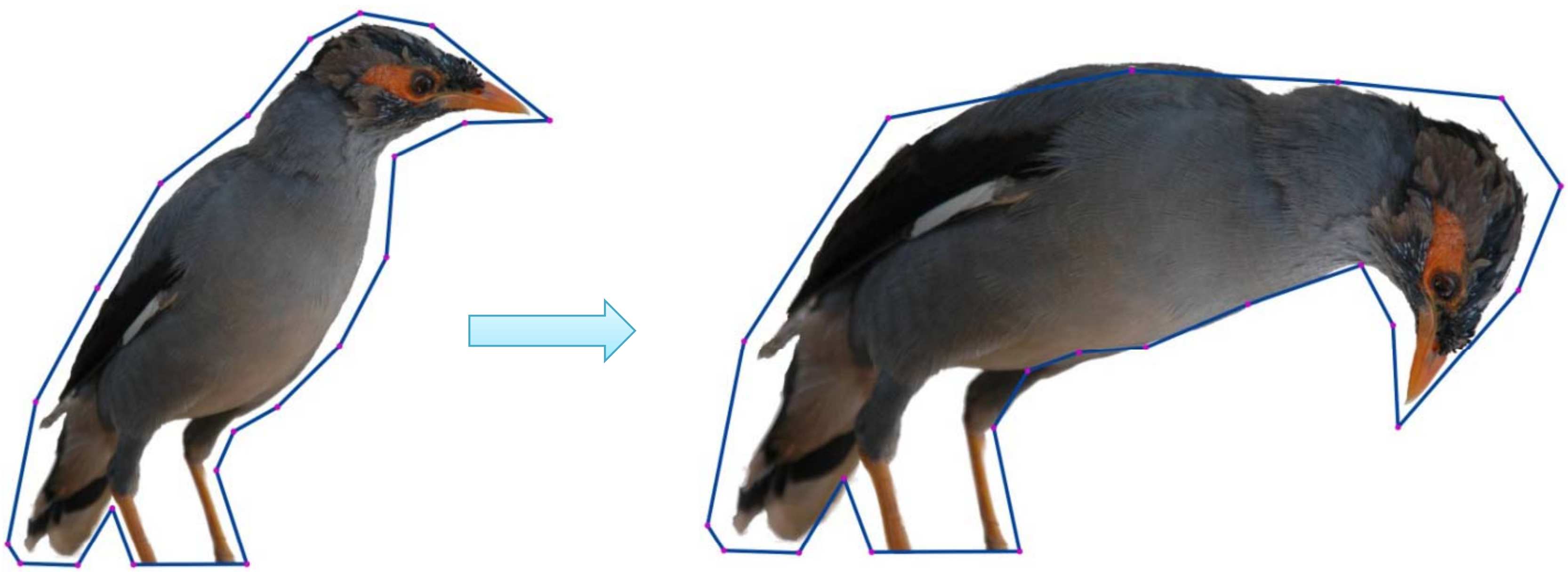
Deformation





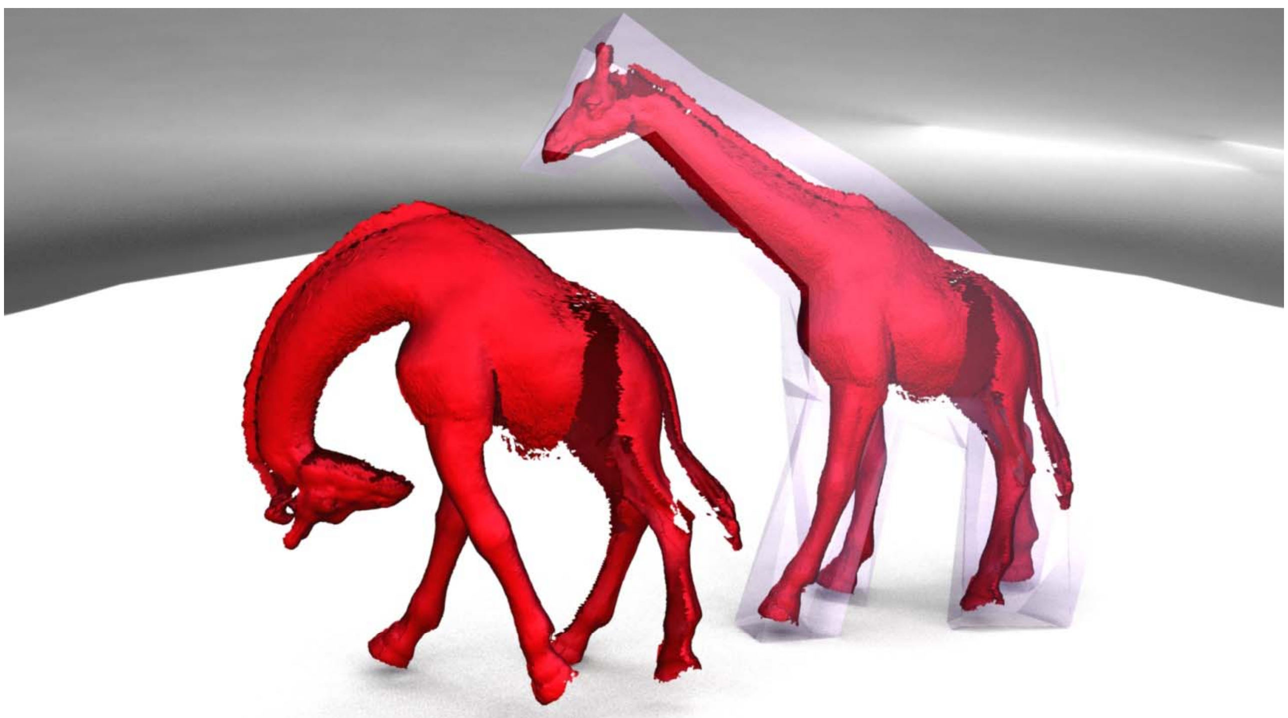
Motivation

Easy modeling – generate new shapes by deforming existing ones



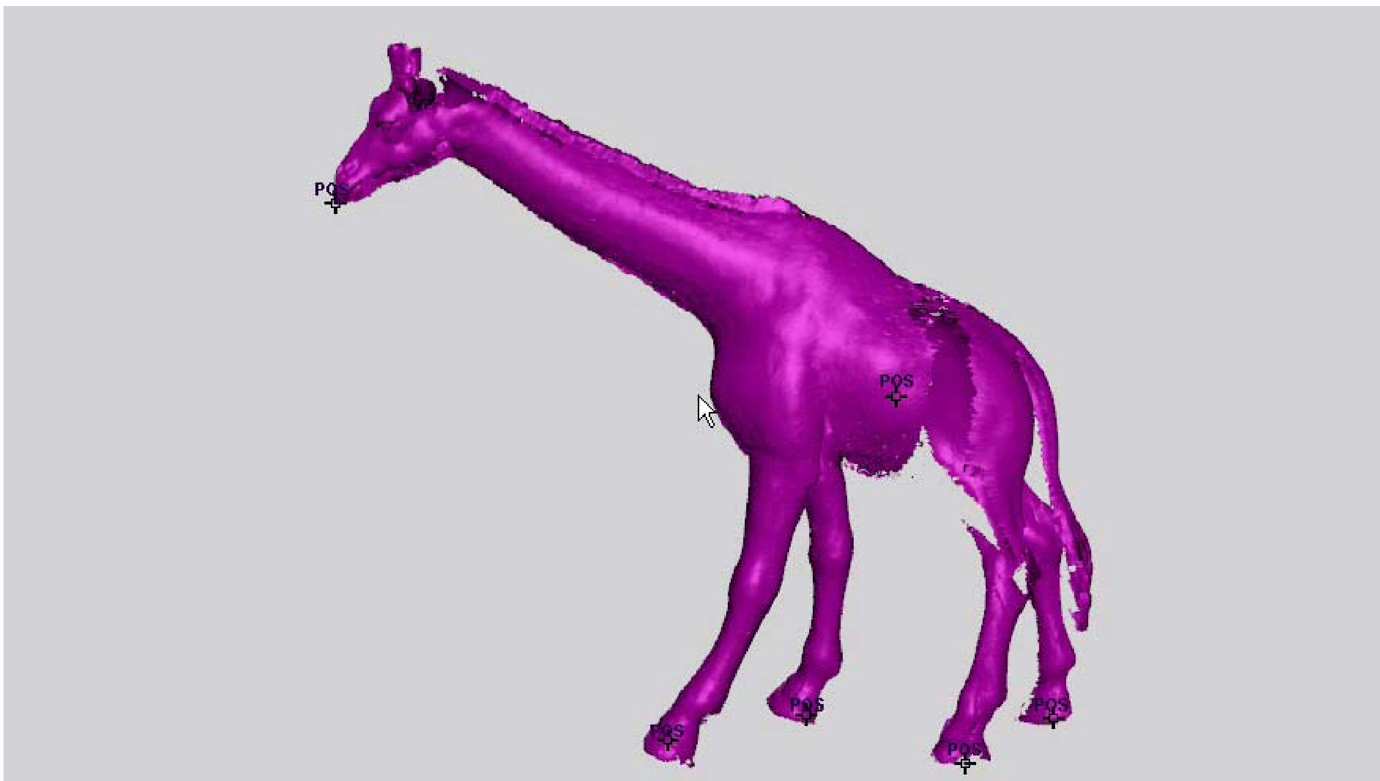
Motivation

Easy modeling – generate new shapes by deforming existing ones



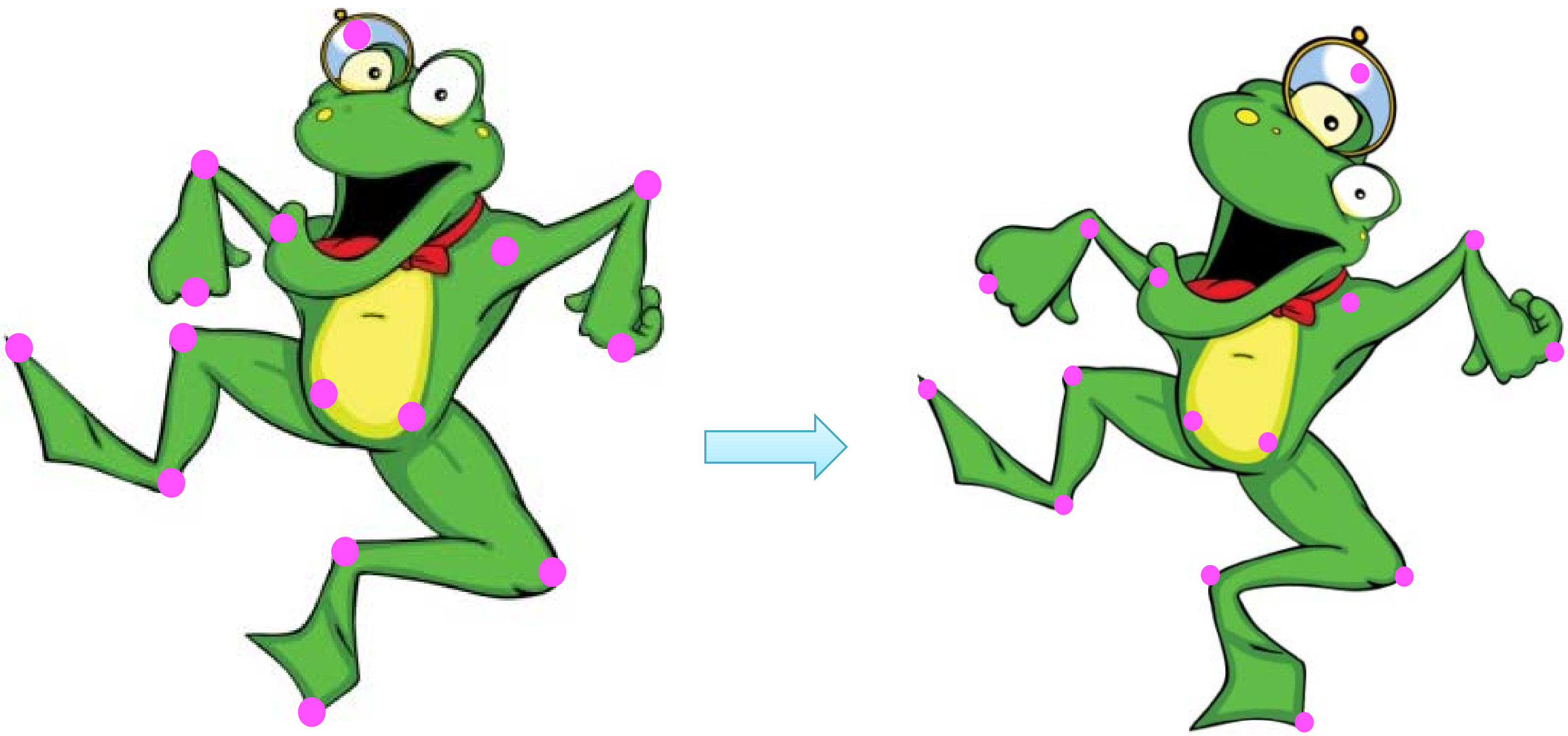
Motivation

Character posing for animation



Challenges

User says as little as possible…

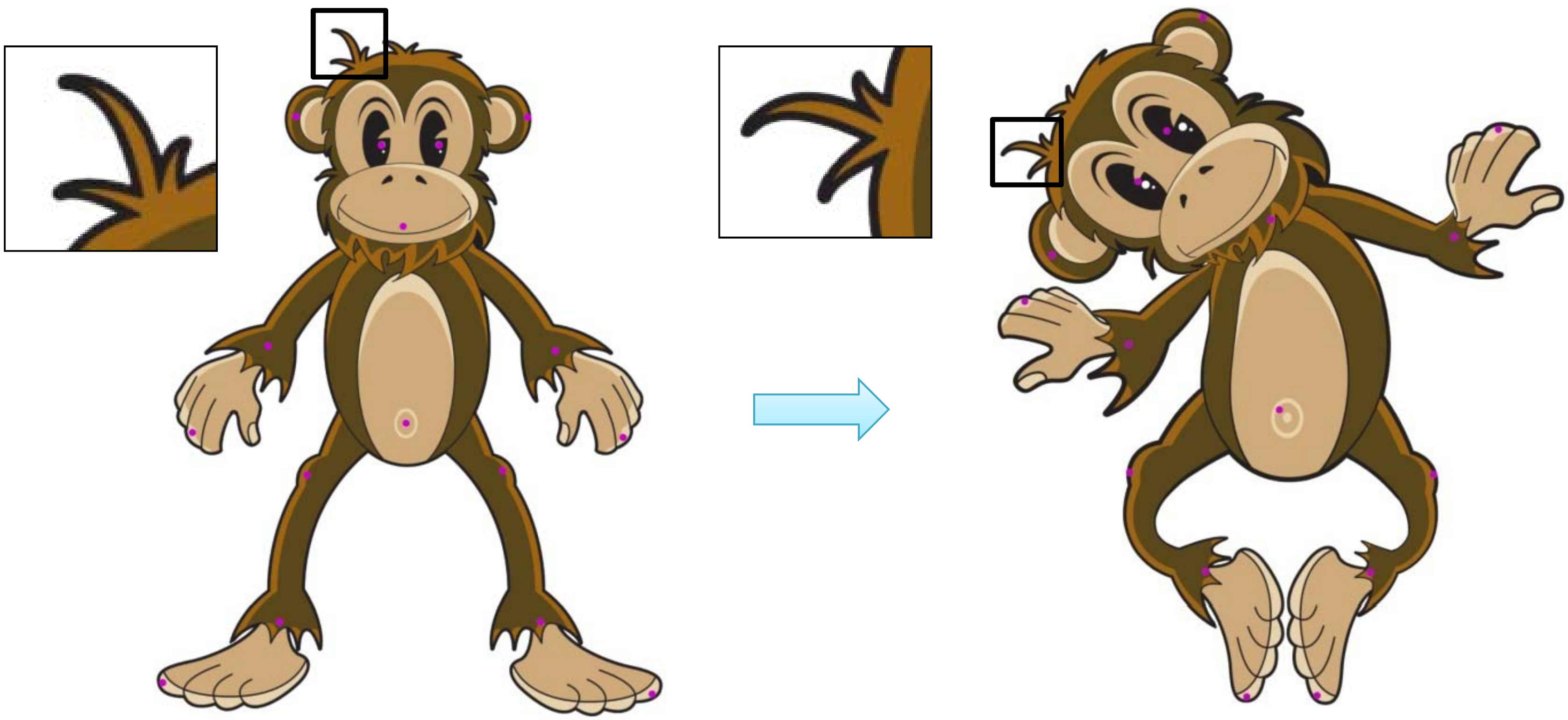


…algorithm deduces the rest

Challenges

"Intuitive deformation"

