



# Why do we need Federated Learning?

- Machine learning algorithms, especially deep learning algorithms, are data hungry.
- Data are generally spread over different devices with different owners and under the protection of privacy restrictions.
- In practice, we cope with isolated data islands and we cannot transfer data









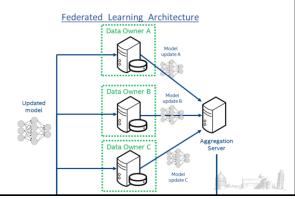
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## Why do we need Federated Learning?

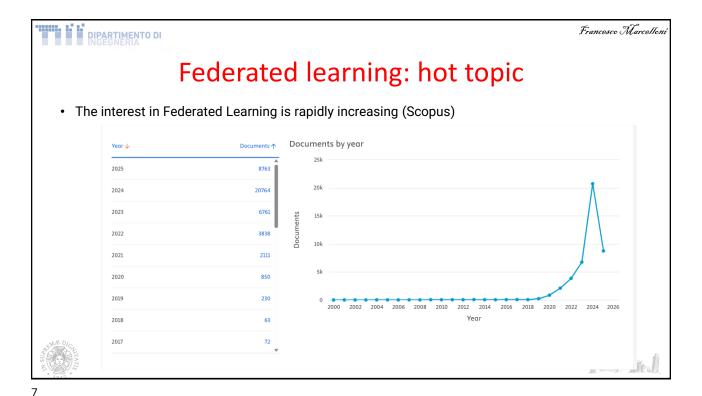
«Federated learning is a machine learning setting where multiple entities (clients) collaborate in solving a Machine learning problem... Each client's raw data is stored locally and not exchanged or transferred; instead, focused updates intended for immediate aggregation are used to achieve the learning objective.»

«Advances and Open Problems in Federated Learning» Foundations and Trends® in Machine Learning, Vol 14

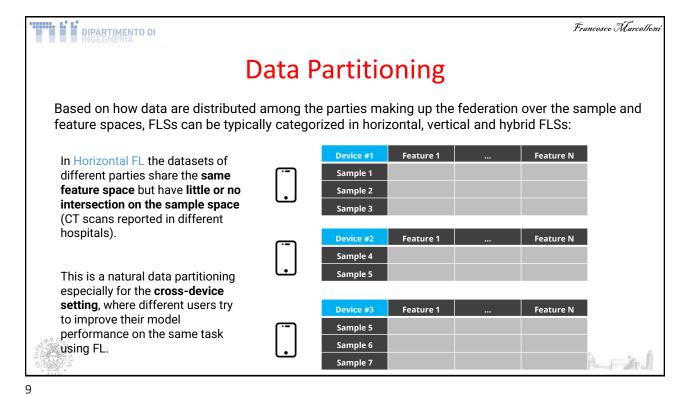
- Federated Learning:
  - to aggregate updates, learned at the edge devices, of the distributed individual versions of the global ML model and to exploit the aggregation for updating the model
  - to broadcast the model to allow edge devices to continuously refine their individual versions







Francesco Marcelloni DIPARTIMENTO DI Federated learning: a taxonomy A Federating Learning System taxonomy according to six main characterizing aspects Federated Learning Systems Privacy Scale of Motivation of ML Model **Data Partitioning** Federation Federation Linear Models Diff. Privacy Centralized Cross-silo Incentive Horizontal Crypto Methods Regulation Vertical **Decision Trees** Non-centralized Cross-device **Neural Networks** Hybrid We are mainly interested in centralized, cross-device FLS. Hence, we will focus on these aspects. Li Q., Wen Z., Wu Z., Hu S., Wang N., Li Y., Liu X., He B.A Survey on Federated Learning Systems: Vision, Hype and Reality for Data Privacy and Protection(2021) IEEE Transactions on Knowledge and Data Engineering



DIPARTIMENTO DI Francesco Marcelloni **Data Partitioning** In Vertical FL the datasets of different parties have the same or similar sample space but differ in the feature space (for instance, municipality registry and hospital data). Feature 3 Feature 4 Feature 5 Device #3 Feature 6 Feature 7 Device #1 Feature 1 Feature 2 Device #2 Sample 1 Sample 1 Sample 1 Sample 2 Sample 2 Sample 2 Sample 3 Sample 3 Sample 3 Sample 4 Sample 4 Sample 4 Sample 5 Sample 5 Sample 5 Sample 6 Sample 6 Sample 6 In Hybrid FL partition of data among the parties may be a hybrid of horizontal partition and vertical partition.



