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Quantum Computing and Quantum Internet

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Introduction

Motivations for the Quantum Internet

- The motivations for building networks are the same for both *quantum* and *classical* networks: the desire to connect people, devices such as computers or sensors, or databases that are in separate locations, for technical, economic, political, and logistical reasons

Applications Running on the Quantum Internet

- The Quantum Internet applications include
  1) quantum forms of distributed tasks such as *leader election* and *Byzantine agreement*
  2) more accurate clock synchronization
  3) long-baseline optical interferometry for telescopes which combining light from distant telescopes improve observations

- As the development of a quantum internet progresses, other useful applications will likely be discovered over the years ahead

Foundational Operations of the Quantum Internet

- The basic operations underpinning the quantum Internet are listed below
  1. *Quantum Teleportation*
  2. *Entanglement Distribution*
  3. *Entanglement Swapping*
  4. *Entanglement Purification/Distillation*

Foundational Operations of the Quantum Internet

- Before we move on to the quantum Internet concepts, let me recall you two topics that I explained in the Quantum Computing lectures:
    1. *Bell states or EPR pairs*
    2. *Quantum teleportation*

Bell States

- *Bell States* or *EPR pairs* are four *maximally entangled* quantum states of *two qubits*

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \rightarrow |\Phi^{+}\rangle$$

$$|\beta_{01}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right) \rightarrow |\Psi^{+}\rangle$$

$$|\beta_{10}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right) \rightarrow |\Phi^{-}\rangle$$

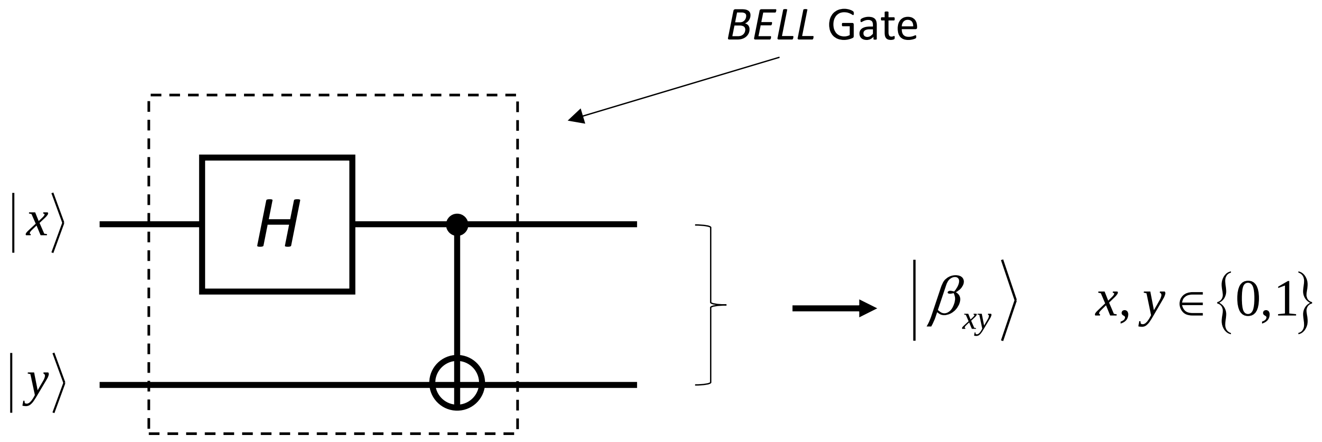
$$|\beta_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \rightarrow |\Psi^{-}\rangle$$

Tangle = 1

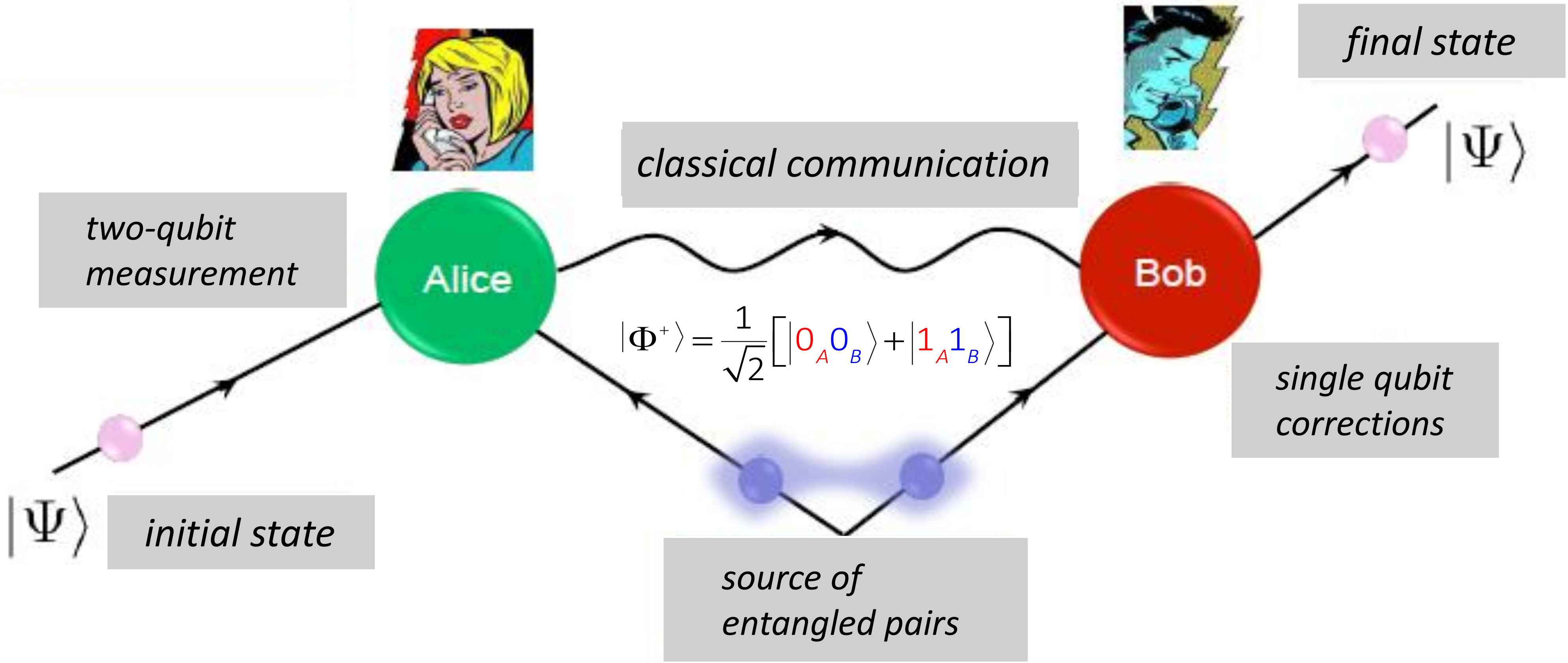
- EPR is an acronym which stands for *Einstein, Podolsky, and Rosen*

The Bell State Synthesizer Circuit

- The BELL gate, sometimes referred to as *Bell State Synthesizer Circuit,* generates EPR pairs by using the standard CBS basis $\{|0\rangle|0\rangle, |0\rangle|1\rangle, |1\rangle|0\rangle, |1\rangle|1\rangle\}$ as inputs

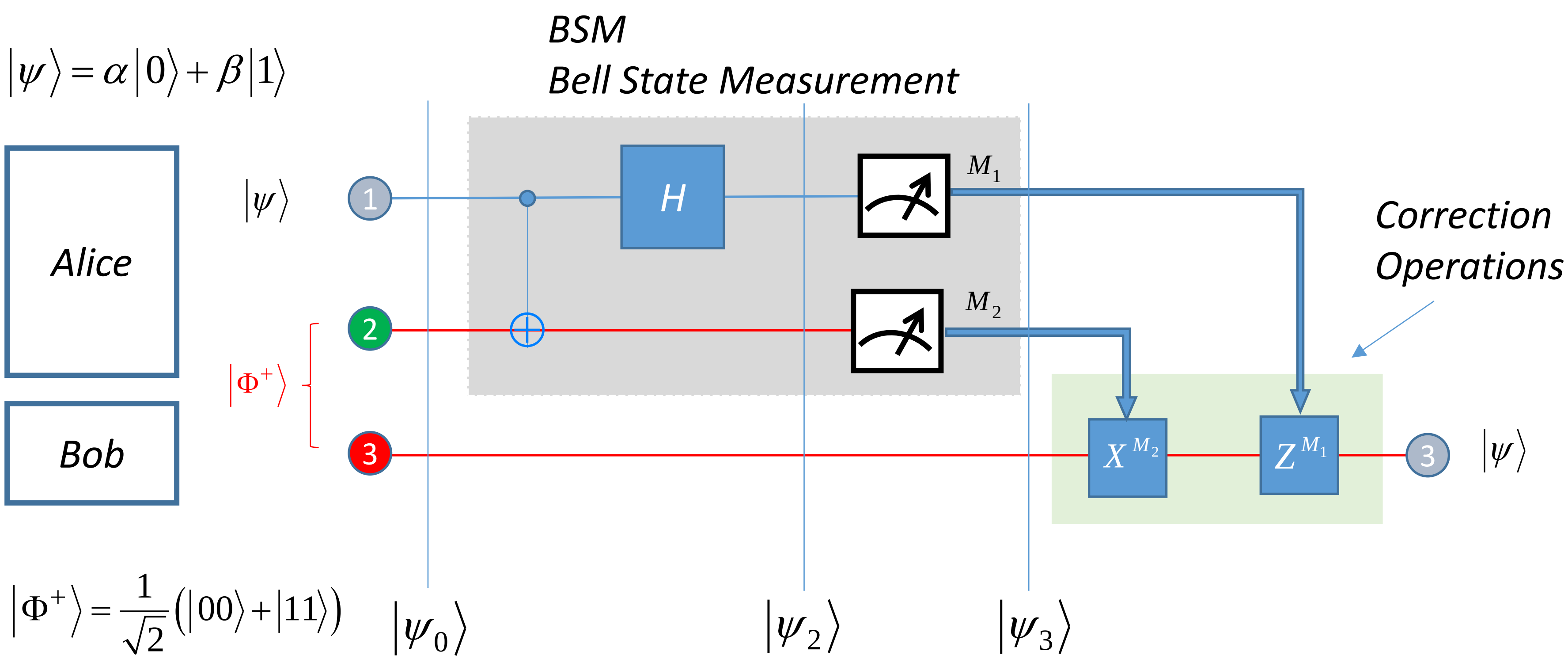


Quantum Teleportation



Quantum Circuit for Teleportation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\psi_0\rangle = |\psi\rangle \otimes |\Phi^+\rangle = \left(\alpha|0_1\rangle + \beta|1_1\rangle\right) \otimes \left(\frac{|0_2 0_3\rangle + |1_2 1_3\rangle}{\sqrt{2}}\right)$$

Quantum Circuit for Teleportation

$$|\psi_2\rangle = \frac{1}{2}\Big[\underbrace{|0_1 0_2\rangle}_{Alice}\underbrace{\left(\alpha|0_3\rangle+\beta|1_3\rangle\right)}_{Bob}+|0_1 1_2\rangle\left(\alpha|1_3\rangle+\beta|0_3\rangle\right)+|1_1 0_2\rangle\left(\alpha|0_3\rangle-\beta|1_3\rangle\right)+|1_1 1_2\rangle\left(\alpha|1_3\rangle-\beta|0_3\rangle\right)\Big]$$

– This expression naturally breaks down into four terms

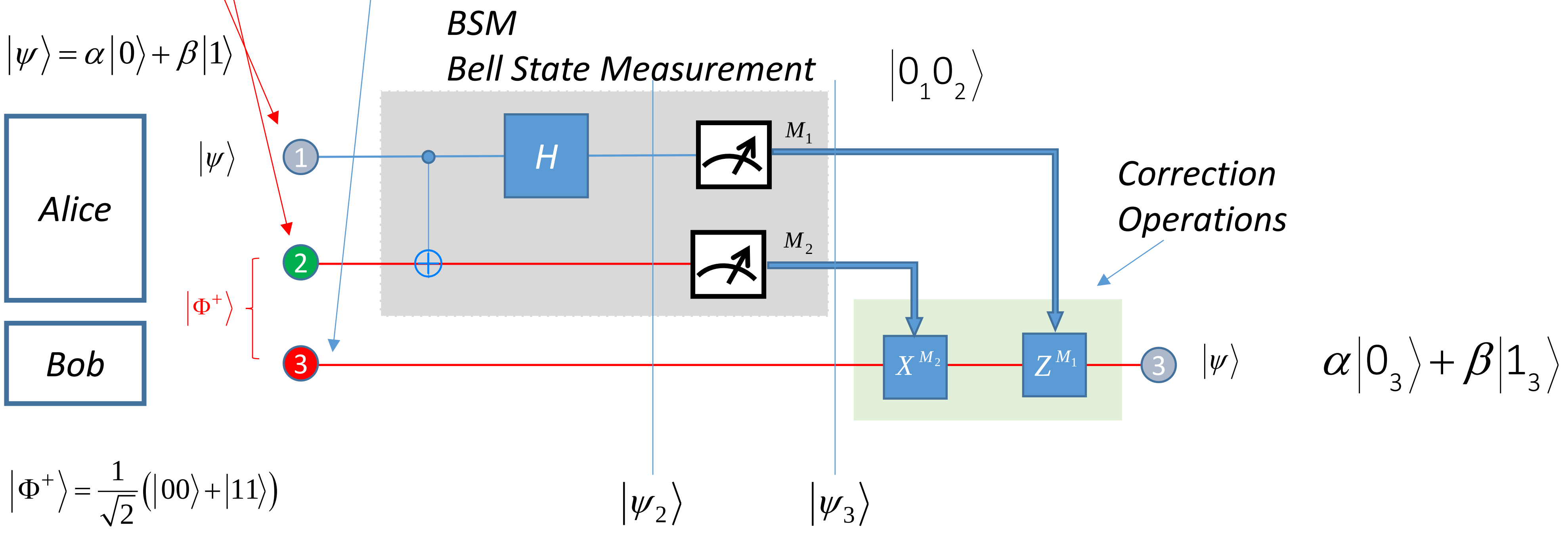
– The first term has Alice's qubits in the state $|0_1 0_2\rangle$, and Bob's qubit in the state $\alpha|0_3\rangle+\beta|1_3\rangle$ - which is the original state $|\psi\rangle$

– If Alice performs a measurement and obtains the result $|0_1 0_2\rangle$ then Bob's system will be in the state $|\psi\rangle$

Quantum Circuit for Teleportation

$$|\psi_2\rangle = \frac{1}{2}\Big[|0_1 0_2\rangle\big(\alpha|0_3\rangle + \beta|1_3\rangle\big) + |0_1 1_2\rangle\big(\alpha|1_3\rangle + \beta|0_3\rangle\big) + |1_1 0_2\rangle\big(\alpha|0_3\rangle - \beta|1_3\rangle\big) + |1_1 1_2\rangle\big(\alpha|1_3\rangle - \beta|0_3\rangle\big)\Big]$$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$

Quantum Circuit for Teleportation

$$|\psi_2\rangle = \frac{1}{2}\Big[|0_1 0_2\rangle\big(\alpha|0_3\rangle + \beta|1_3\rangle\big) + |0_1 1_2\rangle\big(\alpha|1_3\rangle + \beta|0_3\rangle\big) + |1_1 0_2\rangle\big(\alpha|0_3\rangle - \beta|1_3\rangle\big) + |1_1 1_2\rangle\big(\alpha|1_3\rangle - \beta|0_3\rangle\big)\Big]$$

- Depending on Alice's measurement outcome, Bob's qubit will end up in one of these four possible states

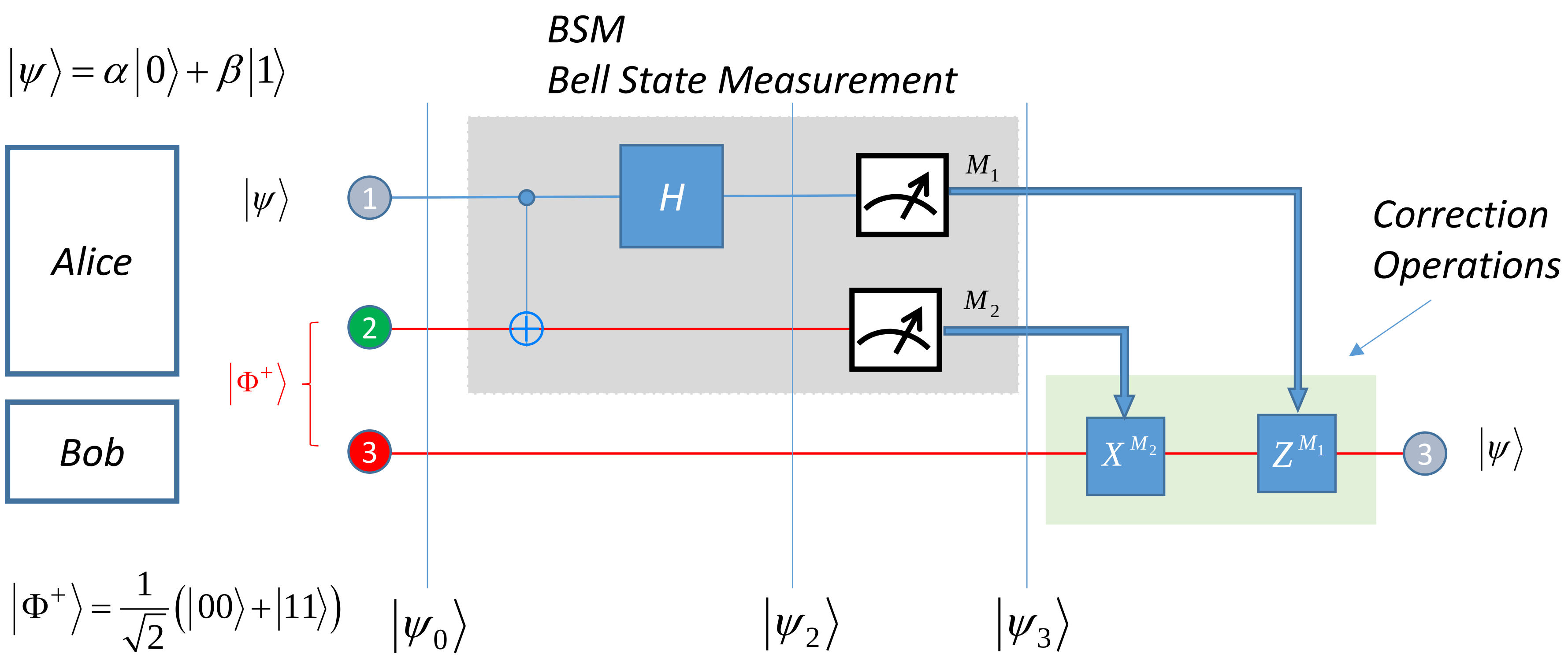
$$0_1 0_2 \rightarrow |\psi_3(0_1 0_2)\rangle = \big[\alpha|0_3\rangle + \beta|1_3\rangle\big] \rightarrow \mathrm{Prob} = 1/4$$

$$0_1 1_2 \rightarrow |\psi_3(0_1 1_2)\rangle = \big[\alpha|1_3\rangle + \beta|0_3\rangle\big] \rightarrow \mathrm{Prob} = 1/4$$

$$1_1 0_2 \rightarrow |\psi_3(1_1 0_2)\rangle = \big[\alpha|0_3\rangle - \beta|1_3\rangle\big] \rightarrow \mathrm{Prob} = 1/4$$

$$1_1 1_2 \rightarrow |\psi_3(1_1 1_2)\rangle = \big[\alpha|1_3\rangle - \beta|0_3\rangle\big] \rightarrow \mathrm{Prob} = 1/4$$

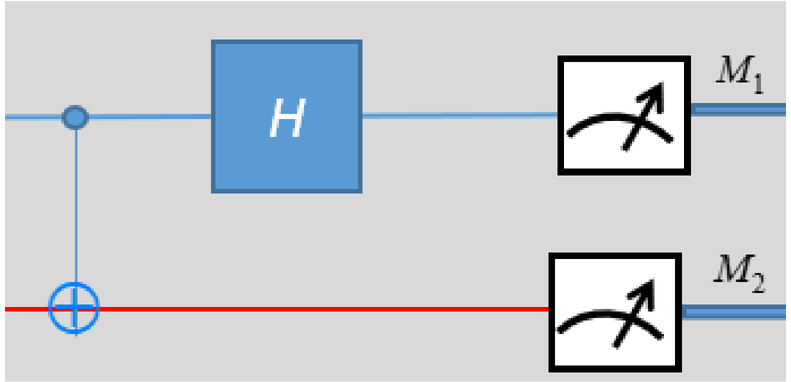
Quantum Circuit for Teleportation



Of course, to know which state it is in, Bob must be told the result of Alice's measurement

The Bell State Analyzer (BSA)

- The device which performs the Bell State Measurement (*BSM*) operation is known as a “**Bell State Analyzer**” (*BSA* for short)



Quantum Challenges

- Light moves substantially slower through optical fiber than air or vacuum, typically at about $c_{\text{fiber}} = 0.7c$
- One-way latency through fiber is about $5\mu s$ per *kilometer*

→ - Unfortunately, photons transmission is limited by losses in the channel, the same issue that affects classical communication

- To recover from losses in the fiber, simple signal amplification provides an elegant solution for the classical world
- However, this is not possible in the quantum world, as the no-cloning theorem forbids qubits to be copied or amplified

Quantum Challenges

- Since copy and resend strategy is fundamentally not allowed in quantum communication, it would seem we have to resort to send the single photons and hope for the best that they arrive at the destination
- Let's do a quick calculation that will demonstrate that this also is not a viable strategy
- The probability that we transmit the photon through a fiber with **attenuation parameter *α*** over distance ***L*** is given by the following,

$$P_{\text{success}} = 10^{-\alpha L/10}$$

Quantum Challenges

$$P_{\text{success}} = 10^{-\alpha L/10}$$

- Let's plug in some numbers to give us some intuition of what this probability is
- Consider a long fiber of 1000 km (which in the context of global communication is not that long), and assume a **best-case scenario** where the fiber has **ultra-low attenuation** of a mere 0.1 dB/km
- The probability that a single photon gets successfully transmitted is $\mathbf{10^{-10}}$
- This is indeed a very small number, but to gain some intuition of how small, consider the case of a source that produces **one photon every second**

Quantum Challenges

$$P_{\text{success}} = 10^{-\alpha L/10}$$

- Every second a single photon gets sent down the fiber
- How long do we need to wait in order for the photon to get successfully transmitted?
- On average, we expect to wait **317 years**!
- We can see that even with ultra-low loss fibers over **moderate** distances, sending a single photon down the fiber and hoping for the best is not very practical!

Quantum Challenges

- Moreover, **loss** is only one source of error that we have to face in long distance quantum communication
- Other sources include **unitary errors** such as Pauli errors, namely X errors that randomly flip the state of our photons, or Z errors where we introduce a phase to the photons

Quantum Challenges

- We do not have to deal with most of these errors in classical communication
- The situation looks dire for quantum communication
- However, quantum problems require quantum solutions, and indeed there are **clever ways** that **address these issues**
- Since signal amplification is *ruled out* as a means to overcome losses, a radically new technological development – the *quantum repeater* – was conceived in order to build the quantum internet

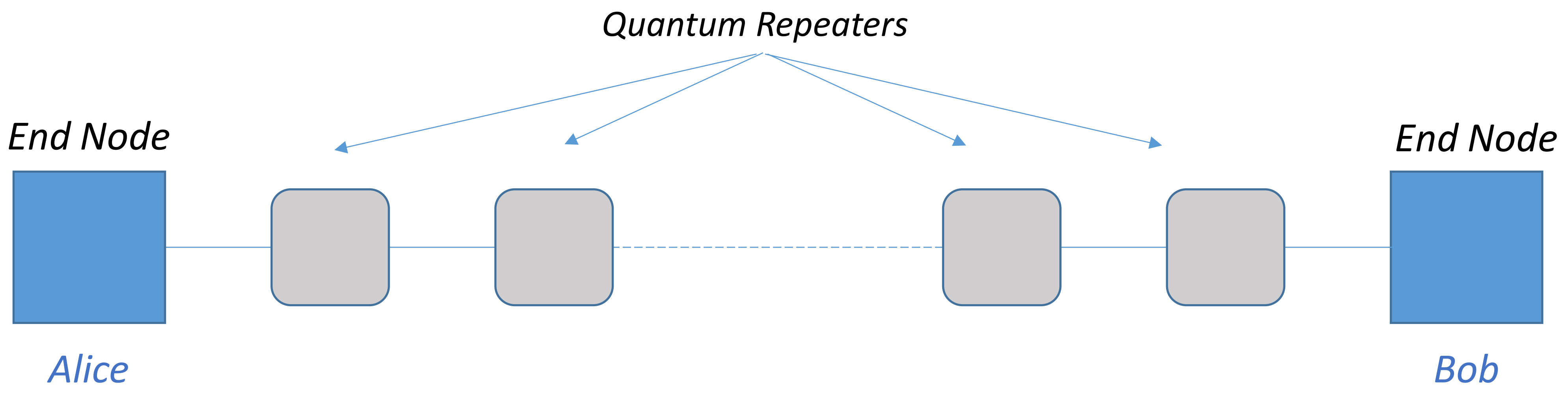
Quantum Repeater

- Realizing a quantum internet demands substantial development to realize *quantum repeaters*

- Today's technology is a long way from a fully-fledged quantum repeater, but research is advancing rapidly…….

