ENPM-662

Homework-5

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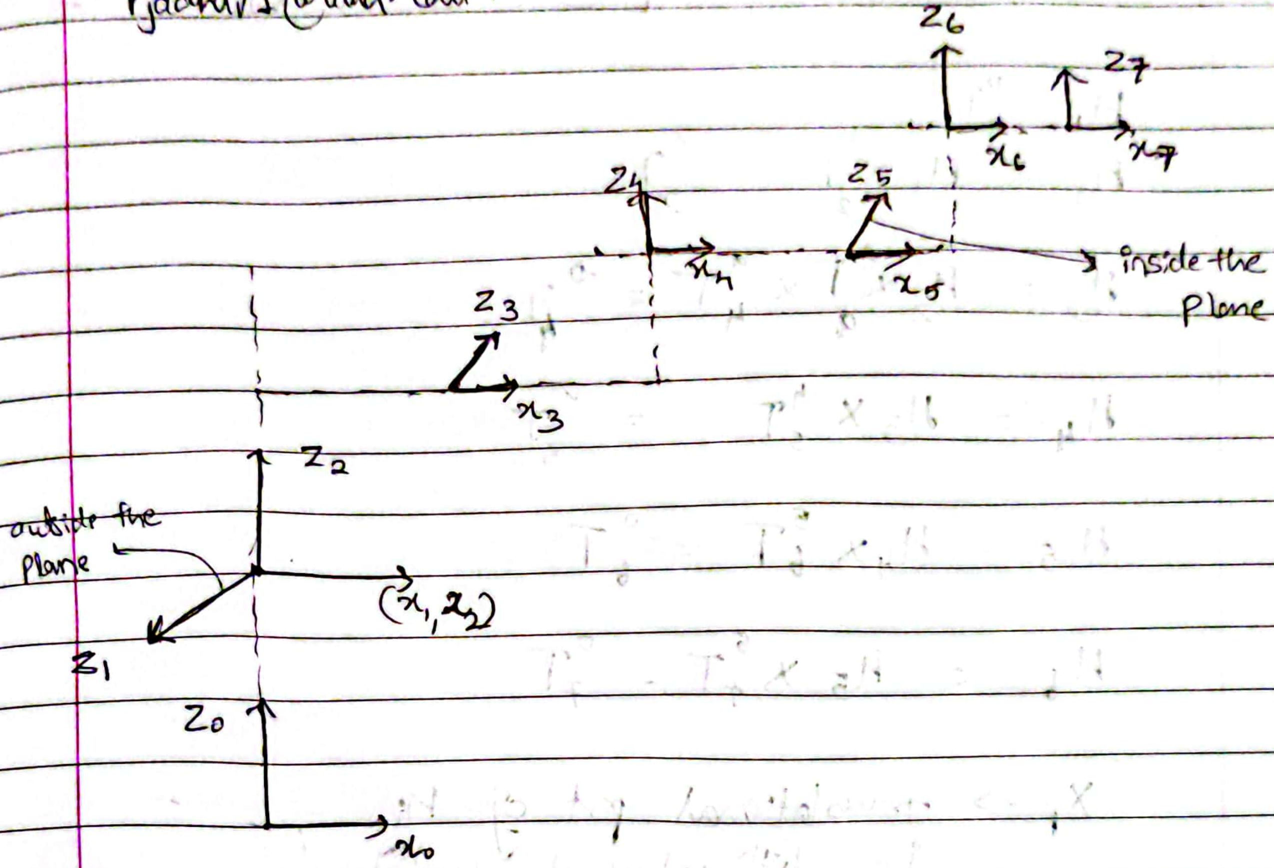


DH table

J.	a	α	d	θ
0-1	0	$\pi/2$	d_1	θ_1
1-2	0	$-\pi/2$	0	θ_2
2-3	$+a_3$	$-\pi/2$	d_3	0
3-4	$-a_3$	$\pi/2$	0	θ_4
4-5	0	$\pi/2$	d_5	θ_5
5-6	a_3	$-\pi/2$	0	θ_6
6-7	0	0	$-d_7-r$	θ_7

a - link length
α - link twist
d - link offset
θ - Joint Angle

Joint 3 is locked, therefore $\theta_3 = 0$

$$q = [0.0 \quad 0.0 \quad \pi/2 \quad 0.0 \quad \pi \quad 0.0]^T$$

Transformation Matrix =
$$\begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\theta_i \times s\alpha_i & a_i c\theta_i \\ s\theta_i & c\alpha_i c\theta_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$H_1 = {}^{0}_{1}T$

$H_2 = H_1 \times {}^{1}_{2}T = {}^{0}_{2}T$

$H_3 = H_2 \times {}^{2}_{d}T \times {}^{d}_{4}T = {}^{0}_{4}T$

$H_4 = H_3 \times {}^{4}_{5}T = {}^{0}_{5}T$

$H_5 = H_4 \times {}^{5}_{6}T = {}^{0}_{6}T$

$H_6 = H_5 \times {}^{6}_{7}T = {}^{0}_{7}T$

$X_p \Rightarrow$ translational part of H_i
i.e 4^{th} column of final transform

${}^{0}_{i}Z \rightarrow$ 3rd column of H_i
$i = 1, 2 \ldots 6$.