## **ML** Models

There have been many efforts in developing new models or reinventing current models to the federated setting. For the sake of brevity **we briefly cite the widely-used models nowadays**:



Neural Networks: there are many studies on federated stochastic gradient descent which can be
used to train NNs.



Decision tree is another widely used model as it is highly efficient to train compared with NNs. (FLSs studies for Gradient Boosting decision trees - GBDTs have been proposed recently).



SVM: there exist a number of examples in which SVM is successfully trained exploiting a federated stochastic gradient descent algorithm.





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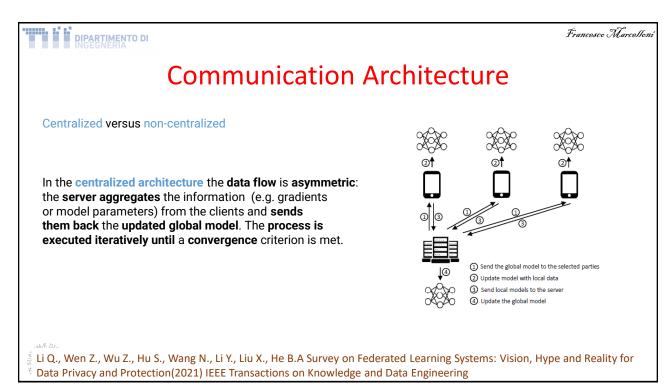
## **Privacy Mechanisms**

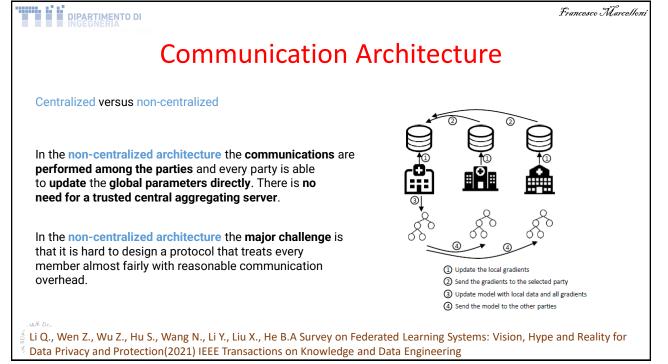
Model parameters exchanged during FL rounds may leak sensitive information about the data. Beyond attacks targeting user privacy, there are also other classes of attacks on federated learning (e.g. an adversary might attempt to bias the model to produce inferences that are preferable to the adversary and much else).

Technology	Main characteristics			
Differential Privacy	Add properly tuned random noise to mask the influence of an individual instance on the output.			
Secure Multi-Party Computation	Enables two or more parties to compute an agreed-upon function of their private inputs in a way that only reveals the intended output to each of the parties, while keeping those inputs private.			
Homomorphic Encryption	Enables parties to perform mathematical operations directly on encrypted data without decrypting them.			
Trusted Execution	TEEs provide the ability to trustably run code on a remote machine, even if you do not trust the machine's owner.			
Environments	TEEs may provide confidentiality, integrity and remote attestation.			











## Scale of Federation

FLSs can be categorized into **two typical types** by the scale of federation: cross-silo FLSs and cross-device FLSs. The **main differences** between them lie on the **number of parties** and the **amount of data stored in each party**.

	Setting	Data Distrubution	Data Availability	Distribution Scale	Primary Bottleneck	Client reliability	Data partition axis
Cross-silo	Training a model on siloed data. Clients are different organizations or geo-ditributed datacenters.	Data is generated locally and remains decentralized. Each client stores its own data and cannot read the data of other clients.	All clients are almost always available.	Typically, 2 – 100 clients.	Might be computation or communication.	Relatively few failures.	Partition is fixed. Could be example- partitioned (horizontal) or feature-partitioned (vertical).
Cross-device	The clients are a very large number of mobile or IoT devices.		Only a fraction of clients are available at any one time, often with diurnal or other variations.	Massively parallel, up to 10 <sup>10</sup> clients.	Communication is often the primary bottleneck. Generally, crossdevice computations use wi-fi or slower connections.	Highly unreliable – 5% or more of the clients participating in a round of computation are expected to fail or drop out.	Fixed partitioning by example (horizontal)

