

RoboSim Sample Robot Plant Model Calculations

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Abstract

An analysis of a simple tank-drive robot is presented. The simple plant model is designed to be a starting point for year-to-year development of more detailed robot models. The device modeled has two sets of motors to drive motion in a tank-drive fashion, along with a normal battery and electrical system.

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Part I

Background Academics Primer

In this section, we will seek to demonstrate the “basic” math and physics required to understand RoboSim. This content is effectively a whirlwind summary of the important parts of AP Calc BC, AP Physics C, basic Chemistry, and any introductory Electronics course.

1 Calculus

Sir Isaac Newton was a smart man. He came up with calculus. Either him or Leibniz. In any event. **Calculus** is a particular field of study within mathematics. It studies functions and properties of functions.

1.1 Functions

A **function** can be thought of as a box which takes one input, and produces some outputs based on that input. For RoboSim, we will deal with many functions of time. This is to say that we will look at how **properties** change over time. The “Box” in this case is the robot itself. The most fundamental input to the robot is time. As time progresses forward, different properties of the robot change. These properties include things like location, speed, torque, rotation, electrical current draw, etc. In a mathematical sense, time is the input, and the various properties are the output. At every given time, each property has a value. Additionally, the physics of the configuration of the parts of the robot determine relationships between the quantities as time progresses forward. For example, a large voltage applied to motors will induce larger torques and speeds and current draws.

1.2 Continuous Functions, Derivatives and Integrals

Continuous calculus is what is usually studied in High School. It is assumed that time is linear and continuous. Valid times are $t = 0s$, $t = 1.2429543987s$, $t = 3.25s$, etc. Any value of time is a defined. An infinitely detailed timeline can be established.

Calculus looks at two primary things about functions: Derivatives and Integrals.

A **Derivative** of a function describes how quickly the function is changing as time goes on. On a curve on a Cartesian (x/y) plot, the derivative is the same as the slope of the curve at any given point, as shown in Figure 1.

$$x = y(t)$$

$$x = \frac{dy}{dt}$$

An **Integral** of a function describes how big or small the function’s value has been historically. On a Cartesian plot, the integral is the same as the area between the curve and the x axis over a given period of time, as shown in Figure 2.

$$x = y(t)$$

$$X = \int y(t)dt + c$$

For the purposes of RoboSim, there is no need to discuss the calculation of integrals or derivatives in the continuous domain.

If a function is getting larger quickly, its derivative will be big at that time. If it is getting smaller in a hurry, its derivative will be negative

If a function has been big for a long time, its integral will be large. If it’s been bouncing around but equally above and below zero, the integral will be very small.

