Пределы

[1] $\lim\limits_{n\to\infty}\dfrac{n\sqrt[3]{7n}-\sqrt[4]{81n^8-1}}{(n+4\sqrt{n})(\sqrt{n^2-5})}=\dfrac{(\infty-\infty)}{\infty}=(*)$

Для устранения неоп-ти делим ч-ль и зн-ль на n^2

$$(*)=\lim_{n\to\infty}\frac{\dfrac{n\sqrt[3]{7n}-\sqrt[4]{81n^8-1}}{n^2}}{\dfrac{(n+4\sqrt{n})(\sqrt{n^2-5})}{n^2}}=\lim_{n\to\infty}\frac{\left(\dfrac{\sqrt[3]{7}\cdot n^{\frac{4}{3}}}{n^2}-\dfrac{\sqrt[4]{81n^8-1}}{\sqrt[4]{n^8}}\right)}{\dfrac{(n+4\sqrt{n})}{n}\cdot\dfrac{\sqrt{n^2-5}}{n^2}}=$$

$$=\lim_{n\to\infty}\frac{\left(\underbrace{\dfrac{\sqrt[3]{7}}{\sqrt[3]{n^2}}}_{\to 0}-\sqrt[4]{81-\underbrace{\dfrac{1}{n^8}}_{\to 0}}\right)}{\left(1+\underbrace{\dfrac{4}{\sqrt{n}}}_{\to 0}\right)\sqrt{1-\underbrace{\dfrac{5}{n^2}}_{\to 0}}}=\frac{0-\sqrt[4]{81}}{1\cdot 1}=-3$$

[2] $\lim\limits_{n\to\infty}\dfrac{3^n-2^n}{3^{n-1}+2^n}=(*)$ разделим ч-ль и зн-ль на 3^n

$$(*)=\lim_{n\to\infty}\frac{\dfrac{3^n-2^n}{3^n}}{\dfrac{3^{-1}\cdot 3^n+2^n}{3^n}}=\lim_{n\to\infty}\frac{1-\overbrace{\left(\frac{2}{3}\right)^n}^{\to 0}}{3^{-1}+\underbrace{\left(\frac{2}{3}\right)^n}_{\to 0}}=\frac{1}{3^{-1}}=3$$

[3] $\lim\limits_{x\to\frac{5}{2}}\dfrac{2x^2-9x+10}{2x-5}=\dfrac{0}{0}=(*)$ разделим ч-ль на зн-ль

$$\begin{array}{r|l} 2x^2-9x+10 & 2x-5 \\ \underline{2x^2-5x} & x-2 \\ -4x+10 & \\ \underline{-4x+10} & \\ 0 & \end{array}$$

$(*)=\lim\limits_{x\to\frac{5}{2}}(x-2)=\dfrac{5}{2}-2=\dfrac{1}{2}$

[4] $\lim\limits_{x\to-2}\dfrac{\sqrt[3]{x-6}+2}{x+2}=\dfrac{0}{0}=(*)$ умн. ч-ль и зн-ль на сопр. выражение

$$(*)=\lim_{x\to-2}\frac{(\sqrt[3]{x-6}+2)(\sqrt[3]{(x-6)^2}-2\sqrt[3]{x-6}+4)}{(x+2)(\underbrace{\sqrt[3]{(x-6)^2}}_{4}-2\underbrace{\sqrt[3]{x-6}}_{-2}+4)}=\lim_{x\to-2}\frac{(x-6+2^3)}{(x+2)(4+4+4)}=$$

$$=\frac{1}{12}\lim_{x\to-2}\frac{(x+2)}{(x+2)}=\frac{1}{12}$$

$\boxed{5}$ $\lim\limits_{x\to0}\frac{\sqrt{4+x}-2}{3\operatorname{arctg}x}=\frac{0}{0}=(*)$ исп. эквив. при $x\to0$ $\operatorname{arctg}x\sim x$

$(*)=\frac{1}{3}\lim\limits_{x\to0}\frac{\sqrt{4+x}-2}{x}=\frac{0}{0}=(**)$ Умн. ч-ль и зн-ль на сопр. выр-е

$(**)=\frac{1}{3}\lim\limits_{x\to0}\frac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\underbrace{\sqrt{4+x}}_{\to2}+2)}=\frac{1}{3}\cdot\frac{1}{4}\lim\limits_{x\to0}\frac{4+x-2^2}{x}=\frac{1}{12}$

$\boxed{6}$ $\lim\limits_{x\to4}\frac{2^x-16}{\sin\pi x}=\frac{0}{0}=(*)$ $\exists\, t=x-4\Rightarrow x=t+4$, если $x\to4$, то $t\to0$

$(*)\lim\limits_{t\to0}\frac{2^{t+4}-16}{\sin(\pi(t+4))}=\lim\limits_{t\to0}\frac{16(2^t-1)}{\sin(\pi t+4\pi)}$ $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$(*)=16\lim\limits_{t\to0}\frac{(2^t-1)t}{t(\sin\pi t\underbrace{\cos4\pi}_{1}+\underbrace{\cos\pi t}_{1}\underbrace{\sin4\pi}_{0})}=16\ln2\cdot\lim\limits_{t\to0}\frac{t}{\sin\pi t}=$

$\to\ln2$ (замеч. пр.)

$=16\ln2\lim\limits_{t\to0}\frac{\pi t}{\pi\sin t\pi}=\frac{16\ln2}{\pi}$

$\to1$ (замеч. пр.)

$\boxed{7}$ $\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{e^{\sin x}-e^{\sin 4x}}=\frac{0}{0}=(*)$ $\sin2\alpha=2\sin\alpha\cos\alpha$

$(*)=\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{e^{\sin x}-e^{2\sin2x\underbrace{\cos2x}_{\to1}}}=\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{e^{\sin x}-e^{2\cdot2\sin x\underbrace{\cos x}_{\to-1}}}=$

$=\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{e^{\underbrace{2\sin\frac{x}{2}}_{\to1}\cos\frac{x}{2}}-e^{-8\underbrace{\sin\frac{x}{2}}_{\to1}\cos\frac{x}{2}}}=\lim\limits_{x\to\pi}\frac{\cos\frac{x}{2}}{e^{2\cos\frac{x}{2}}-e^{-8\cos\frac{x}{2}}}$ $\exists\, t=\cos\frac{x}{2}$

$x\to\pi, t\to0$

$\lim\limits_{t\to0}\frac{t}{e^{2t}-e^{-8t}}=\lim\limits_{t\to0}\frac{t}{e^{2t}(1-e^{-10t})}=-\lim\limits_{t\to0}\frac{t}{(e^{-10t}+1)}=-\left(-\frac{1}{10}\right)\lim\limits_{t\to0}\frac{-10t}{(e^{-10t}-1)}=\frac{1}{10}$

$\to1$ (замеч. пр.)

$\boxed{8}$ $\lim\limits_{x\to0}\frac{e^{2x}-e^x}{\sin3x-\sin5x}=\frac{0}{0}$ $\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\lim\limits_{x\to0}\frac{\overset{\to1}{e^x}(e^x-1)}{2\sin(-x)\underbrace{\cos4x}_{\to1}}=-\frac{1}{2}\lim\limits_{x\to0}\frac{(e^x-1)x}{x\sin x}=-\frac{1}{2}$

$\to1$ (замеч. пр.)

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$\boxed{9}$ $\lim\limits_{x\to 1}\frac{1-x}{\log_2 x}=\frac{0}{0}$ $\quad]\ t=1-x\Rightarrow x=1-t\ ,\ x\to 1,\ t\to 0$

$\lim\limits_{t\to 0}\frac{t}{\log_2(1-t)}=-\lim\limits_{t\to 0}\frac{-t}{\log_2(1+(-t))}=-\lim\limits_{t\to 0}\frac{1}{\underbrace{\frac{\log_2(1+(-t))}{-t}}_{\to\frac{1}{\ln 2}\ (\text{зам. пр.})}}=-\ln 2$

$\boxed{10}$ $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{\ln(1+\sin^2 x)}}=1^{\infty}=\lim\limits_{x\to 0}(1+(\cos x-1))^{\frac{1}{\ln(1+\sin^2 x)}}=$

$=\lim\limits_{x\to 0}\Big(\underbrace{(1+(\cos x-1))^{\frac{1}{\cos x-1}}}_{\to e\ (\text{зам. пр.})}\Big)^{\frac{\cos x-1}{\ln(1+\sin^2 x)}}=e^{-\lim\limits_{x\to 0}\frac{1-\cos x}{\ln(1+\sin^2 x)}}=e^{-\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\sin^2 x}\cdot\underbrace{\frac{\sin^2 x}{\ln(1+\sin^2 x)}}_{\to 1\ (\text{з. п.})}}$

