POLITECNICO DI MILANO

A. MOOC of Economics: Additional slides

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A.1. THEORY OF PRODUCTION AND COSTS

A.1.1. Demand elasticity

DEMAND AND ITS ELASTICITY (1)

The demand curve provides the **demanded quantity** when **price varies**

•**Demand law**: the demand curve has a **negative slope**

•**Direct demand curve**: given the price, how many units are sold at that price?

•**Inverse demand curve**: given the quantity, what is the price at which that quantity is sold?

Key issue: which is the % **decreases** in demand when a % **increase** in **price** occurs?

•If the decrease in demand is (very) limited, an increase in price leads to an increase in firm's revenues

DEMAND AND ITS ELASTICITY (2)

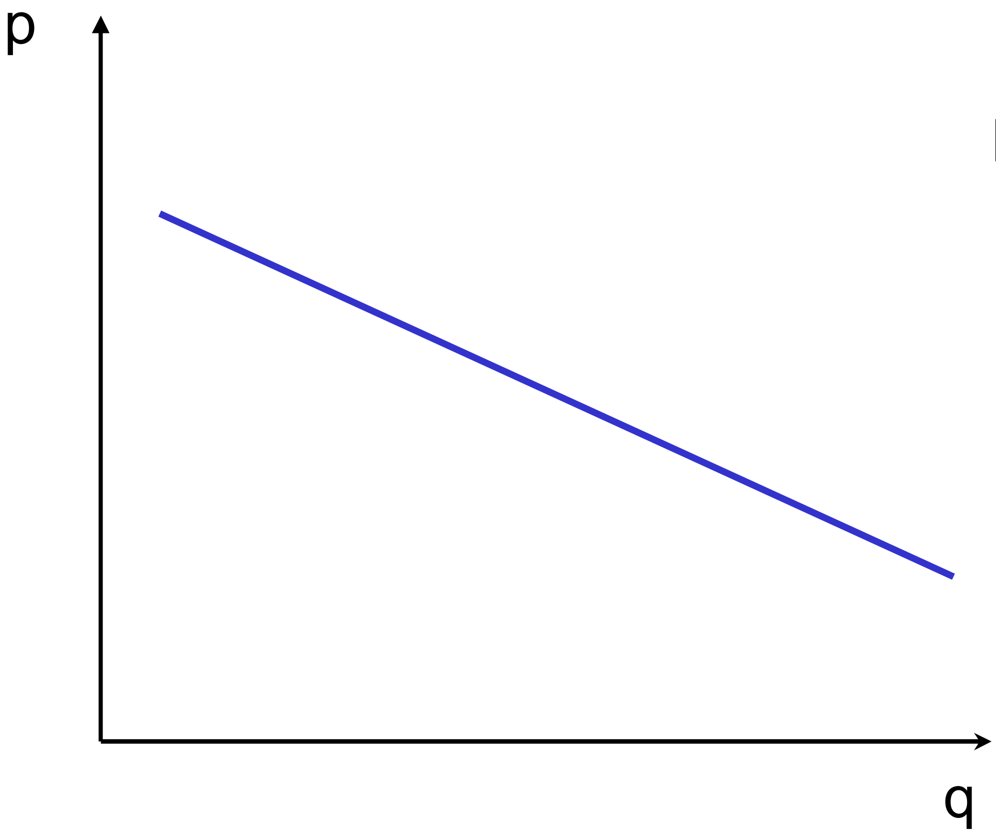
Demand elasticity: % variation of quantity (q) / % variation of price (p)

$$\varepsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \Rightarrow \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} \Rightarrow \frac{\partial q}{\partial p} \cdot \frac{p}{q}$$

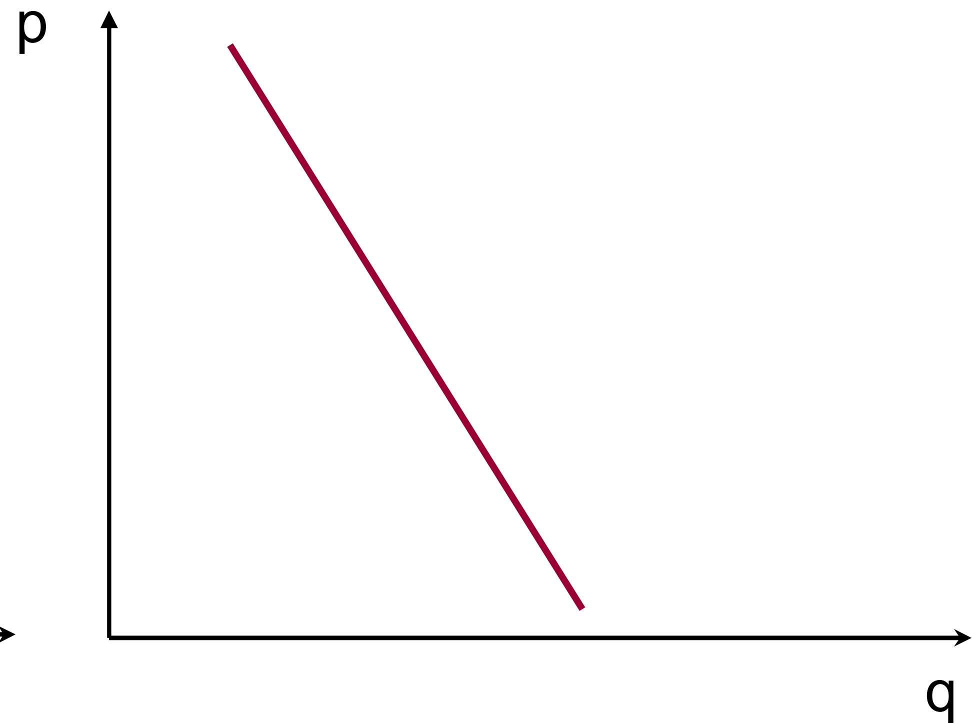
1. **Unelastic demand:** $|\varepsilon| < 1$
- **Low demand elasticity**: an **increase** in **price** causes a **limited reduction** of **demand**
- **Steep** demand curve

2. **Elastic demand:** $|\varepsilon| > 1$
- **High demand elasticity**: an **increase** in **price** causes a **remarkable reduction** of **demand**
  - **Flat** demand curve

ELASTIC AND UNELASTIC DEMAND CURVES

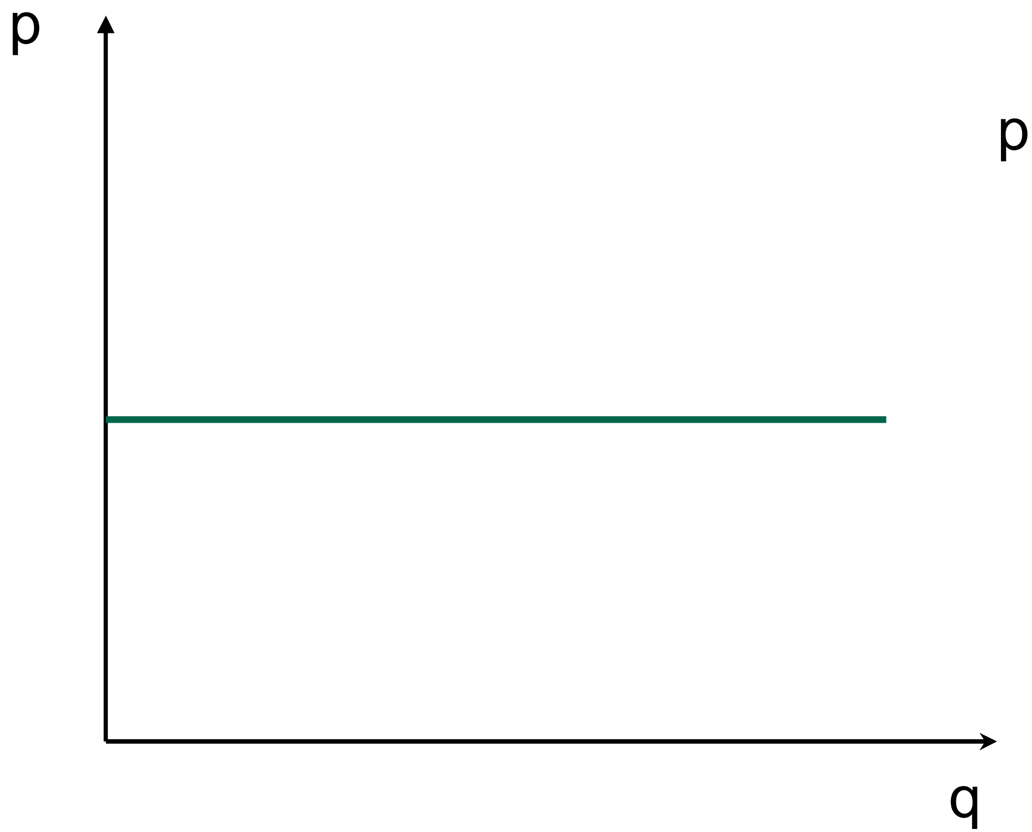


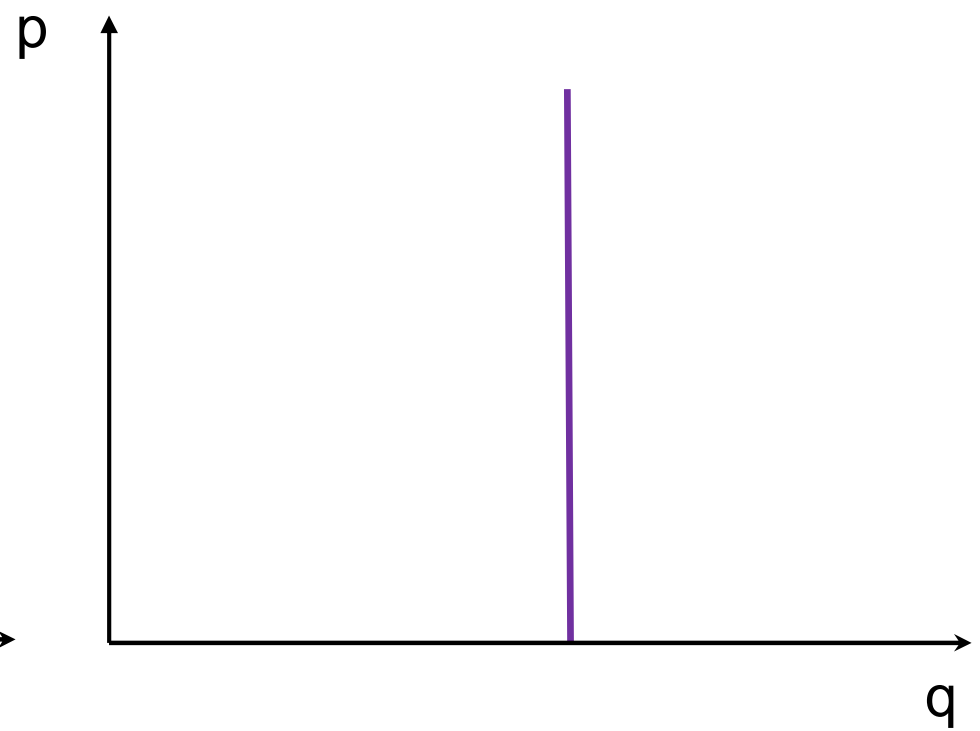
Elastic demand curve



Unelastic demand curve

Perfectly Elastic and perfectly unelastic demand curves





Perfectly elastic demand curve: $|\varepsilon| \to \infty$

Perfectly unelastic demand curve $|\varepsilon| = 0$

Reducing demand elasticity: Product differentiation

Firms aim to reduce the **demand elasticity** of their products so as increases in prices lead to increases in revenues

