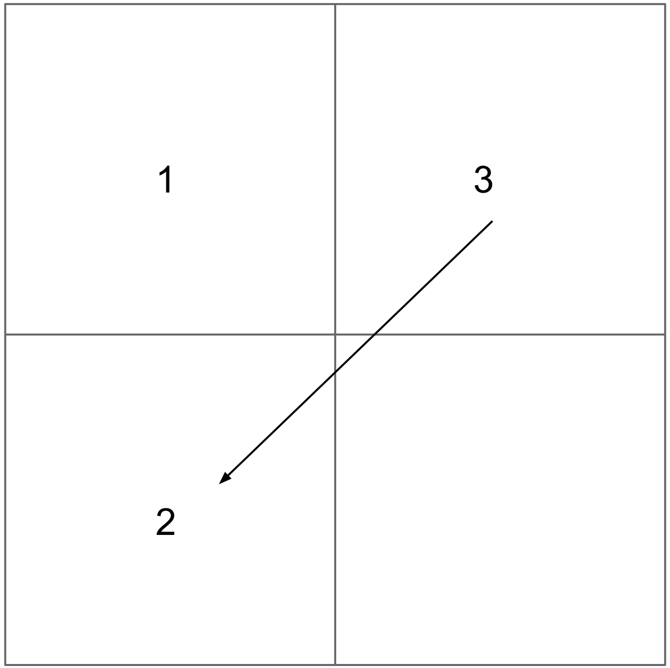
Artificial Intelligence
15 / $n^2 - 1$
Puzzle Solver

Ryan Toner

Introduction – Project Background

- 15 Puzzle is 4x4 matrix with tiles labeled 1 through 16 (last is blank)
- 16!/2 possible states ~1.0461395e+13 (over 10 trillion)
  - “Inversion” problem halves total number of states
- Even playing optimally on hardest configurations takes up to 80 moves
- Solving the 15 puzzle is an NP-Complete Problem
  - This means there is no polynomial time algorithm that can solve this problem
  - However, solutions can be verified in polynomial time
  - Using the boolean satisfiability problem (SAT) as black-box, it can be shown that the 15-puzzle is NP-Complete
- Solving this problem is programmatically difficult
  - <u>This project is fundamentally a research problem</u>

Example of inversion counting



- 3 has a higher value than 2, but 3 comes before 2 in the matrix (when scanning top to bottom and left to right)
- The same notion is generalized to a 4x4 Grid
- This board has exactly 1 inversion

- Since *grid width* is *even*, and *blank tile* is *on the bottom row* (0 moves away), and the *inversion number is odd*, this board is unsolvable

Validity of Board States – “Inversion” Problem

- Inversion Count determines if board is solvable.
- **THEOREM 1**: the puzzle on any n x m board with n, m > 1 has at most (n x m)! /2 legal configurations.
- **THEOREM 2**: Any solvable configuration will remain solvable given valid moves.
- Formula for solvability:
- A board is solvable **if and only if**
  - ((grid width odd) && (# inversions even)) ||
  - ((grid width even) && ((blank on odd row from bottom) == (#inversions even)))

Formula to count inversions

```
private bool validBoard(bool blankevenfrombottom) {

    int inversionCount = 0;

    for (int check = 0; check < Width * Height - 1; check++) //no need to consider last tile.
        for (int checkInvs = check + 1; checkInvs < Width * Height; checkInvs++)
            if (!(board[check].IsBlank || board[checkInvs ].IsBlank) && //ignore blank tile in
calculation.
             board[check].Value > board[checkInvs].Value)
                          inversionCount++;

    return (
        Width % 2 == 1 && inversionCount % 2 == 0 ||
        Width % 2 == 0 && !blankevenfrombottom && inversionCount % 2 == 0 ||
        Width % 2 == 0 && blankevenfrombottom && inversionCount % 2 == 1);
}
```

15 Puzzle Web App (Technologies Used)

- Using a client-server model
- Client runs in browser using HTML, CSS, Bootstrap, Javascript, and AJAX to communicate with server
- Server is IIS Express using ASP.NET with C# and Razor backend
  - Code written using Model-View-Controller architecture
  - Model represents board state
  - Controller handles requests
  - View generates the initial client view data
  - Server calls local python code, including loading the neural network and pathfinding algorithms

