b) State and prove Mean Value Theorem. 7

UNIT - IV

7. a) By using Leibnitz's theorem, show that 7

if $y = \cos(m\sin^{-1}x)$, then

i) $(1 - x^2)\, y_2 - xy_1 + m^2y = 0$

ii) $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y = 0$

b) By using L' Hospital's rule, find the following limits: 2+2+3=7

i) $\lim\limits_{x \to 1} \left\{ \dfrac{x}{x - 1} - \dfrac{1}{\log x} \right\}$

ii) $\lim\limits_{x \to 0} \dfrac{\text{Sin}x - x}{x^3}$

iii) $\lim\limits_{x \to 0} \left\{ \dfrac{1}{x} - \dfrac{2}{x(e^x + 1)} \right\}$

8. a) Define the convergence of the sequence (x_n). Show that the sequence $\left(\dfrac{1}{n^2 + 1}\right)$ converges to zero. 2+5=7

b) Define bounded sequence. Show that every convergent sequence is bounded. 7

2021/ODD/18/24/MCS–302/091

PG ODD SEMESTER (CBCS) EXAM., FEBRUARY 2021

COMPUTER SCIENCE

3rd Semester

COURSE NO. MCSCC - 302

(Mathematics - II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five)

UNIT - I

1. a) Define the E-S, definition of the limit of a function at any point 'a'. If $f : A \to R$ and if 'C' is a cluster point of A, then show that f can have only one limit at 'C'. 2+5=7

b) By using the definition of limit, show that : 7

$$\underset{x \to 1}{\text{Lt}} \frac{x + 5}{2x + 3} = 4$$

2. a) Let $f : A \to R$ and 'C' be a cluster point of A, and let

$\underset{x \to c}{\text{Lt}} f(x) = L$, then show that for every sequence (xn) in A in A that converges to 'C', such that $x_n \neq c$, for all $n \in N$, the sequence (f(xn)) converges to L. 7

b) Evaluate the following limits: 3+4=7

i) $\underset{x \to 0}{\text{Lt}} \dfrac{\sin x - \tan x}{x^3}$

ii) $\underset{x \to 1}{\text{Lt}} \dfrac{x^2 - 3x + 2}{x^2 - 4x + 3}$

UNIT - II

3. Define continuous function. Let $A \subseteq R$, let f and g be functions on A to R, and let $b \in R$. Suppose that $c \in A$ and that f and g are continuous at c. Then show that the functions f+g, f-g, fg and bf are continuous at c. 14

4. Discuss the continuity of the following functions at the points indicated: 3+3+4+4=14

i) $f(x) = \begin{cases} x, & \text{if } 0 < x < 1 \\ 2 - x, & \text{if } 1 \le x \le 2 \\ x - \frac{1}{2}x^2, & \text{if } x > 2 \end{cases}$

at the point x = 2

ii) $f(x) = \begin{cases} \dfrac{\tan^2 x}{3x}, & \text{if } x \neq 0 \\ \dfrac{2}{3}, & \text{if } x = 0 \end{cases}$

at x = 0

iii) $f(x) = \begin{cases} x^2 + x, & \text{if } 0 \le x < 1, \\ 2, & \text{if } x = 1 \\ 2x^3 - x + 1, & \text{if } x > 1 \end{cases}$

at x = 1

iv) $f(x) = \begin{cases} \dfrac{x^4 + 4x^3 + 2x}{\text{Sin } x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

at x = 0

UNIT - III

4. Define $\dfrac{d}{dx}(f(x))$ at x = c. By using the definition of derivative prove that : 2+4+4+4=14

i) $\dfrac{d}{dx}(\tan x) = \sec^2 x$

ii) $\dfrac{d}{dx}(\cot x) = -\text{cosec}^2 x$

iii) $\dfrac{d}{dx}(\phi(x), \psi(x) = \phi(x)\,\psi'(x) + \psi(x)\,\phi'(x)$

where, '/' means first derivative.

6. a) Show that the function f(x) defined by : 7

$f(x) = \begin{cases} 3 + 2x, & \text{if } \frac{-3}{2} < x \le 0 \\ 3 - 2x, & \text{if } 0 < x \le \frac{3}{2} \end{cases}$

is continuous but not differentiable at x = 0

<u>UNIT - V</u>

9. a) If $x_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots.. + \frac{1}{n\,(n+)1}$

then show tht the sequence (x_n) is a convergent sequence. 5

b) Show that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence. 4

c) Define cauchy sequence. Show that every convergent sequence is a cauchy sequence. 5

10. a) Show that $\sum_{n=1}^{\alpha} \frac{1}{n}$ diverges 5

b) Write limit comparison test and rout test. 4

c) Examine the convergence of the following series: 5

$$\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \left(\frac{4}{9}\right)^4 + \ldots\ldots\ldots$$