Thesis Tennis Prediction

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\begin{aligned}
W \sim BIN(n, p_{win}) \approx N(np_{win}, \sqrt{np_{win}q_{win}})\\
p_{win} = \frac{1}{1 + e^{((P_2-P_1)\cdot\beta_p)}}\\
P_i \sim N(S_i, \beta_f)\\
S_i \sim N(\mu_i, \sigma)\\
P_2 - P_1 \sim N(S_2 - S_1, \sqrt{2\beta_f^2})
\end{aligned}
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\begin{aligned}
Pr(P_2 - P_1|S_2 - S_1) &= \frac{1}{\sqrt{2\beta_f^2}}\phi\left(\frac{(P_2-P_1)-(S_2-S_1)}{\sqrt{2\beta_f^2}}\right)\\
&= \frac{1}{\beta_f\sqrt{2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{(P_2-P_1)-(S_2-S_1)}{\sqrt{2\beta_f^2}}\right)^2}\\
&= \frac{1}{2\beta_f\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(P_2-P_1)-(S_2-S_1)}{\sqrt{2\beta_f^2}}\right)^2}\\
Pr(W|P_2-P_1) &= b(n,\ \frac{1}{1+e^{((P_2-P_1)\beta_p)}})\\
&= \binom{n}{w}\left(\frac{1}{1+e^{(P_2-P_1)\beta_p}}\right)^k\left(\frac{e^{(P_2-P_1)\beta_p}}{1+e^{(P_2-P_1)\beta_p}}\right)^{n-k}\\
&= \binom{n}{w}\left(\frac{1}{1+e^{(P_2-P_1)\beta_p}}\right)^n\left(e^{(P_2-P_1)\beta_p}\right)^{n-k}\\
Pr(S_2-S_1) &= \frac{1}{\sqrt{2\sigma^2}}\phi\left(\frac{(S_2-S_1)-(\mu_2-\mu_1)}{\sqrt{2\sigma^2}}\right)\\
&= \frac{1}{\beta_f\sqrt{2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{(S_2-S_1)-(\mu_2-\mu_1)}{\sqrt{2\sigma^2}}\right)^2}\\
&= \frac{1}{2\beta_f\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(S_2-S_1)-(\mu_2-\mu_1)}{\sqrt{2\sigma^2}}\right)^2}
\end{aligned}
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\begin{aligned}
Pr(S_2-S_1|W) &\propto \int_{-\infty}^{\infty} Pr(W|P_2-P_1)\cdot Pr(P_2-P_1|S_2-S_1)d(P_2-P_1)P(S_2-S_1)\\
&= \int_{-\infty}^{\infty}\binom{n}{w}\left(\frac{1}{1+e^{P_2-P_1}}\right)^n\left(e^{(P_2-P_1)\beta_p}\right)^{n-k}\frac{1}{2\beta_f\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(P_2-P_1)-(S_2-S_1)}{\sqrt{2\beta_f^2}}\right)^2}d(P_2-P_1)\frac{1}{2\beta_f\sqrt{\pi}}e^{-\frac{1}{2}\left(\frac{(S_2-S_1)-(\ldots}{\sqrt{2\sigma\ldots}}\right.}\\
&\propto \int_{-\infty}^{\infty}\left(\frac{1}{1+e^{(P_2-P_1)\beta_p}}\right)^n\left(e^{(P_2-P_1)\beta_p}\right)^{n-k}e^{-\frac{1}{2}\left(\frac{(P_2-P_1)-(S_2-S_1)}{\sqrt{2\beta_f^2}}\right)^2}d(P_2-P_1)e^{-\frac{1}{2}\left(\frac{(S_2-S_1)-(\mu_2-\mu_1)}{\sqrt{2\sigma^2}}\right)^2}
\end{aligned}
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\begin{aligned}
Pr(P_2 - P_1 | S_2 - S_1) &= \frac{1}{\sqrt{2\beta_f^2}} \phi \left(\frac{(P_2 - P_1) - (S_2 - S_1)}{\sqrt{2\beta_f^2}} \right) \\
&= \frac{1}{\beta_f \sqrt{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(P_2 - P_1) - (S_2 - S_1)}{\sqrt{2\beta_f^2}} \right)^2} \\
&= \frac{1}{2\beta_f \sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{(P_2 - P_1)^2 + (S_2 - S_1)^2 - 2(P_2 - P_1)(S_2 - S_1)}{2\beta_f^2} \right)} \\