$=e^{-2\lim\limits_{x\to 0}\frac{\sin^2\frac{x}{2}}{\sin^2 x}}$ $\quad]\ \sin\frac{x}{2}\sim\frac{x}{2}\quad \sin x\sim x$

$e^{-2\lim\limits_{x\to 0}\frac{(\frac{x}{2})^2}{x^2}}=e^{-2\lim\limits_{x\to 0}\frac{x^2}{4x^2}}=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$

$\boxed{11}$ $\lim\limits_{x\to 0}[\sin(x+2)]^{\frac{3}{(3+x)}}=(\sin(0+2))^{\frac{3}{(3+0)}}=(\sin 2)^1=\sin 2$

$\boxed{12}$ $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{6\operatorname{tg}x\operatorname{tg}3x}=1^{\infty}$ $\qquad \lim\limits_{x\to a}u(x)^{v(x)}=e^{\lim\limits_{x\to a}[(u(x)-1)v(x)]}$

$e^{\lim\limits_{x\to\frac{\pi}{2}}[6(\sin x-1)\operatorname{tg}x\operatorname{tg}3x]}$

$6\lim\limits_{x\to\frac{\pi}{2}}[(\sin x-1)\operatorname{tg}x\operatorname{tg}3x]=0\cdot\infty=6\lim\limits_{x\to\frac{\pi}{2}}\frac{(\sin x-1)\overset{1}{\sin x}\overset{-1}{\sin 3x}}{\cos x\cos 3x}=-6\lim\limits_{x\to\frac{\pi}{2}}\frac{(\sin x-1)}{\cos x(4\cos^3 x-3\cos x)}$

$=-6\lim\limits_{x\to\frac{\pi}{2}}\frac{(\sin x-1)}{\cos^2 x(4\underbrace{\cos^2 x}_{\to 0}-3)}=2\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin x-1}{1-\sin^2 x}=-2\lim\limits_{x\to\frac{\pi}{2}}\frac{(1-\sin x)}{(1-\sin x)(1+\underbrace{\sin x}_{\to 1})}=-2\cdot\frac{1}{2}=-1$

$e^{-1}=\frac{1}{e}$

$\boxed{13}$ $\lim\limits_{x\to 2}\left(\frac{\sqrt{x+2}-2}{x^2-4}\right)^{\frac{1}{2}}=\lim\limits_{x\to 2}\left(\frac{\sqrt{x+2}-2}{x^2-4}\right)^{\frac{1}{2}}=\frac{0}{0}$

$\lim\limits_{x\to 2}\left(\frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)}{(x^2-4)(\underbrace{\sqrt{x+2}}_{\to 2}+2)}\right)^{\frac{1}{2}}=\lim\limits_{x\to 2}\left(\frac{x+2-4}{4(x-2)(x+2)}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{4\cdot 4}}=\frac{1}{4}$

$\boxed{14}$ $\lim\limits_{x\to 0}\frac{\overbrace{\sqrt[3]{\operatorname{tg}x\operatorname{arctg}\frac{1}{x}}}^{0}+3}{2-\operatorname{tg}(1+\underbrace{\sin x}_{\to 0})}=\frac{3}{2-\operatorname{tg}1}$

$\boxed{1}$ Док-ть: $\lim\limits_{n\to\infty} a_n = a$

$a_n = \dfrac{3n-2}{2n-1}$, $a = \dfrac{3}{2}$

Выбираем произвольное $\varepsilon > 0$

$$\left|\frac{3n-2}{2n-1} - \frac{3}{2}\right| < \varepsilon$$

$$\left|\frac{2(3n-2) - 3(2n-1)}{2(2n-1)}\right| < \varepsilon$$

$$\left|\frac{6n-4-6n+3}{2(2n-1}\right| < \varepsilon$$

$$\frac{1}{2|2n-1|} < \varepsilon$$

$$|2n-1| > \frac{1}{2\varepsilon}$$

$\to 0$ (при $n \in \mathbb{N}$)

$$2n-1 > \frac{1}{2\varepsilon}$$

$$2n > \frac{1}{2\varepsilon} + 1$$

$$n > \frac{1}{2}\left(\frac{1}{2\varepsilon} + 1\right)$$

Вычислить пределы числовых посл-й

$\boxed{2}$ $$\lim_{n\to\infty} \frac{(3-n)^2 + (3+n)^2}{(3-n)^2 - (3+n)^2} = \frac{\infty}{\infty - \infty} = \lim_{n\to\infty} \frac{9-6n+n^2+9+6n+n^2}{9-6n+n^2-9-6n-n^2} = \lim_{n\to\infty} \frac{18+2n^2}{-12n} =$$

$$= \lim_{n\to\infty}\left(-\frac{3}{2n} - \frac{1}{6}n\right) = -\infty$$

$\boxed{3}$ $$\lim_{n\to\infty} \frac{n\sqrt[3]{5n^2} + \sqrt[4]{9n^8+1}}{(n+\sqrt{n})\sqrt{7-n+n^2}} = \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{\dfrac{n\sqrt[3]{5n^2} + \sqrt[4]{9n^8+1}}{n^2}}{\dfrac{(n+\sqrt{n})\sqrt{7-n+n^2}}{n^2}} = \lim_{n\to\infty} \frac{\underbrace{\sqrt[3]{\frac{5}{n}}}_{\to 0} + \sqrt[4]{9 + \underbrace{\frac{1}{n^8}}_{\to 0}}}{\Big(1+\underbrace{\frac{1}{\sqrt{n}}}_{0}\Big)\sqrt{\underbrace{\frac{7}{n^2}}_{0} - \underbrace{\frac{1}{n}}_{0} + 1}} = \frac{\sqrt[4]{9}}{1\cdot 1} = \sqrt[4]{9} = \sqrt{3}$$

$\boxed{4}$ $\lim\limits_{n\to\infty} n(\sqrt{n^2+1}-\sqrt{n^2-1}) = \infty(\infty-\infty)$

$$\lim_{n\to\infty}\frac{n(\sqrt{n^2+1}-\sqrt{n^2-1})(\sqrt{n^2+1}+\sqrt{n^2-1})}{(\sqrt{n^2+1}+\sqrt{n^2-1})} = \lim_{n\to\infty}\frac{n(n^2+1-n^2+1)}{\sqrt{n^2+1}+\sqrt{n^2-1}} = 2\lim_{n\to\infty}\frac{1}{\frac{\sqrt{n^2+1}+\sqrt{n^2-1}}{n}} =$$

$$= 2\lim_{n\to\infty}\frac{1}{\sqrt{1+\underbrace{\frac{1}{n^2}}_{\to 0}}+\sqrt{1-\underbrace{\frac{1}{n^2}}_{\to 0}}} = 2\cdot\frac{1}{2} = 1$$

$\boxed{5}$ $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots+\frac{n-1}{n^2}\right) = \lim\limits_{n\to\infty}\left[\frac{1}{n^2}(1+2+3+\ldots+n-1)\right] = (*)$

$$S_{n-1} = \frac{(a_1+a_{n-1})(n-1)}{2} = \frac{(1+n-1)(n-1)}{2} = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$$

$$(*) = \lim_{n\to\infty}\left(\frac{1}{n^2}\cdot\frac{(n^2-n)}{2}\right) = \frac{1}{2}\lim_{n\to\infty}\left(1-\underbrace{\frac{1}{n}}_{\to 0}\right) = \frac{1}{2}$$

$\boxed{6}$ $\lim\limits_{n\to\infty}\left(\frac{n+1}{n-1}\right)^n = \lim\limits_{n\to\infty}\left(\frac{n-1+2}{n-1}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{2}{n-1}\right)^n = 1^\infty = \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{1}{\frac{n-1}{2}}\right)^{\frac{n-1}{2}}}_{\to e\ (з.\text{-}н)}\right)^{\frac{2n}{n-1}} =$

$$= e^{\lim\limits_{n\to\infty}\frac{\frac{2}{n-1}}{n}} = e^{\lim\limits_{n\to\infty}\frac{2}{1-\underbrace{\frac{1}{n}}_{\to 0}}} = e^2$$

$\boxed{7}$ Найти $\delta(\varepsilon)$, что $\lim\limits_{x\to -3}\frac{2x^2+5x-3}{x+3} = -7$

$\left|\frac{2x^2+5x-3}{x+3}+7\right| < \varepsilon$, $\varepsilon > 0$

$\left|\frac{2x^2+5x-3+7x+21}{x+3}\right| < \varepsilon$

$\left|\frac{2x^2+12x+18}{x+3}\right| < \varepsilon$

$2\left|\frac{x^2+6x+9}{x+3}\right| < \varepsilon$

$\left|\frac{(x+3)^2}{x+3}\right| < \frac{\varepsilon}{2}$

$|x+3| < \frac{\varepsilon}{2}$ $\qquad$ $0 < \delta < \frac{\varepsilon}{2}$

[8] Док-ть: $f(x)$ непр. в x_0, найти $\delta(\varepsilon)$, $f(x)=5x^2-1$ $x_0=6$

