

**Data Mining and Machine Learning**  
**Bioinspired computational methods**  
**Biological data mining**

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## **Getting to Know your Data**

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## **Chapter 2: Getting to Know Your Data**

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- Data Objects and Attribute Types 
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

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# Types of Data Sets

- Record
  - Relational records
  - Data matrix, e.g., numerical matrix, crosstabs
  - Document data: text documents: term-frequency vector
  - Transaction data
- Graph and network
  - World Wide Web
  - Social or information networks
  - Molecular Structures
- Ordered
  - Video data: sequence of images
  - Temporal data: time-series
  - Sequential Data: transaction sequences
  - Genetic sequence data
- Spatial, image and multimedia:
  - Spatial data: maps
  - Image data
  - Video data

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A **cross tab** is a type of **table** in a **matrix** format that displays the (multivariate) **frequency distribution** of the variables.  
Example:

	Right-handed	Left-handed	Totals
Males	43	9	52
Females	44	4	48
Totals	87	13	100

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

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# Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

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## Data Objects

- Data sets are made up of data objects.
- A **data object** represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples*, *examples*, *instances*, *data points*, *objects*, *tuples*.
- Data objects are described by **attributes**.
- Database rows -> data objects; columns -> attributes.

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## Attributes

- **Attribute (or dimensions, features, variables):** a data field, representing a characteristic or feature of a data object.
  - *E.g., customer\_ID, name, address*
- Types:
  - Nominal
  - Binary
  - Numeric: quantitative
    - Interval-scaled
    - Ratio-scaled

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## Attribute Types

- **Nominal:** categories, states, or "names of things"
  - *Hair\_color* = {auburn, black, blond, brown, grey, red, white}
  - marital status, occupation, ID numbers, zip codes
- **Binary**
  - Nominal attribute with only 2 states (0 and 1)
  - Symmetric binary: both outcomes equally important
    - e.g., gender
  - Asymmetric binary: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - *Size* = {small, medium, large}, grades, army rankings

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## Numeric Attribute Types

- Quantity (integer or real-valued)
- **Interval**
  - Measured on a scale of **equal-sized units**
  - Values have order
    - E.g., *temperature in C° or F°, calendar dates*
  - No true zero-point (division makes no sense)
- **Ratio**
  - Inherent **zero-point (natural zero-point such as temperature in Kelvin)**
  - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K° is twice as high as 5 K°).
    - e.g., *temperature in Kelvin, length, counts, monetary quantities*

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## Discrete vs. Continuous Attributes

### ■ Discrete Attribute

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Sometimes, represented as integer variables
- Note: Binary attributes are a special case of discrete attributes


### ■ Continuous Attribute

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables

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## Basic Statistical Descriptions of Data

- Motivation

- To better understand the data: central tendency, variation and spread

- Data dispersion characteristics

- median, max, min, quantiles, outliers, variance, etc.

- Numerical dimensions correspond to sorted intervals

- Data dispersion: analyzed with multiple granularities of precision
- Boxplot or quantile analysis on sorted intervals

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## Measuring the Central Tendency

- Mean (algebraic measure):

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$$

where  $N$  is sample size.

- **Weighted arithmetic mean:**

$$\bar{x} = \frac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i}$$

- **Trimmed mean:** mean obtained by chopping out extreme values (for instance the top and bottom 2% before computing the mean)

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## Measuring the Central Tendency

### ■ Median:

- Middle value if odd number of values, or average of the middle two values otherwise
- **Holistic measure:** must be computed on the entire dataset as a whole
- Estimated by interpolation (for *grouped data*):

$$median = L_1 + \left( \frac{N/2 - (\sum freq)_l}{freq_{median}} \right) width$$

*Lower boundary of the median interval*

age	frequency
1–5	200
6–15	450
16–20	300
21–50	1500
51–80	700
81–110	44

*Sum of the frequencies of all the intervals that are lower than the median interval*

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## Measuring the Central Tendency

### ■ Mode

- Value that occurs most frequently in the data
- Unimodal, bimodal, trimodal
- Empirical formula:

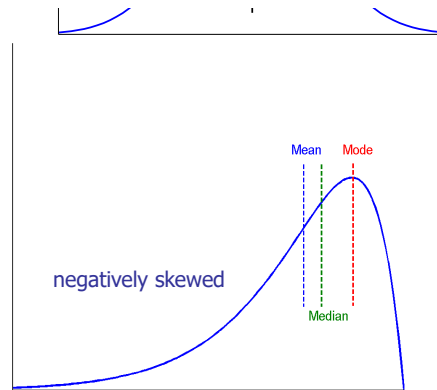
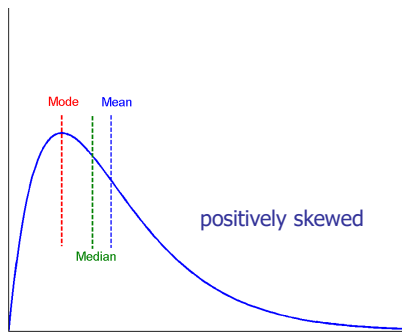
$$mean - mode = 3 \times (mean - median)$$

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## Symmetric vs. Skewed Da

Positively skewed, where the mode occurs at a value that is smaller than the median or negatively skewed, where the mode occurs at a value greater than the median



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## Graphic Displays of Basic Statistical Descriptions

- **Boxplot:** graphic display of five-number summary
- **Histogram:** x-axis are values, y-axis repres. frequencies
- **Quantile plot:** each value  $x_i$  is paired with  $f_i$  indicating that approximately 100  $f_i$ % of data are  $\leq x_i$
- **Quantile-quantile (q-q) plot:** graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot:** each pair of values is a pair of coordinates and plotted as points in the plane

