Introduction to the course "Optimization Methods and Game Theory"

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Optimization Methods and Game Theory
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Preliminary informations

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Question time: by appointment (send e-mail)

Course schedule

- thursday 16.30 - 19.30, Room SI5
- friday 15.30 - 17.30 , Room ETR F5

Course material

- Microsoft Teams platform: Team 696AA 24/25 - Optimization Methods and Game Theory [WAI-LM] (Slides of the lectures)
- https://elearn.ing.unipi.it/course/view.php?id=3049

Aim of the course

Study optimization methods for data analysis and decision problems

Main tool

Optimization problems defined by

$$\min\ (max)\ \{f(x):\ x \in X\} \qquad (P)$$

- $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function
- $X \subseteq \mathbb{R}^n$ is the constraints set or feasible region
- If $X \equiv \mathbb{R}^n$ then (P) is said to be unconstrained

(P) can be considered as a decision problem where X is the set of all the admissible (feasible) decisions x and $f(x)$ is the value of the decision x (for example, a cost or a gain).

In general X is defined by constraint functions

- $X = \{x \in \mathbb{R}^n : g(x) \leq 0, h(x) = 0\}$
- $g(x) = (g_1(x), \dots, g_m(x))$, where $g_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, m$ are the inequality constraints functions
- $h(x) = (h_1(x), \dots, h_p(x))$, where $h_j : \mathbb{R}^n \to \mathbb{R}$, $j = 1, \dots, p$ are the equality constraints functions

$$\begin{cases} \min \ f(x) \\ g(x) \leq 0 \\ h(x) = 0 \end{cases}$$

We will generally consider minimization problems since

$$\max\{f(x) : \ x \in X\} = -\min\{-f(x) : \ x \in X\}.$$

It is of fundamental importance to analyze the properties of (P) under suitable assumptions on X and on the involved functions:

- convexity
- differentiability

An application: Economical Power Dispatch

The production, distribution and consume of electricity are global concerns. In recent years, due to the scarcity of energy resources, it was observed an increasing power generation cost and ever-growing demand for electric energy: in this context, optimal economic dispatch has become an extremely important issue in power systems.

- We consider a power system that has a given electricity load demand P_D;
- The electricity is produced by n different units that can work with different loads. The production is based on a suitable fuel (petroleum, coal, gas, nuclear) and a given fuel cost function is associated with each unit.
- The Economical Power Dispatch (EPD) is modeled as a constrained optimization problem and consists in determining, the load of electricity to be produced by each unit, in a given interval of time T, in order to minimize the total fuel cost and supplying the total load demand and some technical requirements of the system.

The model

- We consider n generating units and we define the variable p_i (in MW) the (unknown) power supply generated by unit $i = 1, .., n$;
- $f_i(p_i)$ is the fuel cost function, in (Euro/T), for the i-th unit, e.g.,

$$f_i(p_i) = a_i p_i^2 + b_i p_i + c_i,$$

  with a_i, b_i, c_i the fuel cost coefficients of unit i, $\quad i = 1, .., n$.
- p_{min_i} and p_{max_i} are the minimum and the maximum output limit of the i-th generating unit, for $i = 1, .., n$;
- P_D is the total load demand of the power system in MW;
- P_L is the amount of the system losses in MW.

EPD problem formulation

$$\begin{cases} \min F(p) := \sum_{i=1}^{n} f_i(p_i) \\ p_1 + p_2 + ... + p_n = P_D + P_L \\ p_{min_i} \leq p_i \leq p_{max_i}, \; i = 1, .., n \end{cases}$$

An extension: Environmental/Economical Power Dispatch

It is of interest to consider the problem of simultaneously minimizing the total fuel cost of meeting the energy requirement of the system and the emissions of pollutants (e.g., fine dust emissions) of the units: this is a multiobjective optimization problem and is called the Environmental/Economical Power Dispatch (EEPD).

Besides the assumptions of problem (EPD) define:

- $e_i(p_i)$ the emission function, in (Kg/T), for the i-th unit, e.g.,

  $$e_i(p_i) = \alpha_i p_i^2 + \beta_i p_i + \gamma_i,$$

  with α_i, β_i, γ_i the fuel emission coefficients of unit i, $\quad i = 1, .., n$.
- $E(p) = \sum_{i=1}^{n} e_i(p_i)$ the total quantity of emissions of the n units.

EEPD problem formulation

$$\begin{cases} \min(F(p), E(p)) \\ p_1 + p_2 + ... + p_n = P_D + P_L \\ p_{min_i} \leq p_i \leq p_{max_i}, \;\; i = 1, .., n \end{cases}$$

Outline of the program of the course

- Preliminaries of convex analysis
- Optimization problems: existence of optima, optimality conditions, duality
- Solution methods for optimization problems:
  - gradient and conjugate gradient method
  - Newton and quasi-Newton methods
  - penalty, logarithmic barrier methods
- Applications to machine learning:
  - Supervised machine learning: optimization models for classification and regression problems
  - Unsupervised machine learning: clustering problems
- Multiobjective (or vector) optimization problems:
  - Pareto and weak Pareto optimal solutions
  - existence, optimality conditions, scalarization approach, goal method
- Non-cooperative game theory:
  - zero-sum finite games: Nash Equilibrium (NE), existence, min-max theorem
  - non zero-sum finite games: existence, optimality conditions, algorithms
  - convex games: existence of NE, optimality conditions, merit functions
- Exercise sessions with MATLAB software

MATLAB

You can download and install MATLAB on your laptop using the Campus License paid by University of Pisa, see:

Link for Matlab installation

- https://unipi.it/matlab
- "Accedi per iniziare"
- Recall that in order to install Matlab it is necessary to use your istitutional mail, namely, ...@studenti.unipi.it, and not any e-mail address.
- In particular, install the optimization toolbox.

Bibliography

- S. Boyd and L. Vandenberghe, Convex optimization, Cambridge University Press, 2004.
- M.S. Bazaraa, H.D. Sherali, C.M. Shetty, Nonlinear Programming: Theory and Algorithms, Wiley-Interscience, 2006.
- J. Nocedal, S. Wright, Numerical Optimization, Springer Series in Operations Research and Financial Engineering, 2006.
- D.T. Luc, Theory of Vector Optimization, Springer, 1989.
- Y. Sawaragi, H. Nakayama, T. Tanino, Theory of Multiobjective Optimization, Academic Press, 1985.
- M.J. Osborne, A. Rubinstein, A Course in Game Theory, MIT press, 1994.