2.187

$3x - 5y = 13$
$2x + 7y = 81$ $\longmapsto \begin{vmatrix} 3 & -5 \\ 2 & 7 \end{vmatrix}$

$\det A = 21 + 10 = 31$

$M = \begin{pmatrix} * & * \\ * & * \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -5 & 3 \end{pmatrix}$ $\quad A_* = \begin{vmatrix} 7 & -2 \\ 5 & 3 \end{vmatrix}$ $\quad A_*^T = \begin{vmatrix} 7 & 5 \\ -2 & 3 \end{vmatrix}$ $\quad A^{-1} = \frac{1}{31}\begin{vmatrix} 7 & 5 \\ -2 & 3 \end{vmatrix}$

$X = A^{-1} \cdot B = \frac{1}{31}\begin{vmatrix} 7 & 5 \\ -2 & 3 \end{vmatrix} \begin{pmatrix} 13 \\ 81 \end{pmatrix} = \frac{1}{31}\begin{vmatrix} 7 \cdot 13 + 5 \cdot 81 \\ -2 \cdot 13 + 3 \cdot 81 \end{vmatrix} = \frac{1}{31}\begin{pmatrix} 496 \\ 217 \end{pmatrix} = \begin{pmatrix} 16 \\ 7 \end{pmatrix} \Rightarrow x_1 = 16,\ x_2 = 7$

2.190

$7x + 2y + 3z = 15$
$5x - 3y + 2z = 15$
$10x - 11y + 5z = 36$ $\longrightarrow \begin{vmatrix} 7 & 2 & 3 \\ 5 & -3 & 2 \\ 10 & -11 & 5 \end{vmatrix}$

$\det A = 7\begin{vmatrix} -3 & 2 \\ -11 & 5 \end{vmatrix} - 2\begin{vmatrix} 5 & 2 \\ 10 & 5 \end{vmatrix} + 3\begin{vmatrix} 5 & -3 \\ 10 & -11 \end{vmatrix} = 7(-15+22) - 2(25-20) + 3(-35+30) = 49 - 10 - 75 = -36$

$M = \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$ $\quad M^{(1,1)} = \begin{vmatrix} -3 & 2 \\ -11 & 5 \end{vmatrix} = 7$ $\quad M^{(1,2)} = \begin{vmatrix} 5 & 2 \\ 10 & 5 \end{vmatrix} = 5$ $\quad M^{(1,3)} = \begin{vmatrix} 5 & -3 \\ 10 & -11 \end{vmatrix} = -25$

$M^{(2,1)} = \begin{vmatrix} 2 & 3 \\ -11 & 5 \end{vmatrix} = 43$ $\quad M^{(2,2)} = \begin{vmatrix} 7 & 3 \\ 10 & 5 \end{vmatrix} = 5$ $\quad M^{(2,3)} = \begin{vmatrix} 7 & 2 \\ 10 & -11 \end{vmatrix} = -97$

$M^{(3,1)} = \begin{vmatrix} 2 & 3 \\ -3 & 2 \end{vmatrix} = 13$ $\quad M^{(3,2)} = \begin{vmatrix} 7 & 3 \\ 5 & 2 \end{vmatrix} = -1$ $\quad M^{(3,3)} = \begin{vmatrix} 7 & 2 \\ 5 & -3 \end{vmatrix} = -31$ $\quad M = \begin{pmatrix} 7 & 5 & -25 \\ 43 & 5 & -97 \\ 13 & -1 & -31 \end{pmatrix}$

$A_* = \begin{pmatrix} 7 & -5 & -25 \\ -43 & 5 & 97 \\ 13 & 1 & -31 \end{pmatrix}$ $\quad A_*^T = \begin{pmatrix} 7 & -43 & 13 \\ -5 & 5 & 1 \\ -25 & 97 & -31 \end{pmatrix}$ $\quad A^{-1} = -\frac{1}{36}\begin{pmatrix} 7 & -43 & 13 \\ -5 & 5 & 1 \\ -25 & 97 & -31 \end{pmatrix}$

$X = A^{-1} \cdot B = -\frac{1}{36}\begin{pmatrix} 7 & -43 & 13 \\ -5 & 5 & 1 \\ -25 & 97 & -31 \end{pmatrix}\begin{pmatrix} 15 \\ 15 \\ 36 \end{pmatrix} = -\frac{1}{36}\begin{pmatrix} 7 \cdot 15 - 43 \cdot 15 + 13 \cdot 36 \\ -5 \cdot 15 + 5 \cdot 15 + 36 \\ -25 \cdot 15 + 97 \cdot 15 - 31 \cdot 36 \end{pmatrix} = \frac{1}{36}\begin{pmatrix} -72 \\ 36 \\ -36 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$x_1 = 2$
$x_2 = -1$
$x_3 = 1$