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## Measuring the Dispersion of Data

- $k$ th percentile of a set of data in numerical order: value  $x_k$  having the property that  $k$  percent of the data entries lie at or below  $x_k$ .
- The median is the 50<sup>th</sup> percentile.
  - **Quartiles:**  $Q_1$  (25<sup>th</sup> percentile),  $Q_3$  (75<sup>th</sup> percentile)
  - **Inter-quartile range:**  $IQR = Q_3 - Q_1$

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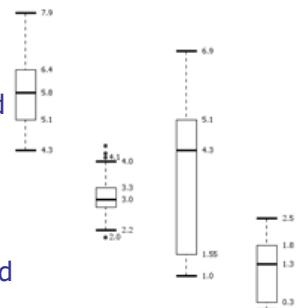
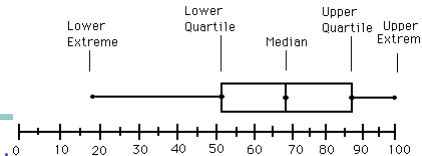
## Boxplot Analysis

- **Five-number summary** of a distribution

- Minimum,  $Q_1$ , Median,  $Q_3$ , Maximum

- **Boxplot**

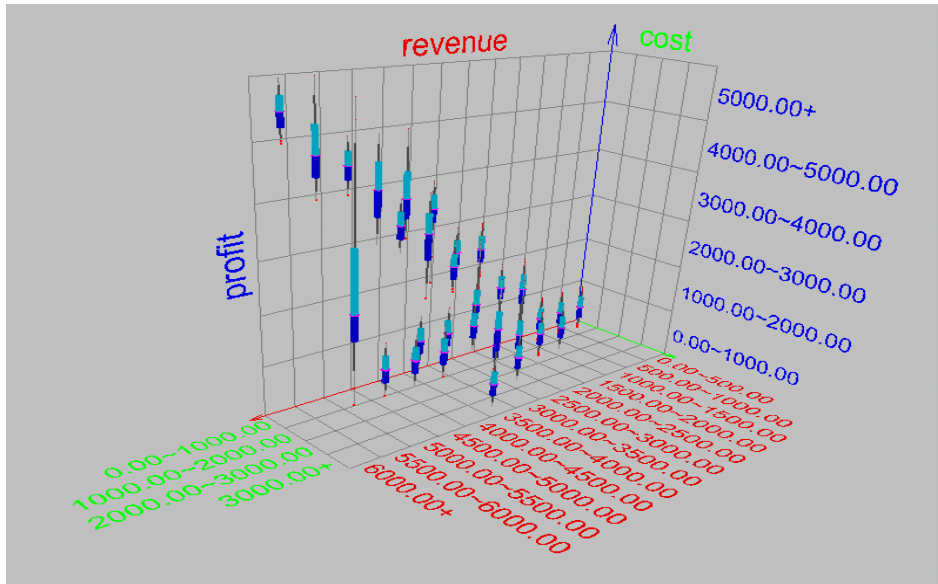
- Data is represented with a **box**
- The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- **Whiskers:** two lines outside the box extended to Minimum and Maximum
- **Outliers:** points beyond a specified outlier threshold, plotted individually (usually, a value higher/lower than  $1.5 \times IQR$ )



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## Visualization of Data Dispersion: 3-D Boxplots



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## Measuring the Dispersion of Data

- Variance and standard deviation  $\sigma$ 
  - **Variance:** (algebraic, scalable computation)

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \frac{1}{N} \sum_{i=1}^N x_i^2 - \bar{x}^2$$

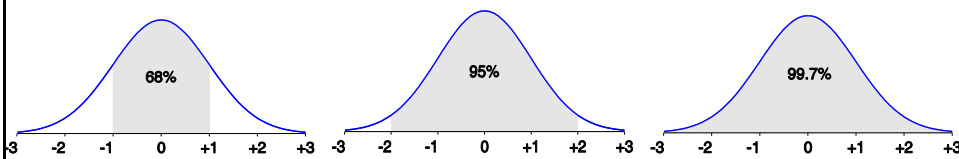
- **Standard deviation**  $\sigma$  is the square root of variance  $\sigma^2$ 
      - Measures the spread about the mean
  - The variance and the standard deviation are algebraic measures because they can be computed from distributive measures.

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## Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From  $\bar{x} - \sigma$  to  $\bar{x} + \sigma$ : contains about 68% of the measurements
  - From  $\bar{x} - 2\sigma$  to  $\bar{x} + 2\sigma$ : contains about 95% of it
  - From  $\bar{x} - 3\sigma$  to  $\bar{x} + 3\sigma$ : contains about 99.7% of it

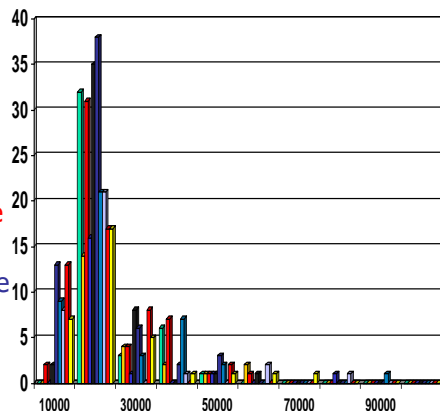


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## Histogram Analysis

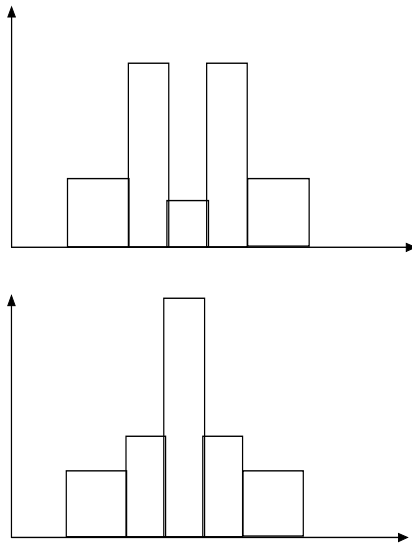
- **Histogram:** Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Differs from a bar chart in that **it is the area of the bar that denotes the value**, not the height as in bar charts, a crucial distinction when the categories are not of uniform width
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



