05 Mapping Diesease

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(05) Mapping Disease

Preamble

Binomial Distribution

A random variable is Binomial if it represents the count of the number of successes from n trials and p is the probability of success.

Simulation and Probabilities

In order to simulate, for example, a coin toss:

```
library(tidyverse)
sample(c("H", "T"), size = 3, replace = TRUE, prob = c(1,1))

## [1] "H" "H" "H"

## Count the number of heads
sum(sample(c("H", "T"), size = 3, replace = TRUE, prob = c(1,1))=="H")

## [1] 3

## This can be automated with `rbinom`
rbinom(n = 2, size = 3, prob = 0.5)
```

```
## [1] 2 3
```

where:

- The output is the number of Successes
- size is the size of teh experiment, i.e. the number of repetitions (i.e. coin flips)
- n is how many numbers you want to get back, each corresponding to a repeated simulation.

We can also use counting formulas like $\binom{m}{n}$ via choose(m,n). See Counting Formulas generally.

Density

The density rather than the count of the binomial distribution can also be simulated.

So for example the probability of getting 0, 1, 2 or 3 heads out of 3 coin tosses is:

```
dbinom(x = 1:3, size = 3, prob = 0.5)
## [1] 0.375 0.375 0.125
```

Mean and Variance

In theory the summary statistics of a binomial distribution are:

	Mean	Variance
Binomial Distribution	np	np(1-p)

Where:

- ullet n is the number of repetitions
 - in **R** this is size because n is already being used for the output vector length.

This can be verified by doing:

```
primes::generate_primes(min = 12, max = 99) # Two primes ensures p, n and (1-p)

## [1] 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97

# are relatively prime

n <- 29
p <- 0.43

(rbinom(n = 100000, size = n, prob = p) %>% mean() /p) %>% signif(2)
```

```
## [1] 29

(rbinom(n = 10000, size = n, prob = p) %>% var() / (n * (1-p))) %>% signif(2)

## [1] 0.43
```

Hypothesis test for a Difference in Proportional

Let's say that we make the following observation:

- *Trump County* has 12/100 with a disease
- Clinton County has 188/1000 with a disease

A Chi-Square Distribution can be used to compare the proportions.

```
## Make a Table
dis_df <- data.frame("Trump" = c(12, 100-12), "Clinton" = c(188, 1000-188))
dis_mat <- as.matrix(dis_df)</pre>
dimnames(dis_mat) <- list(</pre>
   c("Disease", "No Disease"),
   c("Trump", "Clinton")
dis_mat
            Trump Clinton
## Disease
              12
## No Disease 88
## Perform the Chi Test
chisq.test(dis_mat)
## Pearson's Chi-squared test with Yates' continuity correction
## data: dis_mat
## X-squared = 2.3872, df = 1, p-value = 0.1223
chisq.test(t(dis_mat))
##
```

```
## Pearson's Chi-squared test with Yates' continuity correction
## data: t(dis_mat)
## X-squared = 2.3872, df = 1, p-value = 0.1223
```

This shows the probability of rejecting the null hypothesis when it is true (i.e. asserting that there is a difference between counties when there is in fact not) is still too high of a risk at 12%, hence there is not enough evidence to reject the null hypothesis that there is no difference between counties.

Pesticide Question

If a type of pesticide:

- kills 13 out of 20 male budworms
- kills 10 out of 20 female budworks
- 1. The proportion of each gender killed?
- 2. Is there evidence the proportion killed by this pesticide at this dose is different for each gender?

```
female <- c(10, 20-10)

bug <- matrix(c(male, female), ncol = 2)
dimnames(bug) <- list(c("dead", "alive"), c("male", "female"))</pre>
## dead 13 10
chisq.test(bug)
## Pearson's Chi-squared test with Yates' continuity correction
## X-squared = 0.40921, df = 1, p-value = 0.5224
```

- 1. The Proportion of Male Budworms killed is:
- $\frac{13}{20} = 0.65$ for Males $\frac{10}{20} = 0.5$ for Females
- 2. In this case, the probability of concluding that there is a difference between genders when there is in fact no difference is sufficiently small (p=5.2) to reject the null hypothesis and assert that there is indeed a difference between genders.

• The sample size is quite small so a larger sample is justified.

Poisson Distribution

The *Poisson* distribution is appropriate where values are:

- integer values that may occur in interval of time
 - e.g. the number of call outs completed in one day
- Events are independent, i.e. the occurrence of one event does not effect the probability of another event
- The average rate of events occurring is independent from other occurences
- Events cannot overlap
 - So the number of callouts in one day is fine
 - The number of phone calls isn't because if two calls are recieved at the same time they either overlap or the other phone call is rejected in lieu of the first one meaning that the phone calls are not independent.

