DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

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Time Series

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Material

The slides are extracted from the book:
Marco Peixeiro: Time series forecasting in Python
Manning Publications Co.
2022

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Machine Learning Approaches

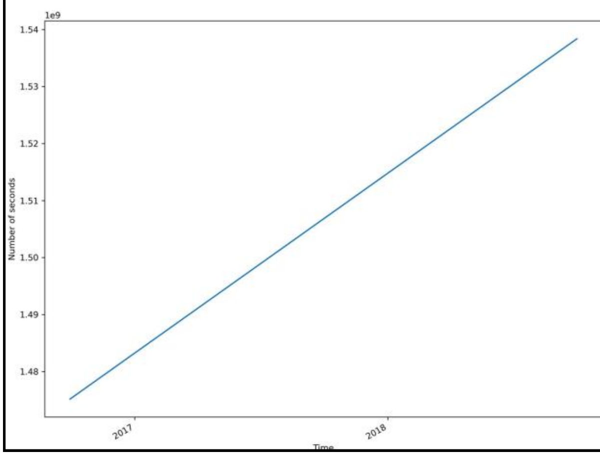
- **Statistical methods work particularly well when you have small datasets (less than 10000 data points). Otherwise, they become very slow**
- **Machine learning** is used either when statistical models take too much time to fit or when they result in correlated residuals that do not approximate white noise.
- Three models:
  - **Single-step model**: outputs a single value representing the prediction for the next timestep. The input can be of any length, but the output remains a single prediction for the next timestep.
  - **Multi-step model:** the output of the model is a sequence of values representing predictions for many timesteps into the future. For example, if the model predicts the next 6 hours, 24 hours, it is a multi-step model.
  - **A multi-output model** generates predictions for more than one target. For example, if we forecast the temperature and wind speed, it is a multi-output model.

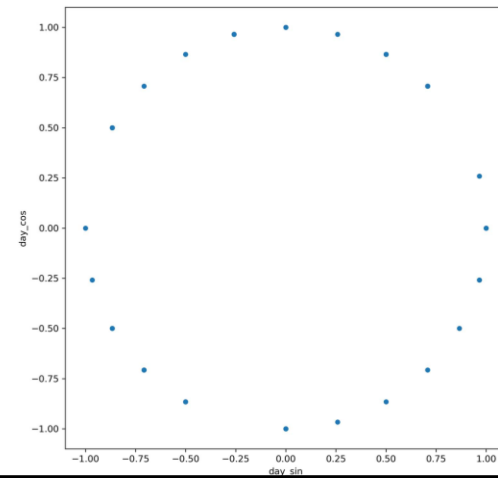
Machine Learning Approaches

- **How can we encode time expressed as YYYY-MM-DD HH:MM:SS?**
  - **Transform date in seconds**
  - **Apply a transformation to recover the cyclical behavior of time**
    - First, we apply a sine transformation
      - day = 24 * 60 * 60
      - df['day_sin'] = (np.sin(timestamp_s * (2*np.pi/day))).values
      - Using this transformation, 12 p.m. is equivalent to 12 a.m.
    - Second, we apply a cosine transformation to avoid the problem
      - df['day_cos'] = (np.cos(timestamp_s * (2*np.pi/day))).values

Machine Learning Approaches

The number of seconds linearly increase with time (left). After transformation, the time is encoded as a numerical value





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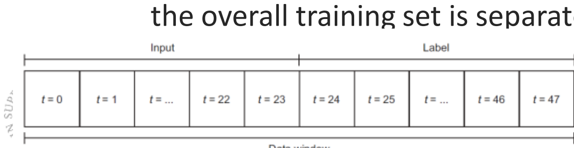
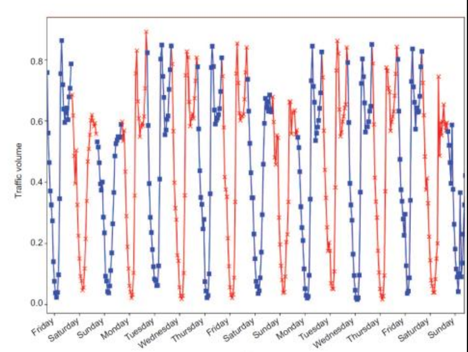
Machine Learning Approaches