It is possible to **reduce** the **demand elasticity** of a product by **differentiating** it from **other products** (of competitors and/or of the firm)

Product differentiation is

- Conducive to **market power**: it allows setting **high prices** without losing **customers**
- A **widespread** competitive strategy
  - **Real differentiation:** by changing the real characteristics of the product, for instance through innovation
  - **Perceived differentiation:** by changing consumers' perception of the characteristics of the product, for instance through advertising

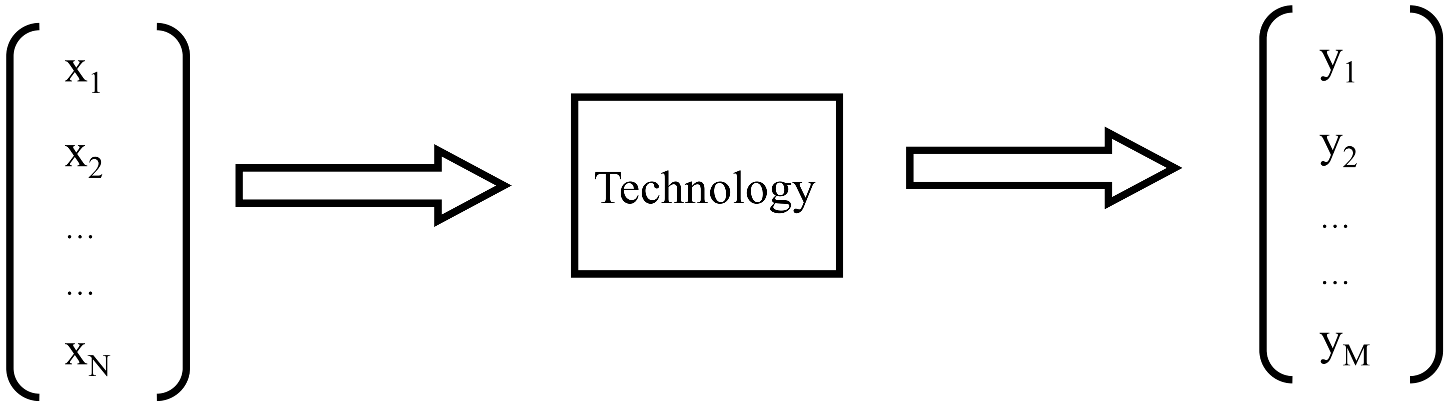
A.1.2. Technology

TECHNOLOGY

When choosing quantities to produce and prices to charge, firms face **constraints**

- Customers (demand), competitors (market structure), nature of the production process (**technology**)

- **Technology**: the set of processes that a firm can use to turn a vector X of **N inputs** into a vector Y of **M outputs**



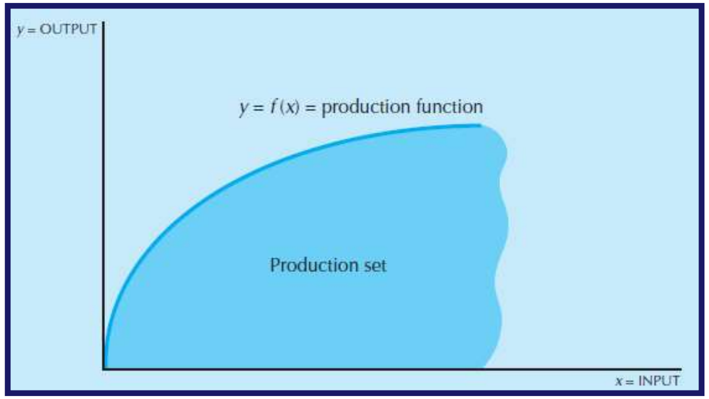
- **Production set**: the set of all combinations of inputs and outputs that comprise a technologically feasible way to produce
- **Production function**: the maximum possible outputs from a given vector of inputs. It is the **boundary** of the **production set**

PRODUCTION FUNCTION AND ISOQUANTS

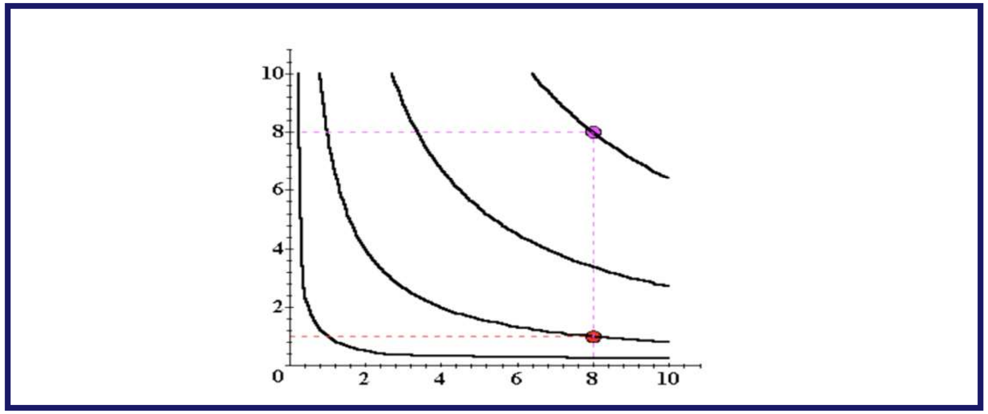
1. If there is 1 input x and 1 output y, the production function is **y=f(x)**

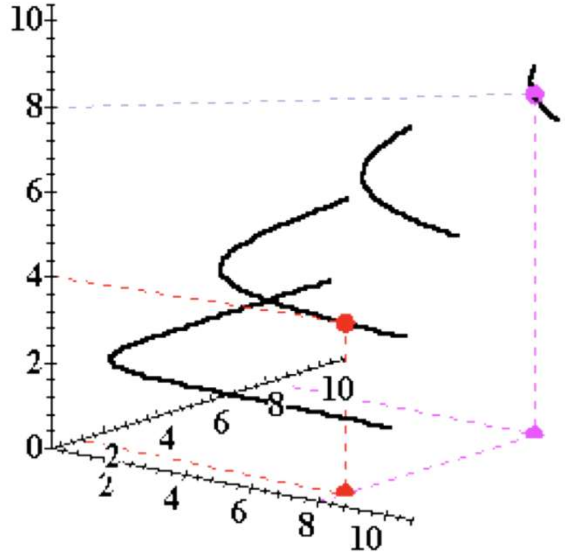
2. With 2 inputs, x_1 and x_2, and 1 output y, the production function is **$y=f(x_1, x_2)$**

- **Isoquant**: set of all input bundles that produce the same output *y*



- Isoquants can be graphed both in a $x_1 0\, x_2$ plan and in 3D

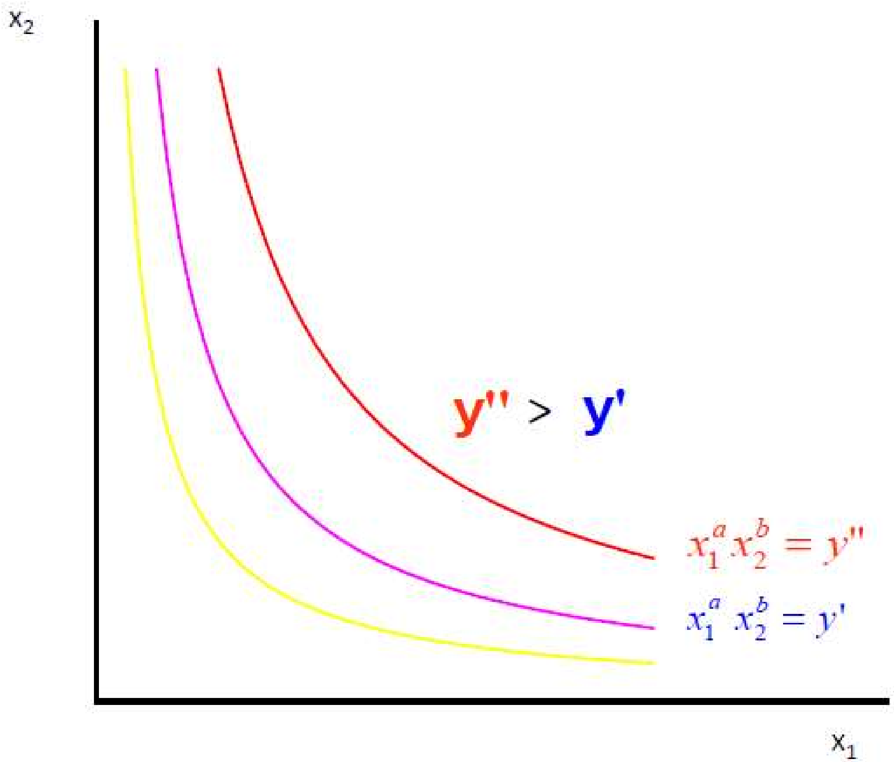




Relevant production functions: Cobb-Douglas Function

$$\boxed{y = f(x_1, x_2) = A x_1^a x_2^b}$$

- *A* measures the **scale of production**, i.e., the output obtained by using 1 unit of each input
  - In the simplest case, A=1
- *a* and *b* measure how the output responds to changes in inputs
- Many production processes are described by Cobb-Douglas functions