Server

- Running C# and Object-Oriented Components that generate the board and run the core of the game
    - Serves the HTML to the client
    - Creates instance of “board” in backend
    - Takes AJAX requests from server e.g. “Click,” “ArrowKey,” and “Solve” that return the valid updated board state
- Runs python code in anaconda environment using command line
    - Reads board state
    - Exports moves to solve the puzzle
    - Using Tensorflow 2.0 GPU with Nvidia CUDA and CuDNN

Training Algorithm

- Created independently of the application
- Start with 4x4 solved board and use deterministic graph “walking” with BFS to generate valid board states
  - Uses Queue to perform BFS
  - Does not have to worry about inversions (since the initial state is solved)
- Moves up to 50 moves away with a limit of 50,000 moves per distance value
- Generates over 2 million boards in ~520mb training.csv file
- Uses data augmentation technique:
  - Some distances (e.g. 0, 1, 2, 3, 4, 5) have less than 50,000 possible boards
  - while(# moves for the distance value < ⅖ (50,000))
    - Cyclically print existing data in modular cycle
  - This technique prevents skewing and over-fitting to the training dataset using boards that are many moves away
- The algorithm ensures that it will not “refind” an existing board by using a hashset

Training Algorithm Encoding

- The board is in 4x4 matrix configuration
- Each tile can occupy 1/16 spots
- Use 1-hot encoding to represent the the tile's location
  - Each tile needs 16 numbers
  - The board has 16 tiles
  - The total board encoding is 256 = (16^2)
- The board encoding is our "features," or input
- The number of moves from the solution is the "target," or output
- [0010000000000000…… (256), 15]
- ^ features ^ target

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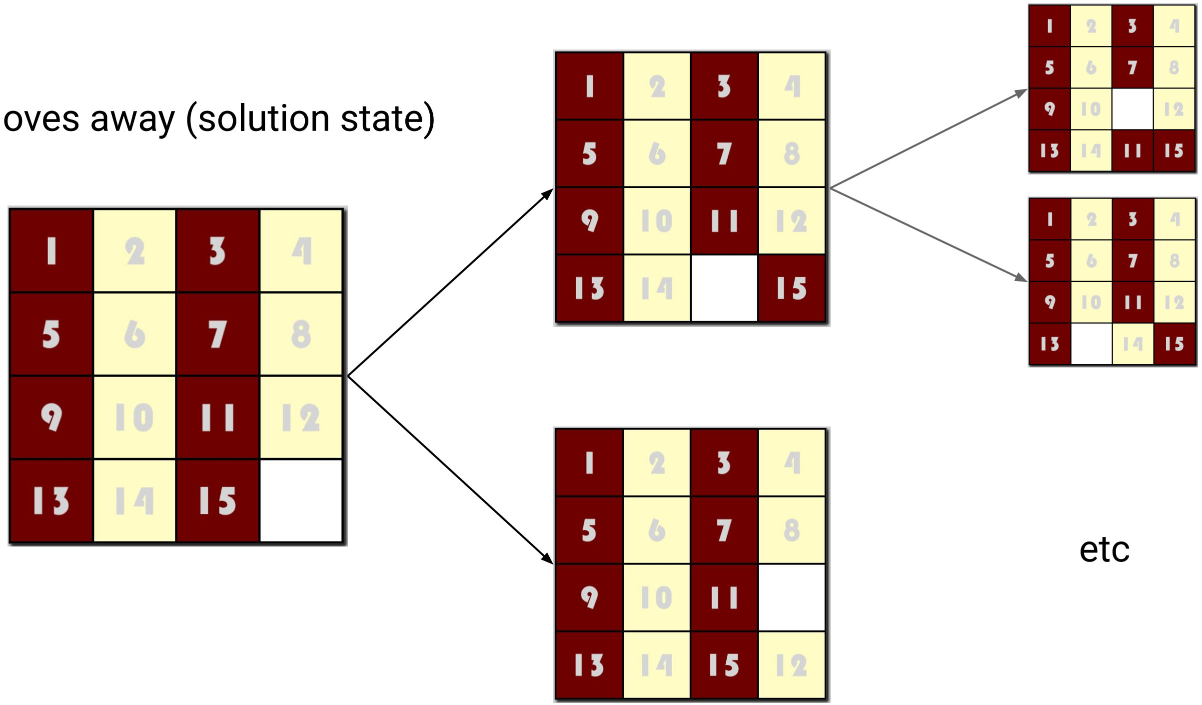
Training Algorithm Diagram