We use a simple 70:20:10 split for the training, validation and test sets

- We scale all the values between 0 and 1 (this decreases the time for training deep learning models)
- **Data windowing process**: we define a sequence of data points on the time series and define which are inputs and which are labels
  - For example, we can adopt a window of 24 hours to predict the next 24 hours. Obviously, the overall training set is separated into multiple





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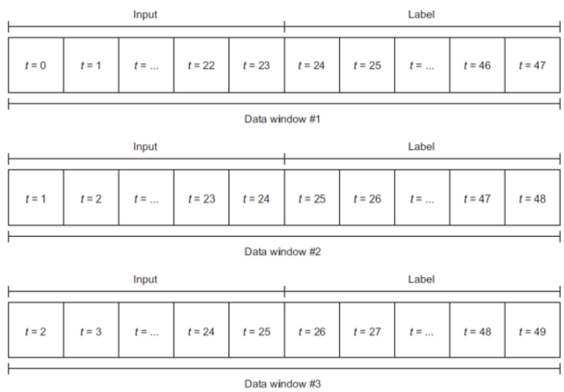
Machine Learning Approaches

- **Data windowing process**
  - With the approach presented in the previous slide, we waste a lot of training data. We can exploit moving windows with step 1

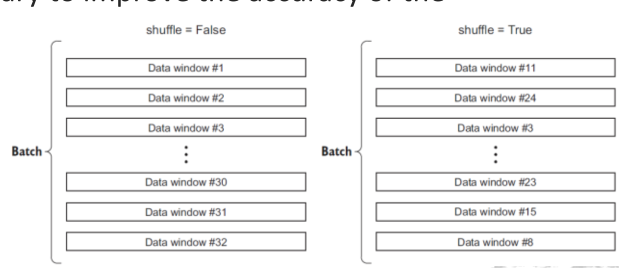


Machine Learning Approaches

- **Data windowing process**
  - Deep learning models are trained with **batches**. A batch is a collection of data windows that are fed to the model for training (update of the parameters after each batch). For example, the batch size is 32.
  - For a training set with 12285 rows, we have 384 batches.
  - Training the model on all the batches is called one epoch.
  - A lot of epochs are normally necessary to improve the accuracy of the predictions
  - **Shuffling** is used at the batch level

Loss function: mean squared error (MSE) (after each batch)

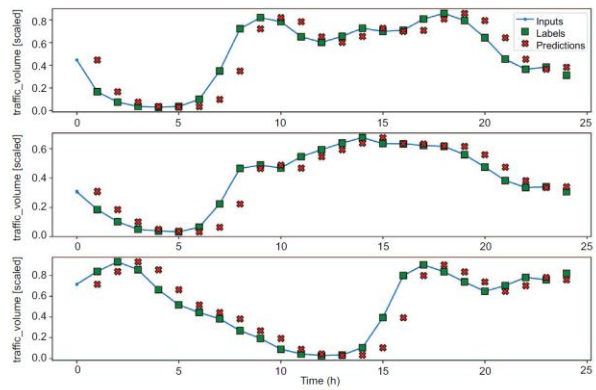
Evaluation metric: mean absolute error (MAE) (after each epoch)



Baseline models

- **Single-step baseline**
  - The input is one timestep and the output is the prediction of the next timestep
  - Window: **input width 1**, **label width 1**, **shift 1**.

Baseline: the simplest prediction we can make is the last observed value. Basically, the prediction is simply the input data point



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Baseline models

- **Multi-step baseline**
  - For a multi-step of N, the input is N timesteps and the output is the prediction of N timesteps (for instance, N = 24 hours)
  - **Window**: input width N, label width N, shift N.

Two possible baselines

1. **Predict the last known value** for the next 24 timesteps.

2. **Predict the last 24 timesteps** for the next 24 timesteps.

