Problem Chosen	2022	Team Control Number
A	MCM/ICM Summary Sheet	2204796

Optimal Power Distribution Based Power Profile of Cyclist

Summary

Cycling road races have a high degree of popularity. For a professional racer, fully understanding the limits of his physical ability to specify a detailed race plan is a prerequisite for winning the race. The aim of this report is to build a mathematical model that can be applied to multiple types of riders and variable road conditions to determine the best energy allocation strategy for riders. We hope to provide race riders with some professional and reliable strategies that will allow riders to reach their full physical potential and gain an advantage in the race.

In order to define different types of riders, we first built the Exertion Model to characterize the power profile of riders. we used three parameters in the model to describe different types of riders. We collected data of different types of riders and different genders. The values of the parameters were determined using **nonlinear least squares**, and solve it by **sequential quadratic programming(SQP)**. The variability in the values of these parameters is a strong indication of the diversity of the rider's abilities.

Dynamic equations are established based on the **Carvallo-Whipple model** to obtain the relationship between power and speed. Considering lateral wind and horizontal sharp turn, analysis the lateral force and extend bicycle model to obtain speed constraint. Take sampling points and set relative power, substitute which into the differential equation to get the speed and time, turning the problem into finding minimum of constrained nonlinear multivariable function. Experiment with the designed route and real scene, and analyze the results to verify the correctness of the model.

By **enumerating** the parameter combinations of different levels of wind speed and different wind direction angles, the parameters are brought into the model to solve, and the fluctuation of the completion track time is compared to analyze the sensitivity of the model to the parameters. The optimal time data obtained by the model indicating that, In the case of complex and long roads, the model has a strong ability to resist interference to the environment. When applied to simple roads, the sensitivity to environmental parameters is high.

To determine how sensitive the results are to rider deviations from the target power distribution, we **Savitzky-Golay method** to simulate real-world power distribution decisions. The Savitzky-Golay-processed power-position curves become smoother and conform to the rider's physical fitness and decision level. In addition, to simulate the randomness of objective world decision making and realization, we further processed the curves using normal distribution to make the curves closer to the real situation. After visual image comparison, it shows that our model is extremely robust to the deviations caused by the rider.

The **drafting effect** taken by teamwork can reduce air resistance and scales with the size of the team. Based on this, we designed two strategies, one is six person ride together, the other is two person take the lead and then separate at exhausting points. Calculate the time separately by modifying the model and substitute the data for selection.

Keywords: nonlinear least squares; SQP ; Carvallo-Whipple model; enumerating; kinetic equation; Savitzky-Golay method

Contents

1 Introduction

1.1 Background

Achieving excellence in cycling depends not only on talent and perseverance, but also on developing a well-thought-out race strategy. The most important basis for developing a race strategy is the power profiling of the cyclist. Power profiling in cycling is most commonly defined as the assessment of field derived power outputs, i.e. values obtained during training and racing. Power profiling can be used for the tracking of longitudinal changes in performance and race analysis. There is no way for a rider to maintain an extraordinary intensity of power output all the time. Extra-intense power output causes the rider's muscle cells to breathe anaerobically, which in turn leads to lactic acid buildup, causing the rider to feel sore and fatigued. After a period of high intensity power output, the player must lower the power output and recover his strength. This allows for better performance and does not place a permanent burden on the body.

In addition to the need to consider the competitors' own factors, it is also necessary to examine the specific road condition information for each race. The route design for a road cycling race is chosen on an outdoor road, and the route includes turns, uphill and downhill. Competitors must make different decisions based on different information about the road conditions. In addition, weather factors, such as wind, humidity, etc., also affect the performance of the competitors. These factors are exactly what we need to consider when modeling.

1.2 Restatement of the Problem

We can only develop a scientific and effective plan if we consider all aspects of the rider's race strategy. Therefore, we were asked to address the following questions:

- A model is created which can be applied to any type of rider. This model determines the relationship between the position of the rider on the track and the power applied. And it takes into account the limitation of the rider's total energy.
- We were asked to define the power profiles for two types of riders, one of which had to be an individual time trial rider, and also to consider the power profiles for different gender riders.
- Apply the model we built earlier to the analysis of a variable time trial course. At a minimum, the ones listed below for each power profile you defined above:
  - 2021 Olympic Time Trial course in Tokyo, Japan,
  - 2021 UCI World Championship time trial course in Flanders, Belgium,
  - Our self-defined route must include at least four sharp turns and at least one steep hill. The end of the course should be near its start point
- Determine how sensitive the results are to rider deviations from the target power distribution.
- Discuss how to extend your model to include the optimal power use for a team time trial of six riders per team, where the team's time is determined when the fourth rider crosses the finish line.

1.3 Literature Review

In terms of cognitive Power Profile, Crtical Power (CP)[1] is a concept that has been widely adopted by many physiologists and coaches in recent years.CP is a power that depends on the rider's physical fitness. The rider's output restores energy when it is below CP, and depletes energy when it is above CP. Based on this concept, there are various mathematical models to simulate the actual power curve of the riders [2].

In considering the optimization problem of power distribution for bicycle riders, the variational method can be used to solve this problem [3]. The application of forward integration of the velocity of the equation in the optimization strategy can reduce the process complexity and improve the accuracy of the calculation [4].

In terms of bicycle dynamics analysis, experiments done by Fintelman revealed the effect of lateral wind on bicycle riding [5]. It has been shown that in team riding, the rider behind has significantly lower wind resistance and consumes less energy due to the shielding effect of the rider in front (BELLOLI201638). During bicycle steering, the maximum speed is determined by the coefficient of friction between the tires and the road [6].

2 Preparation of the Models

2.1 Analysis of Problems

In order to define power profiling for two different types of riders, we want to create a function describing the power-duration relationship in cycling that precisely summarizes many important features of power profiling. This function is based on the physiological CP principle [1]. We have to choose several significant parameters to characterize different types of riders. This allows us to obtain a function that can describe very precisely riders with different physical/physiological characteristics.

In order to apply our model to a diverse range of tracks, we need to solve two problems. The first problem is to perform a dynamics analysis of a rider riding on a complex roadway, which requires consideration of several factors such as ground rolling friction, air resistance, slope, and radius of curvature of the track. Through the kinetic analysis, we will finally determine a power-speed relationship equation. The second issue is to determine the rider's power allocation strategy on the stage. The strategy should aim to consume the least amount of time throughout the course. We use a nonlinear programming approach to solve for this optimal solution. In addition, we use Google Earth to collect data related to the requested route.