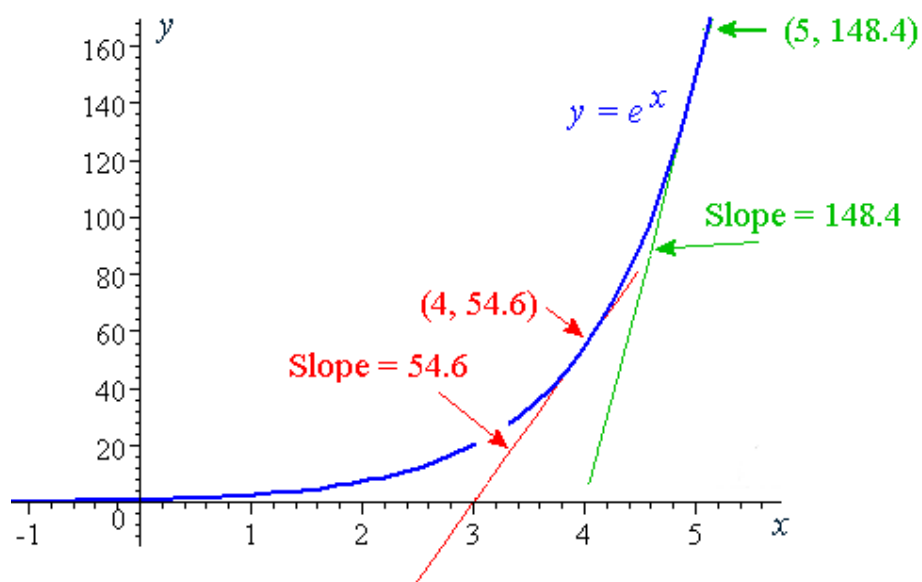


Figure 1: Illustration of a derivative as the “instantaneous slope” of a curve

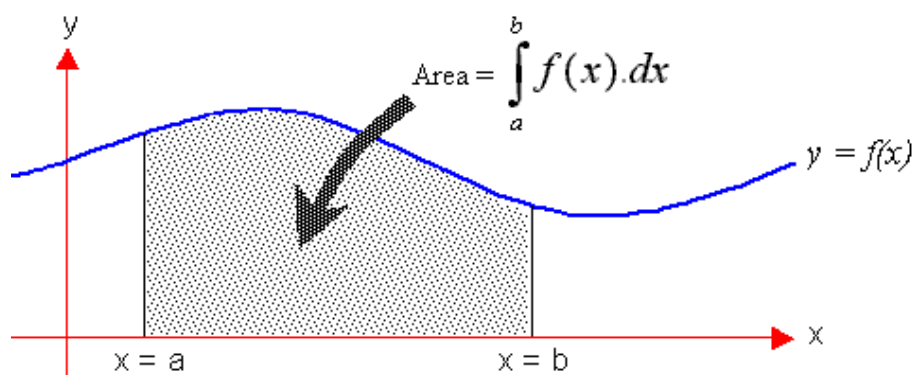


Figure 2: Illustration of a integral as the “area under a curve”

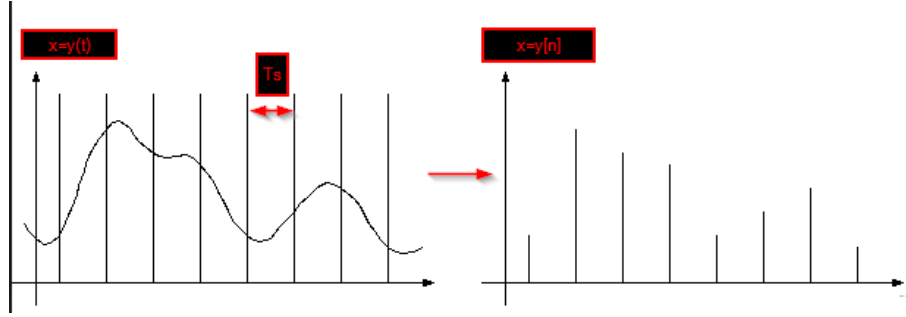


Figure 3: Sampling a continuous function $x=y(t)$ to produce discrete function $x=y[n]$

1.3 Discrete Functions, Derivatives and Integrals

Discrete functions are very similar to normal functions, except they are only defined at certain, regularly-repeating time values. In this way, a discrete function could be thought of as a set of *time, value* pairs.

$$\{0.00, 3\}, \{0.01, 2\}, \{0.02, 1\}, \{0.03, 0\}, \dots$$

We might turn a continuous function into a discrete one by **sampling** it at regular intervals. This is usually the way that sensors and digital systems are integrated - the sensor reads an analog value continuously, and the digital system records the sensor's value at specific points in time.

We usually notate a discrete time function as such

$$x = y[n]$$

For a function y that produces a value x at time index n . For all discrete functions, we must always specify a “**Sample Time**” (notated T_s) to indicate the spacing between samples. For RoboSim, T_s is hard-coded to 50ms.

Note the square brackets which indicate discrete time. Here n is an index, not a measure in seconds. Multiplying the n index by the sample time T_s yields the actual (real-world) time in seconds.

In Discrete time calculus, we define derivatives as follows:

$$x = y[n]$$

$$\frac{dx}{dt} \cong \frac{y[n] - y[n-1]}{T_s}$$

This formula says “A function's derivative is equal to the current value minus the previous value, scaled by the sample time.” You may recognize this form from part of the “Limit” definition of a derivative. Note here that if we force T_s to zero, we have a continuous derivative. However, because computer systems cannot sample infinitely fast, we accept a small but finite T_s as a very close approximation. The conditions under which this approximation is acceptable is the subject of Nyquist Frequency theory, but is a topic out of scope for this discussion.

