UG/PG ODD SEMESTER (CBCS) EXAM., 2020

held in April – 2021

COMPUTER SCIENCE

1st Semester

COURSE NO. MCSCC - 102

(Mathematics - I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

(Answer any five)

UNIT - I

1. a) For the non-empty sets A, B and C, prove the following identities: 8

   i) $A - (B \cap C) = (A - B) \cup (A - C)$

   ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   b) Which of the following relations in the set of real numbers are equivalence relation? 6

   i) aRb if and only if $|a| = |b|$

   ii) aRb if and only if $a \geq b$

   iii) aRb if and only if $|a| > |b|$.

2. a) If $f: A \rightarrow B$ be one-one and onto, then the inverse mapping of f is unique. 5

b) Show that the function $f: R \rightarrow R$, defined by $f(x) = x^2$, is neither one-one nor onto. 4

c) Show that the map $f: Q \rightarrow Q$, defined by $f(x) = 4x + 3$, is both one-one and onto, where 'Q' is the set of rational numbers. 5

UNIT - II

3. Solve the following system of equations by using Gauss-Jordan method:

i) $2x + 3y - z = 5$
$4x + 4y - 3z = 3$ 7+7=14
$2x - 3y + 2z = 2$

ii) $3x + 4y + 5z = 18$
$2x - y + 8z = 13$
$5x - 2y + 7z = 20$

4. a) Define eigenvalue and eigenvector of a matrix. Find all the eigenvalues and eigenvectors of the following matrix: 7

$$A = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$$

b) State Cayby-Hamilton theorem. By using Cayby-Hamilton theorem, find the inverse of the following matrix: 7

$$A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix}$$

UNIT - III

5. a) Find the angle between the following pairs of vectors: 2+2=4

i) $4\hat{i} - 2\hat{j} - 2\hat{k}$, $3\hat{i} + 4\hat{j} - 4\hat{k}$

ii) $2\hat{i} + 3\hat{j} - \hat{k}$, $-2\hat{i} - 3\hat{j} + 5\hat{k}$

b) If $\vec{a} = 2\hat{i} - 10\hat{j} + 2\hat{k}$, 5

$\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{c} = 2\hat{i} + \hat{j} + 3\hat{k}$

Find the value of $\vec{a} \times (\vec{b} \times \vec{c})$

c) If $\vec{a} = xyz\,\hat{i} + x^2z\,\hat{j} - y^3\,\hat{k}$ and

$\vec{b} = x^3\,\hat{i} - xyz\,\hat{j} + x^3z\,\hat{k}$,

Calculate $\dfrac{d^2\vec{a}}{dy^2} \times \dfrac{d^2\vec{b}}{dx^2}$ at the point (1, 1, 0) 5

6. a) If $\vec{F} = xy^2\,\hat{i} + 2x^2yz\,\hat{j} - 3yz^2\,\hat{k}$,
find div ($\vec{F}$) and curl ($\vec{F}$) at (1, -1, 1) 5

b) Compute the following vector products: 4+3+2=9

i) $[(\hat{i} - 2\hat{j} + \hat{k}) \times (-\hat{i} + 3\hat{j} - \hat{k})] \times [(2\hat{i} + \hat{j} + \hat{k}) \times (3\hat{i} - 2\hat{j} - 3\hat{k})]$

ii) $[(\hat{i} + \hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k})] \times [(-\hat{i} - \hat{j} - \hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})]$

iii) $[(\hat{i} - \hat{j}) \times (2\hat{j} + \hat{k})] \times [(\hat{j} - \hat{k}) \times (\hat{i} + \hat{j} + \hat{k})]$

UNIT - IV

7. a) Show that the following equations are exact and hence solve them:

i) $(\sin x \tan y + 1)\, dx - \cos x \sec^2 y\, dy = 0$

ii) $e^y dx + (xe^y + 2y)\, dy = 0$ 4+4=8

b) Verify that the following equations are homogeneous and hence solve them:

i) $(x^2 - 2y^2)\, dx + xy\, dy = 0$

ii) $(x - y)\, dx - (x + y)\, dy = 0$ 3+3=6

8. a) Find the solution of the following linear euqations:

i) $y - x + xy \cot x + xy' = 0$

ii) $\dfrac{dy}{dx} + \dfrac{1}{x}\, y = 3x$ 3+3=6

b) Find the solutions of the following initial value problems: 4+4=8

i) $y'' - 6y' + 9y = 0$, $y(0) = 0$ and $y'(0) = 5$

ii) $y'' + 8y' - 9y = 0$, $y(1) = 2$ and $y'(1) = 0$

UNIT - V

9. a) Find the Laplace transform of the following: 3+4+3=10

i) $\sin^3 2t$

ii) $\dfrac{1 - \cos t}{t}$

iii) $\dfrac{\cos 2t - \cos 3t}{t}$

b) Find the Laplace transform of $f(t)$ defined as 4

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t \geq \pi \end{cases}$$

10. a) Find the inverse Laplace transform of the following: 3+3+3=9

i) $\dfrac{2s-3}{s^2 - s - 3/4}$

ii) $\dfrac{1}{(s+a)^n}$

iii) $\dfrac{s^3}{(s^2+a^2)^2}$

b) Solve the following equation by using Laplace transform: 5

$$y'' + 3y' + 2y = t\,e^{-t}$$

$$y(0) = 1 \text{ and } y'(0) = 0$$