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## Histograms Often Tell More than Boxplots



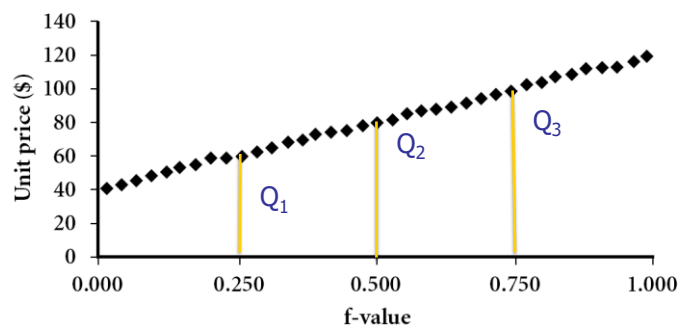
- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

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## Quantile Plot

- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots **quantile** information
  - For a data  $x_i$  data sorted in increasing order,  $f_i$  indicates that approximately 100  $f_i$  % of the data are below or equal to the value  $x_i$ . Note that 0.25, 0.5 and 0.75 quantiles correspond to the quartile  $Q_1$ , the median and the quartile  $Q_3$ , respectively.

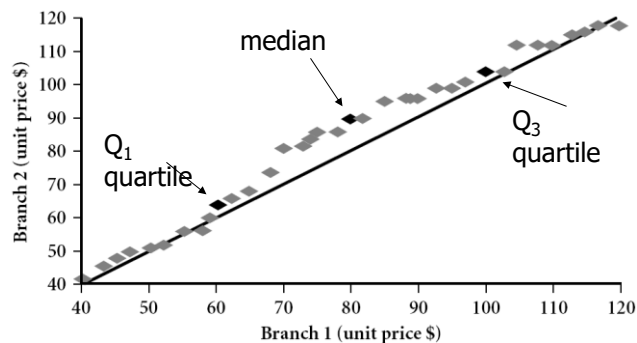


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## Quantile-Quantile (Q-Q) Plot

- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.

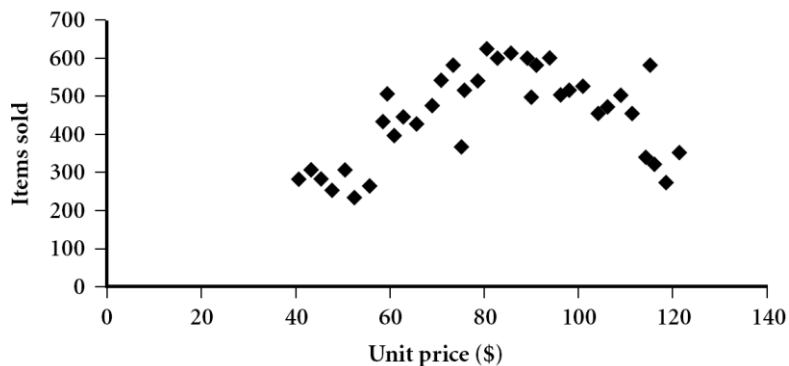


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## Scatter plot

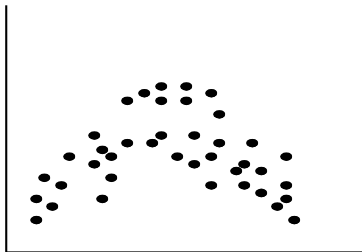
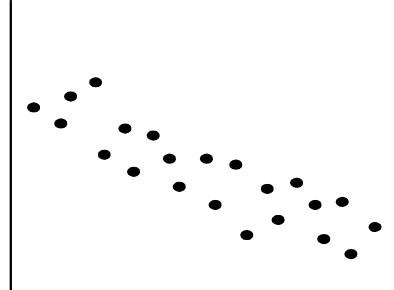
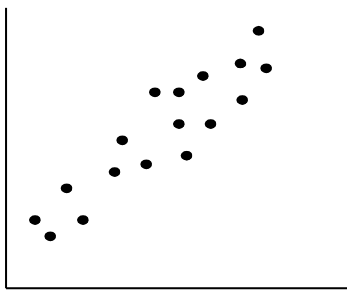
- Provides a first look at bivariate data to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane



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## Positively and Negatively Correlated Data



- The left half fragment is positively correlated
- The right half is negatively correlated

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## Uncorrelated Data




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## Data Visualization

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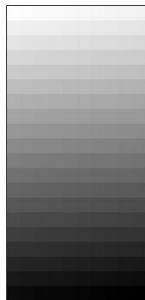
- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
  - Provide a visual proof of computer representations derived from data
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Geometric projection visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

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## Pixel-Oriented Visualization Techniques

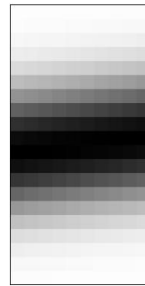
- For a data set of  $m$  dimensions, create  $m$  windows on the screen, one for each dimension
- The  $m$  dimension values of a record are mapped to  $m$  pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values