0 Moves away (solution state)



Training Algorithm Code

- https://github.com/RyanTonerCode/15-Presentation

```
Making Training Dataset
Bad 75526
0: 20000
1: 20000
2: 20000
3: 20000
4: 19992
5: 19980
6: 19795
7: 19716
8: 19624
9: 18920
10: 17532
11: 15752
12: 15616
13: 15544
14: 30820
15: 47777
16: 47808
17: 47841
18: 47750
19: 47944
20: 47821
21: 48069
22: 47932
23: 48167
24: 47960
25: 48193
26: 47992
27: 48127
28: 48095
29: 48114
30: 47856
31: 47929
32: 47946
33: 48092
34: 47923
35: 48061
36: 48039
37: 47990
38: 47837
39: 47958
40: 47970
41: 48091
42: 47935
43: 48087
44: 48033
45: 48013
46: 47834
47: 47967
48: 47955
49: 48099
50: 47929
Finished
```

Neural Network

In [5]:
```
with tf.device('/gpu:0'):

    model = Sequential()
    model.add(Dense(units=256, input_dim=256, activation='relu'))
    model.add(Dense(units=1024, activation='relu'))
    model.add(Dense(units=768, activation='relu'))
    model.add(Dense(units=1, activation='relu'))

    model.compile(optimizer=tf.keras.optimizers.Adam(lr=0.005),
                  loss='mse',
                 metrics=['accuracy'])
```

Neural Network Architecture

- Layers:
    - Input Layer (256 dense units), relu activation
    - Hidden Layer 1 (1024 dense units), relu activation
    - Hidden Layer 2 (768 dense units), relu activation
    - Output Layer (1 dense output unit), relu activation (better performance over linear here)
- Parameters:
    - Adam optimizer with LR = 0.005, using MSE loss function
    - Max epochs set to 50 (only ran ~20)
    - Batch size of 45 to handle large dataset
- Tools:
    - Pandas to read .csv
    - Numpy to handle array manipulation
    - Sklearn for train_test_split
    - Tensorflow 2.0 with Keras backend

Input Data

```
In [2]: data = pd.read_csv(
          'C:\\Users\\Ryan\\Source\\Repos\\15 Puzzle Training Data Generato
           , header=None
        )

        data.shape
        data.iloc[:,0]
```

```
Out[2]: 0          1000000000000000010000000000000000100000000000...
        1          1000000000000000010000000000000000100000000000...
        2          1000000000000000010000000000000000100000000000...
        3          1000000000000000010000000000000000100000000000...
        4          1000000000000000010000000000000000100000000000...
                                         ...
        2020431    0000000000001000000000010000000000000000001000...
        2020432    0000000000001000000000010000000000000000001000...
        2020433    0000000000000001000000100000000000000000001000...
        2020434    0000000000001000000000010000000000000000001000...
        2020435    0000000000001000000000010000000000000000001000...
        Name: 0, Length: 2020436, dtype: object
```

Preparing the training data

```
x_data = data.iloc[:, 0]

x_train = []

for row in x_data:
    a = []
    for s in str(row):
        a.append(int(s))
    x_train.append(a)

dt = np.dtype('i4')

x_train = np.array(x_train, dtype=dt)

y_train = data.iloc[:,1].to_numpy()
```

Preparing the training data

```
x_train1, x_valid, y_train1, y_valid =

    train_test_split(x_train, y_train, test_size=0.05, shuffle= True)
```

Saving the network

```
Cp_callback =
tf.keras.callbacks.ModelCheckpoint(filepath=best_model_filename,
save_weights_only=True, verbose=1)


val_data = (x_valid, y_valid)

model.fit( x=x_train1, y=y_train1, verbose=1,
           epochs=50, shuffle=False, validation_data = val_data,
           callbacks=[cp_callback], batch_size=45
)
```