Isoquants are hyperbolic, asymptotic to the two axes

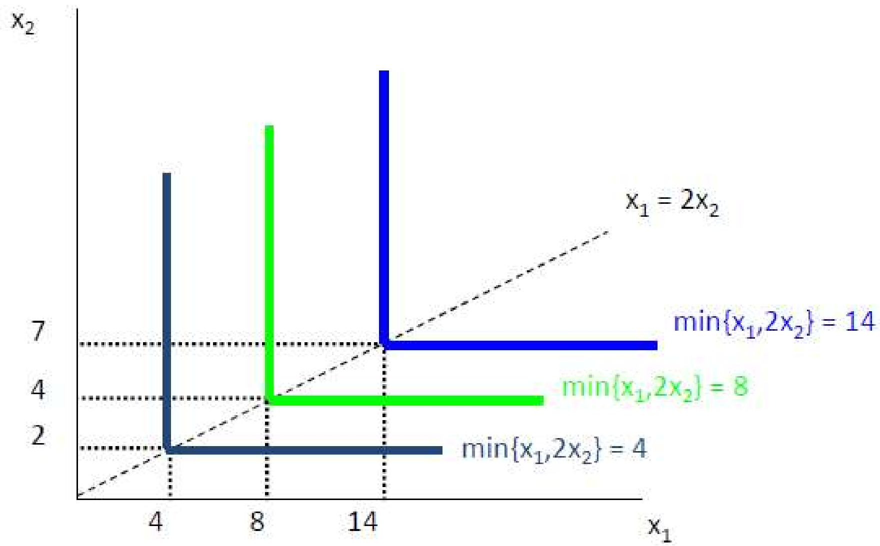
RELEVANT PRODUCTION FUNCTIONS: LEONTIEF FUNCTION (1)

$$\boxed{y = f(x_1, x_2) = \min\{x_1, x_2\}}$$

- To produce the output y, inputs must be used in **fixed proportions**
  - No **substitutability** between inputs

- Example: to produce 4 units of output, the firm needs at least 4 units of input 1 and 2 units of input 2

$$\boxed{y = \min\{x_1, 2x_2\}}$$



Isoquants are L-shaped

RELEVANT PRODUCTION FUNCTIONS: LEONTIEF FUNCTION (2)

+

=

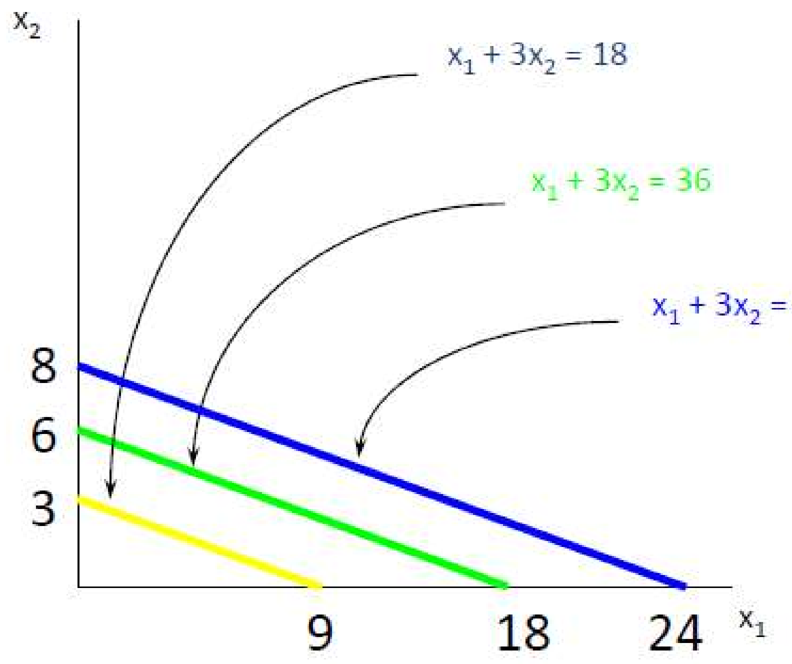
Shovel **Workman** **Hole**

RELEVANT PRODUCTION FUNCTIONS: SUBSTITUTE INPUTS

$$\boxed{y = f(x_1, x_2) = ax_1 + bx_2}$$

- One input can **replace** the other one **in a constant proportion**
  - In the simplest case: *a=b=1*

- Example
  - 1 hamburger can be made by 2 kinds of meat
  - What matters is the final weight (e.g., 100 g)
  - Any combination of the 2 meats produces the hamburger

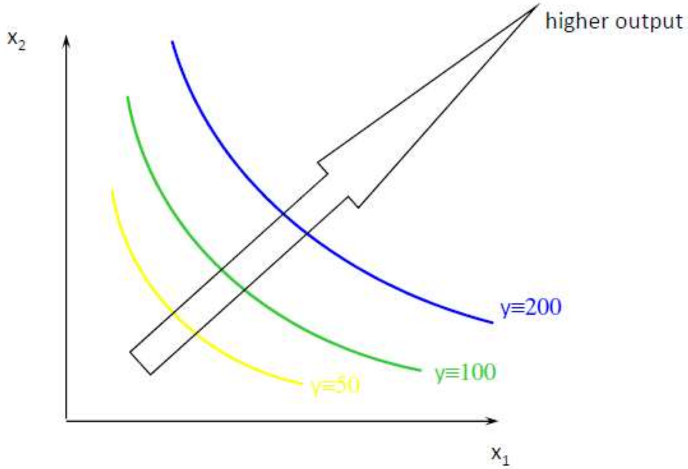


All isoquants are linear and parallel (angular coefficient: *-a/b*)

Some Properties of Well-Behaved Technologies

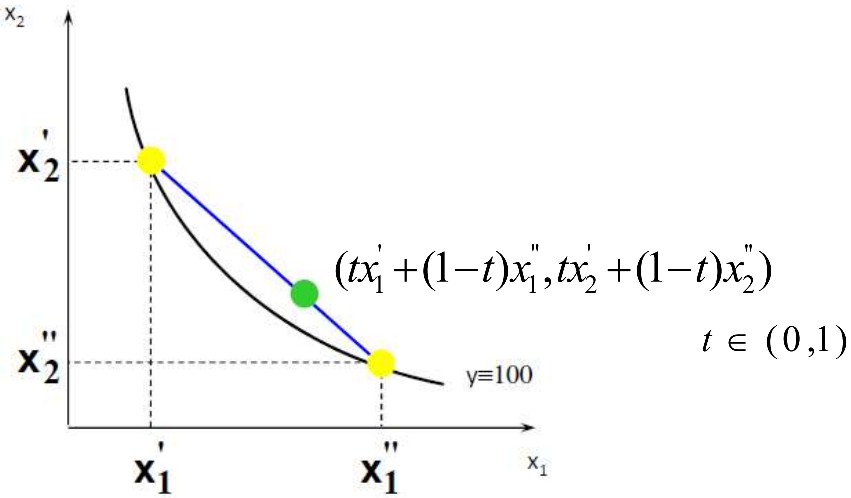
Monotonicity

Production increases if one input increases, while the other input remains constant



Convexity

If two bundles of inputs, (x_1', x_2') and (x_1'', x_2'') produce y units of output, then their weighted average produces at least y units of output



Marginal Product (MP)

The **marginal product (MP)** of an input is the **output variation** due to the variation of **1 unit of this input**, holding **all other inputs constant**

In case of a two-input technology

- The marginal product of **input 1** is $$MP_1 = \lim_{\Delta x_1 \to 0} \frac{f(x_1+\Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1} = \frac{\partial f(x_1, x_2)}{\partial x_1}$$
- The marginal product of **input 2** is $$MP_2 = \lim_{\Delta x_2 \to 0} \frac{f(x_1, x_2+\Delta x_2) - f(x_1, x_2)}{\Delta x_2} = \frac{\partial f(x_1, x_2)}{\partial x_2}$$

- MPs are **positive** because of the monotonicity
- **Law of diminishing marginal product** (common feature of many production processes): the marginal product of an input decreases as the level of the input increases, holding other inputs constant. In general

$$\frac{\partial MP_i}{\partial x_i} = \frac{\partial^2 f(x_i, x_j)}{\partial x_i^{\,2}} < 0$$

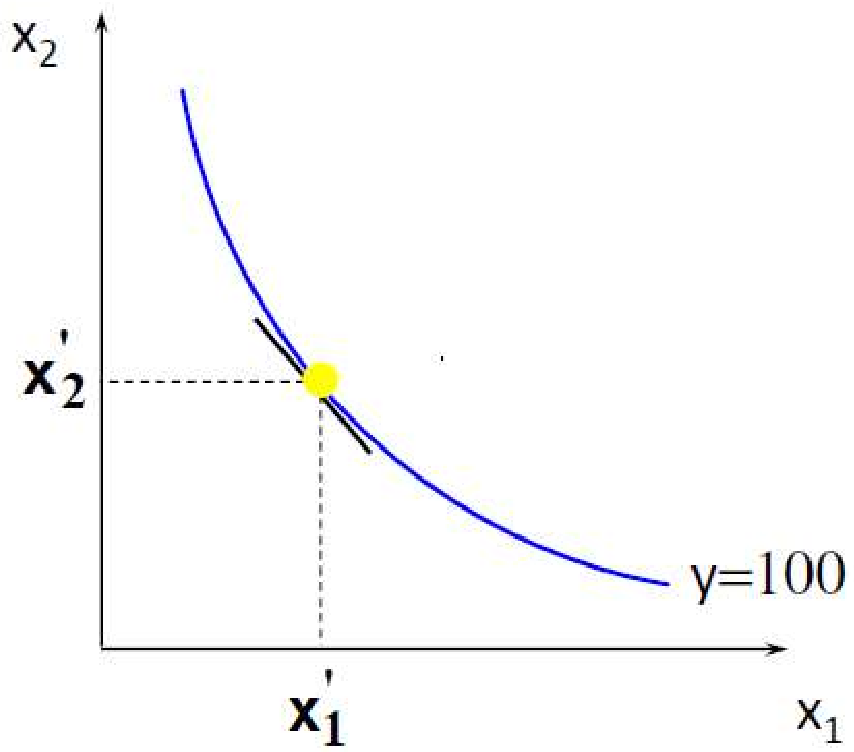
TECHNICAL RATE OF SUBSTITUTION (TRS)

TRS measures the **trade-off between the two inputs**

- It is the rate at which input 2 can be substituted with input 1 to keep the **output level constant**
  - Graphically, it is the **slope** of the isoquant
- Assume a small change (dx_1, dx_2) in the input bundle that keeps output constant

$$\boxed{\begin{aligned} dy &= \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 \\ &= MP_1 dx_1 + MP_2 dx_2 = 0 \end{aligned}}$$

$$\boxed{\Rightarrow = \frac{dx_2}{dx_1} = \left| TRS(x_1, x_2) \right| = \left| \frac{MP_1}{MP_2} \right|}$$