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### **Motivation of Federation**

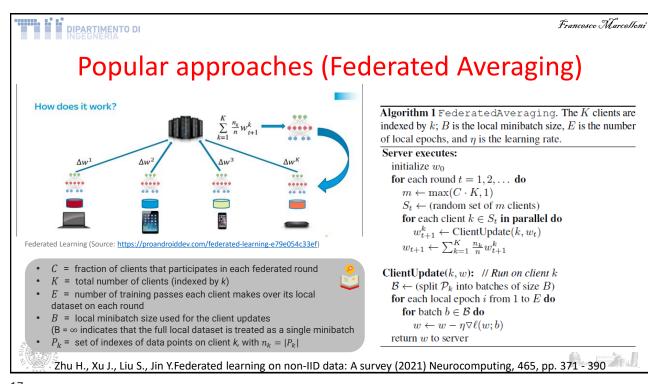
In **real-world applications** of FL, **individual parties need the motivation to get involved** in the FLS. The motivation can be **regulations or incentives**. The parties inside the system can be collaborators as well as competitors.

Take Google Keyboard as an example: Google cannot prevent users who do not provide data from Using Gboard. But those who agree to upload input data may enjoy a higher accuracy of word prediction.











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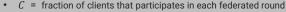
## Popular approaches

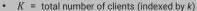
Federated Stochastic Gradient Descent (FedSGD) vs Federated averaging (FedAVG):

In **FedSGD** each client k computes the gradient on its local data at the current model  $w_t$  and the central server aggregates these gradients and updates the global model. Note that FedSGD coincides to FedAvg with

$$C = 1$$
  $B = \infty$   $E = 1$ 

In **FedAVG** each client locally takes one or multiple steps of gradient descent on the current model  $w_t$  using its local data, and the server then takes a weighhed average of the resulting models.





 E = number of training passes each client makes over its local dataset on each round

B = local minibatch size used for the client updates
 (B = ∞ indicates that the full local dataset is treated as a single minibatch

•  $P_k$  = set of indexes of data points on client k, with  $n_k = |P_k|$ 

For the current global model  $w^t$ , the average gradient on its global model is calculated for each client k.

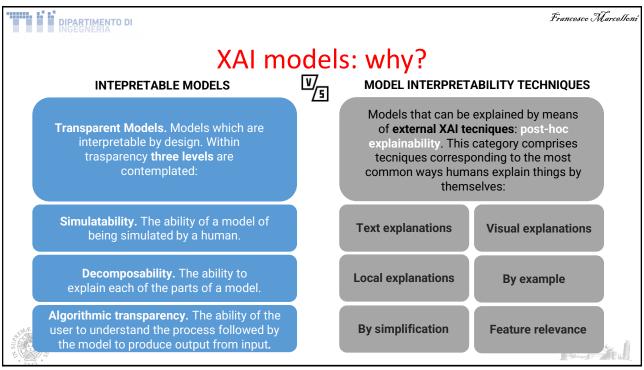
$$F_k(w) = \frac{1}{n_k} \sum_{i \in \mathcal{P}_k} f_i(w)$$

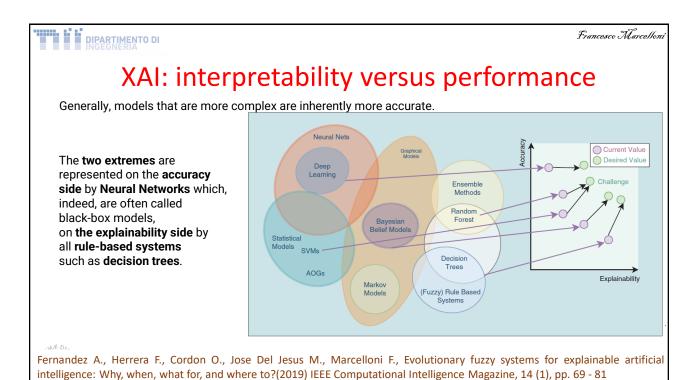
$$g_k = \nabla F_k(w_t)$$

The central server then aggregates these gradients and applies the update.

