

# Finding structures in temporal graphs

**Alain Barrat**

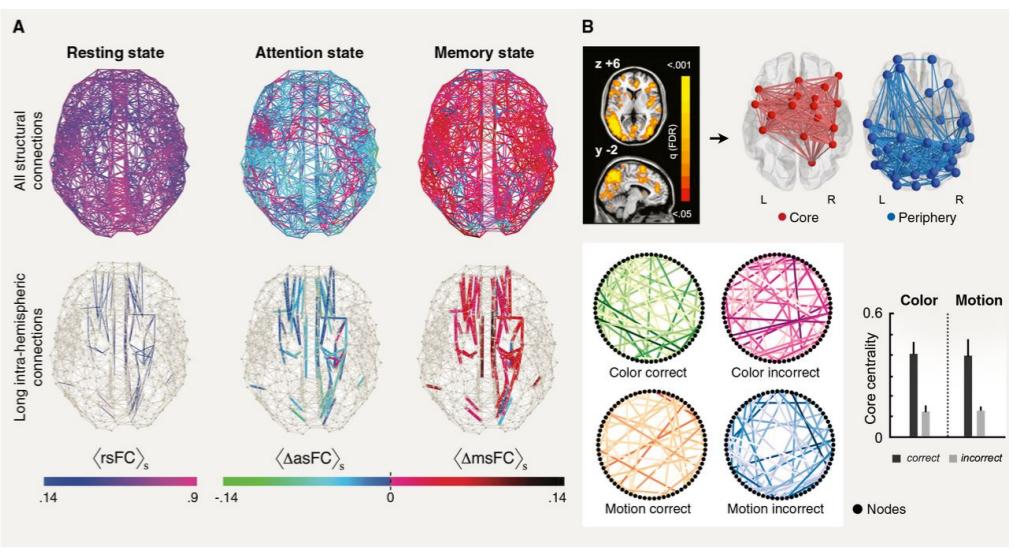
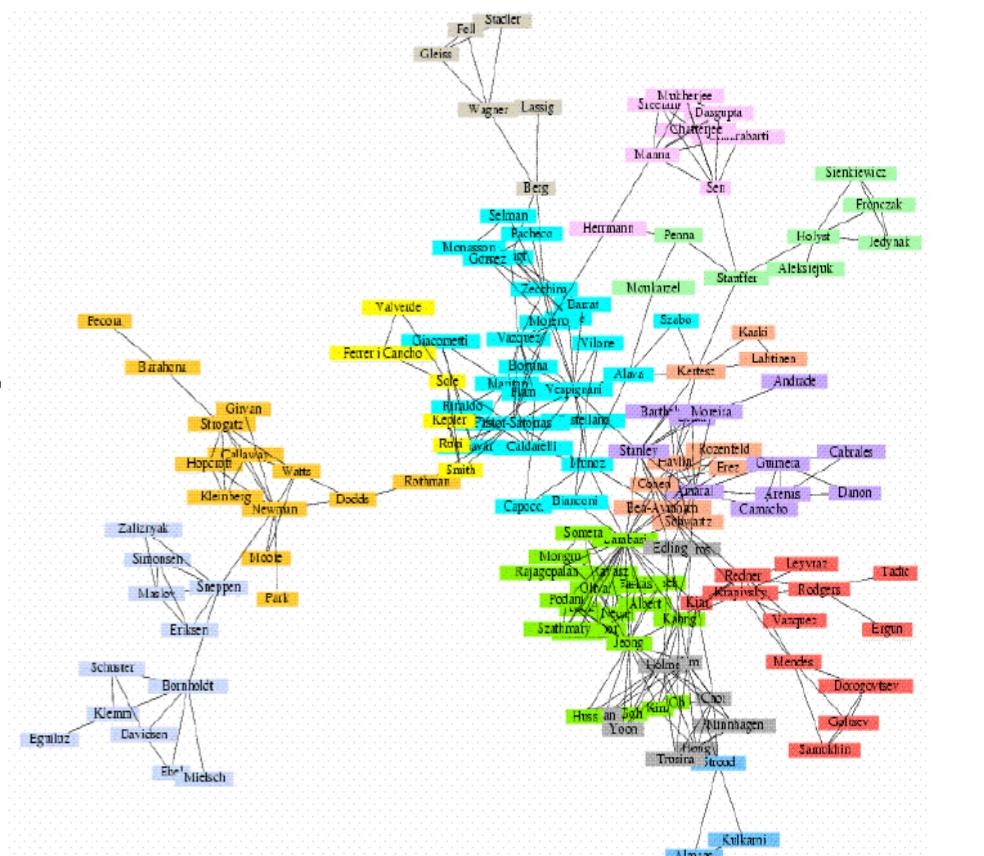
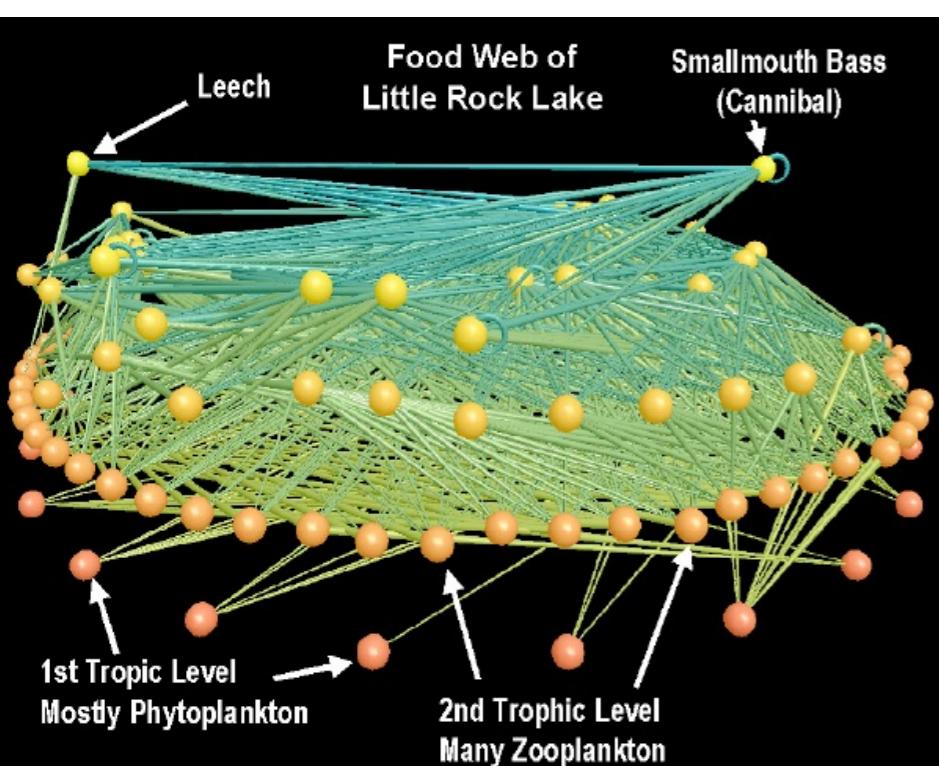
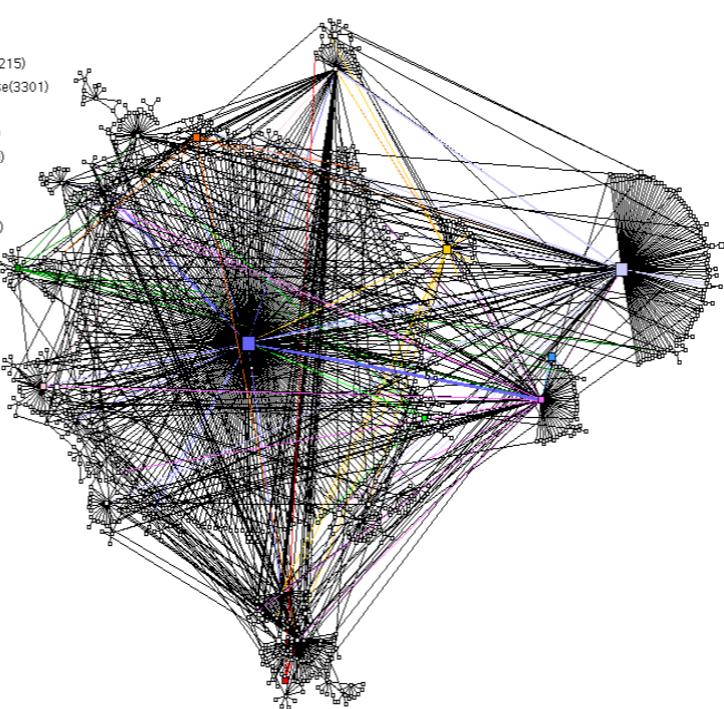
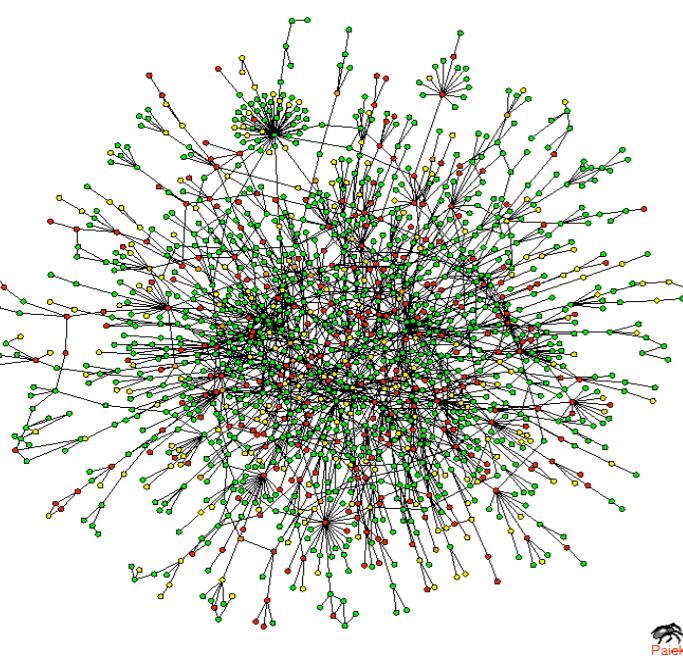
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Infrastructure networks

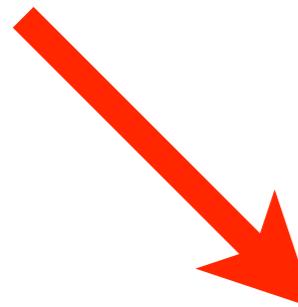
Biological networks

Communication networks

Social networks

Virtual networks

...



many datasets

+

common network representation framework/language



Emergence of **data-driven, interdisciplinary** “network science”



- Empirical study and characterization
- Modeling: understand formation mechanisms
- Consequences of the empirically found properties on dynamical phenomena taking place on the networks (epidemic spreading, resilience, synchronization,...)