$\varepsilon>0$

$|5x^2-1-179|<\varepsilon$

$|5x^2-180|<\varepsilon$

$5|x^2-36|<\varepsilon$

$|(x-6)(x+6)|<\frac{\varepsilon}{5}$

$\delta\ |x+6|<\frac{\varepsilon}{5}$

$0<\delta<\frac{\varepsilon}{5|x+6|}$

[9] $\lim\limits_{x\to-1}\frac{(x^3-2x-1)(x+1)}{x^4+4x^2-5}=\frac{0}{0}$

ч-ль:

$$\begin{array}{l|l} x^3+0x^2-2x-1 & x+1 \\ \hline -\ x^3+x^2 & x^2-x-1 \\ \quad -x^2-2x & \\ -\ -x^2-x & \\ \qquad -x-1 & \\ -\ -x-1 & \\ \qquad 0 & \end{array}$$

зн-ль:

$$\begin{array}{l|l} x^4+0x^3+4x^2+0x-5 & x+1 \\ \hline -\ x^4+x^3 & x^3-x^2+5x-5 \\ \quad -x^3+4x^2 & \\ -\ -x^3-x^2 & \\ \qquad 5x^2+0x & \\ -\ 5x^2+5x & \\ \qquad -5x-5 & \\ -\ -5x-5 & \\ \qquad 0 & \end{array}$$

$$\lim_{x\to-1}\frac{(x+1)(x^2-x-1)(x+1)}{(x+1)(x^3-x^2+5x-5)}=\frac{1\cdot 0}{-12}=0$$

[10] $\lim\limits_{x\to4}\frac{\sqrt{1+2x}-3}{\sqrt{x}-2}=\frac{0}{0}$

$$\lim_{x\to4}\frac{(\sqrt{1+2x}-3)(\sqrt{1+2x}+3)(\underbrace{\sqrt{x}}_{\to2}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)(\underbrace{\sqrt{1+2x}}_{\to3}+3)}=\frac{4}{6}\lim_{x\to4}\frac{1+2x-9}{x-4}=\frac{2}{3}\lim_{x\to4}\frac{2x-8}{x-4}=\frac{4}{3}$$

[11] $\lim\limits_{x\to0}\frac{\ln(1+\sin x)}{\sin 4(x-\pi)}=\frac{0}{0}=\lim\limits_{x\to0}\frac{\sin x\,\ln(1+\sin x)}{\underbrace{\sin x}_{\to1}\,2\sin 2(x-\pi)\underbrace{\cos 2(x-\pi)}_{\to1}}=$

$$=\lim_{x\to0}\frac{\sin x}{2\cdot2\cdot\sin(x-\pi)\underbrace{\cos(x-\pi)}_{\to-1}}=-\frac{1}{4}\lim_{x\to0}\frac{\sin x}{-\sin(\pi-x)}=\frac{1}{4}\lim_{x\to0}\frac{\sin x}{\sin x}=\frac{1}{4}$$

$$\boxed{13}\ \lim_{x\to\frac{\pi}{2}} \frac{2^{\cos^2 x}-1}{\ln\sin x} = \frac{0}{0} = \lim_{x\to\frac{\pi}{2}} \frac{\underbrace{(2^{\cos^2 x}-1)}_{}\cos^2 x}{\underbrace{\cos^2 x}_{\to\ln 2}\cdot\ln\sin x} = \ln 2 \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{\ln\sin x} =$$

$$= \ln 2 \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(1+\sin x)}{\ln(1+(\sin x-1))} = -2\ln 2\cdot\lim_{x\to\frac{\pi}{2}} \frac{\sin x-1}{\ln(1+(\sin x-1))} = -\ln 4$$

$$\boxed{14}\ \lim_{x\to 0} \frac{7^{2x}-5^{3x}}{2x-\operatorname{arctg}3x} = \frac{0}{0} = \lim_{x\to 0}\frac{7^{2x}-1-(5^{3x}-1)}{2x-\operatorname{arctg}3x} = \lim_{x\to 0}\frac{\underbrace{\frac{7^{2x}-1}{2x}}_{\to\ln 7}2x-\underbrace{\frac{5^{3x}-1}{3x}}_{\to\ln 5}3x}{2x-\operatorname{arctg}3x}$$

$$= \lim_{x\to 0}\frac{x(2\ln 7-3\ln 5)}{2x-\operatorname{arctg}3x} = \lim_{x\to 0}\frac{2\ln 7-3\ln 5}{2-\frac{\operatorname{arctg}3x}{x}} \qquad \operatorname{arctg}x\sim x \quad x\to 0$$

$$\lim_{x\to 0}\frac{2\ln 7-3\ln 5}{2-\frac{3x}{x}} = -(2\ln 7-3\ln 5) = 3\ln 5-2\ln 7 = \ln\frac{125}{49}$$

$$\boxed{15}\ \lim_{x\to 0}\frac{e^{x}+e^{-x}-2}{\sin^2 x} = \frac{0}{0} = \lim_{x\to 0}\frac{(e^{\frac{x}{2}}-e^{-\frac{x}{2}})^2}{\sin^2 x} = \lim_{x\to 0}\frac{\left[\underbrace{\frac{(e^{\frac{x}{2}}-1)}{\frac{x}{2}}}_{\to 1}\cdot\frac{x}{2}-\underbrace{\frac{(e^{-\frac{x}{2}}-1)}{-\frac{x}{2}}}_{\to 1}\left(-\frac{x}{2}\right)\right]^2}{\sin^2 x}$$

$$\lim_{x\to 0}\frac{\left(\frac{x}{2}+\frac{x}{2}\right)^2}{\sin^2 x} = \lim_{x\to 0}\frac{x^2}{\sin^2 x} = \lim_{x\to 0}\frac{x}{\sin x}\cdot\frac{x}{\sin x} = 1$$

$$\boxed{16}\ \lim_{x\to 0}\left(1-\ln(1+x^3)\right)^{\frac{3}{x^2\arcsin x}} = 1^{\infty} = \lim_{x\to\infty}\left(\underbrace{(1+(-\ln(1+x^3)))^{\frac{1}{-\ln(1+x^3)}}}_{\to e}\right)^{\frac{-3\ln(1+x^3)}{x^2\arcsin x}}$$

$$e^{-3\lim\limits_{x\to 0}\frac{\ln(1+x^3)}{x^2\arcsin x}} = e^{-3\lim\limits_{x\to 0}\frac{x\ln(1+x^3)}{x^3\arcsin x}} = e^{-3\lim\limits_{x\to 0}\frac{x}{\arcsin x}}, \qquad \arcsin x\sim x \quad x\to 0$$

$$e^{-3\lim\limits_{x\to 0}\frac{x}{x}} = e^{-3}$$

$$\boxed{17}\ \lim_{x\to 0}\left(\frac{\sin 2x}{x}\right)^{1+x} = \lim_{x\to 0}\frac{\sin 2x}{x} = \lim_{x\to 0}\frac{2\sin 2x}{2x} = 2$$

$$\boxed{18}\ \lim_{x\to 1}\left(\frac{3x-1}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-1}} = \lim_{x\to 1}\left(1+\left(\frac{3x-1}{x+1}\right)-1\right)^{\frac{1}{\sqrt[3]{x}-1}} = \lim_{x\to 1}\left(1+\frac{3x-1-x-1}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-1}} =$$

$$= \lim_{x\to 1}\left(1+\frac{2x-2}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-1}} = 1^{\infty} = \lim_{x\to 1}\left(\underbrace{\left(1+\frac{2(x-1)}{x+1}\right)^{\frac{x+1}{2(x-1)}}}_{\to e}\right)^{\frac{2(x-1)}{(x+1)(\sqrt[3]{x}-1)}} = e^{\lim\limits_{x\to 1}\frac{(x-1)}{\sqrt[3]{x}-1}} = e^3$$

$$\boxed{19}\ \lim_{x\to e}\left(\frac{\ln x-1}{x-e}\right)^{\sin\frac{\pi x}{2e}} = \lim_{x\to e}\left(\frac{\ln x-1}{x-e}\right) = \frac{0}{0} \quad \exists\, t=x-e \Rightarrow x=t+e,\ x\to e,\ t\to 0$$

$$\lim_{t\to 0}\left(\frac{\ln(t+e)-1}{t}\right) = \lim_{t\to 0}\frac{\ln(t+e)-\ln e}{t} = \lim_{t\to 0}\frac{\ln(1+\frac{t}{e})}{\frac{t}{e}\,e} = \frac{1}{e}$$

$$\boxed{20}\ \lim_{x\to 0}\left(\sqrt{4\cos 3x}+x\operatorname{arctg}\left(\frac{1}{x}\right)\right) \qquad x\operatorname{arctg}x\to 0$$

$$\lim_{x\to 0}\left(\sqrt{4\underbrace{\cos 3x}_{\to 1}}+\underbrace{x\operatorname{arctg}\left(\frac{1}{x}\right)}_{\to 0}\right) = \sqrt{4} = 2$$