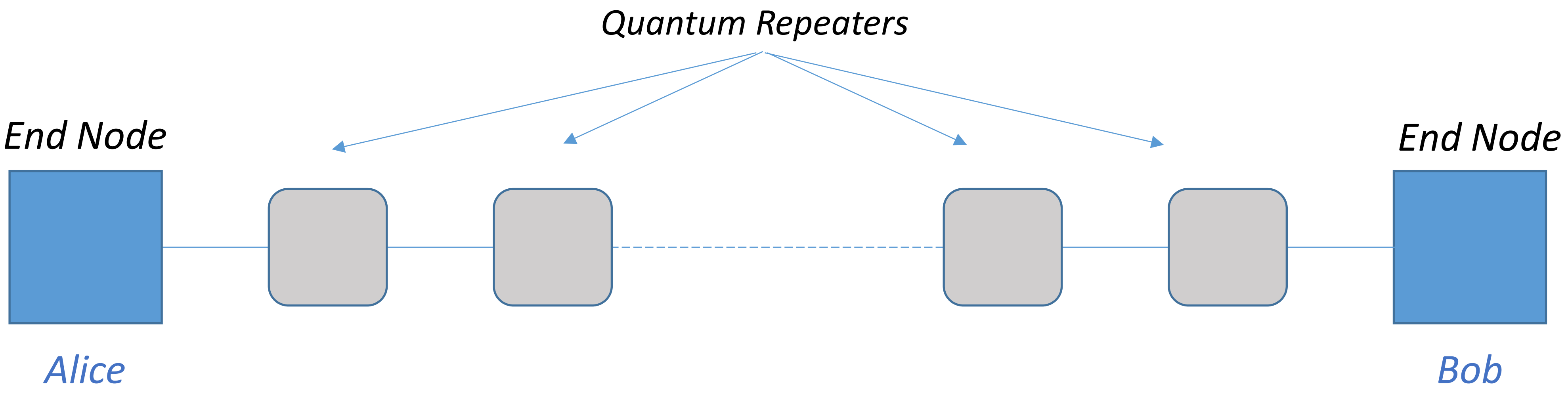
Quantum Repeater

- Quantum repeaters enable one to create a **known maximally entangled** state between two *end nodes* (quite often called *Alice & Bob*) of the network *by first segmenting the fiber optic into small chunks*, and placing a quantum repeater between two consecutive chunks

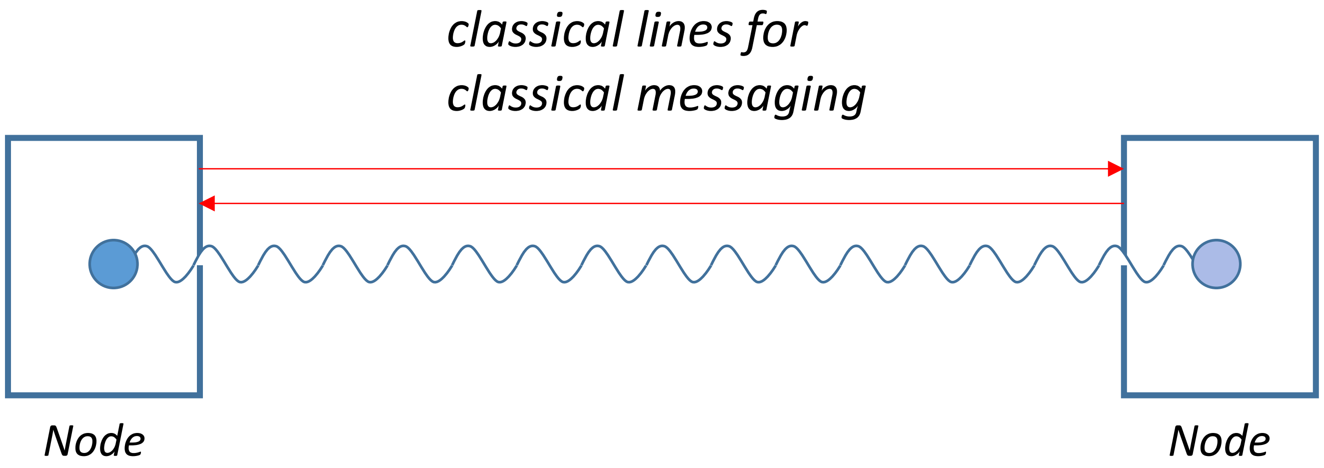


Entanglement Distribution

– Alice and Bob are interspaced with a series of repeaters that are located at a distance of a few tens of kilometers apart from each other



Quantum Repeater

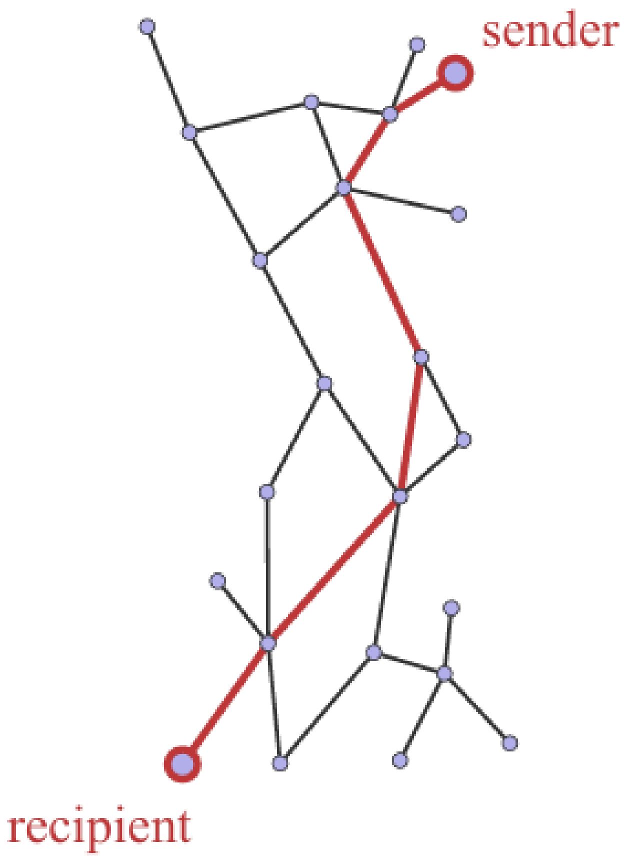


NOTE: Throughout the lectures, I simply use the term *node* when it is not necessary to make a distinction between the quantum *repeater* and the *end node*

- In order to coordinate their quantum operations, two **nodes** may be required to exchange classic messages
- This can be achieved by using classical lines or transport connections provided by the classical Internet infrastructure

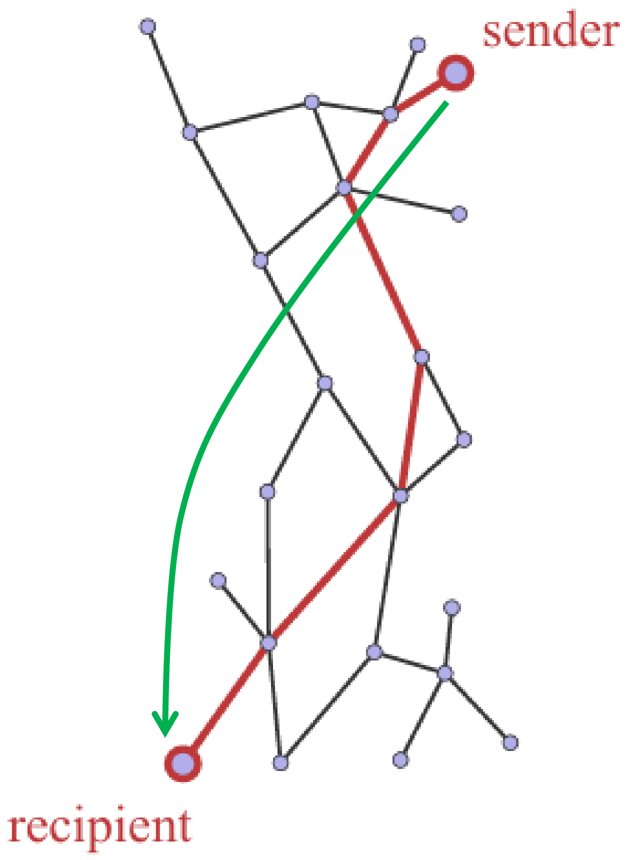
Quantum Repeater

- In quantum networks, information encoded in qubits is not transmitted directly
- Rather, we use entangled pairs of qubits to teleport the state of the qubit
- Each link connecting two devices shares a Bell pair that is used to pass the state of the qubit carrying the quantum message from one device to the next



Quantum Repeater

- The figure illustrates a quantum network, where the quantum sender wishes to pass the state of a qubit to the recipient
- The state of the sender’s qubit can be teleported hop-by-hop along the red path until it reaches the recipient
- The problem with this approach is that the operations required by the teleportation protocol as well as the memories used to store Bell pairs are not perfect and this decreases the fidelity of the teleported qubit



Quantum Repeater

- Repeating the teleportation in this hop-by-hop approach degrades the fidelity of the entangled state, resulting in a garbled quantum state being teleported to the recipient
- One way around this problem is to use the link-level Bell pairs to create a **direct entangled connection** between the **sender** and the **recipient** so that **the quantum state can be teleported in one hop**

Quantum Repeater

From now on, we will tackle this problem by addressing **four requirements**

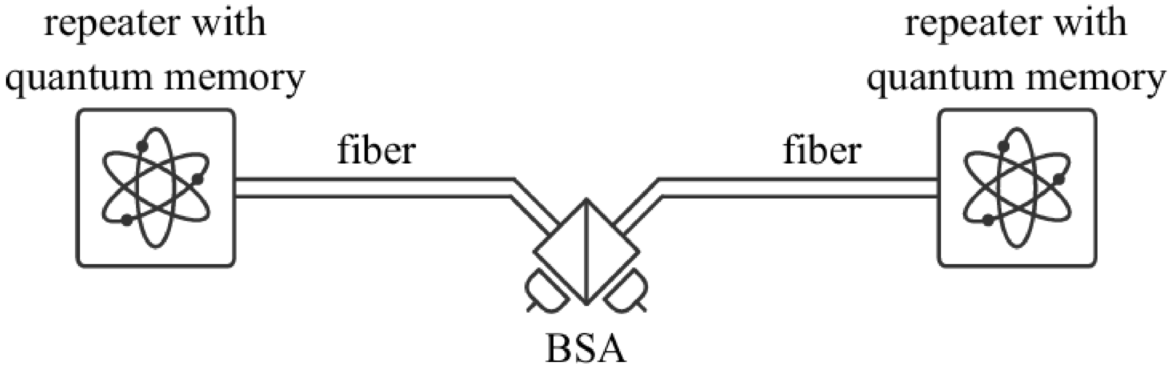
- **First**, we will show how to establish **link-level entanglement** between neighboring quantum repeaters of a quantum network
- The process of creating short-distance entangled qubit-pairs (or **entangled links**) between adjacent nodes is called **Entanglement Distribution**
- **Second**, we will discuss how the *link level entanglement* can be used to create a *long-distance entangled connection* between end nodes using **Entanglement Swapping**

Quantum Repeater

- **Third**, we will deal with the issues presented by the adverse effects of noise by illustrating the process called **Entanglement Purification/Distillation**
- **Fourth**, we will look at management of networks, routing, multiplexing and resource management

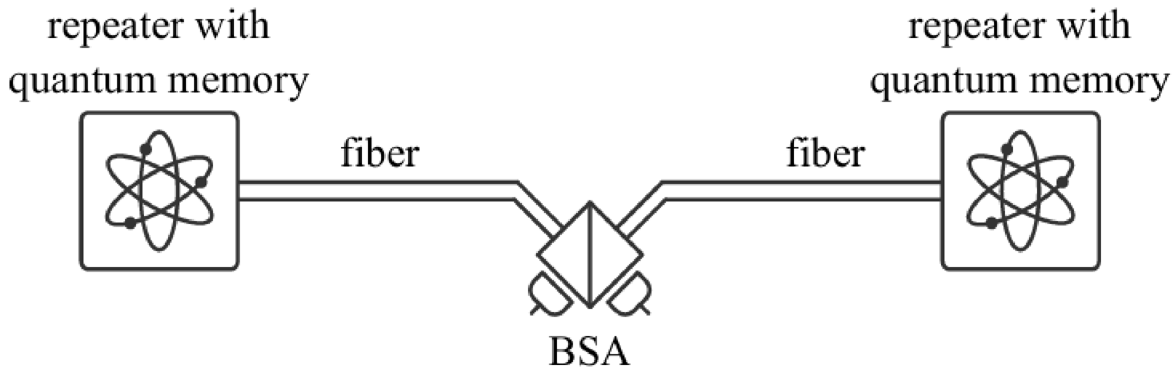
Making Link-Level Entanglement

- To start off, we look at the challenge of establishing **link-level entanglement** between two adjacent repeaters in a quantum network
- One method for entangling two quantum memories and creating link-level entanglement, known as the **memory-interfere-memory** (*MIM*) link architecture, is pictured in figure



Making Link-Level Entanglement

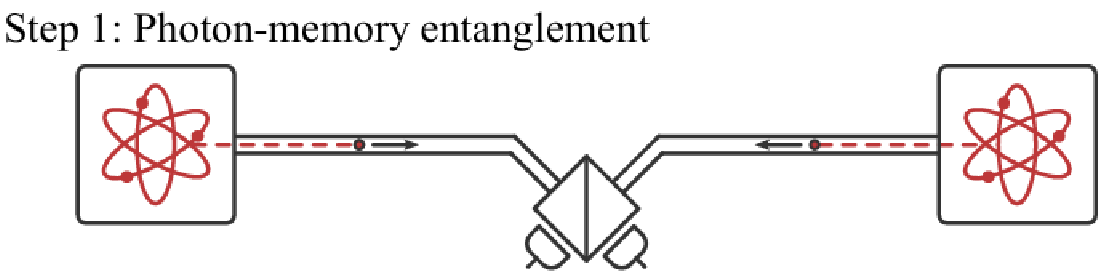
- Another name for this architecture is **“meet-in-the-middle”**
- Each repeater node is equipped with a quantum memory, and is coupled to an optical fiber



- The fibers lead to a Bell state analyzer (*BSA*), an optical device which is capable of measuring two incoming photons in the **Bell basis**
- We will discuss an implementation of the *BSA* very shortly

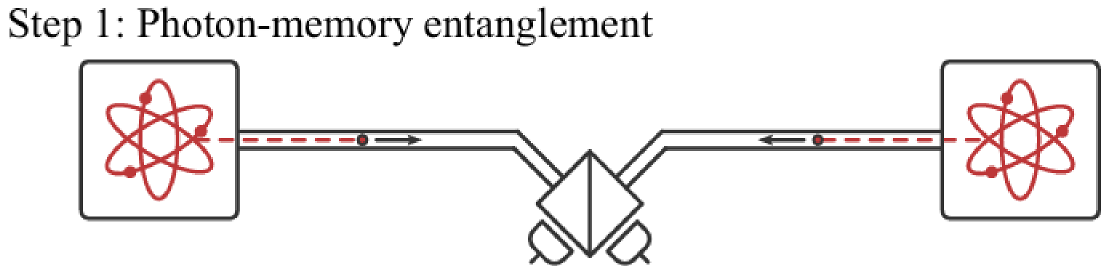
Making Link-Level Entanglement

- The protocol to generate entanglement between the quantum memories is the following
- **Each quantum memory emits a photon that is entangled with it,** represented by the red dashed line in Step 1 of the figure reported below



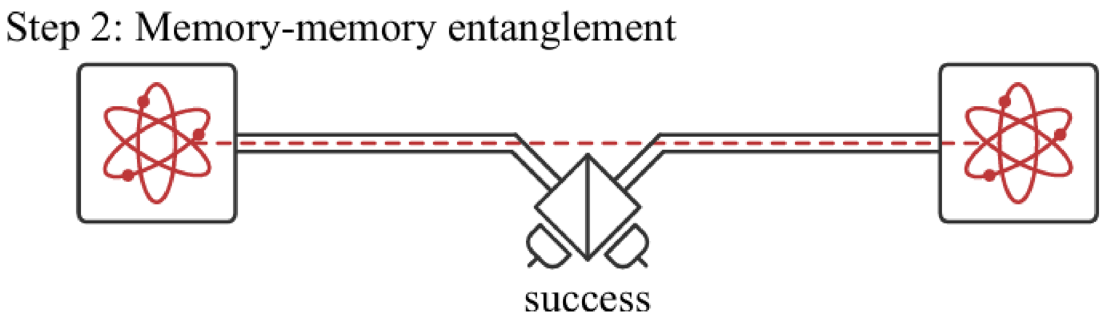
Making Link-Level Entanglement

- These photons are captured and coupled to a fiber, which guides them to the *BSA*
- At the ***BSA*** they **are measured in the Bell basis** and destroyed in the process
- The success probability of the Bell state measurement depends on the implementation of the *BSA*
- Straightforward implementation using only linear optics elements results in success probability of at most 50%.



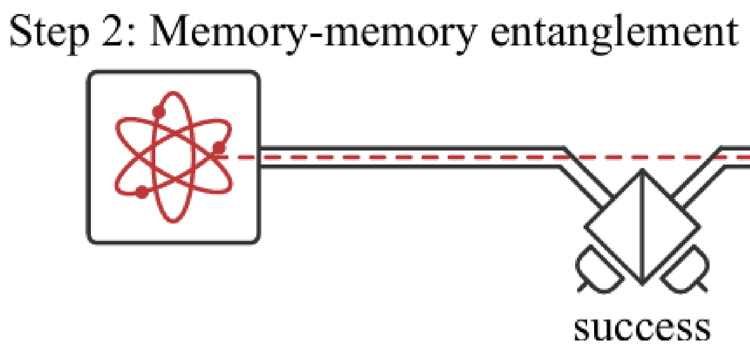
Making Link-Level Entanglement

- This is assuming ideal couplers (to collect photons emitted by the memories), fibers, and detectors at the *BSA*
- Once the measurement is successful, the entanglement between the memory photon pairs is transferred to be between the memories as seen in Step 2
- Straightforward implementation using only linear optics elements results in **success probability of at most 50%.**



Making Link-Level Entanglement

- A crucial point about this scheme, and other link-level architectures, is that the photons arriving at the *BSA* to be measured in the Bell basis must arrive at the *BSA* **simultaneously**
- Even relatively small delays in their arrival times, result in a diminishing success probability of the Bell state measurement

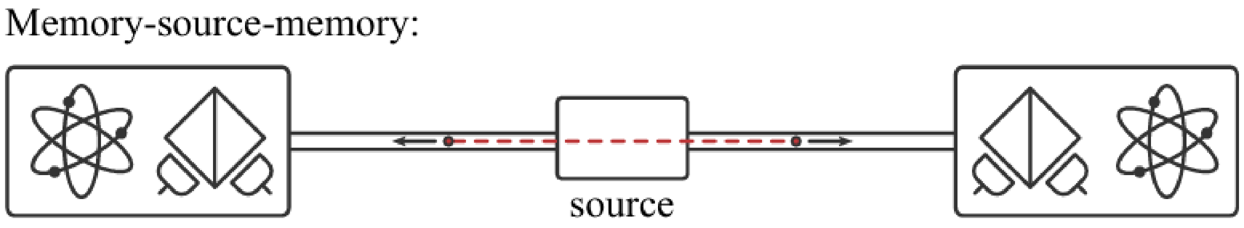


Making Link-Level Entanglement

- The figure below shows one more *link-level architectures* which is called **memory-memory** (MM), where the *BSA* device is included in one of the repeater nodes
- Entanglement is established in a fashion similar to the *MIM* architecture, but this time the photon generated by the quantum memory on the left travels the whole length of the fiber to the right node

Making Link Level Entanglement

- The figure reported below shows the **memory-source-memory** (MSM) architecture
- A **source of entangled photon pairs** is located between the quantum repeaters
- These photons are sent to the quantum repeaters where they interfere with photons emitted locally by the quantum memories



Stationary and Flying Qubits

Implementing these architectures requires addressing two key challenges:

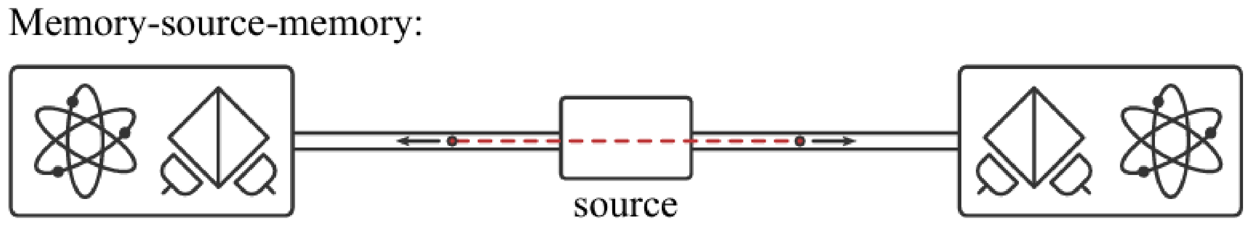
- *Converting or entangling stationary and flying qubits*
- *Enabling the long-distance transmission of flying qubits; i.e., the distance between two adjacent repeaters*

Stationary and Flying Qubits

- The **first** of these is conversion or *entanglement between stationary and flying qubits*
- **Stationary qubits** are those qubits that are sitting in the quantum network nodes, loaded into the **quantum memories (QM)**
- For most common quantum repeater schemes, a *quantum memory (QM)* is a core enabling device, which allows *single photons* to be *temporarily stored* on a *long-lived matter state* and *retrieved on demand*, enabling the storage of entanglement generated for example, by *EPS* over elementary links

Stationary and Flying Qubits

- **Flying qubits** are those qubits that are used for entanglement swapping in the *BSAs* to create link level entanglement between the quantum memories
- We must be able to **entangle photons (the flying qubits) with the stationary qubits** inside the memories, but also, we must be able to use **entanglement swapping to create end-to-end entanglement**



Stationary and Flying Qubits

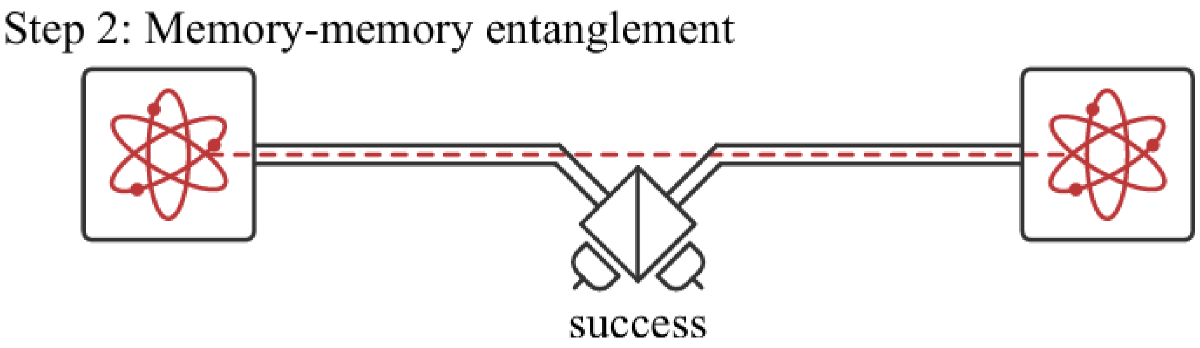
- Finally, we must also be able to transport **flying qubits** between adjacent repeaters or between a source and the two repeaters with which it is communicating
- The physical system used for **flying qubits** are photons

Stationary and Flying Qubits

- The topic we will be discussing next will be **memory lifetime** and **two communication requirements**
- The **ratio** of gate speed to **memory lifetime** is important in quantum computing
- **Longer** memory lifetimes and **shorter** gate speeds let us carry out more complex calculations
- In the context of **quantum communications**, we often store qubits for long periods of time without acting on them, as we await messages from partners in the network

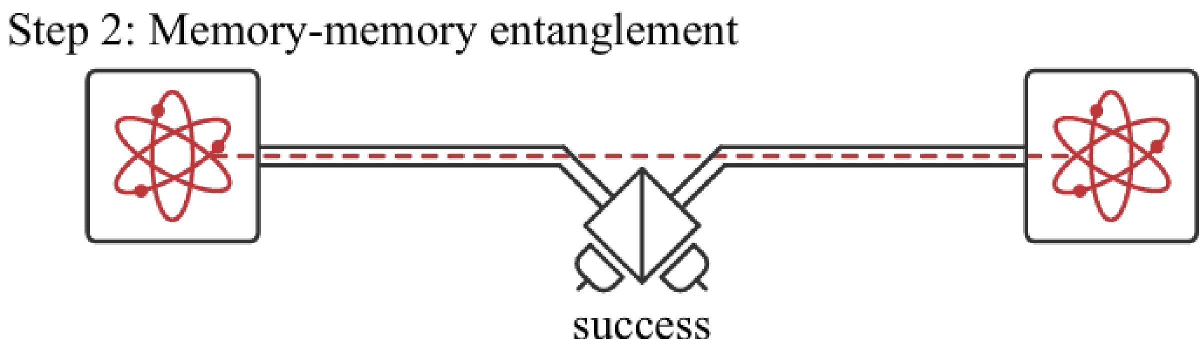
Stationary and Flying Qubits

- Therefore, what is more important is not the gate speed itself, but the **ratio** of **memory lifetime** to **communication time**, provided that the **gate time** is short compared to the ***round trip time* (*RTT*)**
- Let's consider how we establish link level entanglement using the ***MIM*** architecture



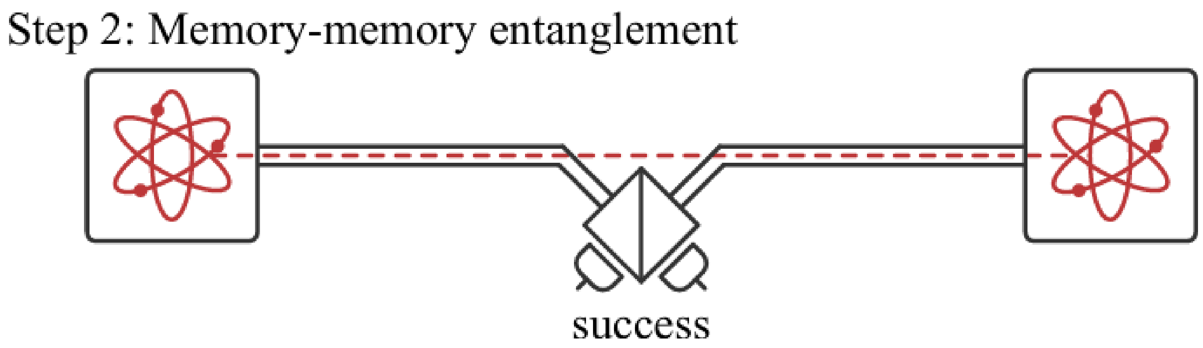
Stationary and Flying Qubits

- The quantum memories emit photons entangled with their respective memory
- These photons are guided by the optical fiber to the *BSA*, where they get measured in the Bell basis
- The *BSA* then communicates the outcome of the measurement back to the network nodes via a classical message



Stationary and Flying Qubits

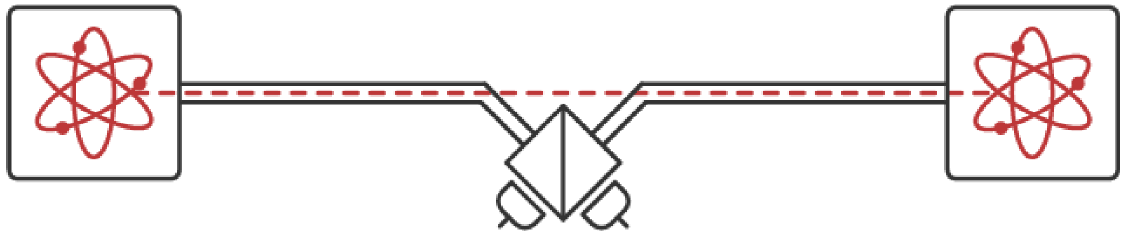
- The **total time** needed for the emitted photon to reach the BSA and the classical message to reach the node holding the quantum memory is the round trip time or *RTT*



Stationary and Flying Qubits

- If the **memory lifetime** is **shorter** than the ***RTT***, then we cannot reliably establish the link-level entanglement
- Even if we successfully perform the Bell-state measurements on the photon pairs at the *BSA*, by the time the return messages are received, our memories will have decohered and are not useful anymore

Step 2: Memory-memory entanglement



Stationary and Flying Qubits

– The following table shows some typical *RTTs* for various length of fiber

distance (km)	RTT in fiber
1	10μsec
10	100μsec
100	1msec
1,000	10msec
10,000	100msec

- The speed of light in a fiber is approximately **0.2 meters per nanosecond**, so if our nodes are one kilometer apart, one round trip from one node to the other and back takes 10 microseconds.

– For a hundred kilometers, it increases to one millisecond, and for ten thousand kilometers it goes all the way up to 100 milliseconds (0.1 seconds) per round trip time.