Horse Kicks

The number of deaths caused by a horse kick in a given regiment per year is (Bortkiewics, 1898):

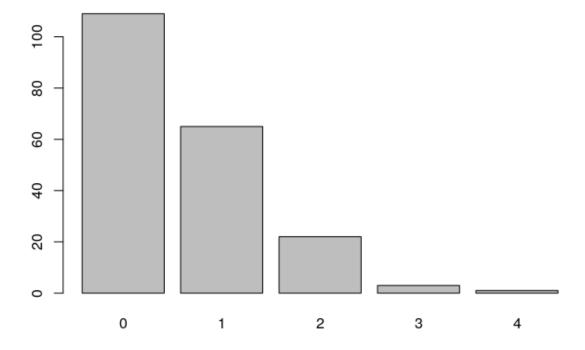
Deaths per Year Per Regiment	Deaths	Poisson Expectation $(\frac{\lambda^k d^{-\lambda}}{k!})$
0	109	108.7
1	65	66.3
2	22	20.2
3	3	4.1
4	1	0.6
5+	0	0.1

This can be plotted in R:

```
horsekick <- c(109, 65, 22, 3, 1)
names(horsekick) <- 0:4
print(horsekick)

## 0 1 2 3 4
## 109 65 22 3 1

barplot(horsekick, col = "grey")
```

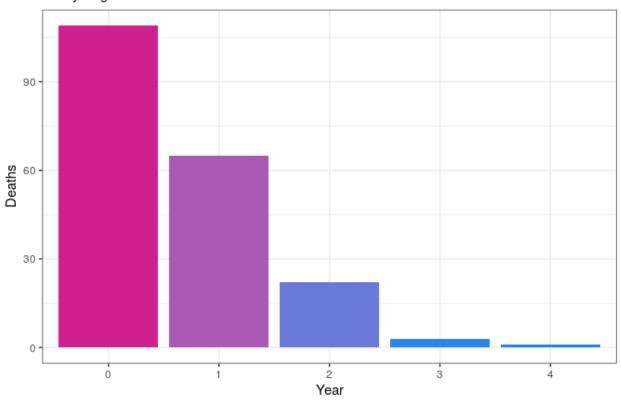


```
hk_tb <- tibble::enframe(horsekick, name = "Year", value = "Deaths")

bp <- ggplot(hk_tb, aes(x = Year, y = Deaths, fill = Deaths)) +
    geom_col() +
    scale_fill_gradient(high = "#D02090", low = "#1c86ee") +
    theme_bw() +
    labs(title = "Deaths by Horsekick", subtitle = "In any Regiment") +
    guides(fill = FALSE)</pre>
```

Deaths by Horsekick

In any Regiment



Mean Value

In order to determine the average number of deaths over the period of years (this is weird because the data set is weird, don't pay mind to it).

```
sum((0:4)*horsekick)/sum(horsekick)
## [1] 0.61
```

Simulation and Poisson Probabilities

Poisson Values can be simulated, If a delivery driver has 3 jobs every day, a month, in no particular order might look like this:

```
rpois(30, lambda = 3)
```

```
## [1] 1 3 4 6 0 6 0 5 2 2 5 5 4 3 4 2 2 1 1 2 1 4 2 1 2 1 4 1 0 0
```

but maybe we would have to consider different days as different poisson distributions?

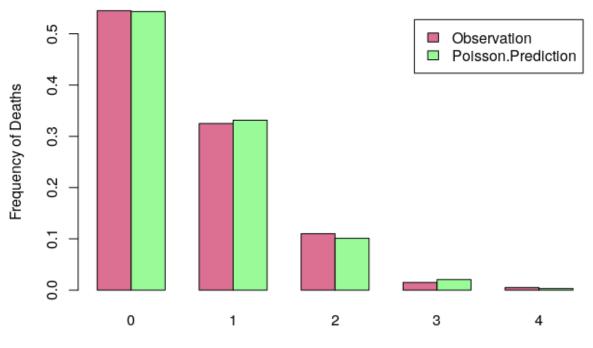
Return Probability density

If we wanted to know the probability of a delivery driver getting a various number of jobs:

```
dpois(0:6, lambda = 3) %>% round(1)
## [1] 0.0 0.1 0.2 0.2 0.2 0.1 0.1
```

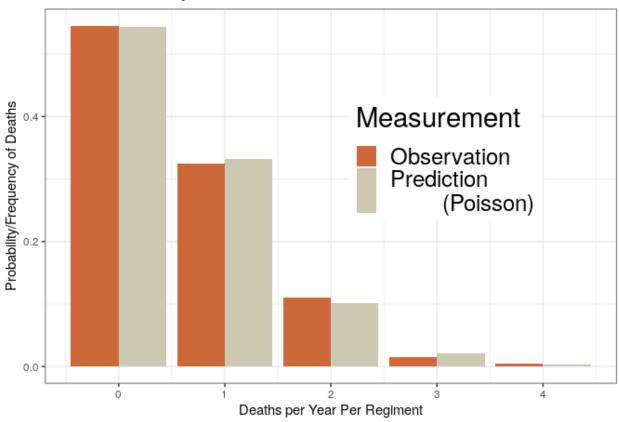
This means that the Horsekick data can be predicted and visualised:

Actual and Predicted Deaths



Men Killed per Year per Regiment

Model of Deaths by Horse-kick



WWII Example

During the Second World war, England was divided into $576\ 0.25*0.25$ km squares, the number of V-bomb strikes per square was:

Number of Strikes	0	1	2	3	4	5
Number of Squares	229	211	93	35	7	1

Hence the Strikes per Square can be calculated:

Number of Strikes	0	1	2	3	4	5	П	15
Number of Squares Strikes per Square				••	7 4/7	_		0.0

And hence the average number of strikes is

Calculate the Average value

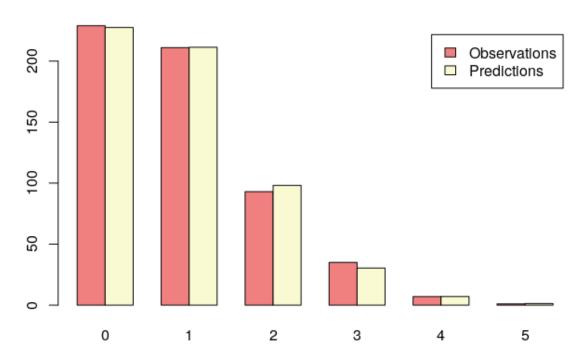
The average value of strikes per square is:

```
strikes_df <- data.frame("N.Strikes" = 0:5, "squares" = c(229, 211, 93, 35, 7, 1))
strikes_mat <- as.matrix(strikes_df)

# Average Strikes per square
lambda <- sum(0:5 * strikes_mat[,2])/576</pre>
```

Plot the Frequencies and Predictions

Rocket Bomb Strikes in London



Using ggplot2:

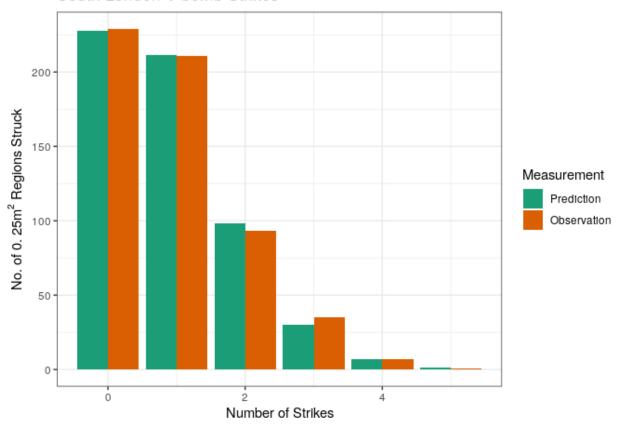
```
strikes_df$pred <- dpois(0:5, lambda)*576

(strikes_tb <- pivot_longer(strikes_df, cols = c(squares, pred)))</pre>
```

```
## # A tibble: 12 x 3
     N.Strikes name
                      value
         <int> <chr>
                      <dbl>
            0 squares 229
            0 pred
                     228.
            1 squares 211
            1 pred
                     211.
            2 squares 93
            2 pred
                      98.1
            3 squares 35
## 8
            3 pred
                      30.4
## 9
            4 squares 7
            4 pred
                       7.06
## 11
            5 squares 1
## 12
            5 pred
                       1.31
```

```
ggplot(strikes_tb, aes(y = value, x = N.Strikes, fill = name)) +
```

South London V-bomb Strikes



Confidence Intervals

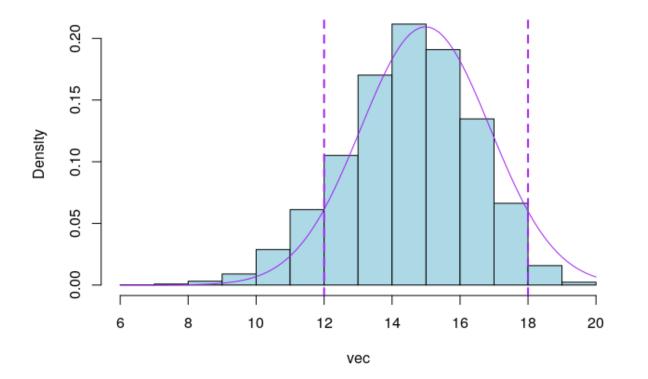
Confidence Intervals can be produced using bootstrapping or a simulating the distribution. ### Bootsrap Confidence Intervals #### Binomial ##### Seeds If it was observed that 15 seeds germinate in a sample of 20, bootstrap a confidence interval for the proportion of seeds germinating.

```
x <- rbinom(n = 10000, size = 20, prob = 15/20)
p <- x/20
quantile(p, c(0.025, 0.975))

## 2.5% 97.5%
## 0.55 0.90</pre>
```

Plot the Histogram

```
dens_hist <- function(vec, main = paste(""), col = "purple", fill = "lightblue") {
## Make the Histogram
   hist(vec, freq = FALSE, col = fill, main = main)
   c_int <- quantile(x = vec, c(0.05, 0.95))
## Plot the Curve
   x <- 1:2000
   curve(dnorm(x = x, mean = mean(vec), sd = sd(vec)), add = TRUE, col = col)
   abline(v = c_int[1], col = col, lwd = 2, lty = 2)
   abline(v = c_int[2], col = col, lwd = 2, lty = 2)
}
dens_hist(x)</pre>
```