2 а) $\lim\limits_{n\to\infty} \frac{n^2-\sqrt{n^3+1}}{\sqrt{n^6+2-n}} = \frac{\infty-\infty}{\infty}$

$\lim\limits_{n\to\infty} \frac{\frac{n^2-\sqrt{n^3+1}}{n^3}}{\frac{\sqrt{n^6+2-n}}{n^3}} = \lim\limits_{n\to\infty} \frac{\frac{n^2}{n^3} - \sqrt{\frac{n^3+1}{n^6}}}{\sqrt{\frac{n^6+2-n}{n^6}}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n} - \sqrt{\frac{1}{n^3}+\frac{1}{n^6}}}{\sqrt{1+\frac{2}{n^6}-\frac{1}{n^5}}} = \frac{0}{1} = 0$

б) $\lim\limits_{n\to\infty} \frac{(n+1)^2+(n-1)^2-(n+2)^3}{(4-n)^3} = \frac{\infty+\infty-\infty}{-\infty} = \lim\limits_{n\to\infty} \frac{n^2+2n+1+n^2-2n+1-(n^3+6n^2+12n}{64-48n+12n^2-n^3}$

$= \lim\limits_{n\to\infty} \frac{2n^2+2-n^3-6n^2-12n-8}{64-48n+12n^2-n^3} = \lim\limits_{n\to\infty} \frac{-n^3-4n^2-12n-6}{64-48n+12n^2-n^3} = \frac{\infty}{\infty}$

$\lim\limits_{n\to\infty} \frac{\frac{-n^3-4n^2-12n-6}{n^3}}{\frac{64-48n+12n^2-n^3}{n^3}} = \lim\limits_{n\to\infty} \frac{-1-\frac{4}{n}-\frac{12}{n^2}-\frac{6}{n^3}}{\frac{64}{n^3}-\frac{48}{n^2}+\frac{12}{n}-1} = \frac{-1}{-1} = 1$

е) $\lim\limits_{x\to-\frac{1}{3}} \frac{3x^2-2x+1}{x+\frac{4}{3}} = \frac{3\cdot\frac{1}{9}+\frac{2}{3}+1}{-\frac{1}{3}+\frac{4}{3}} = \frac{2}{0} = \infty$

3 а) $\lim\limits_{n\to\infty} (n\sqrt{n}-\sqrt{n(n+1)(n+2)}) = \infty-\infty$

$\lim\limits_{n\to\infty} \frac{(n\sqrt{n}-\sqrt{n(n^2+3n+2)})(n\sqrt{n}+\sqrt{n(n^2+3n+2)})}{n\sqrt{n}+\sqrt{n(n^2+3n+2)}} = \lim\limits_{n\to\infty} \frac{n^3-n^3-3n^2-2n}{n\sqrt{n}+\sqrt{n^3+3n^2+2n}} =$

$= \lim\limits_{n\to\infty} \frac{-3n^2-2n}{n\sqrt{n}+\sqrt{n^3+3n^2+2n}} = \frac{\infty}{\infty}$

$\lim\limits_{n\to\infty} \frac{\frac{-3n^2-2n}{n^2}}{\frac{n\sqrt{n}+\sqrt{n^3+3n^2+2n}}{n^2}} = \lim\limits_{n\to\infty} \frac{-3-\frac{2}{n}}{\frac{1}{\sqrt{n}}+\sqrt{\frac{1}{n}+\frac{3}{n^2}+\frac{2}{n^3}}} = \frac{-3}{0} = -\infty$

б) $\lim\limits_{x\to\infty} (\sqrt{x^2+5x}-x) = \infty-\infty = \lim\limits_{x\to\infty} \frac{(\sqrt{x^2+5x}-x)(\sqrt{x^2+5x}+x)}{\sqrt{x^2+5x}+x} =$

$= \lim\limits_{x\to\infty} \frac{x^2+5x-x^2}{\sqrt{x^2+5x}+x} = 5\lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2+5x}+x} = \frac{\infty}{\infty} = 5\lim\limits_{x\to\infty} \frac{1}{\frac{\sqrt{x^2+5x}+x}{x}} =$

$= 5\lim\limits_{x\to\infty} \frac{1}{\sqrt{1+\frac{5}{x}}+1} = \frac{5}{2}$

в) $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{(\sqrt[3]{8+3x+x^2}-2)(\sqrt[3]{(8+3x+x^2)^2}+2\sqrt[3]{8+3x+x^2}+4)}{(x+x^2)(\sqrt[3]{(8+3x+x^2)^2}+2\sqrt[3]{8+3x+x^2}+4)} =$

$= \dfrac{1}{12} \lim\limits_{x\to 0} \dfrac{8+3x+x^2-8}{x+x^2} = \dfrac{1}{12} \lim\limits_{x\to 0} \dfrac{x(3+x)}{x(1+x)} = \dfrac{1}{12}\cdot 3 = \dfrac{1}{4}$

г) $\lim\limits_{x\to 0} \dfrac{2x}{\sqrt{4+x}-\sqrt{4-x}} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{2x(\sqrt{4+x}+\sqrt{4-x})}{(\sqrt{4+x}-\sqrt{4-x})(\sqrt{4+x}+\sqrt{4-x})} = 8\lim\limits_{x\to 0} \dfrac{x}{4+x-4+x} =$

$= 8\lim\limits_{x\to 0} \dfrac{x}{2x} = 8\cdot\dfrac{1}{2} = 4$

$\boxed{4}$ а) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{3x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{\sin 5x}{\frac{3}{5}\cdot 5x} \to 1 = \dfrac{5}{3}$

б) $\lim\limits_{x\to 0} \dfrac{\sin(x+\delta)+\sin(x-\delta)}{2x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{2\cos\left(\frac{x+\delta-(x-\delta)}{2}\right)\sin\left(\frac{x+\delta+x-\delta}{2}\right)}{2x} =$

$= \lim\limits_{x\to 0} \dfrac{2\cos\delta\,\sin x}{2x} \to 1 = 2\cos\delta$

в) $\lim\limits_{x\to 0} \left(\dfrac{1}{\sin^2 x} - \dfrac{1}{4\sin\frac{x}{2}}\right) = \infty - \infty = \lim\limits_{x\to 0} \left(\dfrac{1}{(\frac{1}{2}\sin\frac{x}{2}\cos\frac{x}{2})^2} - \dfrac{1}{4\sin\frac{x}{2}}\right) =$

$= \lim\limits_{x\to 0} \left(\dfrac{1}{\frac{1}{4}\sin^2\frac{x}{2}\underbrace{\cos^2\frac{x}{2}}_{\to 1}} - \dfrac{1}{4\sin\frac{x}{2}}\right) = \lim\limits_{x\to 0} \left(\dfrac{4}{\sin^2\frac{x}{2}} - \dfrac{1}{4\sin\frac{x}{2}}\right) - \lim\limits_{x\to 0} \dfrac{16-\overbrace{\sin\frac{x}{2}}^{\to 0}}{\underbrace{4\sin^2\frac{x}{2}}_{\to 0}} = \dfrac{16}{0} = \infty$

г) $\lim\limits_{x\to 0} \dfrac{1-\cos 6x}{\sin x\cdot tg^3 x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{2\sin^2 3x}{\sin x\cdot\frac{\sin^3 x}{\cos^3 x}} \to 1 = 2\lim\limits_{x\to 0} \dfrac{\sin^2 3x}{\sin^4 x} = \dfrac{0}{0}$ $\quad \sin x \sim x,\ x\to 0$

$2\lim\limits_{x\to 0} \dfrac{(3x)^2}{x^4} = 2\lim\limits_{x\to 0} \dfrac{9x^2}{x^4} = 2\lim\limits_{x\to 0} \dfrac{9}{x^2} \to \infty = \infty$

д) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin(\frac{\pi}{4}-x)}{x^2-\frac{\pi^2}{16}} = \dfrac{0}{0} = \lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin(\frac{\pi}{4}-x)}{(x-\frac{\pi}{4})(x+\frac{\pi}{4})} = \dfrac{1}{\frac{\pi}{2}} \lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin(\frac{\pi}{4}-x)}{-(\frac{\pi}{4}-x)}$ $\quad] \ t = \frac{\pi}{4}-x,\ t\to 0$