In Discrete time calculus, we define Integrals as follows:

$$x = y[n]$$

$$\int x dt \cong \sum_{i=0}^n y[i] * T_s$$

This equation says “A function's integral is equal to the sum of all previous values, scaled by the sample time”. You may recognize this from the Riemann sum approximation of an integral. If we force T_s to zero, we end up with a continuous integral. But again, since computer systems cannot sample infinitely fast, we accept a small but finite T_s and move on with our lives.

2 Physics

Physics is also a thing that Newton did. Told you he was smart. In this section, we will begin to analyze the constraints imposed by mother nature between different quantities on our robot.

2.1 Position, Linear Motion, and Calculus

For physics to work, we must have objects in places. The place the object is at is called its **Position**. Let us define the following function which describes an object's location along one axis at any time. Specifically, it says how far the object is from an arbitrary zero point at any time:

$$Distance = x[n]$$

Velocity is how fast an object is moving. Velocity can be positive or negative. Positive velocity means the distance gets bigger, and negative velocity means the distance gets smaller. Because of physics, velocity is therefore the derivative of distance.

$$Velocity = v[n]$$

$$v[n] = \frac{dx}{dt} = \frac{x[n] - x[n-1]}{Ts}$$

Acceleration is how fast an object is changing in velocity. Acceleration can be positive or negative. A positive acceleration means the object is going faster and faster and faster. A negative acceleration means the object is slowing down. Because physics is still physics, acceleration is the derivative of velocity.

$$Acceleration = a[n]$$

$$a[n] = \frac{dv}{dt} = \frac{v[n] - v[n-1]}{Ts}$$

Note that integration allows you to go the other way. That is to say, velocity is the integral of acceleration, and distance is the integral of velocity.

$$v[n] = \sum_{i=0}^n a[i] * Ts$$

$$x[n] = \sum_{i=0}^n v[i] * Ts$$

Plots of these relationships are shown in Figure 4.

For units, we will always use **metric**. Therefore, distance is in meters (m), velocity is in meters per second (m/s), and acceleration is in meters per second, per second (m/s²)

2.2 Forces and Linear motion

2.2.1 Newton's Law

Force is something pushing on another thing. The more you push, the faster and faster you will go. The larger the object, the harder it is to make it go faster.

Newton's laws of motion allow us to define the following: At any given time, Acceleration is proportional to Force, scaled by an objects mass:

$$Force = mass * acceleration$$

$$F[n] = m * a[n]$$

Note that Force is measured in Newtons (N), mass is measured in kilograms (kg), and acceleration is in meters per second per second (m/s²).

Note also that force changes over time, and acceleration changes over time, but we assume mass is constant.

