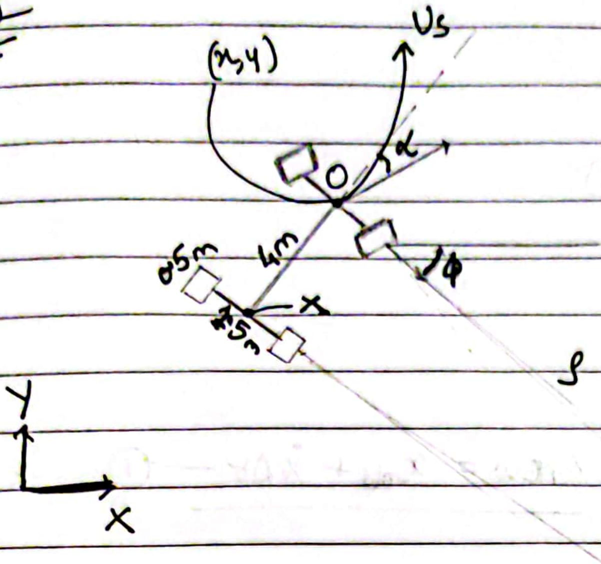
Q 1.1



Given:

initial (x_i, y_i, ϕ_i)

driving speed = 'ω'

Duration = 'T'

steering angle 'α'

$w_{left} + w_{right} = 200$

L = 4 metres

$r = \frac{0.5}{2} = 0.25$ (wheel radius)

α

r

q

In ΔXOq

$$\omega_R = \frac{V_R}{S - 0.725}$$

$$\omega_L = \frac{V_L}{S + 0.75}$$

$$\frac{dy}{dx} = -\tan\phi \quad [\text{IV}^{th} \text{ quadrant}]$$

Point O:

$$\dot{x} = v_s \cos\left(\pi/2 - (\alpha+\phi)\right) = v_s \sin(\alpha+\phi)$$

$$\dot{y} = v_s \sin\left(\pi/2 - (\alpha+\phi)\right) = v_s \cos(\alpha+\phi)$$

Here $v_s = \omega r$

$= 0.25 \times \omega$

classmate
Date
Page
CamScanner

$$dD = \int d(\phi + \alpha)$$

$$\dot{\phi} = \frac{U_s \sin(\alpha)}{L}$$

from formula $x_{new} = x_{old} + \dot{x}\Delta t$ — (1)

$$\Delta t = \frac{T}{N}$$

where

T = Total time (duration)

N = no of readings to plot trajectory.

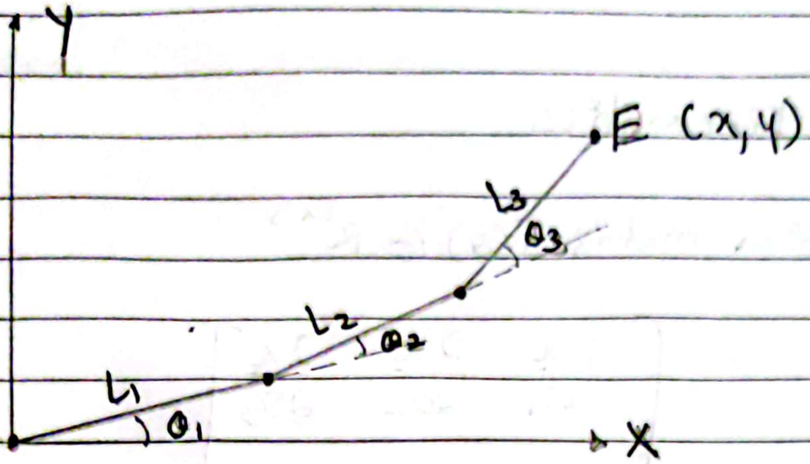
Also $y_{new} = y_{old} + \dot{y}\Delta t$ — (2)

&

$\phi_{new} = \phi_{old} + \dot{\phi}\Delta t$ — (3)

from (1), (2) (3) we can plot trajectory using python,

Q 1.2



i) from the 3-dof robotic arm

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$\cancel{y = -L_1 \sin\theta_1 \dot{\theta}_1 - L_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)} \quad \cancel{1}$$
$$\cancel{- L_3 \sin(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)}$$

$$y \quad L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) + L_3 \sin(\theta_1+\theta_2+\theta_3)$$

$$\dot{x} = -L_1 \sin\theta_1 - L_2 \sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) \quad \text{—— } 1$$
$$- L_3 \sin(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2+\dot{\theta}_3)$$

$$\dot{y} = L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2) + L_3 \cos(\theta_1+\theta_2+\theta_3)(\dot{\theta}_1+\dot{\theta}_2 + \dot{\theta}_3)$$

2

$$\phi = \theta_1 + \theta_2 + \theta_3$$

ii) For inverse Kinematics

Jacobian matrix $J(\theta) \in R^{2\times 3}$

$$= \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \end{bmatrix}$$

$$\frac{\partial x}{\partial \theta_1} = -L_1 \sin(\theta_1) - L_2 \sin(\theta_1 + \theta_2) - L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\frac{\partial x}{\partial \theta_2} = -L_2 \sin(\theta_1 + \theta_2) - L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\frac{\partial x}{\partial \theta_3} = -L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\frac{\partial y}{\partial \theta_1} = L_1 \cos(\theta_1) + L_2 \cos(\phi_1 + \phi_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$\frac{\partial y}{\partial \theta_2} = L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$\frac{\partial y}{\partial \theta_3} = L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

from the following expression

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\end{bmatrix} = J(\theta).\dot{\theta}$$

we get,

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix}\frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3}\\ \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3}\\ 1 & 1 & 1\end{bmatrix} \cdot \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$

Inverse kinematics

$$\Rightarrow \boxed{\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\phi}\end{bmatrix} \begin{bmatrix}\frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3}\\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3}\\ 1 & 1 & 1\end{bmatrix}^{-1}}$$

Using sympy in Python we get, —