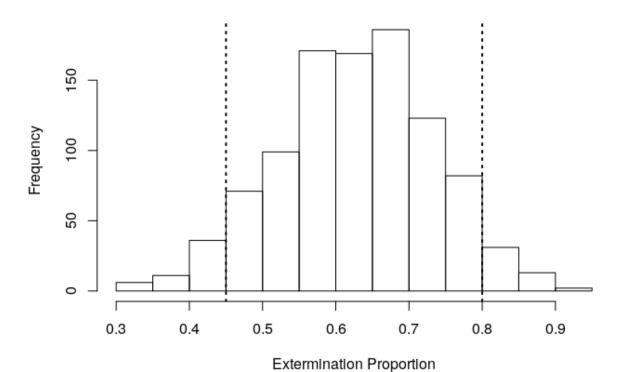


```
| print("test")
| ## [1] "test"
```

Bud Worms A confidence interval for the number of budworms can be generated using rbinom():

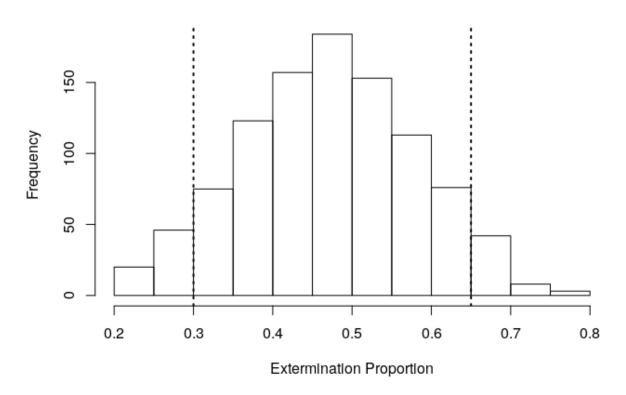
```
## Create a Binomial Sample
x <- rbinom(n = 1000, size = 20, prob = 13/20)
p <- x/20
hist(p, main = "Male Bud Worms", xlab = "Extermination Proportion")
abline(v = quantile(p, 0.05), lwd = 2, lty = 3)
abline(v = quantile(p, 0.95), lwd = 2, lty = 3)</pre>
```

Male Bud Worms



```
x <- rbinom(n = 1000, size = 20, prob = 10/20)
p <- x/20
hist(p, main = "Female Bud Worms", xlab = "Extermination Proportion")
abline(v = quantile(p, 0.025), lwd = 2, lty = 3)
abline(v = quantile(p, 0.875), lwd = 2, lty = 3)</pre>
```

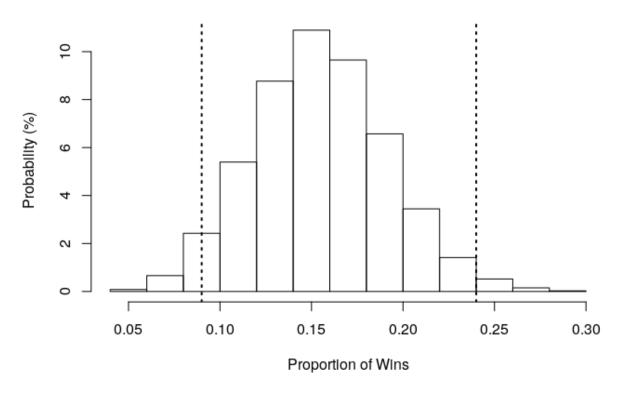
Female Bud Worms



Poker Machine A poker machine is meant to return a win 20% of the time (In NSW the requirement is that the machine gives back 90% of every dollar). Out 100 tries a win is recorded 16 times. Find a 95% confidence interval for the proportion of wins.

Because we should be using a Bar plot not a histogram the scale will be weird, instead set the size of the rbinom to 100 and have the y-axis represent the percentage of success probability. To do this properly it is necessary to use a barplot and/org ggplot + geom_col.

Poker Machine Wins



Poisson

In order to return a confidence interval for the average number of kicks, expand the table into an observation counts, resample from it and then take mean value:

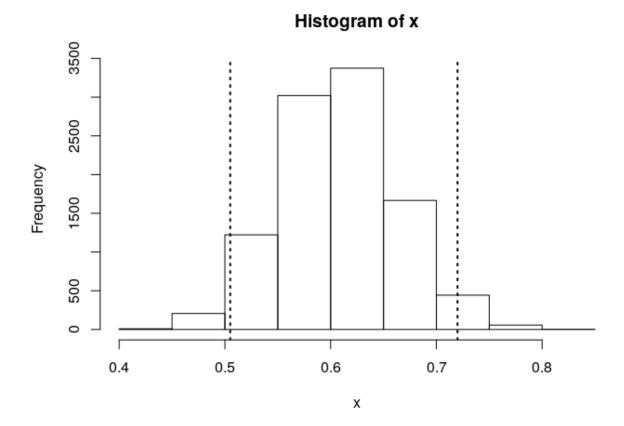
```
## Turn the horsekick table into an observation list
obs <- rep(0:4, horsekick) ## Replicate according to table of counts

n <- 10^4 # size of Bootstrap

x <- replicate(n, {
    resamp <- sample(obs, replace = TRUE)
    mean(resamp)
})

# barplot(table(x)*10/n, col = "lightblue", border = "white", xlab = "Sample Mean
    Value of Death by HorseKick", ylab = "Probability of Population Mean")

hist(x)
abline(v = quantile(x, 0.025), lwd = 2, lty = 3)
abline(v = quantile(x, 0.975), lwd = 2, lty = 3)</pre>
```



