1.35 $a_1=\{-1;2,0\}$ $a_2=\{3,1,1\}$ $a_3=\{2,0,1\}$ $a=a_1-2a_2+\frac{1}{3}a_3$

а) $|a_1|$ - ? $a_{1,0}$ - ?

$|a_1|=\sqrt{1+4}=\sqrt{5}$

$a_{1,0}=\frac{a_1}{|a_1|}=\frac{\{-1,2,0\}}{\sqrt{5}}=\left\{-\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}},0\right\}$

б) $\cos(\widehat{a_1,j})=\frac{a_j}{|a_1|}=\frac{2}{\sqrt{5}}$

в) коорд. X вектора $\bar{a}$ - ?

$\bar{a}=\bar{a}_1-2\bar{a}_2+\frac{1}{3}\bar{a}_3=\left\{-\frac{19}{3};0;-\frac{5}{3}\right\}$

$\bar{a}_1-2\bar{a}_2=\{-1-6;2-2;0-2\}=\{-7;0;-2\}$

$\{-7;0;-2\}+\frac{1}{3}\bar{a}_3=\left\{-7+\frac{2}{3};0+\frac{1}{3}0;-2+\frac{1}{3}\cdot1\right\}=\left\{-\frac{19}{3};0;-\frac{5}{3}\right\}$

Ответ: $-\frac{19}{3}$

г) $Pr_j a=y\cdot j=0\cdot j=\bar{0}$

1.36

$\bar{e}=\{-1,1,\frac{1}{2}\}$ $\bar{a}=\{2,-2,-1\}$ $e\parallel a$ - ? разл. $\bar{a}$ по базису $(\bar{e})$

$\bar{a}=-2\cdot\bar{e}=\{-2\cdot(-1);-2\cdot1;-2\cdot\frac{1}{2}\}=\{2;-2;-1\}\Rightarrow\bar{a}\parallel\bar{e}\Rightarrow$ векторы л.з. $\Rightarrow$ нельзя разл. по базису

1.37

$\bar{e}_1=\{-1,2\}$ $\bar{e}_2=\{2;1\}$ $\bar{a}=\{0,-2\}$ $\bar{e}_1\nparallel\bar{e}_2$ - ? разл. $\bar{a}$ по баз. $(\bar{e}_1,\bar{e}_2)$

сост. матрицу $\bar{e}_1,\bar{e}_2$ $\begin{vmatrix}-1&2\\2&1\end{vmatrix}$ $det(A)=-1-(2\cdot2)=-5\neq0\Rightarrow\bar{e}_1\nparallel\bar{e}_2$

$x\cdot e_1+y\cdot e_2=\bar{a}$

$x\cdot\begin{pmatrix}-1\\2\end{pmatrix}+y\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$

$\begin{cases}-x+2y=0\\2x+y=-2\end{cases}$ $\begin{cases}x=2y\\4y+y=-2\end{cases}$ $\begin{cases}x=-0,8\\y=-0,4\end{cases}$ $\boxed{\bar{a}=-0,8\cdot\bar{e}_1-0,4\cdot\bar{e}_2}$

1.38

$\bar{e}_1=\{1,0,0\}$ $\bar{e}_2=\{1,1,0\}$ $\bar{e}_3=\{1,1,1\}$ $(\bar{e}_1,\bar{e}_2,\bar{e}_3)$ - обр. баз. - ? $a=-2i-k$

$A=\begin{vmatrix}1&1&1\\0&1&1\\0&0&1\end{vmatrix}$ $det(A)=1\cdot\begin{vmatrix}1&1\\0&1\end{vmatrix}-1\begin{vmatrix}0&1\\0&1\end{vmatrix}+1\begin{vmatrix}0&1\\0&0\end{vmatrix}=1\cdot(1-0)-1\cdot(0-0)+1\cdot(0-0)=1\neq0\Rightarrow$ л.нз обр. баз.