2.199

$$\left.\begin{aligned} 2x_1 - 3x_2 + x_3 &= -7 \\ x_1 + 4x_2 + 2x_3 &= -1 \\ x_1 - 4x_2 &= -5 \end{aligned}\right| \rightarrow \begin{pmatrix} 2 & -3 & 1 \\ 1 & 4 & 2 \\ 1 & -4 & 0 \end{pmatrix}$$

$\det A = 2\begin{vmatrix} 4 & 2 \\ -4 & 0 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 1 & -4 \end{vmatrix} = 16 - 6 - 8 = 2$

$M = \begin{pmatrix} * & * & x \\ * & * & x \\ * & * & * \end{pmatrix}$ $M^{(1,2)} = 8$ $M^{(2,2)} = -2$ $M^{(1,3)} = -8$ $M^{(2,1)} = \begin{vmatrix} -3 & 1 \\ -4 & 0 \end{vmatrix} = 4$

$M^{(2,2)} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = -1$ $M^{(2,3)} = \begin{vmatrix} 2 & -3 \\ 1 & -4 \end{vmatrix} = -5$ $M^{(3,1)} = \begin{vmatrix} -3 & 1 \\ -4 & 0 \end{vmatrix} = 4$ $M^{(3,2)} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$

$M^{(3,3)} = \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} = 11$ $M = \begin{pmatrix} 8 & -2 & -8 \\ 4 & -1 & -5 \\ 4 & 3 & 11 \end{pmatrix}$ $A_* = \begin{pmatrix} 8 & 2 & -8 \\ -4 & -1 & 5 \\ 4 & -3 & 11 \end{pmatrix}$ $A_*^T = \begin{pmatrix} 8 & -4 & 4 \\ 2 & -1 & -3 \\ -8 & 5 & 11 \end{pmatrix}$

$A^{-1} = \frac{1}{2}\begin{pmatrix} 8 & -4 & 4 \\ 2 & -1 & -3 \\ -8 & 5 & 11 \end{pmatrix}$ $X = A^{-1} \cdot B = \frac{1}{2}\begin{pmatrix} 8 & -4 & 4 \\ 2 & -1 & -3 \\ -8 & 5 & 11 \end{pmatrix}\begin{pmatrix} -7 \\ -1 \\ -5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -56+4-20 \\ -14+1+15 \\ 56-5-55 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -32 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -16 \\ 1 \\ -2 \end{pmatrix}$

2.204

$x - \sqrt{3}y = 1$

$\sqrt{3}x - 3y = \sqrt{3}$

$\left(\begin{array}{cc|c} 1 & -\sqrt{3} & 1 \\ \sqrt{3} & -3 & \sqrt{3} \end{array}\right)$ $\text{rang } A = \text{rang } A|B$

2.209

$x_1 + x_2 - 6x_3 - 4x_4 = 6$

$3x_1 - x_2 - 6x_3 - 4x_4 = 2$

$2x_1 + 3x_2 + 9x_3 + 2x_4 = 6$

$3x_1 + 2x_2 + 3x_3 + 8x_4 = -7$

$$\left(\begin{array}{cccc|c} 1 & 1 & -6 & -4 & 6 \\ 3 & -1 & -6 & -4 & 2 \\ 2 & 3 & 9 & 2 & 6 \\ 3 & 2 & 3 & 8 & 7 \end{array}\right) \quad \text{rang } A = 4$$

$M_2 = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix}$ x_1, x_2 - базисные x_3, x_4 - свободные

$$\begin{cases} x_1 + x_2 = 6 + 6x_3 + 4x_4 \\ 3x_1 - x_2 = 2 + 6x_3 + 4x_4 \end{cases} \Big| \rightarrow x_3 = C_1, x_4 = C_2$$

$$\begin{array}{l} x_1 = 2 + 3x_3 + 2x_4 \\ x_2 = 4 + 3x_3 + 2x_4 \end{array} \Big| \rightarrow \begin{cases} x_1 = 2 + 3C_1 + 2C_2 \\ x_2 = 4 + 3C_1 + 2C_2 \end{cases}$$

$$X_{[C_1, C_2]} = \begin{pmatrix} 2 + 3C_1 + 2C_2 \\ 4 + 3C_1 + 2C_2 \\ C_1 \\ C_2 \end{pmatrix}$$