Approximate Confidence Intervals

Binomial Approximate confidence intervals can be generated by using the epitools package.

```
library(epitools)
epitools::binom.approx(15, 20) #Approx ConfInt for Binomial for 15 out of 20

## x n proportion lower upper conf.level
## 1 15 20     0.75 0.5602273 0.9397727     0.95

## BudWorms
binom.approx(13,20, conf.level = 0.975)

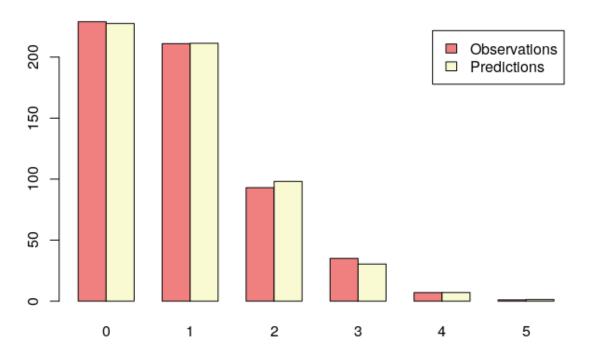
## x n proportion lower upper conf.level
## 1 13 20     0.65 0.4109462 0.8890538     0.975
```

```
binom.approx(10,20)
## x n proportion lower
                                upper conf.level
## 1 10 20
              0.5 0.2808694 0.7191306
binom.approx(20, 100)
   x n proportion lower
                               upper conf.level
## 1 20 100
              0.2 0.1216014 0.2783986
# HorseKick
n_kicks <- sum(0:4*horsekick) # Number of Men killed by a HorseKick</pre>
n_men <- sum(horsekick) # Number of men killed each year per regiment</pre>
epitools::pois.approx(x = sum((0:4)*horsekick), 200)
     x pt rate
                   lower
                            upper conf.level
## 1 122 200 0.61 0.5017575 0.7182425 0.95
# Rocket Bombs
n_strikes <- sum(0:5*strikes_mat[,2])</pre>
n_squares <- sum(strikes_mat[,2])</pre>
pois.approx(n_strikes, n_squares)
                       lower upper conf.level
    x pt
              rate
## 1 535 576 0.9288194 0.8501144 1.007524 0.95
```

Plot the Frequencies and Predictions

```
obs <- strikes_df$squares</pre>
pred <- dpois(0:5, lambda) * 576</pre>
plot_mat <- rbind("Observations" = obs, "Predictions" = pred)</pre>
colnames(plot_mat) <- 0:5</pre>
barplot(plot_mat, beside = TRUE,
        legend.text = TRUE,
        col = c("LightCoral", "LightGoldenrodYellow"),
        main = "Rocket Bomb Strikes in London")
```

Rocket Bomb Strikes in London



Using ggplot2:

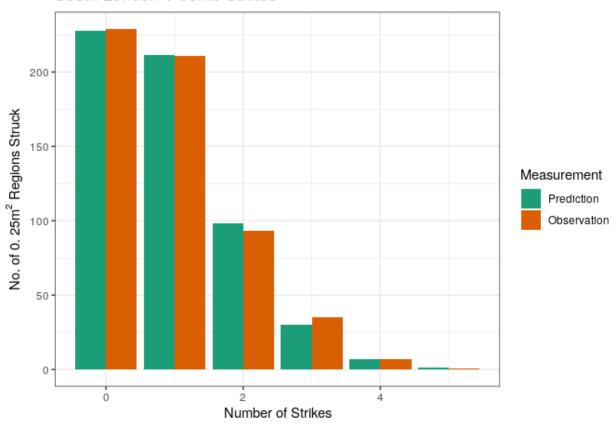
```
strikes_df$pred <- dpois(0:5, lambda)*576

(strikes_tb <- pivot_longer(strikes_df, cols = c(squares, pred)))</pre>
```

```
## # A tibble: 12 x 3
     N.Strikes name
                      value
         <int> <chr>
                      <dbl>
            0 squares 229
            0 pred
                     228.
            1 squares 211
            1 pred
                     211.
            2 squares 93
            2 pred
                      98.1
            3 squares 35
## 8
            3 pred
                      30.4
## 9
            4 squares 7
            4 pred
                       7.06
## 11
            5 squares 1
## 12
            5 pred
                       1.31
```

```
ggplot(strikes_tb, aes(y = value, x = N.Strikes, fill = name)) +
```