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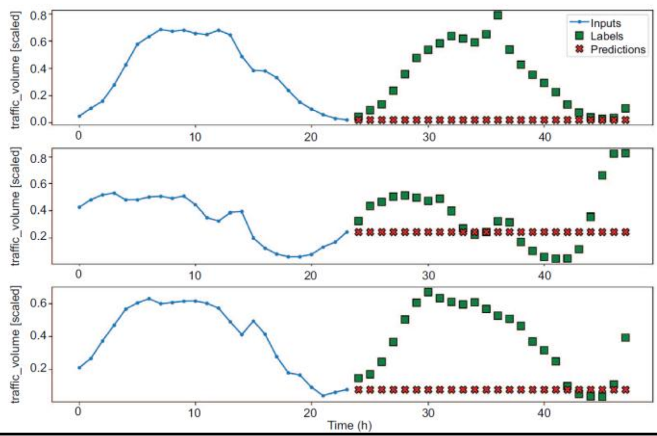
Baseline models

- **Multi-step baseline (predict the last known value)**
  - Simply takes in the input and repeats the last value of the input sequence over N time-steps



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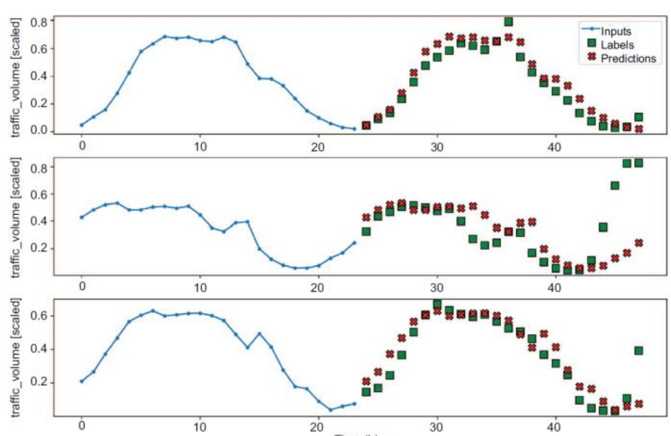
Baseline models

- **Multi-step baseline (repeating the input sequence of N timesteps)**
  - The prediction for the next N timesteps is simply the last known N timesteps of data



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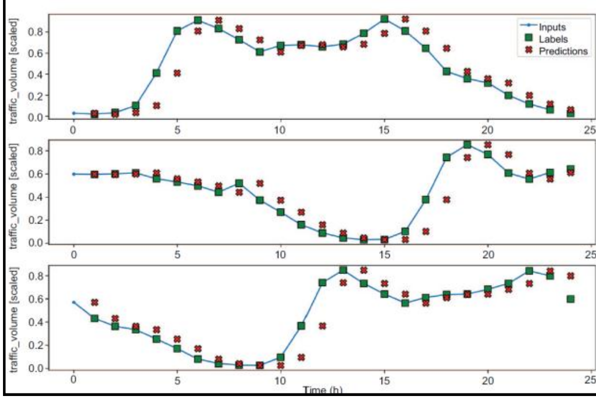
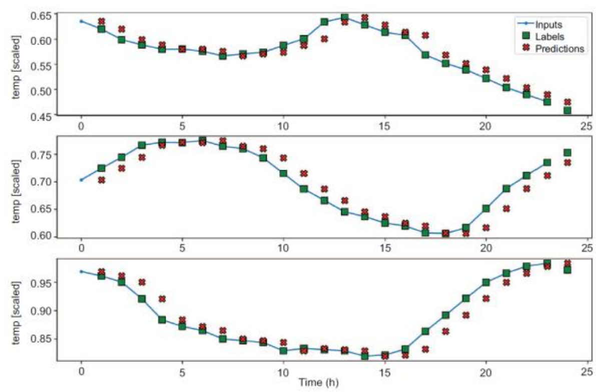
Baseline models

- **Multi-output baseline (single-step model for predicting different outputs)**
  - Let us assume that we have two outputs. We apply a single-step model for predicting both the outputs