$\bar{a}=-2i-k=-2\cdot\{1,0,0\}-1\cdot\{1,1,1\}=\{-2-1;0-1;0\cdot1\}=\{-3,-1;-1\}$

$x_1\cdot\bar{e}_1+x_2\cdot\bar{e}_2+x_2\cdot\bar{e}_3=\bar{a}$ $x_1\cdot\begin{pmatrix}1\\0\\0\end{pmatrix}+x_2\begin{pmatrix}1\\1\\0\end{pmatrix}+x_3\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}-3\\-1\\-1\end{pmatrix}$

$\begin{cases}x_1+x_2+x_3=-3\\x_2+x_3=-1\\\ldots\end{cases}$ $\begin{cases}x_1=-3-x_3-x_2\\x_2=-1-x_3\\\ldots\end{cases}$ $\begin{cases}x_1=-2\\x_2=0\\\ldots\end{cases}$ $\boxed{\bar{a}=-2\cdot\bar{e}_1+0\cdot\bar{e}_2-1\cdot\bar{e}_3}$

1.39

$\bar a = 2\bar i + 3\bar j \quad \bar b = -3\bar j - 2\bar k \quad \bar c = \bar i + \bar j - \bar k$

а) коорд. орта a_0 – ?

$|\bar a| = \sqrt{4+9} = \sqrt{13}$

$a_0 = \frac{\bar a}{|\bar a|} = \frac{\{2,3,0\}}{\sqrt{13}} = \left\{\frac{2}{\sqrt{13}}; \frac{3}{\sqrt{13}}; 0\right\}$

б) коорд. вектора $\bar a - \frac{1}{2}\bar b + \bar c$

$\bar a - \frac{1}{2}\bar b = 2\bar i + (3\bar j + \frac{3}{2}\bar j) + (0\cdot\bar k + 1\cdot\bar k) = 2\bar i + \frac{9}{2}\bar j + 1\cdot\bar k$

$\bar a - \frac{1}{2}\bar b + \bar c = (2\bar i + \bar i) + (\frac{9}{2}\bar j + \bar j) + (1\cdot\bar k - 1\cdot\bar k) = 3\bar i + \frac{11}{2}\bar j \quad \{3; \frac{11}{2}; 0\}$

в) $\bar a + \bar b - 2\bar c = (2\bar i - 2\bar i) + (3\bar j - 3\bar j - 2\bar j) + (-2\cdot\bar k + 2\cdot\bar k) = -2\bar j$

г) $Pr_j(a-b) = Pr_j a - Pr_j b = a_y\cdot j - b_y\cdot j = 3 - (-3) = 6$

1.40

a_0 – ? $\quad \bar a = \{6, 7, -6\}$

$|\bar a| = \sqrt{36+49+36} = \sqrt{121} = 11$

$a_0 = \left\{\frac{6}{11}; \frac{7}{11}; -\frac{6}{11}\right\}$

1.41

$z(\bar a)$ – ? $\quad x(\bar a) = 3 \quad y(\bar a) = -9 \quad |\bar a| = 12$

$|\bar a| = \sqrt{x^2+y^2+z^2} \qquad 144 = 9 + 81 + z^2 \Rightarrow z^2 = 144 - 9 - 81 = 54 \Rightarrow z = \sqrt{54} = \boxed{3\sqrt{6}}$

1.42

$\bar p = 3\bar a - 5\bar b + \bar c \quad \bar a = 4\bar i + 7\bar j + 3\bar k \quad \bar b = \bar i + 2\bar j + \bar k \quad \bar c = 2\bar i - 3\bar j - \bar k$

$\bar p = 12\bar i + 21\bar j + 9\bar k - 5\bar i - 10\bar j - 5\bar k + 2\bar i - 3\bar j - \bar k = 9\bar i + 8\bar j + 3\bar k$

$|\bar p| = \sqrt{81 + 64 + 9} = \sqrt{154}$

$\cos(\widehat{\bar p, \bar i}) = \frac{p_i}{|\bar p|} = \frac{9}{\sqrt{154}} \qquad \cos(\widehat{\bar p, \bar j}) = \frac{p_j}{|\bar p|} = \frac{8}{\sqrt{154}} \qquad \cos(\widehat{\bar p, \bar k}) = \frac{3}{\sqrt{154}}$