•Assumption of **diminishing TRS**: the slope of the isoquant decreases (in absolute value) when moving to the right along the isoquant

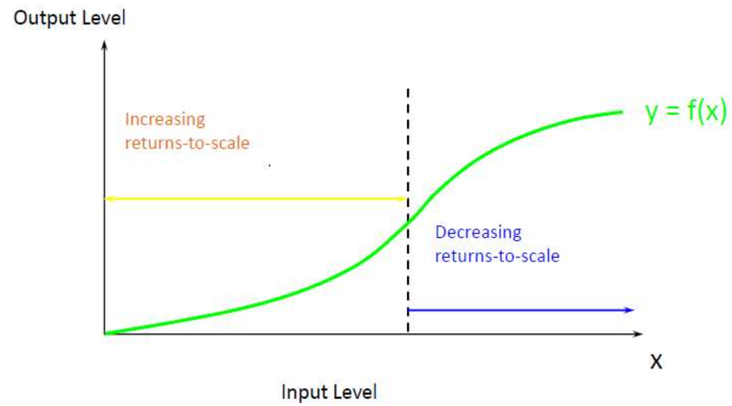
RETURNS TO SCALE

- Marginal products describe the change in output as **one input** changes
- **Returns to scale** describe how output changes as **all inputs change** in the **same proportion** (e.g., all inputs double, or halve) → It is a long run concept

If by scaling all inputs by the amount *t>0,* we obtain

1. $\boxed{f(tx_1, tx_2) = tf(x_1, x_2)}$ we have **constant** returns to scale
2. $\boxed{f(tx_1, tx_2) > tf(x_1, x_2)}$ we have **increasing** returns to scale
3. $\boxed{f(tx_1, tx_2) < tf(x_1, x_2)}$ we have **decreasing** returns to scale

Returns to scale may vary along the production function



A.1.3. Profit maximization and cost minimization

PROFIT MAXIMIZATION IN THE SHORT RUN

- In the short run, input 2 is **constant**
- With p the output price and ϖ_1 and ϖ_2 the input prices, the **profit maximization problem** of the firm is

$$\boxed{\max_{x_1} \; pf(x_1, \bar{x}_2) - \omega_1 x_1 - \omega_2 \bar{x}_2}$$

- The firm maximizes profit by choosing the level of input 1 (and producing the level of output) at which **the marginal product of input 1 equals its price**

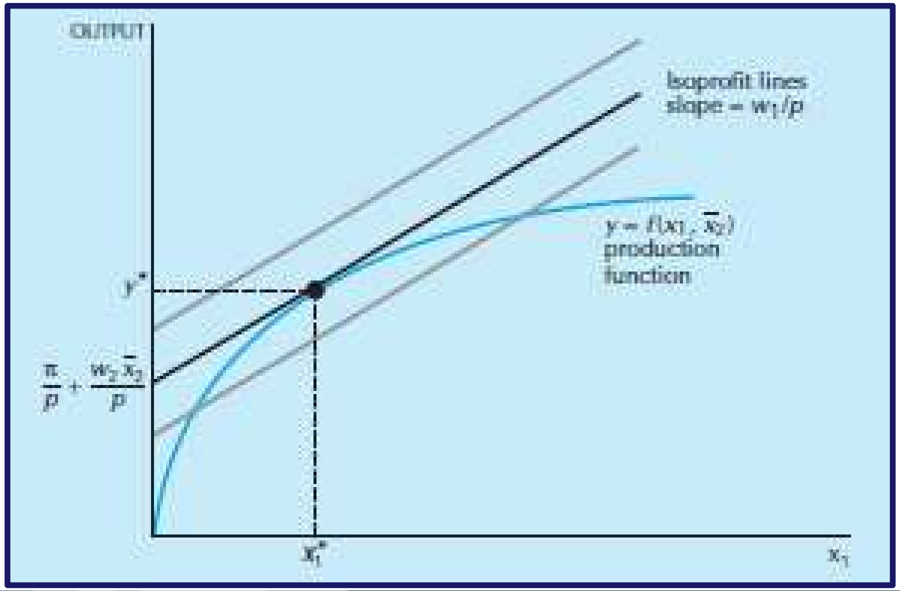
$$\boxed{pMP_1(x_1^*, \bar{x}_2) = \omega_1}$$

- Considering the isoprofit lines, we can solve the problem **graphically**

$$\pi = py - \omega_1 x_1 - \omega_2 \bar{x}_2$$

$$y = \frac{\pi}{p} + \frac{\omega_2}{p}\bar{x}_2 + \frac{\omega_1}{p} x_1$$

$$MP_1 = \frac{\omega_1}{p}$$



Profit Maximization in the Long Run

- In the long run, the firm can choose the level of all inputs, thus the **profit maximization problem** is

$$\boxed{\max_{x_1,x_2} \; pf(x_1,x_2) - \omega_1 x_1 - \omega_2 x_2}$$

- Analogously to the short run case, the firm maximizes profits by choosing the level of inputs (and producing the level of output) at which

$$\boxed{\begin{array}{l} pMP_1(x_1^*,x_2^*) = \omega_1 \\ pMP_2(x_1^*,x_2^*) = \omega_2 \end{array}}$$

- **Factor demand curve**: describes the quantity of **each input**, which maximizes profits as a function of **its price** (given the price of the output and of the other input)
- **Inverse factor demand curve**: describes the same relationship from a different angle; it gives the **input price** at which a **firm demands** a given **quantity of input** (given the price of the output and of the other input)

Cost Minimization in the Long Run

Key question: how to minimize the costs of producing a certain output *y*?

The **cost minimization problem** can be written as

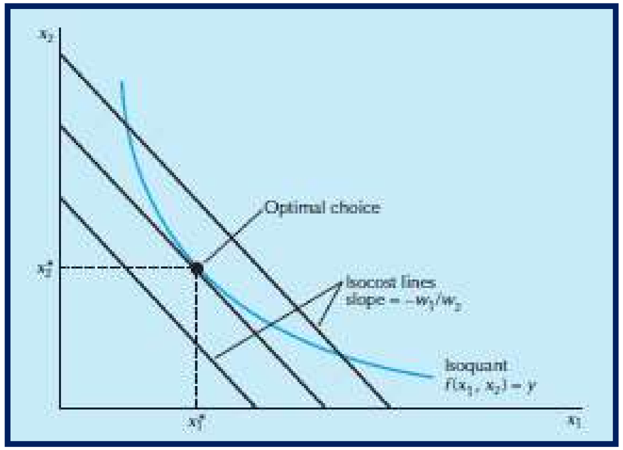
$$\min_{x_1, x_2} \omega_1 x_1 + \omega_2 x_2$$
$$such\ that\ f(x_1, x_2) = y$$

- The solution gives the **cost function**: **minimum cost** of producing *y*, **given input prices** $c(\omega_1, \omega_2, y)$

We obtain the solution **graphically**, by considering **isoquants** and **isocosts**, i.e., set of all the input bundles having the same cost *C*

$$\omega_1 x_1 + \omega_2 x_2 = C \Rightarrow x_2 = \frac{C}{\omega_2} - \frac{\omega_1}{\omega_2} x_1$$

- The solution is such that



$$\left| TRS(x_1^*, x_2^*) \right| = \frac{\omega_1}{\omega_2}$$

- **Conditional factor demand function:** describes the quantity of **each input**, which minimizes the cost as a function of **its price** (given y and the price of the other input)

0.1.4. Cost functions

COST FUNCTIONS: TAXONOMY

In the **short run**, we have

- Total cost function: C(y)
- Fixed cost function: F
- Variable cost function: VC(y)
- Average total cost function: ATC(y)
- Average variable cost function: AVC(y)
- Average fixed cost function: AFC(y)
- Marginal cost function: MC(y)

In the **long run**, we have

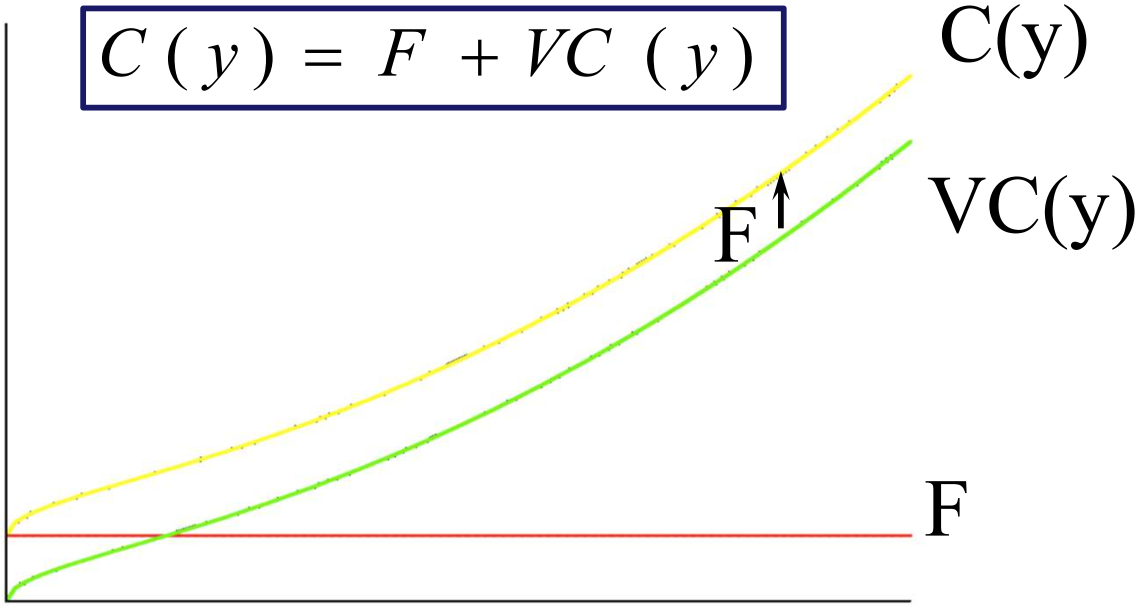
- Average cost function: AC(y)
- Long-run marginal cost function: LMC(y)

- How do these cost functions* relate to each other?
- How do long-run and short-run cost functions relate?

