

RoboSim Sample Robot Plant Model Calculations

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Abstract

An analysis of a simple tank-drive robot is presented. The simple plant model is designed to be a starting point for year-to-year development of more detailed robot models. The device modeled has two sets of motors to drive motion in a tank-drive fashion, along with a normal battery and electrical system.

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Part I

Background Mathematics Primer

In this section, we will seek to demonstrate the “basic” math required to understand RoboSim. This math could be considered at the level of AP Calc BC and AP Physics C.

1 Calculus

Sir Isaac Newton was a smart man. He came up with calculus. Either him or Leibniz. In any event. Calculus is a thing. It studies functions and properties of functions.

1.1 Functions

A function can be thought of as a box which takes one input, and produces some outputs based on that input. For RoboSim, we will deal with many functions of time. This is to say that we will look at how properties (such as speed) change over time. In a mathematical sense, time is the input, and speed is the output. At every given time, there is a speed.

1.2 Continuous

Continuous calculus is what is usually studied in High School. It is assumed that time is linear and continuous. Valid times are $t = 0\text{s}$, $t = 1.2429543987\text{s}$, $t = 3.25\text{s}$, etc. Any value of time is defined. An infinitely detailed timeline can be established.

Calculus looks at two primary things about functions: Derivatives and Integrals.

A derivative of a function describes how quickly the function is changing as time goes on.

$$x = y(t)$$

$$x = \frac{dy}{dt}$$

An Integral of a function describes how big or small the function’s value has been historically.

$$x = y(t)$$

$$X = \int y(t)dt + c$$

For the purposes of RoboSim, there is no need to discuss the calculation of integrals or derivatives in the continuous domain.

If a function is getting larger quickly, its derivative will be big at that time. If it is getting smaller in a hurry, its derivative will be negative.

If a function has been big for a long time, its integral will be large. If it’s been bouncing around but equally above and below zero, the integral will be very small.

1.3 Discrete

Discrete calculus is just like continuous calculus, except we only allow time to take on certain values. Maybe its like:

$$t = \{1, 2, 3, 4, 5...\}$$

Or maybe it’s more like

$$t = \{0.00, 0.01, 0.02, 0.03, 0.04...\}$$

In any case, we only allow time to take on certain values. These values must be regularly spaced for any of the math to work. The spacing between the valid time values is called the Sample Time. The Sample Time will henceforth be referred to as Ts .

Discrete functions are notated as such:

$$x = y[t]$$

Note the square brackets which indicate discrete time. Here t is an index, rather than a measure in seconds. Multiplying t index by the sample time Ts yields the actual (real-world) time in seconds.

For every time value, there is a given output value of the function. For example, consider the following function:

$$y[t] = \{1, 5, 2, 6, 3, 7, 4, 7\}, Ts = 0.1$$

A series of outputs is defined, starting at $t = 0$

Since Ts is 0.1, we can determine the following is true:

$$y[0] = 1$$

$$y[1] = 5$$

$$y[3] = 6$$

And so on.

In Discrete time calculus, we define derivatives as follows:

$$x = y[t]$$

$$\frac{dx}{dt} = \frac{y[t] - y[t-1]}{Ts}$$

In Discrete time calculus, we define Integrals as follows:

$$x = y[t]$$

$$\int x dt = \sum_{i=0}^t y[i] * Ts$$

We define them this way because of reasons.

2 Physics

Physics is also a thing that Newton did. Told you he was smart.

2.1 Things and Calculus

For physics to work, we must have objects in places. The place the object is at is called its Position. Let us define the following function which describes an object's location along one axis at any time. Specifically, it says how far the object is from an arbitrary zero point at any time:

$$Distance = x[t]$$

Velocity is how fast an object is moving. Velocity can be positive or negative. Positive velocity means the distance gets bigger, and negative velocity means the distance gets smaller. Because of physics, velocity is therefore the derivative of distance.

$$Velocity = v[t]$$

$$v[t] = \frac{dx}{dt} = \frac{x[t] - x[t-1]}{Ts}$$

Acceleration is how fast an object is changing in velocity. Acceleration can be positive or negative. A positive acceleration means the object is going faster and faster and faster. A negative acceleration means the object is slowing down. Because physics is still physics, acceleration is the derivative of velocity.

$$Acceleration = a[t]$$

$$a[t] = \frac{dv}{dt} = \frac{v[t] - v[t-1]}{Ts}$$

Note that integration allows you to go the other way. That is to say, velocity is the integral of acceleration, and distance is the integral of velocity.

$$v[t] = \sum_{i=0}^t a[i] * Ts$$

$$x[t] = \sum_{i=0}^t v[i] * Ts$$

For units, we will always use metric. Therefore, distance is in meters (m), velocity is in meters per second (m/s), and acceleration is in meters per second, per second (m/s²).

2.2 Forces and Linear motion

Force is something pushing on another thing. The more you push, the faster and faster you will go.

Newton's laws of motion allow us to define the following: At any given time, Acceleration is proportional to Force, scaled by an object's mass:

$$Force = mass * acceleration$$

$$F[t] = m * a[t]$$

Note that Force is measured in Newtons (N), mass is measured in kilograms (kg), and acceleration is in meters per second per second (m/s²).

Note also that force changes over time, and acceleration changes over time, but we assume mass is constant.

2.3 Torque and Rotational motion

Because symmetry is a thing of life, Newton's laws of motion also have analogs for rotational motion.

Instead of force, we now have Torque. Torque is force that goes in a circle. It is measured in Newton-meters.

Part II

Component Models

Part III

Integrated Model

Part IV

Design Notes

3 Software Architecture

4 RoboSim Hardware