$\dfrac{2}{\pi}\lim\limits_{t\to 0} \dfrac{\sin t}{-t} \to 1 = -\dfrac{2}{\pi}$

$\boxed{5}$ а) $\lim\limits_{x\to 0} \dfrac{1-\cos 5x}{x^2} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{2\sin^2(\frac{5}{2}x)}{x^2} = \lim\limits_{x\to 0} \dfrac{2\sin\frac{5x}{2}\cdot\sin\frac{5x}{2}}{\frac{4}{25}\cdot\frac{5x}{2}\cdot\frac{5x}{2}} = \dfrac{2\cdot 25}{4} = \dfrac{25}{2}$

б) $\lim\limits_{x\to\infty} \left(\dfrac{3x-4}{3x+3}\right)^{2-5x} = \lim\limits_{x\to\infty} \left(\dfrac{3x+3}{3x-4}\right)^{-(2-5x)} = \lim\limits_{x\to\infty} \left(\dfrac{3x-4+7}{3x-4}\right)^{5x-2} = \lim\limits_{x\to\infty} \left(1+\dfrac{7}{3x-4}\right)^{5x-2}$

$= 1^\infty = \lim\limits_{x\to\infty} \Bigg(\underbrace{\left(1+\dfrac{1}{\frac{3x-4}{7}}\right)^{\frac{3x-4}{7}}}_{e}\Bigg)^{\frac{7(5x-2)}{3x-4}} = e^{7\lim\limits_{x\to\infty}\frac{5x-2}{3x-4}} = e^{7\lim\limits_{x\to\infty}\frac{5-\frac{2}{x}}{3-\frac{4}{x}}} = e^{7\cdot\frac{5}{3}} = e^{\frac{35}{3}}$

в) $\lim\limits_{x\to\pm\infty}\left(\frac{x^2-3x+1}{3x^2+x}\right)^x=\left(\frac{\infty}{\infty}\right)^{\pm\infty}=\lim\limits_{x\to\pm\infty}\left(\frac{\frac{x^2-3x+1}{x^2}}{\frac{3x^2+x}{x^2}}\right)^x=\lim\limits_{x\to\pm\infty}\left(\frac{1-\frac{3}{x}+\frac{1}{x^2}}{3+\frac{1}{x}}\right)^x=$

$=\left(\frac{1}{3}\right)^{\pm\infty}=\begin{cases}0, & x\to+\infty\\ +\infty, & x\to-\infty\end{cases}$

г) $\lim\limits_{x\to 0}(1+\operatorname{arctg}2x)^{\frac{1}{3x}}=1^{\infty}=\lim\limits_{x\to 0}\left((1+\operatorname{arctg}2x)^{\frac{1}{\operatorname{arctg}2x}}\right)^{\frac{\operatorname{arctg}2x}{3x}}=$

$=e^{\frac{1}{3}\lim\limits_{x\to 0}\frac{\operatorname{arctg}2x}{x}}=e^{\frac{0}{0}}$ $\quad t=\operatorname{arctg}2x\Rightarrow x=\frac{\operatorname{tg}t}{2},\ t\to 0$

$e^{\frac{1}{3}\lim\limits_{t\to 0}\frac{t}{\frac{\operatorname{tg}t}{2}}}=e^{\frac{2}{3}\lim\limits_{t\to 0}\frac{t\cos t}{\sin t}}=e^{\frac{2}{3}}=\sqrt[3]{e^2}$

д) $\lim\limits_{x\to 0}(1-3\sin x)^{\frac{1}{6x}}=1^{\infty}=\lim\limits_{x\to 0}\left((1+(-3\sin x))^{\frac{1}{(-3\sin x)}}\right)^{-\frac{3\sin x}{6x}}=$

$=e^{-\frac{1}{2}\lim\limits_{x\to 0}\frac{\sin x}{x}}=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$

е) $\lim\limits_{x\to\infty}\left(\frac{x^2+3x+1}{x+x^2}\right)^{x^2+1}=\lim\limits_{x\to\infty}\left(\frac{x^2+x+2x+1}{x^2+x}\right)^{x^2+1}=\lim\limits_{x\to\infty}\left(1+\frac{2x+1}{x^2+x}\right)^{x^2+1}=1^{\infty}=$

$=\lim\limits_{x\to\infty}\left(\left(1+\frac{1}{\frac{x^2+x}{2x+1}}\right)^{\frac{x^2+x}{2x+1}}\right)^{\frac{(2x+1)(x^2+1)}{x^2+x}}=e^{\lim\limits_{x\to\infty}\frac{2x^3+x^2+2x+1}{x^2+x}}=e^{\lim\limits_{x\to\infty}\frac{2+\frac{1}{x}+\frac{2}{x^2}+\frac{1}{x^3}}{\frac{1}{x}+\frac{1}{x^2}}}=$

$=e^{\frac{2}{0}}=e^{\infty}=\infty$

ж) $\lim\limits_{x\to 0}(1+\cos x)^{\frac{x+2}{x-1}}=2^{-2}=\frac{1}{4}$

з) $\lim\limits_{x\to 0}(1+5x)^{\frac{1}{3x}}=1^{\infty}=\lim\limits_{x\to 0}\left((1+5x)^{\frac{1}{5x}}\right)^{\frac{5x}{3x}}=e^{\frac{5}{3}}$

$\boxed{2}$ а) $\lim\limits_{n\to\infty}\frac{n+1}{(n-2)^3}=\frac{\infty}{\infty}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{n^3}}{\frac{(n-2)^3}{n^3}}=\lim\limits_{n\to\infty}\frac{\frac{1}{n^2}+\frac{1}{n^3}}{\left(1-\frac{2}{n}\right)^3}=\frac{0}{1}=0$

б) $\lim\limits_{n\to\infty}\frac{n^3-7}{\sqrt[3]{n^3+2n-3}}=\frac{\infty}{\infty}=\lim\limits_{n\to\infty}\frac{\frac{n^3-7}{n^3}}{\frac{\sqrt[3]{n^3+2n-3}}{\sqrt[3]{n^3}}}=\lim\limits_{n\to\infty}\frac{1-\frac{7}{n^3}}{\sqrt[3]{\frac{1}{n^6}+\frac{2}{n^2}-\frac{3}{n^7}}}=\frac{1}{0}=\infty$

в) $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^8-1}}{(2x-1)^2-x^2}=\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^8-1}}{4x^2-4x+1-x^2}=\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^8-1}}{3x^2-4x+1}=\frac{\infty}{\infty}=$

$=\lim\limits_{x\to\infty}\frac{\frac{\sqrt[4]{x^8-1}}{\sqrt[4]{x^8}}}{\frac{3x^2-4x+1}{x^2}}=\lim\limits_{x\to\infty}\frac{\sqrt[4]{1-\frac{1}{x^8}}}{3-\frac{4}{x}+\frac{1}{x^2}}=\frac{1}{3}$

г) $\lim\limits_{x\to\infty} \frac{4(x+2)^2-(x-1)^2}{3x^2-9} = \lim\limits_{x\to\infty} \frac{4x^2+16x+16-x^2+2x-1}{3x^2-9} = \lim\limits_{x\to\infty} \frac{3x^2+18x+15}{3x^2-9} = \frac{\infty}{\infty} =$

$= \lim\limits_{x\to\infty} \frac{\frac{3x^2+18x+15}{x^2}}{\frac{3x^2-9}{x^2}} = \lim\limits_{x\to\infty} \frac{3+\frac{18}{x}+\frac{15}{x^2}}{3-\frac{9}{x^2}} = \frac{3}{3} = 1$ $\quad \frac{18}{x}\to 0,\ \frac{15}{x^2}\to 0,\ \frac{9}{x^2}\to 0$

д) $\lim\limits_{x\to\infty} \left(\frac{2x^4}{x^3-1} - 2x\right) = \lim\limits_{x\to\infty} \frac{2x^4-2x^4+2x}{x^3-1} = \lim\limits_{x\to\infty} \frac{2x}{x^3-1} = \lim\limits_{x\to\infty} \frac{\frac{2x}{x^3}}{\frac{x^3-1}{x^3}} =$

$= \lim\limits_{x\to\infty} \frac{\frac{2}{x^2}}{1-\frac{1}{x^3}} = \frac{0}{1} = 0$ $\quad \frac{2}{x^2}\to 0,\ \frac{1}{x^3}\to 0$

е) $\lim\limits_{x\to\infty} \frac{\sqrt{x}+1}{x+1} = \frac{\infty}{\infty} = \lim\limits_{x\to\infty} \frac{\frac{\sqrt{x}+1}{x}}{\frac{x+1}{x}} = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}+\frac{1}{x}}{1+\frac{1}{x}} = \frac{0}{1} = 0$ $\quad \to 0,\ \frac{1}{x}\to 0$

ж) $\lim\limits_{x\to 2} \frac{x^2-4}{x^2+4} = \frac{4-4}{4+4} = \frac{0}{8} = 0$

