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CS / CPE 600
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Homework Assignment 12
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Q1. No. 26.6.7

Convert the following linear program into standard form:

$$\begin{aligned} \textbf{minimize:} \quad & z = 3y_1 + 2y_2 + y_3 \\ \textbf{subject to:} \quad & -3y_1 + y_2 + y_3 \geq 1 \\ & 2y_1 + y_2 - y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

Sol.

A linear program in standard form has the following form,

$$\begin{aligned} \textbf{Maximize} \quad & z = \textstyle\sum_{i \in V} c_i x_i \\ \textbf{subject to} \quad & \textstyle\sum_{j \in V} a_{ij} x_j \leq b_i \text{ for } i \in C \\ & x_i \geq 0 \text{ for } i \in V \end{aligned}$$

where V indexes over the set of variables and C indexes over the set of constraints.

The linear program is:

$$\begin{aligned} \text{Minimize:} \quad & z = 3y1 + 2y2 + y3 \\ \text{Subject to:} \quad & -3y1 + y2 + y3 \geq 1 \\ & 2y1 + y2 - y3 \geq 2 \\ & y1, y2, y3 \geq 0 \end{aligned}$$

We can rewrite the linear program into the standard form as:

$$\begin{aligned} \text{Maximize} \quad & z = -3y1 - 2y2 - y3 \\ \text{Subject to:} \quad & 3y1 - y2 - y3 \leq -1 \\ & -2y1 - y2 + y3 \leq -2 \\ & y1; y2; y3 \geq 0 \end{aligned}$$

Q2. No. 26.6.26

Give a linear programming formulation to find the minimum spanning tree of a graph. Recall that a spanning tree T of a graph G is a connected acyclic subgraph of G that contains every vertex of G. The minimum spanning tree of a weighted graph G is a spanning tree T of G such that the sum of the edge weights in T is minimized.

Sol.

Linear programming is a technique for optimizing a mathematical model. Lagrange's inequality and inequality constraints can be applied to an objective function. Convex optimization methods can be used in situations where the objective function or constraint functions are not linear but can be approximated arbitrarily close by linear functions.

A spanning tree T of a graph G is a connected acyclic subgraph of G that contains every vertex of G. The minimum spanning tree of a weighted graph G is a spanning tree T of G such that the sum of the edge weights T is minimized.

With vertices V and edges E, the weighted graph G is constructed. The goal is to find an acyclic subgraph T of G, such as: The subheading T is linked to the rest of the paragraph. T's edge weights are minimized. We need to use linear programming to find the minimum spanning tree of a graph.

We need to follow the following restrictions when writing the code: When an edge connects the two vertices i and j, then xij is 1. Otherwise, xij is 0. S is the owner of () (S being all subsets of E).

As a result, there can never be more than one edge connecting two sets of vertices. If the problem is approached in this way, then a linear programming formulation can be devised. There should be no cycles in the "weighted, undirected graph G = (V,E), so find a subgraph T" of G that contains every vertex in V and has edge weights c e for each e in E.

Assume that w(T) is defined as: we want to minimize weight w(T). the sum in T is equal to w(T)

A linear program can be used to solve this problem:

Maximize x e + y e - z e

where e is a T-expression.
The sum of y u and x v must be less than 1 for each pair (u, v) in V * V where (u!=v). Z v = 2*z v for each pair (u, vi). For each pair (u, vi), there is at least one edge in G between the pair (u, vi). In E, x e > 0, y e > 0, and z e > or equals 0 or 1 for each e in E.

We need to use linear programming to find the minimum spanning tree of a graph. There should be no cycles in the "weighted, undirected graph G = (V, E), so find a subgraph T" of G that contains every vertex in V and has edge weights c e for each e in E.

Q3. No. 26.6.31

A political candidate has hired you to advise them on how to best spend their advertising budget. The candidate wants a combination of print, radio, and television ads that maximize total impact, subject to budgetary constraints, and available airtime and print space.

type	impact per ad	cost per ad	max ads per week
radio	a	10,000	25
print	b	70,000	7
tv	c	110,000	15

Design and solve a linear program to determine the best combination of ads for the campaign.

Sol.

Let there be 'k' ads of radio, 'l' ads of print and 'm' ads of tv.
So, the total impact of the ads would be,
impact = (k * a + l * b + m * c) which is to be maximized

Now, let total budget be 'B'. So, the total cost for the ads would be,
Cost = (10000 * k + 70000 * l + 110000 * m)
which should be less than or equal to total budget
i.e.,
(10000 * k + 70000 * l + 110000 * m) <= B

There is limit to the maximum number for each type of ads per week which depicts
k <= 25
l <= 7
m <= 15

This suggests that the Linear Program would become
maximize: impact = (k * a + l * b + m * c)
subject to: (10000 * k + 70000 * l + 110000 * m) <= B
bounds: k <= 25; l <= 7; m <= 15

The proper solution of this linear program can be found if the values of a, b, and c are provided.