&= \frac{1}{2\beta_f \sqrt{\pi}} e^{-\left(\frac{(S_2 - S_1)^2}{4\beta_f^2} \right)} e^{-\left(\frac{(P_2 - P_1)^2 - 2(P_2 - P_1)(S_2 - S_1)}{4\beta_f^2} \right)} \\
&= \frac{1}{2\beta_f \sqrt{\pi}} e^{-\left(\frac{(S_2 - S_1)^2}{4\beta_f^2} \right)} e^{\left(\frac{2(P_2 - P_1)(S_2 - S_1) - (P_2 - P_1)^2}{4\beta_f^2} \right)} \\
&= \frac{1}{2\beta_f \sqrt{\pi}} e^{-\left(\frac{(S_*)^2}{4\beta_f^2} \right)} e^{\left(\frac{2(P_*)(S_*) - (P_*)^2}{4\beta_f^2} \right)}
\end{aligned}
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\begin{aligned}
Pr(W|P_2-P_1) &= b(n,\ \frac{1}{1+e^{((P_2-P_1)\beta_p)}}) \\
&\approx \frac{1}{\sqrt{\frac{ne^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}\phi\left(\frac{w-\frac{n}{1+e^{((P_2-P_1)\beta_p)}}}{\sqrt{\frac{ne^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}\right) \\
&= \frac{1}{\sqrt{\frac{ne^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{W-\frac{n}{1+e^{((P_2-P_1)\beta_p)}}}{\sqrt{\frac{ne^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}\right)^2} \\
&= \frac{1}{\sqrt{2n\pi}}\frac{1}{\sqrt{\frac{e^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}e^{-\frac{1}{2}\left(\frac{\frac{W(1+e^{((P_2-P_1)\beta_p)})-n}{1+e^{((P_2-P_1)\beta_p)}}}{\sqrt{\frac{ne^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}\right)^2} \\
&= \frac{1}{\sqrt{2n\pi}}\frac{1}{\sqrt{\frac{e^{((P_2-P_1)\beta_p)}}{\left(1+e^{((P_2-P_1)\beta_p)}\right)^2}}}e^{-\frac{1}{2}\left(\frac{\left(W(1+e^{((P_2-P_1)\beta_p)})-n\right)^2}{ne^{((P_2-P_1)\beta_p)}}\right)} \\
&= \frac{1}{\sqrt{2n\pi}}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}e^{-\left(\frac{n^2-2nW(1+e^{((P_2-P_1)\beta_p)})+W^2\left(1+2e^{((P_2-P_1)\beta_p)}+e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}\right)} \\
&= \frac{1}{\sqrt{2n\pi}}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}e^{\left(\frac{2nW(1+e^{((P_2-P_1)\beta_p)})-W^2-n^2-W^2\left(2e^{((P_2-P_1)\beta_p)}+e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}\right)} \\
&= \frac{1}{\sqrt{2n\pi}}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}e^{\left(\frac{2nW-W^2-n^2-W^2\left(e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}+W-\frac{W^2}{n}\right)} \\
&= \frac{e^{\left(W-\frac{W^2}{n}\right)}}{\sqrt{2n\pi}}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}e^{\left(\frac{-(W-n)^2-W^2\left(e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}\right)}
\end{aligned}
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\begin{aligned}
Pr(W|S_2-S_1) &= \int_{-\infty}^{\infty} Pr(W|P_2-P_1)\cdot Pr(P_2-P_1|S_2-S_1)d(P_2-P_1) \\