Network Results

```
Train on 1919414 samples, validate on 101022 samples
Epoch 1/50
1919340/1919414 [============================>.] - ETA: 0s - loss: 3.3606 - accuracy: 0.0188
Epoch 00001: saving model to C:\Users\Ryan\15-neural-network.h5
1919414/1919414 [==============================] - 208s 108us/sample - loss: 3.3605 - accuracy: 0.0188 - val_loss: 2.1416 - va
l_accuracy: 0.0099
Epoch 2/50
1919025/1919414 [============================>.] - ETA: 0s - loss: 1.8086 - accuracy: 0.0193
Epoch 00002: saving model to C:\Users\Ryan\15-neural-network.h5
1919414/1919414 [==============================] - 210s 110us/sample - loss: 1.8086 - accuracy: 0.0193 - val_loss: 1.6673 - va
l_accuracy: 0.0200
Epoch 3/50
1919250/1919414 [============================>.] - ETA: 0s - loss: 1.5607 - accuracy: 0.0193
Epoch 00003: saving model to C:\Users\Ryan\15-neural-network.h5
1919414/1919414 [==============================] - 209s 109us/sample - loss: 1.5607 - accuracy: 0.0193 - val_loss: 1.5844 - va
l_accuracy: 0.0200
Epoch 4/50
1919115/1919414 [============================>.] - ETA: 0s - loss: 1.4347 - accuracy: 0.0193
Epoch 00004: saving model to C:\Users\Ryan\15-neural-network.h5
1919414/1919414 [==============================] - 207s 108us/sample - loss: 1.4348 - accuracy: 0.0193 - val_loss: 1.5738 - va
```

Network Results

```
In [8]: state = "100000000000000001000000000000000010000000000000000100000000000000001000000000000000010000000000000000100000000000000001

        test = []

        for c in str(state):
            test.append(int(c))

        test = [test]

        dt = np.dtype('i4')

        ev = np.array(test, dtype=dt)

        lab = model.predict(x=ev)

        lab

Out[8]: array([[0.]], dtype=float32)
```

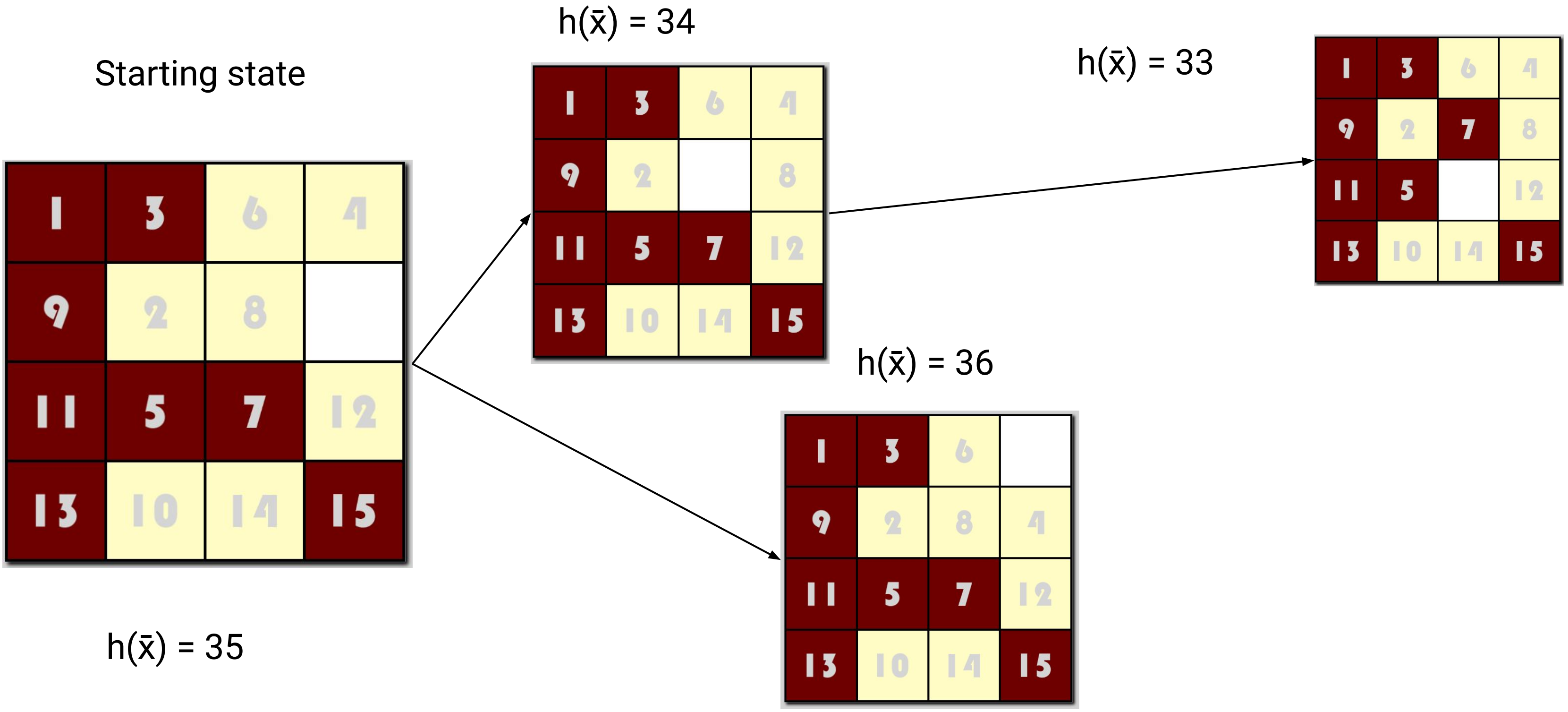
What does this network get us?