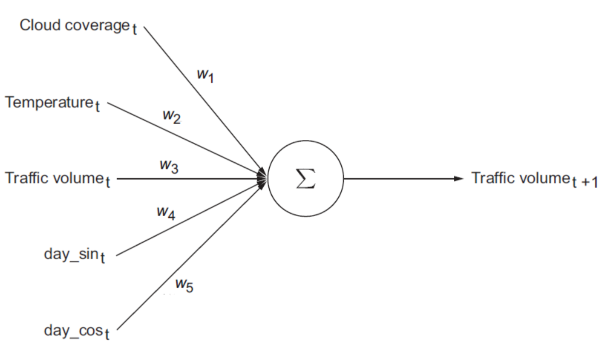


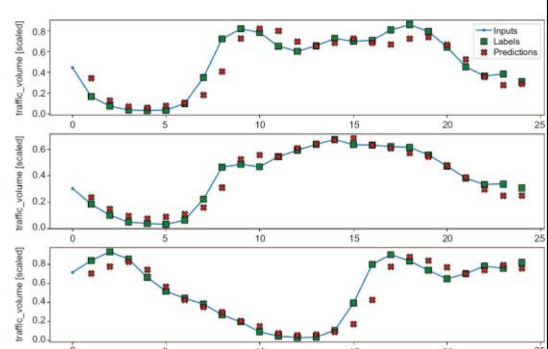
Deep Learning Models

- **Simple linear model**
  - Not actually a deep learning model: A simple multivariate linear regression

$$\text{traffic volume}_{t+1} = w_1 x_{1,t} + w_2 x_{2,t} + w_3 x_{3,t} + w_4 x_{4,t} + w_5 x_{5,t}$$

Single Step Mode





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Deep Learning Models

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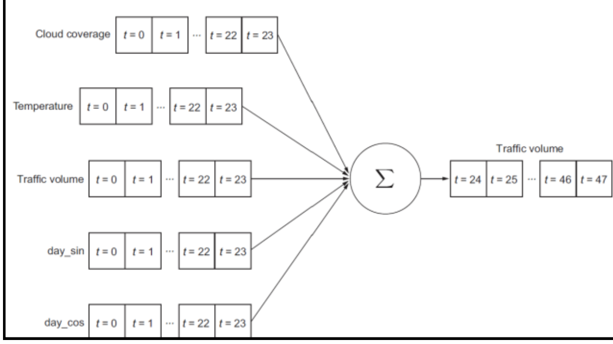
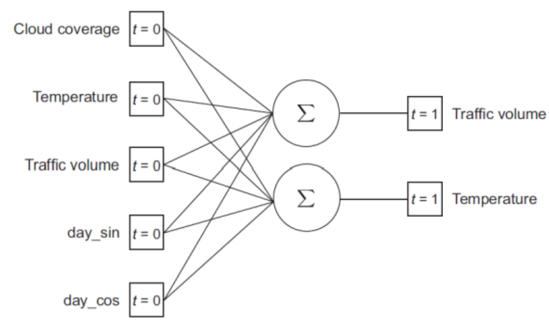
Multi-Step linear Model



Multi-Output linear Model



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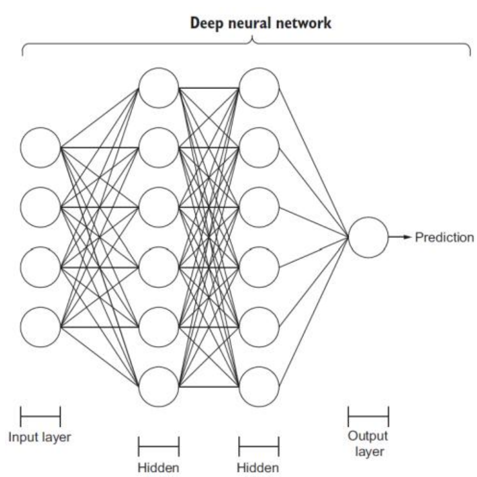
Deep Learning Models

- **Deep neural network**



Activation function: in each neuron of the hidden layers and responsible for generating the output.
If a linear activation function is used, the model will only model linear relationships.
Therefore, to model nonlinear relationships in the data, we must use a nonlinear activation function. Example: Rectified Linear Unit

