1.121

$\begin{pmatrix}1&2\\3&4\end{pmatrix} X = \begin{pmatrix}3&5\\5&9\end{pmatrix}$

$X = \begin{pmatrix}3&5\\5&9\end{pmatrix} \cdot \underbrace{\begin{pmatrix}1&2\\3&4\end{pmatrix}^{-1}}_{=A}$

$\det A = -2$

$M = \begin{pmatrix}*&*\\*&*\end{pmatrix} = \begin{pmatrix}4&3\\2&1\end{pmatrix}$

$A^* = \begin{pmatrix}4&-3\\-2&1\end{pmatrix} \qquad A^{*T} = \begin{pmatrix}4&-2\\-3&1\end{pmatrix}$

$A^{-1} = -\frac{1}{2}\begin{pmatrix}4&-2\\-3&1\end{pmatrix}$

$X = -\frac{1}{2}\begin{pmatrix}3&5\\5&9\end{pmatrix}\begin{pmatrix}4&-2\\-3&1\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}3\cdot4-5\cdot3 & 3\cdot(-2)+5\cdot1\\ 5\cdot4-9\cdot3 & 5\cdot(-2)+9\cdot1\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}-3&-1\\-7&-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3&1\\7&1\end{pmatrix}$

1.122

$X \cdot \begin{pmatrix}3&-2\\5&-4\end{pmatrix} = \begin{pmatrix}-1&2\\-5&6\end{pmatrix}$

$X = \begin{pmatrix}-1&2\\-5&6\end{pmatrix} \cdot \underbrace{\begin{pmatrix}3&-2\\5&-4\end{pmatrix}^{-1}}_{=A}$

$\det A = -12+10 = -2$

$M = \begin{pmatrix}*&*\\*&*\end{pmatrix} = \begin{pmatrix}-4&5\\-2&3\end{pmatrix}$

$A^* = \begin{pmatrix}-4&-5\\2&3\end{pmatrix} \qquad A^{*T} = \begin{pmatrix}-4&2\\-5&3\end{pmatrix} \qquad A^{-1} = -\frac{1}{2}\begin{pmatrix}-4&2\\-5&3\end{pmatrix}$

$X = -\frac{1}{2}\begin{pmatrix}-1&2\\-5&6\end{pmatrix}\begin{pmatrix}-4&2\\-5&3\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}4-10 & -2+6\\ 20-30 & -10+18\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}-6&4\\-10&8\end{pmatrix} = \begin{pmatrix}3&-2\\5&-4\end{pmatrix}$

1.125

$X\begin{pmatrix}5&3&1\\1&-3&-2\\-5&2&1\end{pmatrix} = \begin{pmatrix}-8&3&0\\-5&9&0\\-2&15&0\end{pmatrix} \Rightarrow X = \begin{pmatrix}-8&3&0\\-5&9&0\\-2&15&0\end{pmatrix} \cdot \underbrace{\begin{pmatrix}5&3&1\\1&-3&-2\\-5&2&1\end{pmatrix}^{-1}}_{=A}$

$\det A = 5\begin{vmatrix}-3&-2\\2&1\end{vmatrix} - 3\begin{vmatrix}1&-2\\-5&1\end{vmatrix} + \begin{vmatrix}1&-3\\-5&2\end{vmatrix} = 5(-3+4)-3(1-10)+(2-15) = 5+27-13 = 19$

$M = \begin{pmatrix}*&*&*\\*&*&*\\*&*&*\end{pmatrix} \quad M^{(1,1)} = \begin{vmatrix}-3&-2\\2&1\end{vmatrix} = 1 \quad M^{(1,2)} = -\begin{vmatrix}1&-2\\-5&1\end{vmatrix} = +9 \quad M^{(1,3)} = \begin{vmatrix}1&-3\\-5&2\end{vmatrix} = -13$