2.223

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 9x_2 - 3x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 = x_3 \\ 2x_1 + 9x_2 = 3x_3 \end{cases}$$

$$\begin{cases} x_1 = x_3 - 2x_2 \\ 2x_3 - 4x_2 + 9x_2 = 3x_3 \end{cases}$$

$$\begin{cases} x_1 = x_3 - 2x_2 \\ x_2 = \frac{1}{5} x_3 \end{cases}$$

$$\begin{cases} x_1 = \frac{4}{5} x_3 \\ x_2 = \frac{1}{5} x_3 \end{cases} \Big| \rightarrow \begin{cases} x_1 = \frac{4}{5} C_1 \\ x_2 = \frac{1}{5} C_1 \end{cases} \qquad X_{(C_1)} = \begin{cases} \frac{4}{5} C_1 \\ \frac{1}{5} C_1 \\ C_1 \end{cases}$$

2.231

$$\begin{cases} 5x_1+6x_2-2x_3+7x_4+4x_5=0 \\ 2x_1+3x_2-x_3+4x_4+2x_5=0 \\ 7x_1+9x_2-3x_3+5x_4+6x_5=0 \\ 5x_1+9x_2-3x_3+x_4+6x_5=0 \end{cases} \qquad \text{rang } A = 2$$

$M = \begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}$ x_1, x_2 - баз. $x_3, 4$ - свободные

$$\begin{cases} 5x_1+6x_2 = 2x_3-7x_4-4x_5 \\ 2x_1+3x_2 = x_3-4x_4+2x_5 \end{cases}$$

$$\begin{cases} x_1 = \frac{2}{5}x_3 - \frac{7}{5}x_4 - \frac{4}{5}x_5 - \frac{6}{5}x_2 \\ \frac{4}{5}x_3 - \frac{14}{5}x_4 - \frac{8}{5}x_5 - \frac{12}{5}x_2 + 3x_2 = x_3 - 4x_4 + 2x_5 \end{cases}$$

$$\begin{cases} x_1 = \frac{2}{5}x_3 - \frac{7}{5}x_4 - \frac{4}{5}x_5 - \frac{6}{5}x_2 \\ \frac{3}{5}x_2 = \frac{1}{5}x_3 + \frac{34}{5}x_4 + \frac{18}{5}x_5 \end{cases}$$

$$\begin{cases} 5x_1 + 2x_3 + 68x_4 + 36x_5 = 0 \\ x_2 = \frac{1}{3}x_3 + \frac{34}{3}x_4 + \frac{18}{3}x_5 \end{cases}$$

$$\begin{cases} x_1 = \frac{2}{5}x_3 - \frac{7}{5}x_4 - \frac{4}{5}x_5 - \frac{185}{18}x_3 - \frac{170}{18}x_4 - 10x_5 \\ x_2 = \frac{1}{3}x_3 + \frac{34}{3}x_4 + \frac{18}{3}x_5 \end{cases}$$

$$\begin{cases} x_1 = \frac{11}{90}x_3 - \frac{488}{45}x_4 - \frac{29}{5}x_5 \\ x_2 = \frac{1}{3}x_3 + \frac{34}{3}x_4 + \frac{18}{3}x_5 \end{cases}$$

$$X(c_1, c_2, c_3) = \begin{pmatrix} \frac{11}{90}c_1 - \frac{488}{45}c_2 - \frac{29}{5}c_3 \\ \frac{1}{3}c_1 + \frac{34}{3}c_2 + \frac{18}{3}c_3 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

2.236

$$\begin{cases} 2x_1 + x_2 - x_3 - x_4 + x_5 = 1 \\ x_1 - x_2 + x_3 + x_4 - 2x_5 = 0 \\ 3x_1 + 3x_2 - 3x_3 - 3x_4 + 4x_5 = 2 \\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 = 3 \end{cases}$$

rang $A = 2$

$M = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$ x_1, x_2 - bază. x_3, x_4 - bază.

$$\begin{cases} 2x_1 + x_2 = 1 + x_3 + x_4 - x_5 \\ x_1 - x_2 = -x_3 - x_4 + 2x_5 \end{cases}$$

$$\begin{cases} x_1 = x_2 - x_3 - x_4 + 2x_5 \\ 2x_2 - 2x_3 - 2x_4 + 4x_5 + x_2 = 1 + x_3 + x_4 - x_5 \end{cases}$$

$$\begin{cases} x_1 = x_2 - x_3 - x_4 + 2x_5 \\ 3x_2 = 1 + x_3 + x_4 - x_5 + 2x_3 + 2x_4 - 4x_5 \end{cases}$$

$$\begin{cases} x_1 = x_2 - x_3 - x_4 + 2x_5 \\ 3x_2 = 1 + 3x_3 + 3x_4 - 5x_5 \end{cases}$$

$$\begin{cases} x_1 = x_2 - x_3 - x_4 + 2x_5 \\ x_2 = \frac{1}{3} + x_3 + x_4 - \frac{5}{3}x_5 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{3} + \frac{1}{3}x_5 \\ x_2 = \frac{1}{3} + x_3 + x_4 - \frac{5}{3}x_5 \end{cases}$$

$$X(c_1, c_2, c_3) = \begin{pmatrix} \frac{1}{3} + \frac{1}{3}c_3 \\ \frac{1}{3} + c_1 + c_2 - \frac{5}{3}c_3 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

