Romil V. Shah
20008692

CS / CPE 600
Prof. Reza Peyrovian

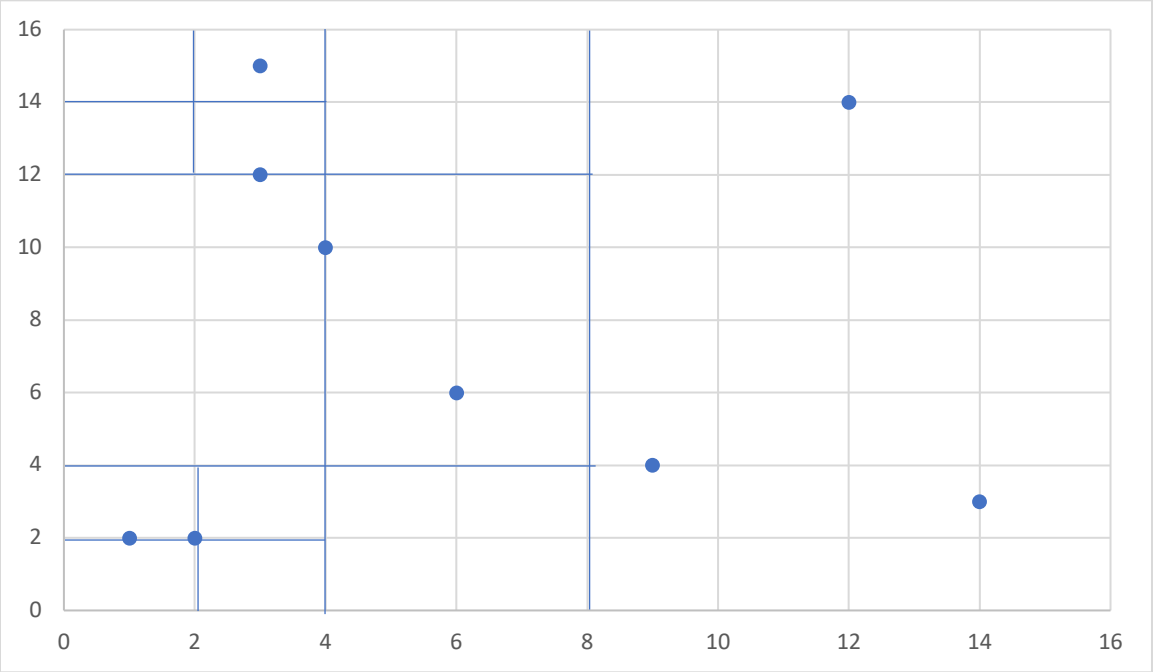
Homework Assignment 11
Submission Date: 12 / 04 / 2022

Q1. No. 21.5.7

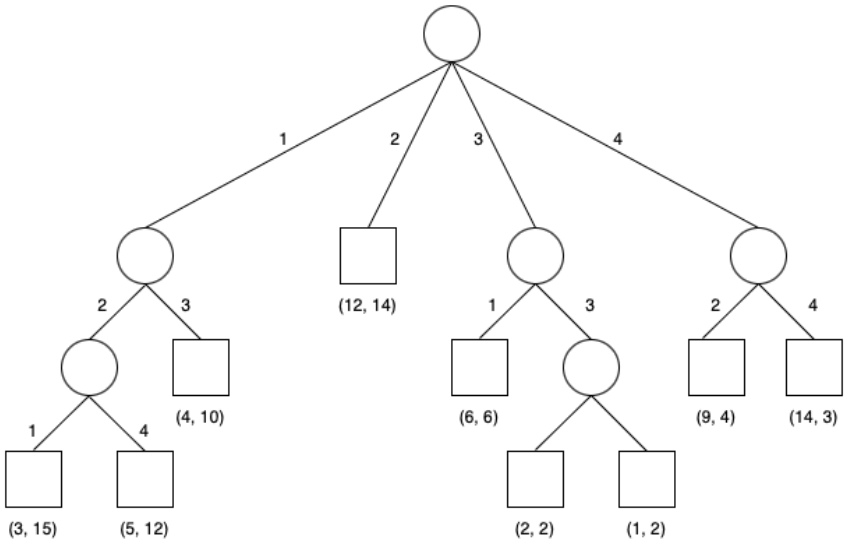
Draw a quadtree for the following set of points, assuming a $16 \times 16$ bounding box:

$$\{(1,2),(4,10),(14,3),(6,6),(3,15),(2,2),(3,12),(9,4),(12,14)\}.$$

Sol.