- From a given board input, we can use this semi-linear regression model to predict the number of moves it should take to solve the board.
- *Okay… that's great, but how does this help you solve the game?*
- We can use a pathfinding algorithm similar to the way we trained the data to solve the puzzle
- Use a modified A* Algorithm with our "neural" heuristic function

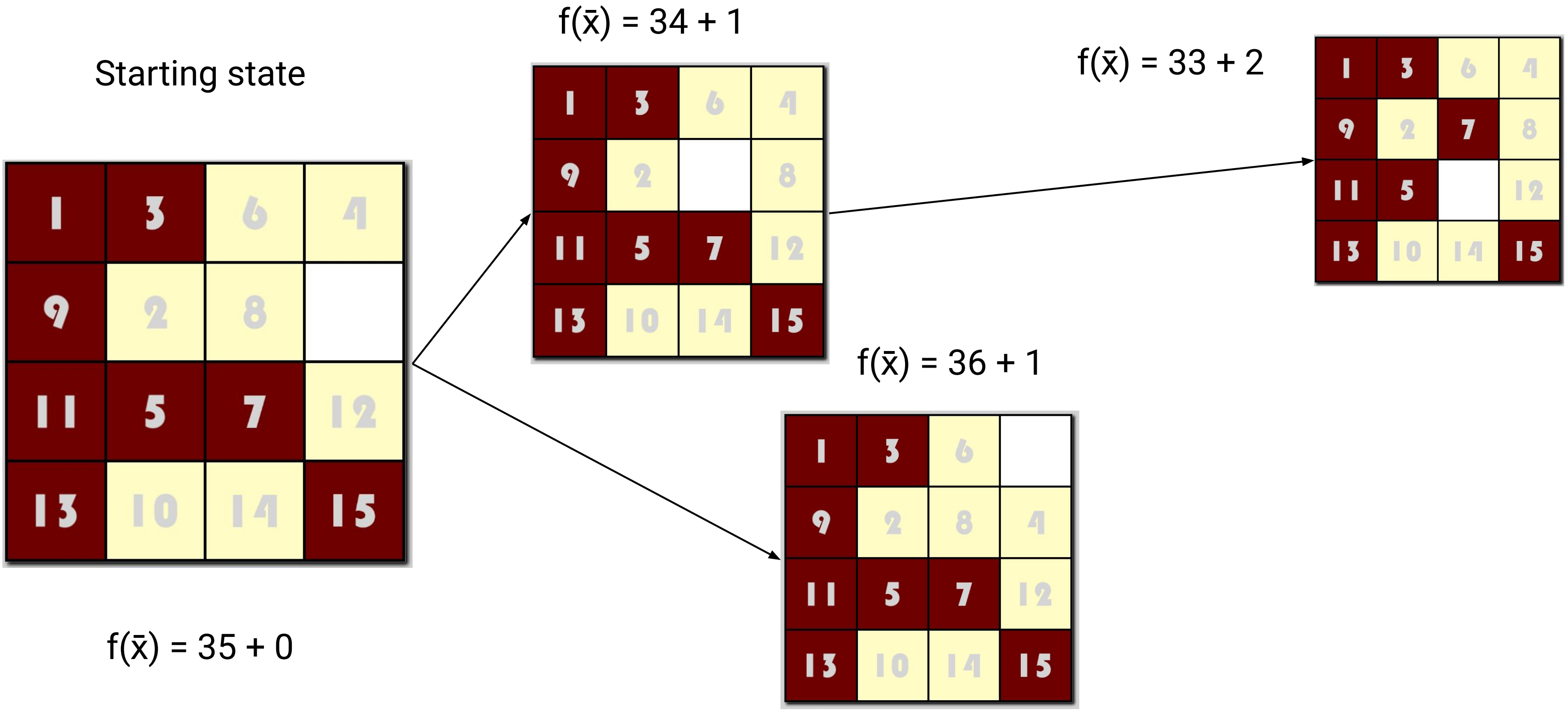
Heuristic Function Explained

- A heuristic function is a function that measures how good something is
- In this case, smaller values mean you are closer to solving the puzzle
- Suppose $\bar{x}$ is a board and *h* is our heuristic function