$$f(x) = x^+ = \max(0, x)$$

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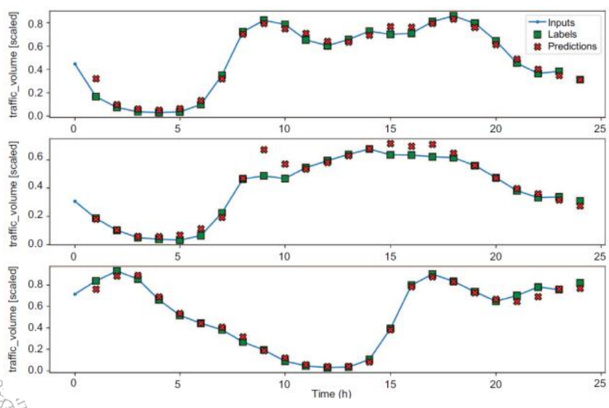
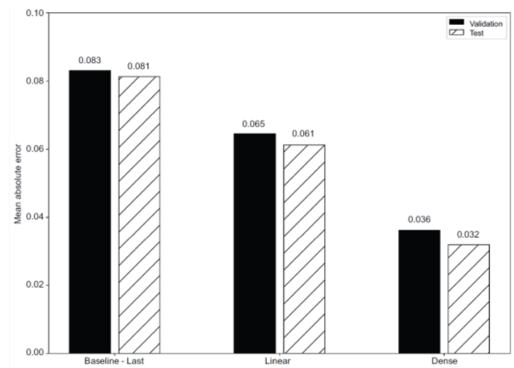
Deep Learning Models

- **Deep neural network (single-step model)**





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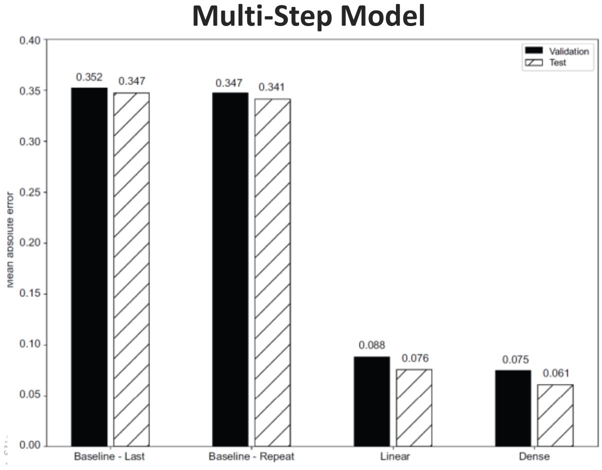
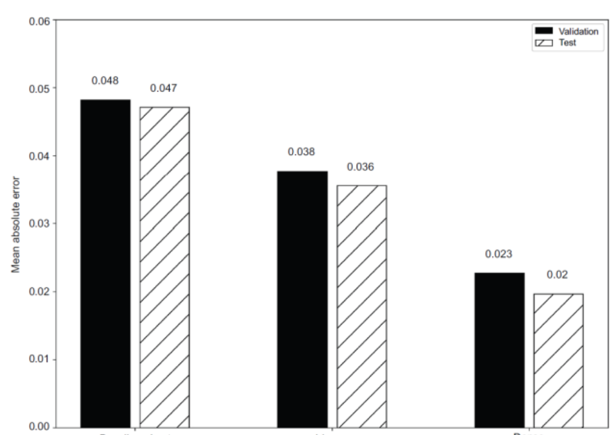
Deep Learning Models

- **Deep neural network**

Multi-Step Model



Multi-Output Model



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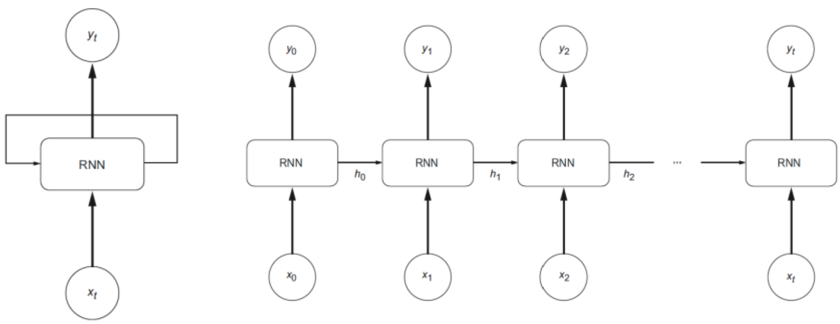
Recurrent Neural Networks

- **A recurrent neural network (RNN) is a deep learning architecture especially adapted to processing sequences of data**
  - It uses a hidden state that is fed back into the network so it can use past information as an input when processing the next element of a sequence.
  - This is how it replicates the concept of memory.