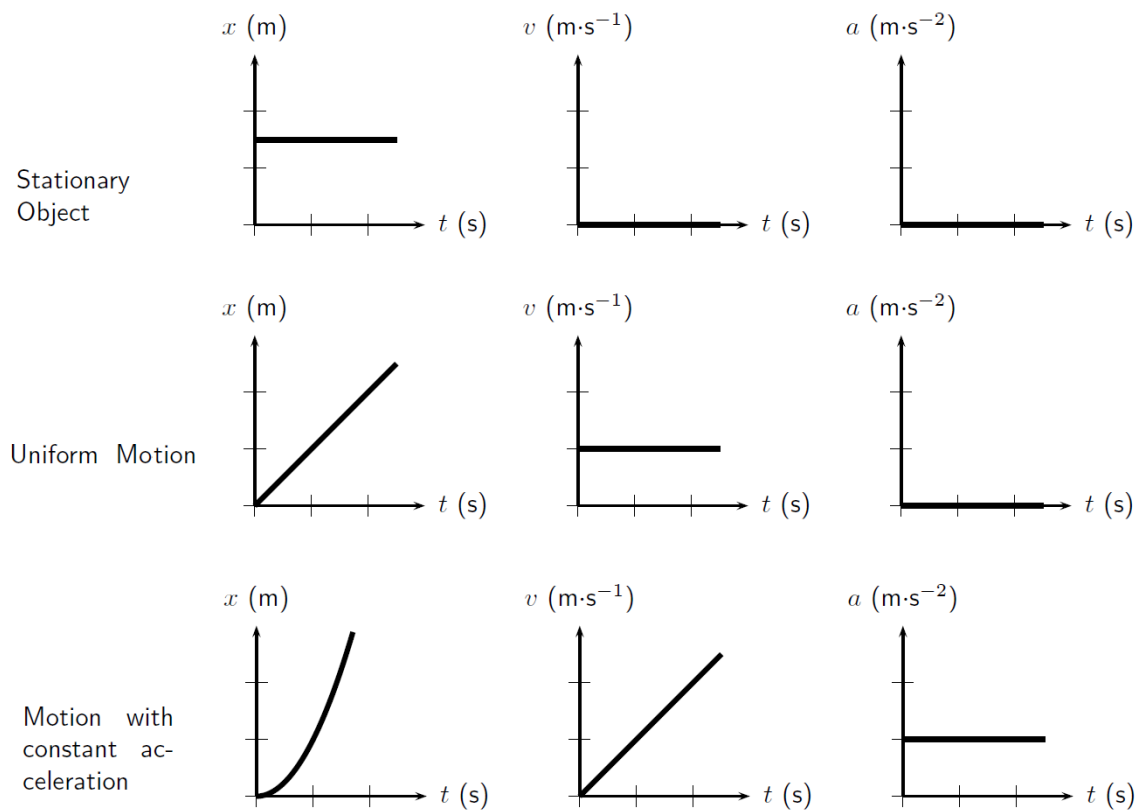


Figure 4: Examples of motion, relating acceleration, velocity, and position



Figure 5: Newton's Second Law

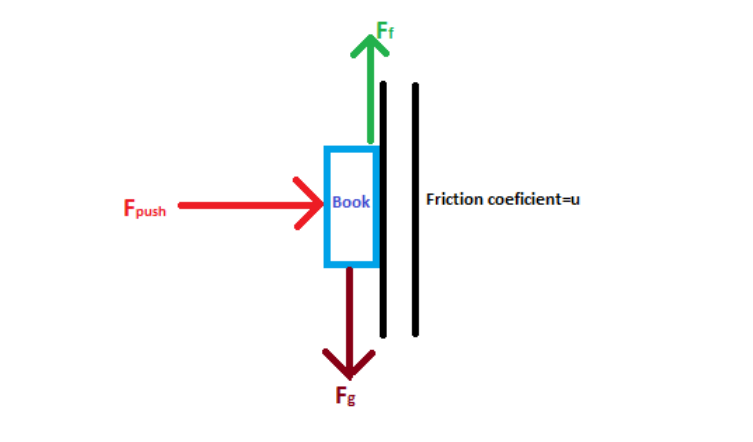


Figure 6: Static friction pinning a book to a wall

2.2.2 Sources of Forces

Forces come from many places. A student pushing on a robot frame is one possible, simple source. A turning wheel in contact with the floor is another. Meshed gear teeth pushing on one another is yet another transfer of force.

Of the list mentioned so far, friction is very important and has some special calculations associated with it.

2.2.3 Frictional Forces

Friction is a force which opposes two contacting surfaces from sliding relative to each other. It comes from microscopic interactions of matter in contact with each other. It can get very complex and is not fully understood, but can be well modeled at a macro level. We will discuss this model, as it is the most useful for FRC robotics applications. There are two main types of friction: Static and Kinetic.

2.2.4 Static Friction

Static friction deals with the case where two surfaces are not moving relative to one another. If you were to pin a book in place on a wall by only pressing on the book toward the wall, you would be using the static friction between the book and the wall to oppose the force of gravity, keeping the book from falling. This is illustrated in Figure 6.

Static frictional force will take on whatever value it needs to counter out other forces, up to a certain point. This certain point is called the **maximum static frictional force**. It is calculated with the following formula:

$$F_{Smax}[n] = \mu_S * F_N[n]$$

Here, F_{Smax} is the maximum static frictional force. It always points in the opposite direction as any force acting along the same axis. F_N is the “normal” force, or the force pinning the two surfaces together. μ_S is the Coefficient of Static Friction, which is derived from how effective the two surfaces are at sticking to each other. Two smooth, slick surfaces (like two pieces of smooth hard plastic) will have a small μ_S , while two rough surfaces (like concrete) will have a large μ_S . Usually, μ_S is just looked up in a table.

Static friction is often informally called “**Stiction**” in the engineering world, since it causes things to stick together. Also, “Stiction” is a portmanteau of “Static Friction”. Engineers are a clever bunch.