- Then $h(\bar{x})$ is the measurement of how close the board $\bar{x}$ is from being solved
- The <u>domain</u> of this function is boards
- The <u>codomain</u> of this function is a subset of the real numbers from around [0,250]
  - Recall 80 move maximum for optimal playing

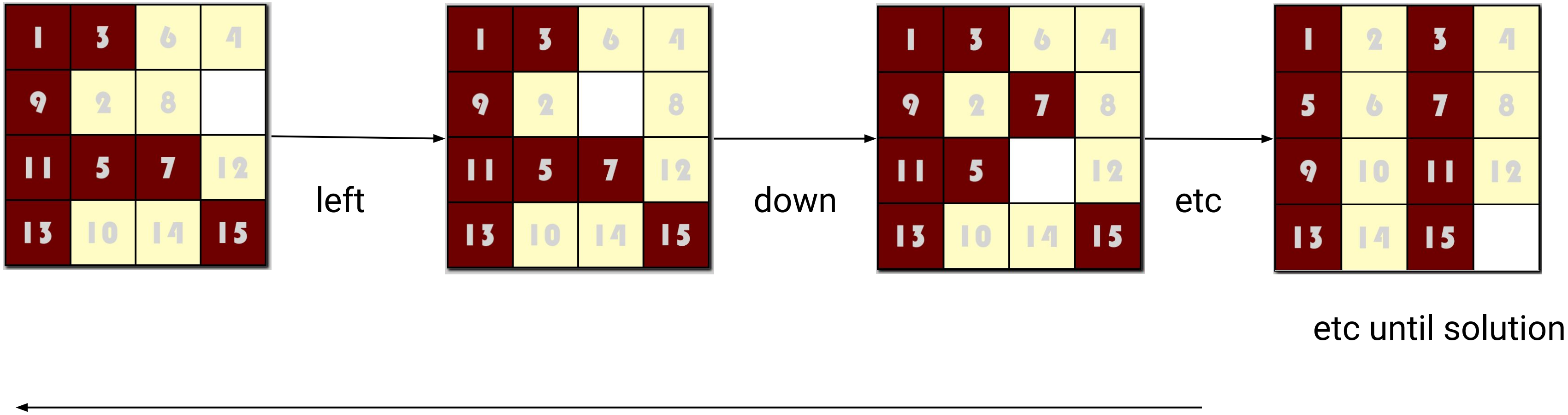
Using pathfinding and heuristic (example)



Using pathfinding and heuristic (example)



Backtracking for solution



To get the solution for the puzzle, work using backtracking from the nodes of the solution path (a subset of the A* Priority Queue Graph) and collect the moves from board to board. Then, reverse this list and output the moves.