RNNs suffer from short-term memory, meaning that information from an early element in the sequence will stop having an impact further into the sequence.

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Long short-term memory

- **Long short-term memory**
  - Long short-term memory (LSTM) is a deep learning architecture that is a subtype of RNN. **LSTM addresses the problem of short-term memory by adding the cell state.** This allows for past information to flow through the network for a longer period of time, meaning that the network still carries information from early values in the sequence.
  - The LSTM is made up of three gates:
    - The **forget gate** determines what information from past steps is still relevant.
    - The **input gate** determines what information from the current step is relevant.
    - The **output gate** determines what information is passed on to the next element of the sequence or as a result to the output layer.

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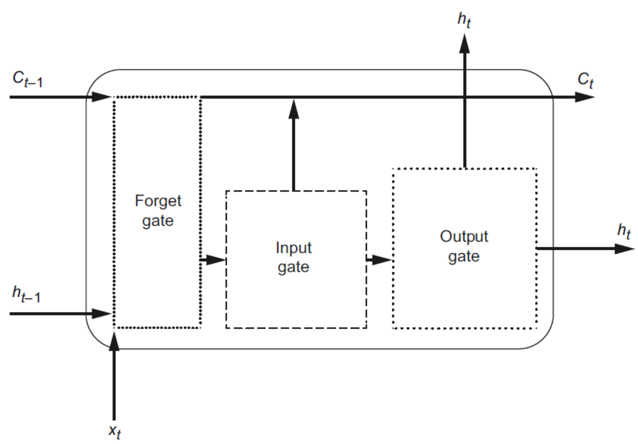
Long short-term memory

- **Long short-term memory**



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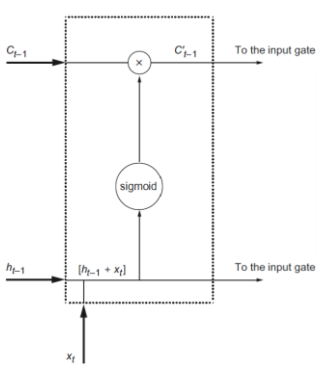
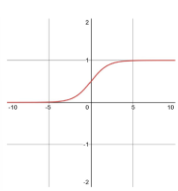
Long short-term memory

- **The forget gate**



$$f(x) = \frac{1}{1 - e^{-x}}$$



The present element of a sequence, x_t, and past information, h_{t-1}, are first combined. They are duplicated, and one is sent to the input gate while the other goes through the sigmoid activation function.

The **sigmoid outputs a value between 0 and 1**, and if **the output is close to 0**, this means that **information must be forgotten**. If **it is close to 1, the information is kept**. The output is then combined with the past cell state using pointwise multiplication, generating an updated cell state C'_{t-1}.

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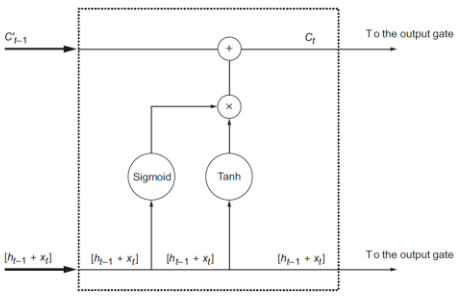
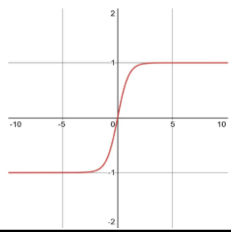
Long short-term memory

- **The input gate**

The **past hidden state and current element of the sequence are duplicated** again and sent through a sigmoid activation function and a hyperbolic tangent (tanh) activation function. Again, the sigmoid determines what information is kept or discarded, while the tanh function regulates the network to keep it computationally efficient.





tanh function

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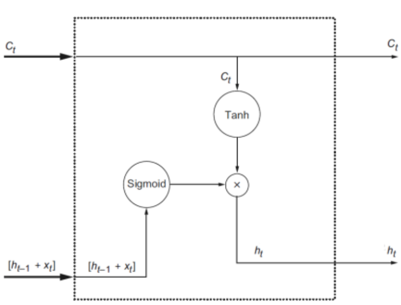
Long short-term memory

- **The output gate**

The past hidden state and current element of a sequence $[h_{t-1} + x_t]$ are passed through the sigmoid function to determine if information will be kept or discarded. Then **the cell state is passed through the tanh function and combined with the output of the sigmoid using pointwise multiplication**. This is the step where past information is used to process the current element of a sequence. We then output a new hidden state h_t, which is passed to the next LSTM neuron or to the output layer. The cell state is also output.