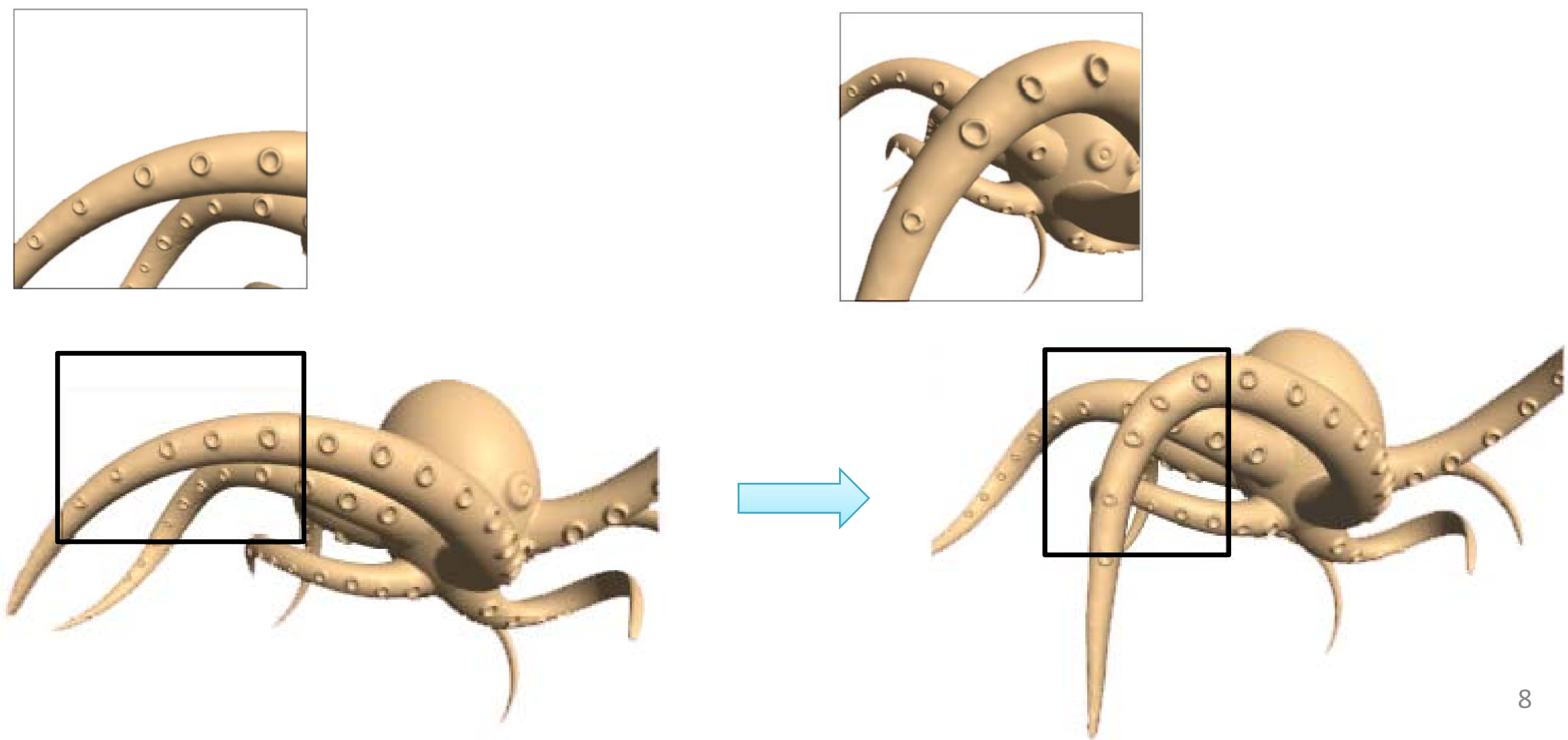
global change + local detail preservation



Challenges

"Intuitive deformation"

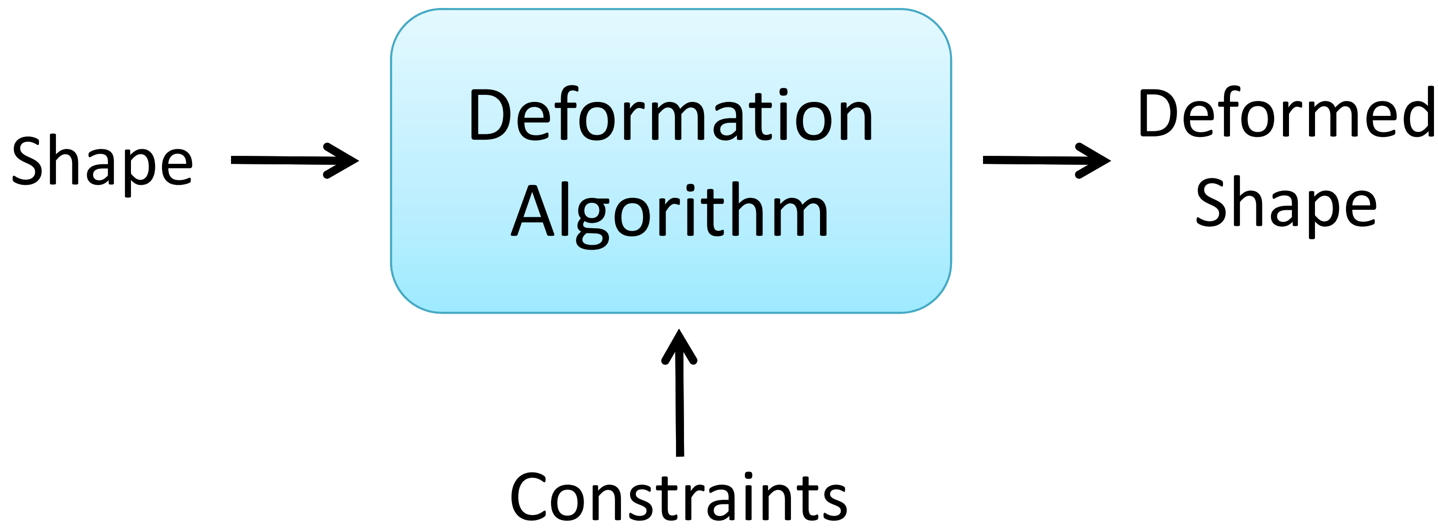
global change + local detail preservation



Challenges

Efficient!

Rules of the Game



Position:
"This point goes there"

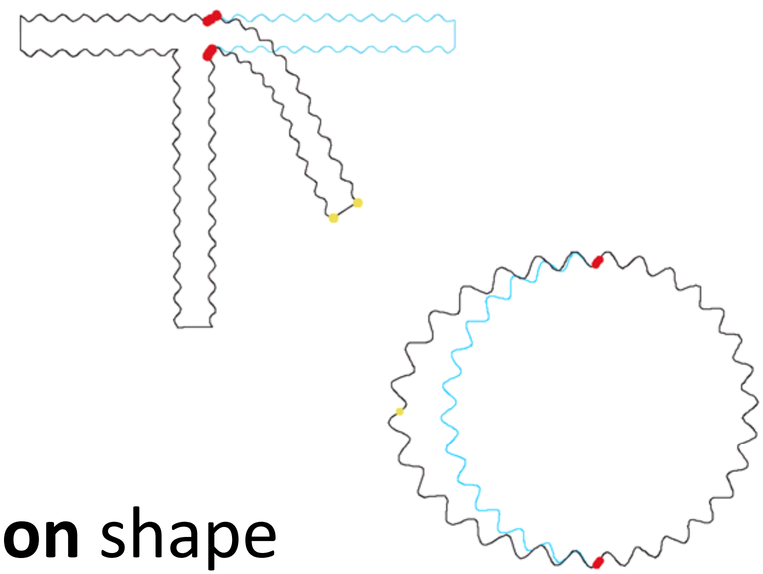
Orientation/Scale:
"The environment of this point should rotate/scale"

Other shape property:
Curvature, perimeter,…

[Parameterization is also "deformation": constraints = curvature 0 everywhere]

Approaches

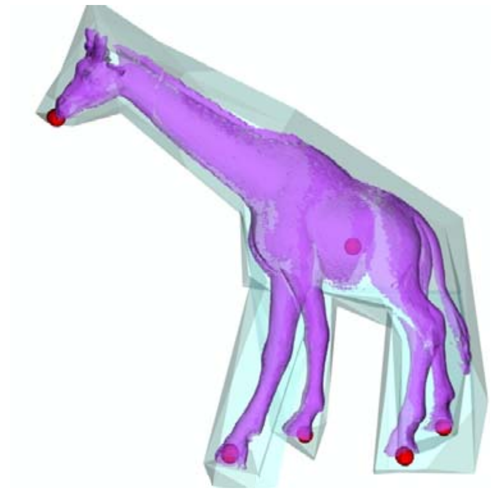
- Surface deformation
  - Shape is empty shell
    - Curve for 2D deformation
    - Surface for 3D deformation



  - Deformation only defined **on** shape
  - Deformation coupled with shape representation

Approaches

- Space deformation
  - Shape is volumetric
    - Planar domain in 2D
    - Polyhedral domain in 3D



  - Deformation defined in neighborhood of shape
  - Can be applied to any shape representation

Approaches

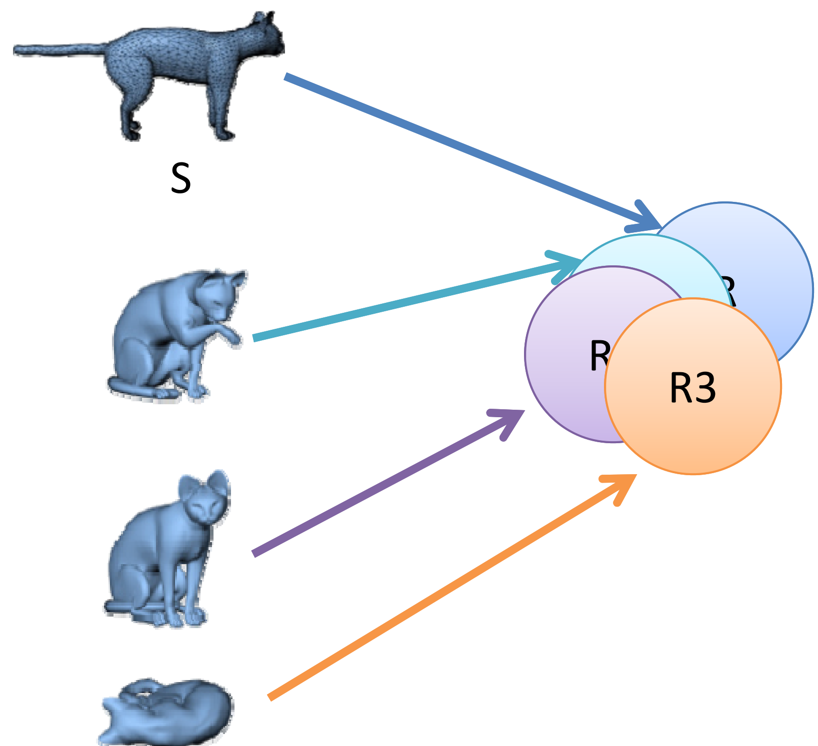
- Surface deformation
  - Find alternative representation which is “deformation invariant”
- Space deformation
  - Find a space map which has “nice properties”

Surface Deformation

Setup:

– Choose alternative representation *f*(S)

– Given S find S' such that
  - Constraints(S') are true
  - *f*(S') = *f*(S)
    (or close)
  - An optimization problem



Shape Representation

How good is the representation?

- Representation should **always** be invariant to:
  - Global translation
  - Global rotation
  - Global scale? Depends on application

- Shapes we want “reachable” should have similar representations
  - Almost isometric deformation
    local translation + rotation
  - Almost “conformal” deformation
    local translation + rotation + scale

Shape Representation

Robustness

- How hard is it to solve the optimization problem?
- Can we find the global minimum?
- Small change in constraints → similar shape?

Efficiency

- Can it be solved at interactive rates?

Shape Representations

Rule of thumb:

If representation is a **linear** function of the coordinates, deformation is:

Robust

Fast

But representation is **not rotation invariant**!

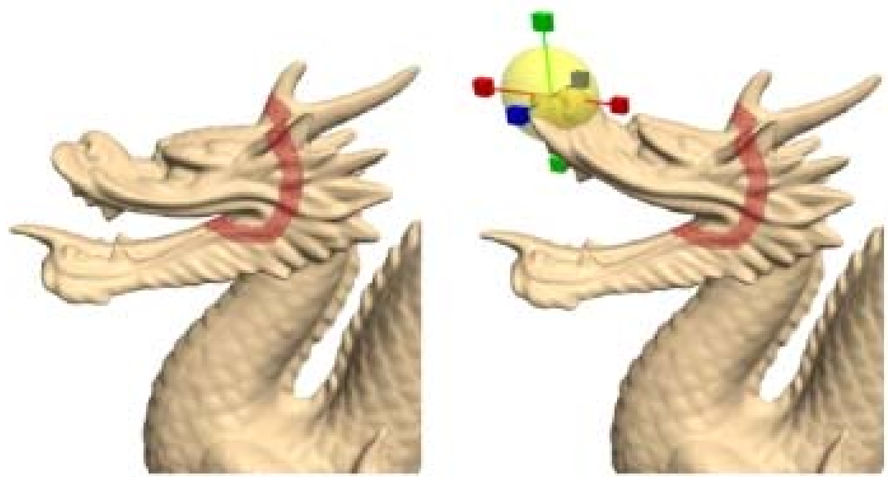
(for large rotations)

Surface Representations

- Laplacian coordinates
- Edge lengths + dihedral angles
- Pyramid coordinates
- Local frames
- ….

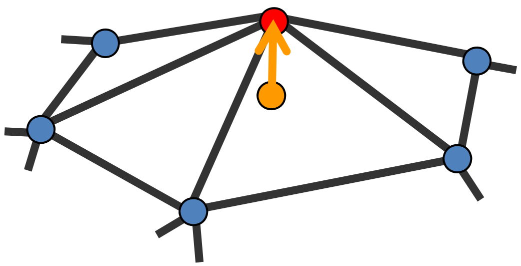
Laplacian Coordinates [Sorkine et al. 04]

- Control mechanism
  - Handles (vertices) moved by user
  - Region of influence (ROI)



Movie

Laplacian Coordinates



$$\boldsymbol{\delta}_i = \mathbf{v}_i - \sum_{j \in N(i)} \frac{1}{d_i} \mathbf{v}_j = \sum_{j \in N(i)} \frac{1}{d_i} \left(\mathbf{v}_i - \mathbf{v}_j \right)$$

$$\delta = LV = (I\text{-}D^{-1}A)V$$

I = Identity matrix

D = Diagonal matrix $[d_{ii} = deg(\boldsymbol{v}_i)]$

A = Adjacency matrix

V = Vertices in mesh

Approximation to normals - unique up to translation

Reconstruct by solving $\boxed{LV = \delta}$ for V, with one constraint

Poisson equation

Deformation

- Pose modeling constraints for vertices $C \subset V$
  - $v'_i = u_i \quad i \in C$

- No exact solution, minimize error

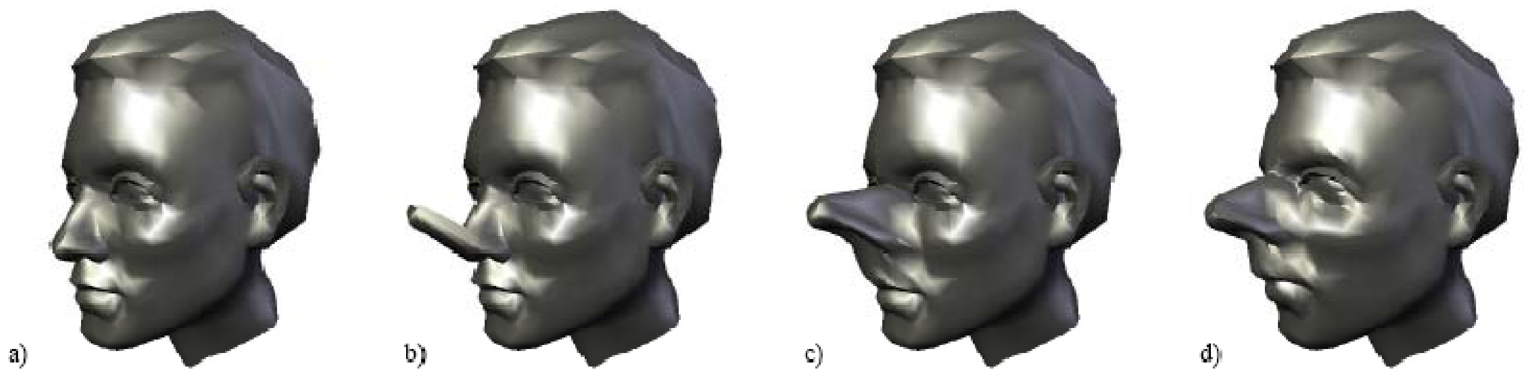
$$\mathbf{V}' = \arg\min_{\mathbf{V}'} \sum_{i=1}^{n} \left\| \delta_i - L(\mathbf{v}'_i) \right\|^2 + \sum_{i \in C} \left\| \mathbf{v}'_i - \mathbf{u}_i \right\|^2$$