South London V-bomb Strikes



Confidence Intervals

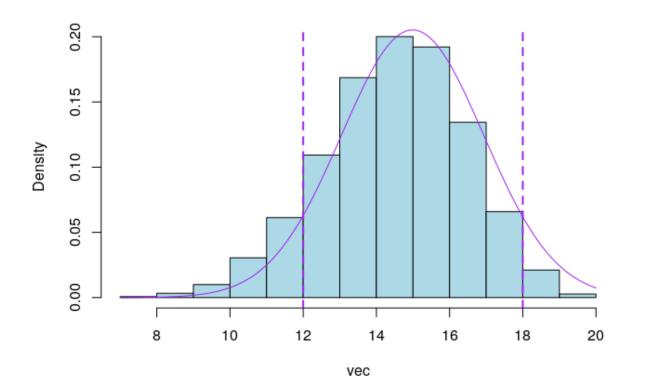
Poisson Confidence Intervals can be produced using bootstrapping or a simulating the distribution. ### Bootsrap Confidence Intervals #### Binomial ##### Seeds If it was observed that 15 seeds germinate in a sample of 20, bootstrap a confidence interval for the proportion of seeds germinating.

```
x <- rbinom(n = 10000, size = 20, prob = 15/20)
p <- x/20
quantile(p, c(0.025, 0.975))

## 2.5% 97.5%
## 0.55 0.90</pre>
```

Plot the Histogram

```
dens_hist <- function(vec, main = paste(""), col = "purple", fill = "lightblue") {
## Make the Histogram
   hist(vec, freq = FALSE, col = fill, main = main)
      c_int <- quantile(x = vec, c(0.05, 0.95))
## Plot the Curve
   x <- 1:2000
   curve(dnorm(x = x, mean = mean(vec), sd = sd(vec)), add = TRUE, col = col)
   abline(v = c_int[1], col = col, lwd = 2, lty = 2)
   abline(v = c_int[2], col = col, lwd = 2, lty = 2)
}
dens_hist(x)</pre>
```

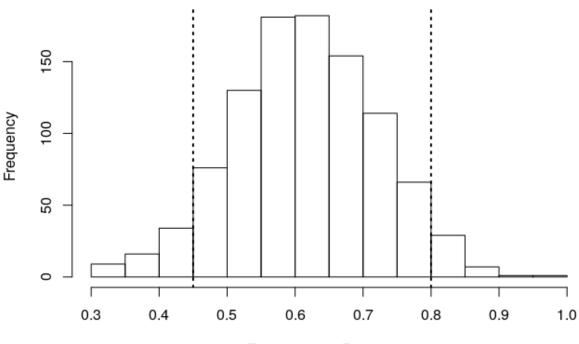


```
print("test")
## [1] "test"
```

Bud Worms A confidence interval for the number of budworms can be generated using rbinom():

```
## Create a Binomial Sample
x <- rbinom(n = 1000, size = 20, prob = 13/20)
p <- x/20
hist(p, main = "Male Bud Worms", xlab = "Extermination Proportion")
abline(v = quantile(p, 0.05), lwd = 2, lty = 3)
abline(v = quantile(p, 0.95), lwd = 2, lty = 3)</pre>
```

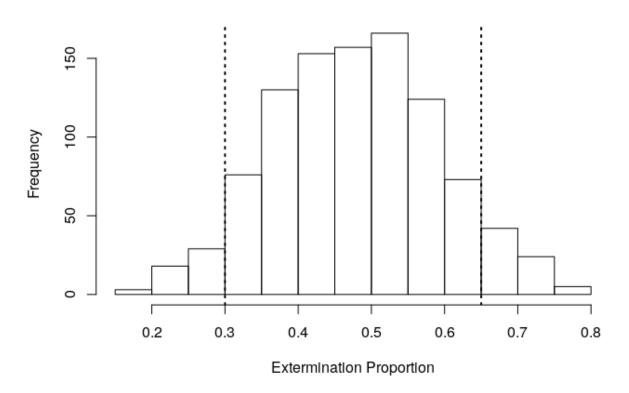
Male Bud Worms



Extermination Proportion

```
x <- rbinom(n = 1000, size = 20, prob = 10/20)
p <- x/20
hist(p, main = "Female Bud Worms", xlab = "Extermination Proportion")
abline(v = quantile(p, 0.025), lwd = 2, lty = 3)
abline(v = quantile(p, 0.875), lwd = 2, lty = 3)</pre>
```

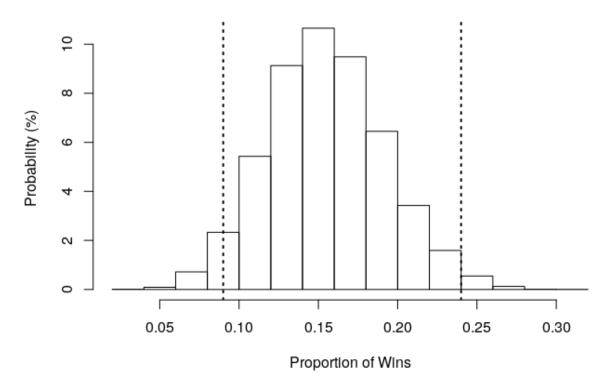
Female Bud Worms



Poker Machine A poker machine is meant to return a win 20% of the time (In NSW the requirement is that the machine gives back 90% of every dollar). Out 100 tries a win is recorded 16 times. Find a 95% confidence interval for the proportion of wins.