$$w_{t+1} \leftarrow w_t - \eta \sum_{k=1}^K \frac{n_k}{n} g_k$$

Zhu H., Xu J., Liu S., Jin Y.Federated learning on non-IID data: A survey(2021) Neurocomputing, 465, pp. 371 - 390







### **Federated Decision trees**

```
Algorithm 1 The ICDTA4FL process
 1: Client's side
        for C_i; i=1,\ldots,n do
             C_i \leftarrow \text{train a decision tree}, Local DT_i, \text{ with its local data } D_i.
            Send Local DT_i to the Server.
 5: Server's side
        Send the received trees to the clients.
 7: Client's side
        for C_i; i=1 to n do
            C_i \leftarrow \text{evaluate the local DTs}, C_k Local DT_i, k = 1, \dots, n; i \neq k
             Send the evaluation metrics to the server.
11: Server's side
        Delete the trees that do not surpass a filter selected for the metrics.
12:
13:
        Extract the rules for selected decision trees.
14:
         Aggregate the rules applying the Cartesian product.
         Build a global decision tree, GlobalDT with the aggregated rules.
15:
        Send GlobalDT and the aggregated rules to the clients.
16:
17: Client's side
        for C_i; i=1 to n do
18:
            Evaluate the GlobalDT with its local data D_i
19:
```

A. Argente-Garrido, C. Zuheros, M.V. Luzón, F. Herrera, An interpretable client decision tree aggregation process for federated learning, Information Sciences, Volume 694, 2025

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### **Federated Decision trees**

Client 1 extracted rules:  $x0 \le 32.5 \longrightarrow Class 1; 2 instances; 1A$  $x0 > 32.5 \longrightarrow Class 2; 4 instances; 2A$  if two rules share a condition, i.e., rule 2A, and rule 4B, the less restrictive condition is selected

```
Client 2 extracted rules:

x3>49; x54<=197085; x64<=0.5; x71<=14.5 —> Class 1; 105 instances; 1B

x3>49; x54>197085; x64<=0.5; x71<=14.5 — Class 1; 1 instances; 2B

x0>39; x74>0.5; x3>22; x71<=10.5 —> Class 2; 31 instances; 3B

x0>35; x74>0.5; x3>22; x71>10.5; x51<=0.5 —> Class 2; 29 instances; 4B

x0>35; x74>0.5; x3>22; x71>10.5; x51>0.5 —> Class 1; 3 instances; 5B
```

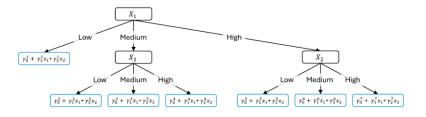
Some aggregated rules x0>32.5; x74>0.5; x3>22; x71<=10.5 Class 2; **2A-3B** x0>32.5; x74>0.5; x3>22; x71>10.5; x51<=0.5 Class 2; **2A-4B** x0>32.5; x74>0.5; x3>22; x71>10.5; x51>0.5 Class 2; **2A-5B** 

The rule 1A has the condition  $x0 \le 32.5$ , while the rule 3B has the condition x0 > 39, making them incompatible.

A. Argente-Garrido, C. Zuheros, M.V. Luzón, F. Herrera, An interpretable client decision tree aggregation process for federated learning, Information Sciences, Volume 694, 2025



### **Federated Decision trees**



The metrics used for determining the variable to be used in the decision node is the variance.

Var(y) = 
$$\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$$
  
where:

$$VarReduction = Var(S) - \left(\frac{n_L}{D}Var(S_L) + \frac{n_R}{D}Var(S_R)\right)$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

José Luis Corcuera Bárcena, Pietro Ducange, Francesco Marcelloni, Alessandro Renda, Increasing trust in Al through privacy preservation and model explainability: Federated Learning of Fuzzy Regression Trees, Information Fusion, Volume 113, 2025,

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## **Federated Decision trees**

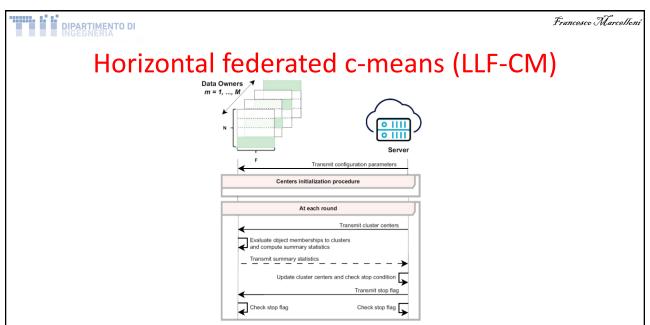
Observation: the variance can be computed as follows

$$Var(y) = \frac{1}{n} \sum y_i^2 - \left(\frac{1}{n} \sum y_i\right)^2$$

For each variable, we compute the first and second sum in each node.