Laplacian coordinates of original mesh

Laplacian coordinates of deformed mesh

User Constraints

Deformation



$$\mathbf{V}' = \arg\min_{\mathbf{V}'} \sum_{i=1}^{n} \left\| \delta_i - L(\mathbf{v}'_i) \right\|^2 + \sum_{i \in C} \left\| \mathbf{v}'_i - \mathbf{u}_i \right\|^2$$

Laplacian coordinates of original mesh

Laplacian coordinates of deformed mesh

User Constraints

Linear Least Squares

- $\boldsymbol{Ax = b}$ with *m* equations, *n* unknowns

- Normal equations: $\boldsymbol{(A^{\mathrm{T}}A)x = A^{\mathrm{T}}b}$
- Solution by pseudo inverse:

$$\boxed{\boldsymbol{x = A^{+}b = [(A^{T}A)^{-1}A^{\mathrm{T}}]b}}$$

If system under determined: $\boldsymbol{x = arg\ min\ \{\ ||x||\ :\ Ax = b\ \}}$

If system over determined: $\boldsymbol{x = arg\ min\ \{\ ||Ax\text{-}b||^{2}\ \}}$

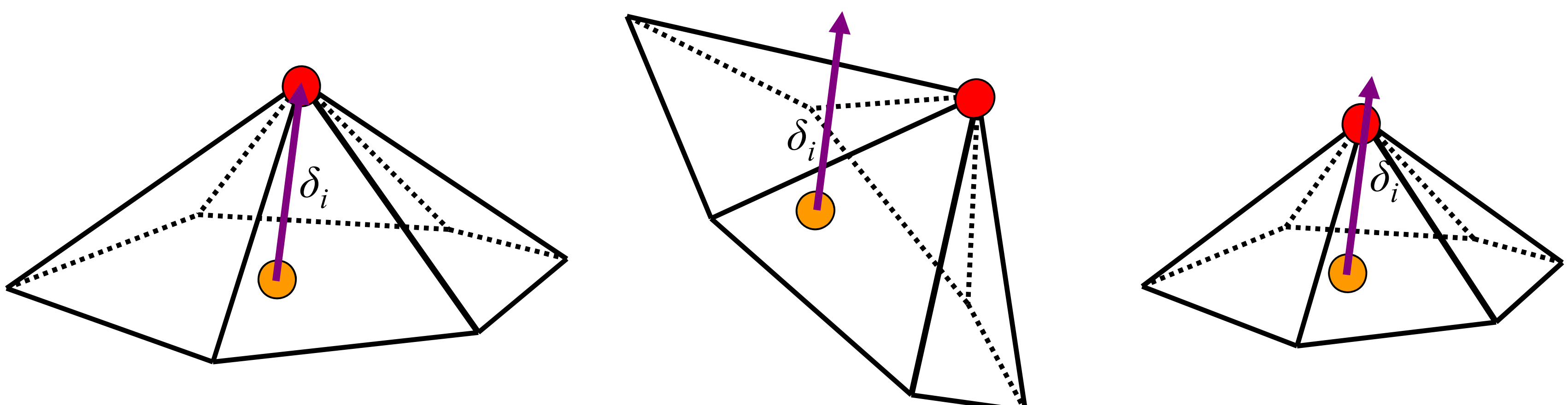
Laplacian Coordinates

Sanity Check

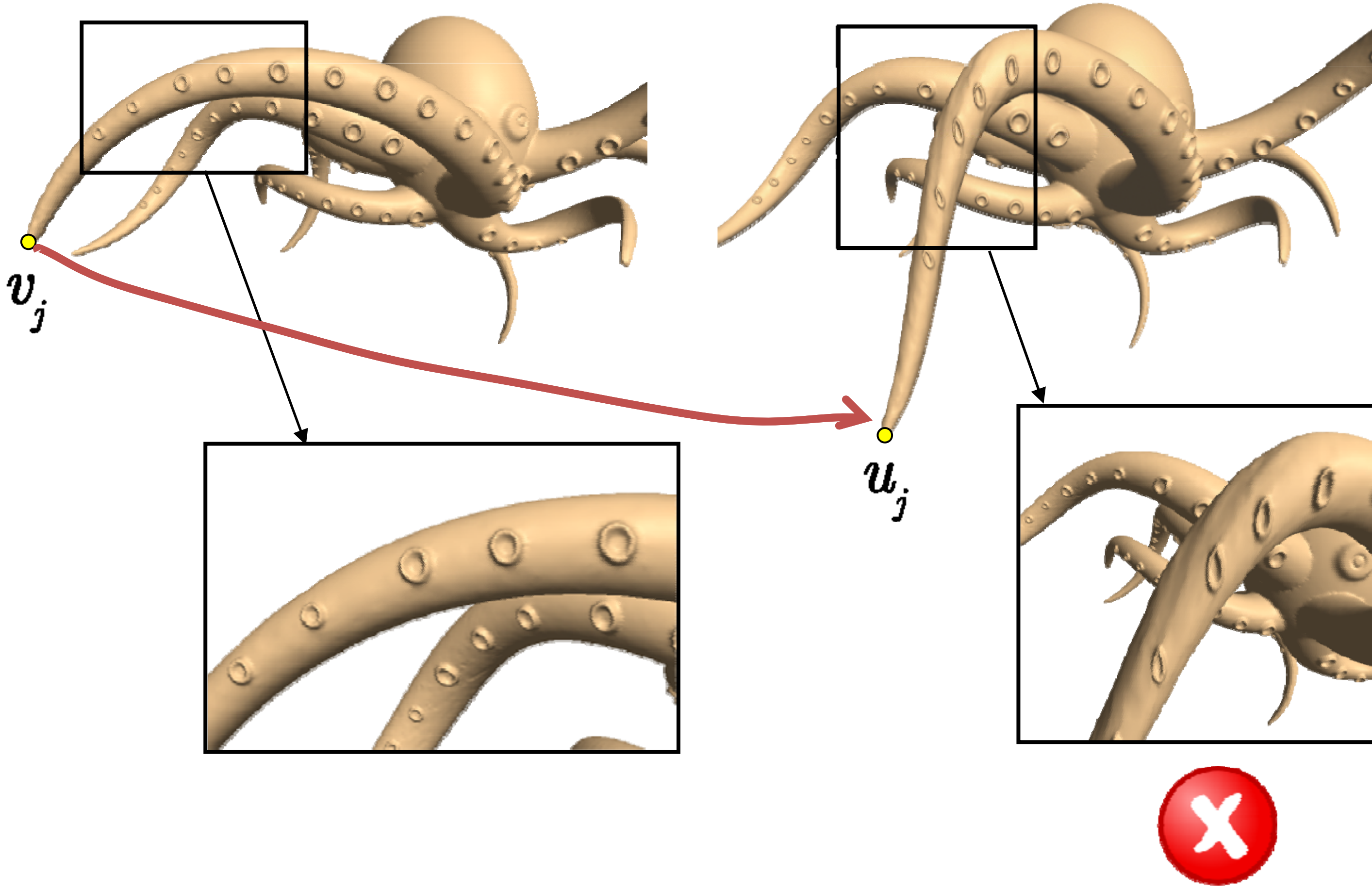
- Translation invariant?

$$\delta_i = L(\mathbf{v}_i) = L(\mathbf{v}_i + \mathbf{t}) \quad \forall \mathbf{t} \in \mathbb{R}^3$$

- Rotation/scale invariant?

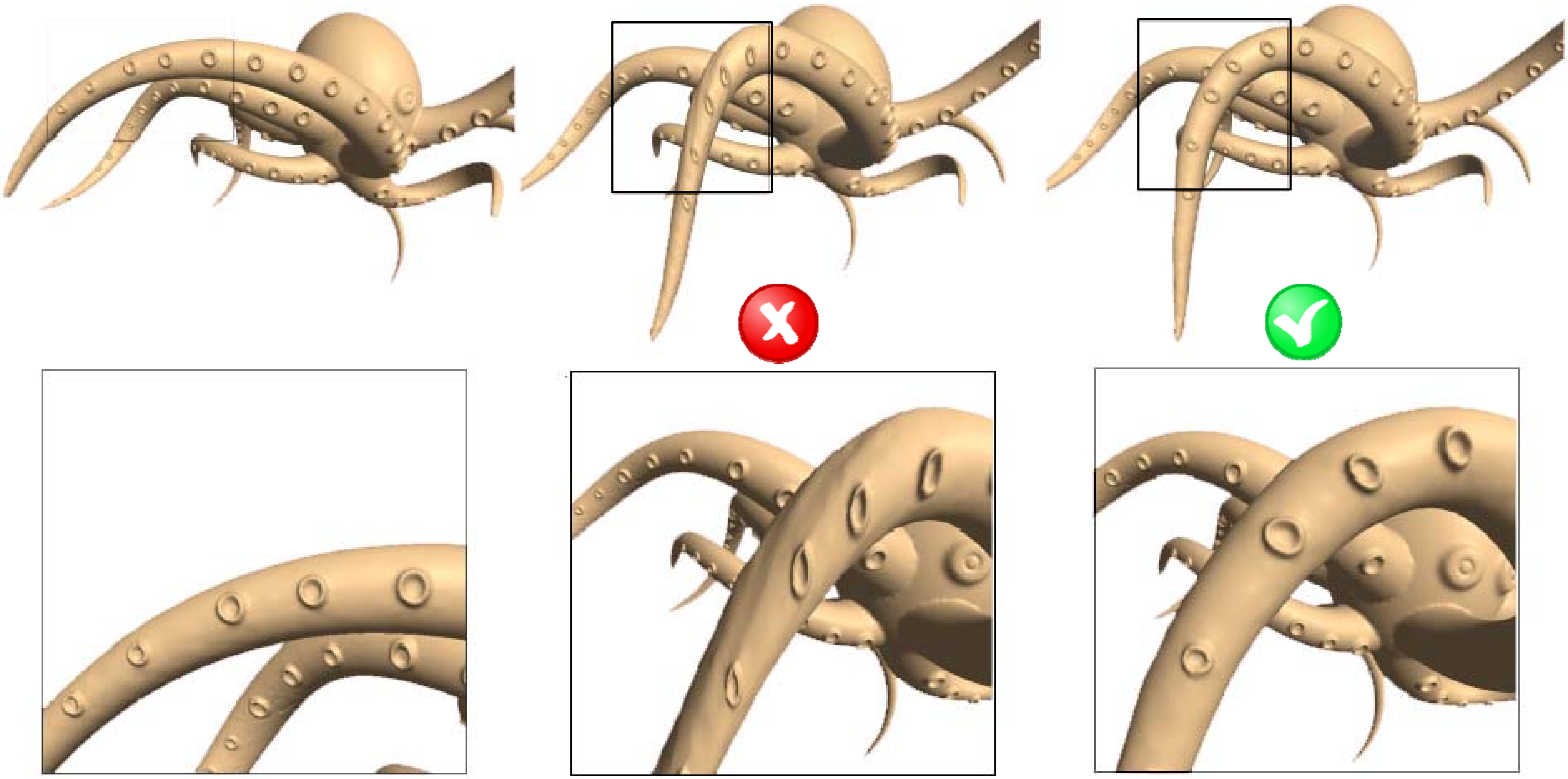


Problem



$$Lv' = \delta$$

$$v'_j = u_j$$

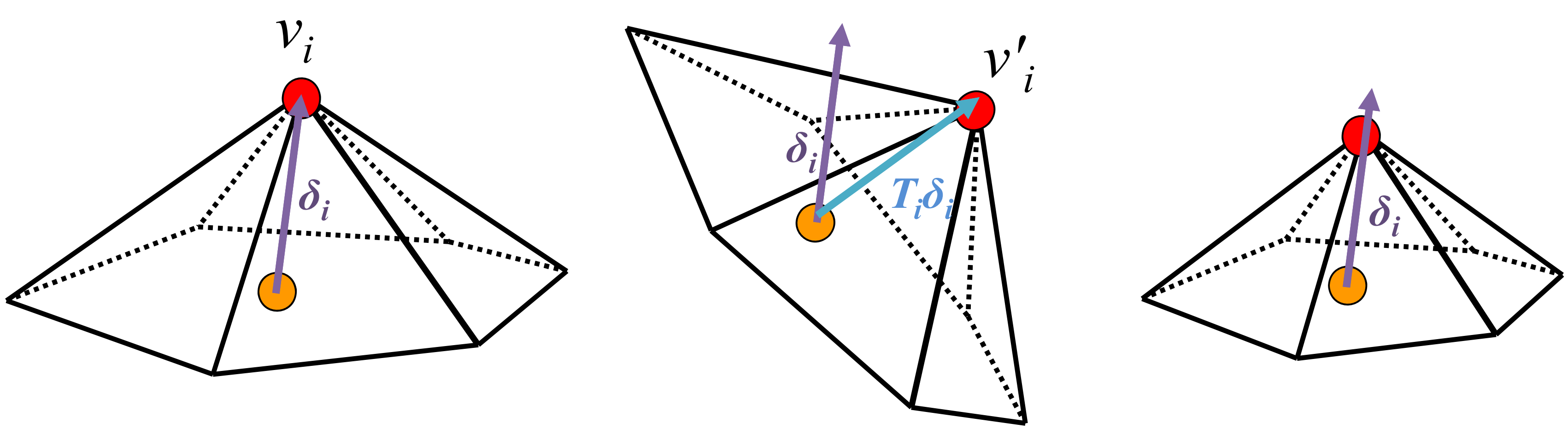


input | Laplacian coords | “Rotation invariant” coords

“Rotation Invariant” Coords

The representation should take into account local rotations + scale

$$\boldsymbol{\delta_i} = L(v_i) \qquad T_i\boldsymbol{\delta_i} = L(v'_i)$$



“Rotation Invariant” Coords

The representation should take into account local rotations + scale

$$\boldsymbol{\delta_i} = \boldsymbol{L}(v_i) \qquad \boldsymbol{T_i}\boldsymbol{\delta_i} = \boldsymbol{L}(v'_i)$$

Problem: $\boldsymbol{T_i}$ depends on **deformed** position v'_i

Solution: Implicit Transformations

Idea: solve for local transformation and deformed surface simultaneously

$$V' = \arg\min_{V'} \left(\sum_{i=1}^{n} \left\| L(\mathbf{v}'_i) - T_i(\boldsymbol{\delta}_i) \right\|^2 + \sum_{j \in C} \left\| \mathbf{v}'_j - \mathbf{u}_j \right\|^2 \right)$$

Transformation of the local frame

Similarities

Restrict T_i to "good" transformations = rotation + scale → similarity transformation

$$V' = \arg\min_{V'} \left(\sum_{i=1}^{n} \left\| L(\mathbf{v}'_i) - T_i(\boldsymbol{\delta}_i) \right\|^2 + \sum_{j \in C} \left\| \mathbf{v}'_j - \mathbf{u}_j \right\|^2 \right)$$

Similarity Transformation

Similarities

- Conditions on T_i to be a similarity matrix?

- Linear in 2D:

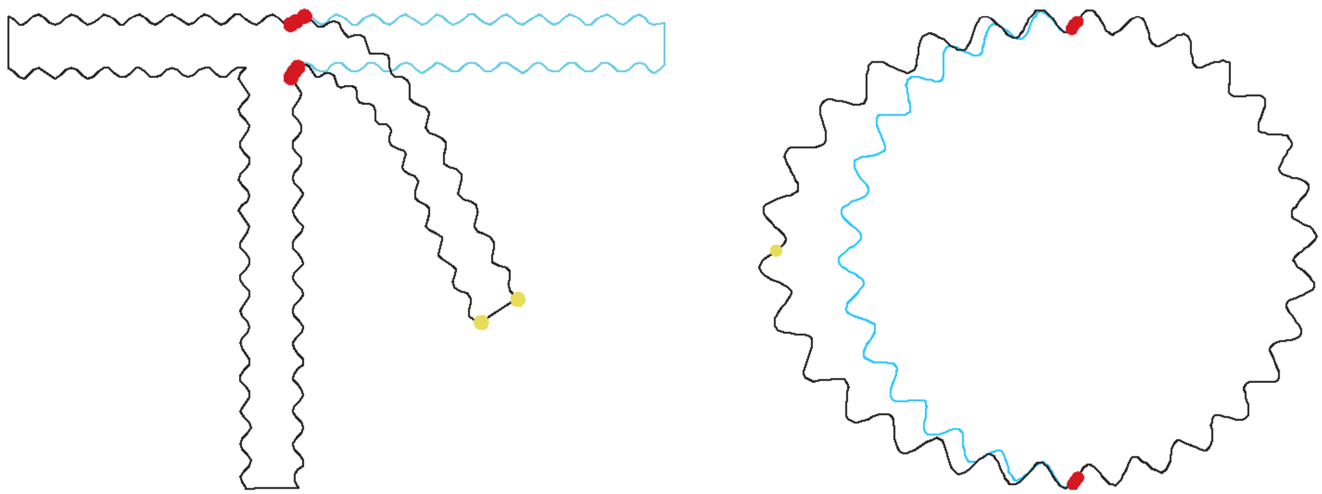
Auxiliary variables

$$T_i = \underbrace{\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Uniform scale}} \underbrace{\begin{pmatrix} \cos\theta & \sin\theta & d_x \\ -\sin\theta & \cos\theta & d_y \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Rotation + translation}} = \begin{pmatrix} w & a & t_x \\ -a & w & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

Uniform scale

Rotation + translation

Similarities 2D



Similarities – 3D case

- Not linear in 3D:

$$\begin{pmatrix}\text{rotation} + \\ \text{uniform scale}\end{pmatrix} = s\ \exp H = s\left(\alpha I + \beta H + \mathbf{h}^T\mathbf{h}\right)$$

H is 3×3 skew-symmetric, $H\mathbf{x} = \mathbf{h}\times\mathbf{x}$

- Linearize by dropping the quadratic term
  - Effectively: only **small** rotations are handled

Laplacian Coordinates

Realtime?