Stationary and Flying Qubits

- What processes degrade the quantum memories?
- The two main processes are **energy relaxation** and **dephasing** that we have analyzed in detail
- They are characterized by two different time scales, referred to as the T_1 time scale and the T_2 time scale, respectively

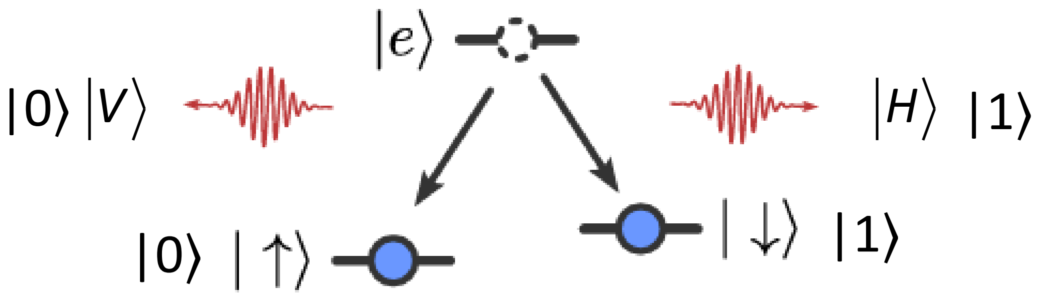
Stationary and Flying Qubits

- Now that we have talked about the lifetimes of memories, why they are important, and given some characteristic time scales for long-distance communication, let's address the question of how *atoms can be entangled with photons at the physical level*

- **First,** we should consider what states of the atom represent the qubit basis states $|0\rangle$ and $|1\rangle$ in the quantum memory

Stationary and Flying Qubits

- The following figure shows the new atomic structure for quantum memory



- The ground state is now degenerate, and spanned by two orthogonal states
- The two ground states are distinguished by their spin
- One is spin up $|\uparrow\rangle$, and the other is spin down $|\downarrow\rangle$

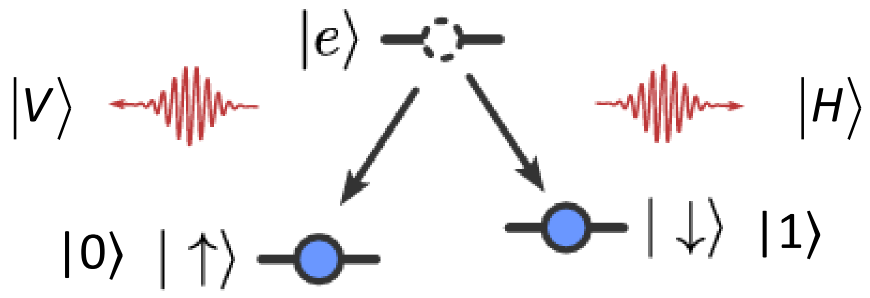
Stationary and Flying Qubits

- Suitable encoding for the flying qubit is the linear polarization of the photon
- State $|0\rangle$ is represented by vertical polarization $|V\rangle$, and $|1\rangle$ is represented by horizontal polarization $|H\rangle$ as we saw at the beginning of this course
- This new representation is summarized in the following table

	quantum memory	**flying qubit**
physical system	ground state spin	linear polarization
$\mathbf{|0\rangle}$	$|\uparrow\rangle$	$|V\rangle$
$\mathbf{|1\rangle}$	$|\downarrow\rangle$	$|H\rangle$

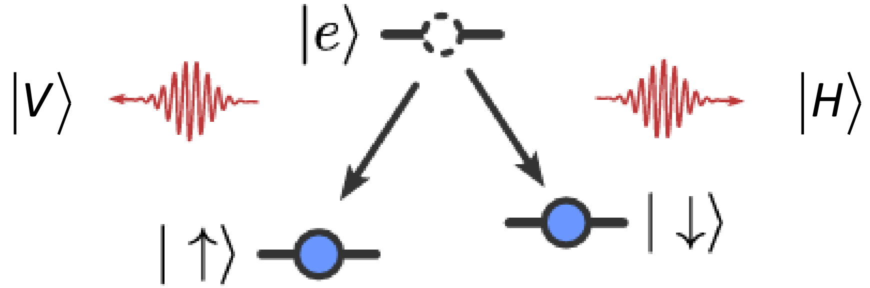
Stationary and Flying Qubits

- Let's see how this representation leads to an entangled memory-photon pair
- We start by preparing the atom in the excited state $|e\rangle$
- The atom has a 50-50 probability of decaying either to the spin up $|\uparrow\rangle$ or the spin down $|\downarrow\rangle$ ground state



Stationary and Flying Qubits

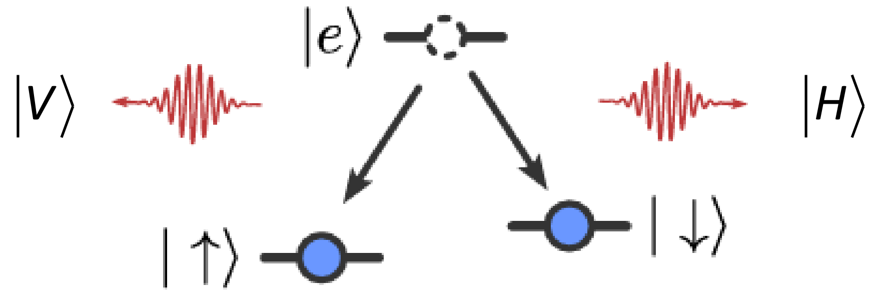
- If the atom decays into the $|\uparrow\rangle$ state, the emitted photon will be polarized in the $|V\rangle$ state
- If the atom decays into the spin $|\downarrow\rangle$ state, the photon will be polarized in the $|H\rangle$ state



Stationary and Flying Qubits

– Since we do not know the polarization of the emitted photon until we measure it, the total memory-photon state is an equal superposition of the two possibilities

– We can write this transformation as



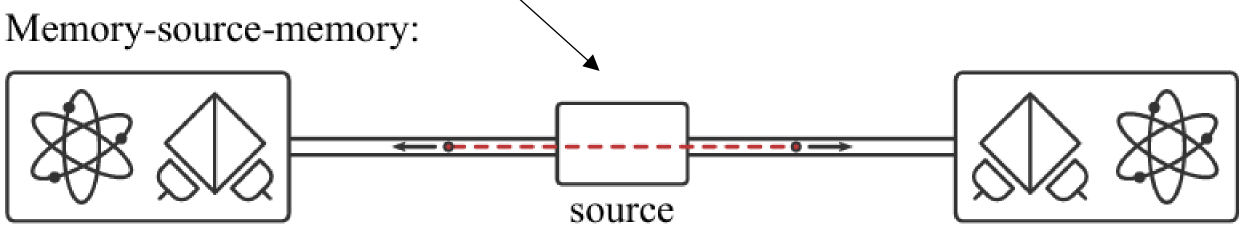
$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|V\rangle + |\downarrow\rangle|H\rangle\right)$$

Stationary and Flying Qubits

- This concludes our discussion of the physical implementation of the *MIM* link architecture
- Furthermore, our previous discussion applies to the *MM* link architecture as well, we just need to place the *BSA* inside one of the nodes

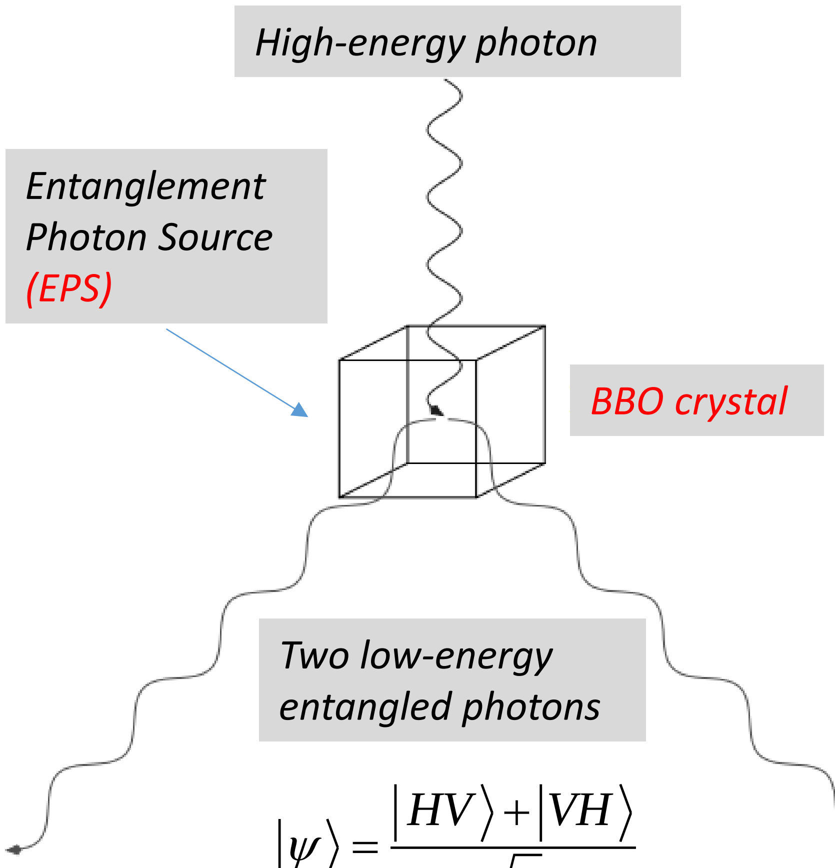
Stationary and Flying Qubits

– Addition of a **source** of entangled photon pairs, such as the *Spontaneous Parameter Down-Conversion* (*SPDC* for short) would cover also the physical implementation of the *MSM* link architecture



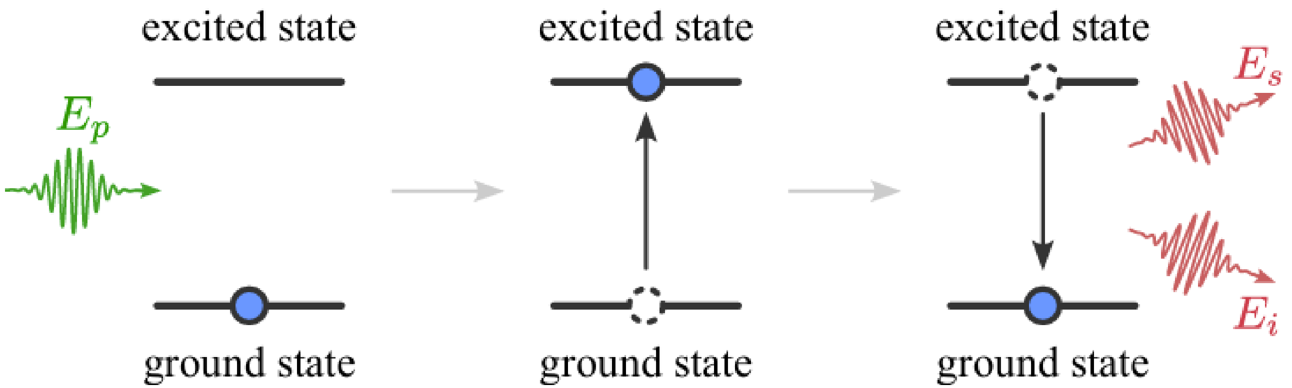
Creating Pairs of Entangled Photons

- The *SPDC* method for generating entangled photon pairs (qubits) involves directing a high-energy photon into a **special crystal**
- This crystal absorbs the single photon and then emits, with some very low probability, a pair of entangled, lower-energy photons, as illustrated in the figure
- The entangled photons are *vertically (V)* and *horizontally (H)* polarized



$$|\psi\rangle = \frac{|HV\rangle + |VH\rangle}{\sqrt{2}}$$

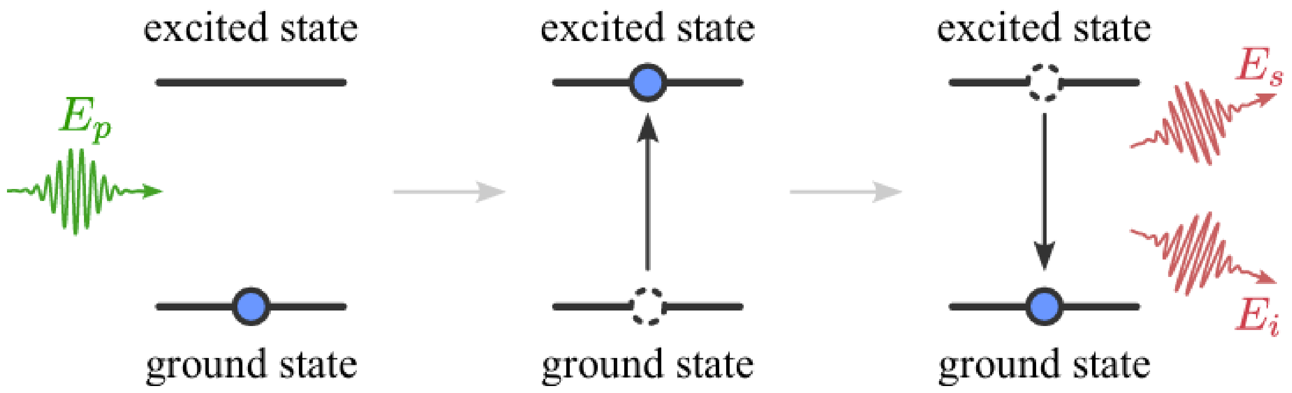
Creating Pairs of Entangled Photons



- The basic idea behind *SPDC* is illustrated in Figure
- A high-energy pump photon, pictured in green, is incident on a nonlinear crystal

Creating Pairs of Entangled Photons

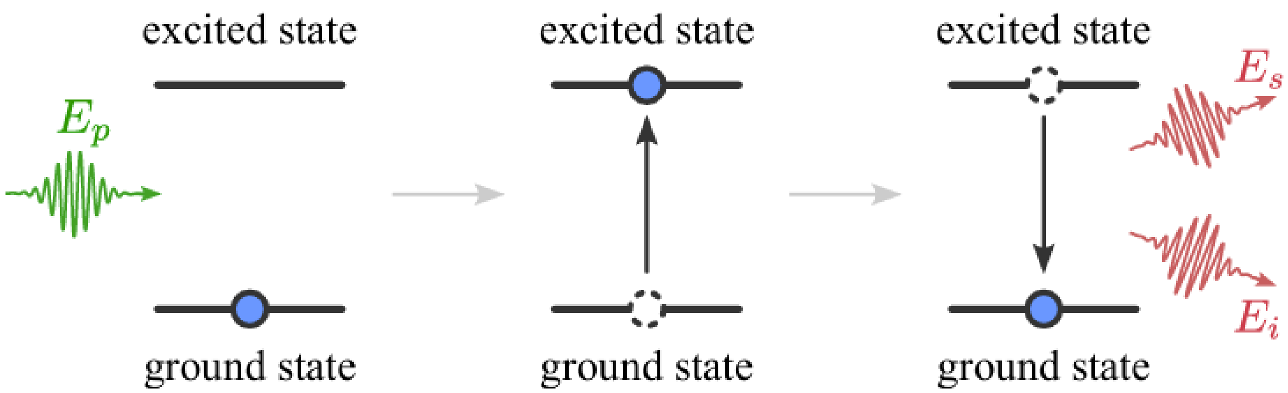
- A popular choice of material for the crystal is **beta barium borate**, or ***BBO*** for short



- We can tune the energy of the pump photons to be resonant with a particular transition frequency of the atoms inside the **BBO crystal**

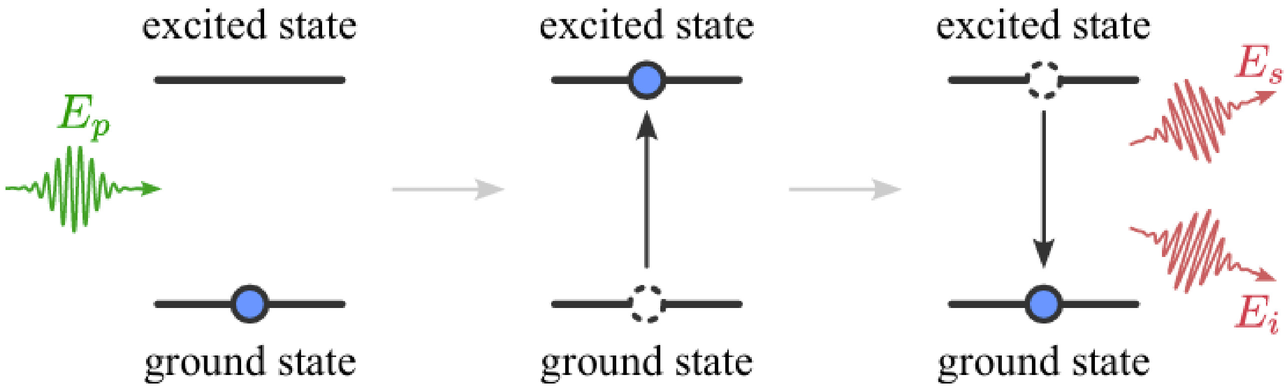
Creating Pairs of Entangled Photons

- Therefore, to a good approximation the pump laser will only affects those two energy levels
- The lower level is called the **ground state** and the upper level is called the **excited state**
- The atom in the *BBO* crystal initially starts in the ground state
- The pump photon carries the right amount of energy to excite the atom



Creating Pairs of Entangled Photons

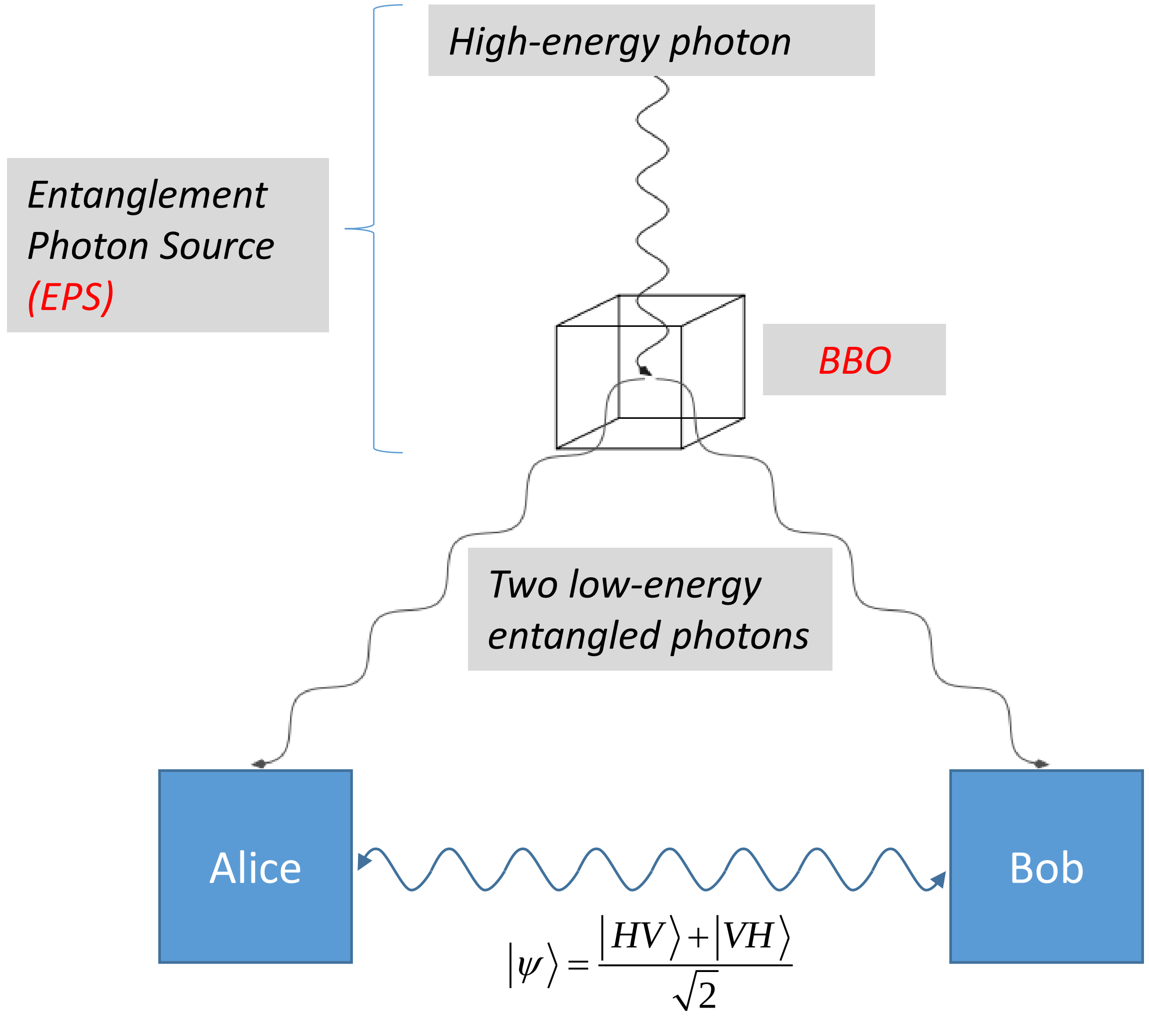
- After a short time the atom de-excites and transitions back into the ground state
- The **vast majority** of the time, this de-excitation is coupled with the emission of a **single**, **high-energy** photon
- With some **very low probability**, however, the atom emits **two entangled photons**



Creating Pairs of Entangled Photons

- Once the entangled photons (qubits) are created, we need to transfer one to Alice and the other to Bob
- We have two options for our optical channels: *free space transmission*, and *optical fibers*

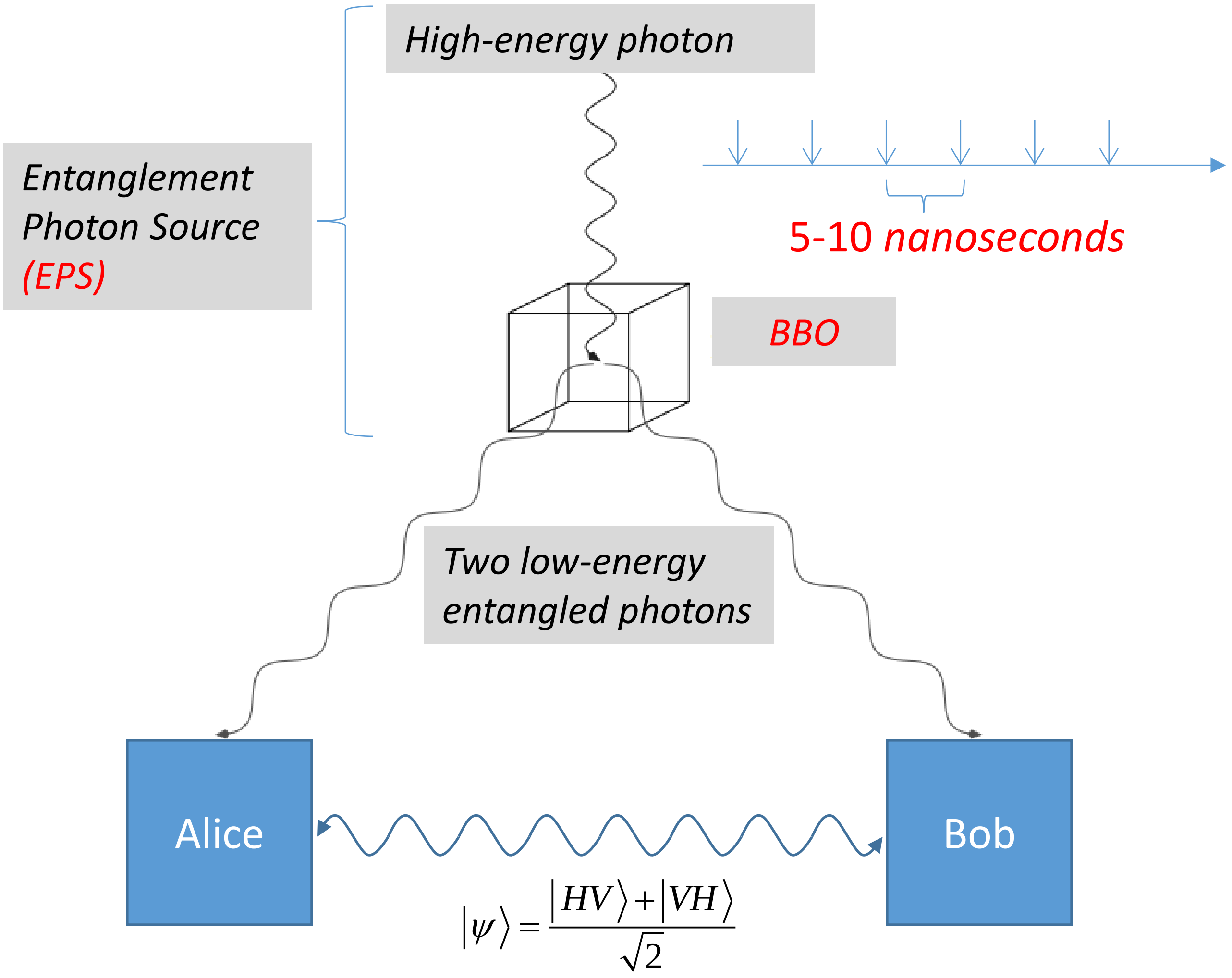
→ - The latter option will be addressed hereafter



Creating Pairs of Entangled Photons

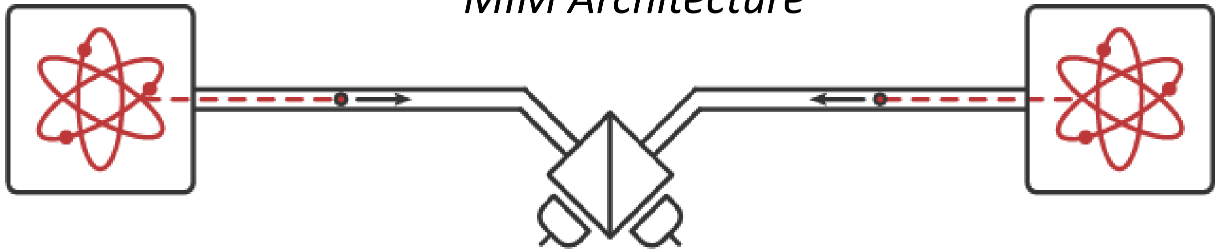
- Using current technology, high-energy photons can be emitted every 5-10 nanoseconds

- The probability of each high-energy photon producing a pair of low-energy entangled photons is between 0.1 and 0.5



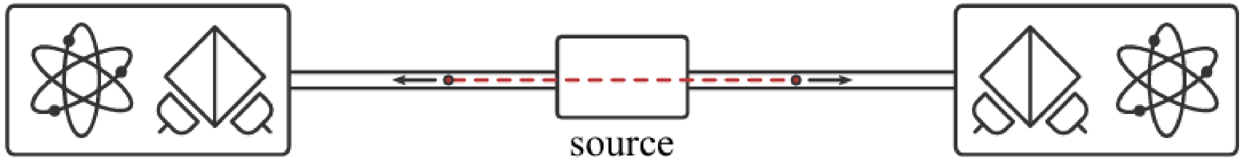
MIM, MM, and MSM Architectures

MIM Architecture



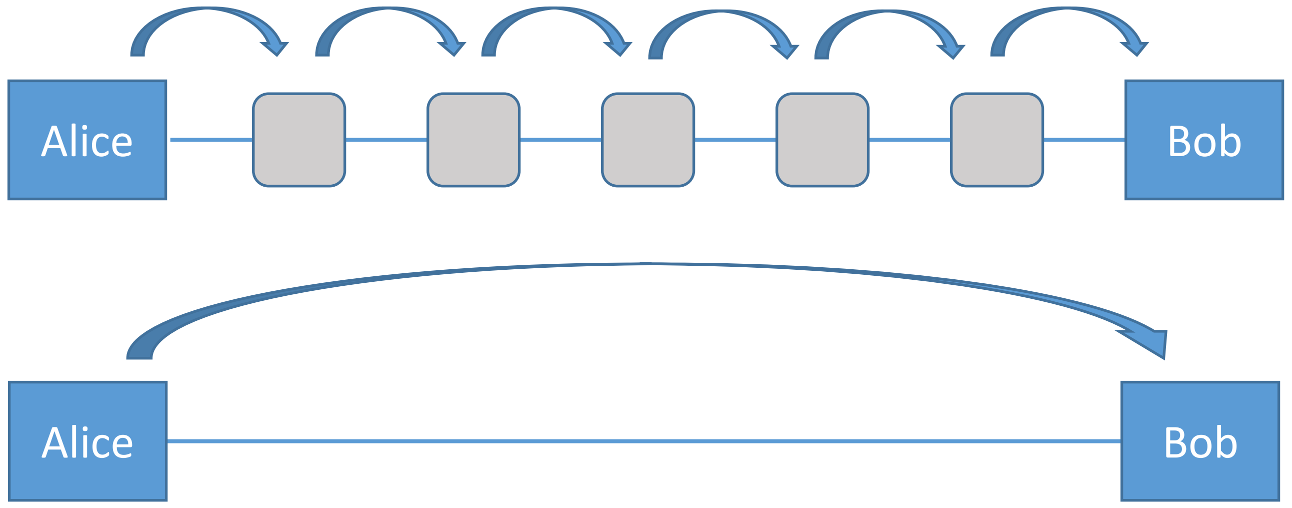
MM Architecture

MSM Architecture



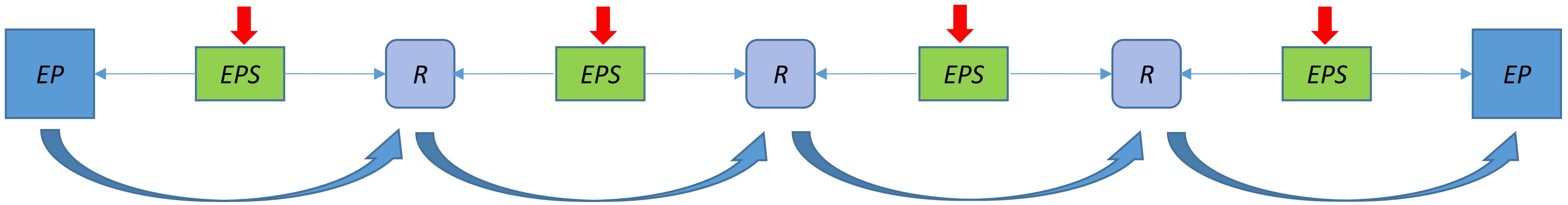
Entanglement Distribution

- *Entanglement Distribution* - the process of creating short-distance entangled qubit-pairs (or entangled links) between adjacent nodes
- Now entanglement is needed only between adjacent nodes and thus the *success probability* for generating the entangled link depends on the *distance of the adjacent nodes*, rather than on the *total distance between end nodes*