1.43

$\bar a = \bar i - 2\bar j - 2\bar k \quad \bar a \parallel \bar x \quad |\bar x| = 15 \quad (\widehat{\bar x, \bar j}) < 90$

$|\bar x| = \sqrt{x^2+y^2+z^2} \Rightarrow \cancel{225 = x_1\cdot\bar i + x_2(-2) + x_3\cdot(-2) \; x_1 = 2}$

$\cos(\bar p, \bar j) = \frac{x}{15} < \frac{\pi}{2} \qquad 225 = \lambda$

найдем методом подбора: $\bar x = 5\bar i - 10\bar j - 10\bar k$

1.46

$\bar{a} = -2\bar{i} + 3\bar{j} + \alpha\cdot\bar{k}$ $\quad \bar{b} = \beta\cdot\bar{i} - 6\bar{j} + 2\bar{k}$ $\quad \alpha, \beta - ?\ ;\ \bar{a} \parallel \bar{b}$

$b_j = \lambda\cdot a_j \Rightarrow -6 = \lambda\cdot 3 \Rightarrow \lambda = -2 \Rightarrow \boxed{\alpha = -1;\ \beta = 4}$

1.66

$|a_1| = 3 \quad |a_2| = 5 \qquad a_1 + \alpha\cdot a_2 \perp a_1 - \alpha\cdot a_2 \quad - \alpha - ?$

вект. $\perp$, если их скал. произв. $= 0$

$((a_1 + \alpha\cdot a_2), (a_1 - \alpha\cdot a_2)) = 0$

$((a_1^2 - \alpha^2\cdot a^2)) = 0$

$|\bar{a}|^2 = (\bar{a}, \bar{a})$

$9 - \alpha^2\cdot 25 = 0$

$\alpha^2\cdot 25 = 9$

$\alpha^2 = \frac{9}{25} \Rightarrow \alpha = \pm\frac{3}{5}$

1.68

$(\bar{a} - \bar{b})^2 + (\bar{a} + 2\bar{b})^2 = 20 \qquad |\bar{a}| = 1 \quad |\bar{b}| = 2 \qquad (\widehat{\bar{a}, \bar{b}}) - ?$

$(\bar{a} - \bar{b})^2 + (\bar{a} + 2\bar{b})^2 = 20$

$\bar{a}^2 - 2\bar{a}\bar{b} + \bar{b}^2 + \bar{a}^2 + 4\bar{a}\bar{b} + 4\bar{b}^2 - 20 = 0$

$2\bar{a}^2 + 2\bar{a}\bar{b} + 5\bar{b}^2 - 20 = 0$

$|\bar{a}| = (a, a) \Rightarrow 2\cdot 1 + 2\bar{a}\cdot\bar{b} + 20 - 20 = 0$

$(\bar{a}\cdot\bar{b}) = |\bar{a}|\cdot|\bar{b}|\cdot\cos\varphi = 2\cos\varphi$

$\Rightarrow 2 + 4\cos\varphi = 0 \Rightarrow 4\cos\varphi = -2 \Rightarrow \cos\varphi = -\frac{1}{2}$

$\Downarrow$

$(\widehat{\bar{a}, \bar{b}}) = 120^\circ$

1.77

$|\bar{a}| = 3 \quad |\bar{b}| = 1 \quad |\bar{c}| = 4 \qquad \bar{a} + \bar{b} + \bar{c} = 0 \qquad \bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a} - ?$

$\cancel{\bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a}} \qquad |\bar{a}|^2 = (a, a)$

$\bar{a} + \bar{b} + \bar{c} = 0 \longrightarrow \bar{a}\bar{c} + \bar{b}\bar{c} + \bar{c}\bar{c} = 0 \rightarrow \bar{a}\bar{c} + \bar{b}\bar{c} + 16 = 0 \quad (1)$

$\downarrow$

$\bar{a}\cdot\bar{a} + \bar{a}\bar{b} + \bar{a}\bar{c} = 0 \qquad \bar{a}\bar{b} + \bar{b}\bar{b} + \bar{b}\bar{c} = 0$