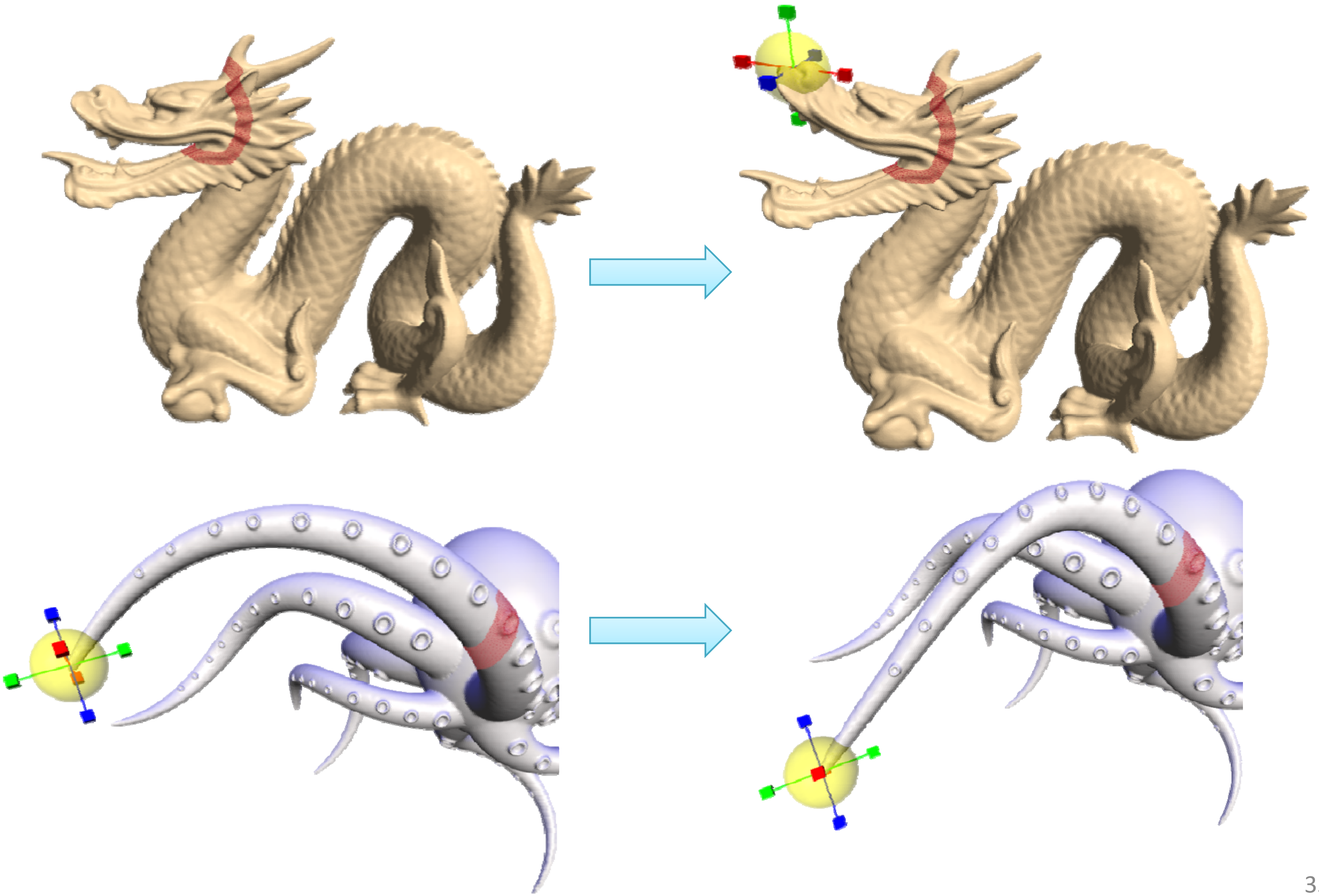
- Need to solve a linear system each frame

$$(A^{\mathrm{T}}A)\boldsymbol{x} = A^{\mathrm{T}}\boldsymbol{b}$$

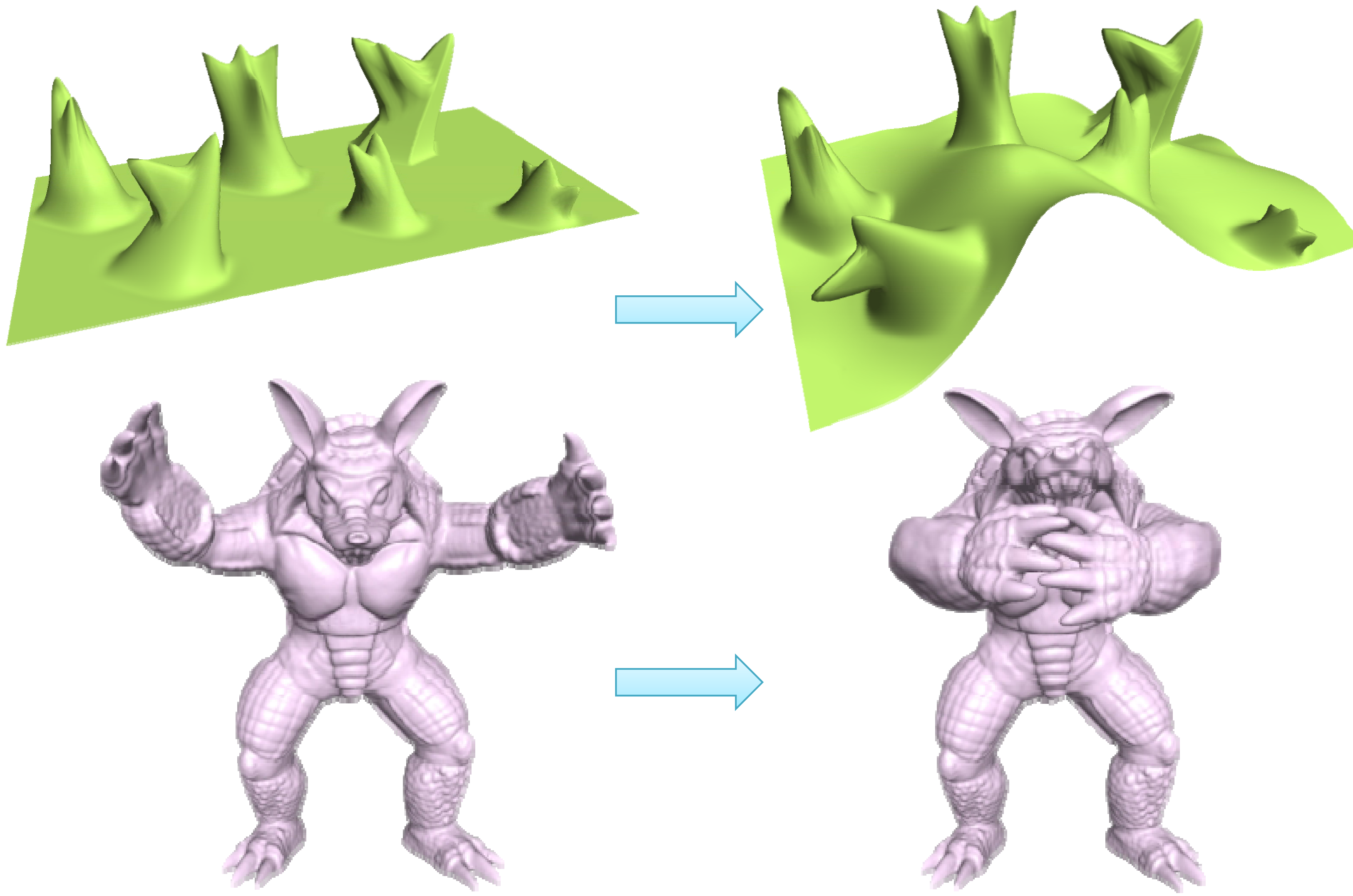
- Precompute sparse Cholesky factorization

- Only back substitution per frame

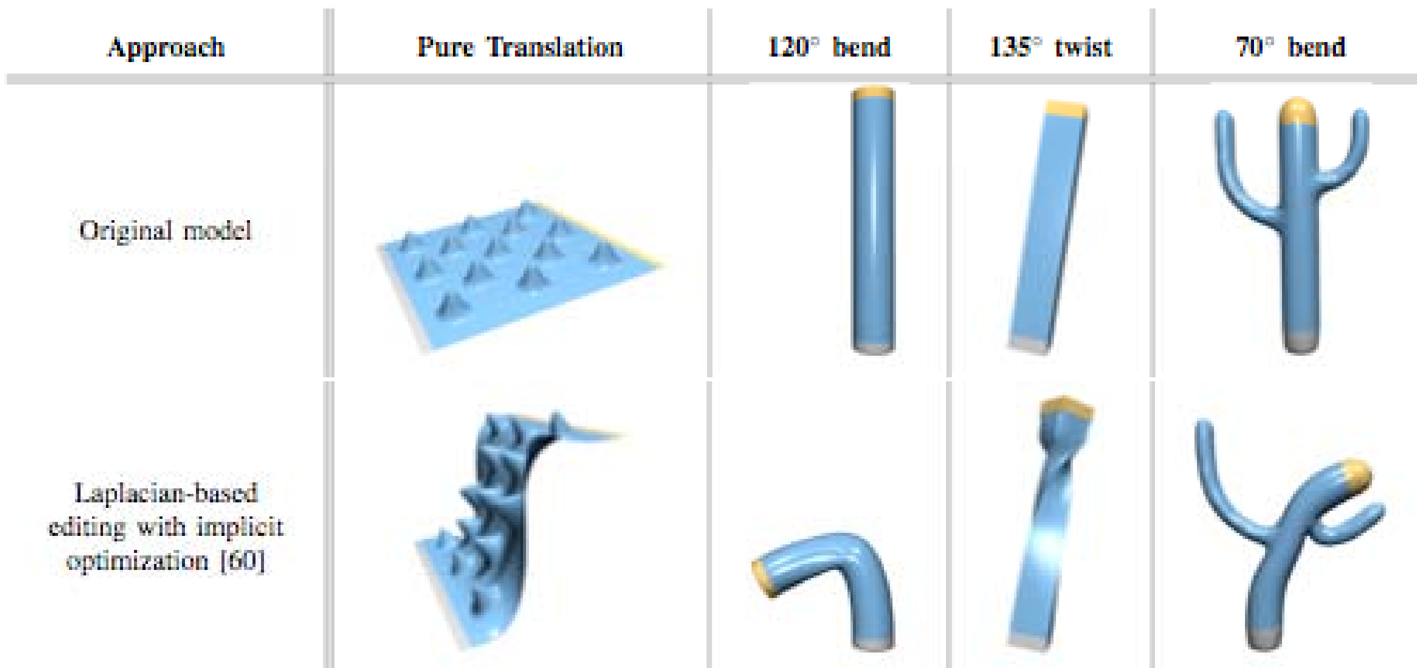
Some Results



Some Results



Limitations: Large Rotations



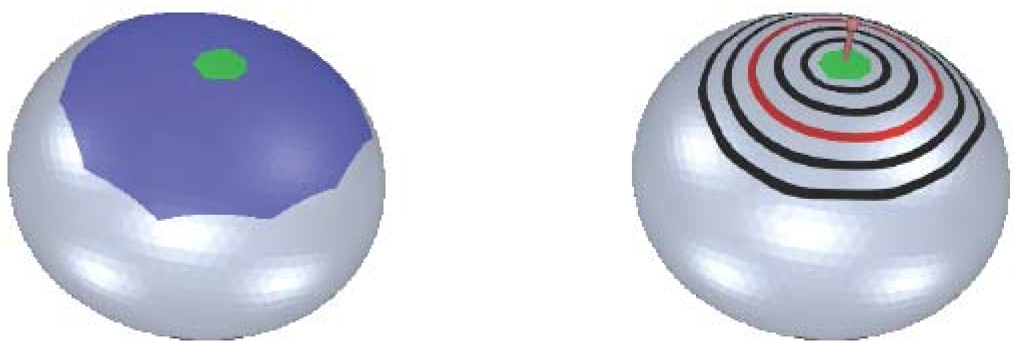
How to Find the Rotations?

- Laplacian coordinates – solve for them
  - Problem: not linear

- Another approach: propagate rotations from handles

Rotation Propagation

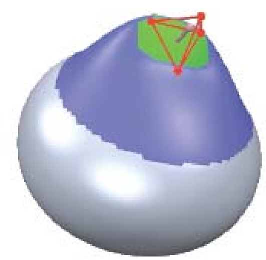
- Compute handle's "deformation gradient"
- Extract rotation and scale/shear components
- Propagate damped rotations over ROI



Deformation Gradient

- Handle has been transformed affinely

$$\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{t}$$



- Deformation gradient is:

$$\nabla \mathbf{T}(\mathbf{x}) = \mathbf{A}$$

- Extract rotation $\mathbf{R}$ and scale/shear $\mathbf{S}$

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad \Rightarrow \quad \mathbf{R} = \mathbf{U}\mathbf{V}^T,\ \mathbf{S} = \mathbf{V}\mathbf{\Sigma}\mathbf{V}^T$$

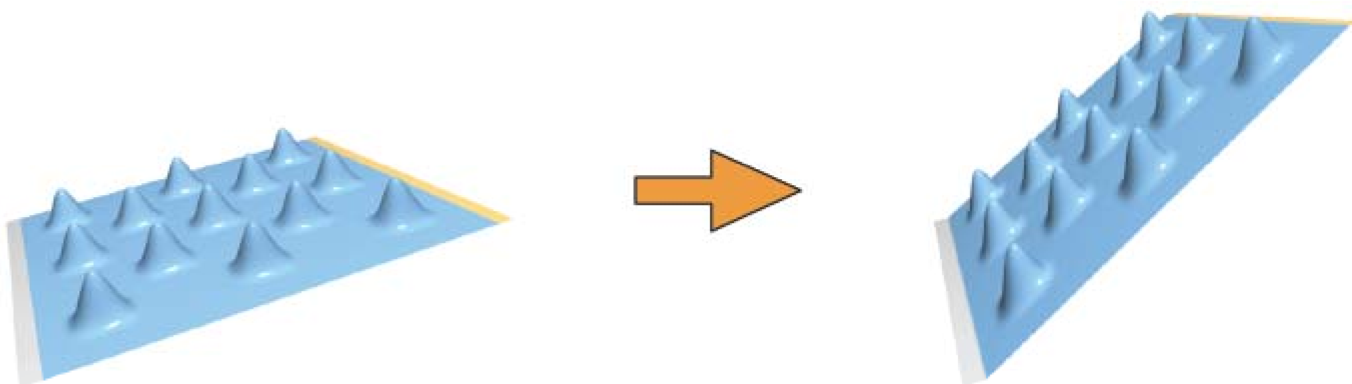
Smooth Propagation

- Construct smooth scalar field [0,1]
  - $\alpha(\mathbf{x})=1$ Full deformation (handle)
  - $\alpha(\mathbf{x})=0$ No deformation (fixed part)
  - $\alpha(\mathbf{x})\in[0,1]$ Damp transformation (in between)

- Linearly damp scale/shear:
  $\mathbf{S}(\mathbf{x}) = \alpha(\mathbf{x})\mathbf{S}(\mathit{handle})$
- Log scale damp rotation:
  $\mathbf{R}(\mathbf{x}) = \exp(\alpha(\mathbf{x})\log(\mathbf{R}(\mathit{handle})))$

Limitations

- Works well for rotations
- Translations don't change deformation gradient
  - "Translation insensitivity"