To explore the potential influence of weather on decision making, we introduce new terms based on the mechanistic model to simulate the effect of wind. Our new model design requires consideration of wind speed and wind direction, and this is used to derive a new strategy for the allocation of the metric. We then perform a sensitivity analysis of the wind speed and direction in this model. This is used to determine how sensitive our model is to slight differences in weather and environment.

In order to give riders a clear and uncluttered ride plan. We consider reducing the number of decisions and avoiding overly aggressive decisions. At the same time, random variables are introduced to simulate the rider's decision failures during the race. The results obtained after the above process are compared with the ideal situation. Based on the comparison results we determine how sensitive the results are to rider deviations from the target power distribution.

In order to expand our model and make it suitable for team time trials, we modified our model according to the concept of "low wind resistance for the latter in the team" . We plan to give a different division of labor to each member of the team. The time taken by the fourth member of the team to cross the line is used as the optimization goal, and the optimization problem will be solved again.

2.2 Assumption and Justification

We make some general assumptions to simplify our model. These assumptions and corresponding justification are listed below:

- The rider maintains the optimal aerodynamic position time-trial position (TTP), i.e. constant windward area, throughout the race. This is because professional riders are professionally trained to maintain the time-trial position during the race. Therefore, in our calculations, only such cases are considered.
- If the road information has not changed much, there is no need to change the power allocation decision. Changing decisions too much is not conducive to a rider achieving good results [1].
- The effect of curve width on the rider's path selection is not considered when making turns. Considering the width of the turn would greatly increase the complexity of model construction and road condition information collection, making the model able to process significantly fewer data points and increasing the granularity.
- Within a single race scenario, the earth's radius, gravitational and magnetic field directions, and wind speed and direction are ignored as the rider's position within the course changes.
- The motion of the bicycle wheel relative to the ground is considered as a pure rolling process, with the premise that no sideslip occurs when turning and subject to lateral wind.

2.3 Notation

3 The Exertion Model of the Power Profiles

3.1 Model Overview

We start our model with the concept of Crtical Power. We believe that the reason why a rider cannot maintain a high level of power output for a long time is that when the power exceeds the critical power, the rider's muscle cells go into an anaerobic state, which greatly depletes the rider's energy and causes the rider to become fatigued, making it difficult to maintain this state. First, we define the "anaerobic energy W" that the rider has, and when this energy reaches zero, there is no way for the rider to maintain this state. Then, we found that the above model does not correspond to

Table 1: Notations

Symbol	Definition
P	output power
CP	critical power
P_m	maximum power
v	velocity
x	position
L	course total distance
t	time
T	total time
τ	duration of effort at a fixed power output
E	exertion
M	mass of cyclist with bicycle
K	air drag coefficient
μ_r	rolling friction coefficient
μ_s	maximum static friction coefficient
θ	velocity direction angle
α	road grade angle
β	wind angle
v_w	wind velocity
ρ_s	slope radius of curvature
ρ_t	horizontal sharp turn radius of curvature
G	gravity
f_w	longitudinal air resistance
F_w	lateral wind force
F_s	forward driving force of the rear wheels
f_r	rolling friction force of the front wheels
N	ground support

a realistic power when maintained for a very short time, so we introduced the concept of "maximum power P_{max}" to correct the equation. Finally, we selected Time Trial Specialist and Sprinter as the subjects of our study. We fit the theoretically derived equations to the actual data we collected for different types and players, using the method nonlinear least squares.

3.2 Quantitative Exertion Model

- The relationship between power and duration can be expressed as followed:

$$P = \frac{W}{t} + CP \tag{1}$$

where,

W represents anaerobic energy, which depends on the fitness of the different riders, and CP represents critical power, which also depends on the fitness of the rider. When the power output exceeds the critical power, it depletes the rider's anaerobic energy. P represents the rider's output power. t represents the maximum time the rider can sustain at an output power of P.

- Considering that there is an upper limit to the rider's output power, we correct the equation (1)

$$P = \frac{W}{t + \frac{W}{P_{max} - CP}} + CP \tag{2}$$

P_{max} is the maximum output power of the rider. When the rider's output power is P_{max}, the rider cannot maintain this state.

The Power Curve can be drawn according to (2), as shown in the Figure 1:

As can be seen from the graph, the higher the output power, the shorter the time the rider can maintain the current power. When the output power is below the dotted line (i.e., when the output power is less than the critical power), when the output power is above the dotted line (i.e., when the output power)

3.3 Parameter Determination of Different types

The function we obtained has three parameters, namely critical power CP, anaerobic energy W, and maximum power P_{max}, which are different for different types of riders and different genders,

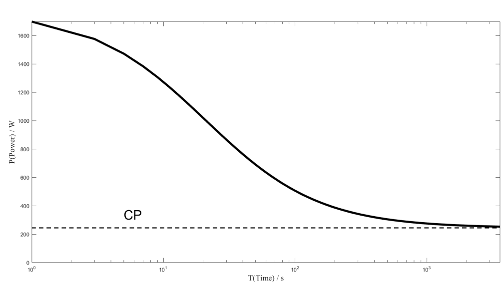

Figure 1: Power Curve

which leads to differences in their power profiling. We collected data related to different types of male cyclists from https://cyklopedia.cc/cycling-tips/power-profile/. Nonlinear least squares was used to determine their parameters. We solved this problem using the method of sequential quadratic programming.

The two types we picked were Time Trial Specialist and Sprinter.

When δ is minimized in the following equation, the parameter is the one we are looking for

$$\delta(CP, W, P_{max}) = \sum^{m}(P_i - \hat{P}_i)^2 \tag{3}$$

3.4 Result

The parameters for different types and different players are listed below

The power curve for different types of riders is shown as Figure 2.

The sprinter can accelerate quickly to a high speed. therefore the sprinter can maintain a high power output for a longer period of time than the time trail specialist. time trail specialist is more suitable for longer distances than the sprinter. The time trail specialist is more suitable for long distance races than the sprinter. As you can see from the graph, the CP of the time trail specialist is significantly higher than that of the sprinter, which means that the time trail specialist can breathe aerobically at a higher power to ensure a longer duration.

The power curve for riders of different genders is shown as Figure 3.