Static networks

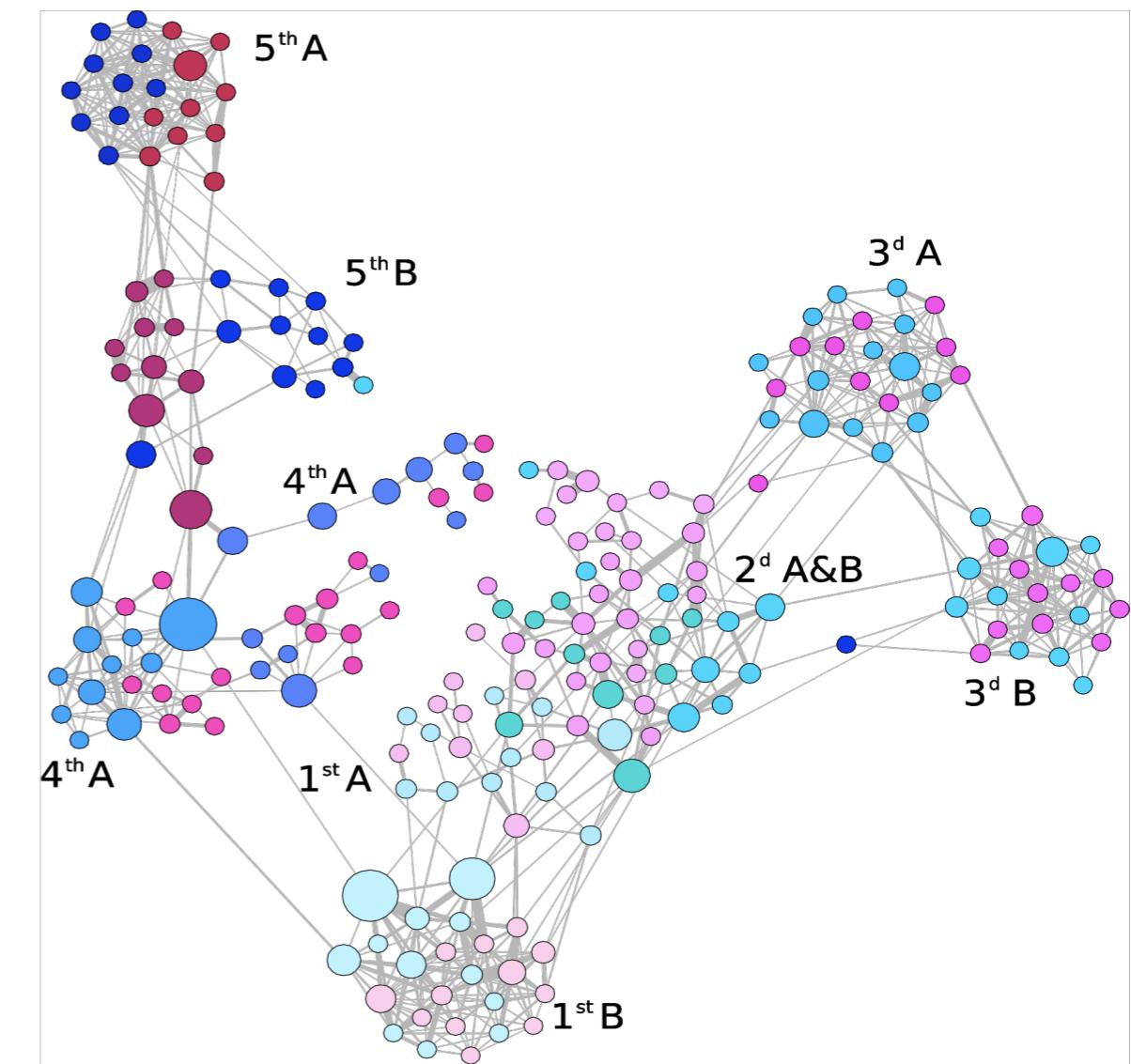
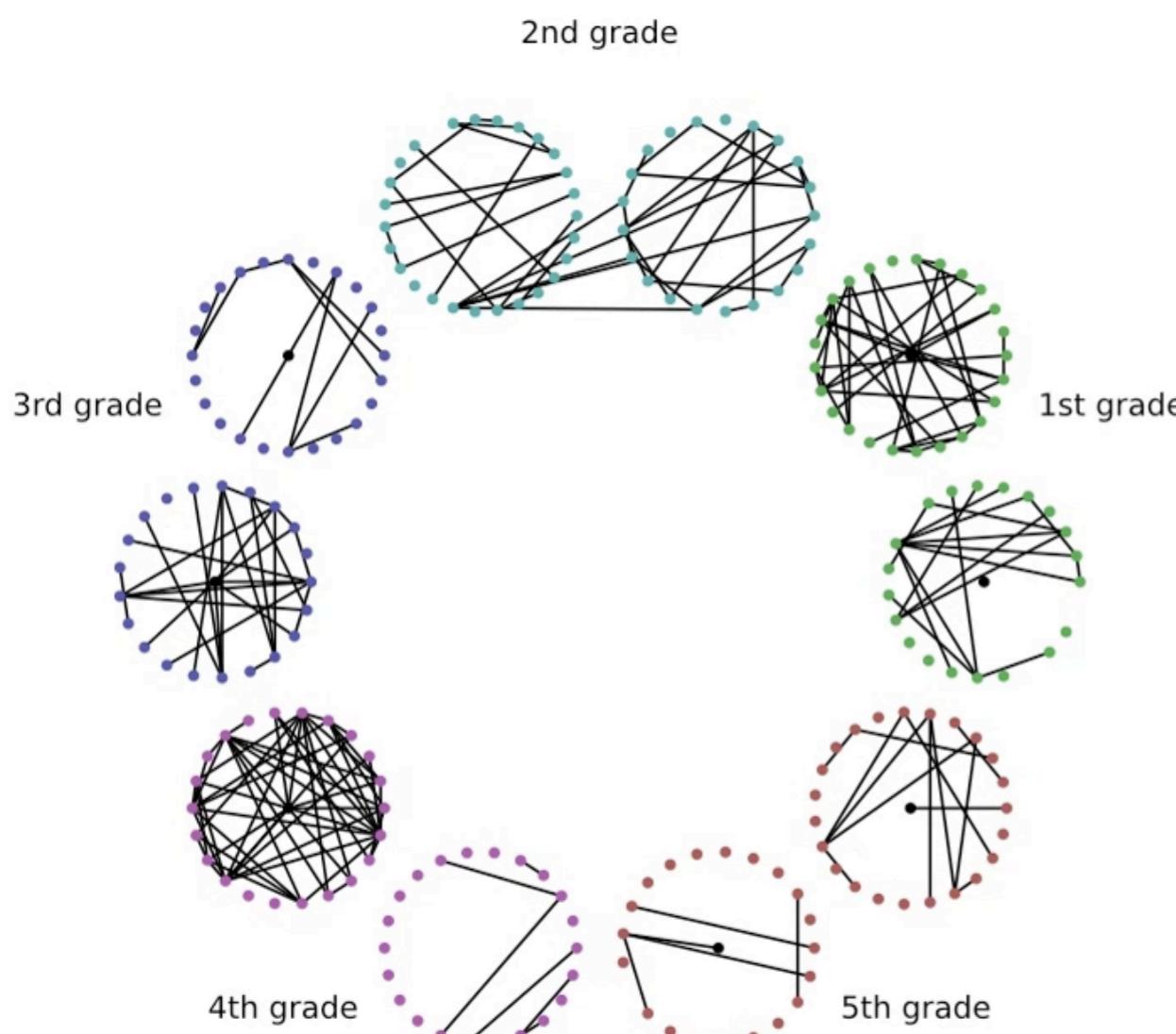
# Structures in static graphs