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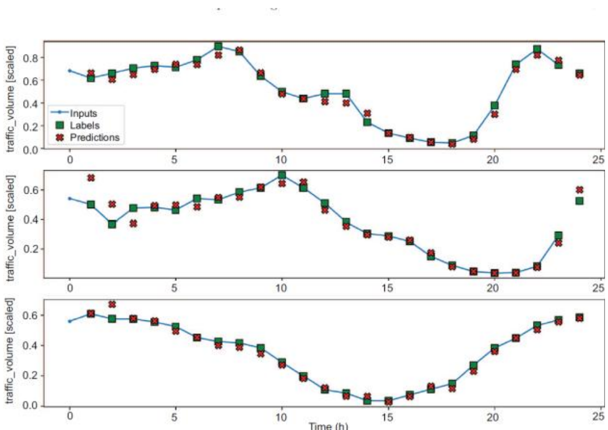
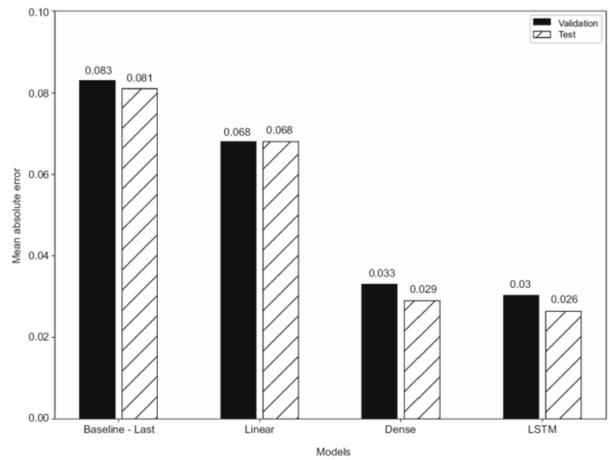
Long short-term memory

- **Single-step model:** we use N timestamps to predict the next timestamp





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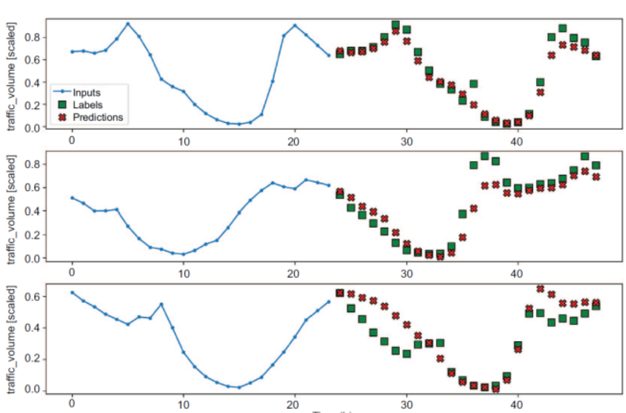
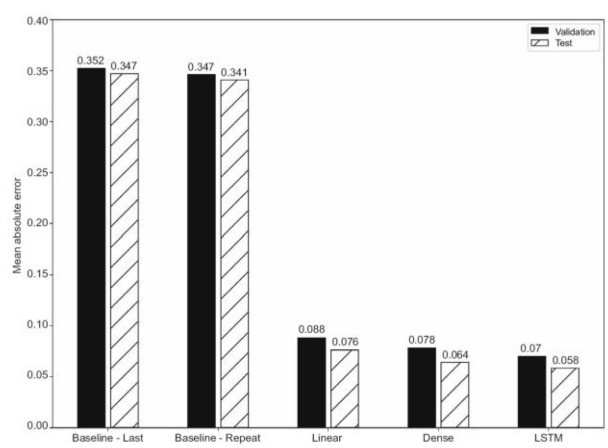
Long short-term memory

- **Multiple-step model:** we use N timestamps to predict N timestamps





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Long short-term memory

- **Multi-output model:** we use N timestamps to predict one timestamp