Male athletes have more aerobic respiration in their muscle cells compared to female athletes, so CP is higher. Male athletes have higher glycogen levels in their bodies than female athletes, and therefore W is also higher, which is reflected in the graph as the area under the curve, which is

Table 2: different parameters of different types

type	W/J	CP/w	P_{max} /w
Sprinter	31556	244	1769
Time Trial Specialist	23658	323	1373
Male	26427	358	1533
Female	15580	276	1077

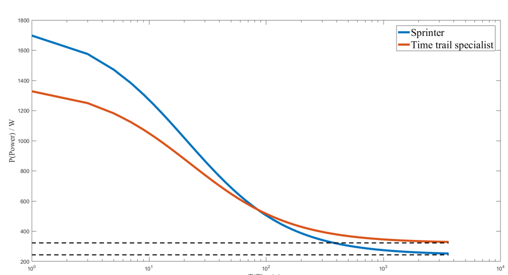

Figure 2: Power Curve of different types

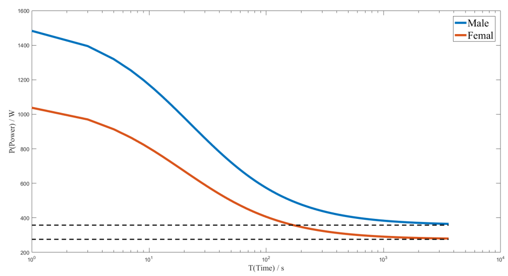

Figure 3: Power Curve of different genders

higher for male athletes than for female athletes.

4 Optimal solution combining 3D geographic roads

4.1 Model Principles

The most widely used physical study on bicycle dynamics is the Carvallo-Whipple bicycle model proposed in 1899, according to which the following basic assumptions are made.

- Ignore the width of the bicycle wheels and assume that the bicycle cannot be balanced at rest.
- Equation of motion of an idealized bicycle, the idea that the leaning of the body can be corrected by steering so that the bicycle can balance stably when moving forward.
- The motion of the bicycle wheel relative to the ground is regarded as a pure rolling process, without considering sliding friction, and the premise that no side slip occurs when turning and being subjected to lateral wind, and that the entire forward force of the bicycle comes from the static friction generated by the rear wheel through the pedal drive.
- Ignore the bike body length when steering, i.e. consider the front wheel orientation as the bike direction

4.2 Longitudinal force analysis

Accordingly, a schematic diagram of the longitudinal forces on the bicycle during travel is shown below 4.

Define the driving force F_s as the static friction force on the rear wheel, the drag force f_r as the rolling friction force on the front wheel, and f_w as the longitudinal air resistance to the relative motion into the wind.

Due to the change in slope angle, the bike needs the ground to provide a centripetal force upward from the vertical slope. That is, the slope support force

$$N = G\,cos(\alpha) + m\frac{v^2}{\rho_s} \tag{4}$$

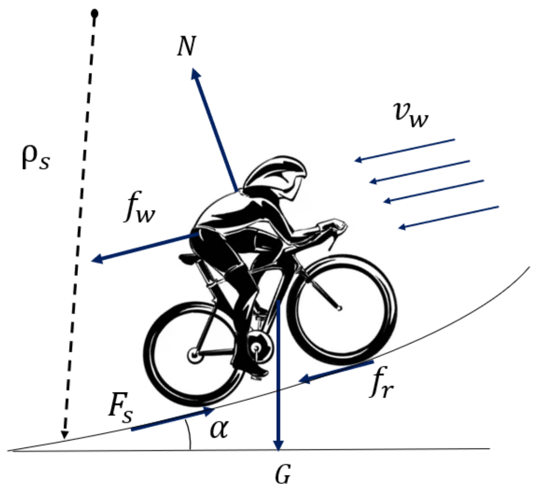

Figure 4: Longitudinal force diagram

where α is the slope inclination angle and $G\ cos(\alpha)$ is the component of gravity perpendicular to the slope.

Then the rolling friction force on the front wheel

$$f_r = \mu_r N = \mu_r (Mg\ cos\alpha + M\frac{v^2}{\rho_s}) \tag{5}$$

where μ_r is the rolling friction coefficient of the ground, M is the overall mass of the rider-bike system, v is the travel speed of the rider at this moment, and ρ_s is the radius of curvature of the slope.

The air resistance is considered to be proportional to the square of the relative velocity

$$f_w = K * (v + v_w)^2 \tag{6}$$

where $K = c\rho A$ is the air drag coefficient, which is related to the aerodynamic posture adopted by the rider, and v_w is the wind speed in a longitudinal headwind.

The kinetic equation is obtained as

$$\begin{aligned} M\frac{dv}{dt} &= F_s - f_r - f_w - Mg\ sin\alpha \\ &= F_s - \mu_r(Mg\ cos\alpha + M\frac{v^2}{\rho_s}) - K * (v + v_w)^2 - Mg\ sin\alpha \end{aligned} \tag{7}$$

Ignoring the gear hinge transmission loss from the rider's pedal to F_s, the F_s work power is the relationship between the rider's output power P and velocity v.

$$\begin{aligned} P &= F_s v \\ &= Mv\frac{dv}{dt} + \mu_r Mv(g\ cos\alpha + \frac{v^2}{\rho_s}) - Kv(v + v_w)^2 - Mgv\ sin\alpha \end{aligned} \tag{8}$$

4.3 Transverse force analysis

Considering the actual situation, the ground static friction is needed to provide horizontal centripetal force pointing to the center of the circle when passing through the horizontal bend, and also needs to be balanced by static friction when subjected to lateral side wind, the lateral dynamics of Whipple is expanded.

The schematic diagram when passing through a horizontal curve is shown in the following figure 5.

Analysis of centripetal forces in the vertical plane.

The body tilts to the left when performing a circular motion with a radius of curvature of ρ_t, causing the wheels to have a tendency to slide to the right, causing the ground to generate a static frictional force f_s against the wheels horizontally to the left, providing the centripetal force required for the circular motion.

Similarly, when the bicycle is subjected to the thrust F_w of the horizontal leftward lateral wind force, it needs the horizontal rightward ground static friction f_s to keep from creating lateral sliding. To balance the moment of F_w, the body needs to be tilted to the right so that gravity G balances the moment generated by F_w.

The above two static friction forces are in opposite directions to obtain the magnitude of the total static friction force

$$|f_s| = |Ma_c - F_w| = |\frac{Mv^2}{\rho_t} - K(v_w \, sin\beta)^2| \tag{9}$$

where β is the angle between the wind direction and the direction of travel, and v_w is the total wind speed.