$9 + \bar{a}\bar{b} + \bar{a}\bar{c} = 0 \quad (3) \qquad \bar{a}\bar{b} + 1 + \bar{b}\bar{c} = 0 \quad (2)$

$(1) + (2) + (3) = \bar{a}\bar{c} + \bar{b}\bar{c} + 16 + \bar{a}\bar{b} + 1 + \bar{b}\bar{c} + 9 + \bar{a}\bar{b} + \bar{a}\bar{c} = 0$

$2\bar{a}\bar{c} + 2\bar{b}\bar{c} + 2\bar{a}\bar{b} + 26 = 0 \ /:2 \qquad \bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a} = -13$

1.78

$\bar{a}_1 = \{4, -2, -4\}$ $\quad \bar{a}_2 = \{6, -3, 2\}$

а) $\bar{a}_1 \bar{a}_2$ - ?

$\bar{a}_1 \bar{a}_2 = 4 \cdot 6 + (-2) \cdot (-3) + (-4) \cdot 2 = 24 + 6 - 8 = 22$

б) $(2\bar{a}_1 - 3\bar{a}_2)(\bar{a}_1 + 2\bar{a}_2)$ - ?

$(2\bar{a}_1 - 3\bar{a}_2)(\bar{a}_1 + 2\bar{a}_2) = -10 \cdot 16 + 5 \cdot (-8) - 14 \cdot 0 = -160 - 40 = -200$

$(2\bar{a}_1 - 3\bar{a}_2) = \{2 \cdot 4 - 3 \cdot 6;\ 2 \cdot (-2) - 3 \cdot (-3);\ 2 \cdot (-4) - 3 \cdot 2\} = \{-10; 5; -14\}$

$(\bar{a}_1 + 2\bar{a}_2) = \{4 + 2 \cdot 6;\ -2 + 2 \cdot (-3);\ -4 + 2 \cdot 2\} = \{16; -8; 0\}$

в) $(\bar{a}_1 - \bar{a}_2)^2$ - ?

$(\bar{a}_1 - \bar{a}_2)^2 = \{-2; 1; -6\}^2 = 4 + 1 + 36 = 41$

г) $|2\bar{a}_1 - \bar{a}_2|$ - ?

$2\bar{a}_1 - \bar{a}_2 = \{8 - 6; -4 - 3; -8 - 2\} = \{2; -7; -10\}$

$|2\bar{a}_1 - \bar{a}_2| = \sqrt{4 + 49 + 100} = \sqrt{153} = 3\sqrt{17}$

д) $Pr_{a_1} a_2$ - ?

$Pr_{a_1} a_2 = \dfrac{(\bar{a}_2, \bar{a}_1)}{|\bar{a}_1|} = \dfrac{24 + 6 - 8}{6} = \dfrac{22}{6} = \dfrac{11}{3}$

$|\bar{a}_1| = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$

е) $Pr_{a_2} a_1 = \dfrac{(\bar{a}_1, \bar{a}_2)}{|\bar{a}_2|} = \dfrac{22}{7}$

$|\bar{a}_2| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$

ж) $\cos(\widehat{\bar{a}_1, \bar{i}})$ - ? $\quad \cos(\widehat{\bar{a}_1, \bar{j}})$ - ? $\quad \cos(\widehat{\bar{a}_1, \bar{k}})$ - ?

$\cos(\widehat{\bar{a}_1, \bar{i}}) = \dfrac{\bar{a}_{1i}}{|\bar{a}_1|} = \dfrac{4}{6} = \dfrac{2}{3}$

$\cos(\widehat{\bar{a}_1, \bar{j}}) = \dfrac{\bar{a}_{1j}}{|\bar{a}_1|} = \dfrac{-2}{6} = -\dfrac{1}{3}$

$\cos(\widehat{\bar{a}_1, \bar{k}}) = \dfrac{\bar{a}_{1k}}{|\bar{a}_1|} = \dfrac{-4}{6} = -\dfrac{2}{3}$