2.2.5 Kinetic Friction

Kinetic friction deals with the case where two surfaces are sliding relative to each other. If you were to slide a book along the ground, the force that opposes your efforts to push the book is the kinetic frictional force of the book in contact with the ground. It is calculated by this formula:

$$F_K[n] = \mu_K * F_N[n]$$

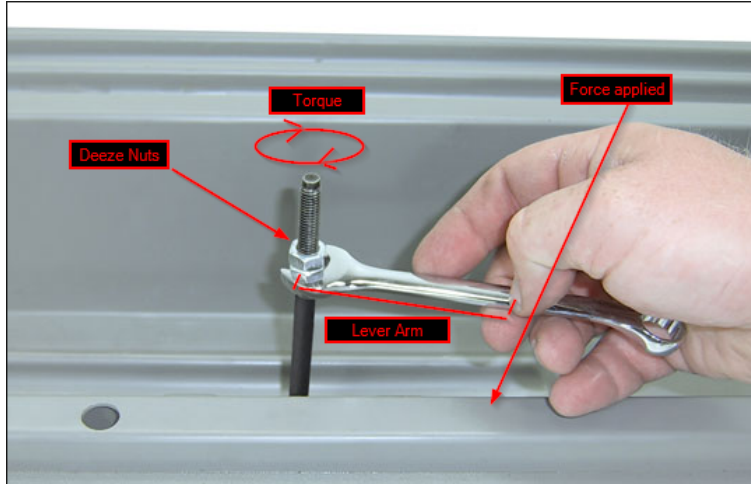


Figure 7: Torque exerted on a nut

Again, F_K is the kinetic frictional force. It always points in the opposite direction of motion of the interface (opposes the motion occurring). F_N is still the “normal” force, or the force pinning the two surfaces together. μ_K is the coefficient of kinetic friction, which again determines how effective the materials can interface with each other to produce an opposing force.

For most materials, μ_K is slightly less than μ_S .

Note that as a wheel is rotating with the ground, there is no relative motion between the ground and the point of contact with the wheel. Indeed, the motion of the robot is due to the rotation of the wheel, and not due to the wheel slipping over the ground. A non-slipping robot’s wheel-ground interface lies in the Static Friction domain. When wheel-ground interfaces start to enter the Kinetic friction domain, the effect is what many young people refer to as “drifting”.

2.3 Torque and Rotation

Because symmetry is a thing of life, Newton’s laws of motion also have analogs for **rotational motion**.

Instead of force, we now have **Torque**. Torque is force that goes in a circle. It is measured in Newton-meters.

All torque acts about a central pivot point. This may be the center of mass of the robot, the center of a gear, the center of a shaft, the center of a wheel, etc.

As indicated by the unit, a torque always has two important things to consider - How much force is pushing in a circle, and how far from the center of the circle is that force acting?

The distance from the central pivot point of the rotation to the point of application of force is called the “**Lever Arm**” of the torque. Bigger lever arms mean more torque for the same force.

This is why wrenches have long handles. They are designed to turn things in a circle with some amount of torque. Assuming a human can only exert a maximum amount of force, a longer handle makes for a longer lever arm, which in turn means a higher max torque applied. If a bolt is stuck, one option is a bigger wrench with a longer handle.

2.3.1 Rotational Motion Parameters

There are rotational measurements which are analogous to the linear motion parameters discussed above. Lower-case Greek letters are used to describe these rotational parameters

The current **angle** a rotating object is pointed in is referred to as theta:

$$Angle = \theta[n]$$

The rate at which that angle is changing is **rotational velocity**, or omega:

$$RotationalVelocity = \omega[n]$$

The rate of change of the rotational velocity is the **rotational acceleration**, or alpha:

$$RotationalAcceleration = \alpha[n]$$

As with linear motion, velocity is the derivative of position, and acceleration is the derivative of velocity.

$$\omega[n] = \frac{d\theta}{dt} = \frac{\theta[n] - \theta[n-1]}{Ts}$$

$$\alpha[n] = \frac{d\omega}{dt} = \frac{\omega[n] - \omega[n-1]}{Ts}$$

Also as with linear motion, velocity is the integral of acceleration, and position is the integral of velocity.

$$\omega[n] = \sum_{i=0}^n \alpha[i] * Ts$$

$$\theta[n] = \sum_{i=0}^n \omega[i] * Ts$$

2.3.2 Torque and Rotational motion

The relationship between torque and rotational motion is highly analogous to force and linear motion:

At any given time, an object's rotational acceleration is proportional to the torque applied to it, scaled by the object's "moment of inertia".

$$torque = moment\ of\ inertia * rotational\ acceleration$$

$$T[n] = I * \alpha[n]$$

An object's "**moment of inertia**" is like its total mass, but also accounts for how far that mass is from the center of rotation. The more mass an object has, and the further that mass is from the center of rotation, the harder it is to get the object rotating at a different speed. The actual value of this is in units of kilogram meters-squared ($kg\ m^2$), and is most easily calculated with a table of shapes and corresponding formulas.

Although we have assumed the moment of inertia to be constant, that is not always the case. Although it is rare for a robot to add or loose mass during a match, it is more common for the location of that mass to change on the robot. For example, if an arm extends out by a foot from the robot, the location of the mass of that arm is now further from the center. This increases the robot's moment of inertia, which makes it harder for the drive-train to start or stop the robot from spinning.

2.4 Ideal Gas Law

Cylinders, tanks, compressors, and valves are all pneumatic components used on FRC robots. **Pneumatic components** on a robot all deal with the movement of compressed air, and the transfer of energy involved therein.

Air is the gas used within these systems. Under the normal temperatures experienced in the robot, and the 125psi pressure restriction of FRC rules, air acts much like an **ideal gas**. Therefore, it is appropriate to model it with the ideal gas law:

$$PV = nrT$$

Here, P refers to the pressure of the gas (in kilo-pascals, or kPa), V refers to the volume of gas considered (in Liters, or L), T refers to the temperature of the gas (in kelvin, or K), and n refers to the number of moles of gas molecules being considered. The remaining symbol r is the "**Gas Constant**", a constant with units $L*kPa/(K*mol)$,

and is numerically equal to 8.3144621. It is determined by the physics of interacting gas particles. Its derivation will not be described in this paper

Note also that n is in moles. A “**mole**” is a unit-less multiplier, like “dozen”. For example, if I have a dozen donuts, it means I have twelve donuts. Similarly, if I have a mole of donuts, I have 6.022×10^{23} donuts.¹ The reason for its use here is that gas molecules are tiny, and usually we have a lot of them. It’s a lot simpler to do math talking about “1 mole of molecules” rather than absurdly large numbers.

What the ideal gas law describes is the relationship between pressure, temperature, amount of gas, the volume the gas occupies. For example, when a compressor kicks on, it forces more moles of air into the same volume. We also assume that temperature does not change. Therefore, based on the above equation, if T , r , and V are unchanged, and n increases, then P must also increase to keep the equation balanced. This makes sense, since anyone who has observed a properly-set-up pneumatic system knows turning on the compressor makes system pressure go up.

2.5 Electrical Laws

Motors, motor controllers, batteries, wires, and circuit breakers are all part of the **electrical system** on a robot. These components obey many laws related to electricity.

2.5.1 Electrical Introduction

For FRC robotics purposes, **electricity** is the movement of electrons through “conductors”. A **conductor** is any substance which has freely-moving charged particles, most commonly electrons. Usually these movements are random, but when they start to go in the same direction, we say there is an “electric current” present. Metals are the most common conductors.

The opposite of a conductor is an insulator. Common insulators include glass, wood, and plastic. These substances do not have freely-moving charged particles, so they cannot sustain an electric current. By wrapping a conductor inside an insulator, you create an isolated conduit to move electrical current in an organized fashion (eg, a wire).

2.5.2 Basic Electrical Quantities and Relationships

When talking about the flow of electrons, there are two important quantities.

Current is the number of electrons flowing past a given point over a given time. It is usually indicated by the lower-case letter i (blame the French). Units are almost always in “Amperes” or “Amps” for short. The Amp unit is denoted “A”. One amp of current is equal to 6.24×10^{18} electrons passing a given point on a wire in one second.

$$Current = i[n]$$

Voltage is the “force” which causes the motion of electrons in a conductor. The more voltage, the more of a punch the electrons will have. It is measured in a unit called “Volts” and denoted with the letter “V”.

$$Voltage = v[n]$$

Note that the word “force” is used very loosely here, as the energy associated with electricity has much more to do with electric and magnetic fields, and not so much the physical impact of tiny electrons on a mass. However, the details of this theory is beyond the scope of this paper.