(a) Income



(b) Credit Limit



(c) Transaction volume



(d) Age

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## Geometric Projection Visualization Techniques

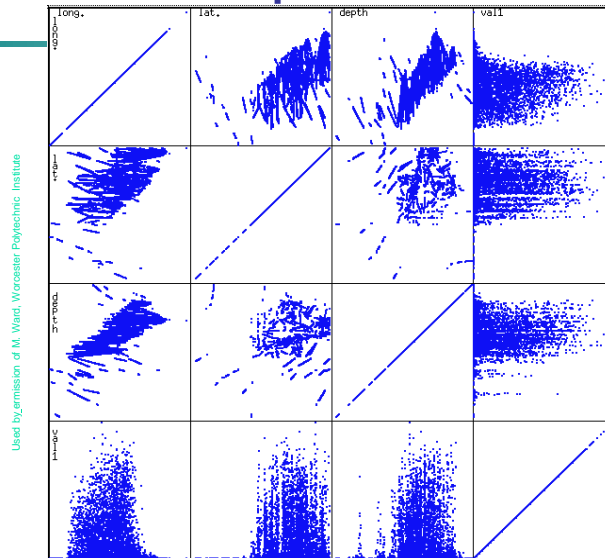
- Visualization of geometric transformations and projections of the data
- Methods
  - **Scatterplot and scatterplot matrices**
  - **Parallel coordinates**
  - **Icon-based**
  - **Projection pursuit technique: Help users find meaningful projections of multidimensional data**

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## Scatterplot Matrices



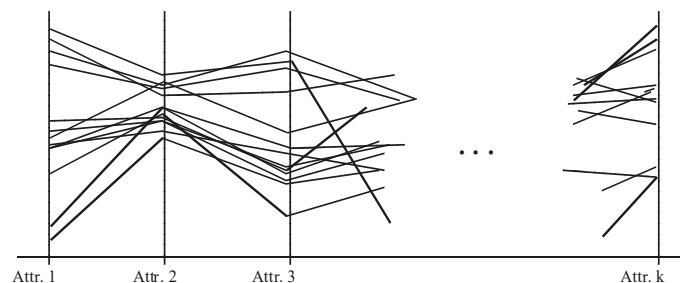
Matrix of scatterplots (x-y-diagrams) of the k-dim. data

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## Parallel Coordinates

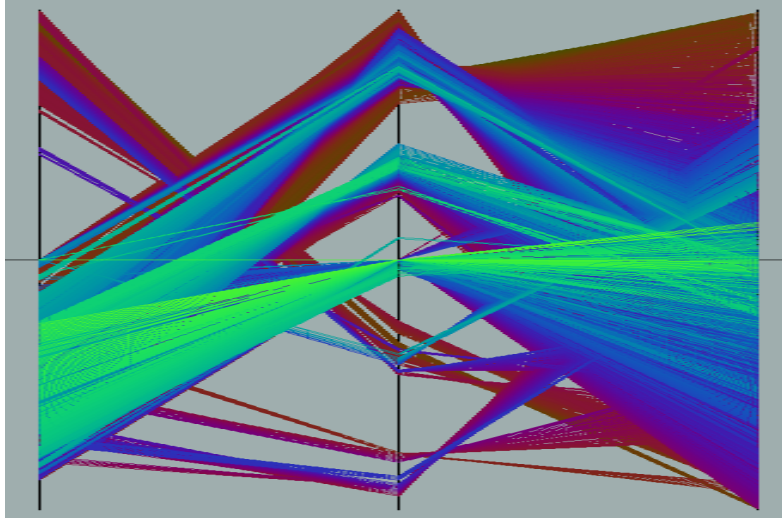
- n equidistant axes which are parallel to one of the screen axes and correspond to the attributes
- The axes are scaled to the [minimum, maximum]: range of the corresponding attribute
- Every data item corresponds to a polygonal line which intersects each of the axes at the point which corresponds to the value for the attribute



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## Parallel Coordinates of a Data Set



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## Icon-Based Visualization Techniques

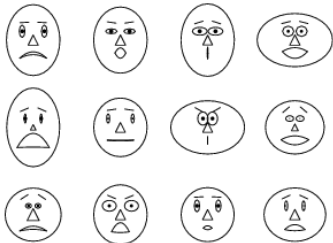
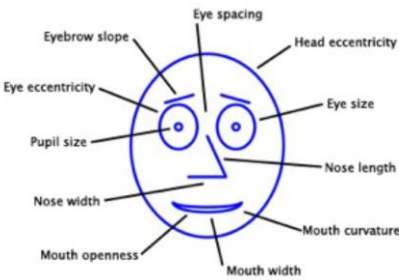
- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding
  - Color icons: Use color icons to encode more information
  - Tile bars: Use small icons to represent the relevant feature vectors in document retrieval

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# Chernoff Faces

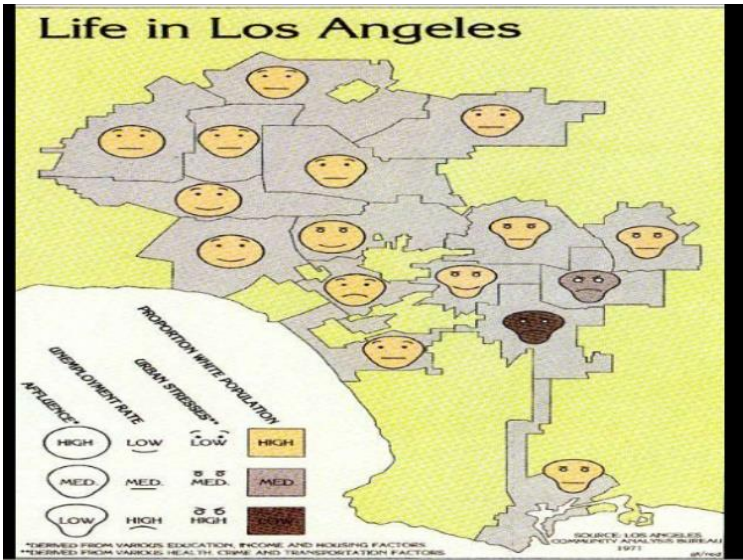
- A way to display variables on a two-dimensional surface, e.g., let x be eyebrow slant, y be eye size, z be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using *Mathematica* (S. Dickson)