\approx \int_{-\infty}^{\infty}\frac{e^{\left(W-\frac{W^2}{n}\right)}}{\sqrt{2n\pi}}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}e^{\left(\frac{-(W-n)^2-W^2\left(e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}\right)}&\frac{1}{2\beta_f\sqrt{\pi}}e^{-\left(\frac{(S_2-S_1)^2}{4\beta_f^2}\right)}e^{\left(\frac{2(P_2-P_1)(S_2-S_1)-(P_2-P_1)^2}{4\beta_f^2}\right)}d(P_2-P_1) \\
= \frac{e^{\left(W-\frac{W^2}{n}\right)}e^{\left(-\frac{(S_2-S_1)^2}{4\beta_f^2}\right)}}{2\pi\beta_f\sqrt{2n}}\int_{-\infty}^{\infty}\frac{1+e^{((P_2-P_1)\beta_p)}}{\sqrt{e^{((P_2-P_1)\beta_p)}}}&e^{\left(\frac{2(P_2-P_1)(S_2-S_1)-(P_2-P_1)^2}{4\beta_f^2}-\frac{(W-n)^2+W^2\left(e^{(2(P_2-P_1)\beta_p)}\right)}{2ne^{((P_2-P_1)\beta_p)}}\right)}d(P_2-P_1)
\end{aligned}
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\begin{aligned}
&Pr(W|P_1-P_2,\\
p=\frac{\beta_p(P_1-P_2)}{2\sqrt{(\beta_p(P_1-P_2))^2+1}}+0.5) &= Pr(W|P_1-P_2, p=\frac{\beta_p(P_1-P_2)+\sqrt{1+(\beta_p(P_1-P_2))^2}}{2\sqrt{(\beta_p(P_1-P_2))^2+1}})\\
&= Pr(W|P_1-P_2, p=\frac{P_*+\sqrt{1+P_*^2}}{2\sqrt{(P_*^2+1)}})\\
&= b(n,\ \frac{P_*+\sqrt{1+P_*^2}}{2\sqrt{1+P_*^2}})\\
&\approx \frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}\phi\left(\frac{w-\frac{n\left(P_*+\sqrt{1+P_*^2}\right)}{2\sqrt{1+(P_*)^2}}}{\sqrt{\frac{n}{4(1+(P_*)^2)}}}\right)\\
&= \frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}\phi\left(\frac{\frac{(2w-n)\sqrt{1+P_*^2}-nP_*}{2\sqrt{1+P_*^2}}}{\sqrt{\frac{n}{4(1+P_*^2)}}}\right)\\
&= \frac{1}{\sqrt{(2\pi)}\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(\frac{\frac{(2w-n)\sqrt{1+P_*^2}-nP_*}{2\sqrt{1+P_*^2}}}{\sqrt{\frac{n}{4(1+P_*^2)}}}\right)^2}\\
&= \frac{1}{\sqrt{(2\pi)}\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(\frac{(2w-n)^2(1+P_*^2)-2nP_*(2w-n)\sqrt{1+P_*^2}+2n^2P_*^2}{n}\right)}\\
&= \frac{1}{\sqrt{(2\pi)}\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(\frac{(4w^2-4nw+n^2)(1+P_*^2)+(-4nwP_*+2n^2P_*)\sqrt{1+P_*^2}+2n^2P_*^2}{n}\right)}\\
&= \frac{1}{\sqrt{(2\pi)}\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(\frac{4w^2-4nw+n^2+P_*^24w^2-P_*^24nw+P_*^2n^2-P_*\sqrt{1+P_*^2}4nw+P_*\sqrt{1+P_*^2}2n^2+P_*^22n^2}{n}\right)}\\
&= \frac{1}{\sqrt{(2\pi)}}e^{-\frac{1}{2}\left(\frac{4w^2}{n}-4w+n\right)}\frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(P_*\left(\frac{P_*4w^2}{n}-P_*4w+P_*n-\sqrt{1+P_*^2}4w+\sqrt{1+P_*^2}2n+P_*2n\right)\right)}\\
&= \frac{1}{\sqrt{(2\pi)}}e^{-\frac{1}{2}\left(\frac{4w^2}{n}-4w+n\right)}\frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}e^{-\frac{1}{2}\left(P_*^2\left(\frac{4w^2}{n}-4w+3n\right)+P_*\sqrt{1+P_*^2}(2n-4w)\right)}\\
&= \frac{1}{\sqrt{(2\pi)}}e^{\left(2w-\frac{n}{2}-\frac{2w^2}{n}\right)}\frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}e^{\left(P_*^2\left(2w-\frac{2w^2}{n}-\frac{3}{2}n\right)+P_*\sqrt{1+P_*^2}(2w-n)\right)}
\end{aligned}
$$