2.240

$$\begin{cases} x_1+2x_2+3x_3+4x_4=0 \\ 7x_1+14x_2+20x_3+27x_4=0 \\ 5x_1+10x_2+16x_3+19x_4=-2 \\ 3x_1+5x_2+6x_3+13x_4=5 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1&2&3&4&0\\ 7&14&20&27&0\\ 5&10&16&19&-2\\ 3&5&6&13&5 \end{array}\right) = \left(\begin{array}{cccc|c} 1&2&3&4&0\\ 0&0&-1&-1&0\\ 0&0&1&-1&-2\\ 0&-1&-3&1&5 \end{array}\right) = \left(\begin{array}{cccc|c} 1&2&3&4&0\\ 0&-1&-3&1&5\\ 0&0&1&-1&-2\\ 0&0&-1&-1&0 \end{array}\right) = \left(\begin{array}{cccc|c} 1&2&3&4&0\\ 0&1&3&-1&-5\\ 0&0&1&-1&-2\\ 0&0&-1&-1&0 \end{array}\right) =$$

$$= \left(\begin{array}{cccc|c} 1&0&-3&6&10\\ 0&1&3&-1&-5\\ 0&0&1&-1&-2\\ 0&0&-1&-1&0 \end{array}\right) = \left(\begin{array}{cccc|c} 1&0&0&3&4\\ 0&1&0&2&1\\ 0&0&1&-1&-2\\ 0&0&0&-2&-2 \end{array}\right) = \left(\begin{array}{cccc|c} 1&0&0&3&4\\ 0&1&0&2&1\\ 0&0&1&-1&-2\\ 0&0&0&1&1 \end{array}\right) = \left(\begin{array}{cccc|c} 1&0&0&0&1\\ 0&1&0&0&-1\\ 0&0&1&0&-1\\ 0&0&0&1&1 \end{array}\right)$$

$x_1=1,\ x_2=-1,\ x_3=-1,\ x_4=1$

2.243

$$\begin{cases} 8x_1+12x_2=20 \\ 14x_1+21x_2=35 \\ 9x_3+11x_4=0 \\ 16x_3+20x_4=0 \\ 10x_5+12x_6=22 \\ 15x_5+18x_6=33 \end{cases}$$

$$\left(\begin{array}{cccccc|c} 8&12&0&0&0&0&20\\ 14&21&0&0&0&0&35\\ 0&0&9&11&0&0&0\\ 0&0&16&20&0&0&0\\ 0&0&0&0&10&12&22\\ 0&0&0&0&15&18&33 \end{array}\right) = \left(\begin{array}{cccccc|c} 1&\frac{3}{2}&0&0&0&0&\frac{5}{2}\\ 0&0&1&\frac{11}{9}&0&0&0\\ 0&0&0&\frac{4}{9}&0&0&0\\ 0&0&0&0&0&0&0\\ 0&0&0&0&10&12&22\\ 0&0&0&0&15&18&33 \end{array}\right) =$$

$$\left(\begin{array}{cccccc|c} 1&\frac{3}{2}&0&0&0&0&\frac{5}{2}\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&\frac{6}{5}&\frac{11}{5}\\ 0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0 \end{array}\right)$$

$$\begin{cases} x_1+\frac{3}{2}x_2=\frac{5}{2} \\ x_3=0 \\ x_4=0 \\ x_5+\frac{6}{5}x_6=\frac{11}{5} \end{cases}$$

2.244

$$\begin{cases} 7x_1 - 5x_2 - 2x_3 - 4x_4 = 8 \\ -3x_1 + 2x_2 + x_3 + 2x_4 = -3 \\ 2x_1 - x_2 - x_3 - 2x_4 = 1 \\ -x_1 + x_3 + 24x_4 = 1 \\ -x_2 + x_3 + 2x_4 = 3 \end{cases}$$

$$\left(\begin{array}{cccc|c} 7 & -5 & -2 & -4 & 8 \\ -3 & 2 & 1 & 2 & -3 \\ 2 & -1 & -1 & -2 & 1 \\ -1 & 0 & 1 & 24 & 1 \\ 0 & -1 & 1 & 2 & 3 \end{array}\right) =$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & -6 & -1 \\ 0 & 1 & -1 & -8 & -3 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 18 & 0 \\ 0 & 0 & 0 & -6 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 - x_3 = -1 \\ x_2 - x_3 = -3 \\ x_4 = 0 \end{cases}$$