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# Chernoff Faces

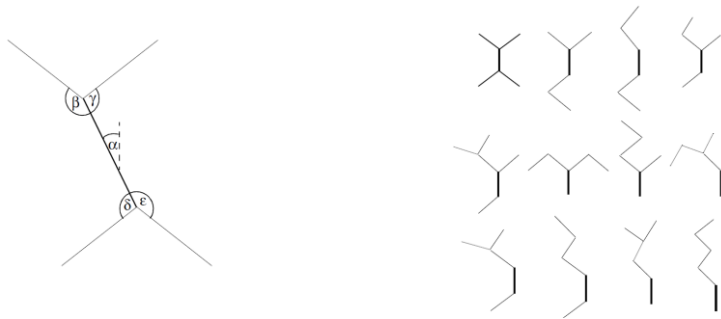


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# Stick Figure

- A very simple type of drawing made of lines and dots, often of the human form or other animals
  - two attributes of the data are mapped to the display axes and the remaining attributes are mapped to the angle and/or length of the limbs
  - texture patterns in the visualization how certain data characteristics



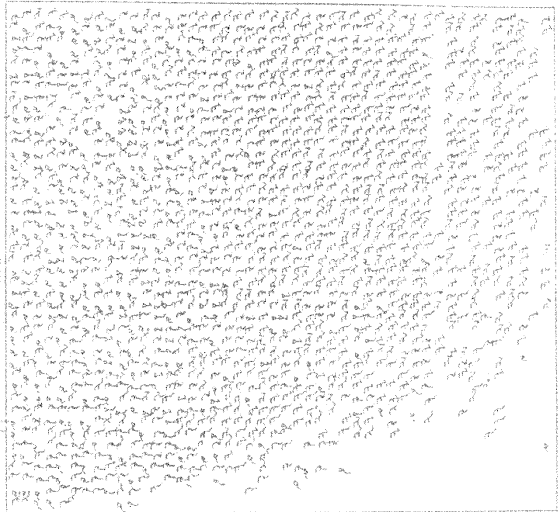
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# Stick Figure

A very simple type of drawing made of lines and dots, often of the human form or other animals.

A census data figure showing age, income, gender, education, etc.



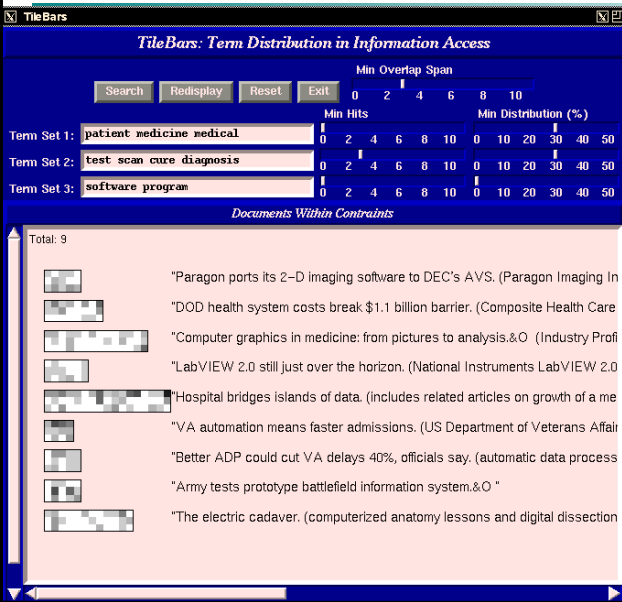
A 5-piece stick figure (1 body and 4 limbs w. different angle/length)

Two attributes mapped to axes, remaining attributes mapped to angle or length of limbs". Look at texture pattern

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# Tile bar



Rectangles correspond to documents.

The query is specified in terms of k topics, one topic per line, called term sets.

Columns in rectangles correspond to document segments.

A square corresponds to a specific term set in a specific text segment.

The darkness of a square indicates the frequency of terms in the segment from the corresponding TermSet. 41

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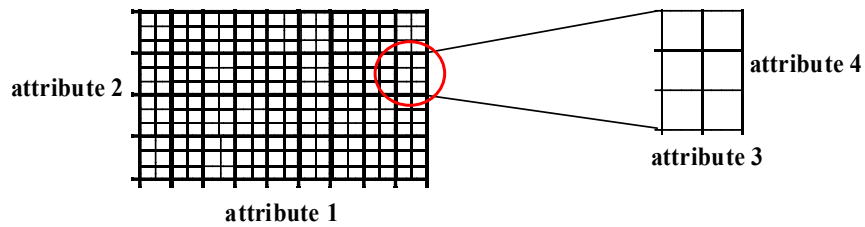
## Hierarchical Visualization Techniques

- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Dimensional Stacking
  - Tree-Map
  - Cone Trees
  - InfoCube
  - ...