Because we should be using a Bar plot not a histogram the scale will be weird, instead set the size of the rbinom to 100 and have the y-axis represent the percentage of success probability. To do this properly it is necessary to use a barplot and/org ggplot + geom_col.

Poker Machine Wins



Poisson

In order to return a confidence interval for the average number of kicks, expand the table into an observation counts, resample from it and then take mean value:

```
## Turn the horsekick table into an observation list
obs <- rep(0:4, horsekick) ## Replicate according to table of counts

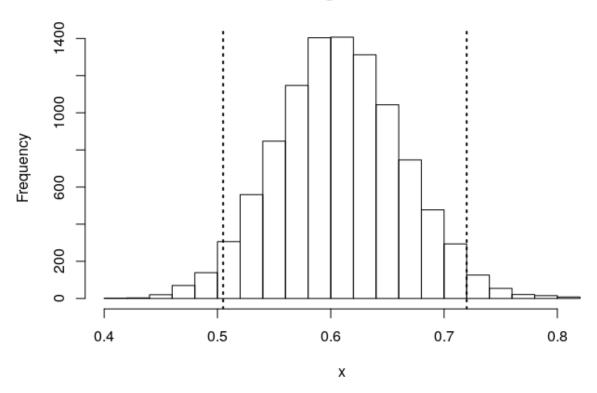
n <- 10^4 # size of Bootstrap

x <- replicate(n, {
    resamp <- sample(obs, replace = TRUE)
    mean(resamp)
})

# barplot(table(x)*10/n, col = "lightblue", border = "white", xlab = "Sample Mean
    Value of Death by HorseKick", ylab = "Probability of Population Mean")

hist(x)
abline(v = quantile(x, 0.025), lwd = 2, lty = 3)
abline(v = quantile(x, 0.975), lwd = 2, lty = 3)</pre>
```





Approximate Confidence Intervals

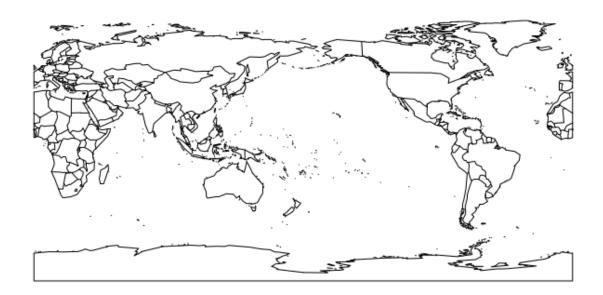
Binomial Approximate confidence intervals can be generated by using the epitools package.

```
binom.approx(10,20)
                              upper conf.level
## x n proportion lower
             0.5 0.2808694 0.7191306
binom.approx(20, 100)
                               upper conf.level
    x n proportion lower
              0.2 0.1216014 0.2783986
## 1 20 100
# HorseKick
epitools::pois.approx(x = (0:4)*horsekick, 200)
## x pt rate
                     lower
                              upper conf.level
## 0 0 200 0.000 0.000000000 0.00000000
## 1 65 200 0.325 0.2459913259 0.40400867 0.95
## 2 44 200 0.220 0.1549953486 0.28500465 0.95
## 3 9 200 0.045 0.0156005402 0.07439946 0.95
## 4 4 200 0.020 0.0004003602 0.03959964 0.95
```

Choropleth maps

Poisson Plots can be drawn of various regions using the maps package, for example a world map focused on the pacific ocean:

```
# help(package = maps)
map("world2")
```



There is also the built in data set state.x77 than we can use:

```
head(state.x77)
```

```
Population Income Illiteracy Life Exp Murder HS Grad Frost Area
## Alabama
                3615 3624
                                2.1
                                      69.05 15.1
                                                          20 50708
## Alaska
                 365
                      6315
                                1.5
                                      69.31 11.3
                                                   66.7 152 566432
                2212 4530
                                1.8
                                      70.55
                                            7.8
                                                   58.1
                                                         15 113417
## Arizona
## Arkansas
                2110 3378
                                1.9
                                      70.66 10.1
                                                   39.9
                                                          65 51945
                21198 5114
                                1.1
                                                          20 156361
## California
                                      71.71 10.3
                                                   62.6
## Colorado
                2541 4884
                                0.7
                                      72.06
                                            6.8
                                                   63.9 166 103766
```

To generate a choropleth map, match names to map names:

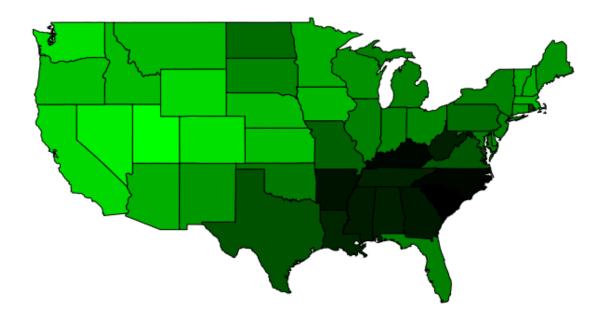
```
state_names <- map("state", plot = FALSE)$name
only_names <- sapply(strsplit(state_names, ":"), "[", 1)
index <- match(only_names, tolower(state.name))
index[8] <- 47</pre>
```