The Curse of Rotations

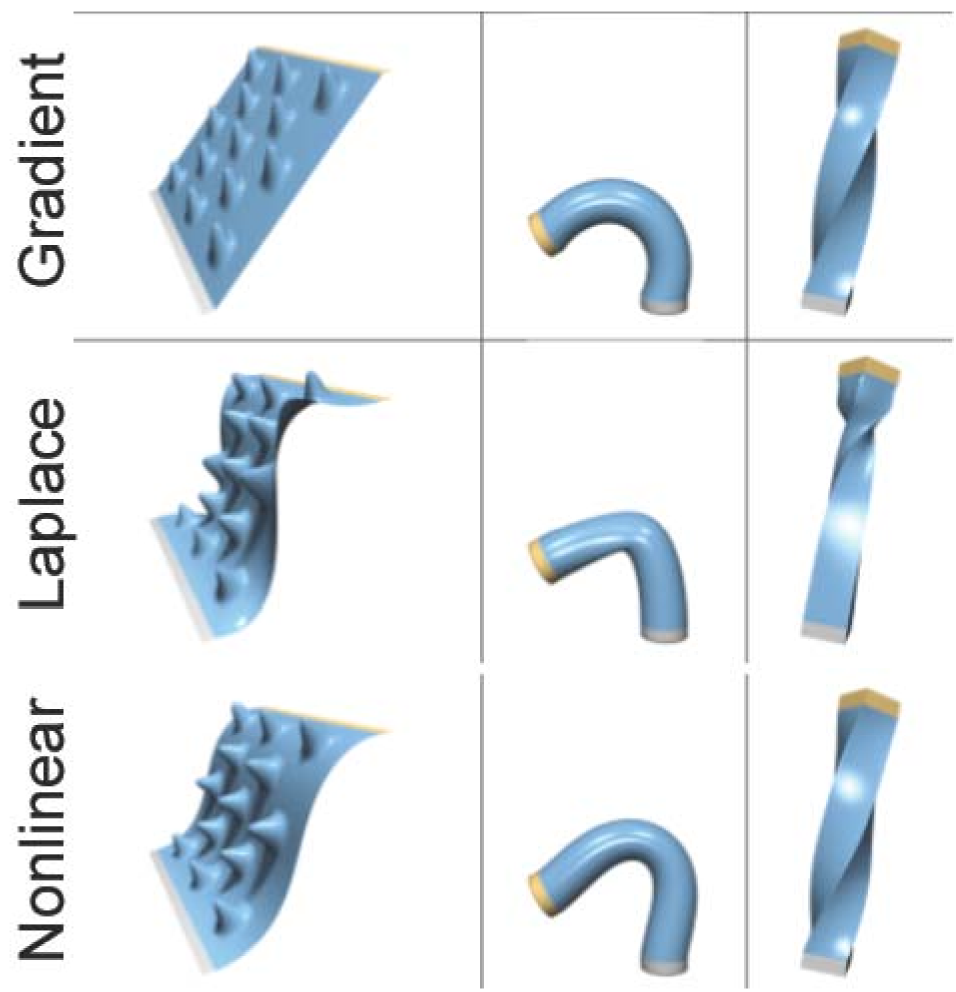
- Can’t solve for them directly using a linear system
- Can’t propagate if the handles don’t rotate

- Some linear methods work for rotations
- Some work for translations
- None work for both

The Curse of Rotations

- Non linear methods work for both large rotations and translation only

- No free lunch: much more expensive



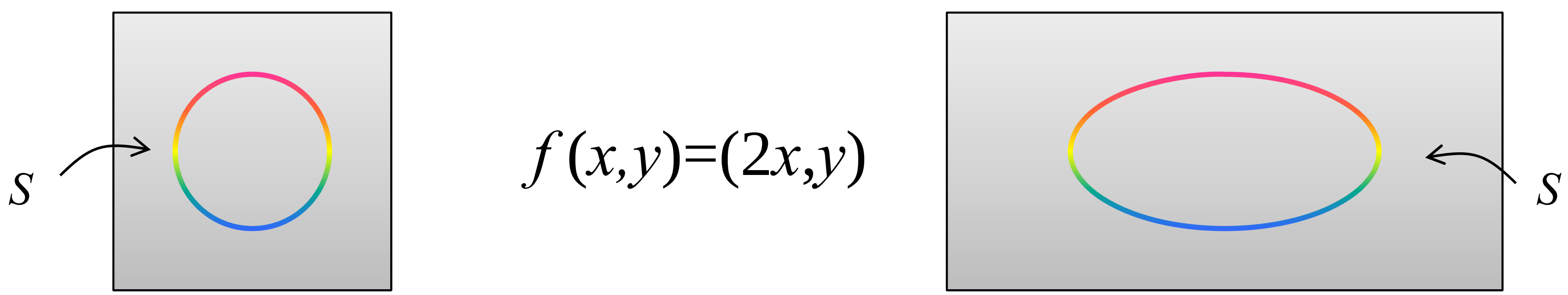
Space Deformation

- Deformation function on ambient space

$$f : \mathbb{R}^n \to \mathbb{R}^n$$

- Shape S deformed by applying f to points of S

$$S' = f(S)$$



Motivation

- Can be applied to any geometry
  - Meshes (= non-manifold, **multiple components**)
  - Polygon soups
  - Point clouds
  - Volumetric data

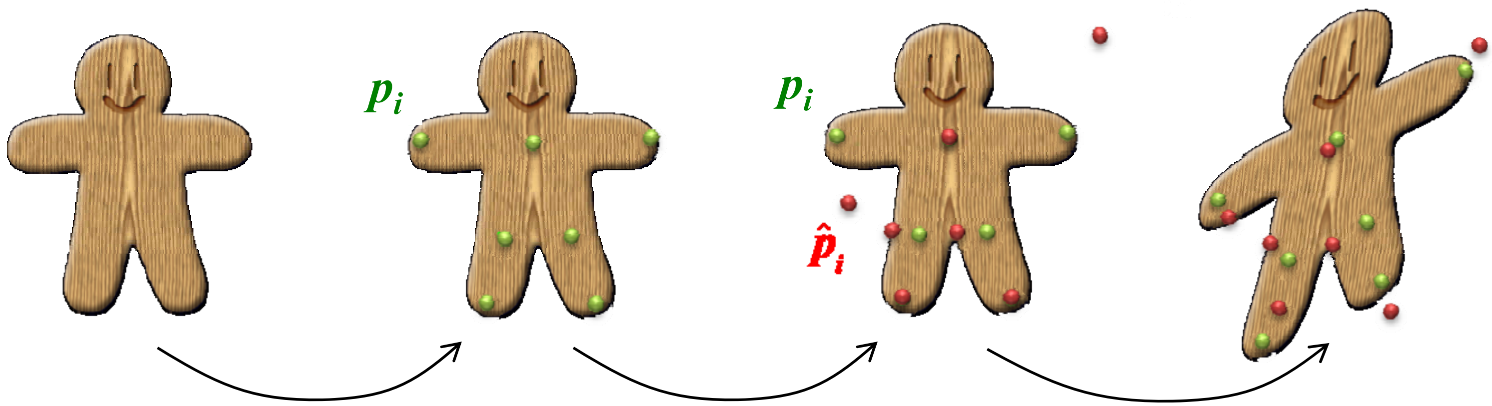
- Complexity decoupled from geometry complexity
  - Can pick the best complexity for required deformation

Required Properties

- Invariant to global operators
  - Global translation
  - Global rotation
- Smooth
- Efficient to compute
- “Intuitive deformation” ?
  - Can pose constraints as in surface deformation

MLS Deformation

[Schaeffer et al. '06]



1. Handles p_i 2. Target locations $\hat{p}_i$

3. Find best affine transformation that maps p_i *to* $\hat{p}_i$

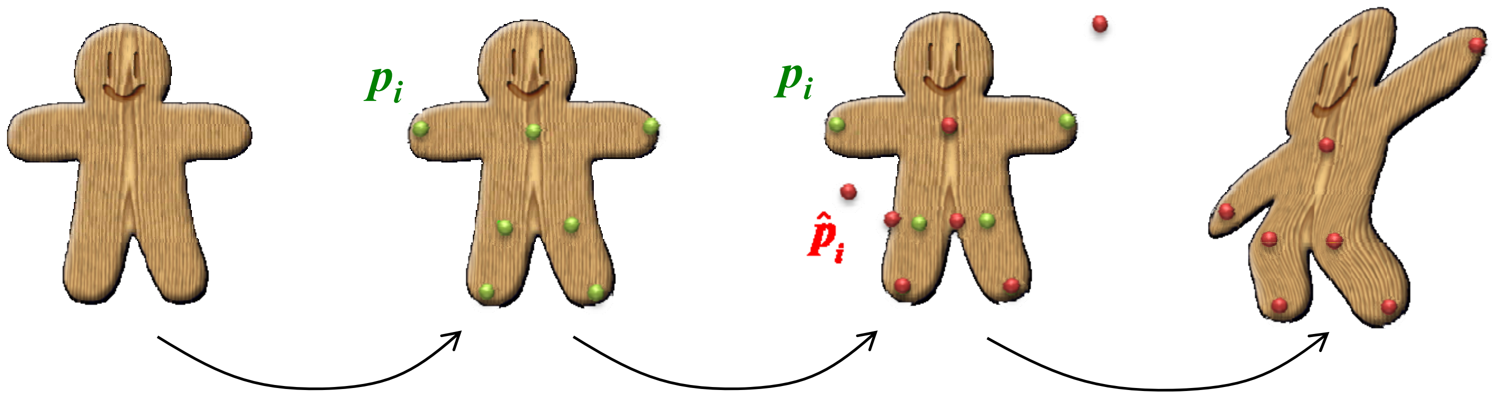
$$\min_{M,T} \sum_i \left| \left(M p_i + T \right) - \hat{p}_i \right|^2$$

4. Deform

$$f(v) = Mv + T$$

MLS Deformation

[Schaeffer et al. '06]



1. Handles p_i

2. Target locations $\hat{p}_i$

3. Find best affine transformation that maps p_i *to* $\hat{p}_i$

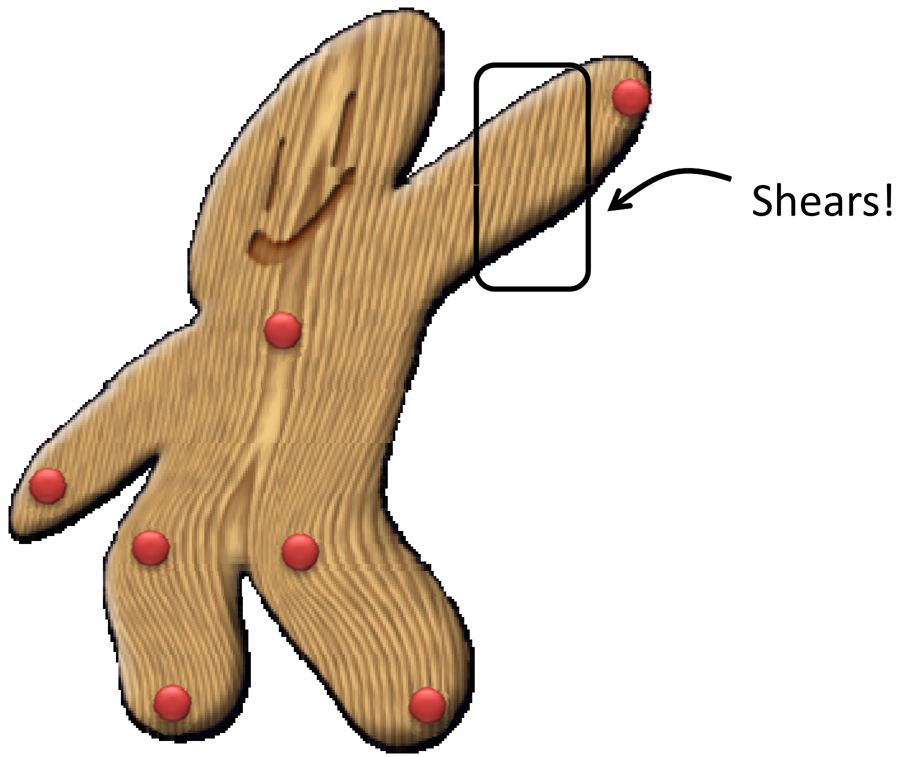
$$\min_{M,T}\sum_i \left|\frac{1}{|p_i - v|}\left(Mp_i + T\right) - \hat{p}_i\right|^2$$

4. Deform

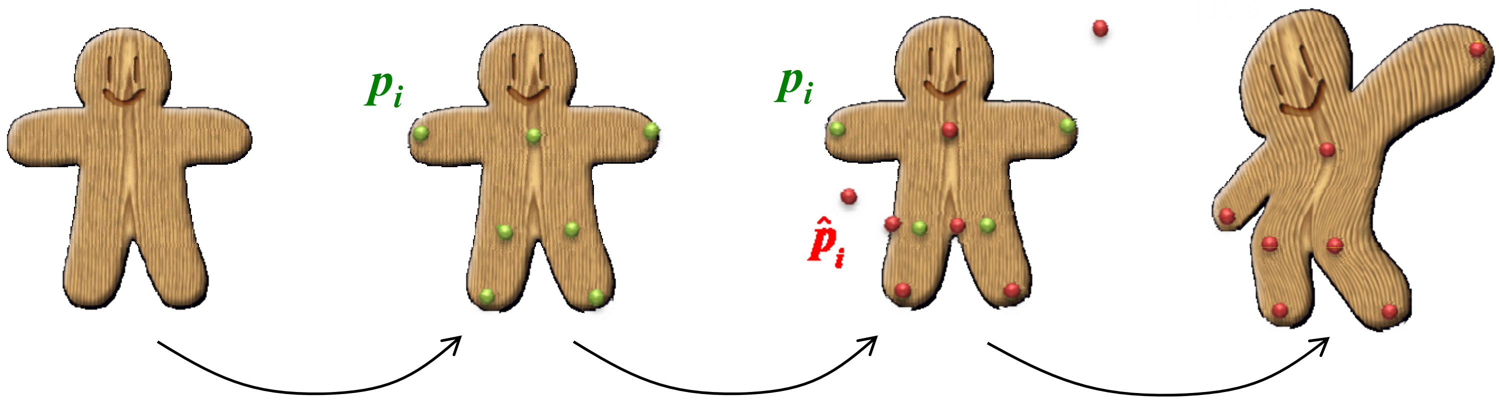
$$f(v) = Mv + T$$

Closed form solution

Similarity ~~Affine~~ Transformations?



Similarity Transformations



1. Handles p_i

2. Target locations $\hat{p}_i$

$$M = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

3. Find best <u>similarity</u> transformation that maps p_i *to* $\hat{p}_i$

$$\min_{M,T} \sum_i \left| \frac{1}{|p_i - v|} \left(\left(M p_i + T \right) - \hat{p}_i \right) \right|^2$$

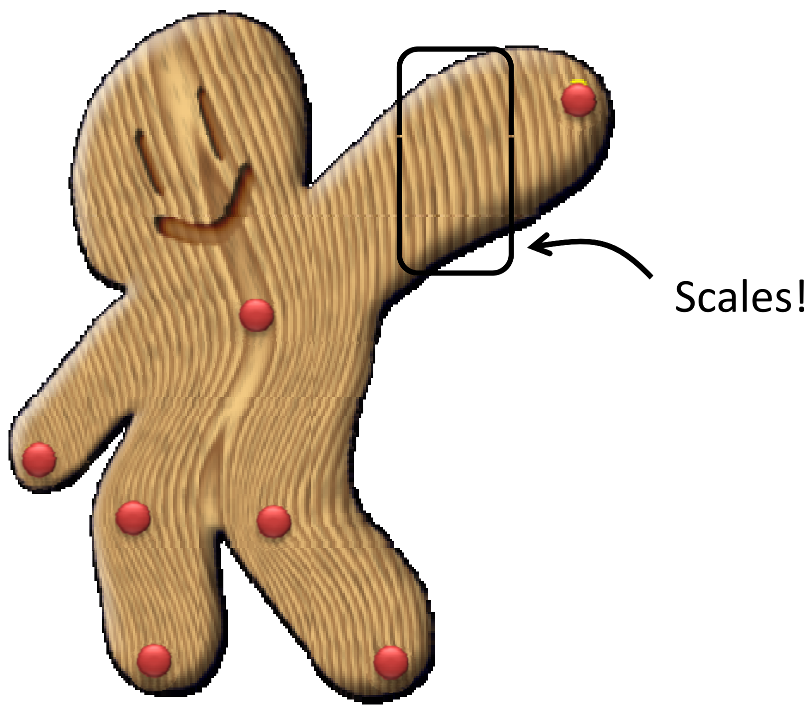
$$\min_{c,s,T} \sum_i \left| \frac{1}{|p_i - v|} \left(c \begin{pmatrix} p_{x,i} \\ p_{y,i} \end{pmatrix} + s \begin{pmatrix} p_{y,i} \\ -p_{x,i} \end{pmatrix} + T \right) - \hat{p}_i \right|^2$$

4. Deform

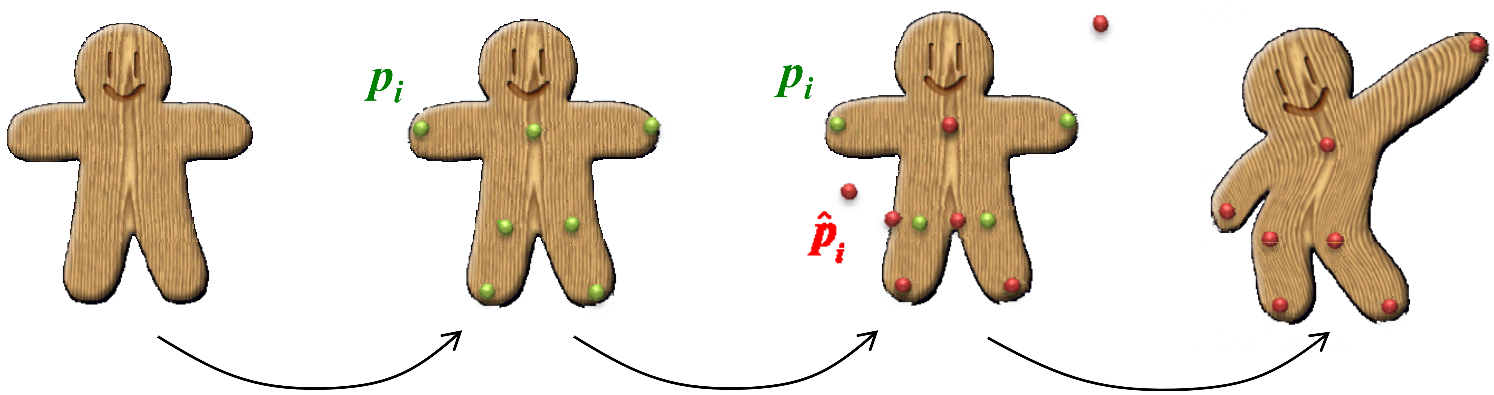
$$f(v) = Mv + T$$

Closed form solution

Rigid ~~Similarity~~ Transformations?