Assuming all the points that lie on the intersection will come in first quadrant whether NE (North East), NW (North West), SE (South East), SW (South West),



Q2. No. 21.5.13

Describe an efficient data structure for storing a set $S$ of $n$ items with ordered keys, so as to support a `rankRange`$(a, b)$ method, which enumerates all the items with keys whose ***rank*** in $S$ is in the range $[a, b]$, where $a$ and $b$ are integers in the interval $[0, n-1]$. Describe methods for object insertions and deletion, and characterize the running times for these and the `rankRange` method.

Sol.

An efficient data structure for storing a set S of n items with ordered keys can be a balanced binary tree(AVL or Red-Black Trees). The rankRange(a, b) method will work by searching for the lower end of a range $x_1$ and upper end of a range $x_2$ where $x_1$ and $x_2$ satisfies ($x1 \leq x \leq x2$ ) and counting all the elements in the search tree that exists between $x_1$ and $x_2$ using in-order ordering.

Insertion in AVL tree:

Algorithm insertAVL(k, e, T ):

Input: A key-element pair, (k, e), and an AVL tree, T
Output: An update of T to now contain the item (k, e)

v ← IterativeTreeSearch(k,T)

if v is not an external node then
    return “An item with key k is already in T ”

Expand v into an internal node with two external-node children
v.key ← k
v.element ← e
v.height ← 1
rebalanceAVL(v, T )

Deletion in AVL tree:

Algorithm removeAVL(k, T ):

Input: A key, k, and an AVL tree, T
Output: An update of T to now have an item (k, e) removed

v ← IterativeTreeSearch(k,T)

if v is an external node then
    return “There is no item with key k in T ”

```
if v has no external-node child then
        Let u be the node in T with key nearest to k
        Move u's key-value pair to v
        v←u

Let w be v's smallest-height child

Remove w and v from T , replacing v with w's sibling, z
rebalanceAVL(z, T )
```

Rebalance Tree:

```
Algorithm rebalanceAVL(v, T ):

Input: A node, v, where an imbalance may have occurred in an AVL tree, T
Output: An update of T to now be balanced

v.height ← 1 + max{v.leftChild().height, v.rightChild().height}

while v is not the root of T do
        v ← v.parent()
        if |v.leftChild().height − v.rightChild().height| > 1 then
                Let y be the tallest child of v and let x be the tallest child of y
                v ← restructure(x) // trinode restructure operation
                v.height ← 1 + max{v.leftChild().height, v.rightChild().height}
```

The rankRange method will take O(log n + k) time where k is the number of points reported in a range and both the deletion and insertion method will take O(log n) time.

Q3. No. 21.5.27

In computer graphics and computer gaming environments, a common heuristic is to approximate a complex two-dimensional object by a smallest enclosing rectangle whose sides are parallel to the coordinate axes, which is known as a ***bounding box***. Using this technique allows system designers to generalize range searching over points to more complex objects. So, suppose you are given a collection of $n$ complex two-dimensional objects, together with a set of their bounding boxes,

$$\mathcal{S} = \{R_1, R_2, \ldots, R_n\}.$$

Suppose further that for some reason you have a data structure, $D$, that can store $n$ four-dimensional points so as to answer four-dimensional range-search queries in $O(\log^3 n + s)$ time, where $s$ is the number of points in the query range. Explain how you can use $D$ to answer two-dimensional range queries for the rectangles in $\mathcal{S}$, given a query rectangle, $R$, would return every bounding box, $R_i$, in $\mathcal{S}$, such that $R_i$ is completely contained inside $R$. Your query should run in $O(\log^3 n + s)$ time, where $s$ is the number of bounding boxes that are output as being completely inside the query range, $R$.