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\begin{aligned}
Pr(W|S_2-S_1) &= \int_{-\infty}^{\infty} Pr(W|P_2-P_1)\cdot Pr(P_2-P_1|S_2-S_1)d(P_2-P_1)\\
&\approx \int_{-\infty}^{\infty}\frac{1}{\sqrt{(2\pi)}}e^{\left(2w-\frac{n}{2}-\frac{2w^2}{n}\right)}\frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}}e^{\left(P_*^2\left(2w-\frac{2w^2}{n}-\frac{3}{2}n\right)+P_*\sqrt{1+P_*^2}(2w-n)\right)}\frac{1}{2\beta_f\sqrt{\pi}}e^{-\left(\frac{(S_*)^2}{4\beta_f^2}\right)}e^{\left(\frac{2(P_*)(S_*)-(P_*)^2}{4\beta_f^2}\right)}\\
&\quad d(P_*)\\
&=
\end{aligned}
$$

$$Pr(P_*|S_*) = \frac{2(P_* - S_* + 50)}{2500}$$

$$\begin{aligned}
Pr(W|S_2 - S_1) &= \int_{-\infty}^{\infty} Pr(W|P_2 - P_1) \cdot Pr(P_2 - P_1|S_2 - S_1) d(P_2 - P_1) \\
&\approx \int_{-\infty}^{\infty} \frac{1}{\sqrt{(2\pi)}} e^{\left(2w - \frac{n}{2} - \frac{2w^2}{n}\right)} \frac{1}{\sqrt{\frac{n}{4(1+P_*^2)}}} e^{\left(P_*^2\left(2w - \frac{2w^2}{n} - \frac{3}{2}n\right) + P_*\sqrt{1+P_*^2}(2w-n)\right)} \frac{2(P_* - S_* + 50)}{2500} d(P_*) \\
&\propto
\end{aligned}$$

option 2, different variances in form of serve and return

$$\begin{aligned}
S_i &\sim N(\mu_i, \beta_S) \\
Pr(S_i) &= \frac{1}{\beta_S}\phi(\frac{S_i - \mu_i}{\beta_S}) \\
Prior = Pr(S_1, S_2) &= \frac{1}{\beta_S^2}\phi(\frac{S_1 - \mu_1}{\beta_S})\phi(\frac{S_2 - \mu_2}{\beta_S}) \\
&\propto e^{-\frac{1}{2}\left(\frac{S_1 - \mu_1}{\beta_S}\right)^2} e^{-\frac{1}{2}\left(\frac{S_2 - \mu_2}{\beta_S}\right)^2}
\end{aligned}$$

$$\begin{aligned}
Likelihood = Pr(W|S_1, S_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\beta_{Serve}\beta_{Return}} \phi(\frac{P_1 - S_1}{\beta_{Serve}})\phi(\frac{P_2 - S_2}{\beta_{Return}}) \binom{n}{w} \left(\frac{1}{1 + e^{(P_2 - P_1)\beta_p}}\right)^n \left(e^{(P_2 - P_1)\beta_p}\right)^{n-k} d_{P_1} d \\
&\propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{P_1 - S_1}{\beta_{Serve}}\right)^2} e^{-\frac{1}{2}\left(\frac{P_2 - S_2}{\beta_{Return}}\right)^2} \left(\frac{1}{1 + e^{(P_2 - P_1)\beta_p}}\right)^n \left(e^{(P_2 - P_1)\beta_p}\right)^{n-k} d_{P_1} d_{P_2}
\end{aligned}$$