Jacobian Calculator.

$$J_i = \begin{bmatrix} \frac{\partial X_p}{\partial q_i} \\ {}^{0}Z_i \end{bmatrix} \quad i = 1 \ldots 6.$$

$$J = [J_1 \, J_2 \, J_3 \ldots J_6]$$

Circle plotting

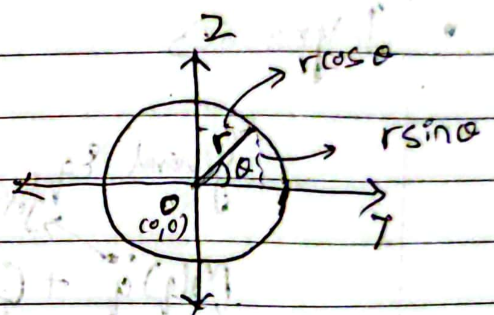
Radius = 10 cm = 0.1 m

Centre co-ordinates wrt robots base frame = (0.679, 0, 0.725)

point S co-ordinates = (0.679, 0, 0.825)

Circle equation in polar co-ordinates system,

$x = 0.679$
$y = 0.1 \cos\theta$
$z = 0.1 \sin\theta + 0.725$



to get velocity, differentiate above eq wrt time.

$V_x = \dot{x} = 0$

$V_y = \dot{y} = -0.1 \sin\theta \cdot \dot{\theta}$

$V_z = \dot{z} = 0.1 \cos\theta \cdot \dot{\theta}$

$\dot{\theta} = \frac{d\theta}{dt} = \frac{2\pi}{200}$ [200 seconds]

$\dot{X} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ $q = \begin{bmatrix} q_1 \rightarrow \theta_1 \\ q_2 \rightarrow \theta_2 \\ \vdots \\ q_6 \end{bmatrix}$ $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_6 \end{bmatrix}$ $\boxed{\dot{X} = J\dot{q}}$

To find joint $\dot{q}$ take inverse of Jacobian.

$$\dot{q} = J^{-1}(q).\dot{X}$$

$$q_i = q_{i-1} + \dot{q}_i \Delta t$$

$$\Delta t = \frac{T}{N} = \frac{5}{50}$$

- Dynamics

General Eq,

$$\underbrace{M(q)}_{\text{Inertia}}\underbrace{\ddot{q}}_{\text{joint acceleration} = 0} + \underbrace{C(q,\dot{q})}_{\text{Coriolis \& centripetal force}}\underbrace{\dot{q}}_{\text{joint velocity} = 0} + \underbrace{g(q)}_{\text{gravity}} = \underbrace{\tau}_{\text{torque}} + J^T(q)\underbrace{F}_{\text{external force}}$$

given

joint angle

Potential Energy for a n-link manipulator,

$$P = \sum_{i=1}^{n} M_i g^T \underbrace{r_{c_i}}_{\text{Centre of mass coordinates}}$$

The lagrangian function, $\boxed{L = \underbrace{K}_{\text{Kinetic energy}} - P}$ $= 0$ as $(\dot{q} = 0)$

The dynamic equations satisfy

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q_k} = Q_k = \frac{\partial P}{\partial q_k}$$

$$\underline{P = mgh}, \quad \underline{K = \frac{1}{2} m (\dot{x} + \dot{y})^2} \quad \dot{x}\dot{y} = 0$$

$\therefore \boxed{K = 0}$

Procedure:

- Get transformation matrices for each link to the base
- Get centre of mass of each link with the help of transformation matrix & the co-ordinates obtained from urdf file.
- Get mass of each link from urdf.
- gravity is along -ve z axis

$$\therefore g = [\,0 \quad 0 \quad -9.8 \quad 0\,]$$

- Calculate Potential Energy using,

$$P = \sum_{i=1}^{n} M[i] * g^T + CM[i]$$

- Calculate gravity matrix in which each term represents P's derivative w.r.t joint.
- Calculate Jacobian
- Calculate Torque using,

$$\tau = G - F_n$$

$$F_n = J^T * F$$

$$F = [-5, 0, 0, 0, 0, 0] \longrightarrow \text{given.}$$