Heuristic evaluated

- Generally finds solutions in only a few hundred boards using A*
  - This is very good and much better than manhattan distance (which may take hundreds of thousands)
- Greedy pathfinding algorithm can also be used, but it finds sub-optimal solutions
- If the move count is not predicted correctly for the initial state, there are two cases:
  - 1. The network underestimates the total number of moves. In this case, A* will have to work harder to catch up and compute greater path lengths until it reaches the prediction
  - 2. The network overestimates the total number of moves. In this case, it may affect the ability for A* to find optimal solutions.
- Case 1 is performance related
- Case 2 is accuracy/ efficiency related
- We can use this to train different networks that change the heuristic goals!

What Changed?

- Using function $f(\bar{x}) = h(\bar{x}) + g(\hat{y})$
  - $h(\bar{x})$ is still the heuristic
  - $g(\hat{y})$ is new function that represents the pathlength for a path $\hat{y}$
- Why?
  - This $f(\bar{x})$is better at finding optimal solutions over $h(\bar{x})$ alone
  - $g(\hat{y})$ metric “punishes” taking long paths and forces algorithm to explore other states
  - $g(\hat{y})$ prevents bias to perform depth-first search in a priority-queue
  - As $h(\bar{x})$ decreases, $g(\hat{y})$ will increase
- Adding 1 for the pathlength?
  - $g(\hat{y})$ adds exactly one for each new board in the path, which makes physical sense

A* Algorithm

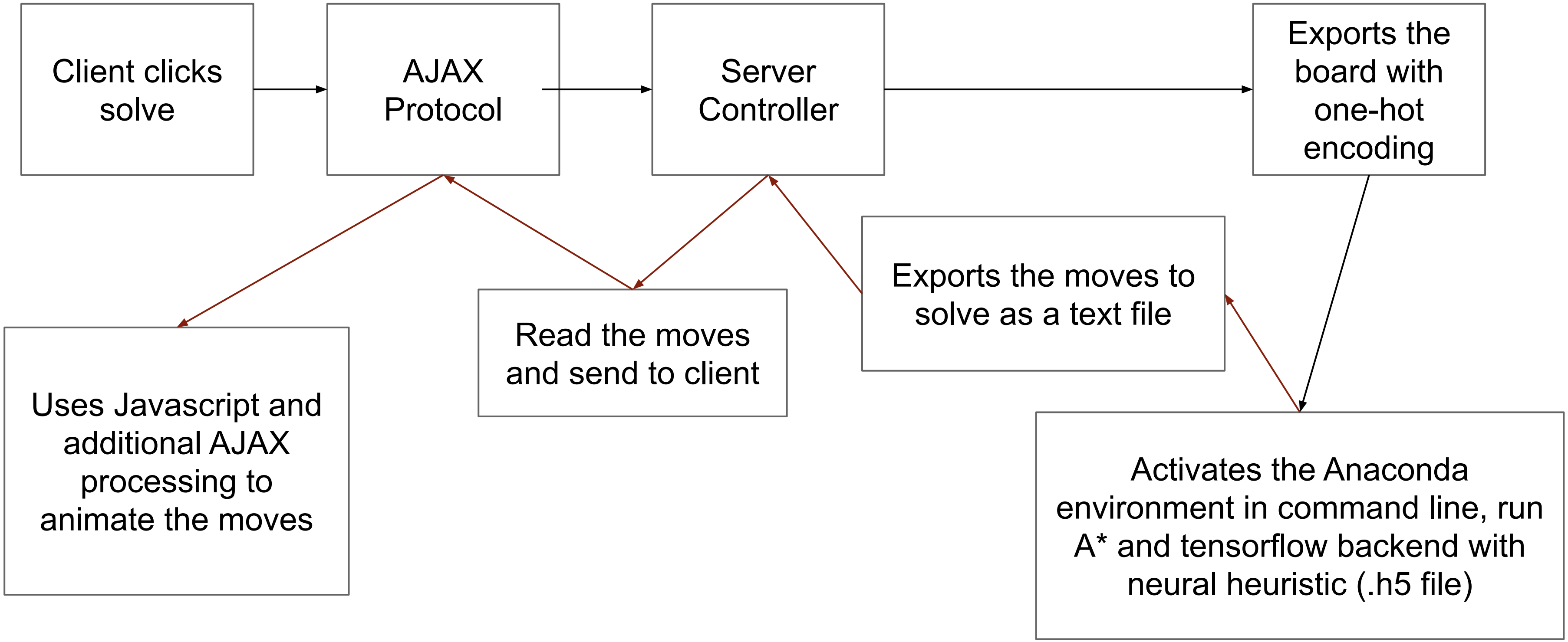
- https://github.com/RyanTonerCode/15-Presentation/blob/master/Python/intelligence.py
- Uses priority queue and graph data structure
- Path (aka the solution to the puzzle) is found using backtracking from the solution board back to the initial unsolved state, and reversing the list
- Prevents re-exploring already found states