Then, each node transmits the two sums to the server which compute the Var(y) and the VarReduction

José Luis Corcuera Bárcena, Pietro Ducange, Francesco Marcelloni, Alessandro Renda, Increasing trust in Al through privacy preservation and model explainability: Federated Learning of Fuzzy Regression Trees, Information Fusion, Volume 113, 2025,



J. L. C. Bárcena, F. Marcelloni, A. Renda, A. Bechini and P. Ducange, "Federated c -Means and Fuzzy c -Means Clustering Algorithms for Horizontally and Vertically Partitioned Data," in IEEE Transactions on Artificial Intelligence, vol. 5, no. 12, pp. 6426-6441, Dec. 2024, doi: 10.1109/TAI.2024.3426408.

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# Horizontal federated c-means (LLF-CM)

• Let  $\mathbf{q}^{(t),m}=\left[q_1^{(t),m},q_2^{(t),m},\dots,q_{N_m}^{(t),m}\right],\ q_j^{(t),m}\in\{1,\dots,C\}$  be

the vector that indicates the cluster for each object in dataset  $\mathsf{P}^{\mathsf{m}}$ 

• Each data owner P<sup>m</sup> assigns each object in the local dataset to the cluster with the nearest center.







## Horizontal federated c-means (LLF-CM)

### Algorithm 1: Horizontal Federated c-means (LLF-CM).

C: number of clusters,  $\varepsilon > 0$ : tolerance value for the stop condition. T: maximum number of rounds

#### Initialization stage

Server:

- 1:  $stop\_flag = FALSE$
- 2: Initialization procedure for C cluster centers  $\mathbf{V}^{(0)} = \left\{ \mathbf{v}_1^{(0)}, \dots, \mathbf{v}_C^{(0)} \right\}$

**Execution stage** 

3: At each round t, with t starting from 0:

#### Cluster assignment

Transmit  $\mathbf{V}^{(t)}$  to each data owner

Each data owner  $P^m$ : Evaluate  $q_i^{(t),m} \in \{1,\ldots,C\}$  for each object j, as its own cluster assignment according to the nearest center  $\mathbf{v}_c^{(t)}$ 





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## Horizontal federated c-means (LLF-CM)

### Centers update

Each data owner  $P^m$ :

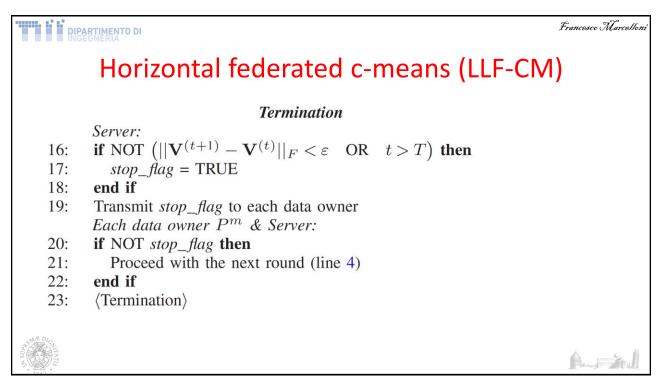
- for each cluster  $\Gamma_c$  do 6:
- $n_c^{(t),m} \leftarrow \text{count of the number of objects in cluster } \Gamma_c$ 7:
- **if**  $n_c^{(t),m} > 1$  **then** 8:
- $\mathbf{L}\mathbf{s}_{c}^{(t),m} = \sum_{\mathbf{x}_{j}^{m} \in \Gamma_{c}} \mathbf{x}_{j}^{m}$ compute  $\mathbf{L}\mathbf{s}_c^{(t),m}$  as per Eq. (1) 9:

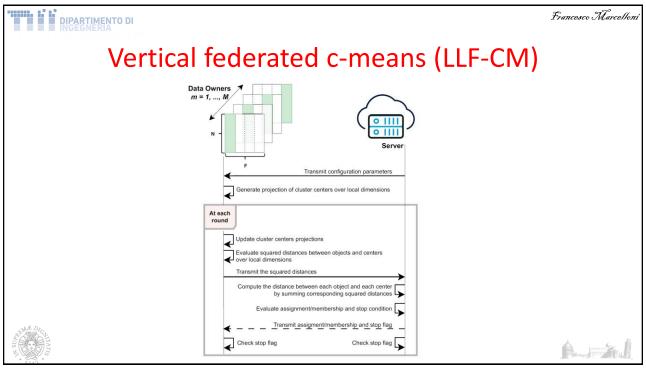
10:

- $(\mathbf{L}\mathbf{s}_c^{(t),m}, n_c^{(t),m}) \leftarrow (\mathbf{0}, 0)$
- 11:
- 12:
- 13: end for
- Transmit to the server all the pairs  $(\mathbf{L}\mathbf{s}_c^{(t),m},n_c^{(t),m})$  calculated above 14:
- Update cluster centers evaluating  $V^{(t+1)}$  as per Eq. (2) 15:



$$\mathbf{v}_{c}^{(t+1)} = \frac{\sum_{m=1}^{M} \mathbf{L} \mathbf{s}_{c}^{(t),m}}{\sum_{m=1}^{M} n_{c}^{(t),m}}, \quad \forall c \in \{1, \dots, C\}$$
 (2)







## Vertical federated c-means (LLF-CM)

#### **Initialization stage**

Server:

- 1: Transmit the number of clusters C to each data owner
- $2: stop\_flag = FALSE$ Each data owner  $P^m$ :
- 3: Randomly generate the projections of the cluster centers over the features defined on  $P^m$ :  $\mathbf{V}_c^{(0),m} = \left\{\mathbf{v}_1^{(0),m},\mathbf{v}_2^{(0),m},\ldots,\mathbf{v}_C^{(0),m}\right\}$





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# Vertical federated c-means (LLF-CM)

### **Execution stage**

4: At each round t, with t starting from 0:

### Centers update

Each data owner  $P^m$ :

- 5:
- 6:

if NOT stop\_flag AND 
$$t > 0$$
 then 5  
Evaluate  $\mathbf{v}_c^{(t),m}$  for each cluster  $\Gamma_c$  
$$\mathbf{v}_c^{(t),m} = \frac{\sum_{\mathbf{x}_j^m \in \Gamma_c} \mathbf{x}_j^m}{n_c^{(t-1)}} \quad \forall c \in \{1, \dots, C\}$$

7: end if







## Vertical federated c-means (LLF-CM)

#### Cluster assignment

Each data owner  $P^m$ :

- 8: Evaluate  $d_{j,c}^{(t),m} = \sum_{f=1}^{F^m} \left( x_{j,f}^m v_{c,f}^{(t),m} \right)^2$  for each object j and cluster  $\Gamma_c$
- 9: Transmit the  $N \times C$  matrix  $\mathbf{D}^{(t),m}$  to the server Server:
- 10: Evaluate  $d_{j,c}^{(t)} = \sqrt{\sum_{m=1}^{M} d_{j,c}^{(t),m}}$  for each object j and cluster  $\Gamma_c$ 11: Evaluate assignment/membership for each object j and cluster  $\Gamma_c$
- 11: Evaluate assignment/membership for each object j and cluster  $\Gamma_c$  i.e.,  $q_j^{(t)}$  and  $n_c^{(t)}$
- 12: **if**  $t \ge 1$  AND  $(||\mathbf{D}^{(t)} \mathbf{D}^{(t-1)}||_F < \varepsilon \text{ OR } t > T)$  **then**
- 13:  $stop\_flag = TRUE$
- 14: **end if**
- 15: Transmit object assignment/membership to clusters and *stop\_flag* to each data owner





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# Vertical federated c-means (LLF-CM)

#### **Termination**

Each data owner  $P^m$  & Server:

- 16: **if** NOT *stop\_flag* **then**
- 17: Proceed with the next round (line 5)
- 18: **end if**
- 19: (Termination)