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## Dimensional Stacking



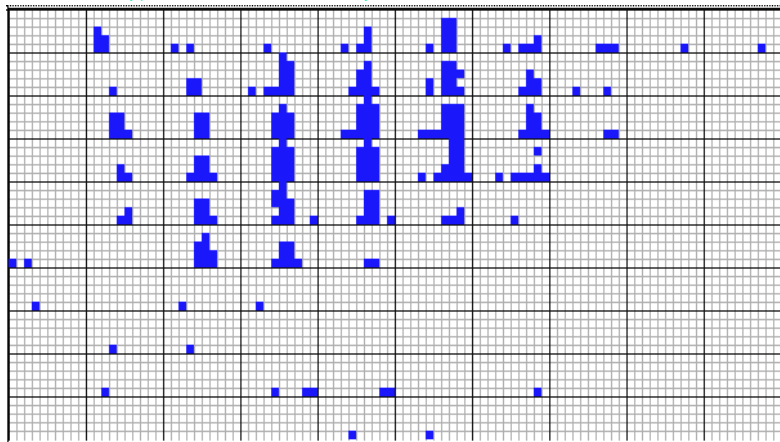
- Partitioning of the n-dimensional attribute space in 2-D subspaces, which are 'stacked' into each other
- Partitioning of the attribute value ranges into classes. The important attributes should be used on the outer levels.
- Adequate for data with ordinal attributes of low cardinality
- But, difficult to display more than nine dimensions
- Important to map dimensions appropriately

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## Dimensional Stacking

Used by permission of M. Ward, Worcester Polytechnic Institute



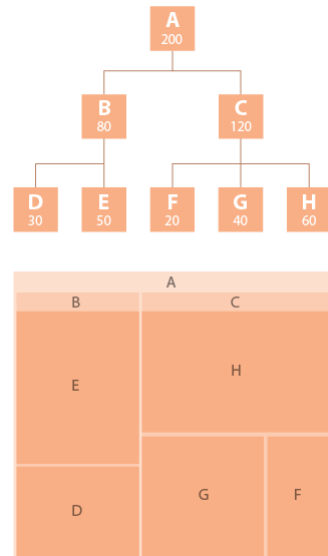
Visualization of oil mining data with longitude and latitude mapped to the outer x-, y-axes and ore grade and depth mapped to the inner x-, y-axes

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## Tree-Map

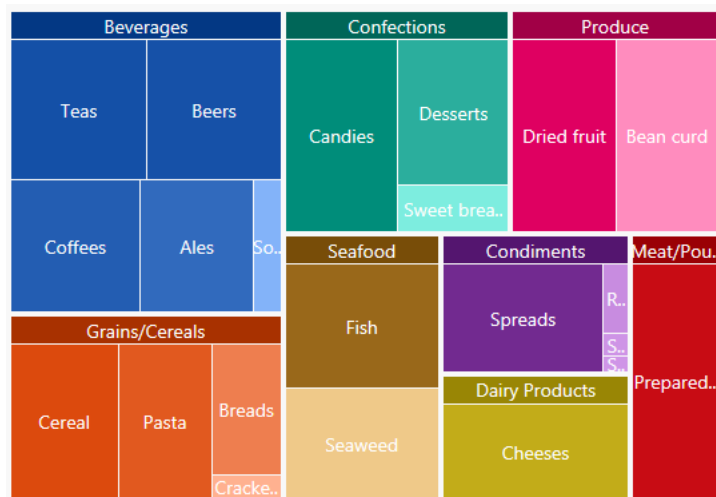
- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values
- The x- and y-dimension of the screen are partitioned alternately according to the attribute values (classes)



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## Tree-Map

- Examples

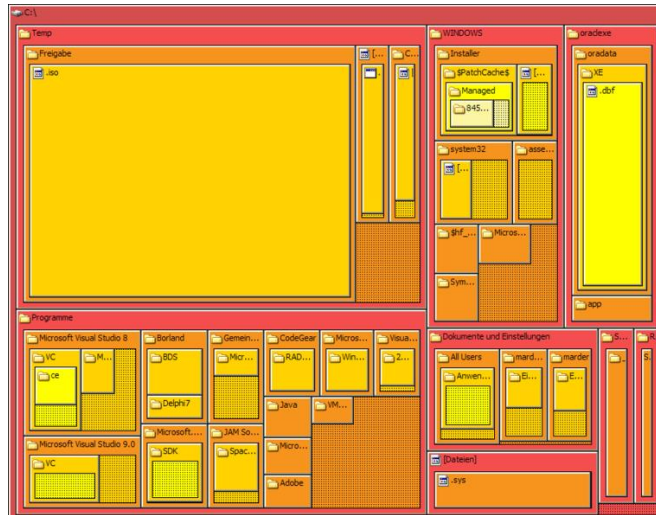


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## Tree-Map

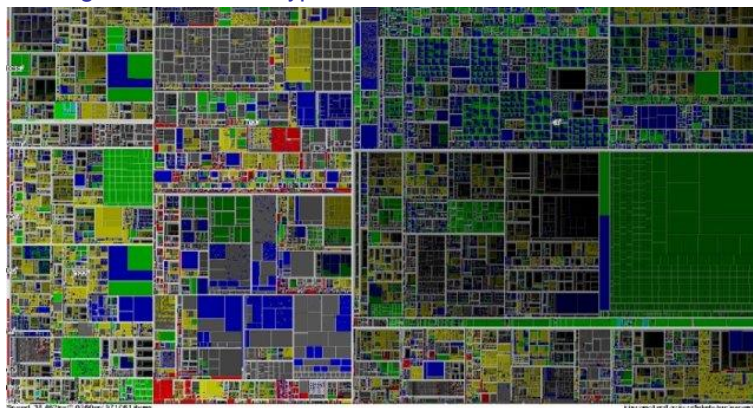
Example: an overview of the organization of file and directory



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## Tree-Map of a File System

The treemap represents each file as a colored rectangle, the area of which is proportional to the file's size. The rectangles are arranged in such a way, that directories again make up rectangles, which contain all their files and subdirectories. So their area is proportional to the size of the subtrees. The color of a rectangle indicates the type of the file.