* Cost functions are also referred as **cost curves** or simply **costs**

SHORT RUN: TOTAL, VARIABLE, AND FIXED COSTS

- **Total cost function, C(y)**: is the minimum cost of all inputs (fixed and variable) when producing y units of output: ***C(y)= F+VC(y)***
- **Fixed cost function, F**: is the cost of inputs, which are fixed in the short run. F does not vary with the firm's output
- **Variable cost function, VC(y)**: is the cost of variable inputs when producing y units of output. It varies with the firm's output and depends on the level of the fixed input



SHORT RUN: AVERAGE COSTS

- **Average total cost function, AC(y)**: is the **total cost of each unit** of output

$$\boxed{AC(y) = \frac{C(y)}{y} = \frac{F}{y} + \frac{VC(y)}{y} = AFC(y) + AVC(y)}$$

- **Average variable cost function, AVC(y)**: is the **variable cost of each unit** of output. If the **MP** is
  - **Increasing**, VC(y) increases at a decreasing rate as y increases: **AVC(y) is decreasing**
  - **Decreasing**, VC(y) increases at an increasing rate as y increases: **AVC(y) is increasing**
  - Usually, along the production function, the MP is first increasing and then decreasing → Thus, **AVC(y) is U-shaped**

- **Average fixed cost function, AFC(y):** is the **fixed cost of each unit** of output. It **decreases** as output increases

AC(y)=AVC(y) +AFC(y). Thus, if AVC(y) is U-shaped, AC(y) is U-shaped

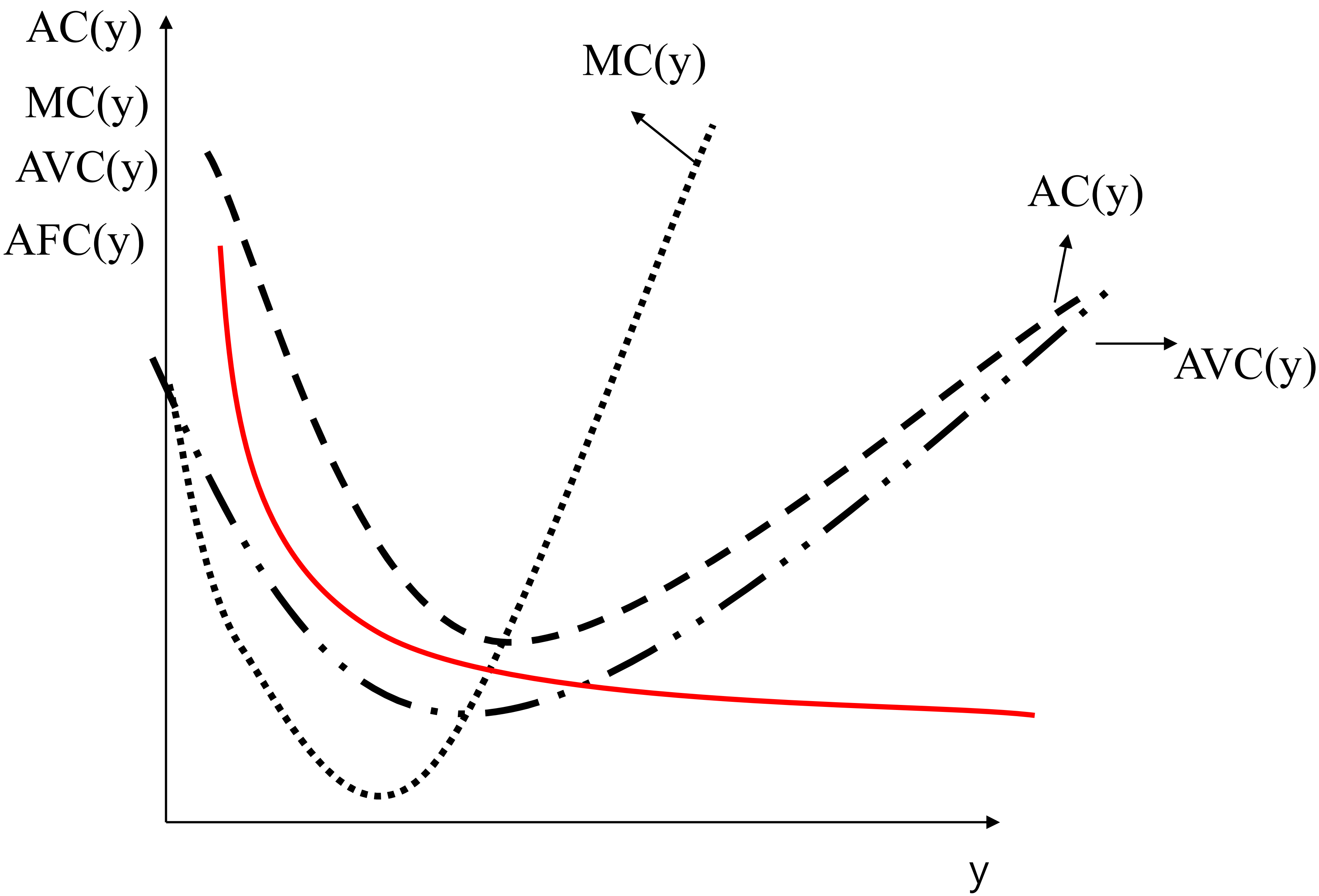
Short run: Marginal Costs

- **Marginal cost function, MC(y)**: is the change in costs associate to a unitary change in output
- Considering infinitesimal changes and noting that fixed cost *F* does not vary with the output level *y*, we have

$$\boxed{MC(y) = \frac{\partial C(y)}{\partial y} = \frac{\partial VC(y)}{\partial y}}$$

- In other words, MC(y) is the derivative of variable and total cost. It gives the **slope** of the C(y) and VC(y) curves as output changes

AVERAGE AND MARGINAL COST FUNCTIONS: GRAPHICAL REPRESENTATION



LONG-RUN: TOTAL COST CURVES

Suppose that there are 3 possible levels of input 2

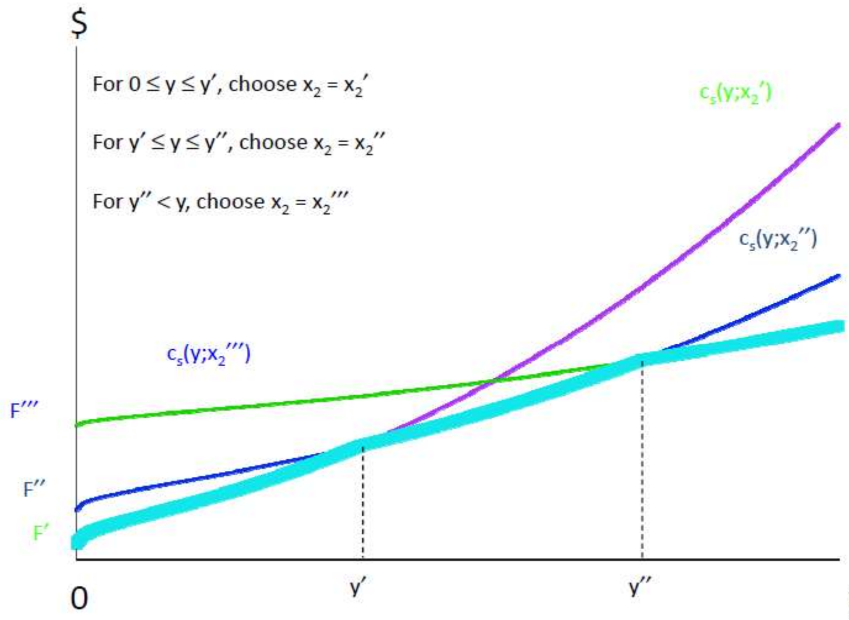
- $x_2 = x_2'$, $x_2 = x_2''$, $x_2 = x_2'''$ with $x_2' < x_2'' < x_2'''$

In the short run, the firm can choose **just one of them** and has a **different short-run total cost curve** for each possible level of input 2 (and thus of fixed costs)

- A larger amount of the fixed input increases the fixed cost (F'<F''<F''')

In the long-run, the firm can choose among the 3 levels of input 2, depending on the quantity *y* to be produced. The result is the long-run total cost curve (light blue line)

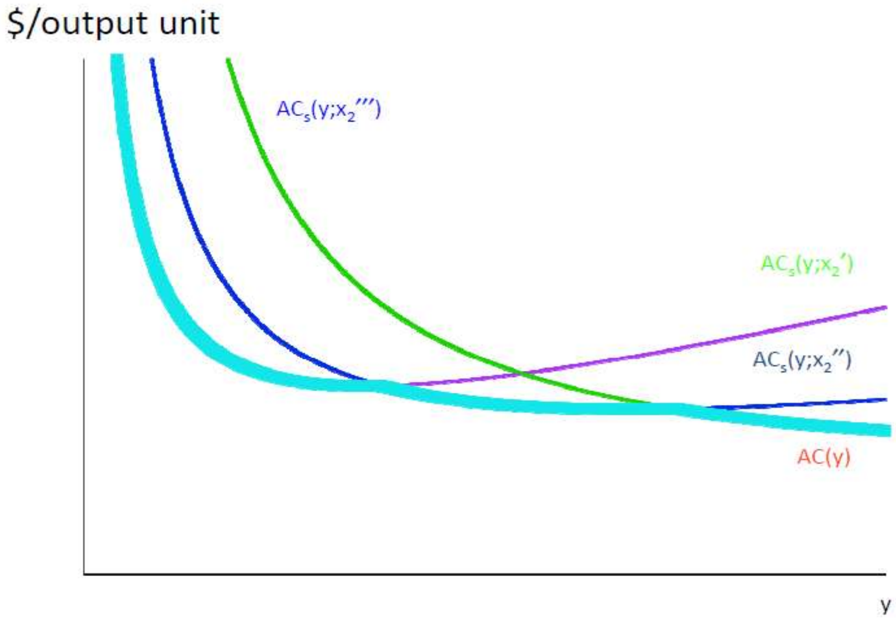
The firm's **long-run total cost curve** is the **lower envelope** of the short-run total cost curves

