**Lecture 12      Prediction for Geostatistical Data; Case Study**

Spatial prediction refers to the prediction of unknown quantities $Z(s_0)$, based on sample data $Z(s_i)$.

## 12.1     Generalized Least Squares

If the model for the geostatistical data is parametric, we can use standard least squares to estimate the coefficients if we do not think the errors are correlated. Once we have the estimated coefficients, we can predict values at any location by simply plugging in the location coordinates into the fitted model. (Caution should be taken against extrapolating outside the range of the predictors.) Figures 10.5-6 were constructed by using standard least squares to fit a quartic model to the benthic index data, predicting values on a grid, and then displaying only those values within the convex hull defined by the observed longitudes and latitudes.

## 12.2    Kriging

An alternative method of predicting values, called ***kriging***, was developed in the early 1960s by G. Matheron, who named this method after D.G. Krige, a South African mining engineer. ***Universal kriging*** is a method of predicting values at any location where the ε term is not assumed to be 0, but is instead predicted as well. ***Ordinary kriging***, usually just called kriging, assumes a constant trend surface, so it is usually performed on residuals from a trend surface model.

For any location $s_0$, the (ordinary) kriging prediction of the response variable is given by:

$$\hat{Z}(s_0) = \sum_{i=1}^{n} \lambda_i z(s_i)$$

where

$$\sum_{i=1}^{n} \lambda_i = 1$$

That is, the predicted value is a weighed average of all of the observed values. The weights, $\lambda_i$, $i = 1, 2, \ldots, n$, depend on the spatial correlation.