Then assign some colours:

```
col_vec <- function(x, lowcol = "white", highcol = "red") {
  rgb(colorRamp(c(lowcol, highcol))((x-min(x))/(max(x)-min(x))),
      maxColorValue = 255)[index]
}</pre>
```

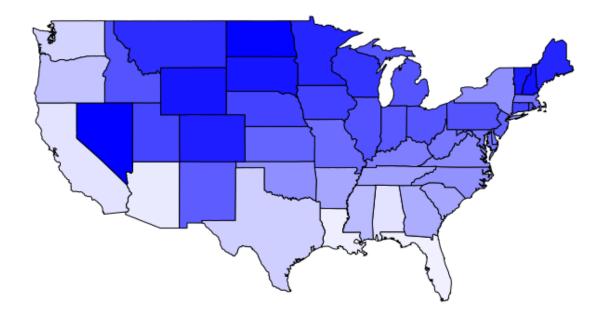
Then draw the map:

```
map('state', fill = TRUE, col = col_vec(state.x77[,"HS Grad"], "black", "green"))
```



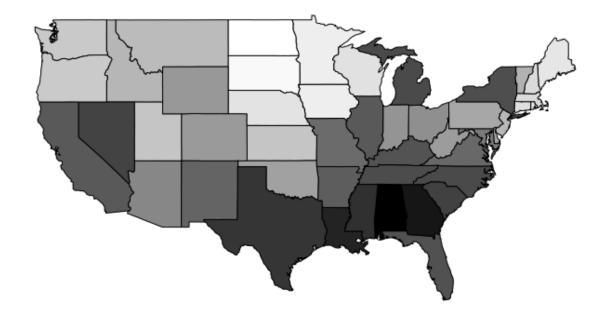
Frost

```
map('state', fill = TRUE, col = col_vec(state.x77[,"Frost"], "white", "blue"))
```

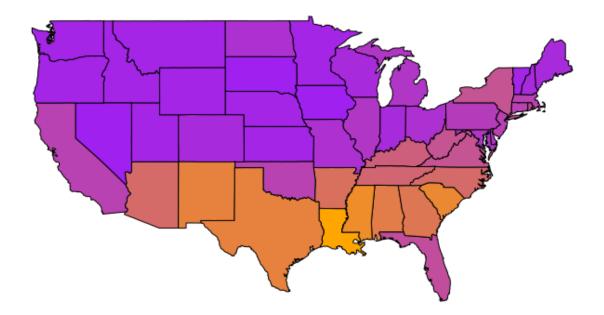


Literacy

```
map("state", fill = TRUE, col = col_vec(state.x77[,"Murder"], "white", "black"))
```



Murder



Using ggmap

```
library("ggmap")

us <- c(left = -125, bottom = 25.75, right = -67, top = 49)
USAMap <- get_stamenmap(us, zoom = 5, maptype = "toner-lite") %>% ggmap()

# USAMap +

# geom_point(aes(x=lon, y=lat), data=mv_num_collisions, col="orange", alpha=0.4, size=mv_num_collisions$collisions*circle_scale_amt) +

# scale_size_continuous(range=range(mv_num_collisions$collisions))
#
```

Explaining Sapply

when using sapply, the notation sapply(list, '['], 1) is using the [function and 1 as a second argument.

When performing subsetting using mtcars[3,1] you are actually performing a different function call, what is being performed is [(mtcars, 2, 3), have a look:

```
mtcars[2,3]
```

```
| ## [1] 160
| `[`(mtcars, 2, 3)
| ## [1] 160
```

So basically \mathbf{R} is similar to LISP and Mathematica in the sense that everything is a function. Accessing the help with ?"["] explains more.

The sapply call before basically iterates over the list of names (which is a long list composed of words seperated by :, sort of like cut -d1 : in bash) and uses the [function with the input item from the list as the first argument and 1 as the second argument, so the previous call to sapply was equivalent to:

```
First_El <- function(x) {
    x[1]
}
sapply(list, First)</pre>
```

GGPlot2 Appendix

BudWorm Confidence Intervals

Simulation of #BudWorms

```
male <- rbinom(10000, 20, 13/20)
female <- rbinom(10000, 20, 10/20)

worms <- tibble("Male" = male, "Female" = female)
worms <- pivot_longer(worms, cols = c("Male", "Female"))

## We should use Columns not a histogram for binomial because it doesn't tend to
## space correctly otherwise
c <- as.data.frame(t(table(worms)))
c$name <- factor(c$name, unique(c$name), ordered = FALSE)
c$value <- factor(c$value, unique(c$value), ordered = TRUE)
c$Freq <- 20*c$Freq/10000

ggplot(data = c, aes(x = value, y = Freq, fill = name)) +
    geom_col(position = "dodge", col = "black") +
    facet_grid(name ~ .) +
    labs(x = "Number of Exterminated") +
    theme_bw()</pre>
```