Rigid Transformations



1. Handles p_i

2. Target locations $\hat{\boldsymbol{p}}_i$

$$c^2 + s^2 = 1$$

3. Find best <u>**rigid**</u> transformation that maps p_i *to* $\hat{\boldsymbol{p}}_i$

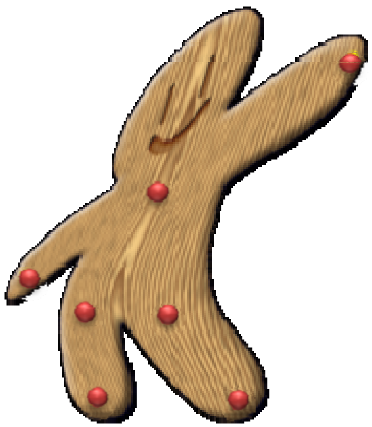
$$\min_{c,s,T} \sum_i \left| \frac{1}{\left| p_i - v \right|} \left(\begin{pmatrix} c & s \end{pmatrix} \begin{pmatrix} p_{x,i} & p_{y,i} \\ p_{y,i} & -p_{x,i} \end{pmatrix} + T \right) - \hat{p}_i \right|^2$$

4. Deform

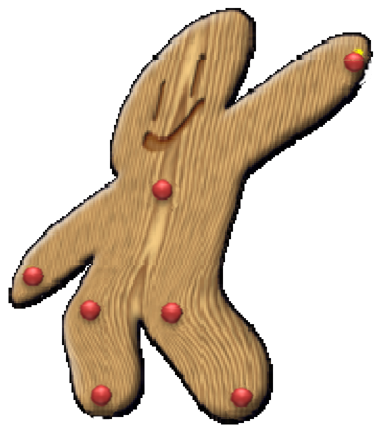
$$f(v) = Mv + T$$

Closed form solution given best similarity

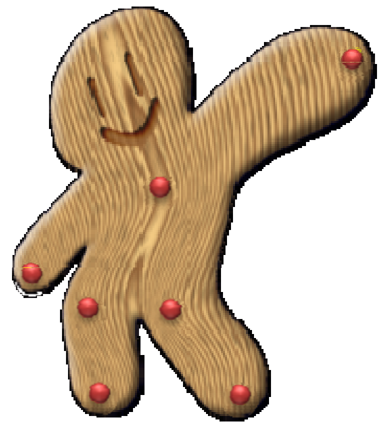
Comparison



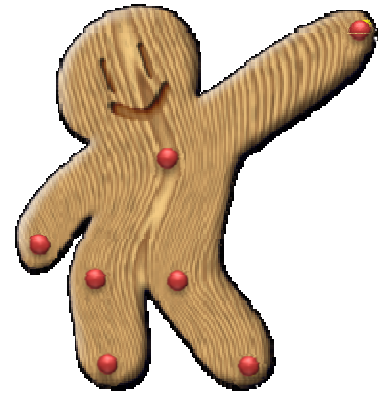
Thin-Plate
[Bookstein ’89]



Affine MLS

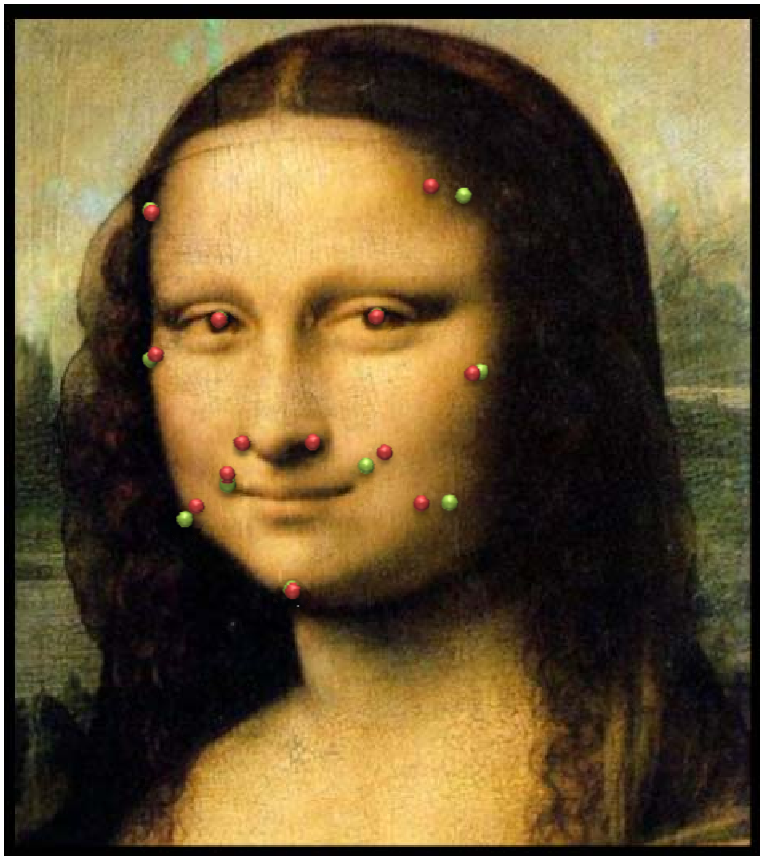


Similarity MLS

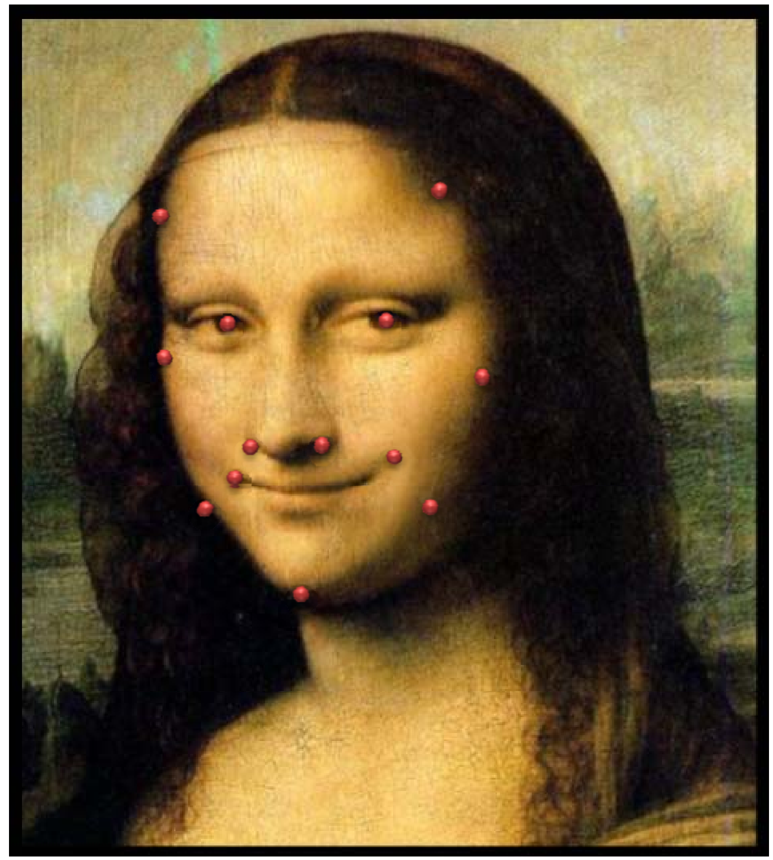


Rigid MLS

Examples



Before



After

Examples

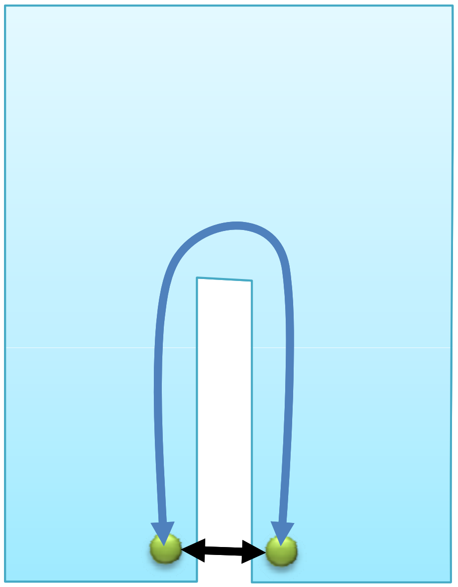
Horse Giraffe

Limitations

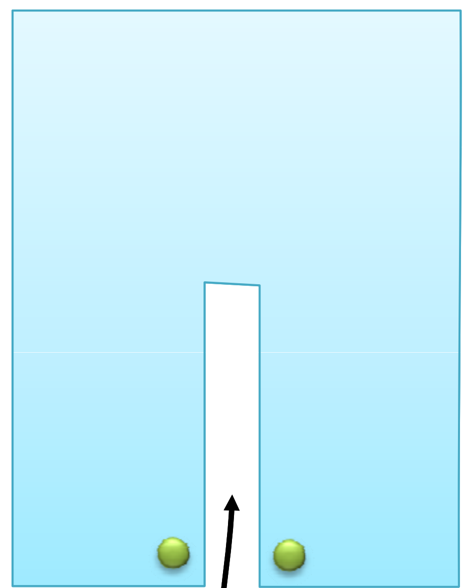
Deforms all space - is not “shape aware”

The "Pants" Problem

Small Euclidean distance
Large **geodesic** distance



The “Pants” Problem



Don’t care about distortion **outside** the shape

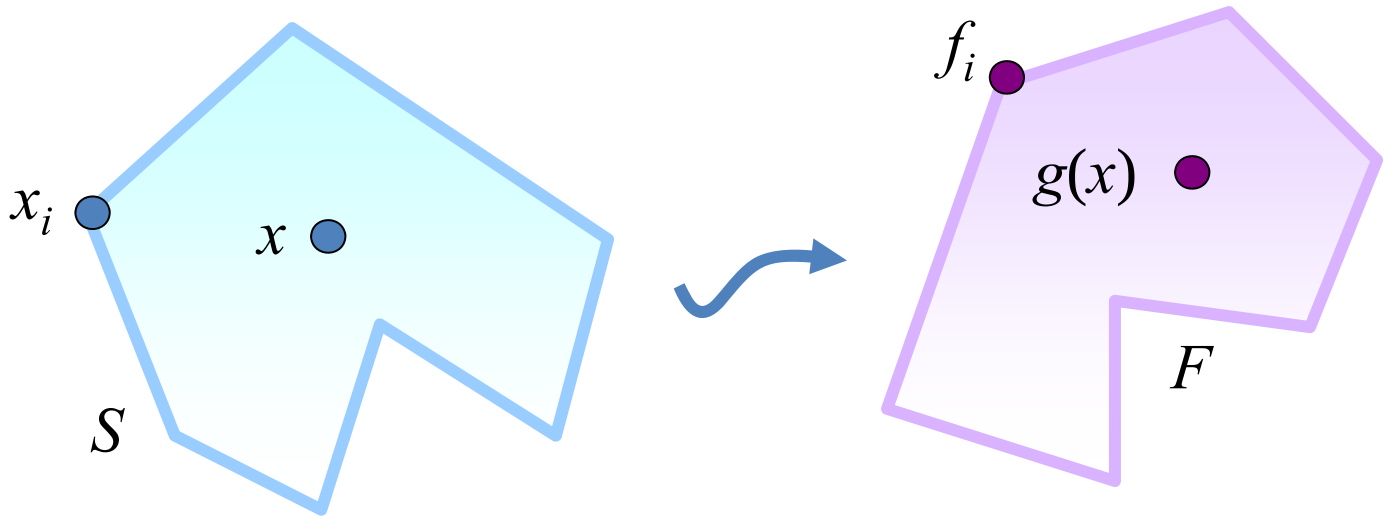
Solution: Cages

- Enclose the shape in a “cage” $\Omega \subset \mathbb{R}^n$
- Deformation function defined only on cage

$$f : \Omega \to \mathbb{R}^n$$

- New problem: how to build the cage?

Deformation with a Cage



$S = \{x_1, x_2, \ldots, x_n\}$

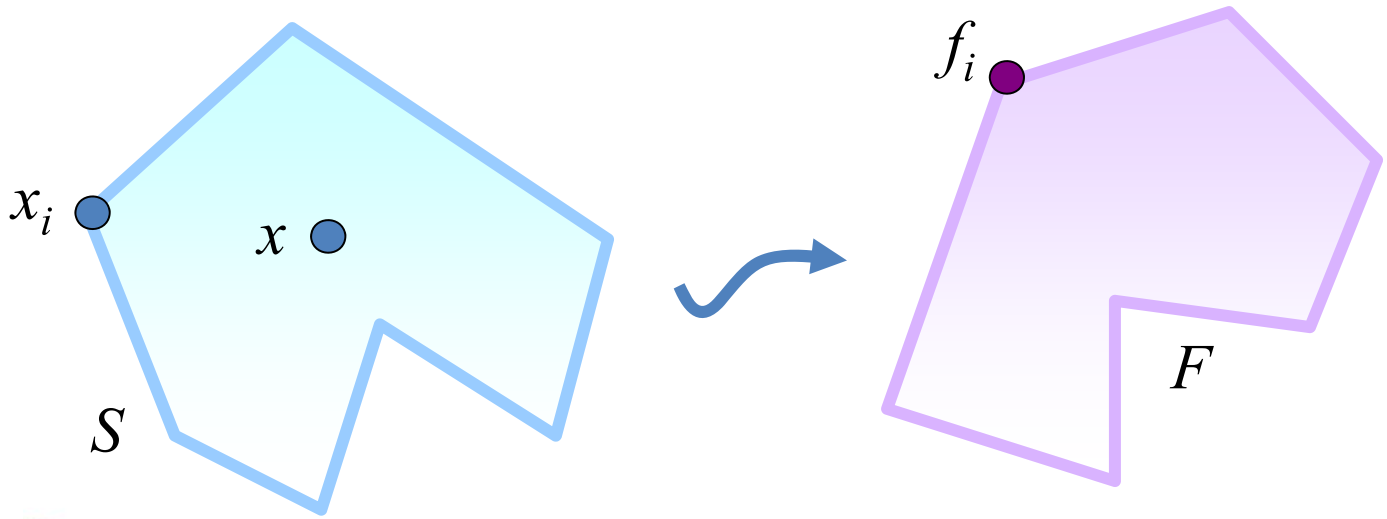
Source polygon

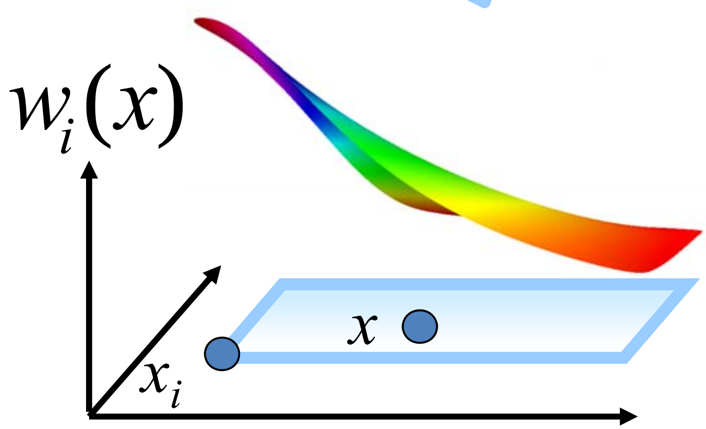
$x_i \rightarrow f_i$

Target polygon

$g(x) = ?$ Interior?