The condition for the wheel not to slide sideways is that the value of f_s is less than the

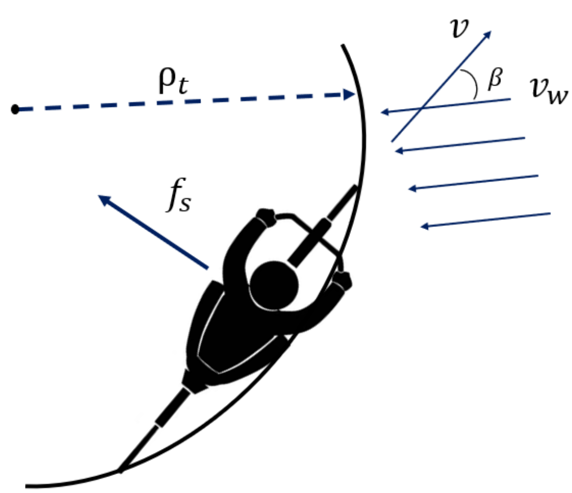

Figure 5: Transverse force diagram

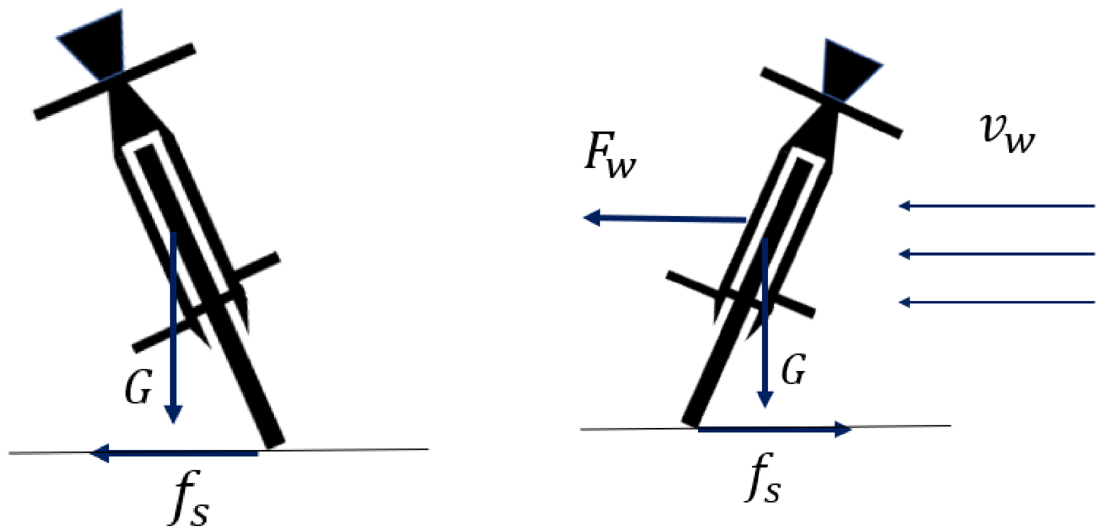

Figure 6: Vertical force analysis

maximum static friction, i.e.

$$|\frac{Mv^2}{\rho_t} - K(v_w\ sin\beta)^2| < \mu_s N \tag{10}$$

where μ_s is the maximum static friction coefficient.

$$\begin{cases} P = Mv\dfrac{dv}{dt} + \mu_r Mv(g\ cos\alpha + \dfrac{v^2}{\rho_s}) - Kv(v + v_w\ cos\beta)^2 - Mgv\ sin\alpha \\ |\dfrac{Mv^2}{\rho_t} - K(v_w\ sin\beta)^2| < \mu_s(G\ cos\alpha + m\dfrac{v^2}{\rho_s}) \end{cases} \tag{11}$$

4.4 Optimization problems

The biological $P \sim \tau$ model is combined with the physical $P \sim v$ model to solve for the $P \sim x$ distribution that minimizes the total time to complete the prescribed route.

Initial mental fullness state $E = 0$, when maintaining a power P for a rated time τ

$$E(t = \tau) = \frac{(P - CP)(P_{max} - CP)}{W(P_{max} - P)} * \tau = 1 \tag{12}$$

Means exhausted When the power is given as a function of time $P(t)$, there is

$$E(t) = \int \frac{(P - CP)(P_{max} - CP)}{W(P_{max} - P)} dt + Const \tag{13}$$

When finding the optimal $P \sim x$ distribution, take n sampling points, for positions $x_1, x_2, \cdots, x_n$, respectively, maintaining constant power $P_1, P_2, \cdots, P_n$ between each two sampling points. Using

$dx = vdt$, for the motion between each two sampling points, the first equation in (12) is transformed into solving the first-order nonlinear differential equation

$$\frac{P_i}{Mv^2} = \frac{dv}{dx} + \mu_r(\frac{g\ cos\alpha}{v} + \frac{v}{\rho_s}) - \frac{K}{Mv}(v + v_w\ cos\beta)^2 - \frac{g}{v}\ sin\alpha \tag{14}$$

Based on P_i, the initial value v_{i-1} and other parameters, the velocity v_i and the time spent at the sampling point t_i can be calculated by integration. The objective function is $f(P_1, P_2, \cdots, P_n) = T = \sum_{i=1}^{n} t_i$. Substituting the above constraint yields

$$\begin{cases} |\frac{Mv_i^2}{\rho_t} - K(v_w\ sin\beta)^2| < \mu_s(G\ cos\alpha + m\frac{v_i^2}{\rho_s}) \\ E_i = \sum_{k=1}^{i} \frac{(P_k - CP)(P_{max} - CP)}{W(P_{max} - P_k)} t_k < 1 \end{cases} \tag{15}$$

Find minimum of constrained nonlinear multivariable function (Find minimum of constrained nonlinear multivariable function)

4.5 Tokyo Olympics

Using Google Earth, a longitude-latitude-elevation 3D model of the Tokyo Olympics roads was created and converted to the XYZ coordinate system as shown in the figure below.