Entanglement Distribution

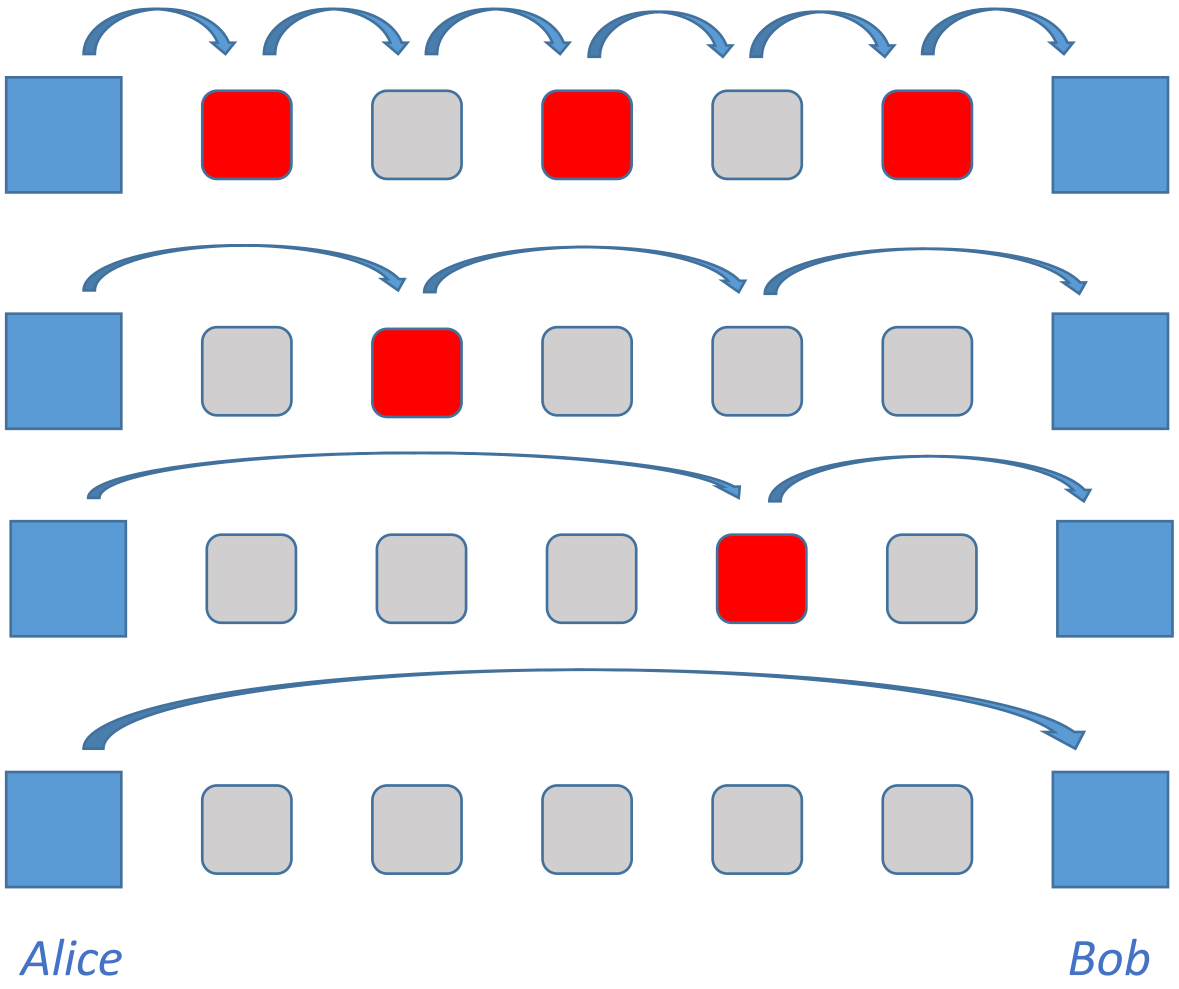
- The *MSM approach,* also called *Midpoint Source (MS)*, is currently the most promising approach for distributing entanglement between two adjacent nodes
- To create entanglement links between shorter-range adjacent nodes, the Entanglement Photon Source (*EPS*) can be situated halfway between the two endpoints in a node without memory of its own



Legenda
EPS: Entanglement Photon Source
EP: End Point
R: Quantum Repeater

Entanglement Swapping

- *Entanglement Swapping* - the process which allows to create a longer entangled link connecting adjacent nodes
- The red nodes are those which carry out the *entanglement swapping*
- Stated differently, quantum repeaters enable the establishment of *long-distance* entanglement connections spanning multiple nodes, by *stitching* together *short-distance* entanglement links

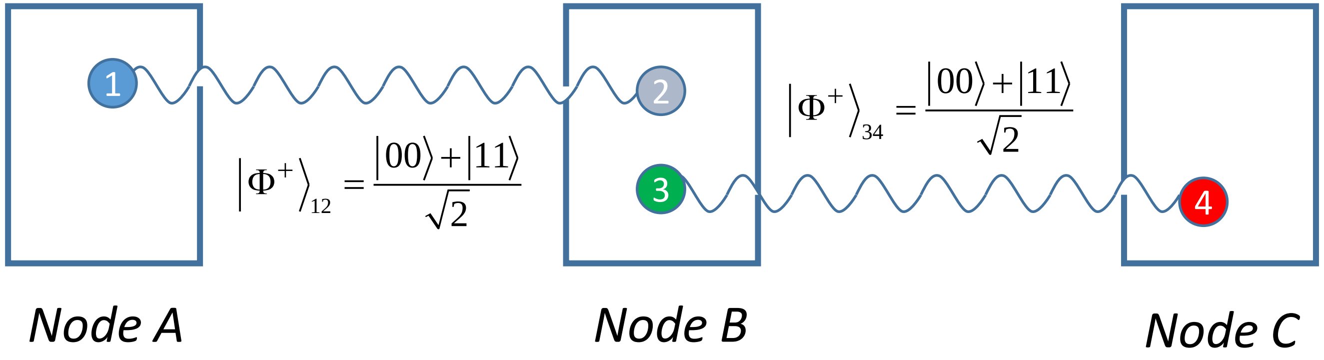


Entanglement Swapping

- So far, we have explained in words the entanglement swapping performed by a quantum repeater
- We shall now describe the entanglement swapping in the language of quantum computing

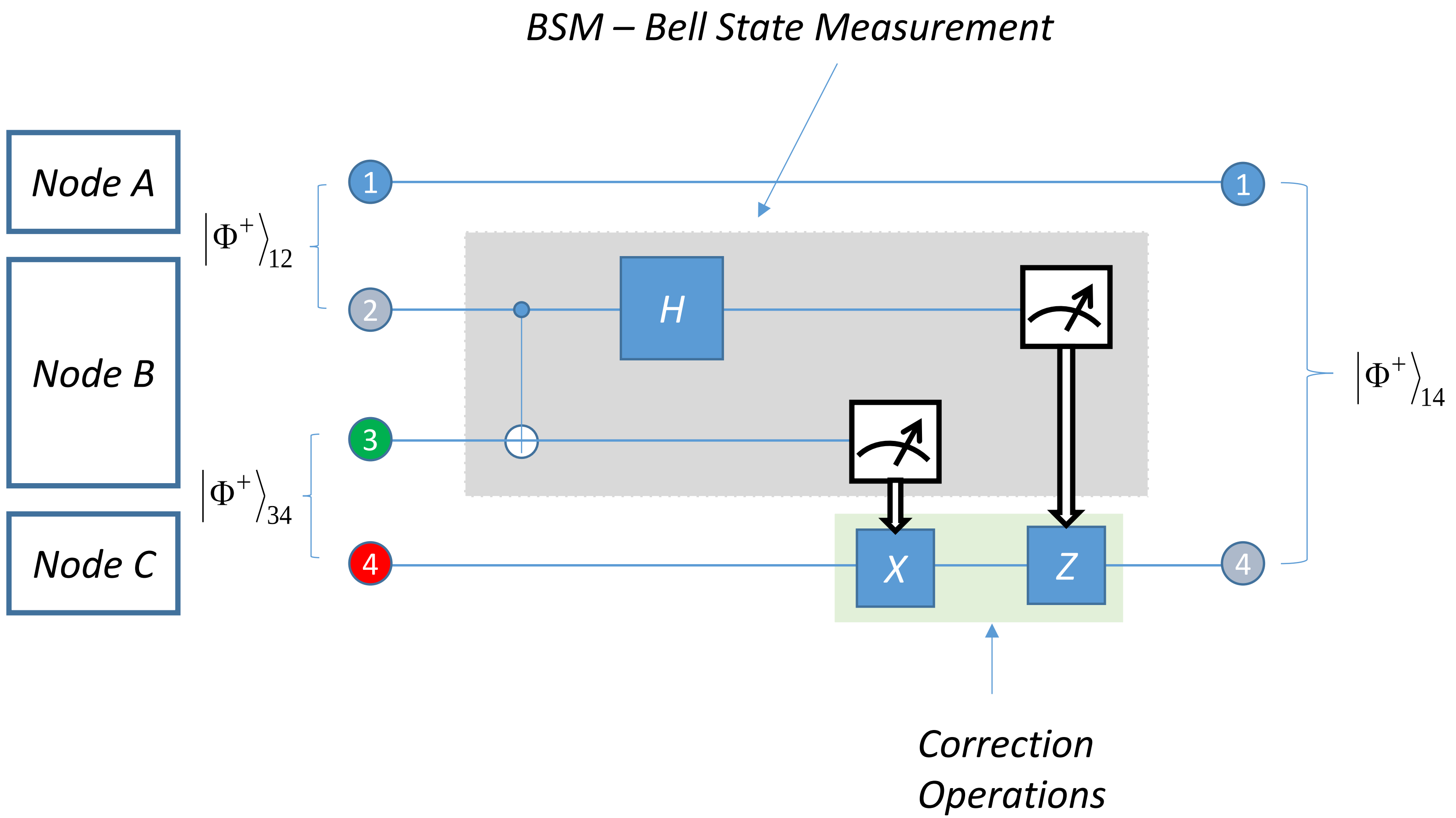
Entanglement Swapping

– Let's start with the following basic configuration of three nodes: *A*, *B*, and *C*



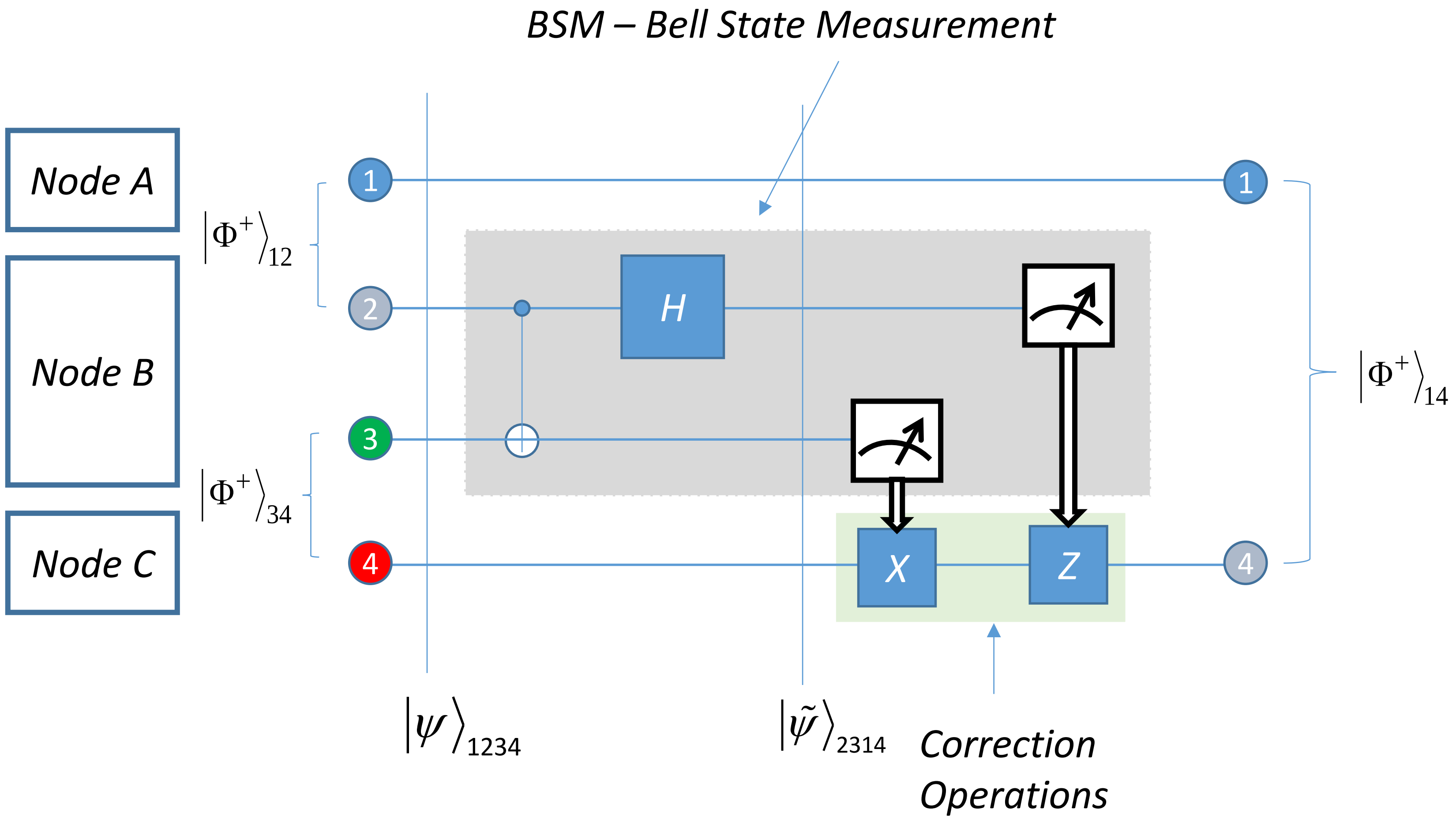
– Nodes *A* and *B* share one pair $|\Phi^+\rangle$ **of maximally entangled qubits** (qubits 1 and 2), and nodes *B* and *C* share another pair $|\Phi^+\rangle$ of **maximally entangled qubits** (qubits 3 and 4)

Entanglement Swapping Circuit



Entanglement Swapping Circuit

To carry out our computation, we use the following quantum circuit to represent the topological configuration shown in the previous slide



Entanglement Swapping Circuit

- The total state of the four qubits can therefore be written as

$$\begin{aligned}|\psi\rangle_{1234} &= |\Phi^+\rangle_{12} \otimes |\Phi^+\rangle_{34} = \frac{1}{\sqrt{2}}\left(|00\rangle_{12} + |11\rangle_{12}\right) \otimes \frac{1}{\sqrt{2}}\left(|00\rangle_{34} + |11\rangle_{34}\right) \\ &= \frac{1}{2}\left(|0000\rangle_{1234} + |0011\rangle_{1234} + |1100\rangle_{1234} + |1111\rangle_{1234}\right) \qquad [1]\end{aligned}$$

- In the next step of our calculation, we will use a little trick
- We rewrite the state of qubits 2 and 3 at Repeater *B*, as superpositions of the Bell states

$$|00\rangle = \frac{1}{\sqrt{2}}\left(|\Phi^+\rangle + |\Phi^-\rangle\right),\ |10\rangle = \frac{1}{\sqrt{2}}\left(|\Psi^+\rangle - |\Psi^-\rangle\right), \qquad [2],\ [3]$$

$$|01\rangle = \frac{1}{\sqrt{2}}\left(|\Psi^+\rangle + |\Psi^-\rangle\right),\ |11\rangle = \frac{1}{\sqrt{2}}\left(|\Phi^+\rangle - |\Phi^-\rangle\right) \qquad [4],\ [5]$$

Entanglement Swapping Circuit

- We can now substitute these identities in Eq. [1], and rewrite the total state of the four qubits as follows,

$$|\psi\rangle_{1234} = \frac{1}{2}\left(|0000\rangle_{1234} + |0011\rangle_{1234} + |1100\rangle_{1234} + |1111\rangle_{1234}\right)$$

$$= \frac{1}{2}\left(|0\rangle\underbrace{\frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}}_{=|00\rangle}|0\rangle + |0\rangle\underbrace{\frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}}_{=|01\rangle}|1\rangle + |1\rangle\underbrace{\frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}}_{=|10\rangle}|0\rangle + |1\rangle\underbrace{\frac{|\Phi^+\rangle - |\Phi^-\rangle}{\sqrt{2}}}_{=|11\rangle}|1\rangle\right) \qquad [6]$$

Entanglement Swapping Circuit

- We can group the qubits that are going to be measured on the left, and the qubits that we are not going to measure on the right,

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\frac{\left|\Phi^{+}\right\rangle+\left|\Phi^{-}\right\rangle}{\sqrt{2}}\left|0\right\rangle\left|0\right\rangle+\frac{\left|\Psi^{+}\right\rangle+\left|\Psi^{-}\right\rangle}{\sqrt{2}}\left|0\right\rangle\left|1\right\rangle+\frac{\left|\Psi^{+}\right\rangle-\left|\Psi^{-}\right\rangle}{\sqrt{2}}\left|1\right\rangle\left|0\right\rangle+\frac{\left|\Phi^{+}\right\rangle-\left|\Phi^{-}\right\rangle}{\sqrt{2}}\left|1\right\rangle\left|1\right\rangle\right) \qquad [7]$$

- We have not really done anything apart from rearranging the qubits
- Stated differently, $\left|\psi\right\rangle_{1234}$ has been re-expressed in the Bell basis by performing a qubit **permutation**

Entanglement Swapping Circuit

- Finally, we collect all terms with the same Bell pair on qubits 2 and 3,

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\left|\Phi^{+}\right\rangle_{23} \underbrace{\frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}}}_{\left|\Phi^{+}\right\rangle_{14}} + \left|\Psi^{+}\right\rangle_{23} \underbrace{\frac{\left|01\right\rangle + \left|10\right\rangle}{\sqrt{2}}}_{\left|\Psi^{+}\right\rangle_{14}} + \left|\Psi^{-}\right\rangle_{23} \underbrace{\frac{\left|01\right\rangle - \left|10\right\rangle}{\sqrt{2}}}_{\left|\Psi^{-}\right\rangle_{14}} + \left|\Phi^{-}\right\rangle_{23} \underbrace{\frac{\left|00\right\rangle - \left|11\right\rangle}{\sqrt{2}}}_{\left|\Phi^{-}\right\rangle_{14}} \right) \quad [8]$$

- Looking at the state of qubits 1 and 4 in Eq. [8], we recognize that the qubits are in fact entangled
- We can make this even more explicit

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\left|\Phi^{+}\right\rangle_{23}\left|\Phi^{+}\right\rangle_{14} + \left|\Psi^{+}\right\rangle_{23}\left|\Psi^{+}\right\rangle_{14} + \left|\Psi^{-}\right\rangle_{23}\left|\Psi^{-}\right\rangle_{14} + \left|\Phi^{-}\right\rangle_{23}\left|\Phi^{-}\right\rangle_{14} \right) \quad [9]$$

Entanglement Swapping Circuit

- We can see that if the Bell-state measurement at Repeater *B* of qubits 2 and 3 results in the outcome $\left|\Phi^+\right\rangle_{23}$, then the state of qubits 1 and 4 at Repeaters *A* and *C* respectively is $\left|\Phi^+\right\rangle_{14}$

- Similarly, if the measurement outcome at Repeater *B* of qubits 2 and 3 is $\left|\Psi^+\right\rangle_{23}$, then the state of qubits 1 and 4 at Repeaters *A* and *C* respectively, share the state $\left|\Psi^+\right\rangle_{14}$, and so on for the other measurement outcomes

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\left|\Phi^+\right\rangle_{23}\left|\Phi^+\right\rangle_{14} + \left|\Psi^+\right\rangle_{23}\left|\Psi^+\right\rangle_{14} + \left|\Psi^-\right\rangle_{23}\left|\Psi^-\right\rangle_{14} + \left|\Phi^-\right\rangle_{23}\left|\Phi^-\right\rangle_{14}\right) \qquad [9]$$

Entanglement Swapping Circuit

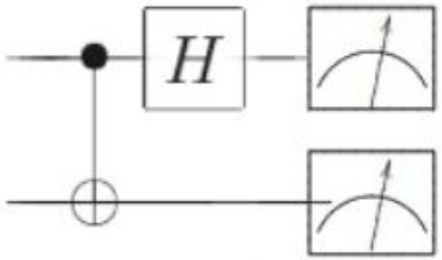
- Hence, in In the “2314” basis representation, we can see immediately that if we perform a **Bell State Measurement** (***BSM***) of qubits 2 and 3 (i.e., if we figure out which Bell state they are in), then qubits 1 and 4 will then be projected into the identical Bell state (up to an overall phase factor), even though qubits 1 and 4 had, at no time, interacted directly

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\left|\Phi^{+}\right\rangle_{23}\left|\Phi^{+}\right\rangle_{14} + \left|\Psi^{+}\right\rangle_{23}\left|\Psi^{+}\right\rangle_{14} + \left|\Psi^{-}\right\rangle_{23}\left|\Psi^{-}\right\rangle_{14} + \left|\Phi^{-}\right\rangle_{23}\left|\Phi^{-}\right\rangle_{14}\right) \qquad [9]$$

Entanglement Swapping Circuit

- By taking into consideration that

$$|00\rangle = (H \otimes I) CNOT |\Phi^+\rangle$$
$$|01\rangle = (H \otimes I) CNOT |\Psi^+\rangle$$
$$|10\rangle = (H \otimes I) CNOT |\Phi^-\rangle$$
$$|11\rangle = (H \otimes I) CNOT |\Psi^-\rangle$$



these equations describe how *BSM* works: read it from right to left: the incoming Bell states are processed by the controlled-Not gate *CNOT* and then *H* acts on the first qubit

- The final state is measured in the computational basis

Entanglement Swapping Circuit

- If we perform a Bell State Measurement (*BSM*) of qubits 2 and 3

$$\left|\tilde{\psi}\right\rangle_{2314} = \frac{1}{2}\left(\left|00\right\rangle_{23}\left|\Phi^{+}\right\rangle_{14} + \left|01\right\rangle_{23}\left|\Psi^{+}\right\rangle_{14} + \left|11\right\rangle_{23}\left|\Psi^{-}\right\rangle_{14} + \left|10\right\rangle_{23}\left|\Phi^{-}\right\rangle_{14}\right) \qquad [10]$$

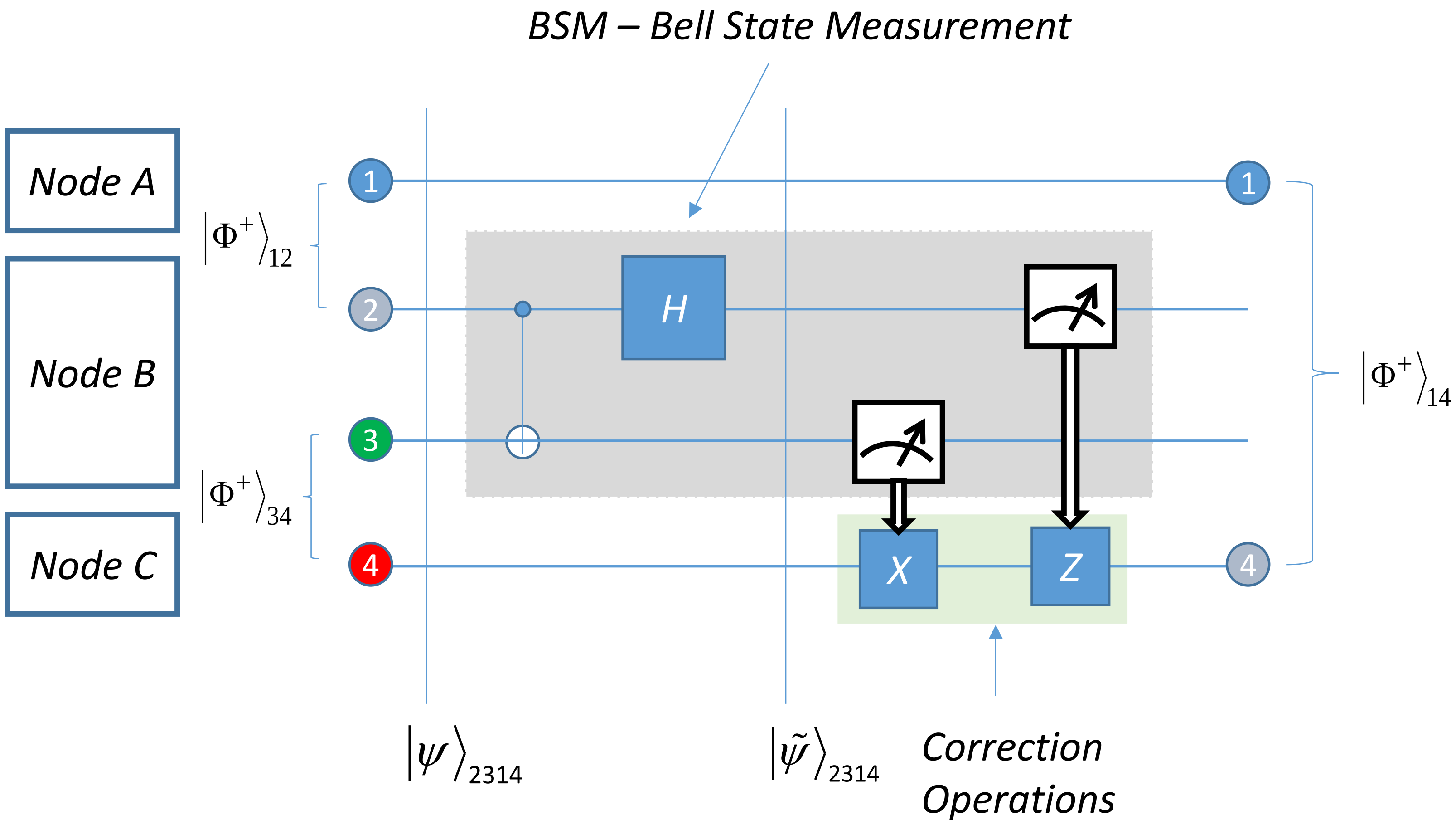
- Equation [10] shares the same structure as the one for teleporting qubit 2's state from node *B* to qubit 4's state at Node *C* (see earlier slide)

$$\left|\psi_2\right\rangle = \frac{1}{2}\left[\left|0_1 0_2\right\rangle\left(\alpha\left|0_3\right\rangle + \beta\left|1_3\right\rangle\right) + \left|0_1 1_2\right\rangle\left(\alpha\left|1_3\right\rangle + \beta\left|0_3\right\rangle\right) + \left|1_1 0_2\right\rangle\left(\alpha\left|0_3\right\rangle - \beta\left|1_3\right\rangle\right) + \left|1_1 1_2\right\rangle\left(\alpha\left|1_3\right\rangle - \beta\left|0_3\right\rangle\right)\right]$$