$M^{(2,1)} = -\begin{vmatrix}3&1\\2&1\end{vmatrix} = -1 \quad M^{(2,2)} = \begin{vmatrix}5&1\\-5&1\end{vmatrix} = 10 \quad M^{(2,3)} = -\begin{vmatrix}5&3\\-5&2\end{vmatrix} = -25 \quad M^{(3,1)} = \begin{vmatrix}3&1\\-3&-2\end{vmatrix} = -3$

$M^{(3,2)} = -\begin{vmatrix}5&1\\1&-2\end{vmatrix} = 11 \quad M^{(3,3)} = \begin{vmatrix}5&3\\1&-3\end{vmatrix} = -18 \quad M = \begin{pmatrix}1&9&-13\\-1&10&-25\\-3&11&-18\end{pmatrix} = A^*$

$A^{*T} = \begin{pmatrix}1&-1&-3\\9&10&11\\-13&-25&-18\end{pmatrix} \qquad A^{-1} = \frac{1}{19}\begin{pmatrix}1&-1&-3\\9&10&11\\-13&-25&-18\end{pmatrix}$

$$X = \frac{1}{19}\begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 & -3 \\ 9 & 10 & 11 \\ -13 & -25 & -18 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} -8+27 & 8+30 & +24+33 \\ -5+81 & 5+90 & +75+99 \\ -2+135 & 2+150 & +6+165 \end{pmatrix} = \frac{1}{19}\begin{pmatrix} 19 & 38 & 57 \\ 76 & 95 & 114 \\ 133 & 152 & 171 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

2.127

$g(x) = x^2 - 3x + 2x^{-1} - x^{-2}$, $A = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

$$g(A) = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}^2 - 3\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}^{-1} - \left(\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}\right)^{-1} =$$

$$= \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix} + \begin{vmatrix} 2 & -2 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -3 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix}$$

$\det A_1 = 1$

$$M_1 = \begin{pmatrix} * & * \\ * & x \end{pmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \quad A_x = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} \quad A_x^T = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} \quad A_x^{-1} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}$$

$\det A_2 = 1$

$$M_2 = \begin{pmatrix} * & * \\ * & x \end{pmatrix} = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \quad A_x = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} \quad A_x^T = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} \quad A_x^{-1} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}$$

2.150

$$\begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix} \quad M_2 = \begin{vmatrix} -1 & 3 \\ -2 & 5 \end{vmatrix} = 1 \neq 0 \quad M_3 = \begin{vmatrix} 2 & -1 & 3 \\ 4 & -2 & 5 \\ 2 & -1 & 1 \end{vmatrix} = 2\begin{vmatrix} -2 & 5 \\ -1 & 1 \end{vmatrix} + \begin{vmatrix} 4 & 5 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 2 & -1 \end{vmatrix} =$$

$$= 2(-2+5) + (4-10) + 3(-4+4) = 6 - 6 = 0$$

$r = 2$

2.153

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix} \quad M_2 = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1 \neq 0 \quad M_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = (15-16) -$$

$$-2(10-12) + 3(8-9) = -1 + 4 - 3 = 0 \Rightarrow r = 2$$

2.154

$$\begin{pmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{pmatrix} = \begin{pmatrix} 75 & 94 & 54 & 134 \\ 75 & 94 & 53 & 132 \\ 25 & 31 & 17 & 43 \\ 25 & 32 & 20 & 48 \end{pmatrix} = \begin{pmatrix} 25 & 31 & 17 & 43 \\ 25 & 32 & 20 & 48 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \end{pmatrix} \begin{matrix} \cdot(-1) \\ \cdot(-1) \\ \cdot(-3) \\ \cdot(-3) \end{matrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{pmatrix} \begin{matrix} \\ \cdot(-1) \\ \cdot(-1) \\ \cdot(-1) \end{matrix} \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{pmatrix} \cdot(-1) \sim \begin{pmatrix} 25 & 31 & 17 & 43 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r = 3$$

