119256534

Rishikesh Jadhav

ENPM-662

Home work 2

Q 1.1
1) Rotate by ϕ about world axis x
2) Translate about y axis by y
3) Rotate by θ about world z-axis
4) Rotate by ψ about current x-axis

1) $\underset{\text{world}}{R_{x,\phi}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2) $\underset{\text{current}}{T_{y,y}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

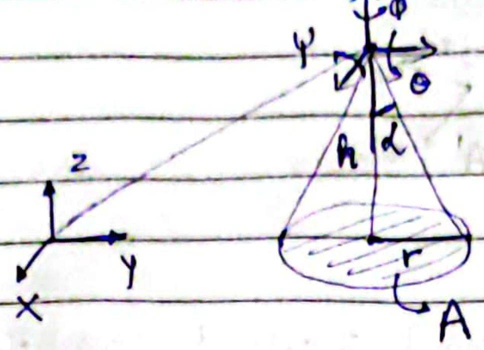
3) $\underset{\text{world}}{R_{z,\theta}} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4) $\underset{\text{current}}{R_{x,\psi}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Ans:- $\boxed{\text{Resulting frame} = R_{z,\theta}\, R_{x,\phi}\, T_{y,y}\, R_{x,\psi}}$

All the rotations & translations done in the homogenous matrix about current frame are post multiplied & about the world frame are pre multiplied

Q 2.2



→ As the 3 rotations of axes x by ψ, y by θ & z by ϕ are about world axis

$$R' = R_z R_y R_x$$

$P = (x, y, z)$

$$H = \begin{bmatrix} & R' & & d_x \\ & & & d_y \\ & & & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad - (1)$$

from equation of circular cone

$\alpha = 45^\circ$

$$x^2 + y^2 = z^2 \qquad \tan\alpha = \frac{r}{h} \qquad r = h\tan\alpha$$

$$\Rightarrow x^2 + y^2 - z^2 = 0 \qquad r = h = z$$

Converting into $P^T A P = 0$ form

where A = coefficient matrix

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

$$R' = \begin{array}{ccc} z & y & x \\ \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \end{array}$$

for simplification purposes $\cos\phi = c\phi$
$\sin\phi = s\phi$ & so on for all angles.

$$R' = \begin{bmatrix} c\phi c\theta & c\phi s\psi s\theta - c\phi s\psi & s\phi s\psi + c\psi c\phi s\theta \\ c\theta s\phi & c\phi c\psi + s\phi s\psi s\theta & c\psi s\phi s\theta - c\phi s\psi \\ -s\theta & c\theta s\psi & c\psi c\theta \end{bmatrix}$$

To get the coverage area wrt world frame :

$$(HP)^T \times (\text{Coefficient matrix}) \times (HP) = 0 \quad — ②$$

After multiplication & removing z axis from above equation

We get; $y c\phi s\psi + y^2 c\phi^2 c\psi^2 - x s\phi s\psi + x^2 c\phi^2 c\theta^2 + y^2 s\phi^2 s\psi^2 s\theta^2$
$- x^2 c\phi\, c\theta s\phi^2 - y^2 c\psi c\theta s\phi^2 - x c\phi c\psi s\theta - y c\psi s\phi s\theta + dy\, y\, c\phi c\psi$
$+ dx.x.\, c\phi c\theta - dy.x.c\psi s\phi + dx\, y \cos\phi - xy c\phi c\psi^2 s\phi + xy c\phi c\theta s\phi^2$
$+ d\, y^2 x^2 c\phi s\psi s\theta + dy.y.s\phi s\psi s\theta + xy c\phi^2 c\psi s\psi s\theta + xy c\phi^2 c\theta s\psi s\theta$
$- xy c\phi s\phi^2 s\psi s\theta + xy c\theta s\phi^2 s\psi s\theta + 2y^2 c\phi c\psi s\phi s\psi s\theta +$
$x^2 c\phi c\theta s\phi s\psi s\theta + xy c\phi s\phi s\psi^2 s\theta^2 + y^2 c\phi c\theta s\phi s\psi s\theta .$

now after comparing coefficients of a, b, c, d, e, f

we can make a matrix out of these.

Area Equation of ellipse is,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0$$

$$\text{Area} = \frac{-\pi}{(ac-b^2)^{3/2}} \begin{vmatrix} a & b & d \\ b & c & e \\ d & e & f \end{vmatrix}$$

here a is coefficient of x^2

b is coefficient of $\frac{xy}{2}$

c is coefficient of y^2

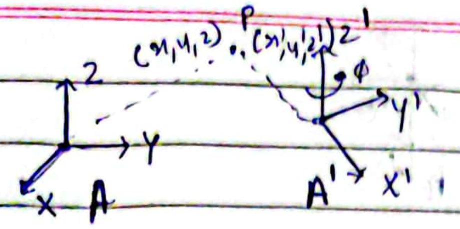
d is coefficient of $\frac{x}{2}$ - y (coefficient of $\frac{xy}{2}$)

e is coff of $\frac{y}{2}$ - x (coefficient of $\frac{xy}{2}$)

f is the constant = equation - $ax^2 + 2bxy + cy^2 + 2dx + 2ey$

The function for finding area of ellipse is provided in code file

Q1.3



→

for rotation about z axis + translation of axis A to A'
Lets assume the translation parameters we want to find as dx, dy, dz

$$A'_H = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & dx \\ \sin\phi & \cos\phi & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A'_H \rightarrow$ Homogeneous transformation matrix