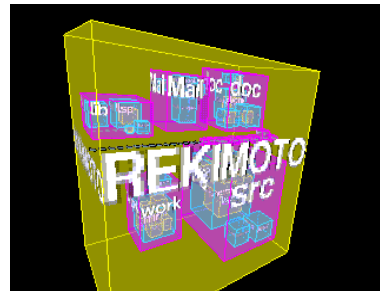
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## InfoCube

- A 3-D visualization technique where hierarchical information is displayed as nested semi-transparent cubes
- The outermost cubes correspond to the top level data, while the subnodes or the lower level data are represented as smaller cubes inside the outermost cubes, and so on



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## Visualizing Complex Data and Relations

- **Visualizing non-numerical data:** text and social networks
- Tag cloud: visualizing user-generated tags
  - The importance of tag is represented by font size/color
- Besides text data, there are also methods to visualize relationships, such as visualizing social networks



Newsmap: Google News Stories in 2005

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## Tag Cloud: Example

- Example of Tag Cloud



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## Similarity and Dissimilarity

### ■ Similarity

- Numerical measure of how alike two data objects are
- Value is higher when objects are more alike
- Often falls in the range [0,1]

### ■ Dissimilarity (e.g., distance)

- Numerical measure of how different two data objects are
  - Lower when objects are more alike
- Minimum dissimilarity is often 0
- Upper limit varies

### ■ Proximity refers to a similarity or dissimilarity

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## Data Matrix and Dissimilarity Matrix

### ■ Data matrix

- n data points with p dimensions
- Two modes (rows and columns represent different entities)

$$\begin{bmatrix} x_{11} & \dots & x_{1f} & \dots & x_{1p} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{if} & \dots & x_{ip} \\ \dots & \dots & \dots & \dots & \dots \\ x_{n1} & \dots & x_{nf} & \dots & x_{np} \end{bmatrix}$$

### ■ Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix
- Single mode

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & \ddots & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

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## Proximity Measure for Nominal Attributes

- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - $m$ : # of matches,  $p$ : total # of variables

$$d(i, j) = \frac{p - m}{p}$$

*Example: Variables: eye color and hair color*

$i = (\text{green}, \text{blond})$

$j = (\text{green}, \text{black})$

$$d(i, j) = \frac{2 - 1}{2} = 0.5$$

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## Proximity Measure for Nominal Attributes

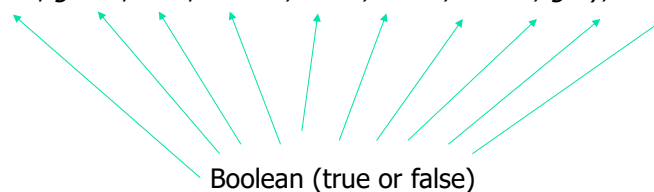
- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the  $M$  nominal states (for instance, for color, create binary attributes red, yellow, blue, green, and so on)

Objects described by eye color and hair color

Eye color = {black, green, blue}

Hair\_color = {auburn, black, blond, brown, grey, red, white}

$i = \{\text{black}, \text{green}, \text{blue}, \text{auburn}, \text{black}, \text{blond}, \text{brown}, \text{grey}, \text{red}, \text{white}\}$



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## Proximity Measure for Binary Attributes

- A contingency table for binary data

Number of variables		Object $j$		
Object $i$	1	0	sum	
	$q$	$r$	$q + r$	
	$s$	$t$	$s + t$	
sum	$q + s$	$r + t$	$p$	

- Distance measure for **symmetric binary variables** (a binary variable is symmetric if both of its states are equally valuable and carry the same weight):

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

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## Proximity Measure for Binary Attributes

- Distance measure for asymmetric binary variables:

$$d(i, j) = \frac{r + s}{q + r + s}$$

- Jaccard coefficient (**similarity** measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$$

- Note: Jaccard coefficient is the same as "coherence":

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q + r) + (q + s) - q}$$

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# Dissimilarity between Binary Variables

## ■ Example

Name	Gender	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	M	Y	N	P	N	N	N
Mary	F	Y	N	P	N	P	N
Jim	M	Y	P	N	N	N	N

- Gender is a symmetric attribute
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N be 0 (use only asymmetric values)

$$D(\text{Jack, Mary}) = \frac{0+1}{2+0+1} = 0.33$$

$$D(\text{Jack, Jim}) = \frac{1+1}{1+1+1} = 0.67$$

$$D(\text{Jim, Mary}) = \frac{1+2}{1+1+2} = 0.75$$

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# Standardizing Numeric Data

- **Z-score** (conversion to unitless variables):  $z = \frac{x-\mu}{\sigma}$ 
  - x: raw score to be standardized,  $\mu$ : mean of the population,  $\sigma$ : standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
- An alternative way: **Calculate the mean absolute deviation**

$$s_f = \frac{1}{N}(|x_{1f} - m_f| + |x_{2f} - m_f| + \dots + |x_{Nf} - m_f|)$$

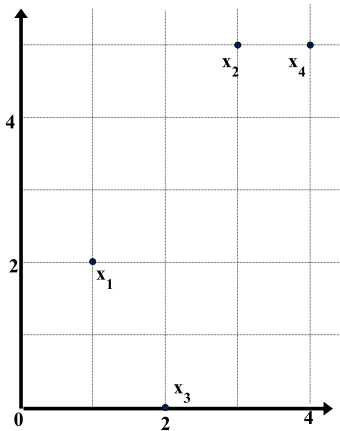
$$\text{where } m_f = \frac{1}{N}(x_{1f} + x_{2f} + \dots + x_{Nf}).$$