2.164

$$\begin{pmatrix} -1 & 3 & 3 & -4 \\ 4 & -7 & -2 & 1 \\ -3 & 5 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 & -3 & 4 \\ 4 & -7 & -2 & 1 \\ -3 & 5 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{pmatrix} \cdot 4 \; +3 \; +2 = \begin{pmatrix} 1 & -3 & -3 & 4 \\ 0 & 5 & 10 & -15 \\ 0 & -4 & -8 & 12 \\ 0 & -3 & -6 & 9 \end{pmatrix} = \begin{pmatrix} 1 & -3 & -3 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 2 & -3 \\ 0 & 1 & 2 & -3 \end{pmatrix} \cdot (-1) \cdot (-1) = \begin{pmatrix} 1 & -3 & -3 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow r = 2$

2.166

$$\begin{pmatrix} 0 & 1 & 10 & 3 \\ 2 & 0 & 4 & -1 \\ 16 & 4 & 52 & 9 \\ 8 & -16 & -7 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 4 & -1 \\ 0 & 1 & 10 & 3 \\ 16 & 4 & 52 & 9 \\ 8 & -16 & -7 \end{pmatrix} \cdot (-8) \; -4 = \begin{pmatrix} 2 & 0 & 4 & -1 \\ 0 & 1 & 10 & 3 \\ 0 & 4 & 20 & 17 \\ 0 & -1 & -10 & -3 \end{pmatrix} \cdot (-4) \; +1 = \begin{pmatrix} 2 & 0 & 4 & -1 \\ 0 & 1 & 10 & 3 \\ 0 & 0 & -20 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r = 3$$

2.170

$$\begin{pmatrix} 0 & -1 & 0 & 1 & 3 & 1 \\ 1 & 0 & -2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ -1 & -3 & -1 & 2 & 2 & 0 \\ 3 & 2 & -1 & -1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 1 & 0 & -2 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 & 3 & 1 \\ 3 & 2 & -1 & -1 & -1 & -1 \end{pmatrix} \cdot (-) \cdot (-) \cdot (-3) = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & -3 & -3 & 2 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & 3 & 3 & 1 \\ 0 & -1 & 0 & 1 & 3 & 1 \\ 0 & -7 & 2 & 5 & 5 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -3 & -3 & 2 & 1 & 1 \\ 0 & -3 & -1 & 3 & 3 & 1 \\ 0 & -1 & 0 & 1 & 3 & 1 \\ 0 & -7 & 2 & 5 & 5 & -1 \end{pmatrix} \cdot (-3) \cdot (-3) \cdot (-1) \cdot (-7)$$

$$= \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 5 & 1 & 1 \\ 0 & 0 & -1 & 6 & 3 & 1 \\ 0 & 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 2 & 12 & 5 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & -3 & -1 \\ 0 & 0 & -3 & 5 & 1 & 1 \\ 0 & 0 & 2 & 12 & 5 & -1 \\ 0 & 0 & 0 & 2 & 3 & 1 \end{pmatrix} \cdot (-3) \cdot 2 = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & -3 & -1 \\ 0 & 0 & -13 & -8 & -2 \\ 0 & 0 & 0 & 24 & 11 & 1 \\ 0 & 0 & 0 & 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & -3 & -1 \\ 0 & 0 & 0 & -13 & -8 & -2 \\ 0 & 0 & 0 & 24 & 11 & 1 \end{pmatrix} \cdot (-7) \cdot (-12) =$$

$$= \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & -3 & -1 \\ 0 & 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 & 13 & 5 \\ 0 & 0 & 0 & 0 & -25 & -11 \end{pmatrix} \cdot 2 = \begin{pmatrix} 1 & 3 & 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & -3 & -1 \\ 0 & 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 & 7 & 3 \\ 0 & 0 & 0 & 0 & -25 & -11 \end{pmatrix}$$

$r = 6$