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & dx \\ \sin\phi & \cos\phi & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ from $x' = A'_H . x$

→ $dx = x' - x\cos\phi + y\sin\phi$
$dy = y' - x\sin\phi - y\sin\phi$
$dz = z' - z$

$\Rightarrow \therefore$

Ans: $\therefore$ $H = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & x' - x\cos\phi + y\sin\phi \\ \sin\phi & \cos\phi & 0 & y' - x\sin\phi - y\sin\phi \\ 0 & 0 & 1 & z' - z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Q.2.1

For Finding Rotation matrix

$\rightarrow$ $R' = R_{z,\phi}\, R_{y,\theta}\,, R_{\psi,x}$ [According to world Axis]

$$= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\theta & -\sin\theta\cos\psi + \cos\phi\sin\theta\sin\psi & \sin\theta\sin\psi + \cos\phi\sin\theta\cos\psi \\ \sin\phi\cos\theta & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{bmatrix}$$

for K & θ we use the formulae:-

$$K = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{Tr(R') - 1}{2}\right)$$

Given values $\theta_g = 15^\circ$ $\psi_g = 35^\circ$ $\phi_g = 20^\circ$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{Tr(R') - 1}{2}\right)$$

$\Rightarrow$ After using θ, ψ & ϕ values in R'

$$\alpha = \boxed{\theta = 40.56^\circ}$$

$$K = \frac{1}{2\sin\theta}\begin{bmatrix} \cos\theta\sin\psi + \cos\phi\sin\psi - \sin\phi\sin\theta\cos\psi \\ \sin\theta\sin\psi + \cos\phi\sin\theta\cos\psi + \sin\theta \\ \sin\phi\cos\theta + \sin\theta\cos\psi - \cos\phi\sin\theta\sin\psi \end{bmatrix}$$

$$= \frac{1}{2\sin(40.56)}\begin{bmatrix} 1.0205 \\ 0.6064 \\ 0.4028 \end{bmatrix}$$

$$\Rightarrow \quad K = \begin{bmatrix} 0.7847 \\ 0.5030 \\ 0.360 \end{bmatrix} \begin{matrix} \rightarrow & K_x = 0.7847 \\ \rightarrow & K_y = 0.5030 \\ \rightarrow & K_z = 0.360 \end{matrix}$$

Assuming $\omega_x = \omega_{max} = 1 \text{ deg/sec}$.

$$\omega_x = K_x \times \dot{\alpha}$$

as K_x is highest $\omega_{max} = \omega_x = 1$

$$\Rightarrow \dot{\alpha} = \frac{1}{0.7847} = \underline{1.2743^{\circ}} = \underline{\underline{\overset{0.0222}{\cancel{0.6409}}}} \text{ radians}$$

Similarly $\omega_y = K_y \times 1.2743 = 0.6409 \text{ deg/sec}$

$$\omega_z = K_y \times 1.2743 = 0.4587 \text{ deg/sec}$$

$$\dot{\alpha} = \frac{\overset{40.56}{\Delta \alpha}}{\Delta t}$$

$$\Rightarrow 1.2743 = \frac{40.56}{\Delta t}$$

$$\Rightarrow \boxed{\Delta t = 31.8292 \text{ sec}} \rightarrow \text{shortest time.}$$

2.1 cont'

We know that Rotation matrix $R \in (SO(3))$ is repesented as a single rotation matrix about a certain suitable axis by an appropriate angle.

$$R = R_{k,\alpha}$$

Where
$$R_{k,\alpha} = \begin{bmatrix} k_x^2 v_\alpha + c_\alpha & k_x k_y v_\alpha - k_z s_\alpha & k_x k_y v_\alpha + k_y s_\alpha \\ k_x k_y v_\alpha + k_z s_\alpha & k_y^2 v_\alpha + c_\alpha & k_y k_z v_\alpha - k_x s_\alpha \\ k_x k_z v_\alpha - k_y s_\alpha & k_y k_z v_\alpha + k_x s_\alpha & k_z^2 v_\alpha + c_\alpha \end{bmatrix}$$

$$v_\alpha = 1 - c_\alpha$$

from above matrix, comparing with the rotation matrix we get,

$$\theta = \tan^{-1}\left(-r_{31}, \pm\sqrt{r_{32}^2 + r_{33}^2}\right)$$

$$\phi = \tan^{-1}\left(\frac{r_{21}}{\cos\theta}, \frac{r_{11}}{\cos\theta}\right)$$

$$\psi = \tan^{-1}\left(\frac{r_{21}}{\cos\theta}, \frac{r_{33}}{\cos\theta}\right)$$

The above equations can be written w.r.t. R'

To get plots of θ, ϕ & ψ wrt time we change the angle α wrt time

The trajectory of w_x, w_y, w_z are kept constant.