Barycentric Coordinates

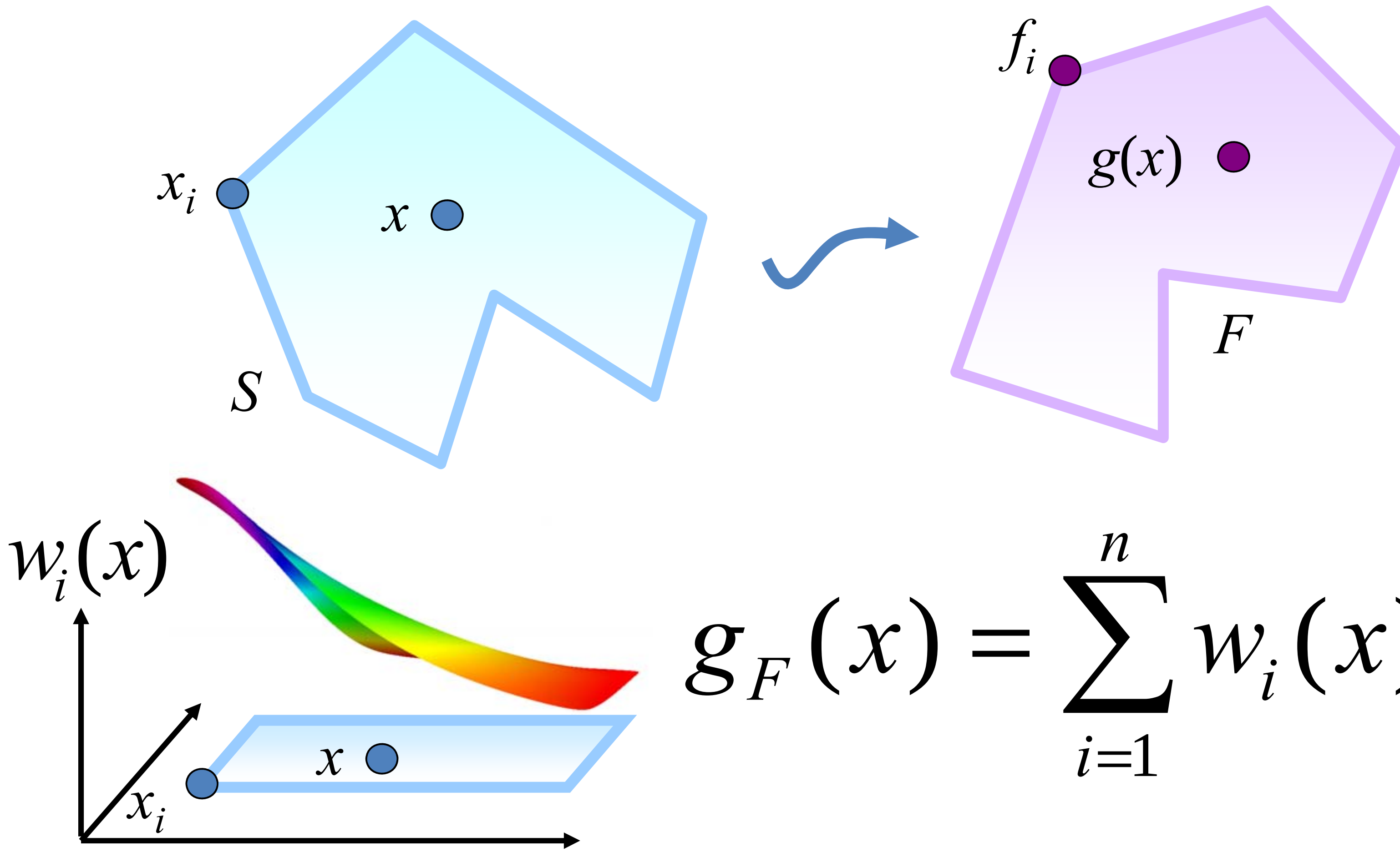




$$w_i(x) : \Omega \rightarrow \mathbb{R}^n$$

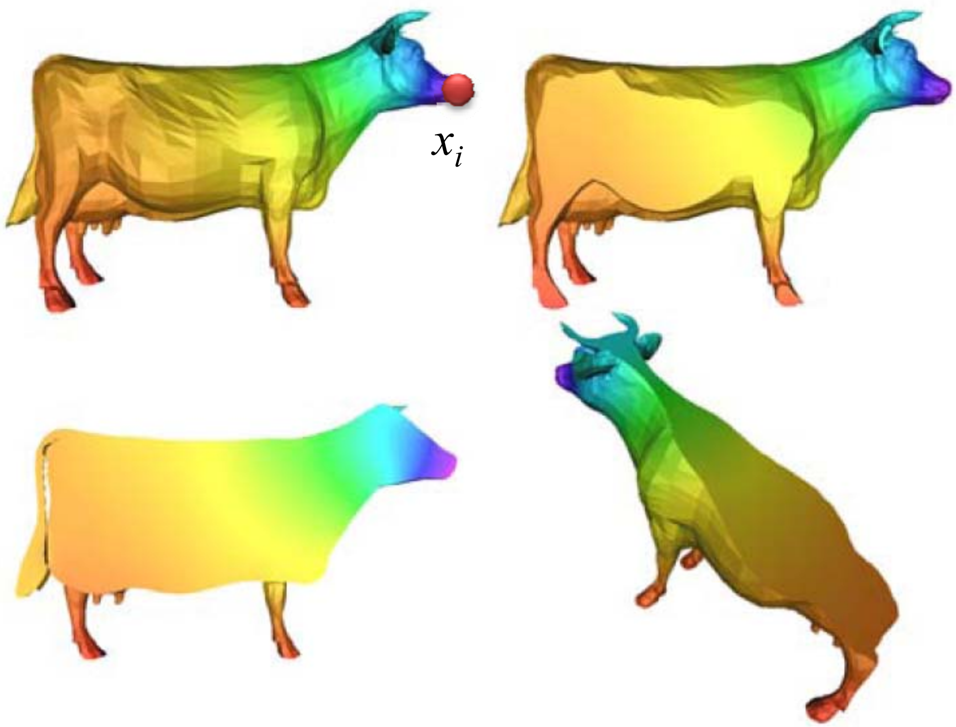
Barycentric Coords Function

Barycentric Coordinates



$$g_F(x) = \sum_{i=1}^{n} w_i(x) f_i$$

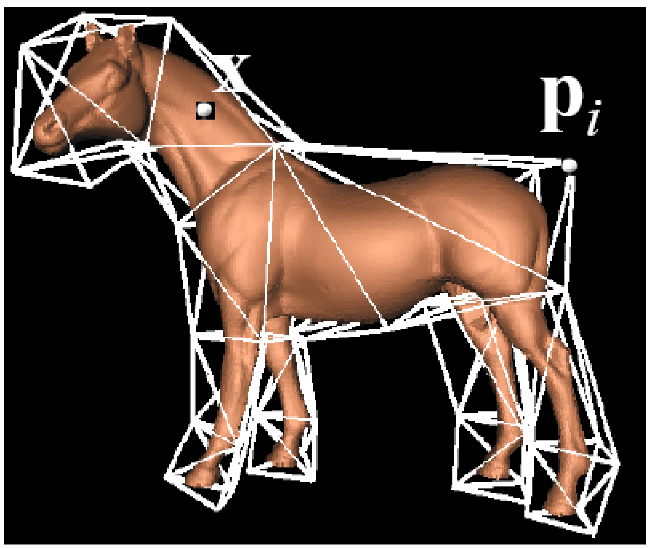
Example

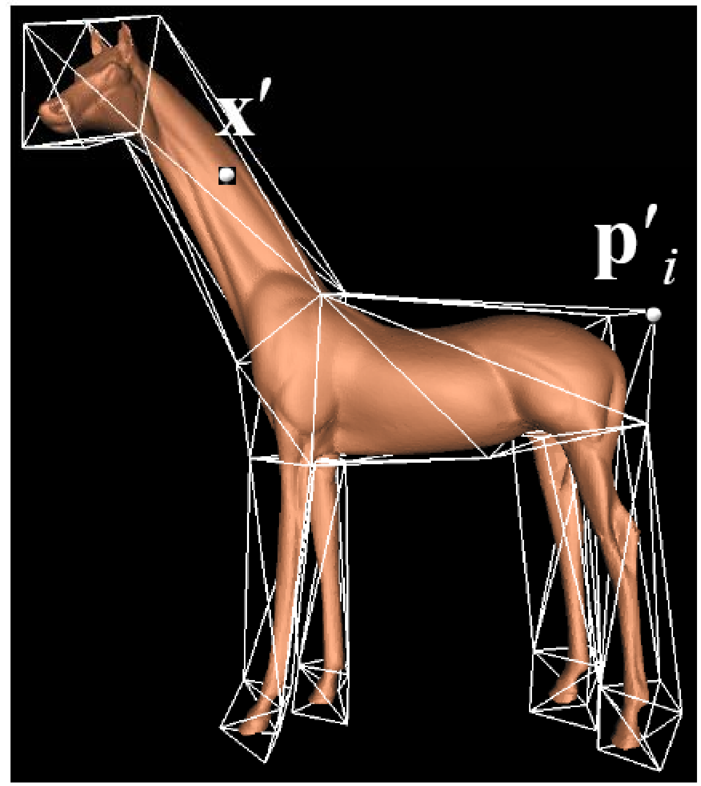


$w_i(x)$

Example

$$\mathbf{x}' = \sum_{i=1}^{k} w_i(\mathbf{x})\mathbf{p}'_i$$





Barycentric Coordinates

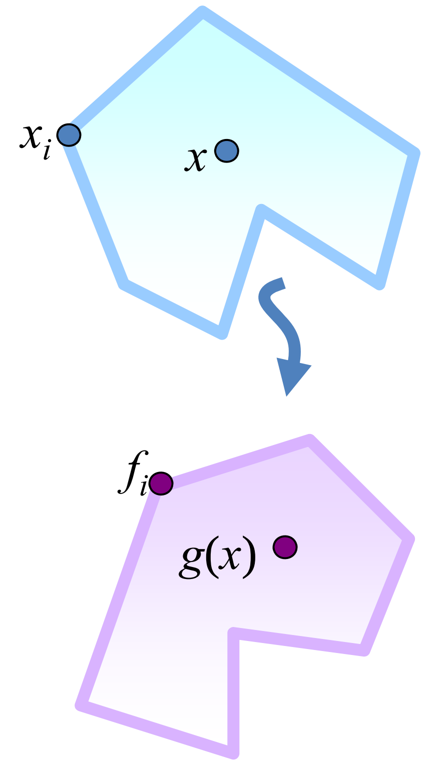
Required properties

- Translation invariance (constant precision)

$$\sum_{i=1}^{n} w_i(x) = 1$$

- Reproduction of identity (linear precision)

$$\sum_{i=1}^{n} w_i(x) x_i = x$$



$$g(x) = \sum_{i=1}^{n} w_i(x) f_i$$

Barycentric Coordinates

Constant + linear precision = affine invariance

$$g_{Ax_i+T}(x) =$$

$$x \qquad\qquad 1$$

Barycentric Coordinates

Required properties

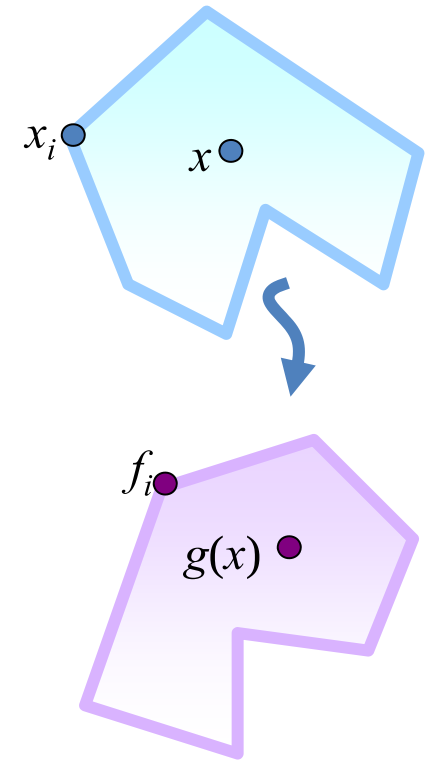
- Smoothness – at least C1

- Interpolation (Lagrange property)

$$f(x_j) = f_j$$

$$\Updownarrow$$

$$w_i(x_j) = \delta_{ij}$$



$$g(x) = \sum_{i=1}^{n} w_i(x) f_i$$

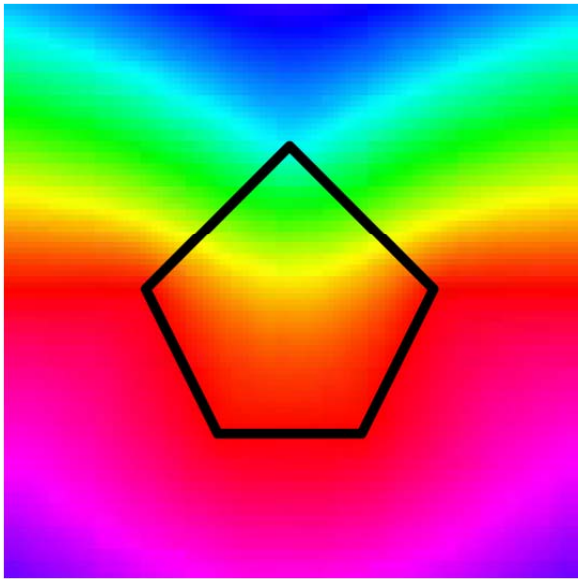
Example: Mean Value Coords

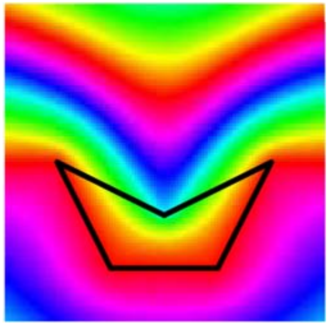
[3D: Ju et al '05]

$$k_i(x) = \frac{\tan\left(\frac{\alpha_{i-1}}{2}\right) + \tan\left(\frac{\alpha_i}{2}\right)}{\left|x_i - x\right|}$$

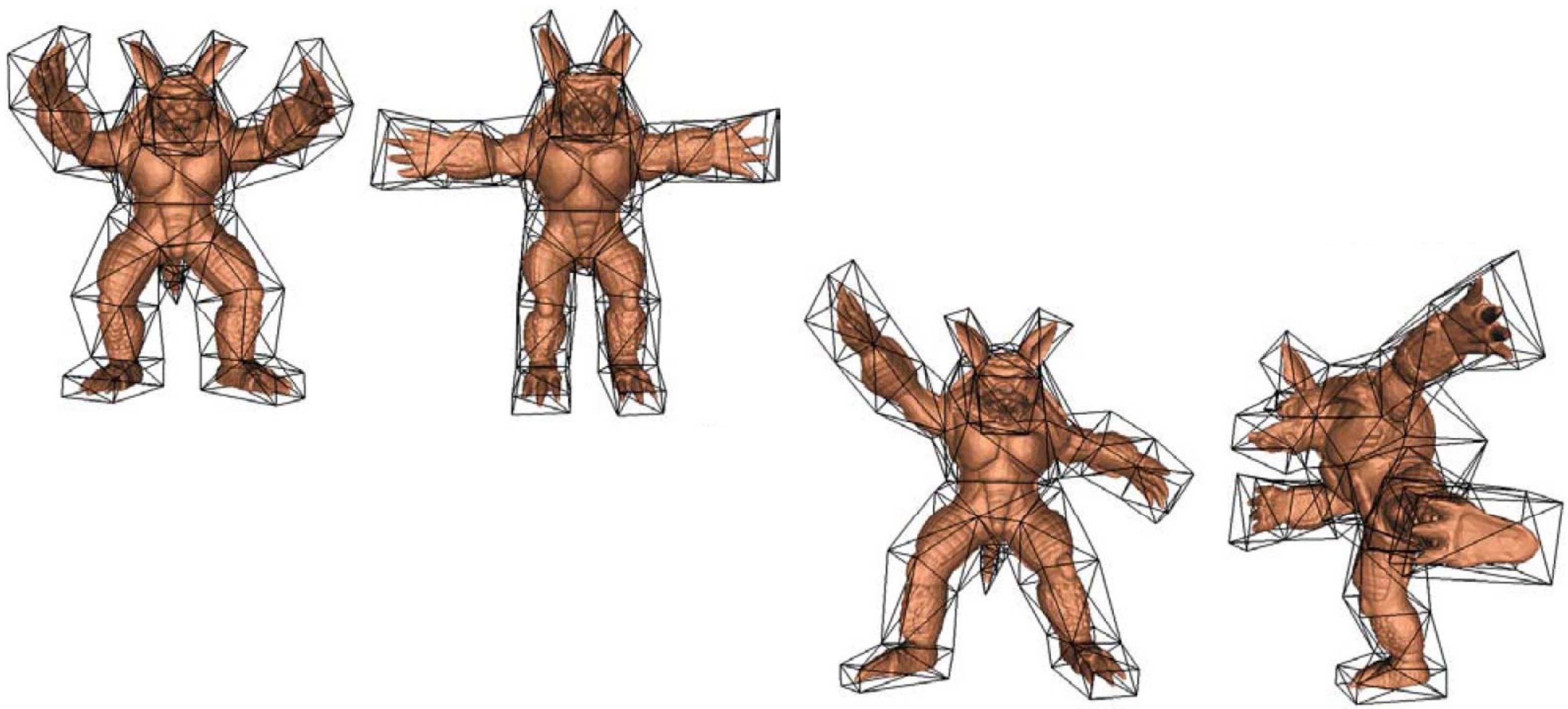
$$w_i(x) = \frac{k_i(x)}{\sum_i k_i(x)}$$

Closed form!