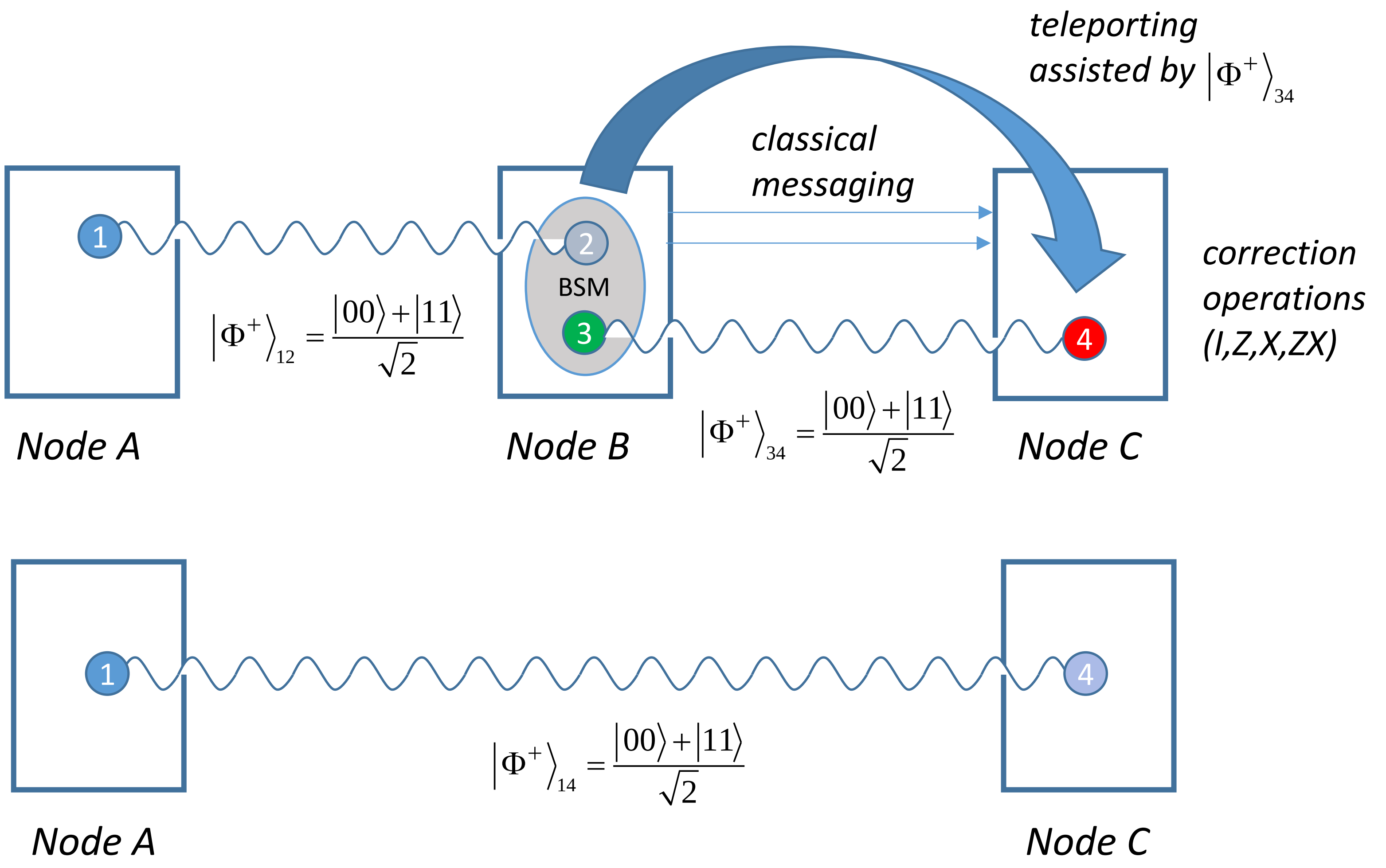
- Consequently, we can use all the considerations that were performed when talking about teleportation

$$\left|\psi\right\rangle_{2314} = \frac{1}{2}\left(\left|\Phi^{+}\right\rangle_{23}\left|\Phi^{+}\right\rangle_{14} + \left|\Psi^{+}\right\rangle_{23}\left|\Psi^{+}\right\rangle_{14} + \left|\Psi^{-}\right\rangle_{23}\left|\Psi^{-}\right\rangle_{14} + \left|\Phi^{-}\right\rangle_{23}\left|\Phi^{-}\right\rangle_{14}\right)$$

$$\left|\tilde{\psi}\right\rangle_{2314} = \frac{1}{2}\left(\left|00\right\rangle_{23}\left|\Phi^{+}\right\rangle_{14} + \left|01\right\rangle_{23}\left|\Psi^{+}\right\rangle_{14} + \left|11\right\rangle_{23}\left|\Psi^{-}\right\rangle_{14} + \left|10\right\rangle_{23}\left|\Phi^{-}\right\rangle_{14}\right)$$

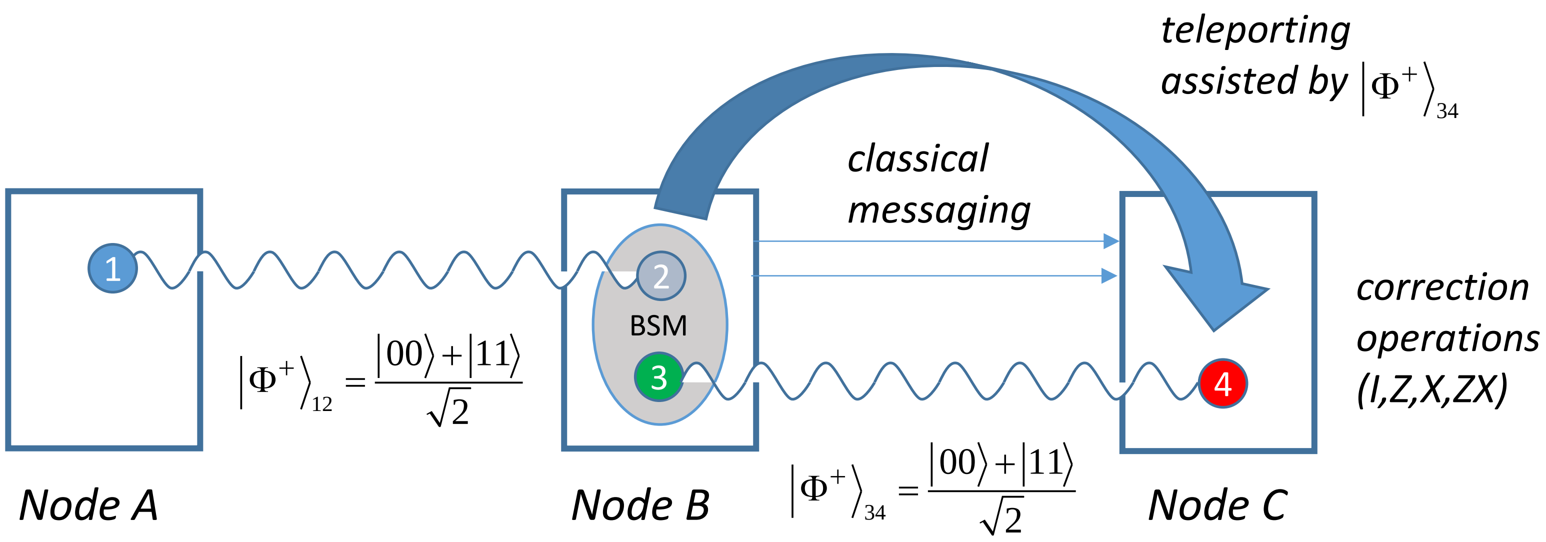


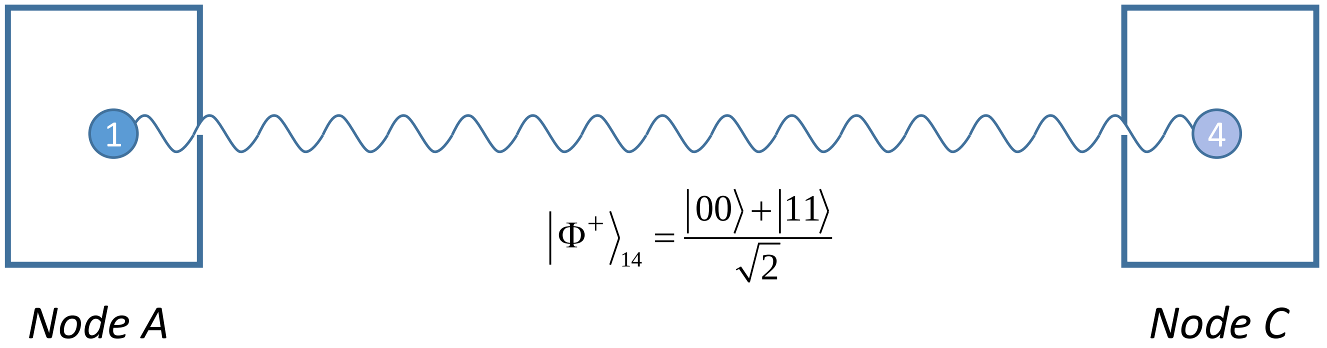
Entanglement Swapping Circuit



Node *B* transfers (by quantum teleportation) the state of qubit 2 to qubit 4 owned by Node *C* by using the pair of entangled qubits (qubits 3 and 4) between Nodes *B* and *C*

Entanglement Swapping Circuit

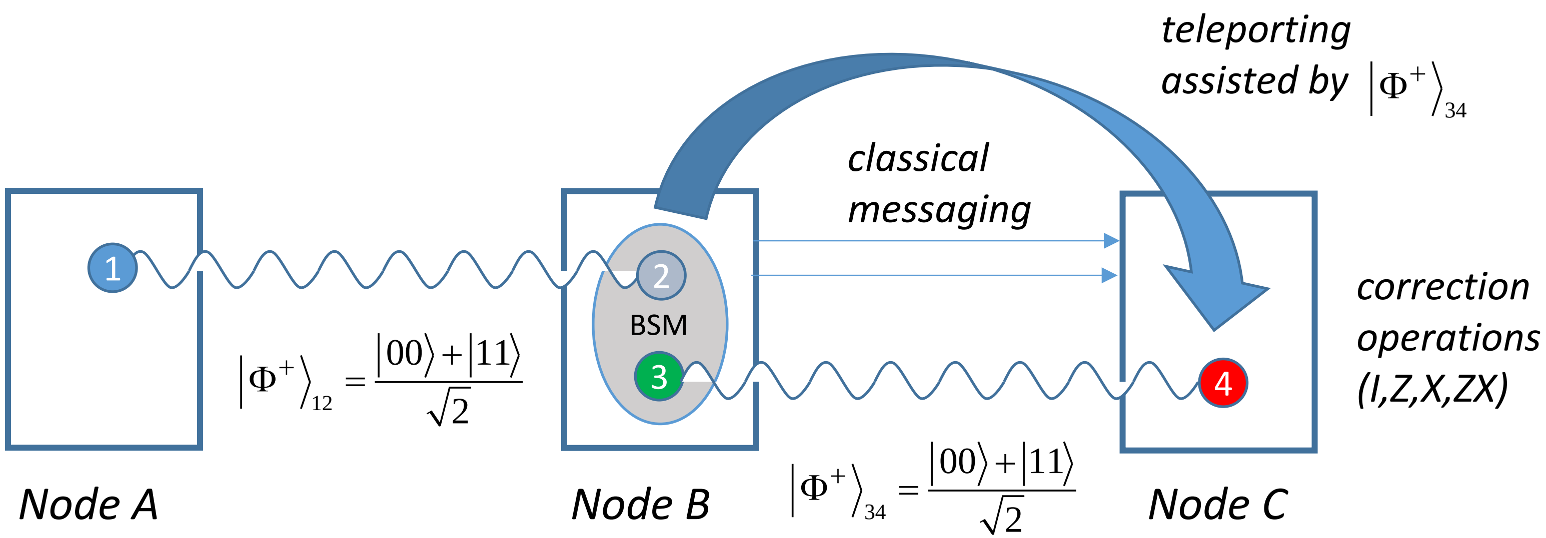




Qubit 1 at Node *A* is now entangled to the qubit 4 at Node *C*, resulting in a longer entanglement

Entanglement Swapping Circuit

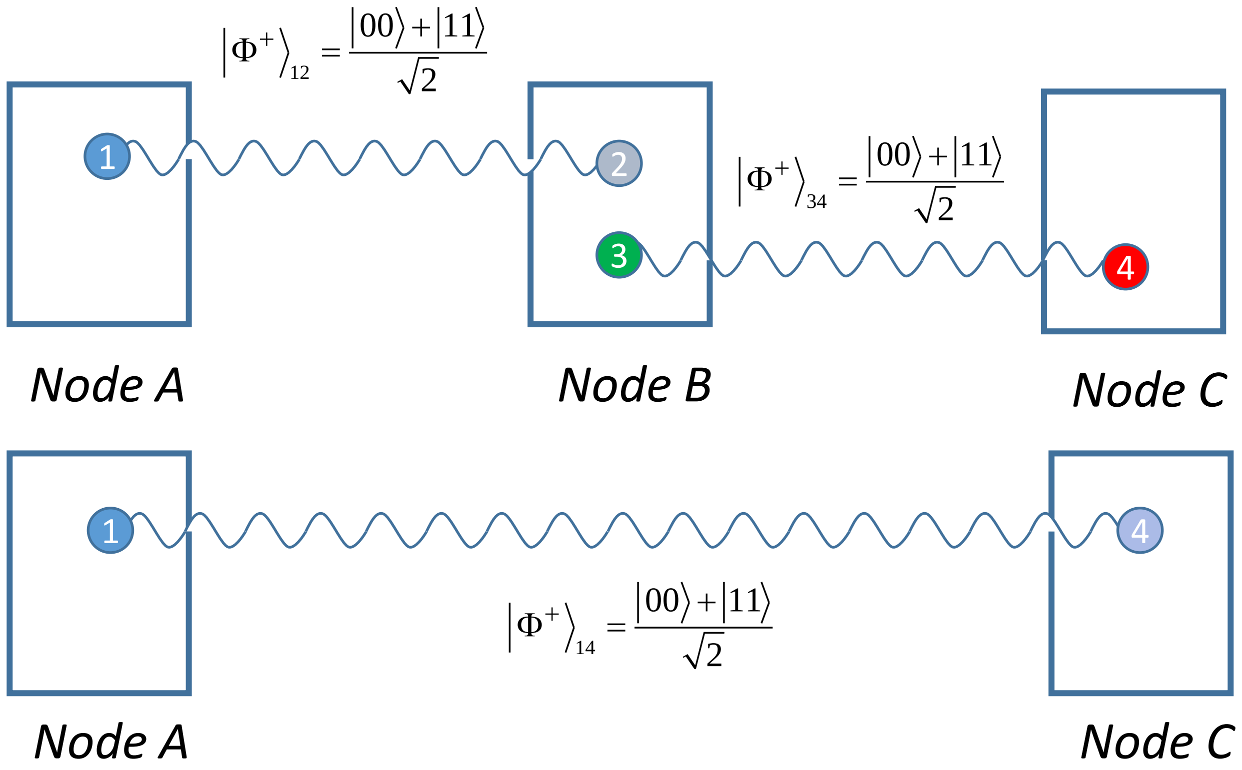
- In theory, *Node A* never needs to be told that the entanglement swapping operation has occurred



- Although *Node C* must apply corrective operations to complete the reconstruction, *Node A* is entirely passive, merely storing its half of the Bell pair in a quantum memory
- However, *Node A* is very likely waiting on the completion of the swapping operation in order to perform some other action; at the very least, an application at *Node A* is waiting to use the end-to-end Bell pair

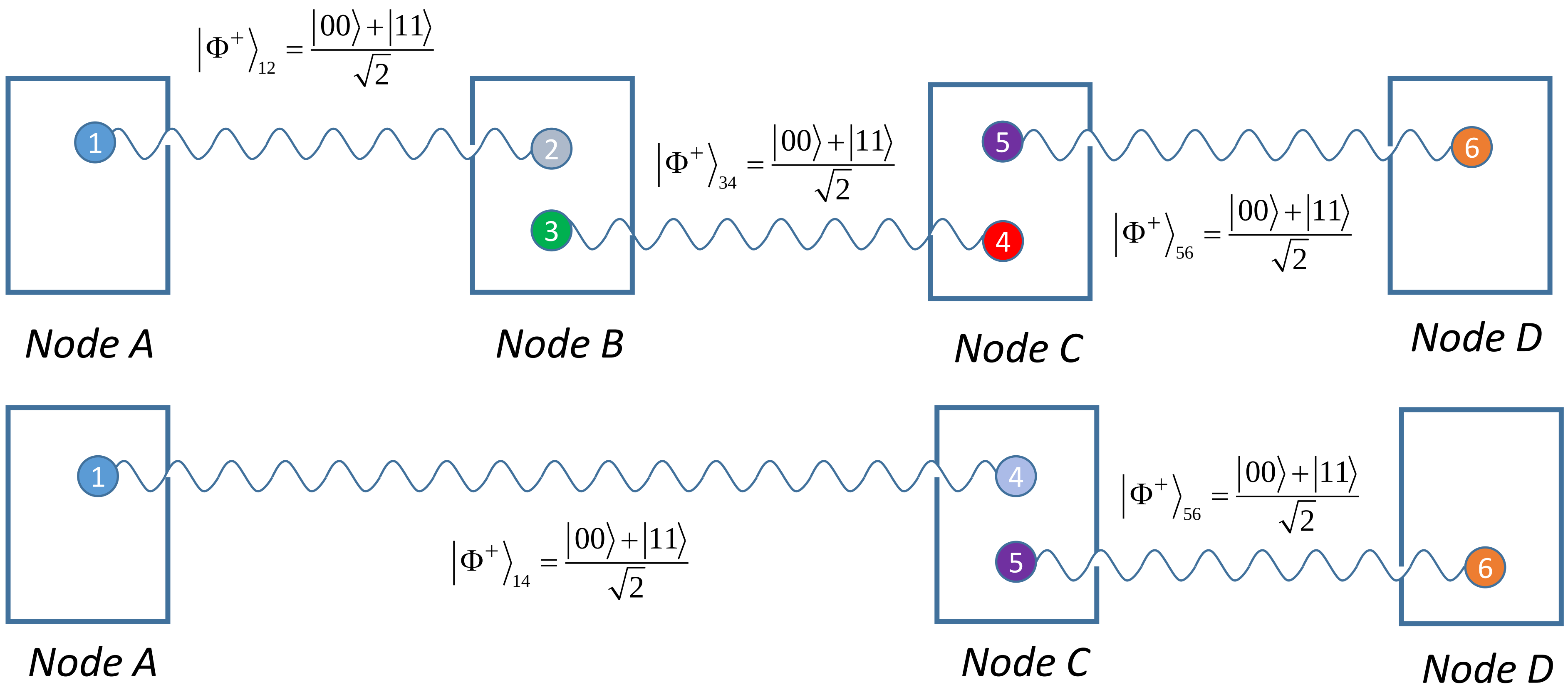
Entanglement Swapping Circuit

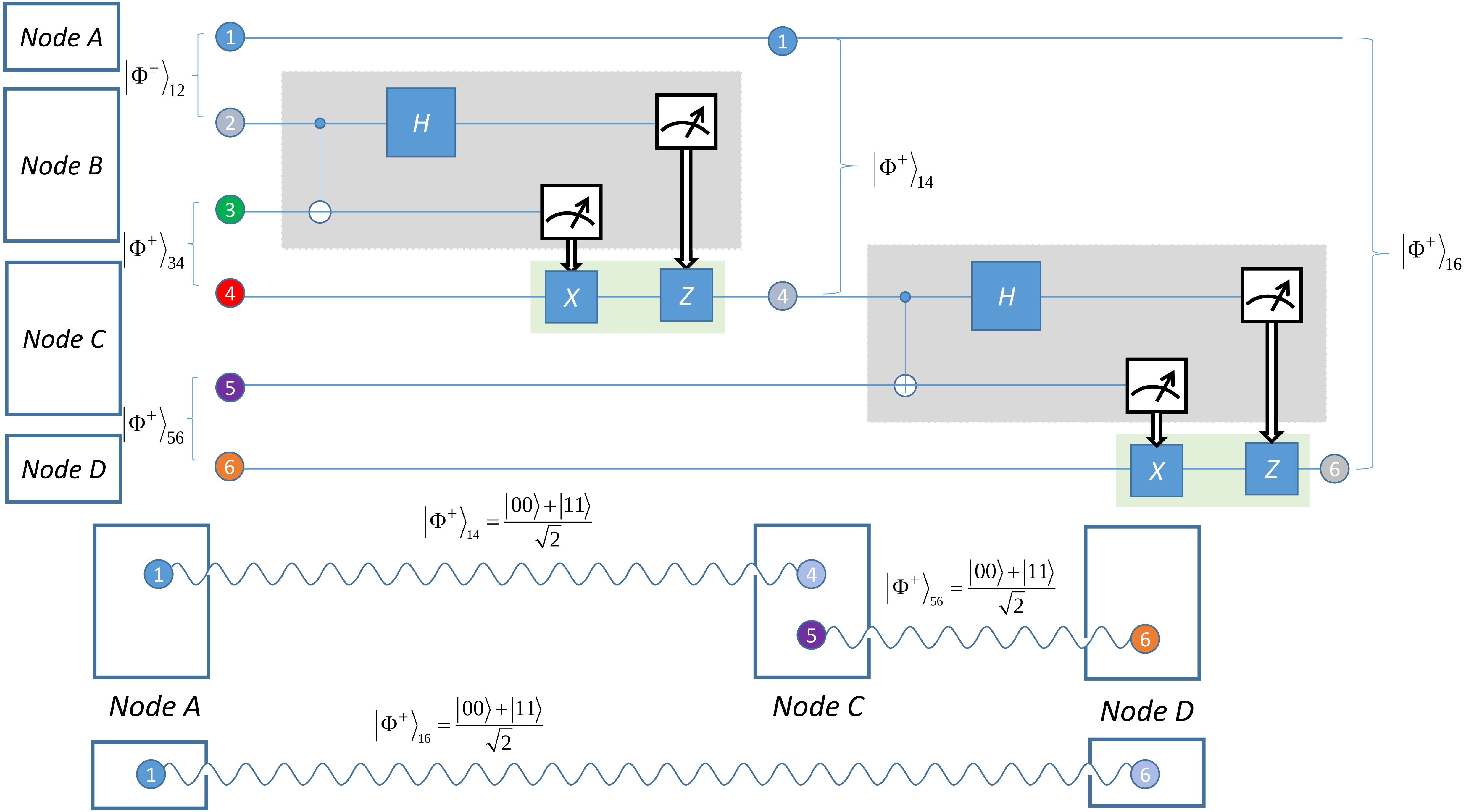
– Qubits 1 and 4, which are located on Node *A* and Node *C*, are **maximally entangled**, just as the original *EPR* pairs between nodes *A*, *B*, and nodes *B*, *C*



Entanglement Swapping

Let's now add *Node D* which share with *Node C* one pair of maximally entangled qubits (qubits 5 and 6)





Node A
Node B
Node C
Node D
$|\Phi^+\rangle_{12}$
$|\Phi^+\rangle_{34}$
$|\Phi^+\rangle_{56}$
$|\Phi^+\rangle_{14}$
$|\Phi^+\rangle_{16}$
H
X
Z
1
2
3
4
5
6
$|\Phi^+\rangle_{14} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$
$|\Phi^+\rangle_{56} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$
$|\Phi^+\rangle_{16} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

Beam Splitters

- In a **polarizing beam splitter**, the transmitted beam and the diverted beam will have orthogonal polarizations
- If you input polarized light, then the relative amount of transmitted/diverted power will depend on the input polarization
- A **non-polarizing beam splitter** will have a fixed transmitted/diverted ratio regardless of input polarization, and the output beams will have **the same polarization as the input beam**

BSM Of Photonic Qubits

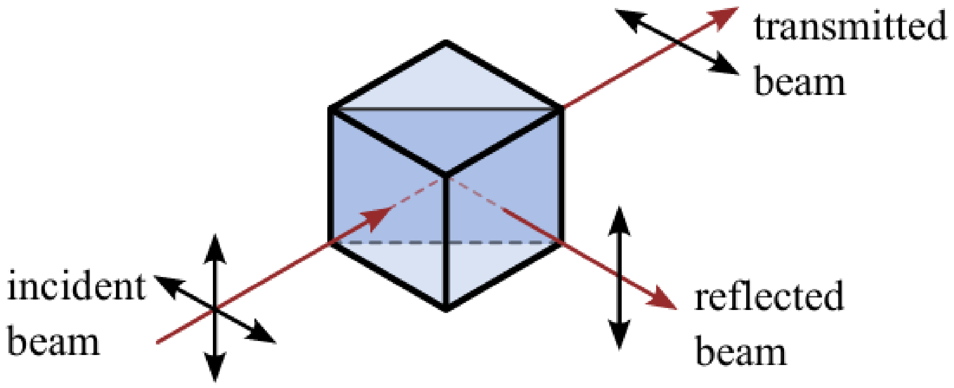
- In this section, we will turn our attention to Bell-state measurement of photonic qubits
- We will see that even though the abstract function of the *BSA* is the same as entanglement swapping, the implementation differs substantially from the case of quantum memories previously discussed
- The implementation scheme depends on the particular encoding chosen
- We will consider the case of encoding the state of a qubit using the photon's polarization

BSM Of Photonic Qubits

- In this encoding, a qubit in the state $|0\rangle$ is represented as a single photon that is polarized in the **horizontal** direction, which we write as $|H\rangle$
- The other computational basis state $|1\rangle$ is represented as a single photon polarized in the **vertical** direction, and we write $|V\rangle$

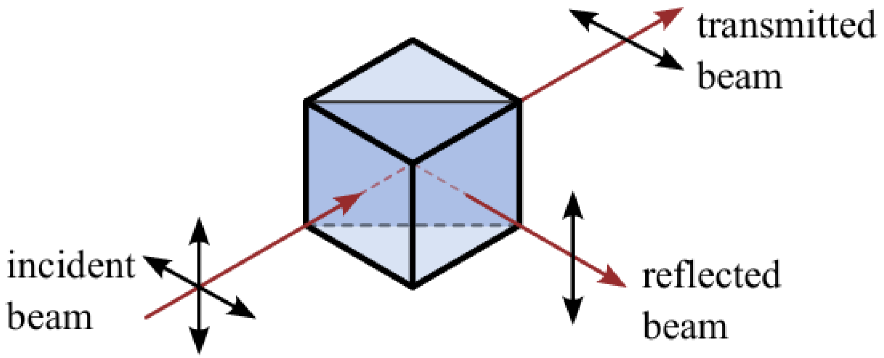
BSM Of Photonic Qubits

- The first question that we should ask is, “how do we implement a Pauli Z measurement with this encoding?”
- We need to distinguish *horizontal* and *vertical* polarizations
- This can be done with a piece of **crystal** called a **polarizing beam splitter** (*PBS*), as shown in figure
- A *PBS* transmits light of **horizontal** polarization only, and **reflects** vertically-polarized light