Python backend

- Loads the saved neural-network model
- Loads the board state saved by the server
- Like the training algorithm, capable of generating valid moves
- Processes these moves using A* to find solution
- Prints the solution moves as a text file

Demo

Solve Button Mechanics



Technical Challenges

- Requires many components to get this to work
- Not using inter-process communication, so instead I am using text files as the memory communication between the various stages of the pipeline
- Dataset takes time to generate
- Network takes long time to train

Recap: High-Level Overview

Training Algorithm
(BFS module)

Generates training.csv

Network Trainer
Dense regression model

Generates model.h5

Client/ Server Architecture

Save board state to final

A* and neural heuristic

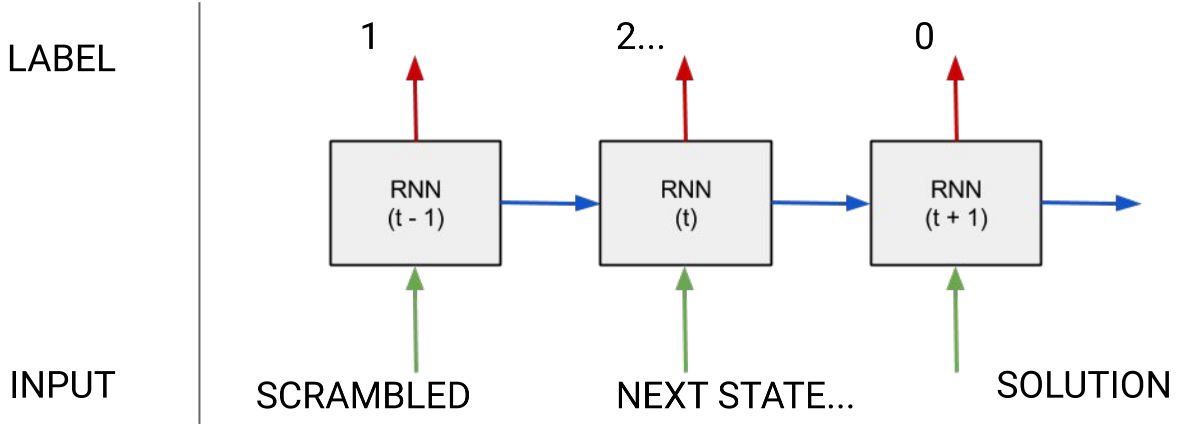
Loads board and saves moves to solution

Conclusions and Lessons Learned

- As a proof-of-concept, this project exceeded my expectations
- The network runs extremely quickly and efficiently given random board states
- The neural-network completely outperforms other heuristic functions such as Manhattan-Distance
- Solutions found by the network exhibit “creativity”
    - Uses the higher-dimensional space of the neural network to find extremely clever solutions
    - Does not solve boards like a human (it clearly bested me)
- Proves that although solving the 15-puzzle is NP-complete, we can use AI, graph-theory, and machine-learning to solve difficult configurations
- A basic neural network architecture is sufficient as a proof-of-concept
    - We could easily add additional training data up to 80 moves and improve the performance

Future Research (RNN)

- We could try using time series with a recurrent neural networks with dynamic, variable-length output sequences for a research project
- In essence, this technique could map moves (up, down, left, right) with boards like a “sentence”
- We can use our existing work to generate more sophisticated data sets



Cited Sources

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2. http://kevingong.com/Math/SixteenPuzzle.html
3. https://medium.com/breathe-publication/solving-the-15-puzzle-e7e60a3d9782
4. https://github.com/prestoj/15-puzzle