## Various scales

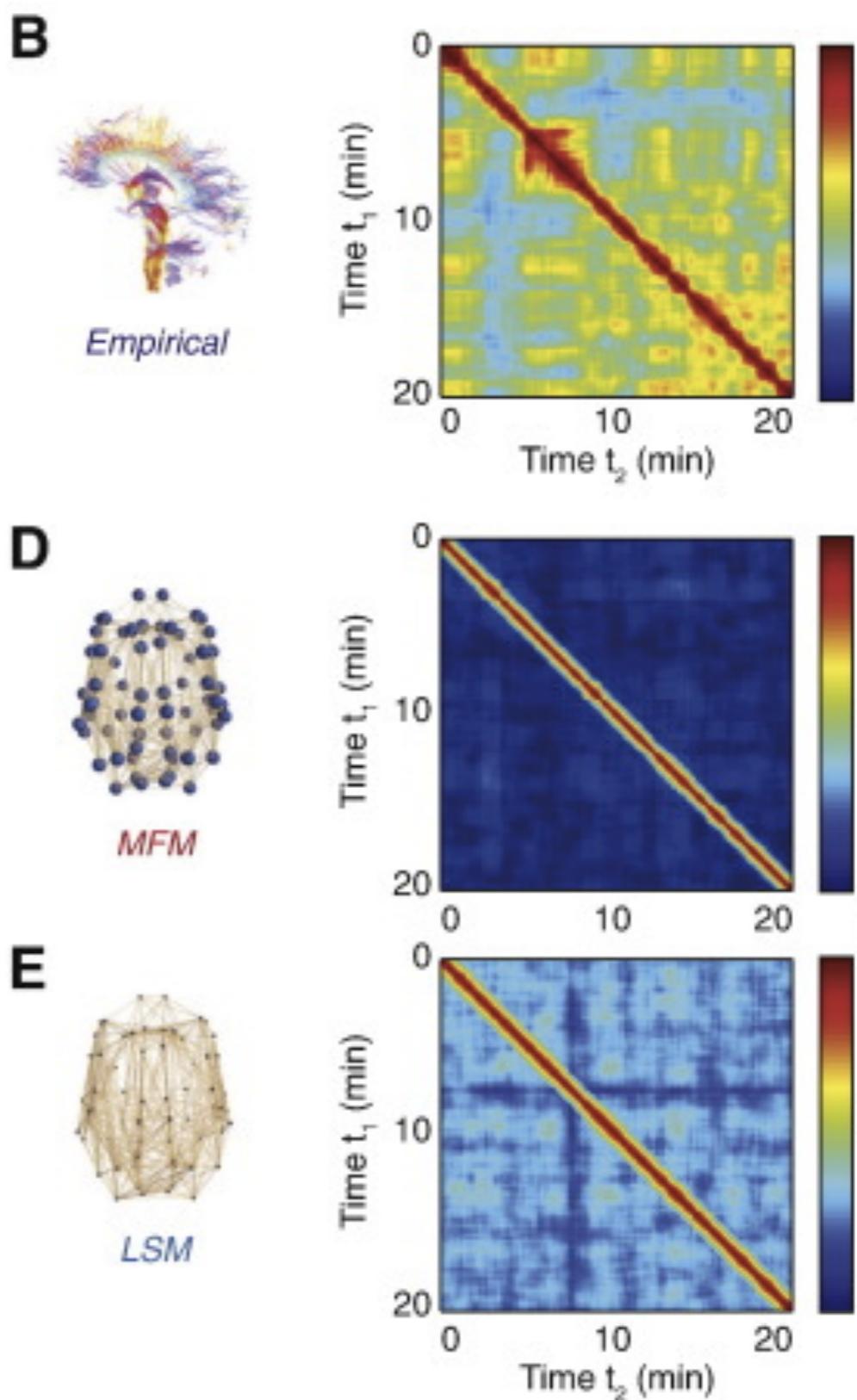
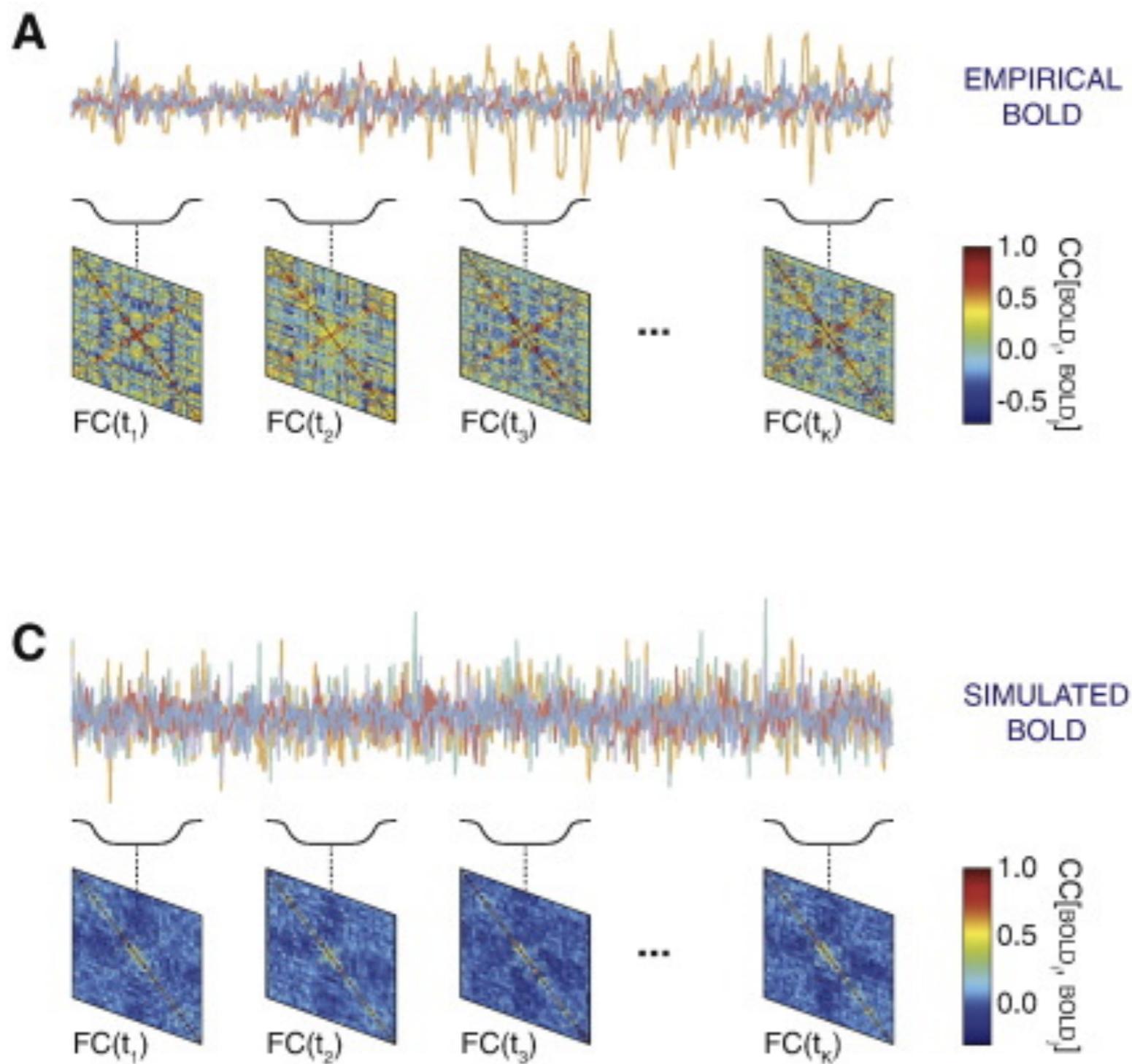
- Clustering coefficient, local cohesiveness, small-world
- Heterogeneities (degree distribution, hubs...)
- Motifs (small subgraphs more frequent than expected)
- Communities
- Backbones (most “relevant” parts of a network)
- Hierarchies (e.g., k-core decomposition)
- ...

>Networks change over time

# Example: contacts in a primary school,



# Example: brain functional connectivity



# Beyond static networks

Infrastructure networks

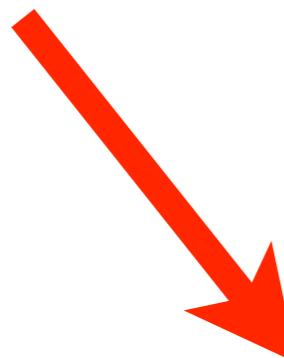
Biological networks

Communication networks

Social networks

Virtual networks

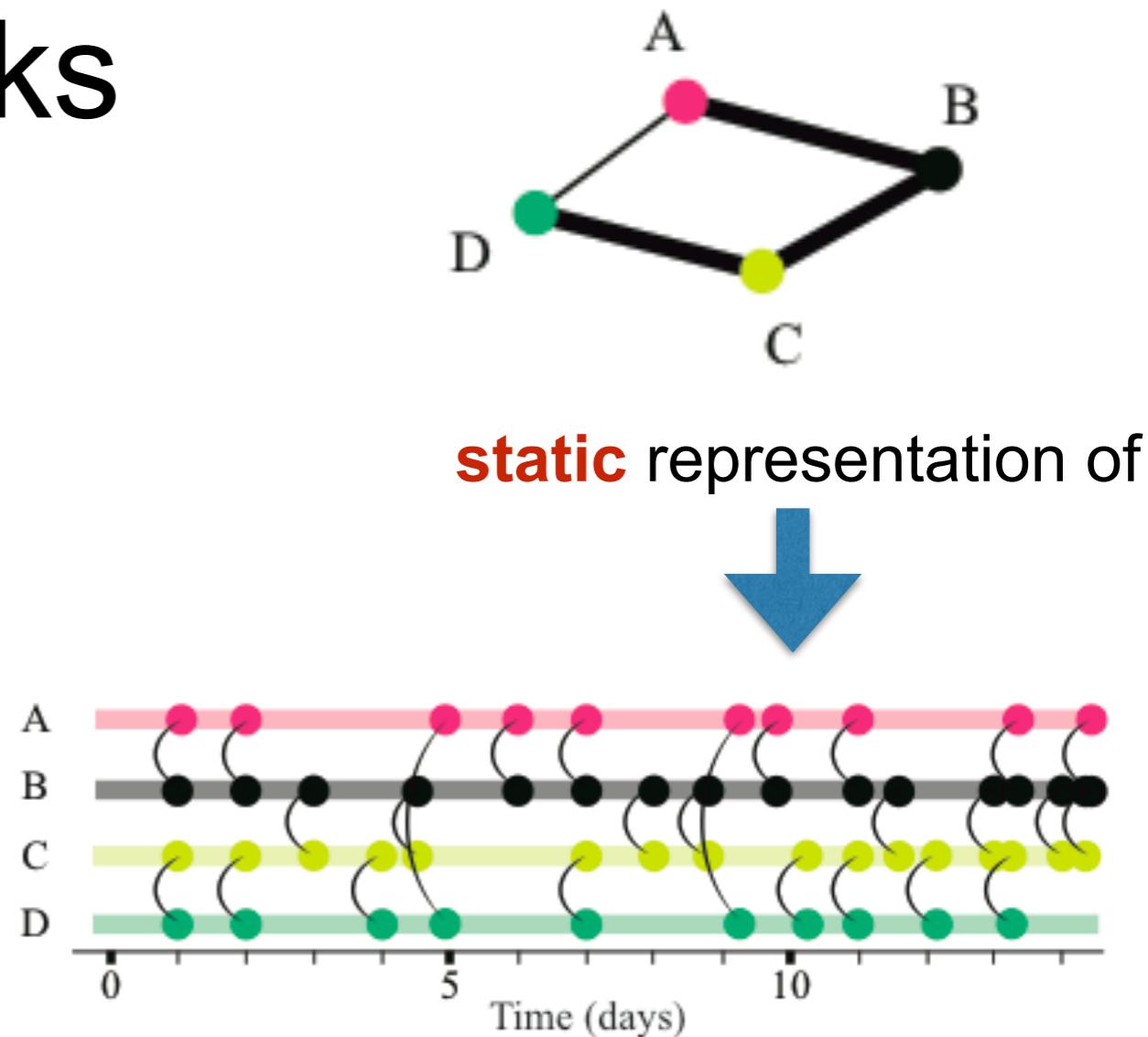
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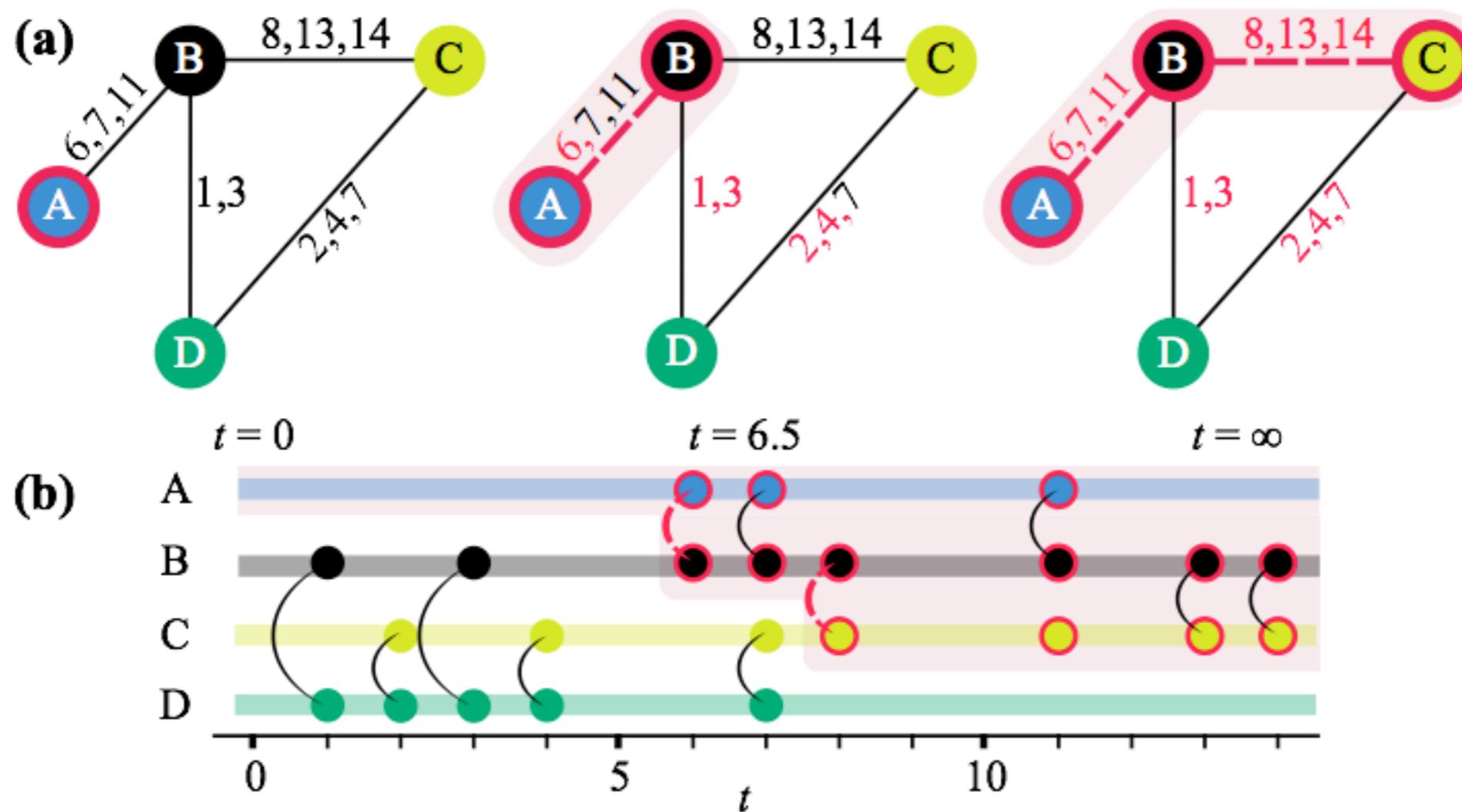
many datasets:

**static** network representation framework

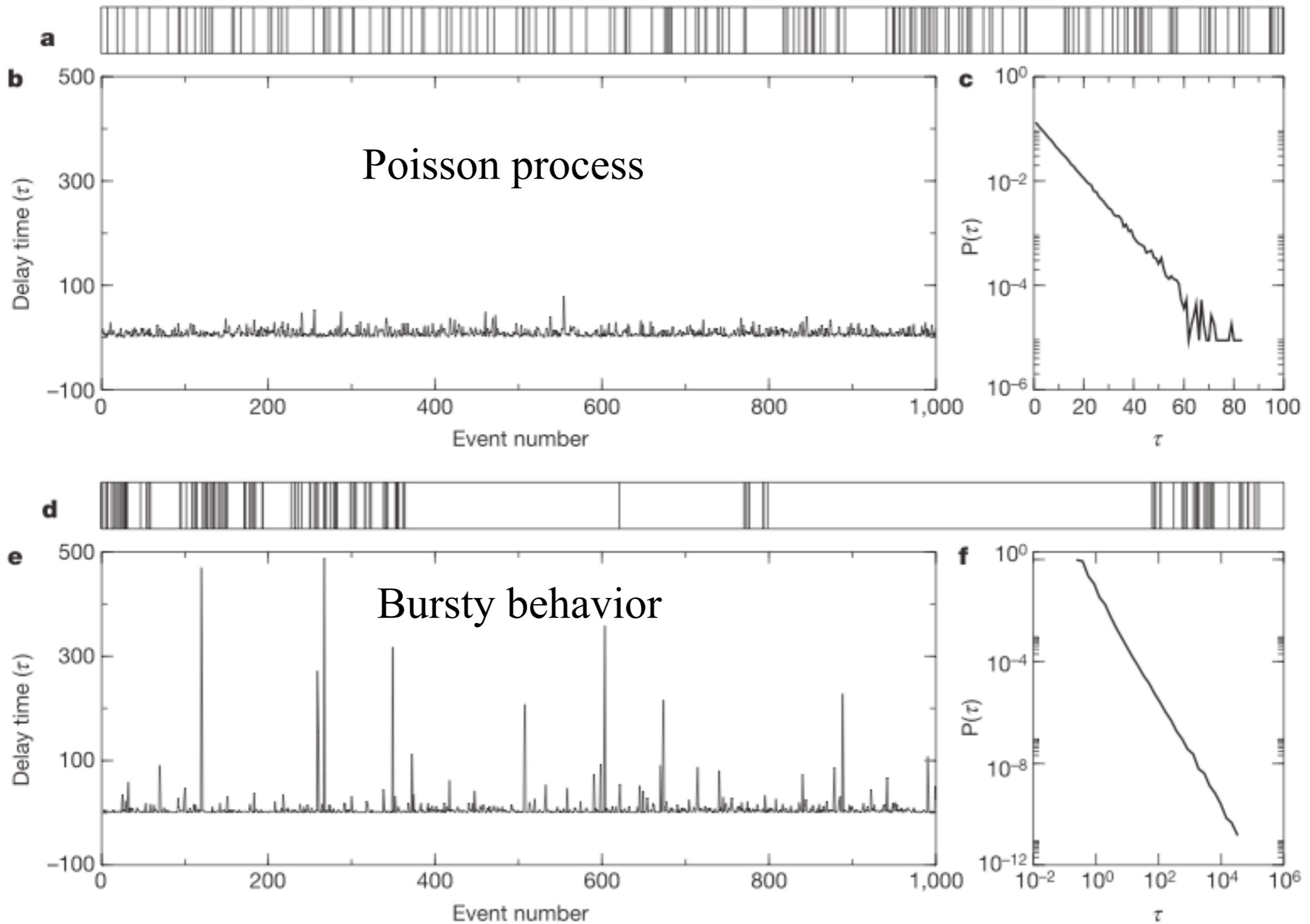
- As first approach
- If timescale of process << timescale of network reconfiguration



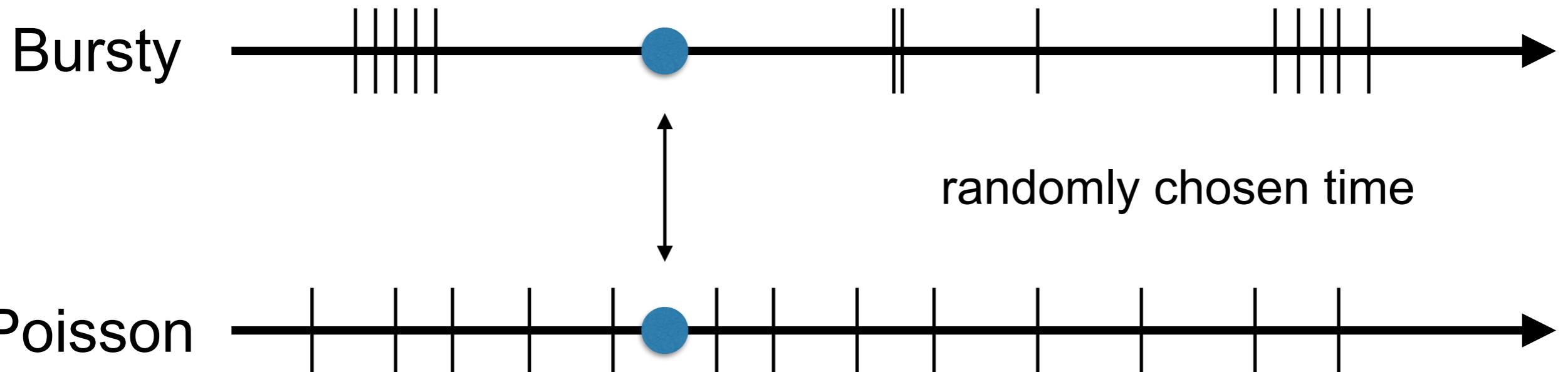
# Impact of temporality: Reachability issue



# Burstiness



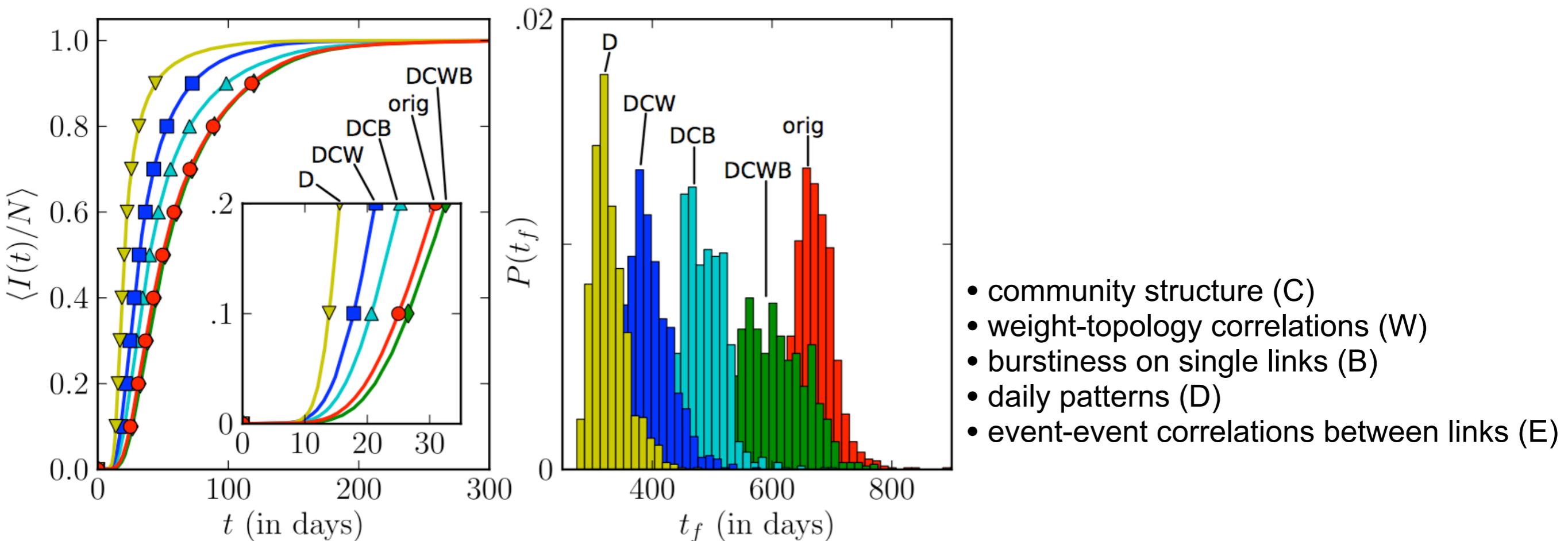
# Consequence of burstiness



Bursty timeline implies larger waiting time with higher probability  
=> typically slows down diffusion (if no correlations)

