2.1

$$\begin{vmatrix} -1 & 4 \\ 5 & 2 \end{vmatrix} = -1\cdot 2 - 5\cdot 4 = -22$$

2.2

$$\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix} = (a+b)^2 - (a-b)^2 = (a+b-a+b)(a+b+a-b) = 2b\cdot 2a = 4ab$$

2.3

$$\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -\cos^2\alpha + \sin^2\alpha = -\cos 2\alpha$$

2.4

$$\begin{vmatrix} a+bi & b \\ 2a & a-bi \end{vmatrix} = (a+bi)(a-bi) - 2ab = (a^2 - b^2 \underbrace{i^2}_{=-1}) - 2ab = a^2 + b^2 - 2ab = (a-b)^2$$

2.5

$$\begin{vmatrix} \cos\alpha + i\cdot\sin\alpha & 1 \\ 1 & \cos\alpha - i\sin\alpha \end{vmatrix} = (\cos\alpha + i\cdot\sin\alpha)(\cos\alpha - i\sin\alpha) - 1 = (\cos^2\alpha - i^2\cdot\sin^2\alpha) - 1 = \cos^2\alpha + \sin^2\alpha - 1 = 1 - 1 = 0$$

2.6

$$\begin{vmatrix} 2\sin\varphi\cos\varphi & 2\sin^2\varphi - 1 \\ 2\cos^2\varphi - 1 & 2\sin\varphi\cos\varphi \end{vmatrix} = \begin{vmatrix} \sin 2\varphi & -\cos 2\varphi \\ \cos 2\varphi & \sin 2\varphi \end{vmatrix} = \sin^2 2\varphi + \cos^2 2\varphi = 1$$

$\boxed{2.7}$

$$\begin{vmatrix} \frac{1-t^2}{1+t^2} & \frac{2t}{1+t^2} \\ -\frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{vmatrix} = \frac{(1-t^2)^2}{(1+t^2)^2} + \frac{4t^2}{(1+t^2)} = \frac{1-2t^2+t^4+4t^2}{(1+t^2)^2} = \frac{(1+t^2)^2}{(1+t^2)^2} = 1$$

$\boxed{2.8}$

$$\begin{vmatrix} x & x+1 \\ -4 & x+1 \end{vmatrix} = 0 \qquad \det|A| = x^2+x+4x+4 = x^2+5x+4=0$$

$$D = 25-16=9$$

$$\left[\begin{array}{l} x_1 = \frac{-5+3}{2} = -1 \\ x_2 = \frac{-5-3}{2} = -4 \end{array}\right.$$

Ответ: $-1; 4$

$\boxed{2.9}$

$$\begin{vmatrix} \cos 8x & -\sin 5x \\ \sin 8x & \cos 5x \end{vmatrix} = \cos 8x \cdot \cos 5x + \sin 8x \cdot \sin 5x = 0$$

$$\cos 3x = 0 \qquad 3x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{6}$$

$$3x = \frac{\pi}{2} + 2\pi k,\ k \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow x = \frac{\pi}{6} + \frac{2}{3}\pi k,\ k \in \mathbb{Z}$$

$\boxed{2.12}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 45-48-2\cdot(36-42)+3\cdot(32-35) =$$

$$= -3+12-9=0$$

$\boxed{2.13}$

$$\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix} = 3\begin{vmatrix} 7 & -2 \\ -1 & 8 \end{vmatrix} - 4\begin{vmatrix} 8 & -2 \\ 2 & 8 \end{vmatrix} - 5\begin{vmatrix} 8 & 7 \\ 2 & -1 \end{vmatrix} = 3\cdot(56-2)-4(64+4)-5(-8-14) =$$

$$= 3\cdot 54 - 4\cdot 68 + 5\cdot 22 = 162-272+110 = 0$$

$\boxed{2.14}$

$$\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix} = a+x \begin{vmatrix} b+x & x \\ x & c+x \end{vmatrix} - x \begin{vmatrix} x & x \\ x & c+x \end{vmatrix} + x \begin{vmatrix} x & b+x \\ x & x \end{vmatrix} = (a+x)((b+x)(c+x)-x^2)-$$

$$-x(x\cdot c+x^2-x^2)+x(x^2-bx-x^2) = (a+x)(bc+bx+xc+x^2-x^2)-$$

$$-x^2c-x^2b = abc+abx+acx+xbc+\cancel{bx^2}+\cancel{x^2c}-\cancel{x^2c}-\cancel{x^2b} =$$