**Example 12.1**        Predictions of benthic index based on ordinary kriging with quartic

model:

```r
library(sp)
library(EnvStats)
library(gstat)
data(Benthic.df)
coordinates(Benthic.df)=~Longitude+Latitude
vg.benthic <- variogram(Index ~ Longitude + Latitude +
                        Longitude^2 + Longitude*Latitude + Latitude^2 +
                        Longitude^3 + Longitude^2*Latitude +
                        Longitude*Latitude^2 + Latitude^3 + Longitude^4+
                        Longitude^3*Latitude + Longitude^2*Latitude^2 +
                        Longitude*Latitude^3 + Latitude^4, data=Benthic.
df)
vg.fit.benthic<-fit.variogram(vg.benthic, model=vgm(1,"Exp", 0.5,1))
lat <- Benthic.df$Latitude
lon <- Benthic.df$Longitude
Latitude <- seq(min(lat), max(lat), length=50)
Longitude <- seq(min(lon), max(lon), length=50)
predict.list <- list(Longitude=Longitude,Latitude=Latitude)
predict.grid <- expand.grid(predict.list)
coordinates(predict.grid) = ~Longitude+Latitude
gridded(predict.grid) = TRUE
krige.fit.benthic <- krige(Index ~1,Benthic.df,predict.grid,model= vg.fit.
benthic)
spplot(krige.fit.benthic["var1.pred"],main="Figure 12.1  Predictions of Be
nthic Index - ordinary kriging")
index.chull <- chull(lon, lat)
inside <- point.in.polygon(point.x = predict.grid$Longitude,point.y = pred
ict.grid$Latitude,pol.x = lon[index.chull],pol.y = lat[index.chull])
krige.fit.benthic[inside == 0] <- NA
contour(Longitude, Latitude, data.matrix(krige.fit.benthic), levels=seq(1,