4.6 2021 UCI

The same for modeling and calculations:

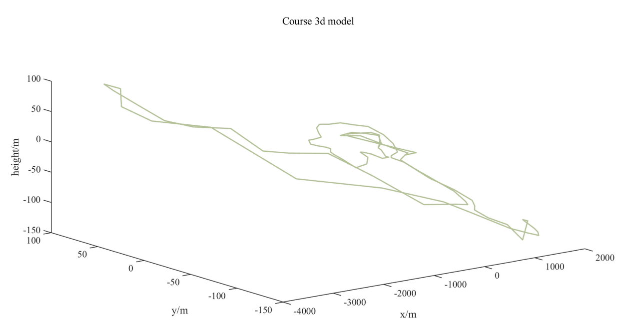

Figure 7: Tokyo Olympics

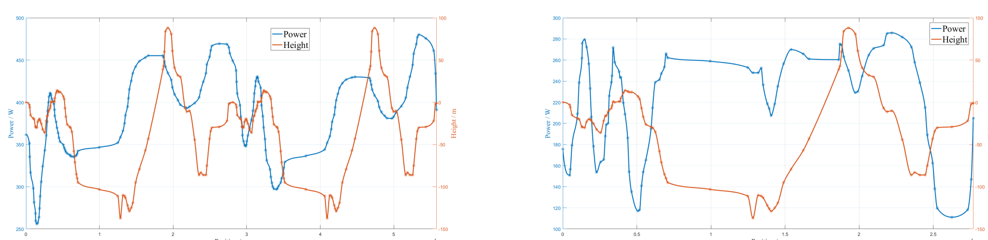

Figure 8: Tokyo Olympics different gender

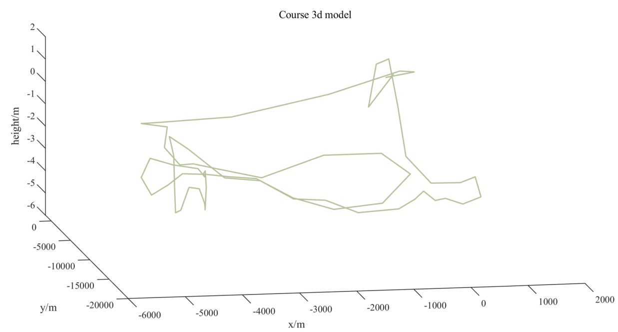

Figure 9: 2021 UCI

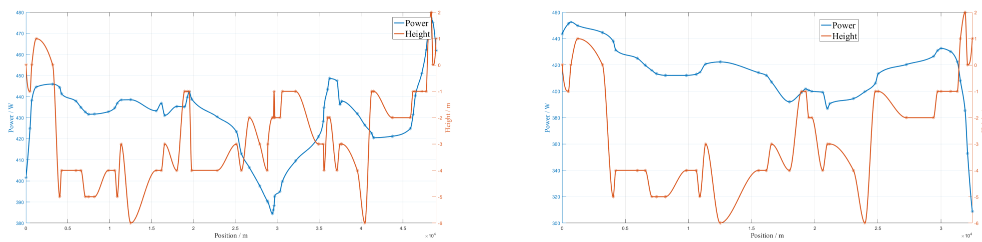

Figure 10: 2021 UCI different gender

4.7 Customized tracks

5 The Impact of Wind Condition

5.1 Model Overview

The above mathematical model can be used to find the minimum time Tmin for the time trial specialist to complete the track for different wind speed parameters (v) and wind direction parameters (β). The sensitivity of the model to the v β parameter can be measured by the degree of change in the Tmin value with the variables v

The following is the distribution of T_{min} obtained after the experimental calculation. The experimental tracks selected in Fig. 15, Fig. **??** and Fig. **??** are simple straight, simple curve and custom long distance track respectively.