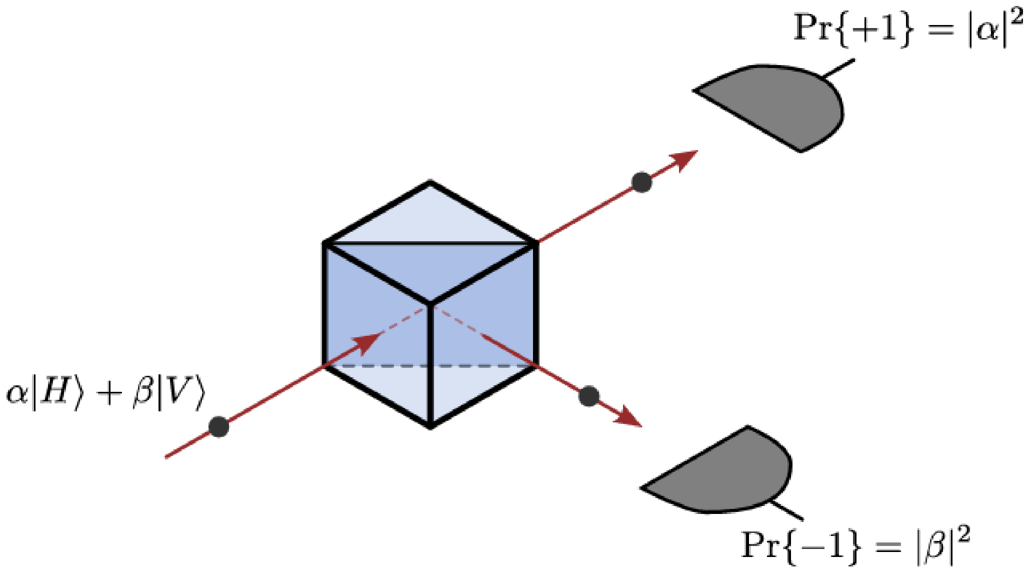
BSM Of Photonic Qubits

- An incident beam of **arbitrary polarization** gets split into **two beams** by the *PBS*
- The relative strength of the two beams depends on the polarization of the input light
- The *PBS* is not creating or changing polarization, but rather sorting the input light into the two categories
- Thinking in terms of computational states, we now have two beams, one for our $|0\rangle$ and one for our $|1\rangle$



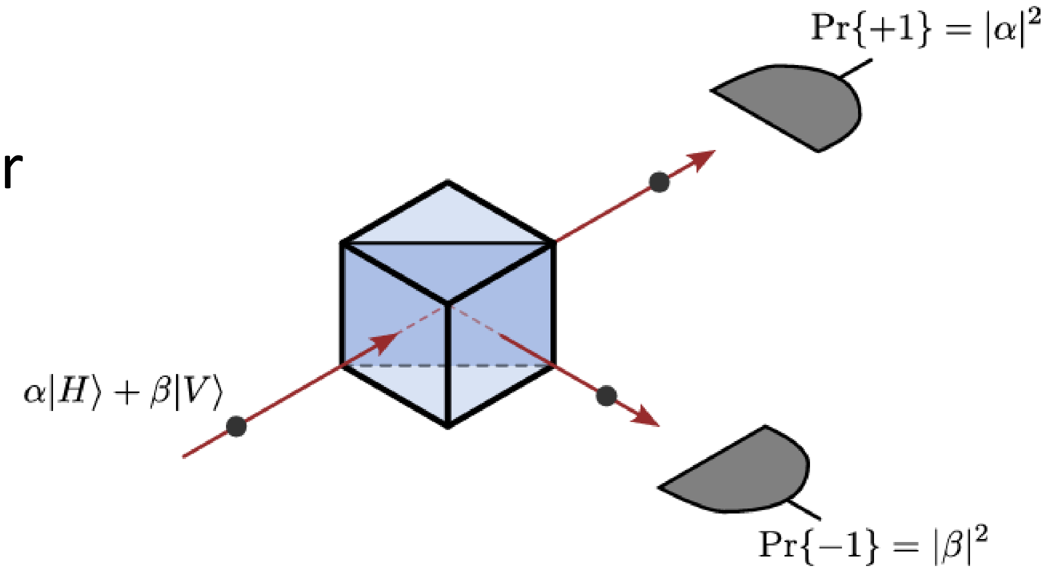
BSM Of Photonic Qubits

- Assume that we have a single photon incident onto the *PBS*
- We can determine which output path was taken by the photon by placing two detectors into the two possible output paths, as shown in the figure
- If the photon is **horizontally** polarized, it gets **transmitted** through the polarizing beam splitter
- It has no chance of being reflected and gets detected by the detector placed in the transmitted path with probability one



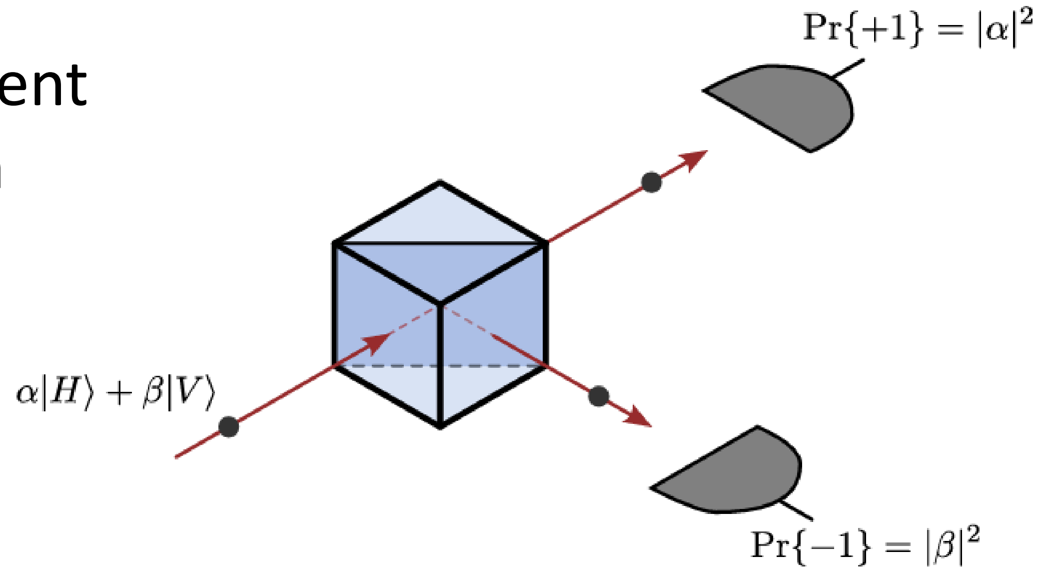
BSM Of Photonic Qubits

- This represents our measurement outcome of +1
- On the other hand, if the initial photon is **vertically** polarized, it always gets **reflected** and travels down into the bottom detector
- In that case, the probability of the outcome **−1** is 1, and the probability of the outcome **+1** is always 0
- What happens if we put in a superposition of the two linear polarizations, as in figure?



BSM Of Photonic Qubits

- Our input state is given by $\alpha|H\rangle + \beta|V\rangle$
- The photon has a chance to get transmitted, with probability given by $|\alpha|^2$, and it also has a chance to get reflected and travel down into the other detector corresponding to the measurement outcome -1 (bottom), with probability of $|\beta|^2$
- This shows how this arrangement implements a measurement in the Pauli Z basis

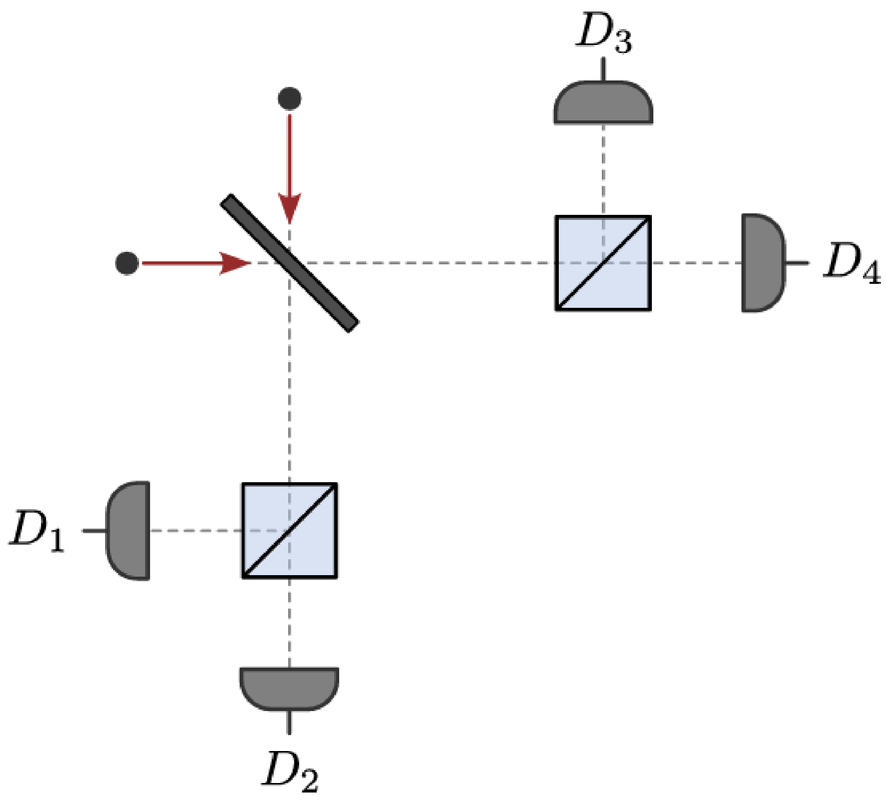


BSM Of Photonic Qubits

- We have seen in the previous step that for a Bell-state measurement, we need two measurements in the *Z* basis and a **suitable unitary** preceding the measurements
- We have just learned that *Z* measurements can be implemented with a single *PBS* and two detectors
- Two *Z* measurements will therefore require two *PBS* and four detectors
- The last remaining ingredient that is needed is the **unitary** transformation changing the basis of the measurements

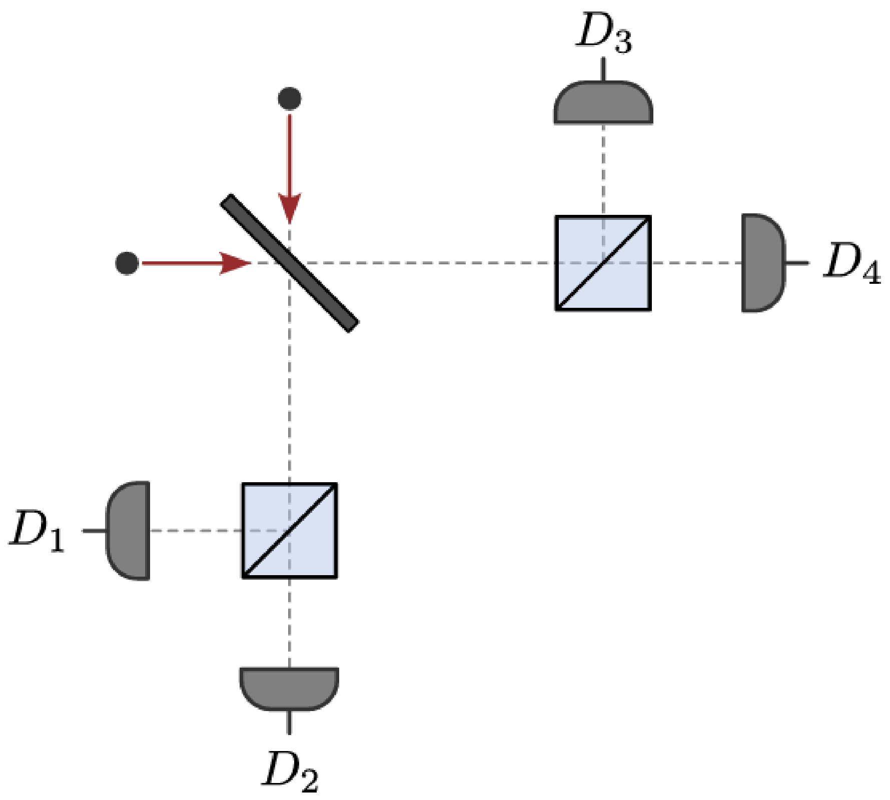
BSM Of Photonic Qubits

- This **unitary** is given by a regular, **non-polarizing, beam splitter** as pictured in the figure
- Let's investigate the behavior of the optical arrangement in the figure
- Assume we have two incoming photons, one coming from the top and one coming from the left, arriving at the regular beam splitter simultaneously



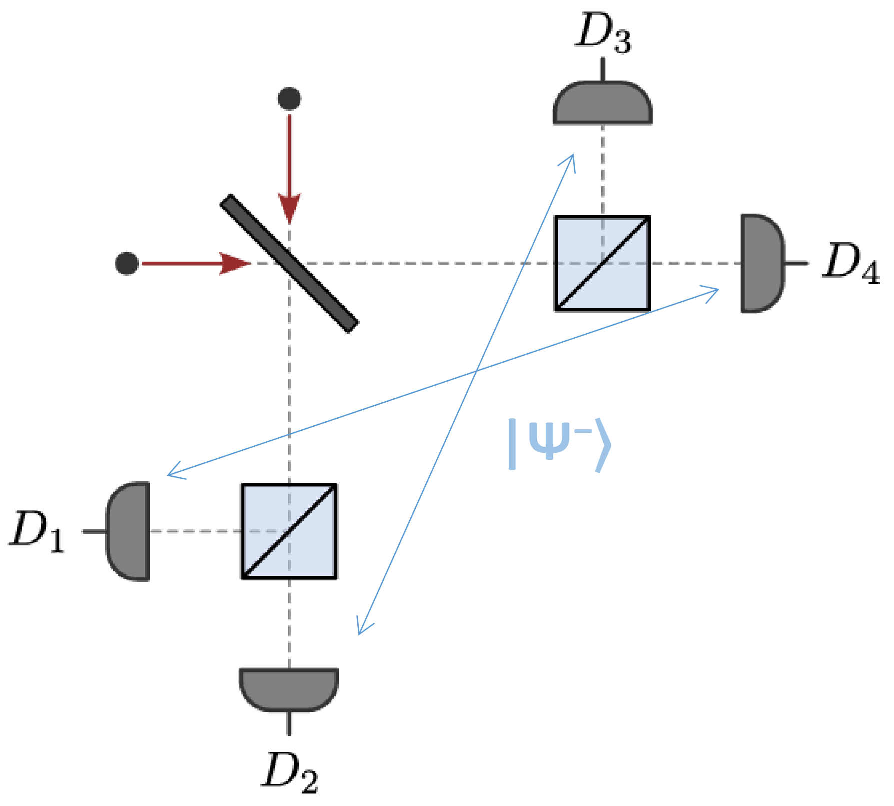
BSM Of Photonic Qubits

- A complete analysis requires extensive quantum optics, which is beyond the scope of this course
- Therefore, instead of presenting the full derivation, we will provide only the result
- Depending on which of the four detectors click, we may learn which Bell-state has been measured
- Let's see what the different patterns are



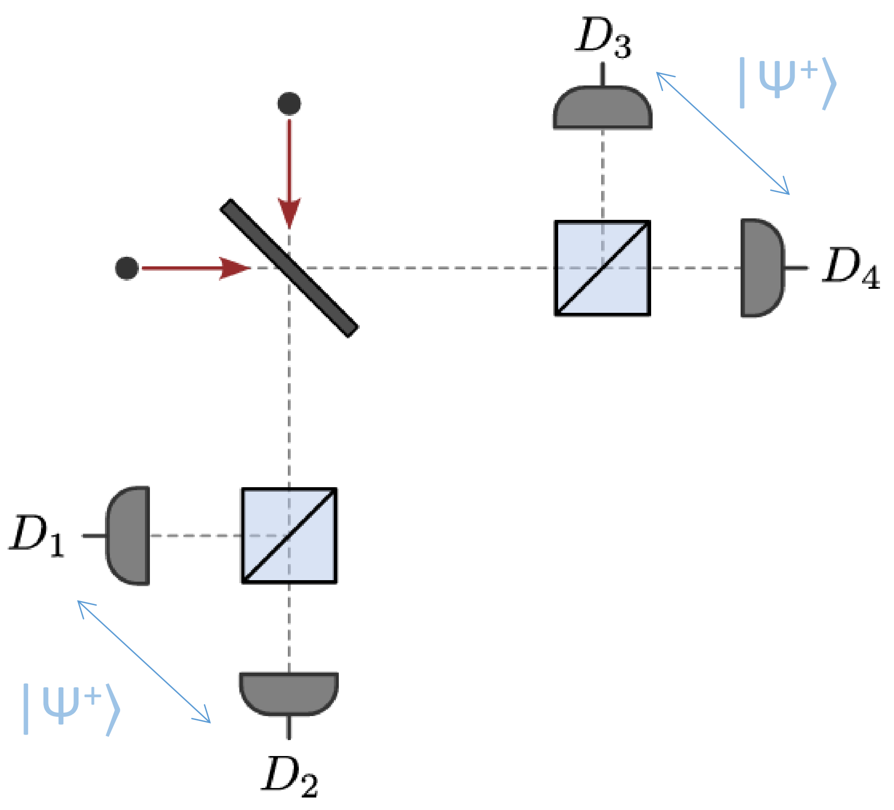
BSM Of Photonic Qubits

- If we get a joint detection in detectors D_1 and D_4, meaning that both detectors click, then we have implemented a successful Bell measurement, and the outcome corresponds to the projection onto the state $|\Psi^-\rangle$
- If we get a joint detection in D_2 and D_3, then we can also say that we have implemented a successful Bell measurement, and the result corresponds to the state $|\Psi^-\rangle$



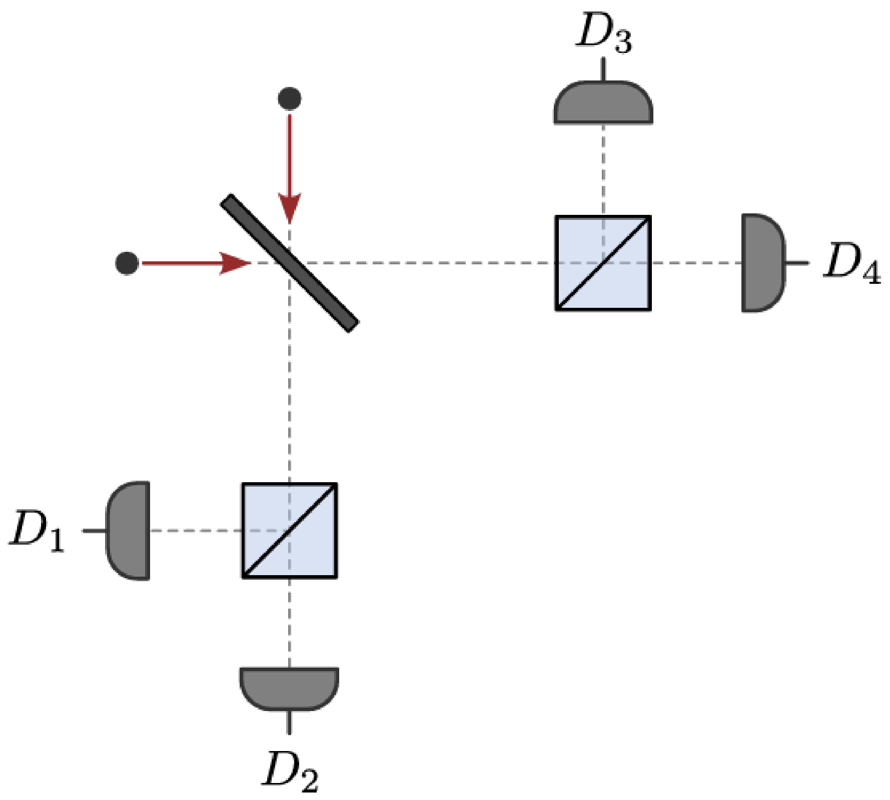
BSM Of Photonic Qubits

- A different pattern is a joint detection at the two detectors in the lower branch of our Bell-state analyzer (both D_1 and D_2 click), from which we can conclude that we have a state $|\Psi^+\rangle$, corresponding to another successful Bell measurement
- Equally, if both detectors in the right branch of our Bell-state analyzer (D_3 and D_4) click, then we can also conclude that we have a Bell-state $|\Psi^+\rangle$



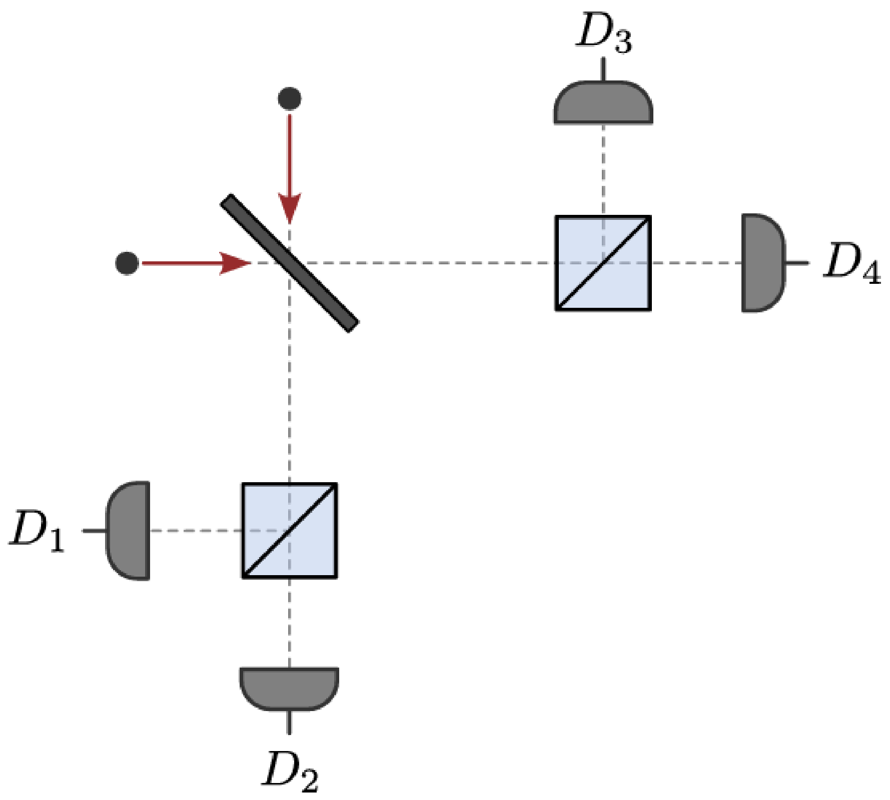
BSM Of Photonic Qubits

- It is also possible that both photons travel into a **single** detector
- Detection in any of the four detectors is equally probable
- However, because **more than one Bell state can result in this happening**, the answer we get is ambiguous: we cannot say that whether we have $|\Phi^{+}\rangle$ or $|\Phi^{-}\rangle$



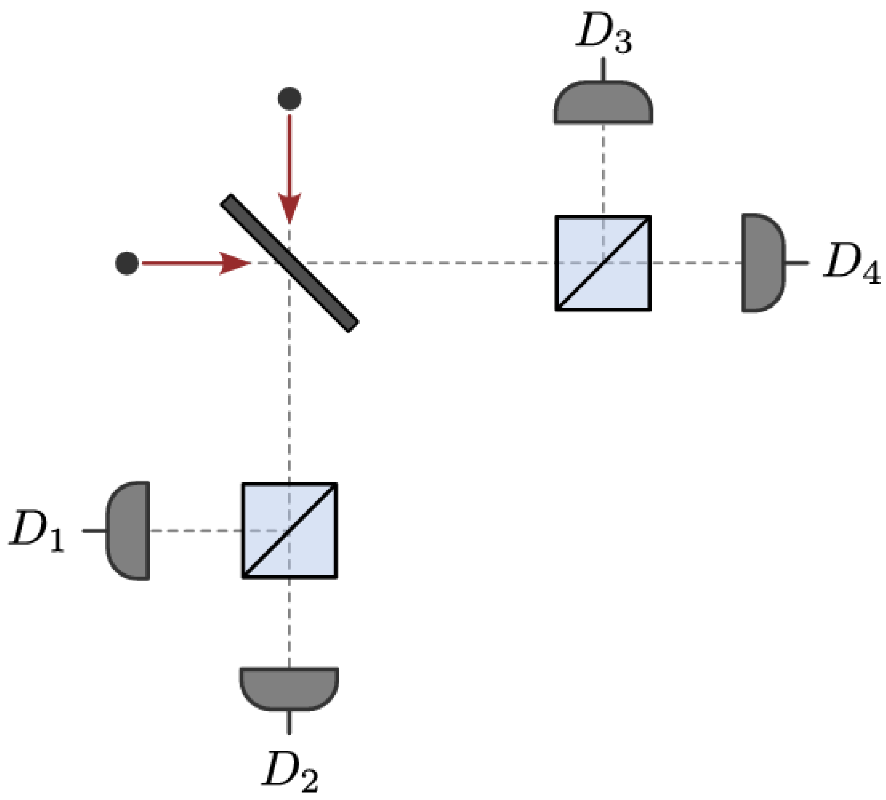
BSM Of Photonic Qubits

- This is a problem because we cannot fully implement a Bell-state measurement
- We cannot distinguish all four Bell states, only two of them, $|\Psi^+\rangle$ and $|\Psi^-\rangle$
- Using the arrangement in the figure, we can only implement a partial Bell-state measurement



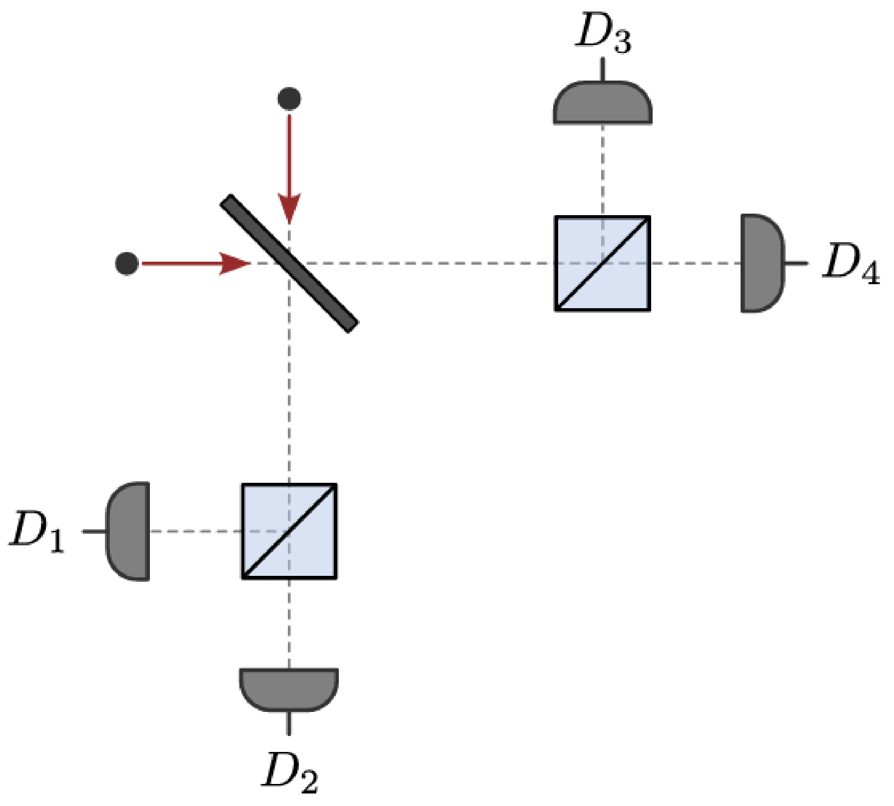
BSM Of Photonic Qubits

- A complete, unambiguous Bell-state measurement cannot always be successfully implemented with linear optics
- In fact, even with 100% probability of receiving both photons, the maximum probability of a successful Bell measurement is limited to only 50% at most