 5, by=0.5), labcex=0.75,xlab="-Longitude (degrees West)", ylab="Latitude
(degrees North)")
title(main=paste("Figure 12.2  Contour Plot of Benthic Index", "ordinary k
riging", sep="\n"))
persp(Longitude, Latitude, data.matrix(krige.fit.benthic), xlim = c(-77.3,
 -75.9), ylim = c(38.1, 39.5), zlim = c(0, 6), theta = -45, phi = 30, d =
0.5, xlab="-Longitude (degrees West)",
     ylab="Latitude (degrees North)",
     zlab="Benthic Index", ticktype = "detailed")
title(main=paste("Figure 12.3  Surface Plot of Predictions of Benthic Inde
x", "Based on Kriging", sep="\n"))
```
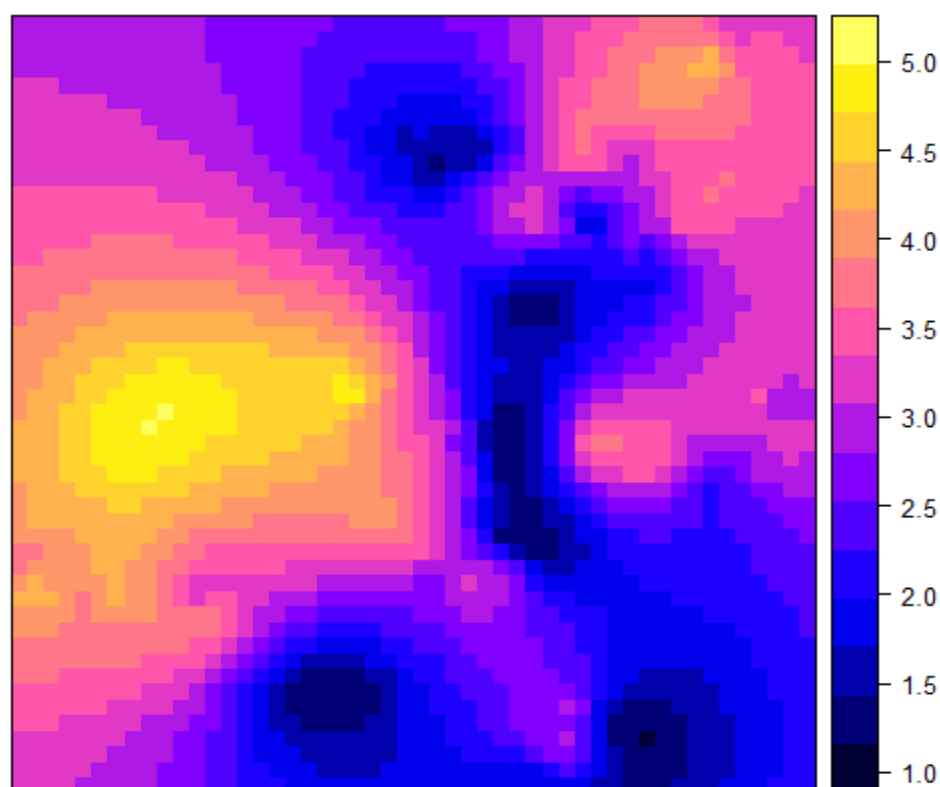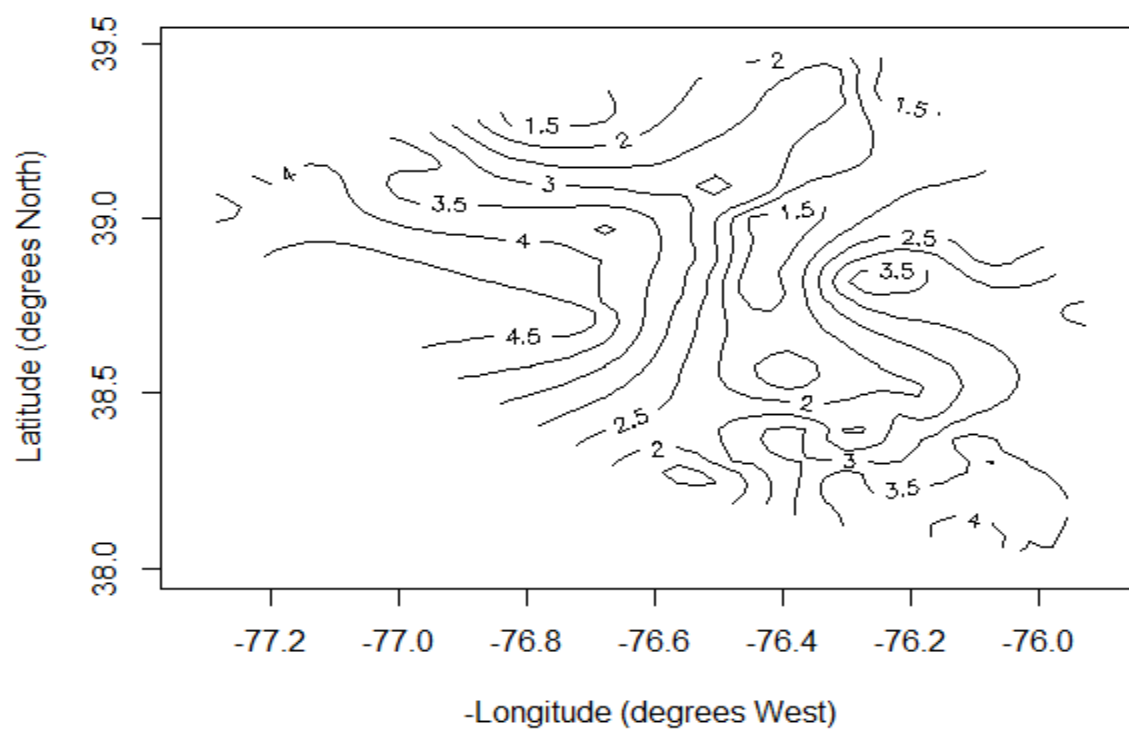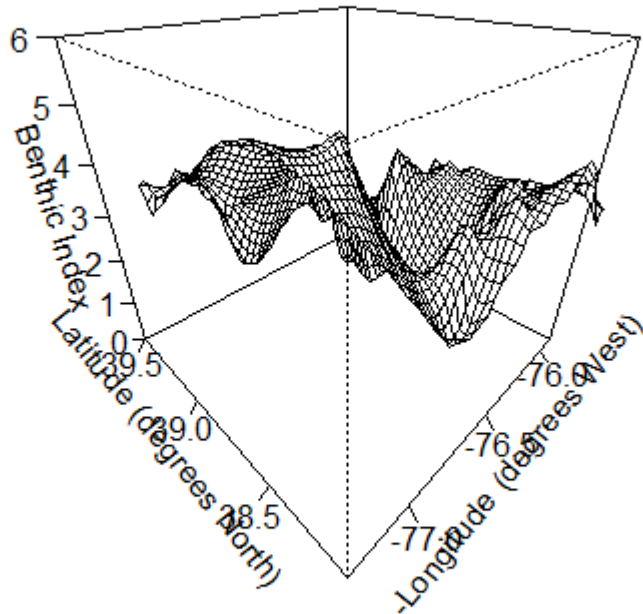
**Figure 12.1 Predictions of Benthic Index - ordinary kriging**



**Figure 12.2 Contour Plot of Benthic Index ordinary kriging**

Figure 12.3 Surface Plot of Predictions of Benthic Index Based on Kriging

## 12.3 A case study

Paper attached: *The meuse data set: a brief tutorial for the* **gstat** *R package*, by E. Pebesma (Sections 1-6).

**Exercises**

12.1    Repeat the example in this talk.

12.2    Re Example 12.1, make predictions of benthic index based on ordinary kriging with quadratic model.

12.3    Re Case Study, predict the lead concentration using ordinary kriging. Is the log-transformation necessary?

**References**

- Bivand, R. S., Pebesma, E. and Gómez-Rubio, V., (2013), *Applied Spatial Data Analysis with R*, SPRINGER

- Millard, S.P. and Neerchal, N. K. (2000), *Environmental Statistics with S-PLUS*, Chapman & Hall.