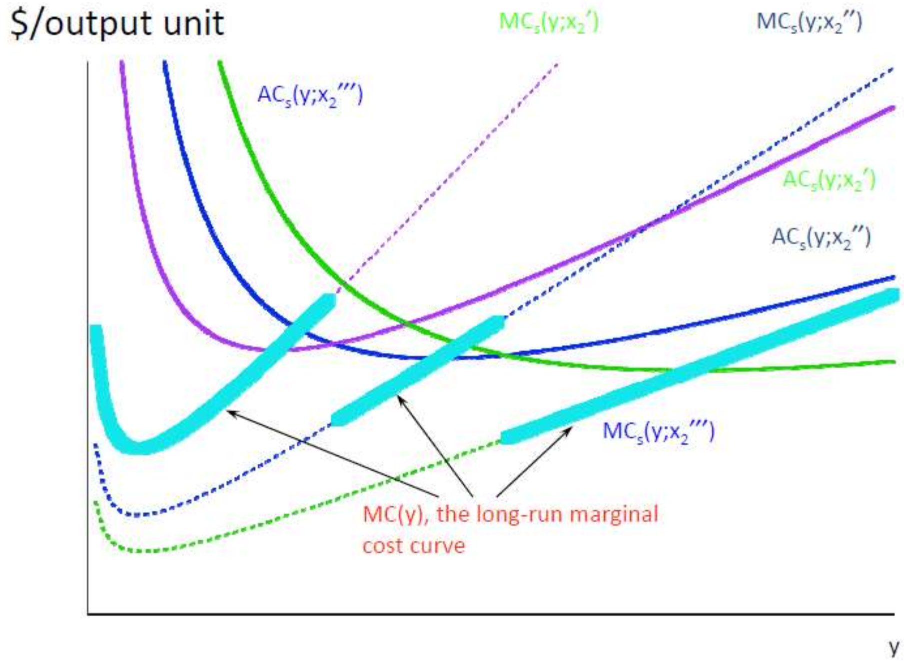


LONG-RUN: AVERAGE AND MARGINAL COST CURVES (1)

For **any output level *y***, the long-run total cost curve always gives the **lowest possible total production cost**

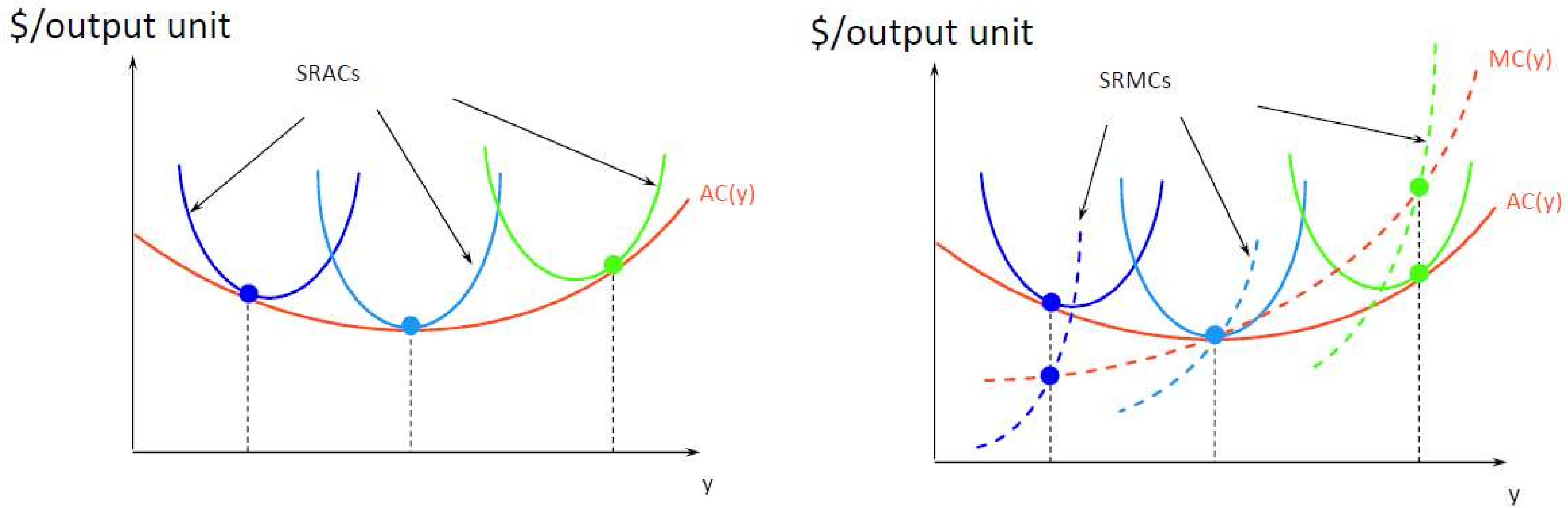
- The **long-run average total cost curve** must always give the **lowest possible average total cost** → The **long-run average total cost curve** is the **lower envelope** of all the short-run average total cost curves
- The **long-run marginal cost curve** must always give the **lowest possible marginal cost** → The **long-run marginal cost curve** is the **lower envelope** of all of the short-run marginal cost curves





LONG-RUN: AVERAGE AND MARGINAL COST CURVES (2)

If we assume that the input, which is **fixed** in the **short run**, can vary in the long-run in a continuous way, we have



A.2. BASICS OF MARKET STRUCTURES

INTRODUCTION

It is possible to define different **market structures** depending on

- The **number of firms** operating in a market
- The **ways in which these firms interact** when they make their pricing and output decisions

- **Perfect competition**
- Monopoly
- Monopolistic competition
- Oligopoly

$\left.\begin{array}{l}\text{Monopoly}\\\text{Monopolistic competition}\\\text{Oligopoly}\end{array}\right\}$ **Imperfect competition**

0.2.1. Perfect competition

Perfect Competition: Main Hypotheses

In perfect competition

1. There are **many (infinite) firms on the market**, $N \rightarrow \infty$, where N is the number of firms

2. Firms produce **a homogeneous good**, having access to the same technology and thus having the same cost curves

In consequence of **1** and **2**

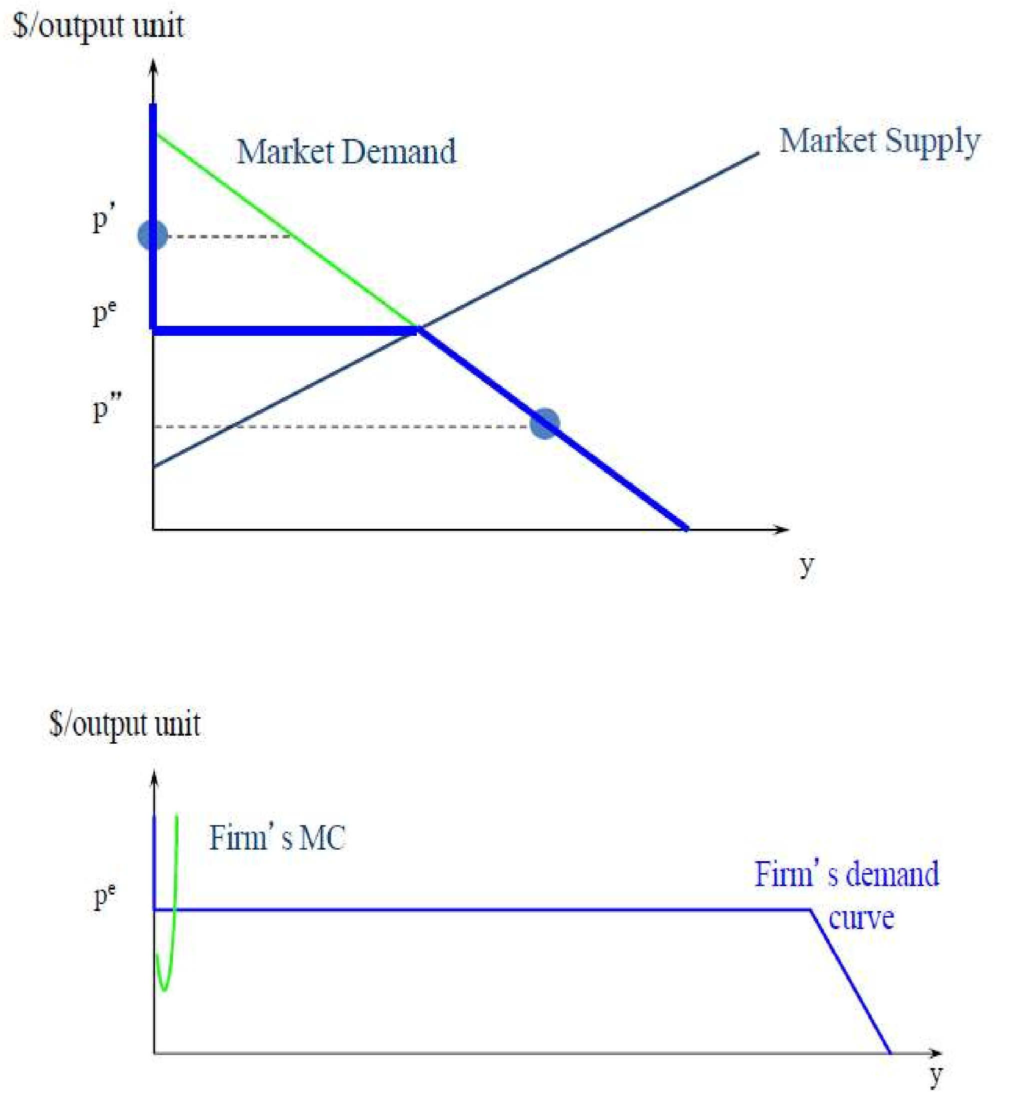
3. Firms have **no influence** over the market price → They are **price-takers**

4. There are **free entry and exit**

5. Information is **perfect**

THE DEMAND CURVE OF A FIRM IN PERFECT COMPETITION

- If $p' > p^e$, the firm has no demand
- At the equilibrium price p^e , market demand is equal to market supply
- If $p'' < p^e$, the firm faces the entire market demand
  - But it cannot serve it due to its very limited **productive capacity**
  - **Atomistic hypothesis**: the firm's technology allows it to **supply just a small part of the market demand**