- standardized measure (*z-score*):  $z_{if} = \frac{x_{if} - m_f}{s_f}$
- Using mean absolute deviation is more robust to outliers than using standard deviation

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## Example: Data Matrix and Dissimilarity Matrix



Data Matrix

point	attribute1	attribute2
$x1$	1	2
$x2$	3	5
$x3$	2	0
$x4$	4	5

Dissimilarity Matrix  
(with Euclidean Distance)

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

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## Distance on Numeric Data: Minkowski Distance

- **Minkowski distance:** A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where  $i = (x_{i1}, x_{i2}, \dots, x_{ip})$  and  $j = (x_{j1}, x_{j2}, \dots, x_{jp})$  are two  $p$ -dimensional data objects, and  $h$  is the order (the distance so defined is also called L- $h$  norm)

- Properties
  - $d(i, j) > 0$  if  $i \neq j$ , and  $d(i, i) = 0$  (Positive definiteness)
  - $d(i, j) = d(j, i)$  (Symmetry)
  - $d(i, j) \leq d(i, k) + d(k, j)$  (Triangle Inequality)
- A distance that satisfies these properties is a **metric**

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# Special Cases of Minkowski Distance

- $h = 1$ : **Manhattan** (city block,  $L_1$  norm) **distance**
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

- $h = 2$ : ( $L_2$  norm) **Euclidean** **distance**

$$d(i, j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

- $h \rightarrow \infty$ : **“supremum”** ( $L_{\max}$  norm,  $L_{\infty}$  norm) **distance**.
  - This is the maximum difference between any component (attribute) of the vectors

$$d(i, j) = \lim_{h \rightarrow \infty} \left( \sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f |x_{if} - x_{jf}|$$

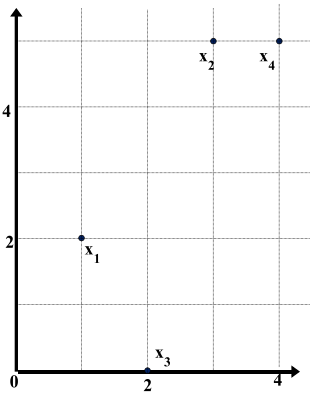
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# Example: Minkowski Distance

## Dissimilarity Matrices

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



## Manhattan ( $L_1$ )

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

## Euclidean ( $L_2$ )

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

## Supremum

$L_{\infty}$	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

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## Ordinal Variables

- An ordinal variable is similar to a categorical variable. **The difference between the two is that there is a clear ordering of the variables.** For example, suppose you have a variable, economic status, with three categories (low, medium and high).
- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank  $r_{if} \in \{1, \dots, M_f\}$        $\{small, medium, large\}$
- Can be treated like interval-scaled       $\{1, 2, 3\}$ 
  - replace  $x_{if}$  by their rank
  - map the range of each variable onto  $[0, 1]$  by replacing  $i$ -th object in the  $f$ -th variable by
$$z_{if} = \frac{r_{if} - 1}{M_f - 1} \quad \begin{matrix} \{small, medium, large\} \\ \{0, 0.5, 1\} \end{matrix}$$
  - compute the dissimilarity using methods for interval-scaled variables

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## Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i, j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- $f$  is binary or nominal:
  - $d_{ij}^{(f)} = 0$  if  $x_{if} = x_{jf}$ , or  $d_{ij}^{(f)} = 1$  otherwise
- $f$  is numeric: use the normalized distance
- $f$  is ordinal
  - Compute ranks  $r_{if}$  and
  - Treat  $z_{if}$  as interval-scaled       $z_{if} = \frac{r_{if} - 1}{M_f - 1}$

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## Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	team	coach	hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If  $d_1$  and  $d_2$  are two vectors (e.g., term-frequency vectors), then
 
$$\cos(d_1, d_2) = (d_1 \bullet d_2) / (||d_1|| ||d_2||),$$
 where  $\bullet$  indicates vector dot product,  $||d||$ : the length of vector  $d$

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## Example: Cosine Similarity

- $\cos(d_1, d_2) = (d_1 \bullet d_2) / (||d_1|| ||d_2||)$ ,  
where  $\bullet$  indicates vector dot product,  $||d||$ : the length (norm) of vector  $d$

- Ex: Find the **similarity** between documents 1 and 2.

$$d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0)$$

$$d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$$

$$d_1 \bullet d_2 = 5*3 + 0*0 + 3*2 + 0*0 + 2*1 + 0*1 + 0*1 + 2*1 + 0*0 + 0*1 = 25$$

$$||d_1|| = (5*5 + 0*0 + 3*3 + 0*0 + 2*2 + 0*0 + 0*0 + 2*2 + 0*0 + 0*0)^{0.5} = (42)^{0.5} = 6.481$$

$$||d_2|| = (3*3 + 0*0 + 2*2 + 0*0 + 1*1 + 1*1 + 0*0 + 1*1 + 0*0 + 1*1)^{0.5} = (17)^{0.5} = 4.12$$


$$\cos(d_1, d_2) = 0.94$$

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## Chapter 2: Getting to Know Your Data

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- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary 

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## Summary

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- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing.
- Many methods have been developed but still an active area of research.

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