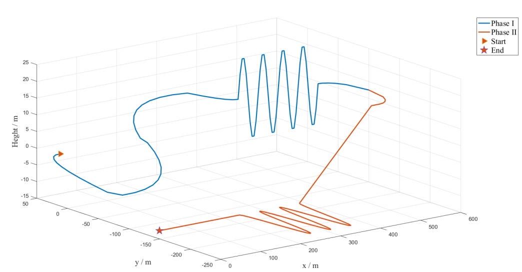

Figure 11: Customized tracks

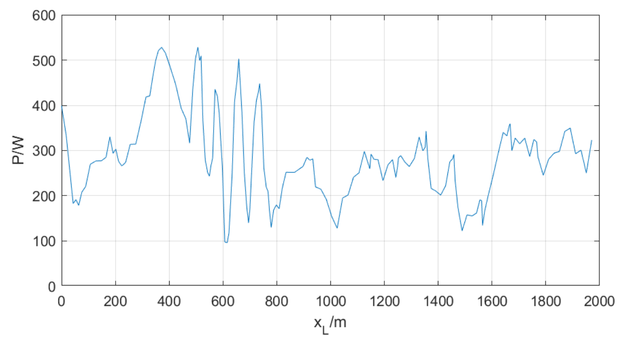

Figure 12: Customized tracks power distribution

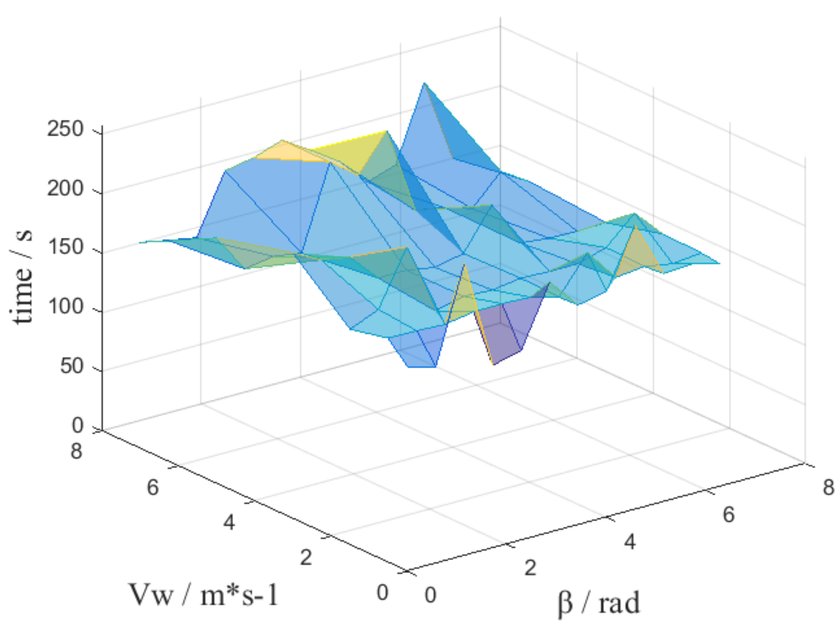

Figure 13: Simple Straight Course

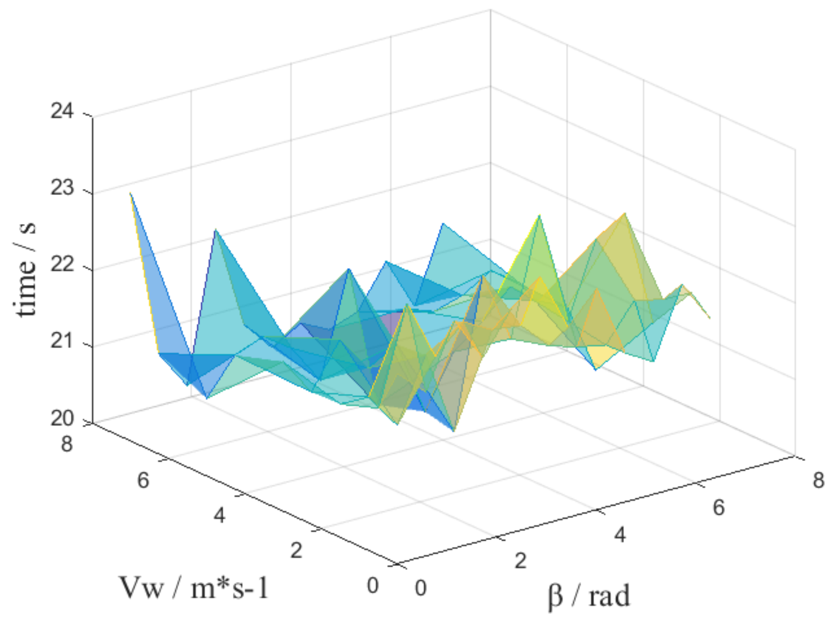

Figure 14: Simple Curve Course

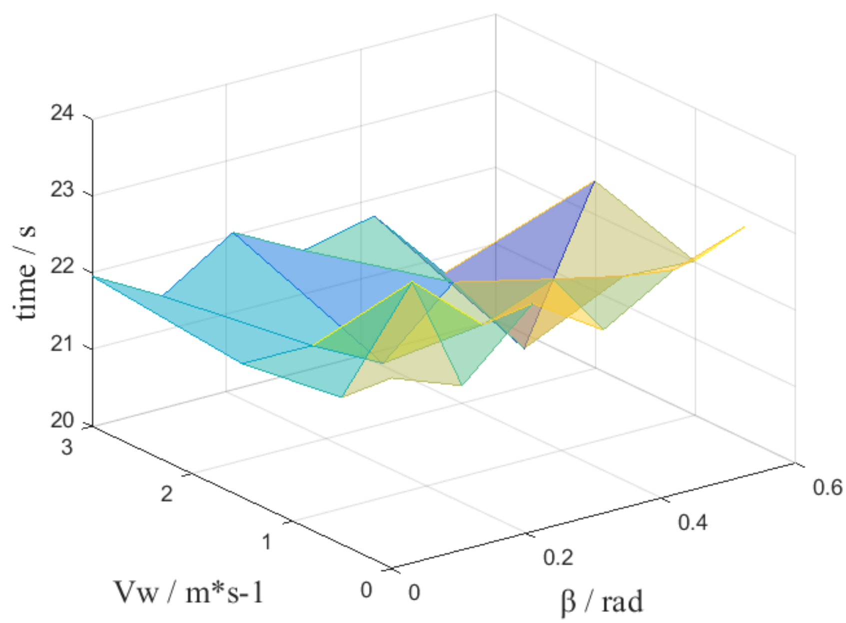

Figure 15: Custom Long Course

5.2 Results

It can be found that the value of Tmin is more sensitive to v β in simple straight roads, and in simple curves. In the simple straight road, the air air resistance generated by the relative motion of human and wind will continue to act steadily on the rider because the direction of travel is constant. The amount of air resistance is roughly linearly related to the square of the wind speed (v_M), so the resistance changes rapidly as the wind speed increases, which in turn affects the rider's time to complete the course to a greater extent.

And in the corner course, the wind will produce a continuous lateral force on the person, affecting the maximum centripetal force that the corner can provide, which in turn affects the maximum speed limit of the course, thus affecting the rider's performance to a greater extent.

For custom long distance courses, as this kind of course contains a variety of sections, the start and finish are closer together, so the rider may gain as well as lose from the wind speed in each part of the road, and the impact of wind speed or direction is not as obvious when averaged over the whole section of the course.

6 Modeling Rider Deviations with the Savitzky-Golay Method

6.1 Model Overview

It is impossible for a coach in the objective world to perform calculations of high complexity like a computer. It is also impossible for an objective-world rider to execute a plan with too high a level of detail. To test whether our our model can still work in the objective world, we applied the Savitzky-Golay Method to simulate the effects of real-world execution.

Savitzky-Golay filter is a special type of low-pass filter, also known as Savitzy-Golay smoother. The obvious use of a low-pass filter is to smooth noisy data.The Savitzky-Golay Method is a polynomial-based, best-fit method in the time domain using least squares by shifting the window. This method is better at preserving distribution characteristics such as relative maxima, minima and widths than other similar averaging methods. By using this method to deal with the original ideal power distribution, we can obtain a power distribution scheme similar to that of the objective world.

In addition, in order to simulate the randomness of objective world decisions, we further process the curves using normal distribution to make the curves closer to the real situation.

6.2 Steps of Algorithm

Step 1 Obtaining the power-position curve for the ideal case.

Step 2 The curve was processed using different levels of Savitzky-Golay Method.

Step 3 Further processing of the curve using normal distribution.

Step 4 Calculate the velocity-position image and minimum time based on the new power-position curve obtained.

Step 5 Comparison of velocity-position images and minimum time for the ideal and new cases.

6.3 Results

Using the Savitzky-Golay method for 3 and 5 times, respectively, we obtained Fig

The comparison graph shows that the Savitzky-Golay method and the normal distribution serve well to simulate the objective world. After processing, the curve changes the output frequency less often and deviates from the ideal target. But the overall shape is again similar to the ideal image. This is exactly the curve produced by a human rider who can only execute a rough strategy.

When we put the three speed-position graphs together for comparison, we will see that there is no significant difference between the three. The resulting calculated race times are also very similar. This indicates that our proposed model is insensitive to rider deviations from the target power distribution. It can be applied to the objective world with good results.

7 Six-person team time trial strategy

7.1 Theoretical basis

In terms of bicycle dynamics analysis, experiments done by Fintelman revealed the effect of lateral wind on bicycle riding [5]. It has been shown that in team riding, the rider behind has significantly lower wind resistance and consumes less energy due to the shielding effect of the rider in front (BELLOLI201638). During bicycle steering, the maximum speed is determined by the coefficient of friction between the tires and the road [6].