з) $\lim\limits_{x\to 3} \frac{x^3-27}{x^2-9} = \frac{0}{0} = \lim\limits_{x\to 3} \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} = \frac{9+9+9}{6} = \frac{27}{6}$

и) $\lim\limits_{x\to 2} \frac{x^2+x-6}{3x^2-2x-8} = \frac{0}{0} = \frac{(x-2)(x+3)}{(x-2)(3x+4)} = \frac{2+3}{6+4} = \frac{5}{10} = \frac{1}{2}$

к) $\lim\limits_{x\to 0} \frac{x^2-3}{x^2-3x} = \frac{-3}{0} = \infty$

$\boxed{3}$ а) $\lim\limits_{x\to 2} \left(\frac{2}{2-x} - \frac{4}{x^2-4}\right) = (\infty-\infty) = \lim\limits_{x\to 2} \left(\frac{2}{2-x} + \frac{4}{(2-x)(2+x)}\right) =$

$= \lim\limits_{x\to 2} \frac{4+2x+4}{(2-x)(2+x)} = \lim\limits_{x\to 2} \frac{2x+8}{(2-x)(2+x)} = \frac{12}{0\cdot 4} = \frac{12}{0} = \infty$

б) $\lim\limits_{x\to 1} \frac{\sqrt[3]{x}-1}{x^3-1} = \frac{0}{0} = \lim\limits_{x\to 1} \frac{(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(x^3-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)} = \frac{1}{3}\lim\limits_{x\to 1} \frac{x-1}{(x-1)(x^2+x+1)} = \frac{1}{3}\cdot\frac{1}{3} = \frac{1}{9}$ $\quad \to 1,\ \to 1$

в) $\lim\limits_{x\to \frac{1}{5}} \frac{\sqrt{10x-1}-1}{\sqrt{5x}-1} = \frac{0}{0} = \lim\limits_{x\to \frac{1}{5}} \frac{(\sqrt{10x-1}+1)(\sqrt{10x-1}-1)(\sqrt{5x}+1)}{(\sqrt{10x-1}+1)(\sqrt{10x-1}-1)(\sqrt{5x}+1)} = \frac{2}{2}\lim\limits_{x\to \frac{1}{5}} \frac{10x-1-1}{5x-1} =$ $\quad \to 1,\ \to 1$

$= \lim\limits_{x\to \frac{1}{5}} \frac{10x-2}{5x-1} = \lim\limits_{x\to \frac{1}{5}} \frac{2(5x-1)}{5x-1} = 2$

г) $\lim\limits_{x\to\infty} (\sqrt{x^2-3}-x)x = (\infty-\infty)\infty = \lim\limits_{x\to\infty} \frac{(\sqrt{x^2-3}-x)(\sqrt{x^2-3}+x)x}{(\sqrt{x^2-3}+x)} = \lim\limits_{x\to\infty} \frac{(x^2-3-x^2)x}{\sqrt{x^2-3}+x} =$

$= -3\lim\limits_{x\to\infty} \frac{x}{\sqrt{x^2-3}+x} = \frac{\infty}{\infty} = -3\lim\limits_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2-3}+x}{x}} = -3\lim\limits_{x\to\infty} \frac{1}{\sqrt{1-\frac{3}{x^2}}+1} = -3\cdot\frac{1}{(1+1)} = -\frac{3}{2}$

$\boxed{4}$ а) $\lim\limits_{x\to0}\dfrac{x^2}{\sin 5x} = \dfrac{0}{0} = \lim\limits_{x\to0}\dfrac{x\cdot\frac{1}{5}\cdot 5x}{\sin 5x} = 0\cdot\dfrac{1}{5} = 0$

б) $\lim\limits_{x\to0}\dfrac{1-\cos 4x}{x^2} = \lim\limits_{x\to0}\dfrac{2\sin^2 2x}{x^2} = \dfrac{0}{0} = 2\lim\limits_{x\to0}\dfrac{\sin 2x\cdot\sin 2x}{\frac{1}{2}\cdot\frac{1}{2}\cdot 2x\cdot 2x} = 2\cdot 2\cdot 2 = 8$

в) $\lim\limits_{x\to\frac{\pi}{10}}\dfrac{tg(x-\frac{\pi}{10})}{x^3-(\frac{\pi}{10})^3} = \dfrac{0}{0} = \lim\limits_{x\to\frac{\pi}{10}}\dfrac{\sin(x-\frac{\pi}{10})}{\underbrace{\cos(x-\frac{\pi}{2})}_{1}(x-\frac{\pi}{10})(x^2+\frac{\pi}{10}x+\frac{\pi^2}{100})} = \dfrac{1}{\frac{\pi^2}{100}+\frac{\pi^2}{100}+\frac{\pi^2}{100}} =$

$= \dfrac{1}{\frac{3\pi^2}{100}} = \dfrac{100}{3\pi^2}$

г) $\lim\limits_{x\to0}\dfrac{2tg7x}{\sin(x^2-x)} = \dfrac{0}{0}$ $\quad \sin x \sim x,\ x\to0$, $\ tg x \sim x,\ x\to0$

$\lim\limits_{x\to0}\dfrac{2\cdot7x}{x^2-x} = 14\lim\limits_{x\to0}\dfrac{x}{x(x-1)} = 14\cdot\dfrac{1}{(-1)} = -14$

д) $\lim\limits_{x\to0} x\cdot ctg\dfrac{x}{3} = 0\cdot\infty = \lim\limits_{x\to0}\dfrac{x\cos\frac{x}{3}}{\sin\frac{x}{3}} = \dfrac{0}{0} = \lim\limits_{x\to0}\dfrac{3\cdot\frac{x}{3}}{\sin\frac{x}{3}} = 3$

$\boxed{5}$ ж) $\lim\limits_{x\to0}\left(1-\dfrac{\sin 5x}{5}\right)^{\frac{2}{x}} = 1^\infty = \lim\limits_{x\to0}\left(\left(1-\dfrac{\sin 5x}{5}\right)^{-\frac{5}{\sin 5x}}\right)^{-\frac{\sin 5x}{5}\cdot\frac{2}{x}} =$

$= e^{-2\lim\limits_{x\to0}\frac{\sin 5x}{5x}} = e^{-2}$

з) $\lim\limits_{x\to0}(1+x)^{\frac{5}{arctg x}} = 1^\infty = \lim\limits_{x\to0}\left((1+x)^{\frac{1}{x}}\right)^{\frac{5x}{arctg x}} = e^{5\lim\limits_{x\to0}\frac{x}{arctg x}} = e^{\frac{0}{0}} =$

$\exists\ t = arctg x \Rightarrow x = tg t,\ t\to0$

$e^{5\lim\limits_{t\to0}\frac{tg t}{t}} = e^{5\lim\limits_{t\to0}\frac{\sin t}{t\cos t}} = e^5$

6.1 $\lim\limits_{n\to\infty}\left(\frac{1}{3}+\frac{1}{3^2}+\dots+\frac{1}{3^n}\right)$ $\qquad S=\frac{b_1}{1-q}$

$$\frac{\frac{1}{3}}{(1-\frac{1}{3})}=\frac{1}{3}\cdot\frac{3}{2}=\frac{1}{2}$$

6.2 $\lim\limits_{n\to\infty}\frac{\sin n!}{n}=\lim\limits_{n\to\infty}\left(\sin n!\cdot\frac{1}{n}\right)=0$

6.3 $\lim\limits_{x\to+\infty}\frac{2x^2-3x-4}{\sqrt{x^4+1}}=\frac{\infty}{\infty}=\lim\limits_{x\to+\infty}\frac{\frac{2x^2-3x-4}{x^2}}{\frac{\sqrt{x^4+1}}{x^2}}=\lim\limits_{x\to+\infty}\frac{2-\underset{\to 0}{\frac{3}{x}}-\underset{\to 0}{\frac{4}{x^2}}}{\sqrt{\frac{x^4+1}{x^4}}}=$

$=2\lim\limits_{x\to+\infty}\frac{1}{\sqrt{1+\underset{\to 0}{\frac{1}{x^4}}}}=2$

6.4 $\lim\limits_{x\to+\infty}\frac{3x^2+2x+3}{4x^3-5x+6}=\frac{\infty}{\infty}=\lim\limits_{x\to+\infty}\frac{\frac{3x^2+2x+3}{x^3}}{\frac{4x^2-5x+6}{x^3}}=\lim\limits_{x\to+\infty}\frac{\underset{\to 0}{\frac{3}{x}}+\underset{\to 0}{\frac{2}{x^2}}+\underset{\to 0}{\frac{3}{x^3}}}{4-\underset{\to 0}{\frac{5}{x^2}}+\underset{\to 0}{\frac{6}{x^3}}}=\frac{0}{4}=0$