з) $Pr_{\bar{a}_1 + \bar{a}_2}(\bar{a}_1 - 2\bar{a}_2)$ - ? $\quad a_1 + a_2 = \{10; -5; -2\}$ $\quad |\bar{a}_1 + \bar{a}_2| = \sqrt{100 + 25 + 4} = \sqrt{129}$

$Pr_{\bar{a}_1 + \bar{a}_2}(\bar{a}_1 - 2\bar{a}_2) = Pr_{a_1 + a_2} \bar{a}_1 - Pr_{a_1 + a_2} 2\bar{a}_2 = \dfrac{(\bar{a}_1, \bar{a}_1 + \bar{a}_2)}{|\bar{a}_1 + \bar{a}_2|} - \dfrac{(2\bar{a}_2, \bar{a}_1 + \bar{a}_2)}{|\bar{a}_1 + \bar{a}_2|} =$

$= \dfrac{4 \cdot 10 + (-2) \cdot (-5) + (-4) \cdot (-2)}{\sqrt{129}} - \dfrac{12 \cdot 10 + (-6) \cdot (-5) + 4 \cdot (-2)}{\sqrt{129}} = \dfrac{58}{\sqrt{129}} - \dfrac{142}{\sqrt{129}} = -\dfrac{84}{\sqrt{129}}$

и) $\cos(\widehat{\bar{a}_1, \bar{a}_2})$ - ?

$\cos(\widehat{\bar{a}_1, \bar{a}_2}) = \dfrac{(\bar{a}_1, \bar{a}_2)}{|\bar{a}_1| \cdot |\bar{a}_2|} = \dfrac{22}{42} = \dfrac{11}{21}$

$\boxed{1.89}$

$\bar{a}_1 = \{2, 3, -1\}$ $\quad \bar{a}_2 = \{1, -2, 3\}$ $\quad \bar{x} \perp \bar{a}_1 \perp \bar{a}_2$ $\quad \bar{x}(2i - j + k) = -6$ $\quad$ коорд. $\bar{x}$ - ?

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 0 \\ x_1 - 2x_2 + 3x_3 = 0 \\ 2x_1 - x_2 + x_3 = -6 \end{cases} \quad A = \left|\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 3 & -2 & -1 & 0 \\ -1 & 3 & 1 & -6 \end{array}\right|$$

$$\det(A) = 2\begin{vmatrix} -2 & -1 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 3 & -2 \\ -1 & 3 \end{vmatrix} =$$

$$= 2(-2+3) - 1(3-1) + 2(9-2) = 2 - 2 + 14 = 14$$

$\Delta = 14$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 0 & -2 & -1 \\ -6 & 3 & 1 \end{vmatrix} = 0 - 1\begin{vmatrix} 0 & -1 \\ -6 & 1 \end{vmatrix} + 2\begin{vmatrix} 0 & -2 \\ -6 & 3 \end{vmatrix} = 6 - 24 = -18$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 & 2 \\ 3 & 0 & -1 \\ -1 & -6 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 0 & -1 \\ -6 & 1 \end{vmatrix} - 0 + 2\begin{vmatrix} 3 & 0 \\ -1 & -6 \end{vmatrix} = -12 - 36 = -48$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 0 \\ 3 & -2 & 0 \\ -1 & 3 & -6 \end{vmatrix} = 2 \cdot \begin{vmatrix} -2 & 0 \\ 3 & -6 \end{vmatrix} - 1\begin{vmatrix} 3 & 0 \\ -1 & -6 \end{vmatrix} + 0 = 24 + 18 = 42$$

$$x_1 = \frac{\Delta_1}{\Delta} = -\frac{18}{14} = -\frac{9}{7}$$

$$x_2 = \frac{\Delta_2}{\Delta} = -\frac{48}{14} = -\frac{24}{7} \qquad \bar{x} = \left\{-\frac{9}{7}; -\frac{24}{7}; 3\right\}$$

$$x_3 = \frac{\Delta_3}{\Delta} = \frac{42}{14} = \frac{21}{7} = 3$$