7.2 Strategy Design

Considering a six-person team time trial, with the result of the fourth cyclist in the team to reach the finish line as the team result, we devised the following two team strategies and compared

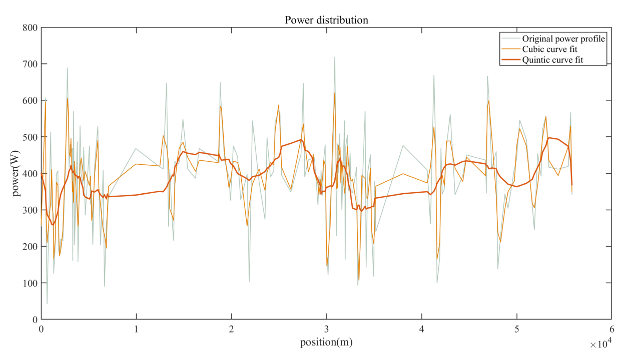

Figure 16: Contrast Chart

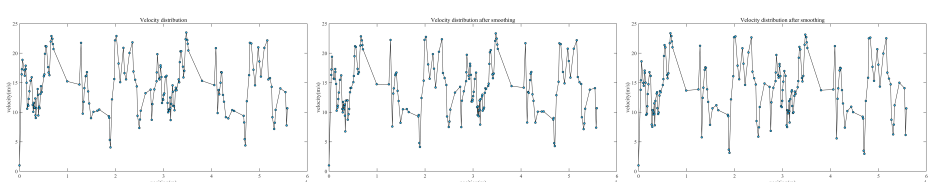

Figure 17: three speed-position graphs

them based on actual data.

Option 1.

The simpler way: the six-person team never separates and reaches the finish line sequentially before and after

Option 2.

Considering the fourth person hitting the line time as the team time, refer to the leader of the marathon who only participates in the first half of the race, making two people take turns to block the wind in front and bear most of the air resistance, and separate at some exhausted point of time, the original four people in the back continue to maintain a very close distance until they reach the finish line, without considering the other two people.

7.3 Results

First calculate the six-rider independent race arrival time to the finish line as a reference value.

Substitute the air resistance coefficients corresponding to 6 people into the model and calculate the results of option 1

Considering that the front 2 people are subject to the greatest air resistance, the speed of the front 2 people needs to prevail before separation, that is, the front two people in the section before the separation point to modify the K value of future generations into the model to obtain the velocity distribution

The speed distribution is substituted into the model modified by the next four people, and the exertion value at the separation point is calculated, which is used as the initial value and then optimized for the separated section.

As you can see, in this example, the optimal solution is for the six people to stay together and keep a close distance, taking turns to take the lead until they reach the finish line.

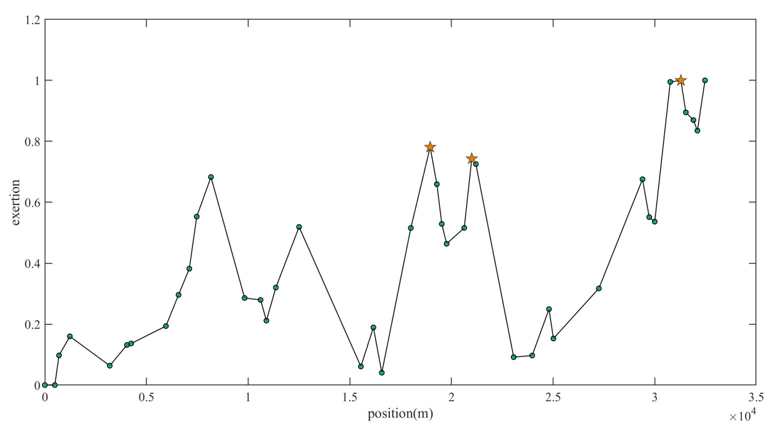

Figure 18: Q5 result

8 Strengths and Weaknesses

8.1 Strengths

- Our models are highly specific, taking into account the characteristics of different types of riders, of different genders.
- The development of our model has a solid theoretical foundation. We have reviewed a large amount of literature, all of which is based on accepted theories for model building. This greatly increases the persuasiveness of our model.
- Our model has an extremely large input data set. When we modeled the tracks in 3D, we sampled nearly 200 points per track using Google Earth.
- Our model has undergone a rigorous mathematical derivation. This makes our results more instructive to reality. And it can help us to better quantify our decisions.
- Our model takes full account of the effects of realistic factors, such as weather and geography. This makes our model very close to reality. Our predicted results are very similar to the results of real riders.
- Our model simplifies the influence of unimportant factors. There are many factors that influence rider decisions in reality. Our model selects the important factors and simplifies the unimportant ones. This gives our model the advantage of both accuracy and simplicity.
- Our models are extremely scalable. Our model is not only applicable to the analysis of individual cycling time trials. The same ideas and models can be used with minor modifications to solve optimization problems such as large group cycling races or car races.
- After sensitivity analysis, our model has good robustness.

8.2 Weaknesses

- Our covariates maintain a constant value during the simulation, while they may change during the real game.
- We only considered the effect of wind on the ride, but there are actually other weather factors that can have an impact on the race.
- For the definition of sprinter, we only have less data and there is no way to guarantee the accuracy of the sprinter definition.

8.3 Promption

Our model currently has only two types of riders. And in order to study the most suitable strategy, we should accumulate data on more types of riders and model them. This way we can analyze the most favorable and convincing strategy. Our model is very computationally intensive, so the computation takes a long time. It is possible to use algorithms with lower time complexity so that we can reduce the computing time and analyze more data.

9 Conclusion

This paper presents a mathematical model to solve the optimal power distribution curve of bicycle racer in time trial. The model can be applied to different types of riders, different wind speed and direction, as well as the track with changing road conditions. Starting with p-V model and P-T model, the mathematical model for solving the optimal power distribution is established step by step. From the physiological point of view, p-T model defines several ability indicators of riders, describes the relationship between indicators, and distinguishes different types of riders. After simplifying the complexity of the problem, the relationship model between power distribution and finish time is established by using p-V kinematics model and differential equation obtained from mechanical analysis. SQP algorithm is introduced to solve the relational model because it conforms to nonlinear programming form in mathematics. It is proved by practical application that the model can give the power distribution curve in accordance with experience when the road section is short and there are many sampling points. For example, increase the power when going uphill, reduce the power when going downhill, decelerate in advance when entering the curve, and maintain the power until the gentle slope. However, a large number of sufficient sampling points need to be added to accurately restore the road information in the long road segment, which leads to the increase of solving complexity of the model. Therefore, it is difficult to give consideration to algorithm complexity and accuracy when using this model to analyze long distance track. In the sensitivity analysis stage, this paper focuses on the analysis of wind speed and wind direction environmental parameters, as well as the influence of power distribution offset on the shortest finish time, the results show that the deviation caused by these parameters interference is still within the acceptable range, indicating that the model has a certain robustness. In the team competition problem, two ways of solving the problem are proposed on the basis of reasonable assumptions of experience, and tested by the model proposed in this paper.