SHORT RUN: PROFIT MAXIMIZATION PROBLEM

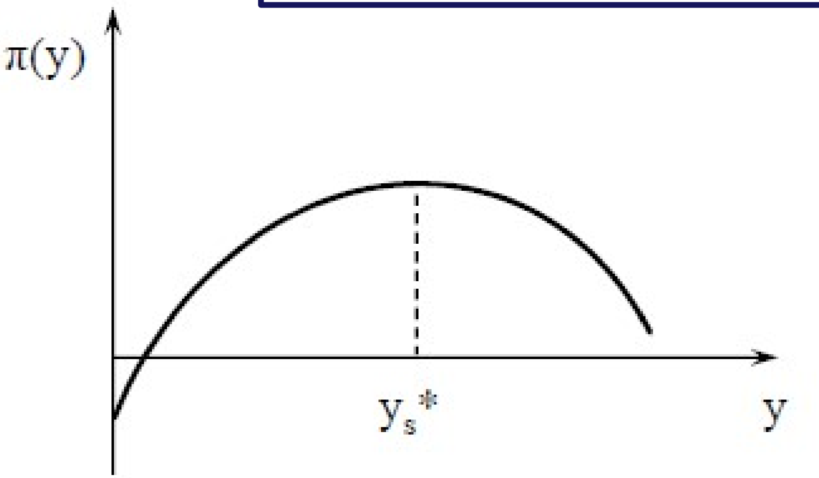
- Given p (price-taking assumption), each firm offers the **quantity** that **maximizes** its **profit**

$$\boxed{\max_{y \geq 0} \Pi(y) = R(y) - C(y) = py - C(y)}$$

- **First** and **second order** conditions for profit maximisation

$$(i) \quad \frac{\partial \Pi(y)}{\partial y} = p - MC(y) = 0 \quad \Rightarrow \quad \boxed{p = MC(y^*)}$$

$$(ii) \quad \frac{\partial^2 \Pi(y)}{\partial y^2} = \frac{d}{dy}\big(p - MC(y)\big) = -\frac{dMC(y)}{dy} < 0 \quad at\ y = y_s^* \quad \Rightarrow \quad \boxed{\frac{dMC(y^*)}{dy} > 0}$$



The firm offers the quantity at which the **marginal revenue** of producing that quantity equals the **marginal costs**

- **In perfect competition, marginal revenue is equal to price**

SHORT RUN: THE SHUT DOWN

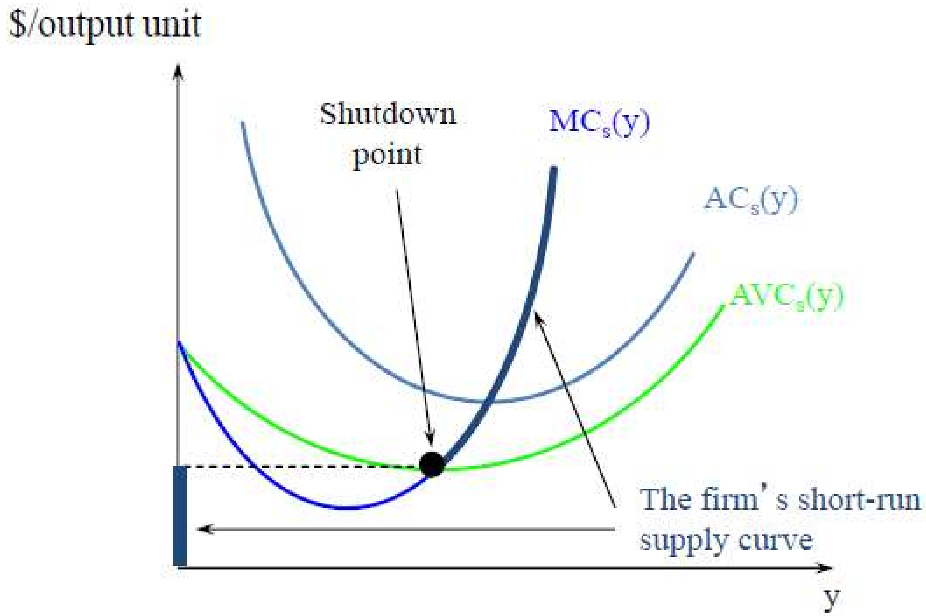
We have to **compare** the $y^*>0$ **solution with the no production case** ($y=0$)

- The firm's profit function is $\boxed{\Pi_s(y) = py - C(y) = py - F - VC(y)}$
- If the firm chooses $y = 0$ then its profit is $\boxed{\Pi(y) = 0 - F - VC(0) = -F}$

- The firm chooses an output $y>0$ only if

$$\boxed{\Pi(y) = py - F - VC(y) \geq -F} \Rightarrow \boxed{py - VC(y) \geq 0} \Rightarrow \boxed{p \geq \frac{VC(y)}{y} = AVC(y)}$$

- If $\boxed{p > AVC(y) \rightarrow y^* > 0}$ the firm has a positive production level
- If $\boxed{p < AVC(y) \rightarrow y^* = 0}$ the firm **shut-down**, producing no output

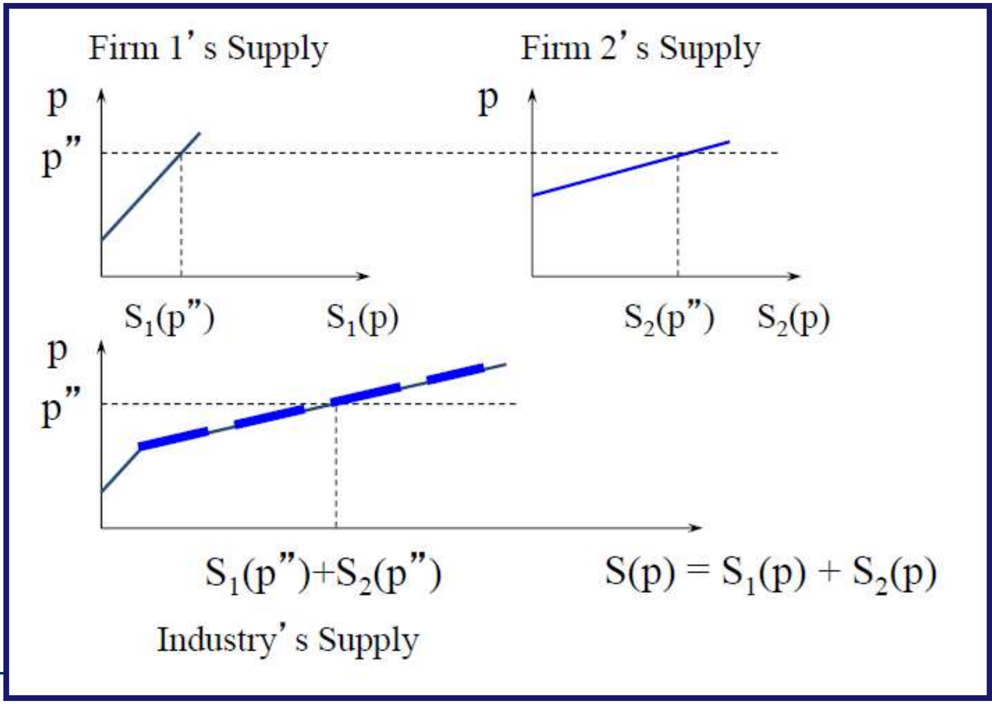


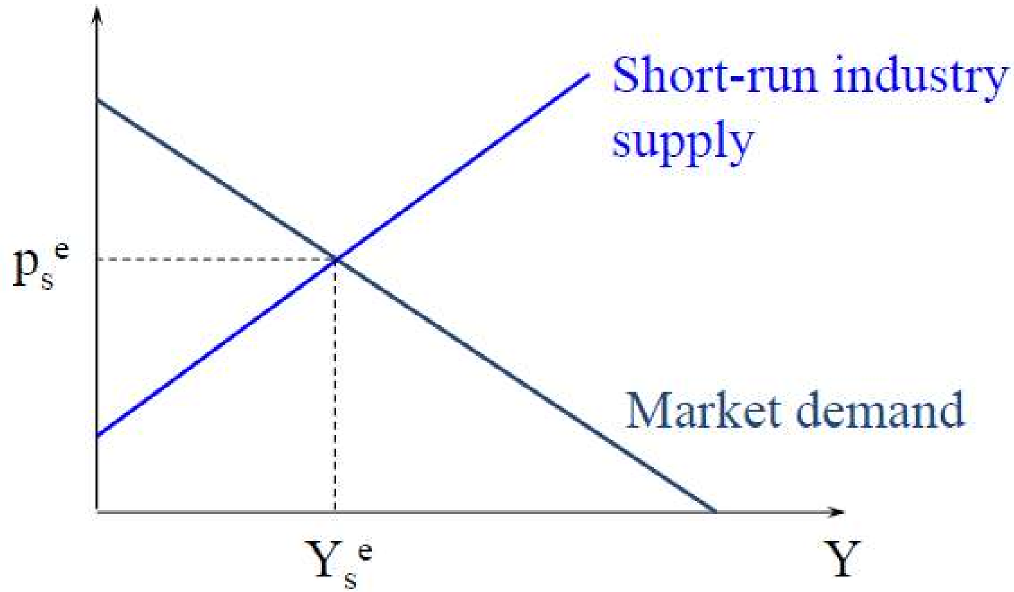
The **total quantity supplied** in the industry at the market price is the **sum of quantities supplied** at that price by each firm