Resistance is how easily a substance allows electricity to pass through. Conductors have a very low resistance, while insulators have a very high resistance. Resistance to electrical flow can be affected by many factors, including the temperature and physical shape of the device. Resistance is measured in a unit called an “Ohm”, and is denoted with a capital Greek letter Omega (Ω).

The resistance of a device determines how much voltage it pushes back with (ie, the “voltage drop”) when a current runs through it. This relationship is linear, and is expressed with Ohm’s Law:

¹Interestingly enough, the average jelly donuts contains 10^6 Joules of energy. This means that one mole of jelly donuts would have about 6×10^{29} Joules of energy. The rotational energy stored in Earth’s rotation is about 2×10^{29} Joules. This means that if you had a 100% efficient rocket powered by jelly donuts, and attached it to the earth’s equator, you could stop the earth from spinning, and still have 2/3rds of your donuts left over for eating! The more you know...

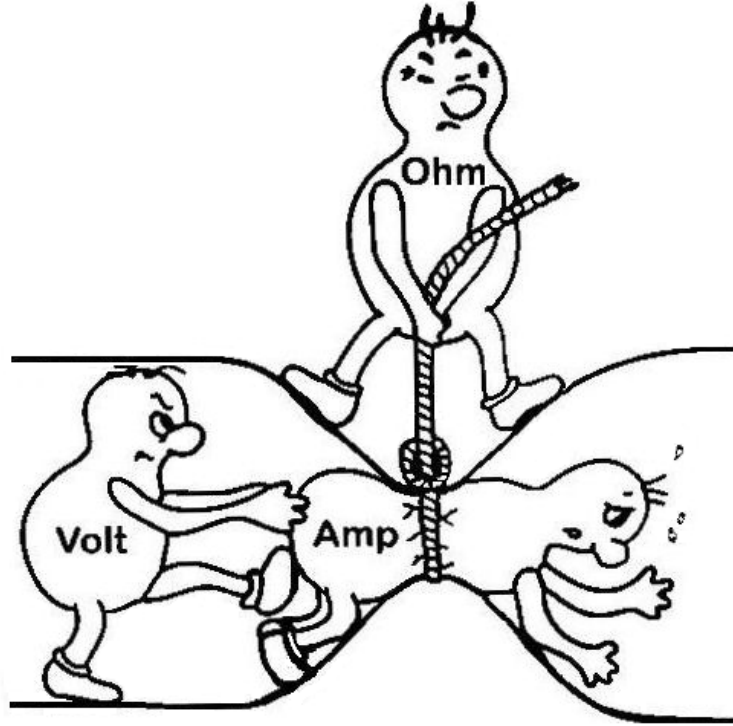


Figure 8: Ohm's Law, with various electrical quantities modeld as anthropomorphic peanuts.

$$v[n] = R * i[n]$$

This equation states that for a given amount of current running through a device, that device will push back on that flow with a proportional amount of voltage, scaled by the device's resistance.

For example, Ohm's law explains why system voltage drops when you drive a robot very hard. All batteries have some small amount of internal resistance (something like 0.012Ω). They usually supply around 12V of voltage. However, as motors and solenoids and compressors draw large amounts of current, the battery internally starts to fight itself due to its internal resistance. The voltage actually supplied by the battery is then some quantity less than 12V, and the amount of drop is roughly proportional to the amount of current pulled from the battery.

Inductance and **Capacitance** are properties of devices, describing their ability to store electrical energy and release it over time. The math associated with these is beyond the scope of this paper.

Power is not specifically an electrical topic, but relates strongly to electrical quantities. Electrical Power refers to the total energy absorbed or dissipated by an electrical component over time. Positive power in an electrical component implies energy is leaving the system (via motion, heat, sound, etc.), while negative power means energy is entering the system (Battery, power supply, hand-cranked generator, etc.)

Power is calculated in many ways. In general, it can be found by multiplying voltage and current:

$$P[n] = v[n] * i[n]$$

Power will be measured in units of Watts (denoted "W").

Part II

Component Models

3 Electric Motor

4 Main Battery

5 Main Circuit Breaker

6 Compressor and Tank

7 Traction Wheel

Part III

Integrated Model

Part IV

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8 Software Architecture

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