

1.124

$a_1 \perp a_2$ $a_1 \perp a_3$ $a_2 \perp a_3$ ~~или~~ a_1, a_2, a_3 - попарно перпендикулярны $|a_1|=4$ $|a_2|=2$ $|a_3|=3$
 $(a_1, a_2, a_3) = ?$

$$(a_1, a_2, a_3) = [a_1, a_2] a_3$$

$$[a_1, a_2] = |a_1| \cdot |a_2| \cdot \sin(\widehat{a_1, a_2}) = 4 \cdot 2 \cdot \sin \frac{\pi}{2} = 8 \quad / \quad [a_1, a_2] \perp a_1, [a_1, a_2] \perp a_2 \Rightarrow [a_1, a_2] \perp a_3$$

$$[a_1, a_2] a_3 = |[a_1, a_2]| \cdot |a_3| \cdot \cos(\widehat{[a_1, a_2], a_3}) = 8 \cdot 3 \cdot \cos 0 = 24$$

1.125

a, b, c - попарно перпендикулярны $|a|=1$ $|b|=2$ $|c|=3$ $(a, b) = 30^\circ$ $c \perp a$ $c \perp b$ $(a, b, c) = ?$

$$(a, b, c) = -[a, b] c$$

$$[a, b] = |a| \cdot |b| \cdot \sin(\widehat{a, b}) = 1 \cdot 2 \cdot \sin \frac{\pi}{6} = 1$$

$$(a, b, c) = -([a, b] \cdot |c| \cdot \cos(\widehat{[a, b], c})) = -1 \cdot 3 = -3$$

1.135

a) $a = -2i + j + k$ $b = i - 2j + 3k$ $c = 14i - 13j + 7k$

$(a, b, c) = 0$ - ?

$$(a, b, c) = \begin{vmatrix} -2 & 1 & 14 \\ 1 & -2 & -13 \\ 1 & 3 & 7 \end{vmatrix} = -2 \begin{vmatrix} -2 & -13 \\ 3 & 7 \end{vmatrix} - 1 \begin{vmatrix} 1 & -13 \\ 1 & 7 \end{vmatrix} + 14 \begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix} = -2 \cdot 25 - 1 \cdot 20 + 14 \cdot 5 = -50 - 20 + 70 = 0$$

a, b, c - kollinear

b) $a = 2i + j - 3k$ $b = 3i - 2j + 2k$ $c = i - 4j + k$

$(a, b, c) = 0$ - ?

$$(a, b, c) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & -4 \\ -3 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} -2 & -4 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & -4 \\ -3 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -2 \\ -3 & 2 \end{vmatrix} = 2 \cdot 6 - 3 \cdot (-11) + 1 \cdot (-4) \neq 0$$

a, b, c - nie kollinear

1.136

a) $a = \{\lambda, 3, 1\}$ $b = \{5, -1, 2\}$ $c = \{-1, 5, 4\}$

$(a, b, c) = 0$: λ - ?

$$(a, b, c) = \begin{vmatrix} \lambda & 5 & -1 \\ 3 & -1 & 5 \\ 1 & 2 & 4 \end{vmatrix} = \lambda \begin{vmatrix} -1 & 5 \\ 2 & 4 \end{vmatrix} - 5 \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} - 1 \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = \lambda \cdot (-14) - 5 \cdot 7 - 1 \cdot 7 = -14\lambda - 42 \Rightarrow$$

$$\Rightarrow -14\lambda - 42 = 0$$

$$-14\lambda = 42 \Rightarrow \lambda = -3$$

b) $a = \{1, 2, \lambda, 1\}$ $b = \{1, \lambda, 0\}$ $c = \{0, \lambda, 1\}$

$(a, b, c) = 0$: λ - ?

$$(a, b, c) = \begin{vmatrix} 1 & 1 & 0 \\ 2 & \lambda & \lambda \\ 1 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} \lambda & \lambda \\ 0 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & \lambda \\ 1 & 1 \end{vmatrix} = \lambda - 1 \cdot \lambda = 0 \Rightarrow \lambda - \text{mod } 2 \text{ equals}$$