BSM Of Photonic Qubits

- Of course, as we saw when discussing the loss of photons in fiber, the loss of one or both photons is highly probable, increasing the ambiguity of interpreting the result of one click



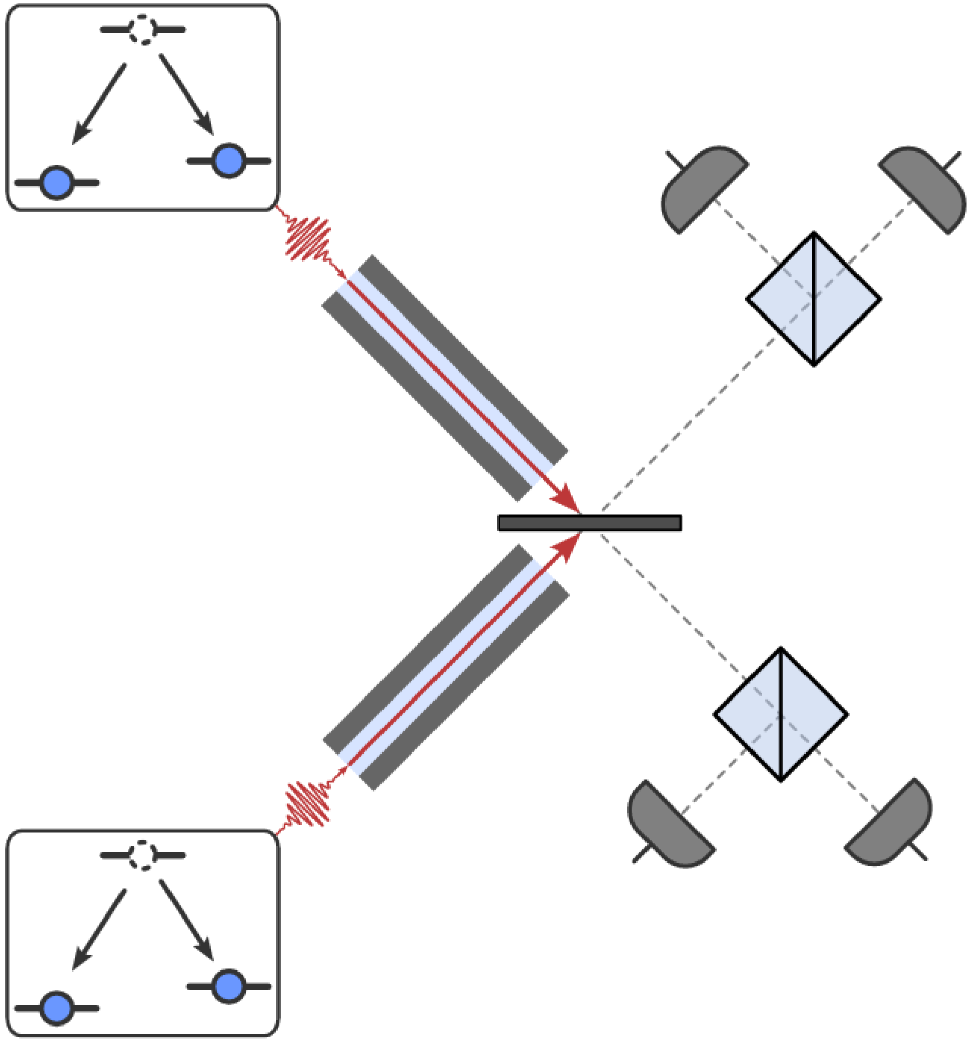
BSM Of Photonic Qubits

- These cases (along with the case where something goes wrong in the hardware) are summarized in the following table

Pattern	Result	Reason	Action
D_1 and D_4	$\|\Psi^-\rangle$		keep
D_2 and D_3	$\|\Psi^-\rangle$		keep
D_1 and D_2	$\|\Psi^+\rangle$		keep
D_3 and D_4	$\|\Psi^+\rangle$		keep
single click	$\|\Phi^\pm\rangle$	two photons together/only one arrived	discard
no click	N/A	photons lost/detection failure	discard
other pattern	N/A	detection error	discard

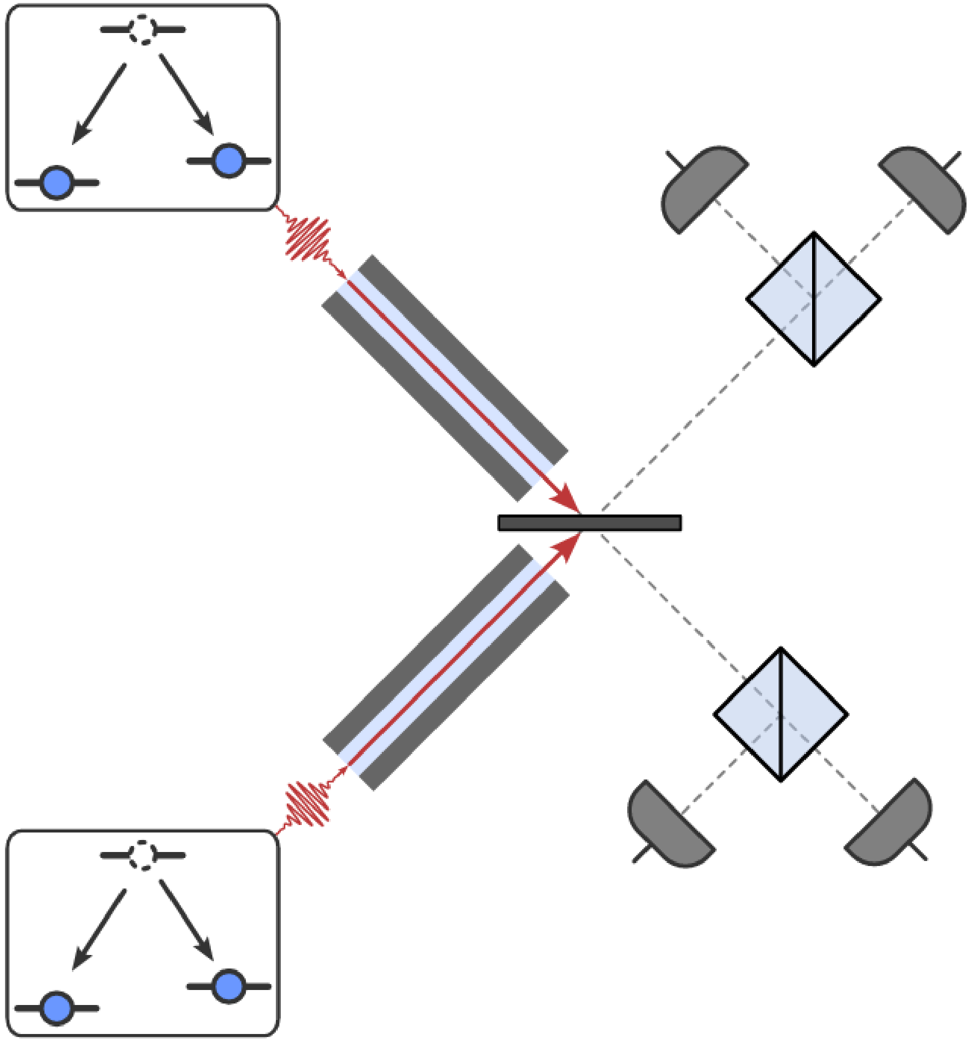
BSM and MIM Architecture

- So far, we have discussed the *BSA* only abstractly but now we have a much better idea how to make it work in practice
- The following figure brings this all back to a concrete representation of our Bell-state analyzer
- In the middle are the two single-mode fibers



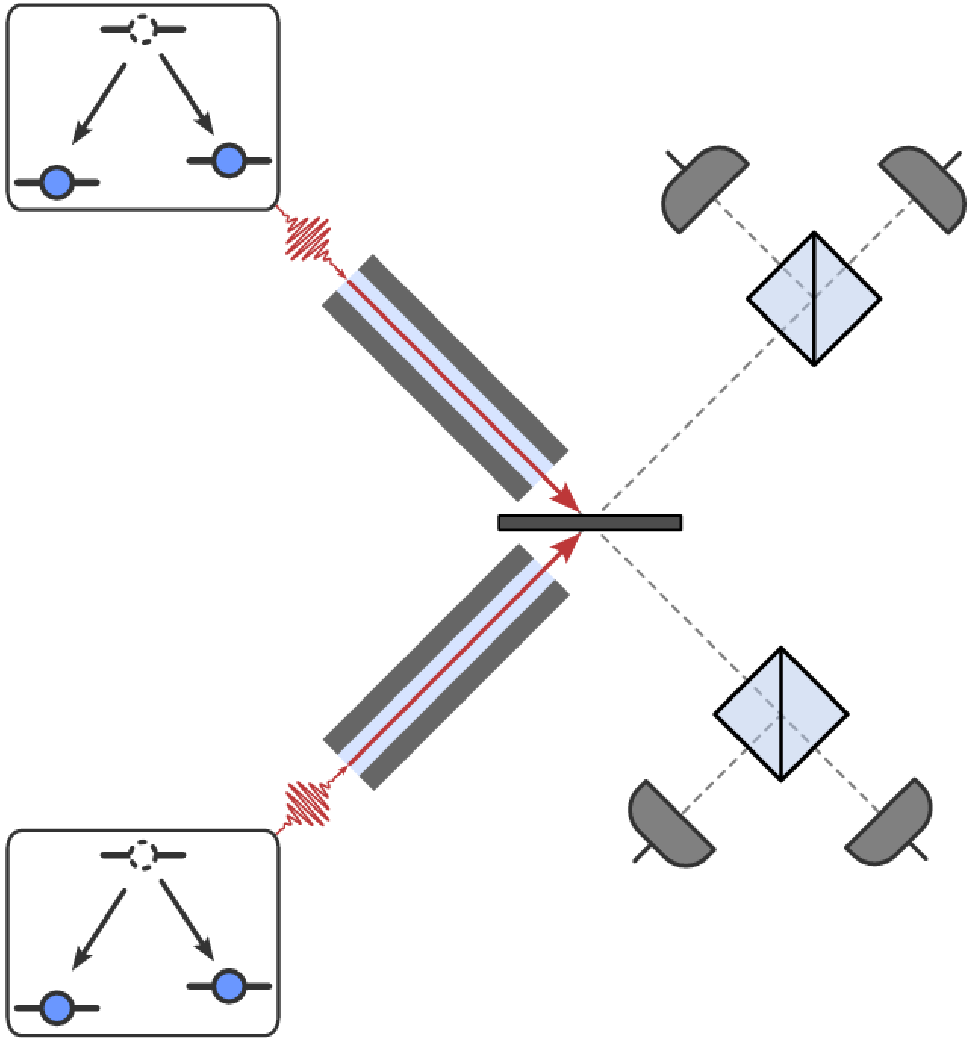
BSM and MIM Architecture

- On the left are the two quantum memories at our two repeater nodes, separated by some distance, represented by their energy level diagrams
- Each memory is prepared initially in the excited state $|e\rangle$, which decays into one of its ground states, either spin up $|\uparrow\rangle$ or spin down $|\downarrow\rangle$
- We do not know which state it decays into, leaving us with a flying photon that is entangled with its respective memory



BSM and MIM Architecture

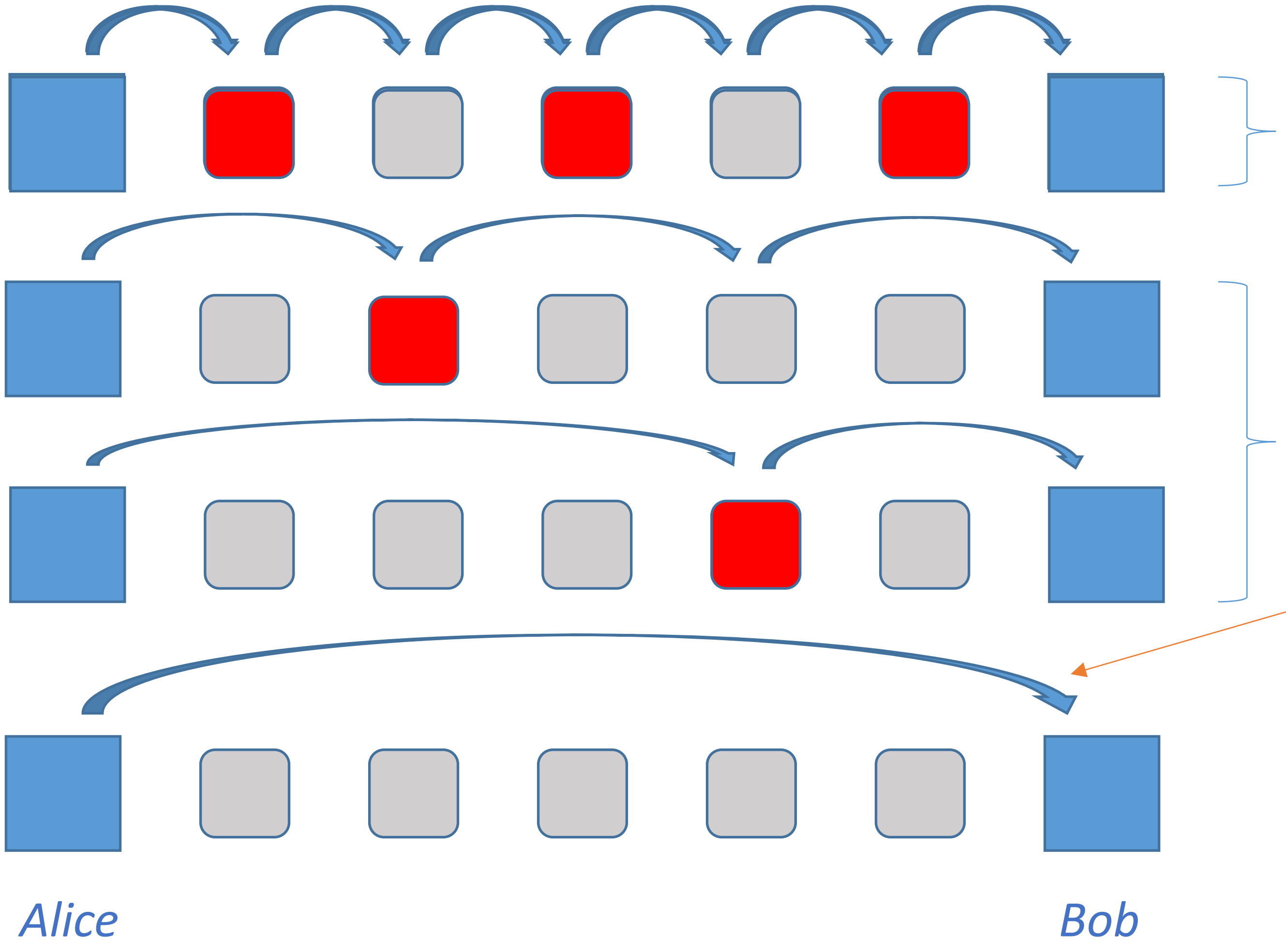
- These flying photons travel through the single mode fibers, then hit the central beam splitter, where (if all goes well) they interfere, and we perform a Bell-state measurement using the setup we discussed earlier
- In this way, we can establish link-level entanglement between the atomic memories sitting at the ends of the link



BSM and MIM Architecture

- This concludes our discussion of the physical implementation of the MIM link architecture
- In fact, our discussion applies to the MM link architecture as well, we just need to place the BSA inside one of the nodes
- Addition of a source of entangled photon pairs, such as the one we discussed earlier (Spontaneous Parameter Down-Conversion) would cover also the physical implementation of the MSM link architecture

To Conclude….



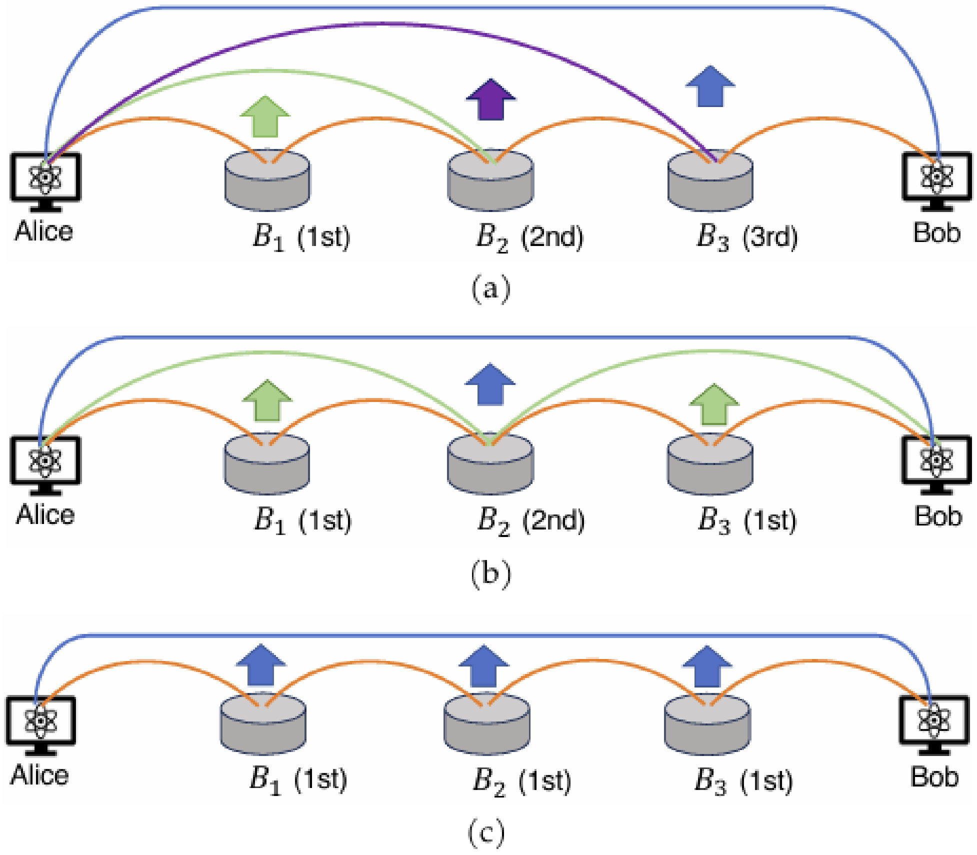
EPSs create entanglement between adjacent repeaters called *short-range entanglement (entangled link or entangled connection)*

Entanglement swapping creates the required *long-range (or e2e) entanglement*

However, many other entanglement swapping polices can be conceived as shown in the next slide

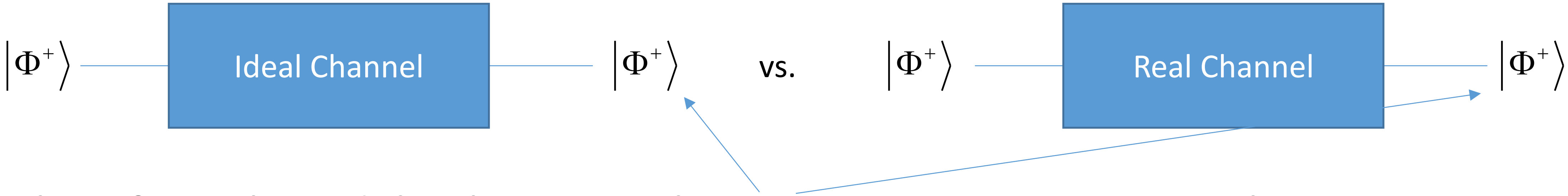
To Conclude….

- Three entanglement swapping policies on a five-nodes chain (two end nodes and three repeaters):
  a) Sequential;
  b) Nested;
  c) As Soon As Possible (ASAP).
- The arcs represent entanglement between the two connected nodes, whereas the arrows and the legend specify the entanglement swapping order



Entanglement Purification

- By repeatedly employing entanglement swapping, we can theoretically establish entangled connections of any length, theoretically infinite in scope
- Since we have assumed ideal gates and are using maximally entangled states, the output state of the real system (real channel) matches the output state of the ideal system (ideal channel)



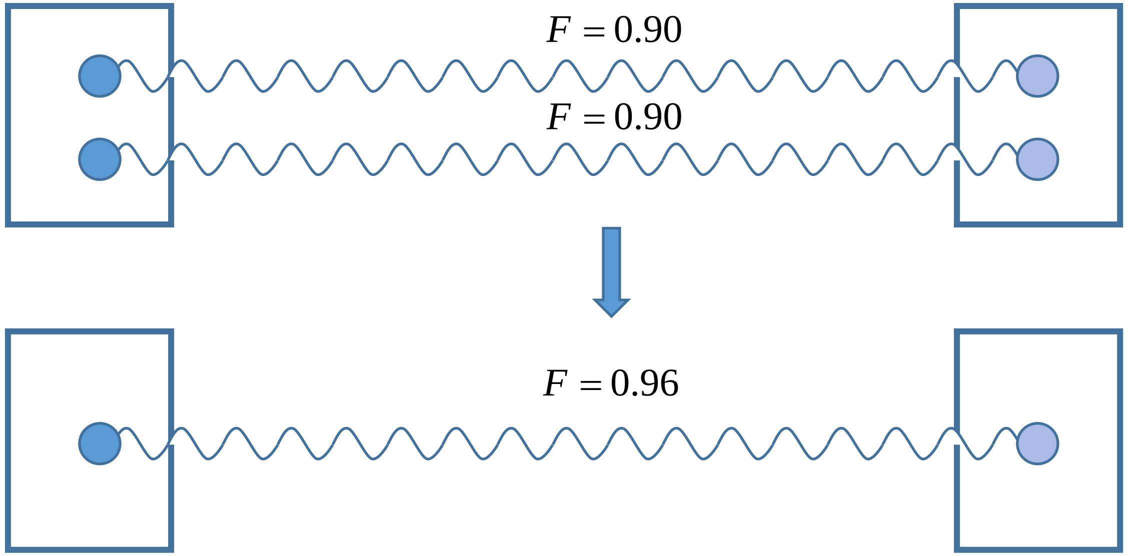
- Therefore, the Fidelity between the **two** output states is equal to one

Entanglement Purification

- However, because the gates, EPR pairs, and measurements are not ideal, the resulting entangled connection after entanglement swapping typically exhibits lower quality (i.e., fidelity $F<1$) compared to the quality of the originally established entangled connections.

Entanglement Purification

- *Entanglement Purification/Distillation* - the process where we create *higher-quality* entangled connections from a number of *low-quality* entangled connections

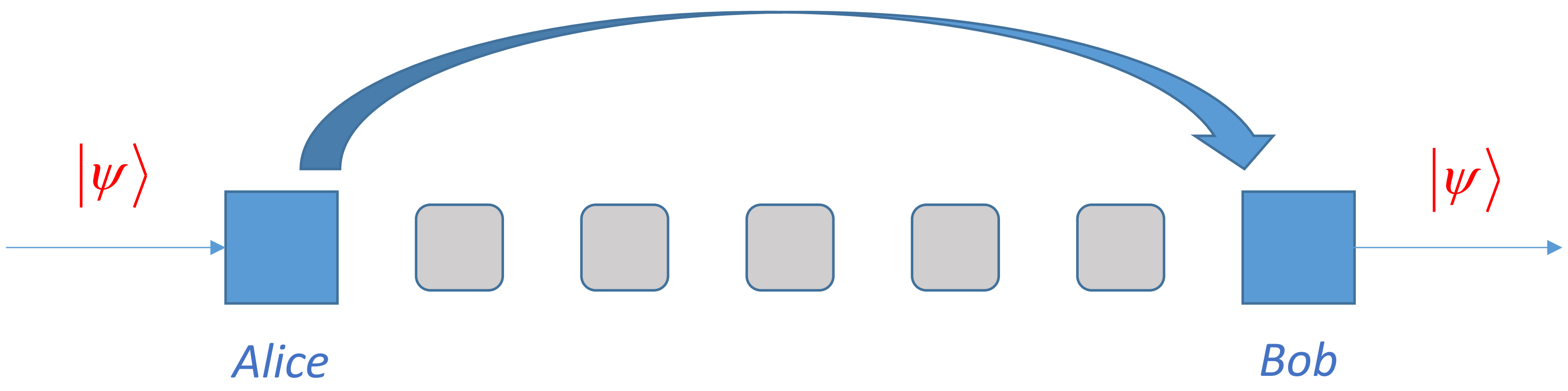


Quantum Repeater Operations

- *Entanglement swapping* and *entanglement purification* need to be analyzed independently as they can be quite different in nature and the operational details will show us the limitations each has
- However, they should cooperate for reasons outlined later on

Quantum Repeater Operations

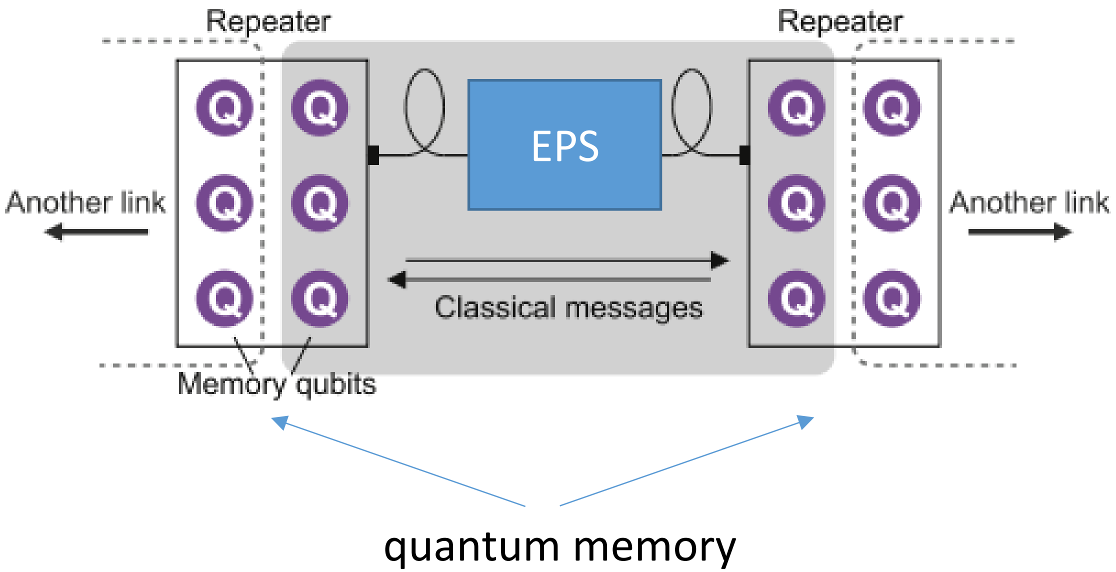
- *Quantum teleportation* then allows the state of an unknown quantum qubit to be transferred between them using the *long-range entangled connection*
- This form of quantum communication will be at *the heart of the future quantum Internet*



Quantum Repeater Memory

– Quantum repeaters use their quantum memory to carry out entanglement/distribution/swapping/purification operations

– At a highly abstract level, quantum repeaters have the following basic structure

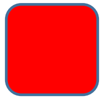


- As shown in the figure, each repeater will devote some of its memory qubits to each active link