6.5 $\lim\limits_{x\to-\infty}\frac{4x^2+2x-3}{2x+21}=\frac{\infty}{-\infty}=\lim\limits_{x\to-\infty}\frac{\frac{4x^2+2x-3}{x^2}}{\frac{2x+21}{x^2}}=\lim\limits_{x\to-\infty}\frac{4+\frac{2}{x}-\frac{3}{x^2}\to 0}{\underset{\to 0}{\frac{2}{x}+\frac{21}{x^2}}}=\frac{4}{0}=-\infty$

6.6 $\lim\limits_{x\to-2}\frac{x^2+5x+6}{x^2+x-2}=\frac{0}{0}=\lim\limits_{x\to-2}\frac{(x+2)(x+3)}{(x-1)(x+2)}=\lim\limits_{x\to-2}\frac{x+3}{x-1}=\frac{1}{-3}=-\frac{1}{3}$

6.7 $\lim\limits_{x\to 8}\left(\frac{1}{x-8}-\frac{16}{x^2-64}\right)=(\infty-\infty)=\lim\limits_{x\to 8}\left(\frac{1}{x-8}-\frac{16}{(x+8)(x-8)}\right)=\lim\limits_{x\to 8}\frac{x+8-16}{(x-8)(x+8)}=$

$=\lim\limits_{x\to 8}\frac{x-8}{(x-8)(x+8)}=\lim\limits_{x\to 8}\frac{1}{x+8}=\frac{1}{16}$

6.8 $\lim\limits_{x\to 0}\frac{x^3}{\sqrt{4+x^3}-2}=\frac{0}{0}=\lim\limits_{x\to 0}\frac{x^3(\sqrt{4+x^3}+2)}{(\sqrt{4+x^3}-2)(\sqrt{4+x^3}+2)}=\lim\limits_{x\to 0}\frac{x^3(\sqrt{4+x^3}+2)}{4+x^3-4}=$

$=\lim\limits_{x\to 0}(\sqrt{4+\underset{\to 0}{x^3}}+2)=2+2=4$

6.9 $\lim\limits_{x\to+\infty}\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)=(\infty-\infty)=\lim\limits_{x\to+\infty}\frac{\left(\sqrt[3]{(x+1)^2}-\sqrt[3]{(x-1)^2}\right)\left(\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}\right)}{\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}}$

$=\lim\limits_{x\to+\infty}\frac{x^2+2x+1-(x^2-2x+1)}{\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}}=\lim\limits_{x\to+\infty}\frac{4x}{\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}}=\frac{\infty}{\infty}$

$=\lim\limits_{x\to+\infty}\frac{\frac{4x}{\sqrt[3]{x^4}}\to 0}{\frac{\sqrt[3]{(x+1)^4}+\sqrt[3]{(x+1)^2(x-1)^2}+\sqrt[3]{(x-1)^4}}{\sqrt[3]{x^4}}}=\lim\limits_{x\to+\infty}\frac{\frac{4x}{\sqrt[3]{x^4}}\to 0}{\sqrt[3]{(1+\underset{\to 0}{\frac{1}{x}})^4}+\sqrt[3]{(1+\underset{\to 0}{\frac{1}{x}})^2(1+\underset{\to 0}{\frac{1}{x}})^2}+\sqrt[3]{(1+\underset{\to 0}{\frac{1}{x}})^4}}=$

$=\frac{0}{(1+1+1)}=\frac{0}{3}=0$

6.10 $\lim_{x\to 0}\frac{\sin 5x}{4x}=\frac{0}{0}=\lim_{x\to 0}\frac{\sin 5x}{5x\cdot\frac{4}{5}}=\frac{1}{4/5}=\frac{5}{4}$

6.11 $\lim_{x\to 0}\frac{8x^3}{tg^3 7x}=\frac{0}{0}$ $\quad tgx\sim x,\ x\to 0$

$\lim_{x\to 0}\frac{8x^3}{(7x)^3}=\frac{8}{343}$

6.12 $\lim_{x\to 0}\frac{\cos 3x-\cos 5x}{x^2}=\frac{0}{0}=-2\lim_{x\to 0}\frac{\sin 4x\cdot\sin(-x)}{x^2}=2\lim_{x\to 0}\frac{\sin 4x\cdot\sin x}{x^2}=$

$=2\lim_{x\to 0}\frac{\sin 4x}{4x\cdot\frac{1}{4}}\cdot\frac{\sin x}{x}=2\cdot 4=8$

6.13 $\lim_{x\to\frac{\pi}{4}}\frac{x(1-tgx)}{\cos 2x}=\frac{0}{0}$ $\quad 3t=x-\frac{\pi}{4}\Rightarrow x=t+\frac{\pi}{4},\ t\to 0$

$\lim_{t\to 0}\frac{(t+\frac{\pi}{4})(1-tg(t+\frac{\pi}{4}))}{\cos(2(t+\frac{\pi}{4}))}=\frac{\pi}{4}\lim_{t\to 0}\frac{1-tg(t+\frac{\pi}{4})}{\cos(2t+\frac{\pi}{2})}=\frac{\pi}{4}\lim_{t\to 0}\frac{1-\frac{(tgt+tg\frac{\pi}{4})}{(1-tgt\cdot tg\frac{\pi}{4})}}{-\sin 2t}=$

$=\frac{\pi}{4}\lim_{t\to 0}\frac{1-\frac{(tgt+1)}{(1-tgt)}}{-\sin 2t}=\frac{\pi}{4}\lim_{t\to 0}\frac{1-tgt-tgt-1}{-(1-tgt)\sin 2t}=\frac{\pi}{4}\lim_{t\to 0}\frac{-2tgt}{-2(1-tgt)\cdot\sin t\cos t}=$

$=\frac{\pi}{4}\lim_{t\to 0}\frac{\sin t}{(1-tgt)\cdot\sin t\cos^2 t}=\frac{\pi}{4}$

6.14 $\lim_{x\to 0}\frac{\sin x}{\arcsin x}=\frac{0}{0}$ $\quad \sin x\sim x,\ \arcsin x\sim x,\ x\to 0$

$\lim_{x\to 0}\frac{x}{x}=1$

6.15 $\lim_{x\to+\infty}(1+\frac{2}{x})^{3x}=\lim_{x\to+\infty}\left(\left(1+\frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}}\right)^{\frac{2}{x}\cdot 3x}=e^6$

6.16 $\lim_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{3x}=\left(\frac{\infty}{\infty}\right)^{\infty}=\lim_{x\to\infty}\left(\frac{2x-3+3}{2x-3}\right)^{3x}=\lim_{x\to\infty}\left(1+\frac{3}{2x-3}\right)^{3x}=$

$=1^{\infty}=\lim_{x\to\infty}\left(\left(1+\frac{1}{\frac{2x-3}{3}}\right)^{\frac{2x-3}{3}}\right)^{\frac{3\cdot 3x}{2x-3}}=e^{9\lim_{x\to\infty}\frac{x}{2x-3}}=e^{9\lim_{x\to\infty}\frac{1}{2-\frac{3}{x}}}=e^{\frac{9}{2}}$

6.17 $\lim_{x\to 0}(\cos^2 x)^{\frac{1}{\sin^2 x}}=1^{\infty}=\lim_{x\to 0}(1-\sin^2 x)^{\frac{1}{\sin^2 x}}=\lim_{x\to 0}\left((1+(-\sin^2 x))^{\frac{1}{(-\sin^2 x)}}\right)^{-1}=$

$=e^{-1}$

17.1 $\lim\limits_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{5n^2}\right)=\frac{\infty}{\infty}=\lim\limits_{n\to\infty}\frac{\frac{(1+n)n}{2}}{5n^2}=\frac{1}{10}\lim\limits_{n\to\infty}\frac{n+n^2}{n^2}=$

$=\frac{1}{10}\lim\limits_{n\to\infty}\left(\frac{1}{n}+1\right)=\frac{1}{10}$

17.2 $\lim\limits_{n\to\infty}\frac{9^{n+1}-4^{n+1}}{9^n+4^n}=\frac{\infty-\infty}{\infty}=\lim\limits_{n\to\infty}\frac{\frac{9^{n+1}-4^{n+1}}{9^n}}{\frac{9^n+4^n}{9^n}}=\lim\limits_{n\to\infty}\frac{9-4\cdot\left(\frac{4}{9}\right)^n}{1+\left(\frac{4}{9}\right)^n}=\frac{9}{1}=9$

17.3 $\lim\limits_{x\to-\infty}\frac{(x-1)^2(x+2)^3}{3x(x+3)^4}=\frac{\infty}{\infty}=\lim\limits_{x\to-\infty}\frac{\frac{(x-1)^2(x+2)^3}{x^5}}{\frac{3x(x+3)^4}{x^5}}=\frac{1}{3}\lim\limits_{x\to-\infty}\frac{\left(\frac{x-1}{x}\right)^2\left(\frac{x+2}{x}\right)^3}{\left(\frac{x+3}{x}\right)^4}=$

