MM 409: Colloids and Interface Science

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Specific Surface area of powders

$$\boxed{A_{sp} = \frac{3}{\rho R_s}} \tag{1}$$

Where R_s is the radius of the equivalent sphere and ρ is the density.

Laplace's Equation

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{-\rho_e}{\epsilon_r\epsilon_0} \tag{2}$$

The permittivity can change locally.

Boltzmann distribution of ion concentration

$$\boxed{C_i = C_i^0 e^{\frac{-w_i}{k_B T}}} \tag{3}$$

Where the C_i denotes the concentration of the i^{th} ion and C_i^0 is the concentration at the surface of the particle. w_i denotes the work required to bring an ion in solution from infinity to a position closer to the surface.

Poisson-Boltzmann Equation in 1-D:

$$\frac{\partial^2\phi}{\partial x^2} = \frac{C_0 Ze}{\epsilon_r\epsilon_0}\left(e^{\frac{Ze\phi}{k_B T}} - e^{\frac{-Ze\phi}{k_B T}}\right) \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2} = \left(\frac{2C_0 Ze^2}{\epsilon_0\epsilon_r k_B T}\right)\phi \quad \text{for } |e\phi| << k_B T \tag{5}$$

General Solution to P-B equation:

$$\phi(x) = C_1 e^{\kappa x} + C_2 e^{-\kappa x} \tag{6}$$

$$\implies \phi(x) = \phi_0 e^{-\kappa x} \tag{7}$$

where,

$$\boxed{\kappa = \sqrt{\frac{2C_0 Ze^2}{\epsilon_r\epsilon_0 k_B T}}.} \tag{8}$$

So the potential decays faster for electrolytes of higher charge(Z) and concentration(C_0) at the surface.

Debye Length

$$\lambda_D = \frac{1}{\kappa} \tag{9}$$

Potential at the Debye length is,

$$\phi(\lambda_D) = \frac{\phi_0}{e} \approx \frac{\phi_0}{2.72}$$

P-B equation in non-linearised case:

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{C_0 Z e}{\epsilon_r \epsilon_0} \left(e^{\dfrac{e\phi}{k_B T}} - e^{\dfrac{-e\phi}{k_B T}} \right)$$

Let $y = \frac{Ze\phi}{k_B T}$,