- The **short-run industry supply** is

$$\boxed{S(p) = \sum_{i=1}^{N} S_i(p)}$$

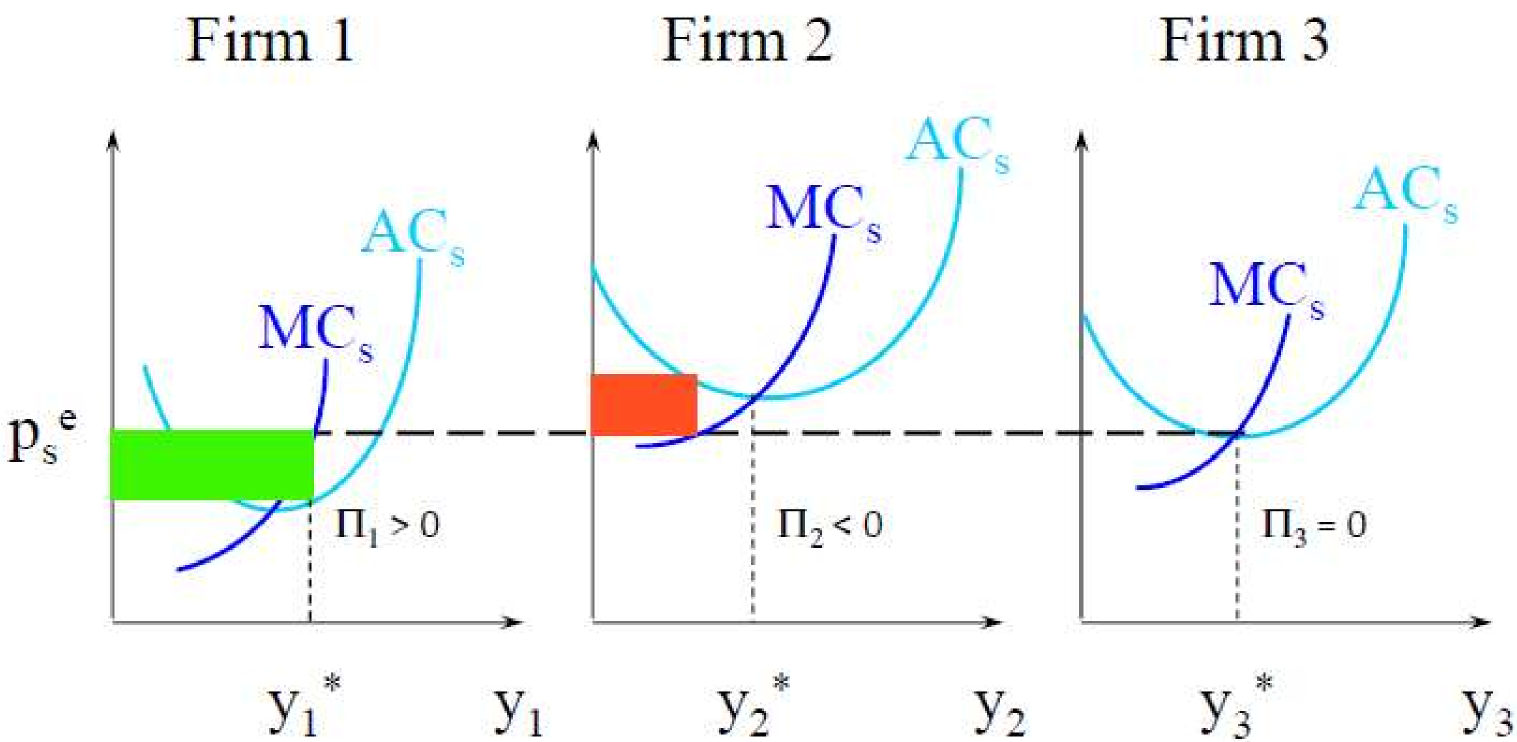
Where N is the number of firms, $i = 1, \dots, N$, which is temporarily **fixed** in the short-run, and $S_i(p)$ is the firm i' s supply function





SHORT RUN: ECONOMIC PROFITS AND LOSSES

In the short run, firms in perfect competition can make **economic profits** or **report losses**



Firm 1 wishes to remain in the industry

Firm 2 wishes to exit from the industry

Firm 3 is indifferent

LONG RUN: PROFIT MAXIMIZATION PROBLEM

- In the long-run all inputs are variable, thus, the cost *C(y)* of producing y units of output consists only of **variable costs**. The firm's **long-run profit function** is

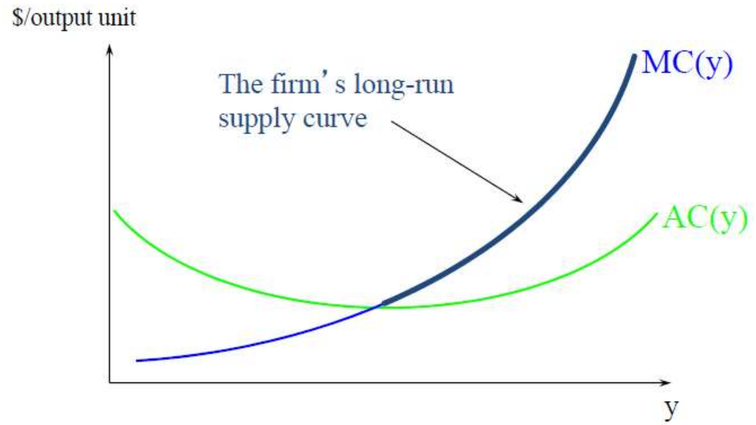
$$\boxed{\Pi(y) = py - C(y)}$$

- The profit maximization problem is $\boxed{\max_{y \geq 0} \Pi(y) = py - C(y)}$

- The 1st and 2nd-order conditions are
$$\boxed{\begin{array}{l} p = MC(y) \quad and \\ \dfrac{dMC(y)}{dy} > 0 \end{array}}$$

- Additionally, the firm must not report losses otherwise it would **exit** the industry. So

$$\boxed{\begin{array}{l} \Pi(y) = py - C(y) \geq 0 \\ \Rightarrow p \geq \dfrac{C(y)}{y} = AC(y) \end{array}}$$

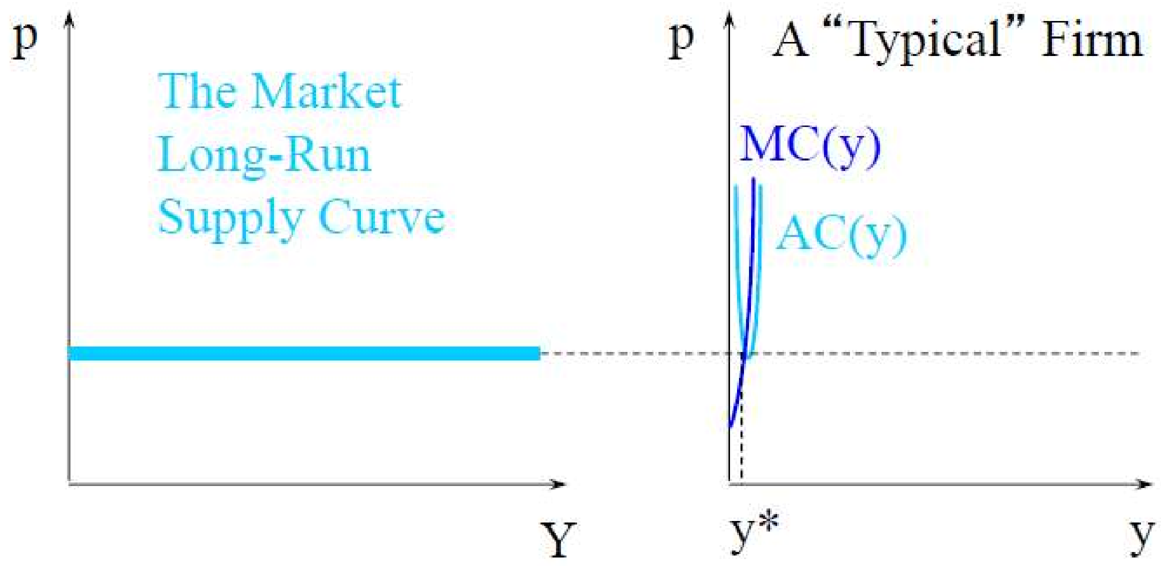


LONG RUN: INDUSTRY SUPPLY

The **industry long run supply function** is the **sum** of **firms' supply function.** In the long run, **firms in the industry** are **free to exit** and **firms outside the industry** are **free to enter**, this dynamic causes N to vary. Specifically

- Firms outside industry **enter** when firms in the industry gain economic profits and this happens when ***p> min AC(y)*** → **N increases**
- Entry increases industry supply, *p* falls causing some firms to make losses and exit the industry → **N decreases**

The process ends when the **long-run market equilibrium price** emerges ***p^e=min AC(y)***, which also defines the long-run **number of firms** in the industry



0.2.2. Monopoly

MONOPOLY

- There is **one firm** in the industry, which faces the market demand as its unique constraint
- **What causes monopolies**
  - A legal fiat; e.g. the salt monopoly
  - A patent; e.g. on a new drug
  - Sole ownership of a resource; e.g. a toll highway
  - The formation of a cartel; e.g. OPEC
  - A peculiar cost structure; e.g. local utility companies

The monopolist wants to maximize its economic profit

$$\boxed{\max_{y} \Pi(y) = r(y) - C(y) = p(y)y - C(y)}$$

- It produces the output y^*, at which marginal revenue equals marginal cost

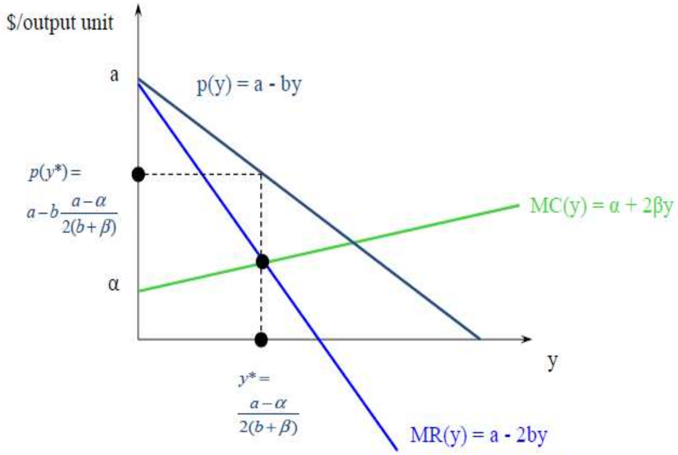
$$\boxed{MR\left(y^*\right) = MC\left(y^*\right)}$$

An Example: Linear Demand

Marginal revenue *MR(y)* is the change in revenues for one unitary change in output

$$\boxed{MR(y) = \frac{\partial(p(y)\cdot y)}{\partial y} = p(y) + y\frac{\partial p(y)}{\partial y} < p(y)}$$

- If *p(y) = a - by* (linear demand) then *R(y) = p(y)y = ay - by²* ($R(y) = p(y)y = ay - by^2$)
- Then, $MR(y) = a - 2by$
- If $C(y) = F + \alpha y + \beta y^2$ then $MC(y)= \alpha y + 2\beta y$
- At the profit-maximizing output y^*, $MR(y^*) = MC(y^*) \rightarrow a - 2by=\alpha y + 2\beta y$
- The **profit-maximizing output** is $\boxed{y^* = \frac{a-\alpha}{2(b+\beta)}}$



The **profit-maximizing price** is $\boxed{p(y^*) = a - by^* = a - b\frac{a-\alpha}{2(b+\beta)}}$

MONOPOLY PRICE AND DEMAND ELASTICITY

It is possible to express the **profit maximizing condition** in **monopoly** in **terms of demand elasticity**

$$\boxed{\frac{p - MC}{p} = \frac{1}{\varepsilon}}$$

The monopolist

- Can charge a **higher price** the **lower** the **demand elasticity**
- Has **market power** (as p>CM) and this **market power** is **higher the lower** the **demand elasticity**

Monopoly vs. Perfect competition

Compared to perfect competition (where the price is equal to marginal cost), the monopoly offers a **lower quantity** at a **higher price**

- This leads to the **MONOPOLY DEAD-WEIGHT LOSS**

- **HP**: Linear production costs, and thus average and marginal costs equal and constant