Quantum Repeater Memory

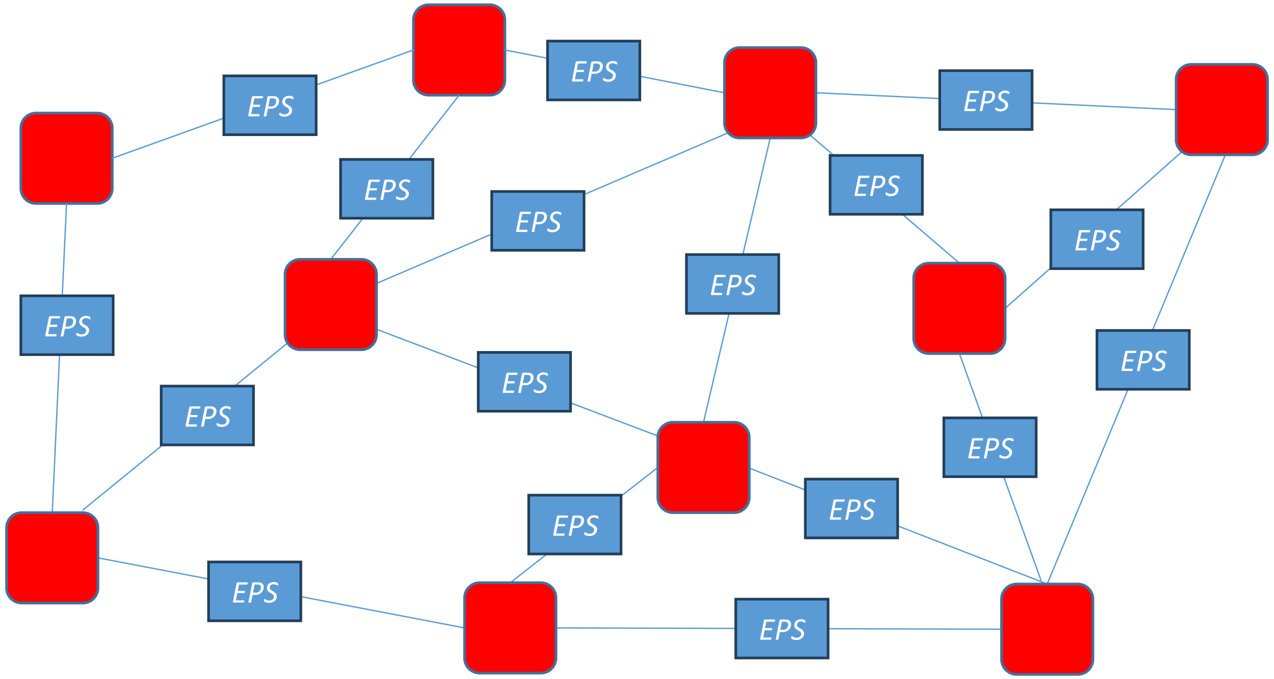
- Each quantum repeater in a network has *multiple optical links* to its neighbors

Legend

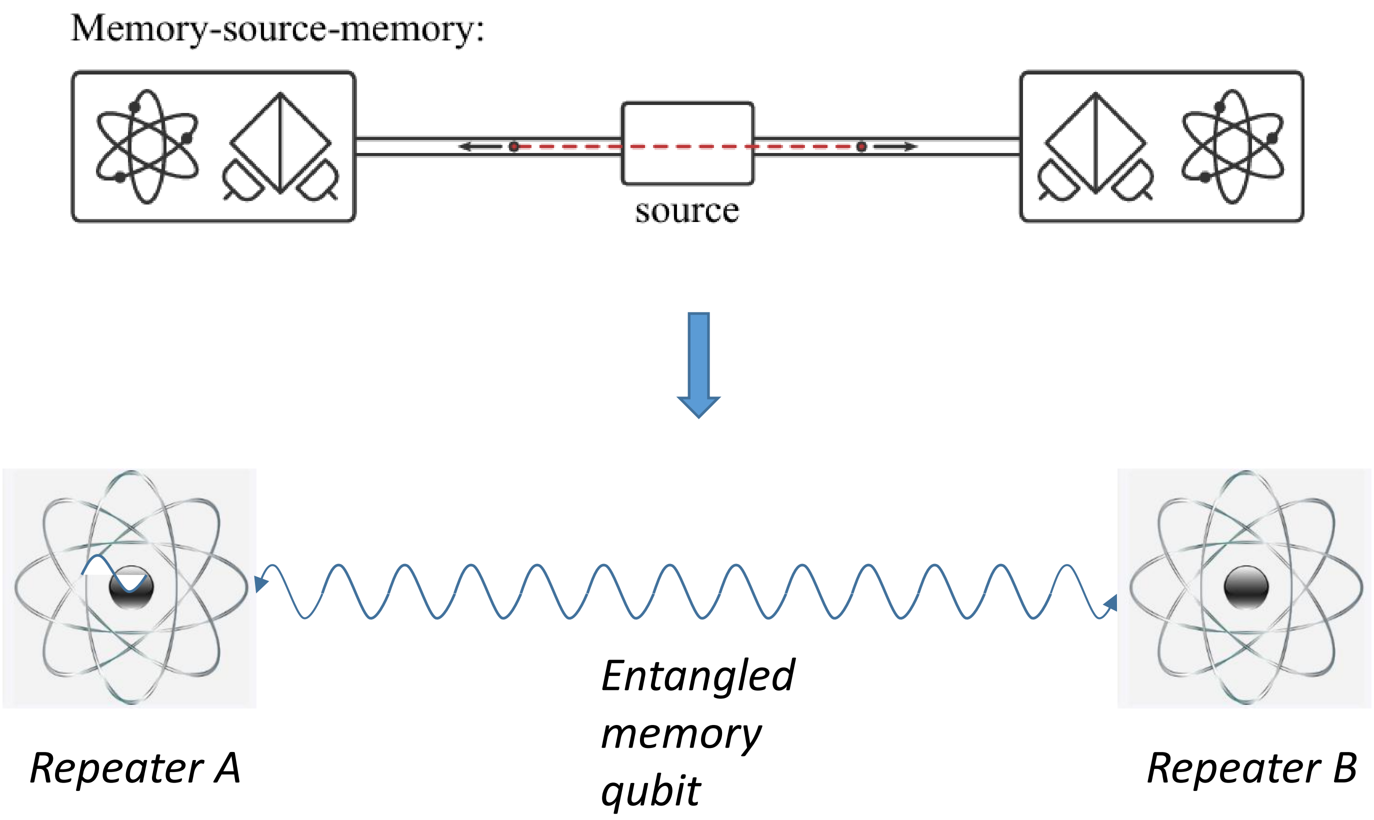


Repeater

Entanglement Photon Source

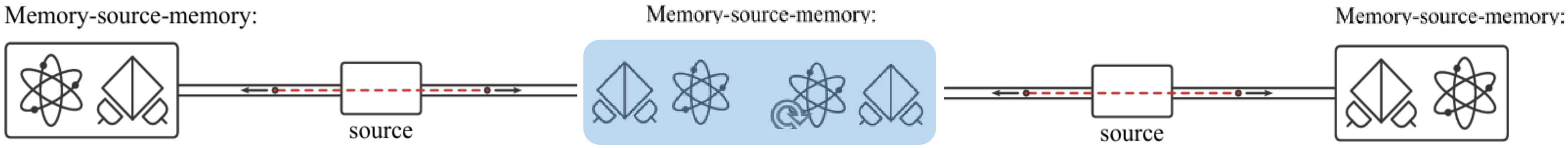


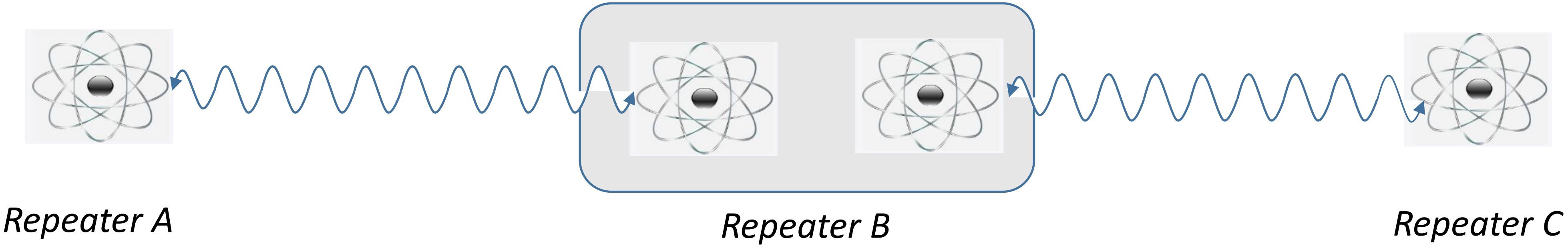
System For Performing Entanglement Swapping



At the end of the process, two entangled photons (i.e., qubits) generated by the source will result in two entangled memory qubits

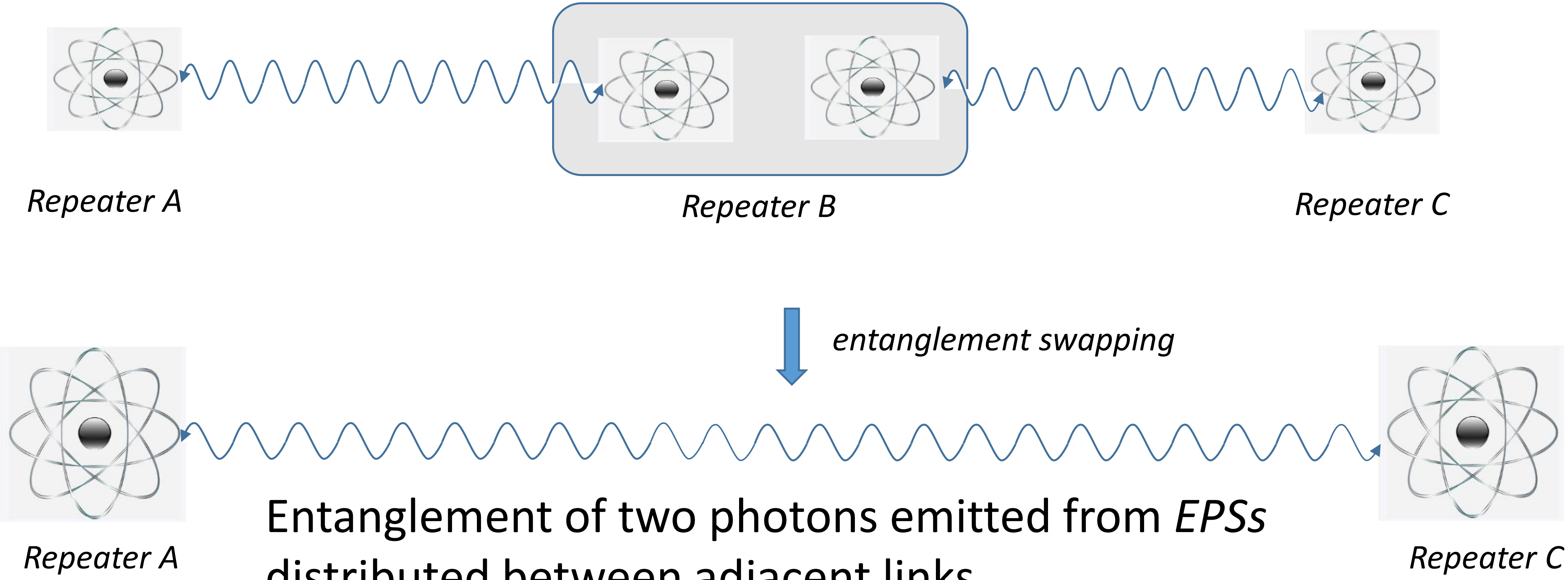
System For Performing Entanglement Swapping





- Once the quantum states have been preserved by the quantum memories in the repeater, **entanglement swapping can be performed**

System For Performing Entanglement Swapping



Entanglement of two photons emitted from *EPSs* distributed between adjacent links

Quantum Repeater Memory

- Thus, each repeater has some number of controllable *memory qubits* that must have *long coherence times* (of order 10 ms)
- During the last decade, several systems have been used to implement quantum memories, including
  - laser-cooled atomic ensembles
  - hot atomic vapors
  - single atoms in high-finesse cavities and
  - atomic ensembles in rare-earth-ion doped crystals

and the field continues to move forward tremendously

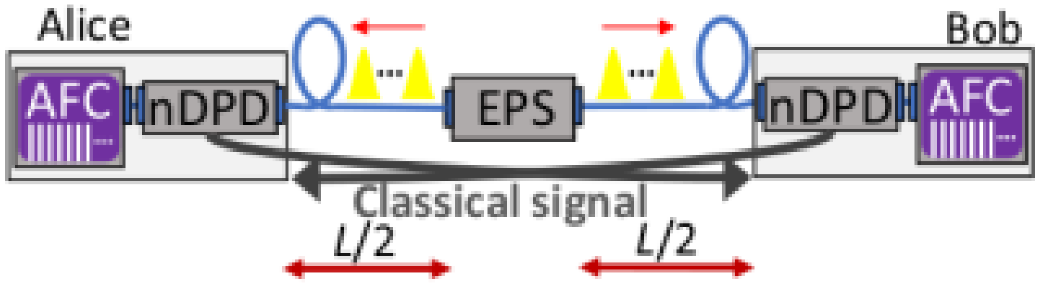
Quantum Repeater Memory

- In recent years, a technique called the *Atomic Frequency Comb (AFC)* using an atomic ensemble (especially rare-earth ion ensembles in a solid) has been proposed and developed as a quantum memory since it **outperforms the other quantum memory technologies**

- Thus, in the examples developed below, we make reference to the *AFC quantum memory*

System For Creating Short-Range Entanglement

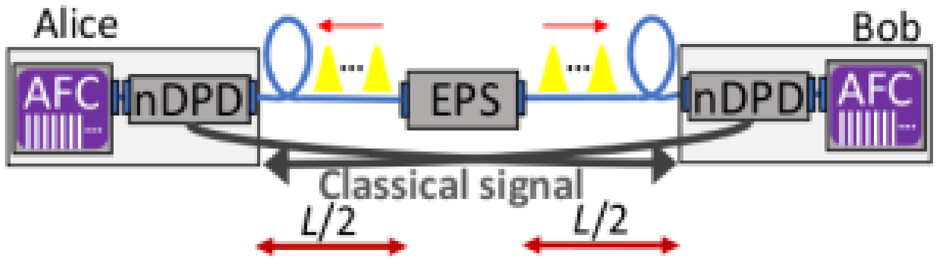
- The resulting system shown in the figure involves the entanglement of two photon qubits emitted from *Entangled Photon Sources (EPSs)* distributed between adjacent links



- The entangled photon qubits are sent to the quantum repeater nodes through optical fibers
- Upon arriving at a repeater node, the quantum state of photons is transferred to the node's *AFC* quantum memory which acts as an *absorbing* quantum memory

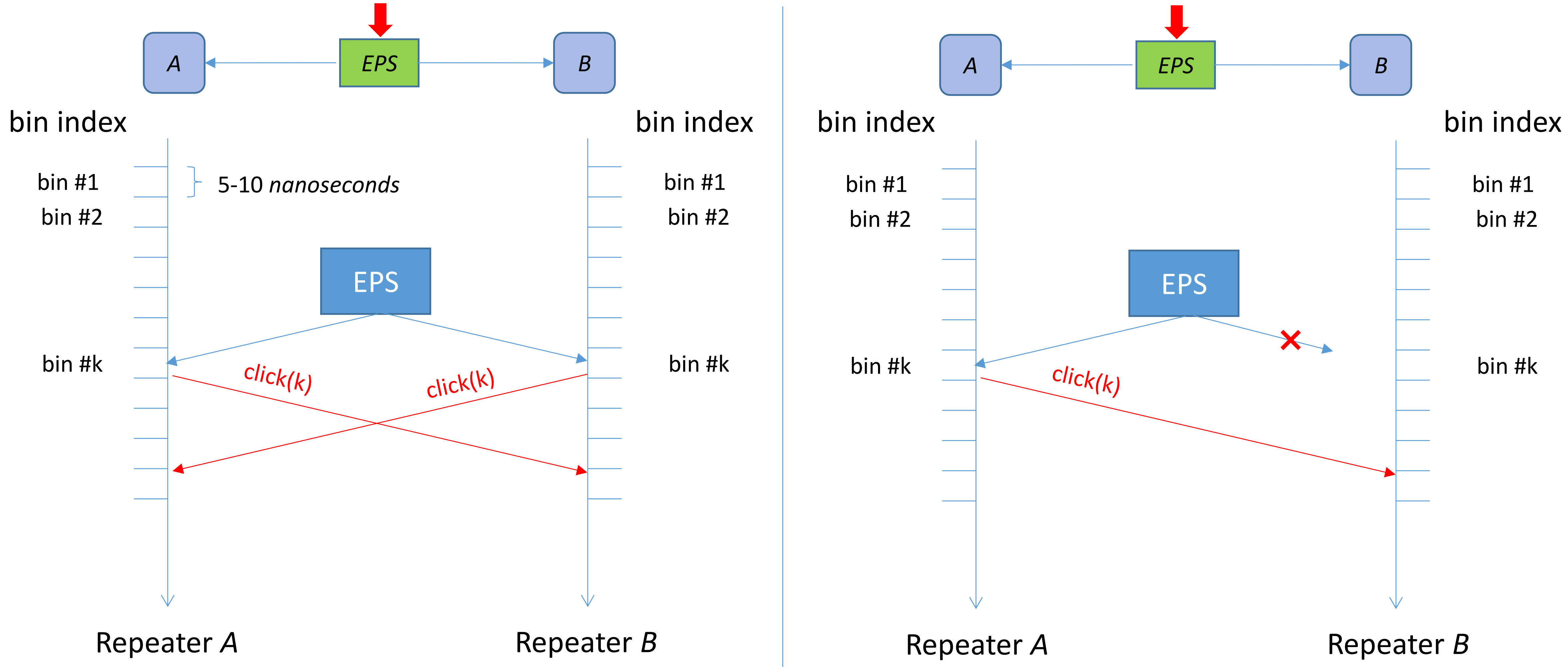
System For Creating Short-Range Entanglement

- By placing a *non-Destructive Photon Detectors (nDPD)* before the *AFC*, the *nDPD* **click** functions as a *weak heralding signal (ACK)*



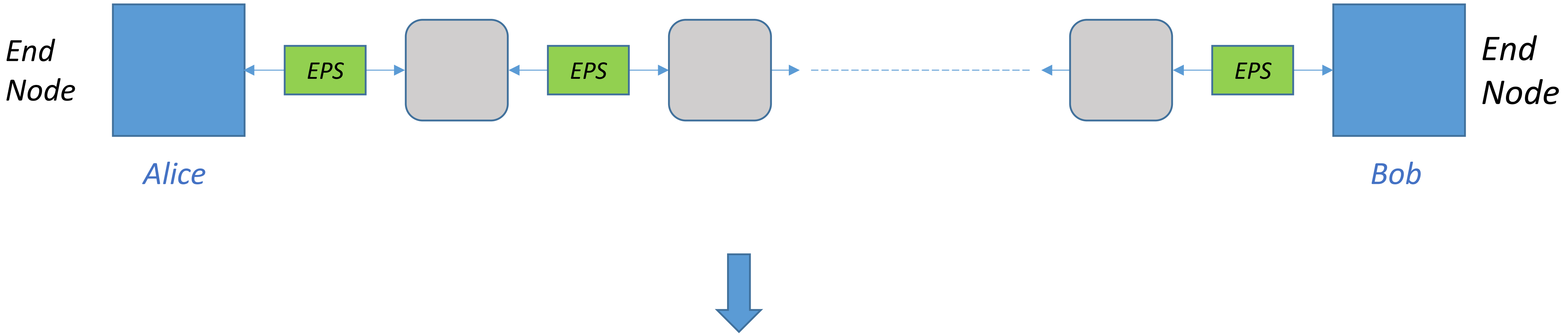
- With the above scenario for the *AFC-MS (Midpoint Source)*, it is not guaranteed that a photon will be reliably absorbed but does confirm the presence of a photon immediately before the *AFC*

Exploitation of the Click Function



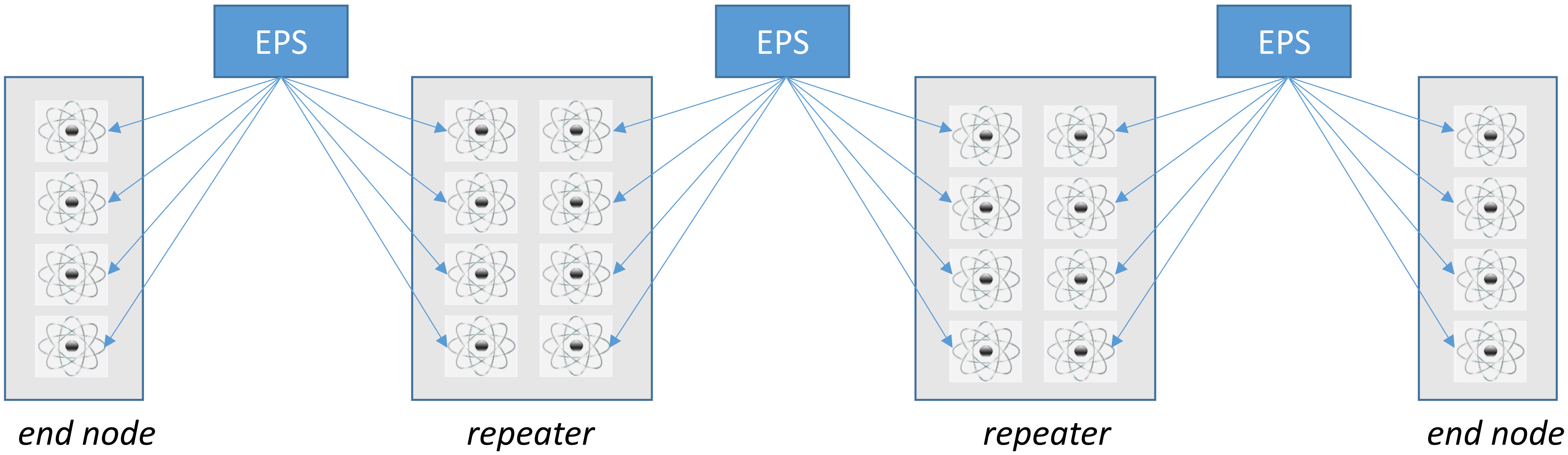
Line of Repeaters

Generic view of a line of repeaters



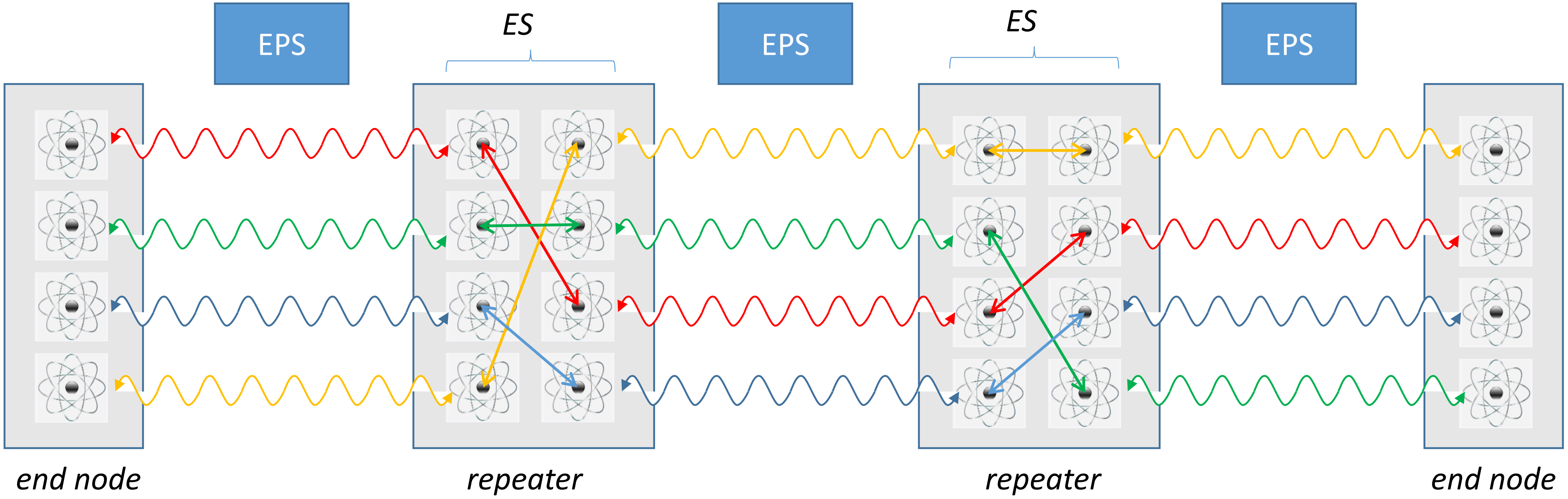
Line of Repeaters

Qubit memories are represented by the atom symbol, regardless of physical device type



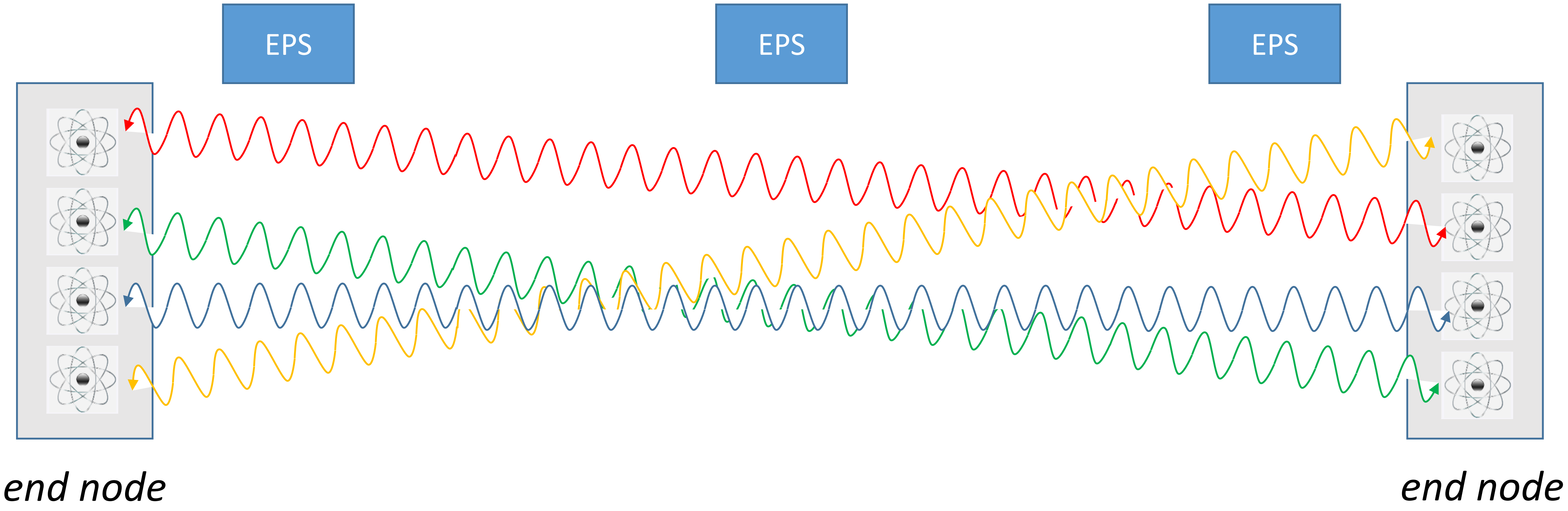
Line of Repeaters

– By performing entanglement swapping (ES) on the repeaters…..



Line of Repeaters

………… we obtain



Line of Repeaters

However, from a graphic perspective, we prefer to use the following representation