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Get your rider to the top

This rider's race guide is for the Men's Time Trial Specialist category of the Road Cycling Individual Time Trial at the 2021 Tokyo Olympics.

Know Your Rider

The first thing to do is to make sure the rider is a Time Trial Specialist type. To identify a Time Trial Specialist type of rider, you can judge from the following angles.

- **Explosive power**: Time Trial Specialist-type riders are not the most plucky, and their maximum output is about 1373J/s
- **anaerobic capacity**: Time Trial Specialist riders have good anaerobic capacity, which means that they can perform high-intensity exercise for a certain amount of time with little muscle soreness.
- **VO2 max capability**: Time Trial Specialist riders have excellent VO2 max capability, which determines the maximum power that can be provided by aerobic breathing, also known as critical power (CP). time trial Specialist riders last longer and burn more energy than amateurs when training over longer distances.

Of course, if you want to determine the type of rider more accurately, you can have the rider perform a fitness test and construct a power profile of the rider. If the shape of rider's power-time curve is similar to the figure below, then it means he is a Time Trial Specialist type of rider. Of course, it can also be compared directly with the following table.

Familiarize yourself with the track and environment

The course analyzed in this guide is the men's individual road cycling time trial course for the 2021 Tokyo Olympics. The men's individual time trial course for the Tokyo Olympics is 44.2km long and starts at the Fuji International Circuit. Men will complete two laps of the same course with a total elevation gain of 846m. The exact route map is Fig 19

As you can see from the Fig 20, this race has a lot of turns, especially near the start and finish. The presence of a large number of curves increases the uncertainty and danger of the race. So the players must be guided to be familiar with the course in advance. The practice of curves should also be introduced in the daily training.

Table 3: the standard of Time Trial Specialist

type	W/J	CP/w	P_{max} /w
Time Trial Specialist	23658	323	1373

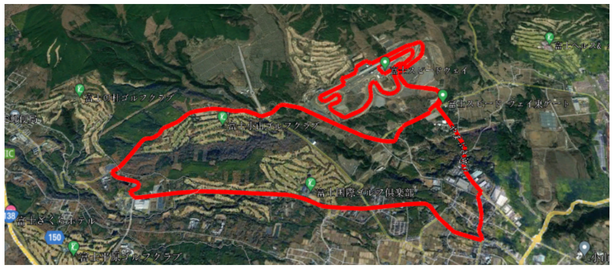

Figure 19: TokyoRoute

If you only know this track from satellite aerial photos, it is easy to ignore another difficult point of this track. It is the gradient. The total elevation of the race will increase by 846m, which can be clearly seen in the picture below. The gradient change of the course will increase the energy consumption of the riders, which is an opportunity for the Time Trial Specialist type with excellent physical strength.

Finally, it is also important to note the weather on the day of the race. The date of the men's race is July 28, 2021. According to our research, the wind will have a significant impact on the outcome of the race. Please be sure to consider this in detail when developing your race strategy.

Scientific Competition Skills

How to face the curve?This is because the rider will be affected by centrifugal force when cornering. The greater the speed, the greater the centrifugal force, the more likely it is that a rollover will occur. To avoid a safety accident, the rider should slow down when facing a corner. But slowing down is obviously not conducive to a good result. So the best strategy is to provide power output as early as possible to get a faster exit speed and to take as straight a track as possible. To achieve an effect of lower entry speed but faster exit speed.

How to face the ramp?The strategy to be adopted in the face of a ramp should be discussed according to the length of the ramp. When it is a short uphill, the speed should be increased very fast. When it is a short downhill, you should recover your strength and reduce the output power to a more appropriate size. When facing a long ramp, either uphill or downhill, one should not quickly increase or decrease one's output power, as it is easy to get tired or slow down. The correct approach is to change one's power tentatively and maintain the range of power change in a smooth range.

How to face strong winds? The body has to lean in the direction the wind is blowing. This is because the wind will provide a moment to the bicycle, and the effect of this moment will make the bicycle tilt easily. Tilting the body counteracts the effect of this moment and reduces the probability of tipping. Riders should reduce their speed when they feel a tire slip. This is because when a tire skids, it can both seriously affect the rider's speed, waste a lot of the rider's energy output, and cause safety problems. So be sure to slow down in time to ensure life safety.

Figure 20: TokyoAltitude

Appendices

```
x_3d = zeros(n,3);
x_3d(:,1) = x_L;
x_3d(:,3) = h;
v0 = 1;
L = 40000;
vmax = 20;
r = 1000*ones(1,n);
rho = 1000*ones(1,n);
miu = 0.0035*ones(1,n);
beta = zeros(1,n);

V0 = 1;
route();
vw = 10;
beta0 = pi/6;
filename = ['wind_', num2str(vw), '_' , num2str(beta0), '.xlsx'];
beta = beta0-theta;
miu = 0.0035*ones(1,n);
miu_s = 0.02*ones(1,n);
vmax = 15;
vlimit = ones(1,n) * vmax;

options=optimoptions(@fmincon,'Algorithm','sqp','MaxFunEvals',100000,'MaxIter',10000,'(
[P,fval] = fmincon('func_P',CP*rand(n,1),[],[],[],[],zeros(1,n),Pm*ones(n,1),'nonlcon_F
fval
v = P2v(P);

figure
plot(x_L, v, 'ok-', 'linewidth', 1.1, 'markerfacecolor', [36, 169, 225]/255)v-x_L
xlabel('position(m)');
ylabel('velocity(m/s)');
set(gca, 'linewidth', 1.1, 'fontsize', 16, 'fontname', 'times')
figure
plot(x_L, P, 'ok-', 'linewidth', 1.1, 'markerfacecolor', [29, 191, 151]/255)
xlabel('position(m)');
ylabel('power(W)');
title(['Power distribution with Total time ',num2str(sum(det_T)), '(s)']);
set(gca, 'linewidth', 1.1, 'fontsize', 16, 'fontname', 'times')

ontname', 'times')

exertion = 0;
E = [];
for i=1:n
    exertion = exertion + (Pm-CP) * (P(i)-CP) / W / (Pm-P(i)) * det_T(i);
    exertion = max(0, exertion);
    E = [E exertion];
end
```