# Impact on processes on networks

Mobile phone data



Burstiness slows down spread  
Correlations slightly favour spread

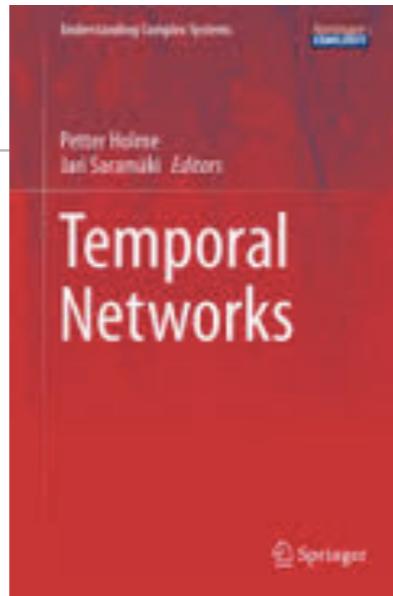


## Temporal networks

Petter Holme<sup>a, b, c</sup>, , Jari Saramäki<sup>d</sup>

[+ Show more](#)

<http://dx.doi.org/10.1016/j.physrep.2012.03.001>



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[The European Physical Journal B](#)

September 2015, 88:234

## Modern temporal network theory: a colloquium

Authors

[Authors and affiliations](#)

Petter Holme

Colloquium

First Online: 21 September 2015

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Part of the following topical collections:

- [Topical issue: Temporal Network Theory and Applications](#)

## Many challenges:

- Definitions and **representations** at various levels of temporal information
- **Characterization/measures/signatures** of temporality/temporal features
- **Modeling**
- Consequences on **dynamical phenomena** (e.g. epidemics, information propagation...)
- Link temporal+topological structure and function
- **Structures**
- ...

# >What kind of structures in temporal networks?

- Small scale: **motifs** (small temporal subgraphs more frequent than expected)
- **Decomposing** a temporal network into “components”: non-negative tensor factorisation
- Most “relevant” part: **Backbones**
- Well connected structures: temporal **cores**
- Finding relevant **timescales**, “**states**” of a temporal network
- ...

- knowledge on structure/evolution of system
- filtering out noise
- relevance for dynamical processes on temporal networks

# Definition: temporal network

Temporal network:  $T=(V,S)$

- $V$ =set of nodes
- $S$ =set of event sequences assigned to pairs of nodes

$$s_{ij} \in S : s_{ij} = \{(t_{ij}^{s,1}, t_{ij}^{e,1}) \cdots (t_{ij}^{s,\ell}, t_{ij}^{e,\ell})\}$$

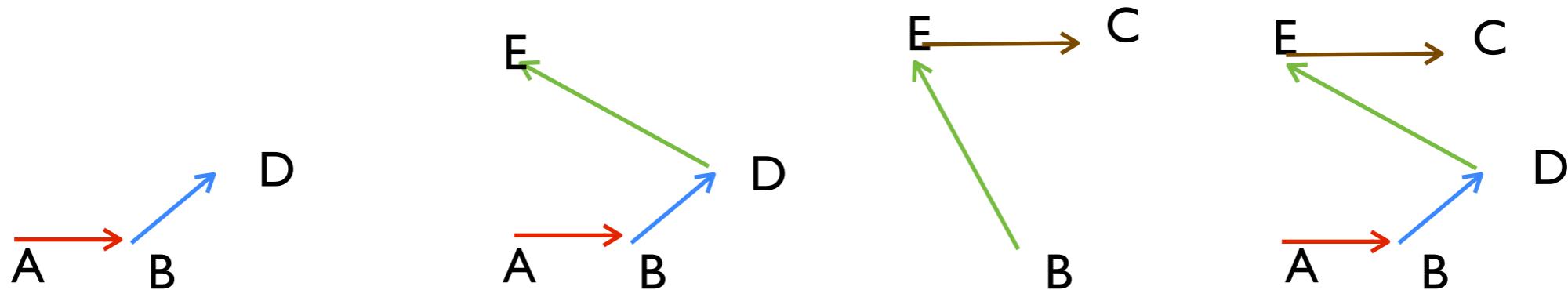
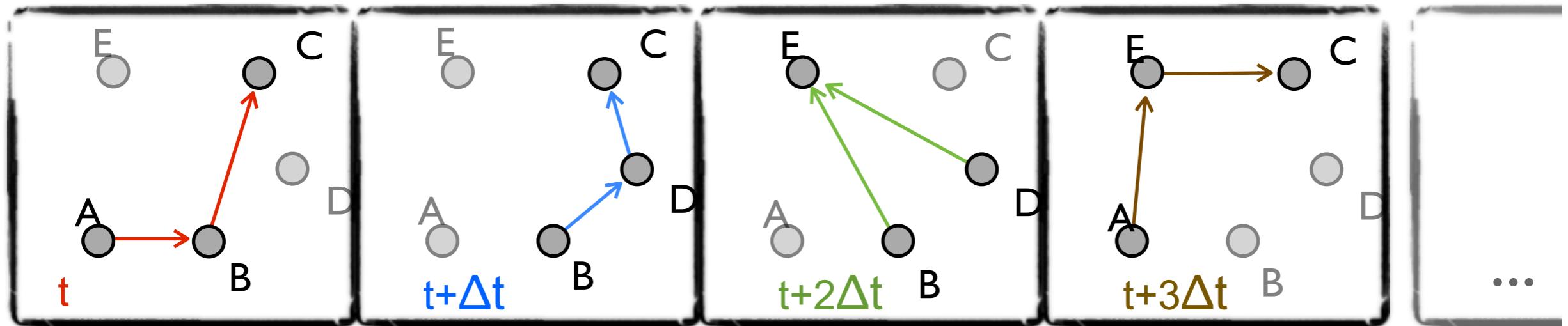
Other representation:

time-dependent adjacency matrix:

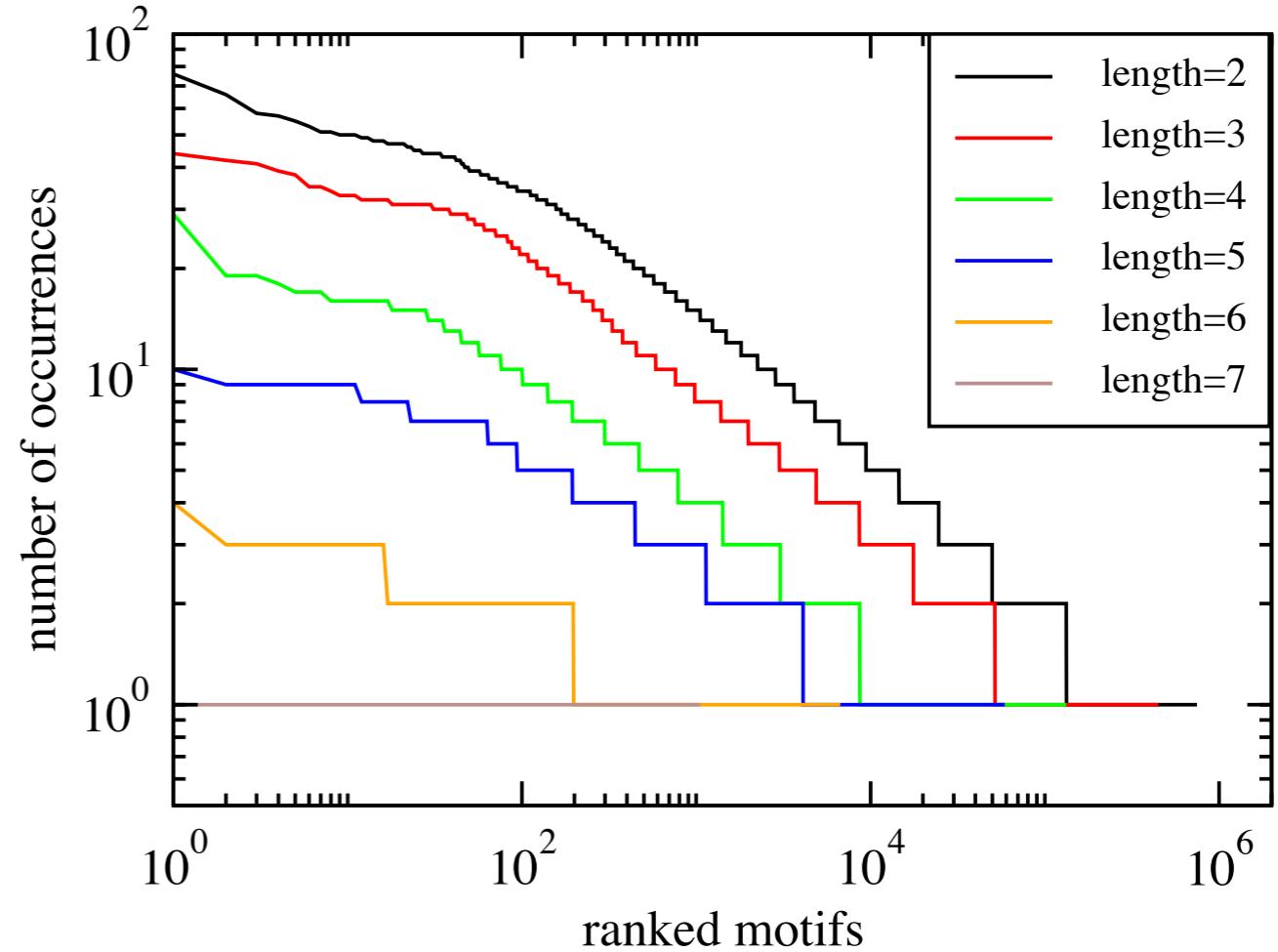
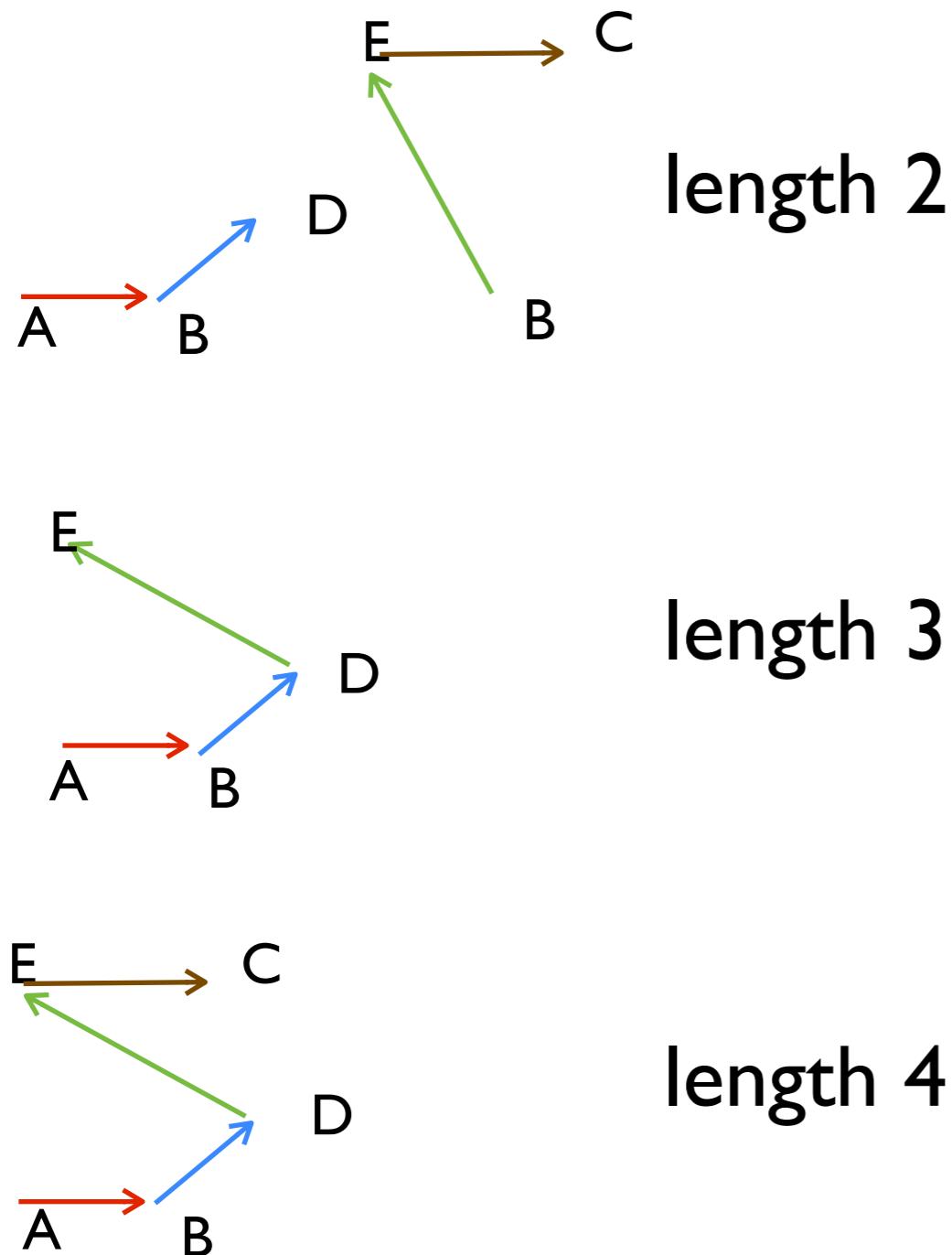
$a(i,j,t) = 1 \Leftrightarrow i \text{ and } j \text{ connected at time } t$