$$= abc+abx+acx+xbc$$

$\boxed{2.15}$

$$\begin{vmatrix} \alpha^2+1 & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2+1 & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2+1 \end{vmatrix} = \alpha^2+1 \begin{vmatrix} \beta^2+1 & \beta\gamma \\ \beta\gamma & \gamma^2+1 \end{vmatrix} - \alpha\beta \begin{vmatrix} \alpha\beta & \beta\gamma \\ \alpha\gamma & \gamma^2+1 \end{vmatrix} + \alpha\gamma \begin{vmatrix} \alpha\beta & \beta^2+1 \\ \alpha\gamma & \beta\gamma \end{vmatrix} =$$

$$=(\alpha^2+1)(\cancel{\beta^2\gamma^2}+\beta^2+\gamma^2+1-\cancel{\beta^2\gamma^2})-\alpha\beta(\cancel{\alpha\beta\gamma^2}+\alpha\beta-\cancel{\alpha\beta\gamma^2})+$$

$$+\alpha\gamma(\cancel{\alpha\beta^2\gamma}-\cancel{\alpha\gamma\beta^2}-\alpha\gamma) = \cancel{\alpha^2\beta^2}+\cancel{\alpha^2\gamma^2}+\alpha^2+\beta^2+\gamma^2+1-\cancel{\alpha^2\beta^2}-\cancel{\alpha^2\gamma^2} = \alpha^2+\beta^2+\gamma^2+1$$

$\boxed{2.16}$

$$\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix} = \sin\alpha \begin{vmatrix} \cos\beta & 1 \\ \cos\gamma & 1 \end{vmatrix} - \cos\alpha \begin{vmatrix} \sin\beta & 1 \\ \sin\gamma & 1 \end{vmatrix} + 1 \begin{vmatrix} \sin\beta & \cos\beta \\ \sin\gamma & \cos\gamma \end{vmatrix} =$$

$$= \sin\alpha(\cos\beta-\cos\gamma)-\cos\alpha(\sin\beta-\sin\gamma)+\sin\beta\cos\gamma-\sin\gamma\cos\beta =$$

$$= \underline{\sin\alpha\cos\beta} - \underline{\underline{\sin\alpha\cos\gamma}} - \underline{\sin\beta\cos\alpha} + \underline{\underline{\sin\gamma\cos\alpha}} + \sin\beta\cos\gamma - \sin\gamma\cos\beta =$$

$$= (\sin\alpha\cos\beta-\sin\beta\cos\alpha)+(\sin\gamma\cos\alpha-\sin\alpha\cos\gamma)+(\sin\beta\cos\gamma-\sin\gamma\cos\beta) =$$

$$= \sin(\alpha-\beta)+\sin(\gamma-\alpha)+\sin(\beta-\gamma)$$

$\boxed{2.17}$ $\quad \varepsilon = \cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}$

$$\begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & \varepsilon^2 \\ \varepsilon & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & \varepsilon^2 \\ \varepsilon^2 & 1 \end{vmatrix} + \varepsilon\begin{vmatrix} 1 & 1 \\ \varepsilon^2 & \varepsilon \end{vmatrix} = 1 - \varepsilon^3 - 1 + \varepsilon^4 + \varepsilon(\varepsilon - \varepsilon^2) =$$

$$= 1 - \varepsilon^3 - 1 + \varepsilon^4 + \varepsilon^2 - \varepsilon^3 = \varepsilon^4 + \varepsilon^2 - 2\varepsilon^3$$

$$\left(\cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}\right)^4 + \left(\cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}\right)^2 - 2\cdot\left(\cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}\right)^3$$

$\boxed{2.18}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \end{vmatrix} = 1\begin{vmatrix} \beta & \beta^2 \\ \beta^2 & \beta \end{vmatrix} - 1\begin{vmatrix} 1 & \beta^2 \\ 1 & \beta \end{vmatrix} + 1\begin{vmatrix} 1 & \beta \\ 1 & \beta^2 \end{vmatrix} = \beta^2 - \beta^4 - \beta + \beta^2 + \beta^2 - \beta =$$

$$= 3\beta^2 - 2\beta - \beta^4$$

$\boxed{2.19}$

$$\begin{vmatrix} 3 & x & -x \\ 2 & -1 & 3 \\ x+10 & 1 & 1 \end{vmatrix} = 0 = 3\begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} - x\begin{vmatrix} 2 & 3 \\ x+10 & 1 \end{vmatrix} - x\begin{vmatrix} 2 & -1 \\ x+10 & 1 \end{vmatrix} = 3\cdot(-1-3) - x(2-3x-30) -$$

$$- x(2+x+10) = -12 + 28x + 3x^2 - 12x - x^2 =$$

$$= 2x^2 + 16x - 12 = x^2 + 8x - 6$$

$$D = 64 + 36 = 100$$

$$\left[\begin{array}{l} x_1 = \dfrac{-8+10}{2} = 1 \\ x_2 = \dfrac{-8-10}{2} = -9 \end{array}\right.$$