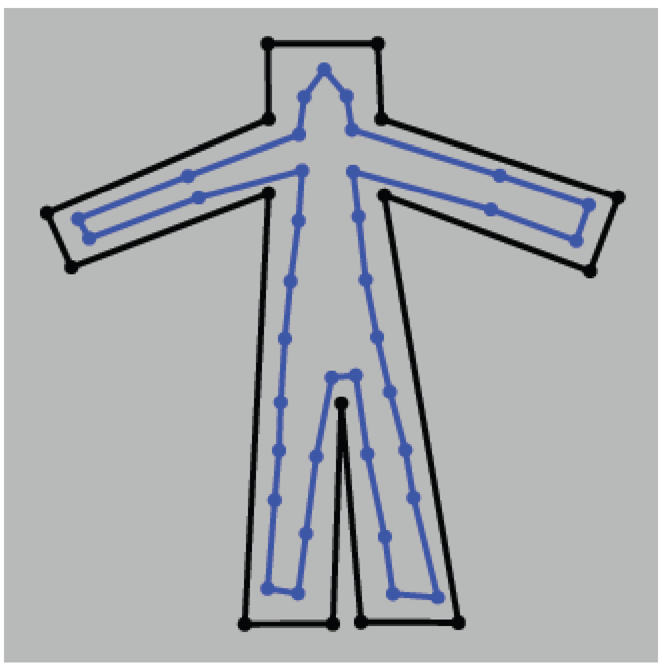
Example: Mean Value Coords

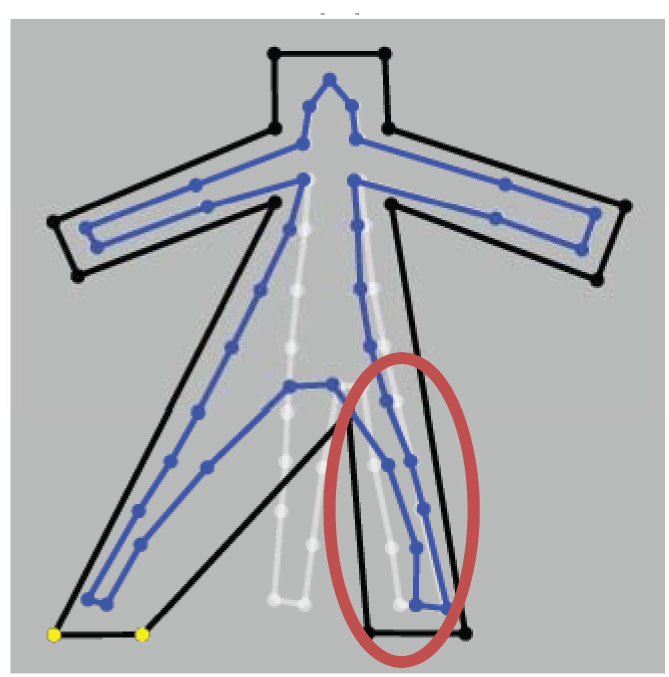


MV - Limitations

Back to the pants problem

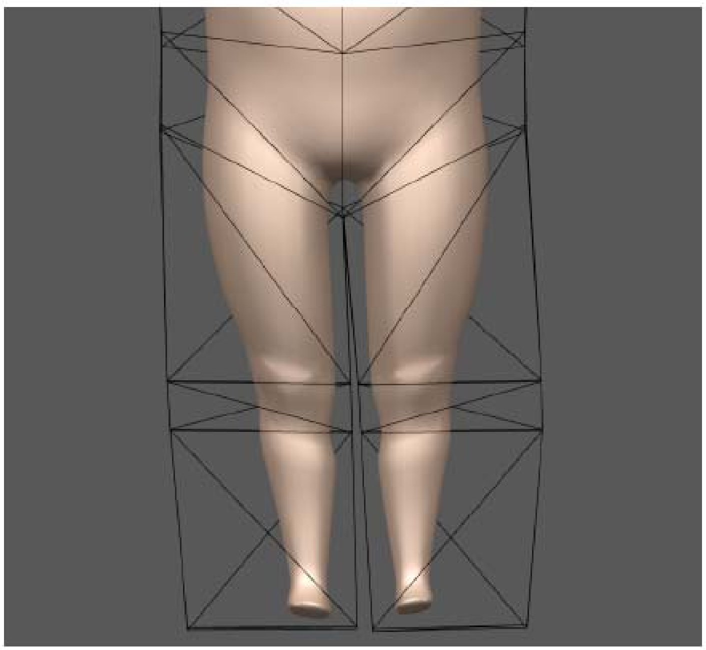
- MV **negative** on concave polygons

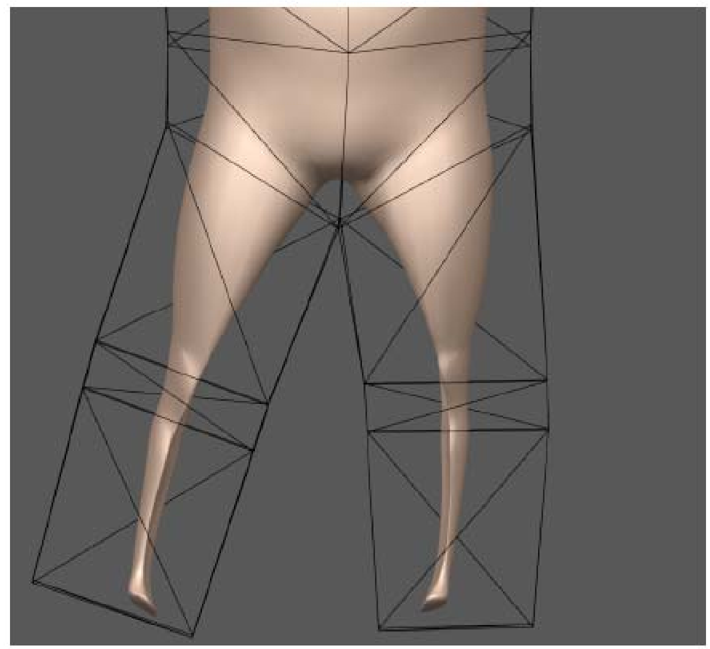




MV - Limitations

Other leg moves in opposite (!) direction





Barycentric Coords

Additional property required:

$$w_i(x) \geq 0$$

Mean value coords only positive
on convex polygons

Harmonic Coordinates

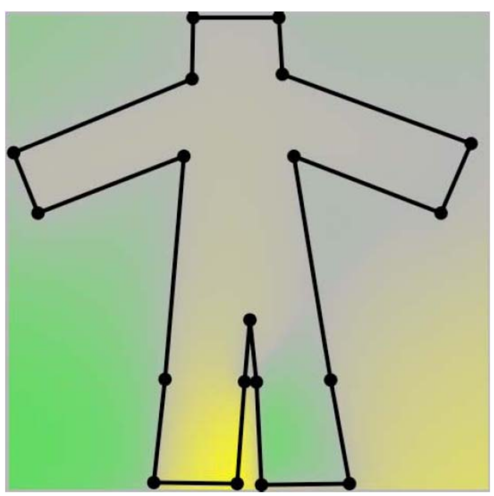
[Joshi et al '07]

Solve for $w_i(x)$:

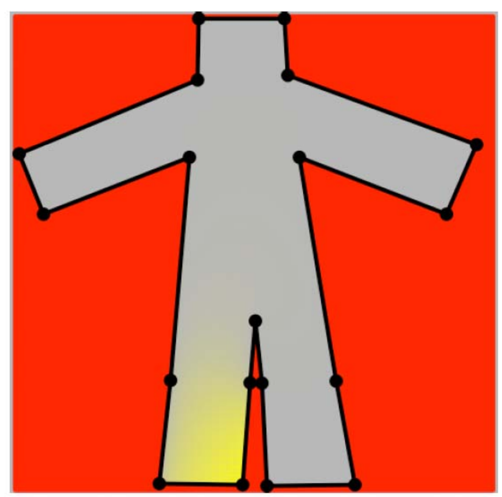
$$\nabla^2 w_i(x) = 0$$

subject to: w_i linear on the boundary and

$$w_i(x_j) = \delta_{ij}$$

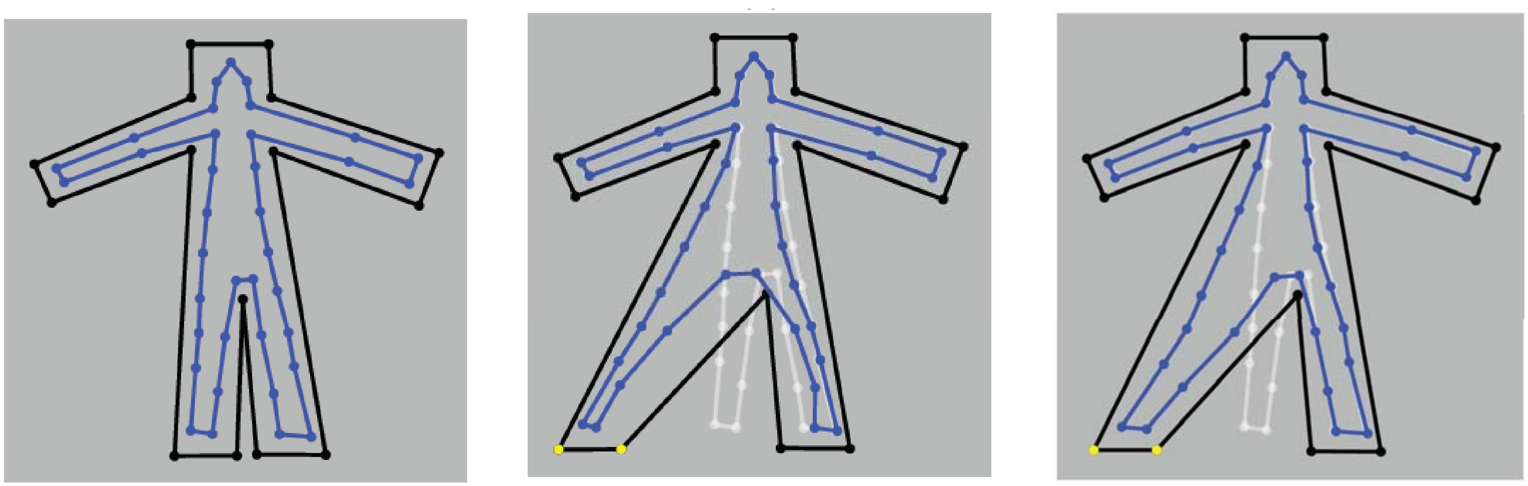


MVC



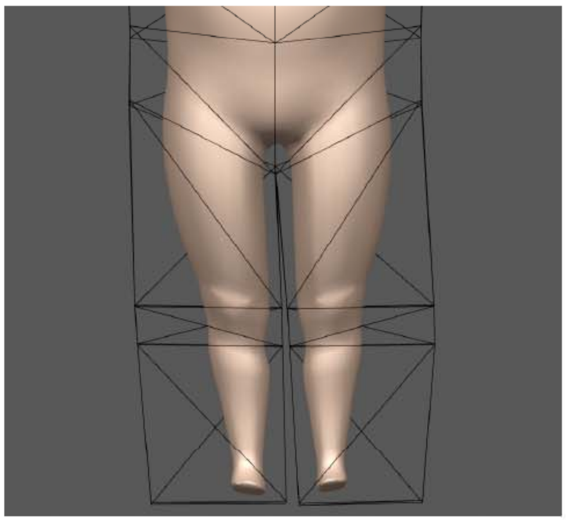
HC

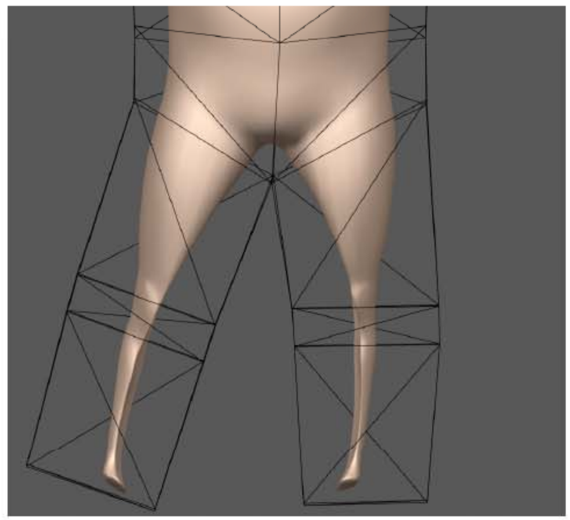
Harmonic Coordinates



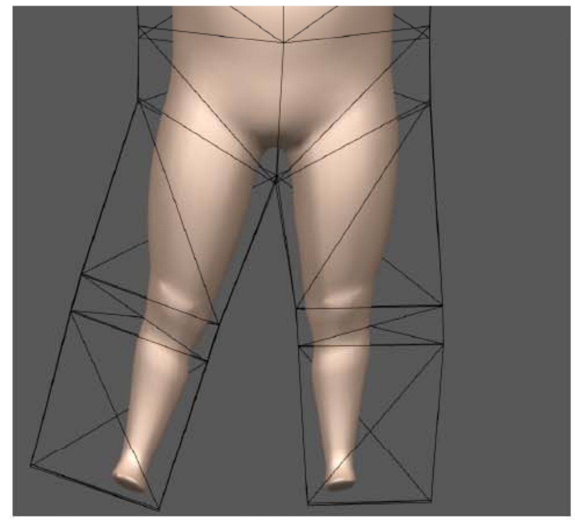
MVC HC

Harmonic Coordinates





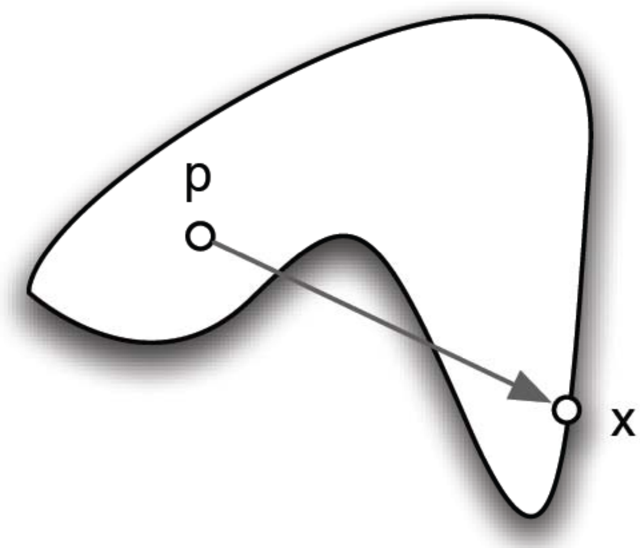
MVC



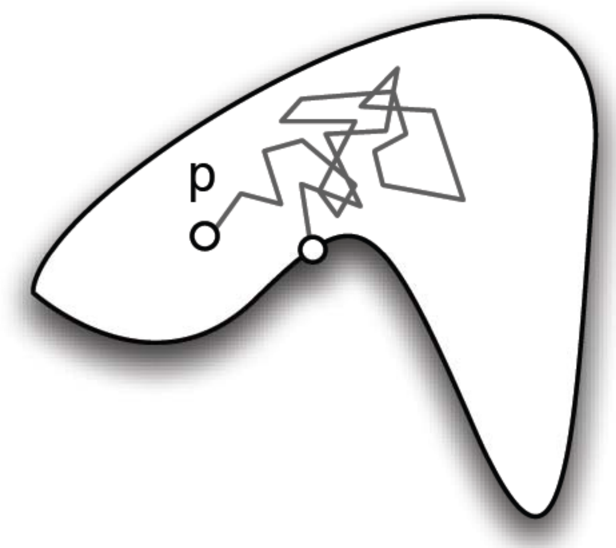
HC

Harmonic Coordinates

Why does it work?



MVC use
Euclidean distances



HC use
resistance distances

Harmonic Coordinates

Properties:

- All required properties
  - Smooth, translation + rotation invariant
- Positive everywhere
- No closed form, need to solve a PDE

References

- “On Linear Variational Surface Deformation Methods” [Botsch & Sorkine ‘08]
- Tutorial: “Interactive Shape Modeling and Deformation” [Sorkine & Botsch ’09]
- “Image deformation using moving least squares” [Schaefer et al ’06]
- “Mean Value Coordinates for Closed Triangular Meshes” [Ju et al ’05]
- “Harmonic coordinates for character articulation” [Joshi et al ’07]
- Excellent webpage on barycentric coordinates: http://www.inf.usi.ch/hormann/barycentric/