>Finding structures:  
Temporal motifs

# Dynamical motifs

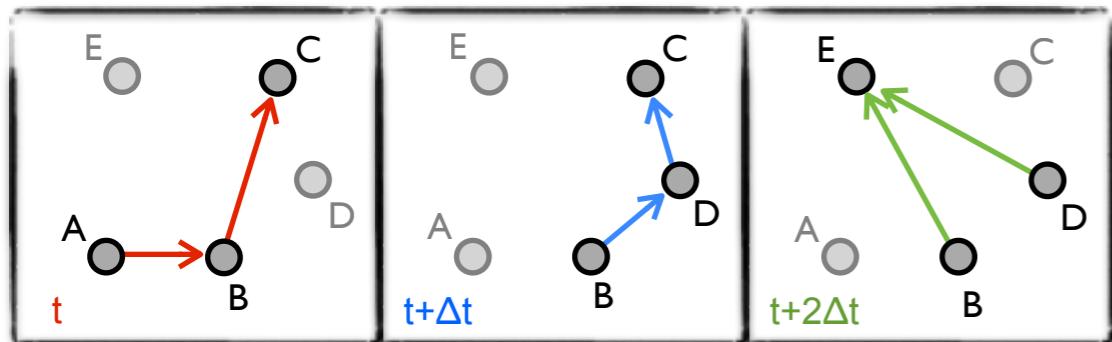


# Dynamical motifs

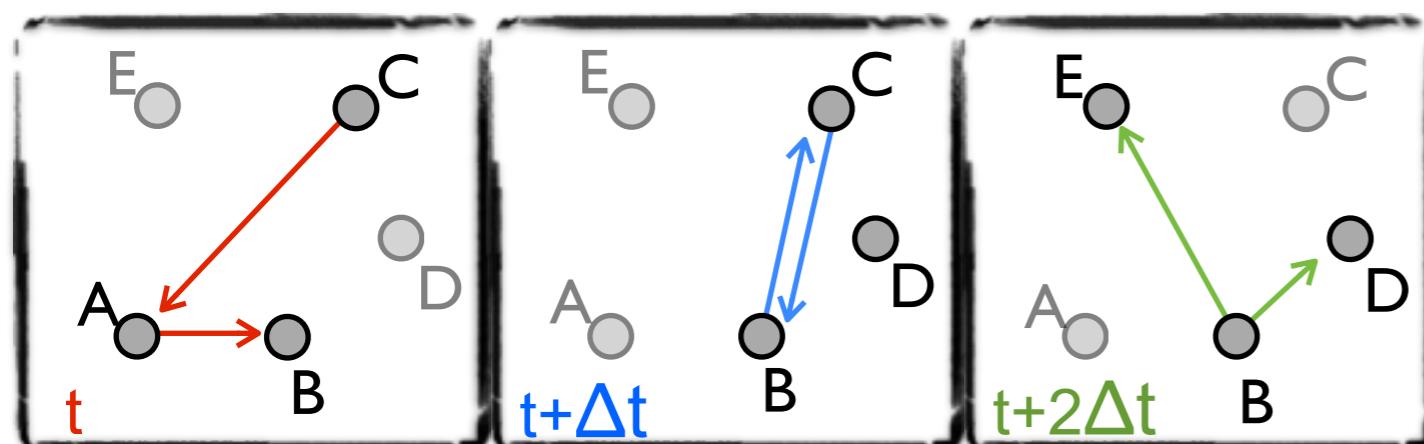


# Dynamical motifs

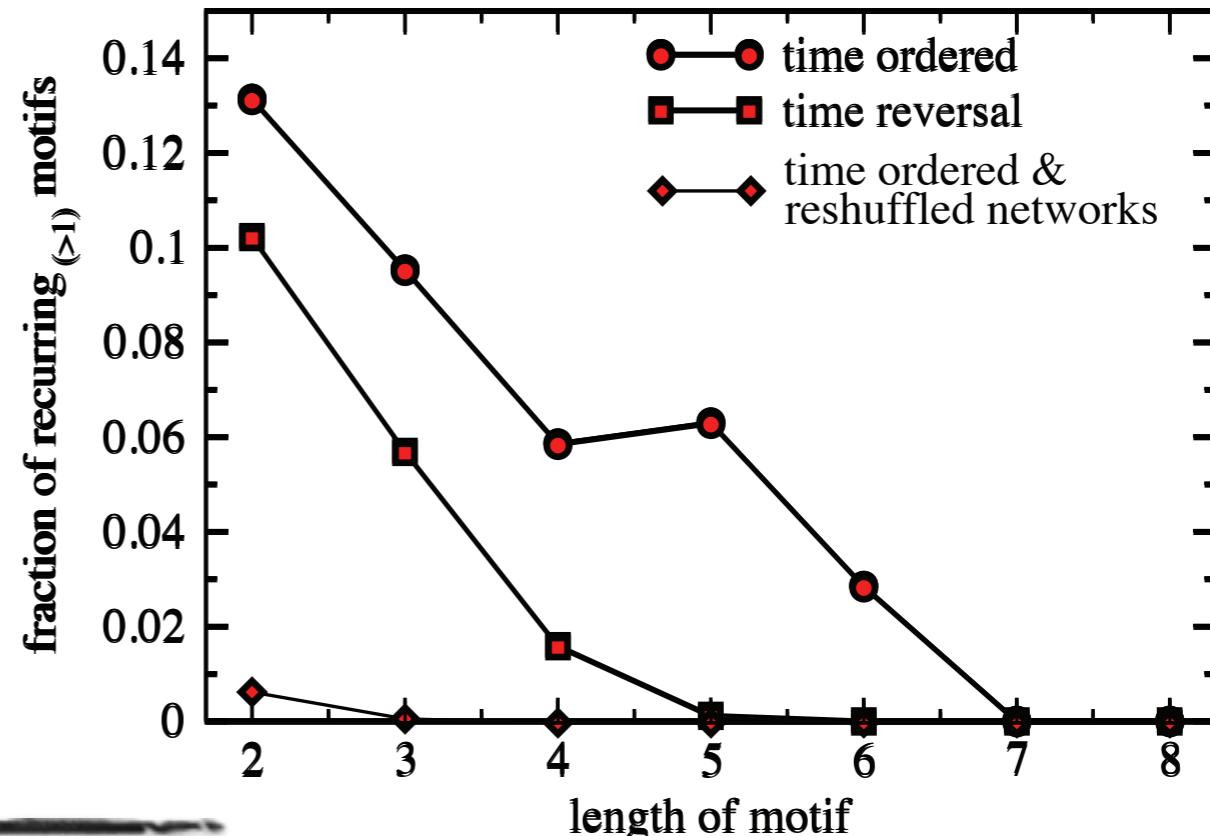
Original network:



Null model:  
reshuffled networks

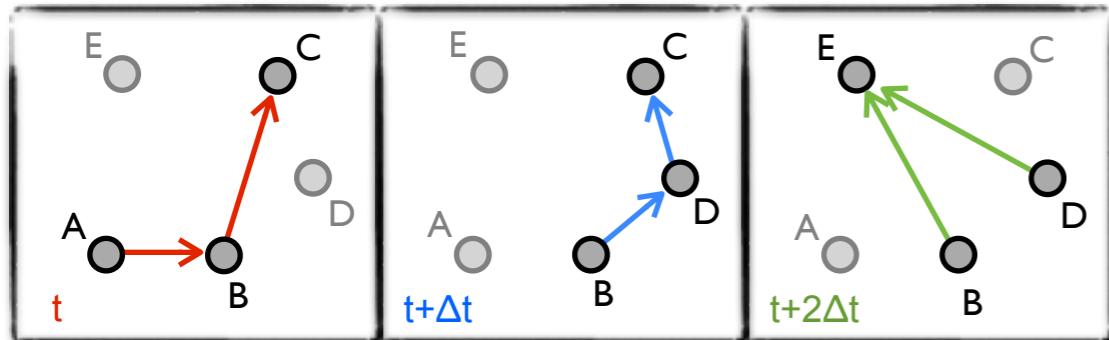


time

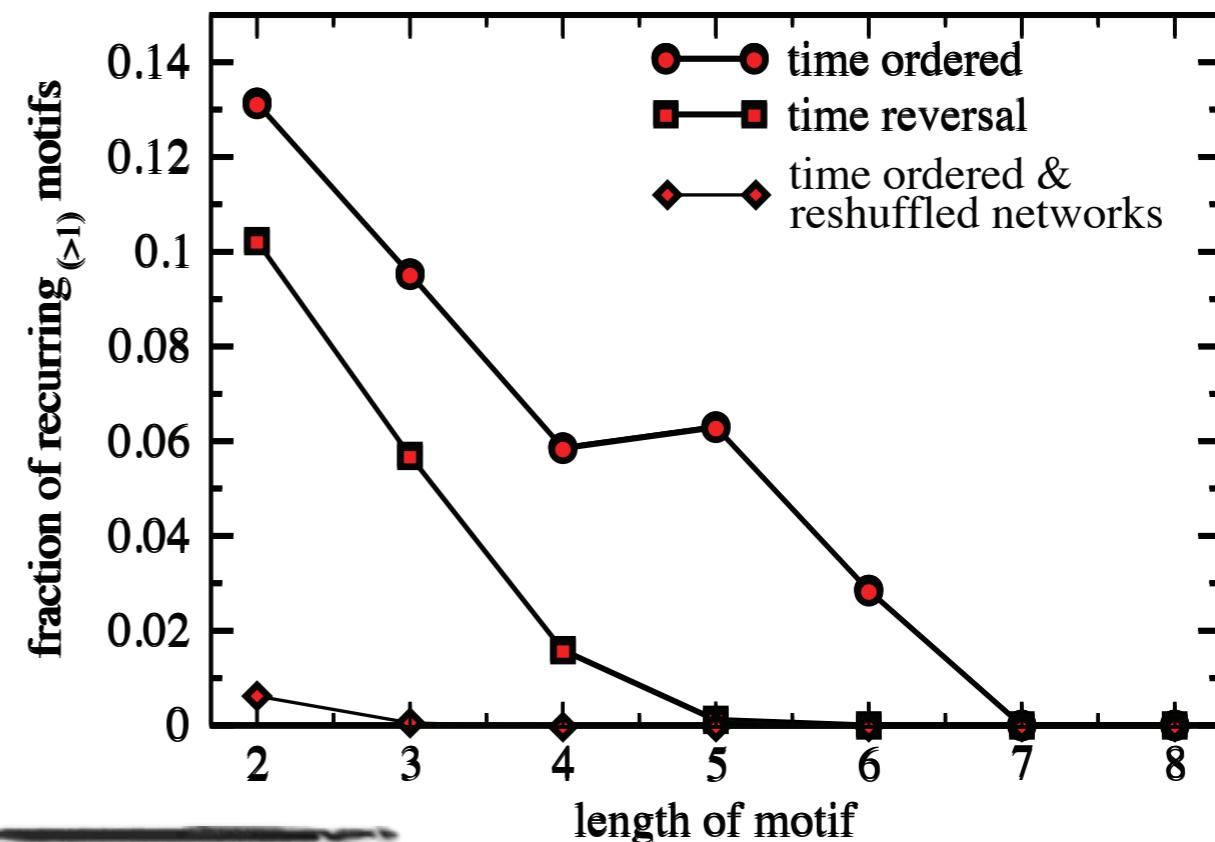
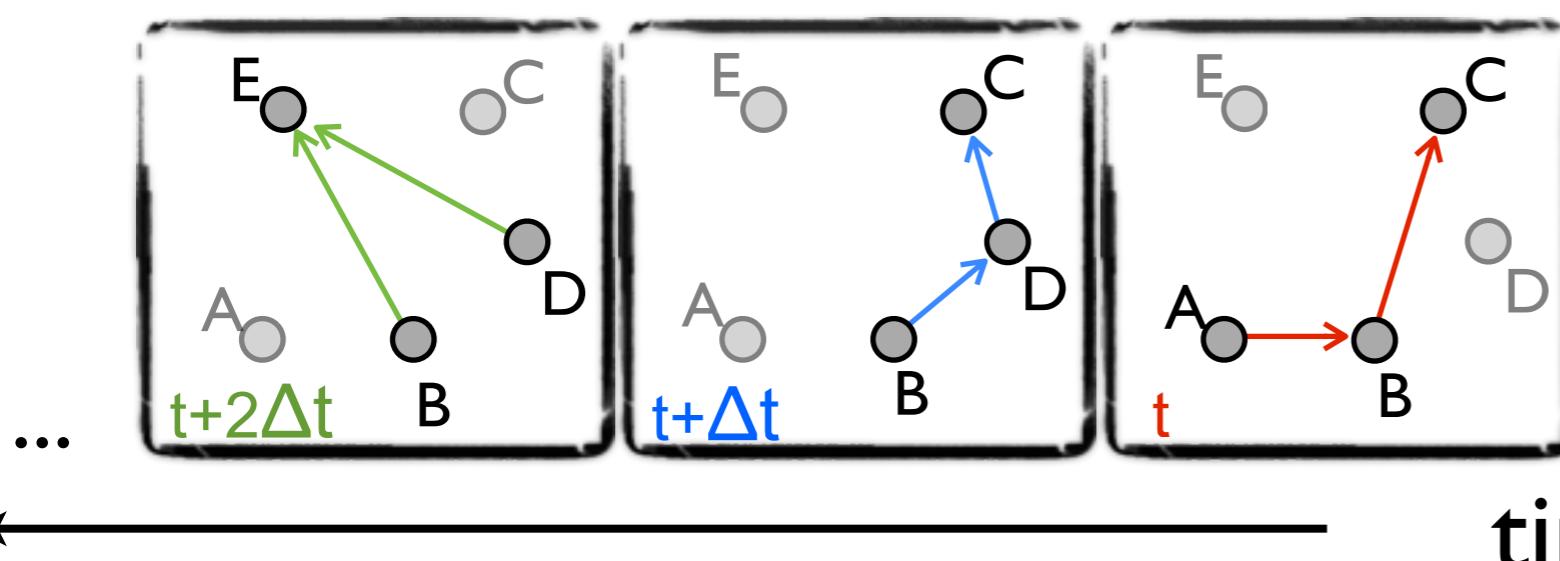


# Dynamical motifs

Original network:

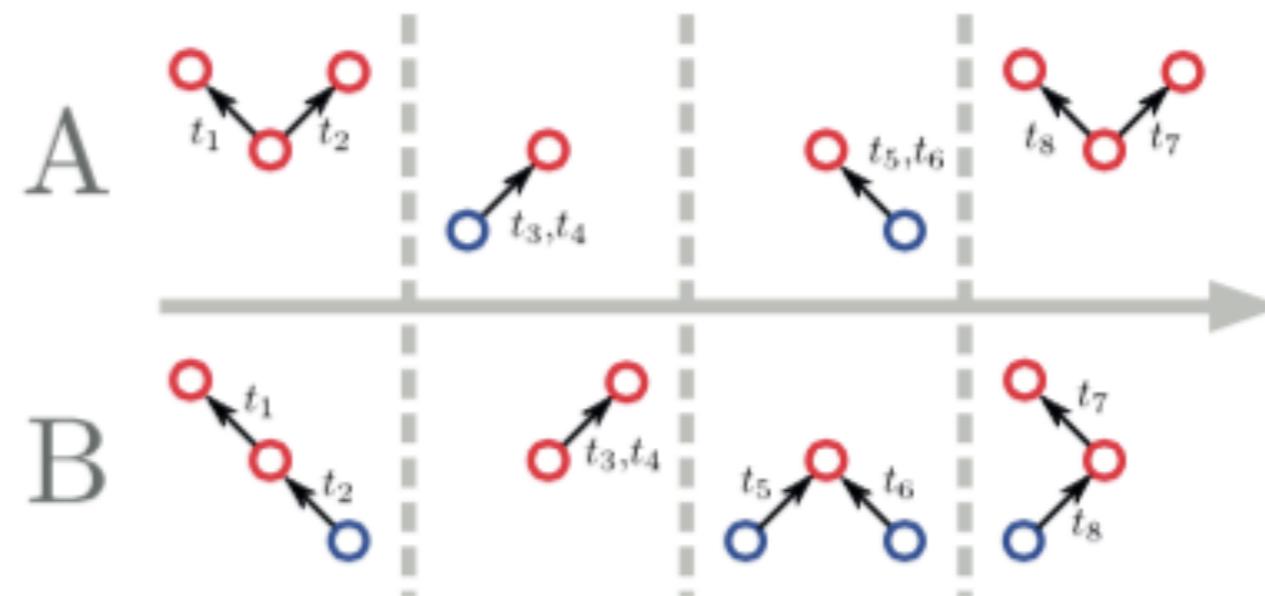


Null model:  
time reversal



# Temporal motifs

Same aggregated network, different temporal sequences



$$\sum_{t_1, \dots, t_8} =$$

The aggregated network is shown as a central red node connected to four blue nodes. Each edge is labeled with the value 2, indicating the frequency or strength of the connection across all time steps.

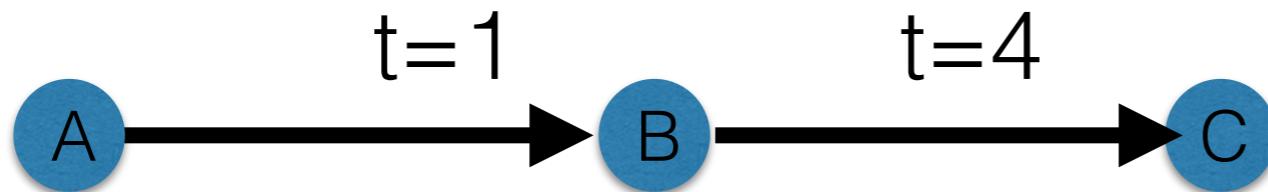


detection of temporal patterns?