Sol.

We can use Range Trees data structure for querying two-dimensional data. Instead of an auxiliary tree, we can store an array, sorted by Y-coordinates. At $x_{split}$ we will do binary search for y1 As, we continue to search for x1 and x2, we can use pointers to keep track of the result of binary search for y1 in each of the arrays along the path. This method is also known as fractional cascading search.

The run time of the algorithm for d dimension is $O(\log^{d-1} n + s)$ and for 4 dimension it will be $O(\log^3 n + s)$.

Q4. No. 22.6.7

Give a pseudocode description of the plane-sweep algorithm for finding a closest pair of points among a set of $n$ points in the plane.

Sol.

Let S be a set of n points.
For finding the minimum distance between two points P and Q it can be calculated as
dist(a, b) =
d = dist(a, b)

For any point p in S, x(p) and y(p) denote the x and y coordinates.

Consider a sweep line SL is the vertical line through point p of S.

Algorithm closestPair()

Input: Set S of n points in the plane
Output: Finding Closest pair (p, q) of points

Let X be the structure in an array A[1, … ,n] containing set S points sorted by x-coordinates.
    δ := dist(A[1], A[2])  (minimum distance among all points to the left of SL)

Let Y be the empty dictionary.
while (point p <= n)
    p = p+1
    when new point is found

    if (dist (p, q) < δ)
    then
        A[1] ← p,
        A[2] ← q
        d ← dist(p, q)

    Insert p into dictionary Y
    Search closest point q to the left of p to points in Y.
    for( all points whose y coordinates lie in [y(p) - □, y(p) + □ ] )
        find q point closest to p
return (p,q)

Sorting of elements will take O(n log n)time. Insertion and deletion for an element will be done once in the dictionary takes O(log n) time and each range query in S takes O(log n).
So, the total running time of the algorithm is O(n log n).

Q5. No. 22.6.16

Let $C$ be a collection of $n$ horizontal and vertical line segments. Describe an $O(n \log n)$-time algorithm for determining whether the segments in $C$ form a simple polygon.

Sol.

It is given that in a collection C of n horizontal and vertical line segments to form a simple polygon we can use.

1. Using plane sweep, determine all pairs of intersecting segments with common coordinates.
2. As seep line SL move from left to right find the coordinates which have common horizontal and vertical points in the plane.
3. If a common coordinate is found then store it in dictionary, whenever we find another line segment check in the dictionary if they have common endpoints.
4. In this form we can return all the coordinated in the dictionary and check if they form the closed loop. It means there exists a polygon.

Number of points in collection C is n. Plane Sweep algorithm will take O(n log n) times. Moving for coordinates for the line segments will cover each line will take O(n) time.
So, algorithm will take O(n log n) time in total.

Q6. No. 22.6.30

In machine learning applications, we often have some kind of condition defined over a set, $S$, of $n$ points, which we would like to characterize—that is, "learn"—using some simple rule. For instance, these points could correspond to biological attributes of $n$ medical patients and the condition could be whether a given patient tests positive for a given disease or not, and we would like to learn the correlation between these attributes and this disease. Think of the points with positive tests as painted "red," and the tests with negative tests as painted "blue." Suppose that we have a simple two-factor set of attributes; hence, we can view each patient as a two-dimensional point in the plane, which is colored as either red or blue. An ideal characterization, from a machine learning perspective, is if we can separate the points of $S$ by a line, $L$, such that all the points on one side of $L$ are red and all the points on the other side of $L$ are blue. So suppose you are given a set, $S$, of $n$ red and blue points in the plane. Describe an efficient algorithm for determining whether there is a line, $L$, that separates the red and blue points in $S$. What is the running time of your algorithm?

Sol.

- To determine a line L which divides the blue and red points into two separate sets we can use convex hull property.
- Forming a convex hull around blue and red points separately.
- Then checking if both the convex hull intersects each other not.
- If intersection between convex hull exists, then there is a line which divides red and blue points separately.

Creating a convex hull using Graham Scan Algorithm will takes O(n log n) time.