UG Odd Semester (CBCS) Exam., December—2019

COMPUTER SCIENCE

(1st Semester)

Course No. : MCSCC–102

(Mathematics—I)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* For the non-empty sets A, B, C, prove the following identities : 4+4=8

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) If $n(A)$ and $n(B)$ denote the number of elements in the finite sets A and B respectively, then prove, by the Venn diagram, that

$$n(A) + n(B) = n(A \cup B) + n(A \cap B)$$ 4

(c) If $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 4, 5\}$, then find $(A \times B) \cap (A \times C)$. 2

2. (a) In a class of 50 students, 30 like language, 20 like science and 10 like neither. Show, by a Venn diagram, that 10 students like both language and science. 4

(b) If R be a relation in the set of integers Z defined by the open sentence $(x-y)$ is divisible by 6, that is

$$R = \{(x, y) : x \in z, y \in z, (x-y) \text{ is divisible by } 6\}$$

then prove that R is an equivalence relation. 5

(c) Show that $f : R \to R$, defined by $f(x) = 3x + 4$, $x \in R$ is one-one and onto. 4+1=5

UNIT—II

3. (a) Solve the following system of equations by using Gauss-Jordan method : 5+5=10

(i)
$$2x + 3y - z = 5$$
$$4x + 4y - 3z = 3$$
$$2x - 3y + 2z = 2$$

(ii)
$$3x + 4y + 5z = 18$$
$$2x - y + 8z = 13$$
$$5x - 2y + 7z = 20$$

(Continued)

(b) Define rank of a matrix. Find the rank of the following matrix : 4

$$A = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

4. (a) Define eigenvalue and eigenvector of a matrix. Find all the eigenvalues and eigenvectors of the following matrix : 5

$$A = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$$

(b) State Cayley-Hamilton theorem. By using Cayley-Hamilton theorem, find the inverse of the following matrix : 5

$$A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix}$$

(c) Define linearly independent and linearly dependent vectors. Show that the vectors (1, 1, 1), (1, 2, 0) and (0, −1, 2) are linearly dependent. 4

(Turn Over)

UNIT—III

5. (a) Given two vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. Find a unit vector $\vec{c}$ perpendicular to the vector $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$. 5

(b) Find the angle between the following pairs of vectors : 2+2=4

(i) $3\hat{i} - \hat{j} + 3\hat{k},\ 2\hat{i} - 4\hat{j} + 3\hat{k}$

(ii) $\hat{i} - 2\hat{j} + 2\hat{k},\ -\hat{i} + 2\hat{j} - 2\hat{k}$

(c) Find the volume of the tetrahedron, whose Cartesian coordinates of vertices are (0, 1, 2), (3, 0, 1), (4, 3, 6) and (2, 3, 2). 5

6. (a) Find the area of the triangle whose vertices are the points with rectangular Cartesian coordinates (1, 2, 3), (–2, 1, –4) and (3, 4, –2). 5

(b) Compute the following vector products : 3+3=6

(i) $[(\hat{i} - 2\hat{j} + 3\hat{k}) \times (2\hat{i} + \hat{j} - 3\hat{k})] \times [(-\hat{i} + \hat{j} - 2\hat{k}) \times (2\hat{j} - \hat{k})]$

(ii) $[(3\hat{i} - \hat{j} + \hat{k}) \times (-\hat{i} + \hat{k})] \times [(\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} - \hat{j} + \hat{k})]$

(c) If $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that

$$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$ 3

UNIT—IV

7. (a) Define order and degree of an ordinary differential equation with examples. 2

(b) Verify that the following equations are homogeneous and hence solve them : 3+3=6

(i) $x^2 y' - 3xy - 2y^2 = 0$

(ii) $(x + y)dx - (x - y)dy = 0$

(c) Show that the following equations are exact and hence solve them : 3+3=6

(i) $(y + y\cos xy)\,dx + (x + x\cos xy)dy = 0$

(ii) $\cos x \cos^2 y\, dx - 2\sin x \sin y \cos y\, dy = 0$

8. (a) Find the solution of the following linear equations : 3+3=6

(i) $(1 + x^2)dy + 2xy\,dx = \cot x\, dx$

(ii) $y' + y\cot x = 2x \operatorname{cosec} x$

(b) Show that $y = c_1 e^{x} + c_2 e^{2x}$ is the general solution of

$$y'' - 3y' + 2y = 0$$ 2

(c) Find the solutions of the following initial value problems : 3+3=6

(i) $y'' - 6y' + 5y = 0$, $y(0) = 3$ and $y'(0) = 11$

(ii) $y'' - 5y' + 6y = 0$, $y(1) = e^2$ and $y'(1) = 3e^2$

UNIT—V

9. (a) Find the Laplace transform of the following : 3+2+2=7

(i) $\dfrac{(1-\cos t)}{t}$

(ii) $\sin 2t \cdot \sin 3t$

(iii) $te^{-4t} \cdot \sin 3t$

(b) State and prove the shifting property in Laplace transform. 4

(c) Let $f_1(s)$ and $f_2(s)$ be the Laplace transforms of $F_1(t)$ and $F_2(t)$ respectively. Prove that

$$L(c_1 F_1(t) + c_2 F_2(t)) = c_1 f_1(s) + c_2 f_2(s)$$ 3

10. (a) Find the inverse Laplace transform of the following : 4+5=9

(i) $\dfrac{s+2}{s^2 - 4s + 3}$

(ii) $\dfrac{4s^2 - 3s + 5}{(s+1)(s^2 - 2s + 2)}$

(b) Solve the following equation by using Laplace transform : 5

$$y'' + 4y' + 8y = \cos 2t, \quad y(0) = 2$$

and $y'(0) = 1$

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