2019

UG Even Semester (CBCS) Exam., May—2019

COMPUTER SCIENCE

(4th Semester)

Course No. : MCSCC–402

(Discrete Mathematics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Without using truth table, show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. 5

(b) By using truth table show that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent. 5

(c) Write the converse and inverse of the following statements : 4

(i) "If it is cold, he wears a hat."

(ii) "If it snows, then they do not drive the car."

2. (a) Determine the validity of the following arguments : 5+5=10

(i) If I study, then I will pass.
If I do not go to a movie, then I will study.
I failed
____
Therefore, I went to a movie.

(ii) If I like Biology, then I will study it.
Either I study Biology or I fail the course
____
If I fail the course then I do not like Biology.

(b) Define universal quantifier and existential quantifier. What are the truth values of $\forall x P(x)$ and $\exists x P(x)$, where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse is the set $\{1, 2, 3\}$? 4

UNIT—II

3. (a) Define cyclic group. Show that every subgroup of a cyclic group is itself a cyclic group. 5

(b) Let G be the set of all 2×2 matrices, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d are real numbers, such that $ad - bc \neq 0$. Show that G is a group with respect to matrix multiplication. 6

(c) Let $S = N \times N$. Let $*$ be the operation on S defined by $(a, b) * (a', b') = (aa', bb')$.

(i) Show that S is a semigroup w.r.t. $*$.

(ii) Define $f : (S, *) \to (Q, X)$ by $f(a, b) = a/b$. Show that f is a homomorphism. 3

4. (a) Define subgroup of a group. Let G be the group of all 2×2 real matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, with $ad - bc \neq 0$, under matrix multiplication. Let

$$H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in G / ad \neq 0 \right\}$$

Then show that H is a subgroup of G. 5

(b) Define Abelian group. Give an example of a group which is not Abelian group with justification. 3

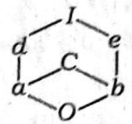
(c) Define ring, field, integral domain, zero divisor, division ring. 6

UNIT—III

5. (a) Define Hasse diagram. If D_m denotes the set of divisors of m ordered by divisibility, then draw the Hasse diagram of D_{36}. 4

(b) Define distributive lattice. Let L be a bounded distributive lattice, then show that complements are unique if they exist. 6

(c) Consider the lattice L in the figure below :



(i) Which non-zero elements are join irreducible?

(ii) Which elements are atoms?

(iii) Is L distributive?

(iv) Find the complements if they exist for the elements a, b and c. 4

6. (a) Prove that the following are equivalent in a Boolean algebra :

(i) $a+b=b$ (ii) $a*b=a$

(iii) $a'+b=1$ (iv) $a*b'=0$

where a, b are any elements in the Boolean algebra and +, *, ′, 0, 1 are of usual meanings. 6

(b) Express each of the following Boolean expressions as a sum of products and then in its complete sum of products form : 8

(i) $E(x, y, z) = z(x'+y)+y'$

(ii) $E(x, y, z) = (x'+y)'+x'y$

(iii) $E(x, y, z) = y(x+yz)'$

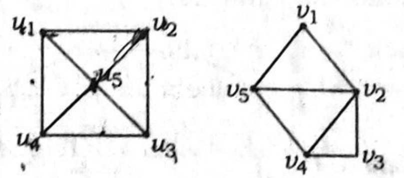
UNIT—IV

7. (a) Define incidence matrix and adjacency matrix. Draw the undirected graphs represented by the following adjacency matrix : ~~2+2+2=6~~ 3+3=6

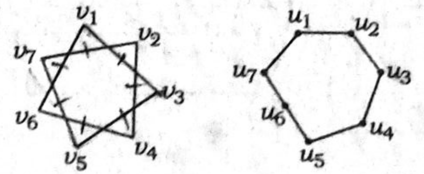
(i) $\begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

(b) Define isomorphic graphs. Check whether the following pairs of graphs are isomorphic or not : 1+4=5

(i)

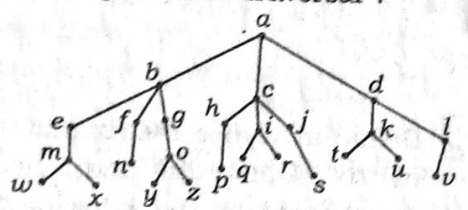


(ii)



(c) Define complete and regular graphs with example. 3

8. Write the preorder, inorder and postorder traversal algorithms for a rooted tree, find the order in which the vertices of the tree will be visited by preorder traversal, inorder traversal and postorder traversal : 14

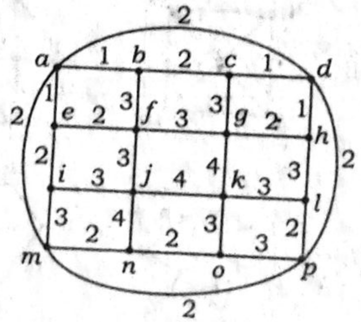


UNIT—V

9. (a) Define spanning tree and minimum spanning tree. Write the Prim's algorithm for finding minimum spanning tree of a weighted graph. 5

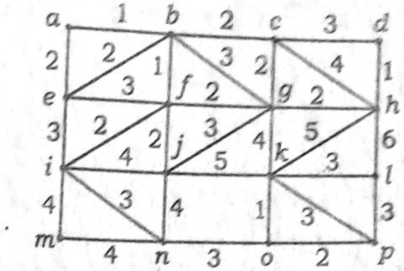
(b) By using Kruskal algorithm, find a minimum spanning tree for the following graphs : 5+4=9

(i)



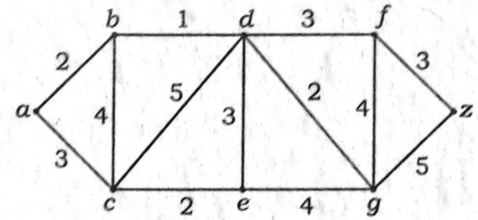
(Continued)

(ii)

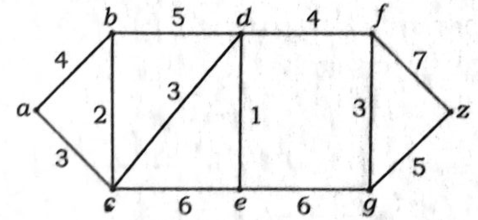


10. Write Dijkstra's algorithm for finding the shortest path of a weighted graph. Using Dijkstra's algorithm, find the shortest path between the vertices a to z for the following graphs : 14

(a)



(b)



★★★