Ответ: 1; -9

$\boxed{2.20}$

$$\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = 0 = x\begin{vmatrix} x+4 & x+5 \\ x+7 & x+8 \end{vmatrix} - (x+1)\begin{vmatrix} x+3 & x+5 \\ x+6 & x+8 \end{vmatrix} + (x+2)\begin{vmatrix} x+3 & x+4 \\ x+6 & x+7 \end{vmatrix} =$$

$$= x((x+4)(x+8)-(x+5)(x+7)) - (x+1)((x+3)(x+8)-(x+5)(x+6)) +$$

$$+ (x+2)((x+3)(x+7)-(x+4)(x+6)) = x(\cancel{x^2}+\cancel{8x}+\cancel{4x}+32-\cancel{x^2}-\cancel{7x}-\cancel{5x}-35) -$$

$$-(x+1)(\cancel{x^2}+\cancel{8x}+\cancel{3x}+24-\cancel{x^2}-\cancel{6x}-\cancel{5x}-30)+(x+2)(\cancel{x^2}+\cancel{7x}+\cancel{3x}+21-\cancel{x^2}-\cancel{6x}-\cancel{4x}-24) =$$

$$= x(-3)-(x+1)(-6)+(x+2)(-3) = -3x+6x+6-3x-6 = 0$$

x- любое число

$\boxed{2.21}$

$$\begin{vmatrix} 3 & -2 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{vmatrix} < 0 = 3\begin{vmatrix} x & -2 \\ 2 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & x \\ -1 & 2 \end{vmatrix} = 3(-x+4)+2(-1-2)+$$

$$+1(2+x) = -3x+12-6+2+x = -2x+8$$

$$-2x+8<0$$

$$-2x<-8$$

$$x>4$$

Ответ: $(-4;+\infty)$

$\boxed{2.22}$

$$\begin{vmatrix} 2 & x+2 & -1 \\ 1 & 1 & -2 \\ 5 & -3 & x \end{vmatrix} > 0 = 2\begin{vmatrix} 1 & -2 \\ -3 & x \end{vmatrix} - (x+2)\begin{vmatrix} 1 & -2 \\ 5 & x \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 5 & -3 \end{vmatrix} =$$

$$= 2(x-6)-(x+2)(x+10)-1(-3-5) =$$

$$= 2x-12-(x^2+10x+2x+20)+8 = \cancel{2x}-12-x^2-10x-\cancel{2x}-20+8>0$$

$$-x^2-10x-24>0$$

$$D = 100-96 = 4 \qquad \left[\begin{array}{l} x_1 = \frac{10+2}{-2} = -6 \\ x_2 = \frac{10-2}{-2} = -4 \end{array}\right.$$

Ответ: $(-6;-4)$

2.24

$$\begin{vmatrix} a_1+b_1x & a_1-b_1x & c_1 \\ a_2+b_2x & a_2-b_2x & c_2 \\ a_3+b_3x & a_3-b_3x & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_1-b_1x & c_1 \\ a_2 & a_2-b_2x & c_2 \\ a_3 & a_3-b_3x & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1-b_1x & c_1 \\ b_2x & a_2-b_2x & c_2 \\ b_3x & a_3-b_3x & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & a_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & b_1x & c_1 \\ a_2 & b_2x & c_2 \\ a_3 & b_3x & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1 & c_1 \\ b_2x & a_2 & c_2 \\ b_3x & a_3 & c_3 \end{vmatrix} - \begin{vmatrix} b_1x & b_1x & c_1 \\ b_2x & b_2x & c_2 \\ b_3x & b_3x & c_3 \end{vmatrix} =$$

$$= \begin{vmatrix} a_1 & b_1x & c_1 \\ a_2 & b_2x & c_2 \\ a_3 & b_3x & c_3 \end{vmatrix} + \begin{vmatrix} b_1x & a_1 & c_1 \\ b_2x & a_2 & c_2 \\ b_3x & a_3 & c_3 \end{vmatrix} = - \begin{vmatrix} a_1 & b_1x & c_1 \\ a_2 & b_2x & c_2 \\ a_3 & b_3x & c_3 \end{vmatrix} - \begin{vmatrix} a_1 & b_1x & c_1 \\ a_2 & b_2x & c_2 \\ a_3 & b_3x & c_3 \end{vmatrix} =$$

$$= -2 \begin{vmatrix} a_1 & b_1x & c_1 \\ a_2 & b_2x & c_2 \\ a_3 & b_3x & c_3 \end{vmatrix} = -2x \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$