$=\frac{1}{3}\lim\limits_{x\to-\infty}\frac{\left(1-\frac{1}{x}\right)^2\left(1+\frac{2}{x}\right)^3}{\left(1+\frac{3}{x}\right)^4}=\frac{1}{3}\cdot 1=\frac{1}{3}$

17.4 $\lim\limits_{x\to+\infty}\frac{3x^3+2x-3}{4x^2-3x^2+8}=\lim\limits_{x\to+\infty}\frac{3x^3+2x-3}{x^2+8}=\frac{\infty}{\infty}=\lim\limits_{x\to+\infty}\frac{\frac{3x^3+2x-3}{x^3}}{\frac{x^2+8}{x^3}}=$

$=\lim\limits_{x\to+\infty}\frac{3+\frac{2}{x^2}-\frac{3}{x^3}}{\frac{1}{x}+\frac{8}{x^3}}=\frac{3}{0}=+\infty$

17.5 $\lim\limits_{x\to+\infty}\frac{5x^4-2x+8}{3x^6-2x+5}=\frac{\infty}{\infty}=\lim\limits_{x\to+\infty}\frac{\frac{5x^4-2x+8}{x^6}}{\frac{3x^6-2x+5}{x^6}}=\lim\limits_{x\to+\infty}\frac{\frac{5}{x^2}-\frac{2}{x^5}+\frac{8}{x^6}}{3-\frac{2}{x^5}+\frac{5}{x^6}}=\frac{0}{3}=0$

17.6 $\lim\limits_{x\to 2}\frac{x^2-3x+2}{2x^2-5x+2}=\frac{0}{0}=\lim\limits_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(2x-1)}=\frac{1}{4-1}=\frac{1}{3}$

17.7 $\lim\limits_{x\to 4}\left(\frac{8}{x^2-16}-\frac{1}{x-4}\right)=(\infty-\infty)=\lim\limits_{x\to 4}\left(\frac{8}{(x-4)(x+4)}-\frac{1}{(x-4)}\right)=\lim\limits_{x\to 4}\frac{8-(x+4)}{(x-4)(x+4)}=$

$=\lim\limits_{x\to 4}\frac{4-x}{(x-4)(x+4)}=-\lim\limits_{x\to 4}\frac{x-4}{(x-4)(x+4)}=-\lim\limits_{x\to 4}\frac{1}{x+4}=-\frac{1}{8}$

17.8 $\lim\limits_{x\to 5}\frac{\sqrt{5x}-x}{x-5}=\frac{0}{0}=\lim\limits_{x\to 5}\frac{(\sqrt{5x}-x)(\sqrt{5x}+x)}{(x-5)(\sqrt{5x}+x)}=\frac{1}{10}\lim\limits_{x\to 5}\frac{5x-x^2}{x-5}=$

$=\frac{1}{10}\lim\limits_{x\to 5}\frac{x(5-x)}{x-5}=-\frac{1}{10}\lim\limits_{x\to 5}\frac{x(x-5)}{x-5}=-\frac{1}{10}\cdot 5=-\frac{1}{2}$

17.9 $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+1}-\sqrt{x^2+2x}\right)=\infty-\infty=\lim\limits_{x\to+\infty}\frac{\left(\sqrt{x^2+1}-\sqrt{x^2+2x}\right)\left(\sqrt{x^2+1}+\sqrt{x^2+2x}\right)}{\sqrt{x^2+1}+\sqrt{x^2+2x}}=$

$=\lim\limits_{x\to+\infty}\frac{x^2+1-x^2-2x}{\sqrt{x^2+1}+\sqrt{x^2+2x}}=\lim\limits_{x\to+\infty}\frac{1-2x}{\sqrt{x^2+1}+\sqrt{x^2+2x}}=\frac{\infty}{\infty}=\lim\limits_{x\to+\infty}\frac{\frac{1-2x}{x}}{\frac{\sqrt{x^2+1}+\sqrt{x^2+2x}}{\sqrt{x^2}}}=$

$=\lim\limits_{x\to+\infty}\frac{\frac{1}{x}-2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{2}{x}}}=\frac{-2}{(1+1)}=-1$

17.10 $\lim_{x\to\frac{\pi}{8}} \frac{tg\, 2x}{\sin 4x} = \frac{tg\frac{\pi}{4}}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1$

17.11 $\lim_{x\to0} \frac{x^3}{tg^3\frac{x}{3}} = \frac{0}{0} = \lim_{x\to0} \frac{x^3}{(\frac{x}{3})^3} = \lim_{x\to0} \frac{x^3}{\frac{x^3}{27}} = \lim_{x\to0} \frac{x^3\cdot 27}{x^3} = 27$

17.12 $\lim_{x\to0}\frac{\cos x - \cos^3 x}{x-\sin 2x} = \frac{0}{0} = \lim_{x\to0} \frac{\underbrace{\cos x}_{\to1}(1-\cos^2 x)}{x - 2\sin x\underbrace{\cos x}_{\to1}} = \lim_{x\to0} \frac{\sin^2 x}{x-2\sin x} =$

$= \lim_{x\to0} \frac{\sin x}{\frac{x-2\sin x}{\sin x}} = \lim_{x\to0} \frac{\sin x}{\underbrace{\frac{x}{\sin x}}_{\to1}-2} = \frac{\sin 0}{1-2} = \frac{0}{-1} = 0$

17.13 $\lim_{x\to-\frac{\pi}{2}} \frac{ctg\, x}{\frac{\pi}{2}+x} = \frac{0}{0}$ // $]\, t = \frac{\pi}{2}+x \Rightarrow x = t-\frac{\pi}{2},\ t\to0$

$\lim_{t\to0}\frac{ctg(t-\frac{\pi}{2})}{t} = \lim_{t\to0} \frac{ctg(-(\frac{\pi}{2}-t))}{t} = -\lim_{t\to0} \frac{ctg(\frac{\pi}{2}-t)}{t} = -\lim_{t\to0} \frac{tg\, t}{t} =$

$= -\lim_{t\to0} \frac{\sin t}{\underbrace{\cos t}_{\to1}\cdot t} = -1$

17.14 $\lim_{x\to0}\frac{\arcsin x}{5x} = \frac{0}{0}$ // $]\, x=\sin t \Rightarrow t=\arcsin x,\ t\to0$

$\lim_{t\to0} \frac{t}{5\underbrace{\sin t}_{\to1}} = \frac{1}{5}$

17.15 $\lim_{x\to+\infty}(1-\frac{2}{x})^x = 1^\infty = \lim_{x\to+\infty} \Big(\underbrace{(1+\frac{1}{-\frac{x}{2}})^{-\frac{x}{2}}}_{\to e}\Big)^{-\frac{2}{x}\cdot x} = \lim_{x\to+\infty} e^{-\frac{2x}{x}} = e^{-2}$

17.16 $\lim_{x\to\infty}\left(\frac{x+3}{x+2}\right)^{2x+1} = \lim_{x\to\infty} \left(\frac{x+2+1}{x+2}\right)^{2x+1} = \lim_{x\to\infty}\left(1+\frac{1}{x+2}\right)^{2x+1} = 1^\infty =$

$= \lim_{x\to\infty} \Big(\underbrace{(1+\frac{1}{x+2})^{x+2}}_{\to e}\Big)^{\frac{2x+1}{x+2}} = e^{\lim_{x\to\infty}\frac{2x+1}{x+2}} = e^{\frac{\infty}{\infty}} =$

$= e^{\lim_{x\to\infty} \frac{\frac{2x+1}{x}}{\frac{x+2}{x}}} = e^{\lim_{x\to\infty} \frac{2+\frac{1}{x}}{1+\frac{2}{x}}} = e^{2}$

17.18 $\lim_{x\to0}(\cos x)^{\frac{1}{x^2}} = 1^\infty = \lim_{x\to0}(1+(\cos x-1))^{\frac{1}{x^2}} = \lim_{x\to0}\left((1+(\cos x-1))^{\frac{1}{\cos x-1}}\right)^{\frac{\cos x-1}{x^2}}$

$= e^{\lim_{x\to0} \frac{\cos x-1}{x^2}} = e^{-\lim_{x\to0}\frac{1-\cos x}{x^2}} = e^{-\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{x^2}} = e^{-\lim_{x\to0}\frac{2(\frac{x}{2})^2}{x^2}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

17.19 $\lim_{x\to1-0} 5^{\frac{1}{1-x}} = 5^{\frac{1}{1-(1-0)}} = 5^{\frac{1}{1-1+0}} = 5^{\frac{1}{+0}} = 5^{+\infty} = +\infty$