L. Kovanen et al., J. Stat. Mech. (2011) P11005

L. Kovanen et al., PNAS (2013)

## $\Delta t$ -adjacent events



events  $\Delta t$ -adjacents for  $\Delta t=4$

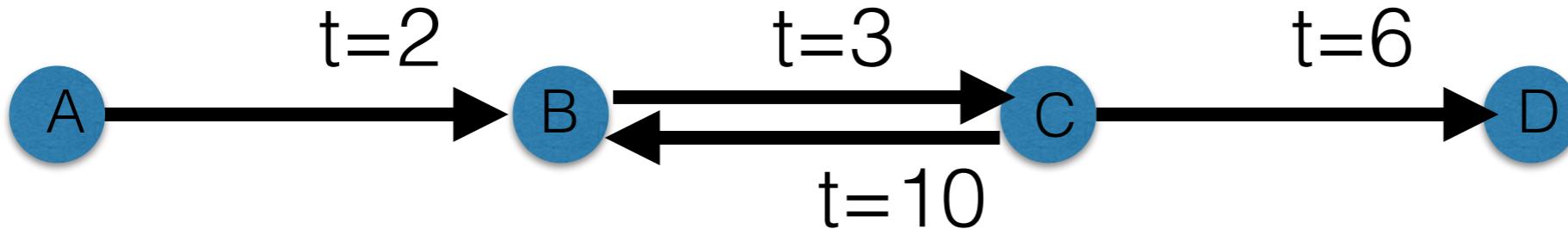
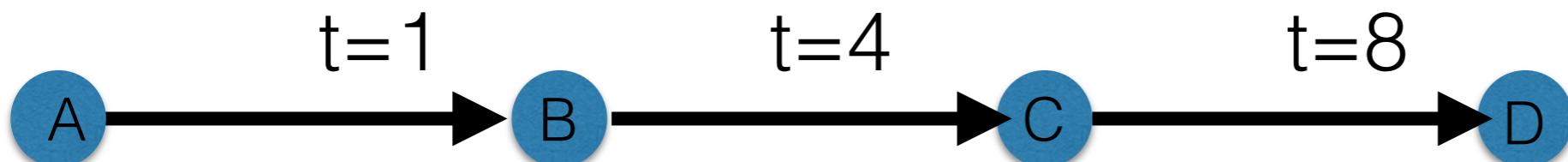
events  $\Delta t$ -adjacent:

- at least one node in common
- at most  $\Delta t$  between end of 1st event and start of 2nd event

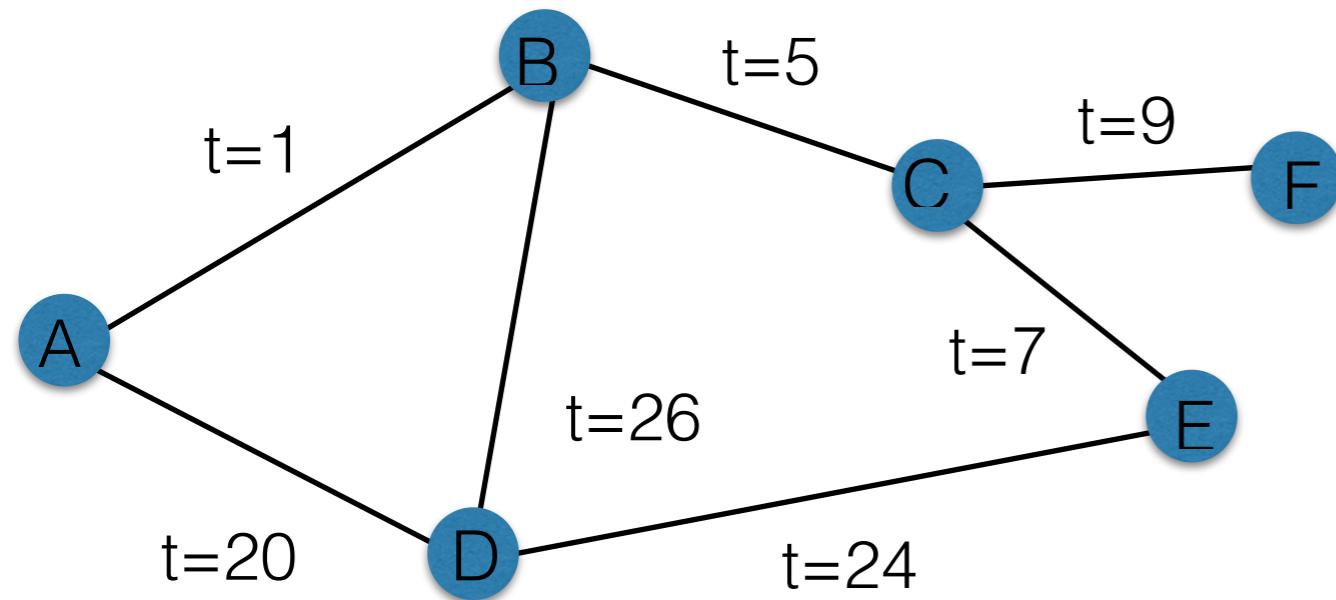
## $\Delta t$ -connected events

=events connected by a chain of  $\Delta t$ -adjacent events

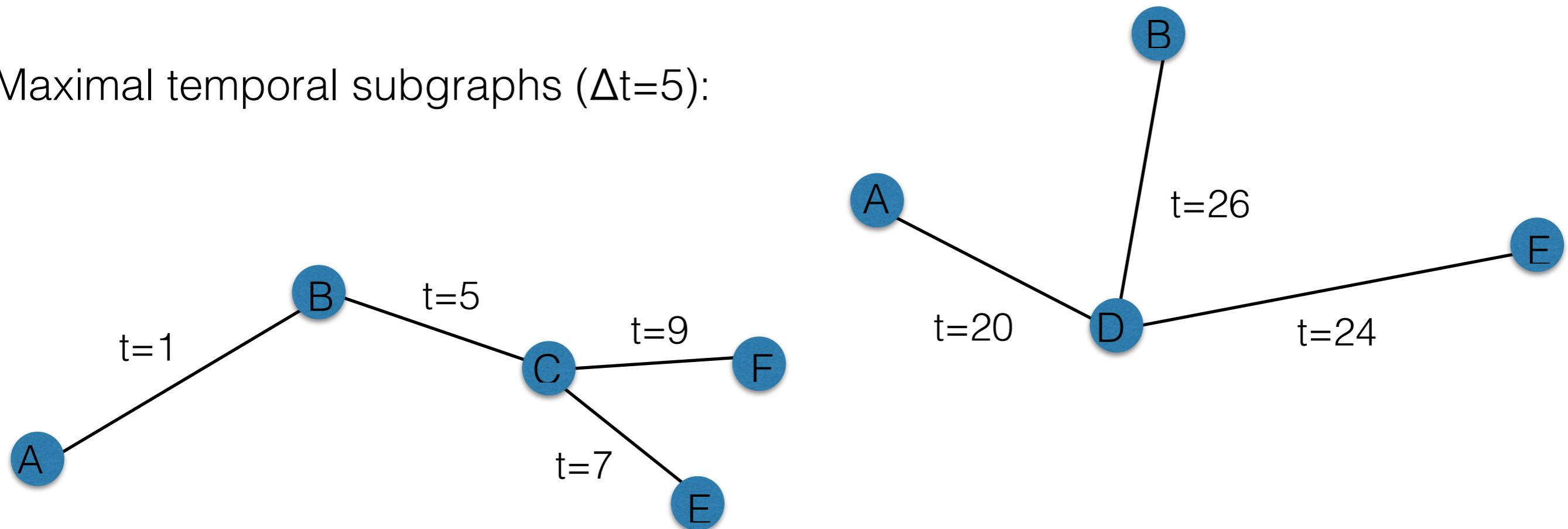
Examples ( $\Delta t=5$ ):



Temporal subgraph = set of  $\Delta t$ -connected events



Maximal temporal subgraphs ( $\Delta t=5$ ):

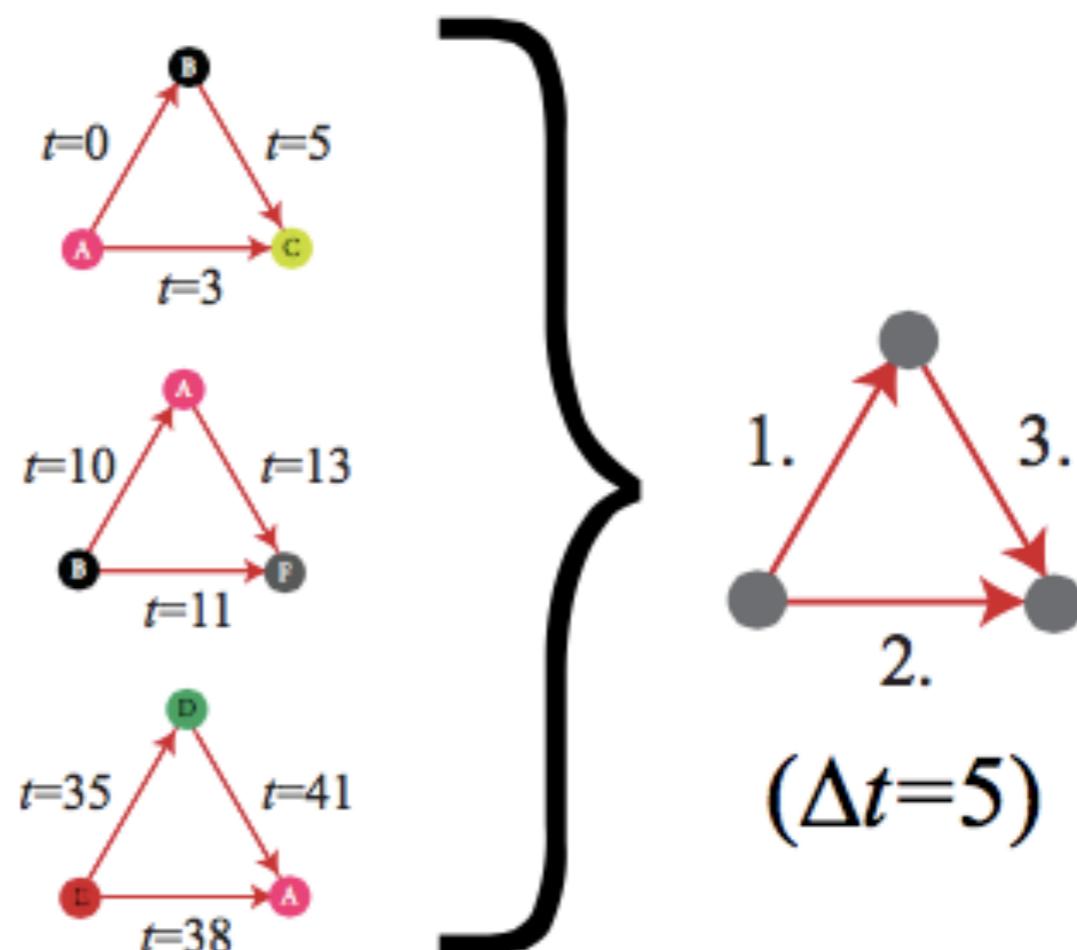


# Temporal motifs

= Equivalence classes of valid temporal subgraphs

Valid: no events are skipped at each node

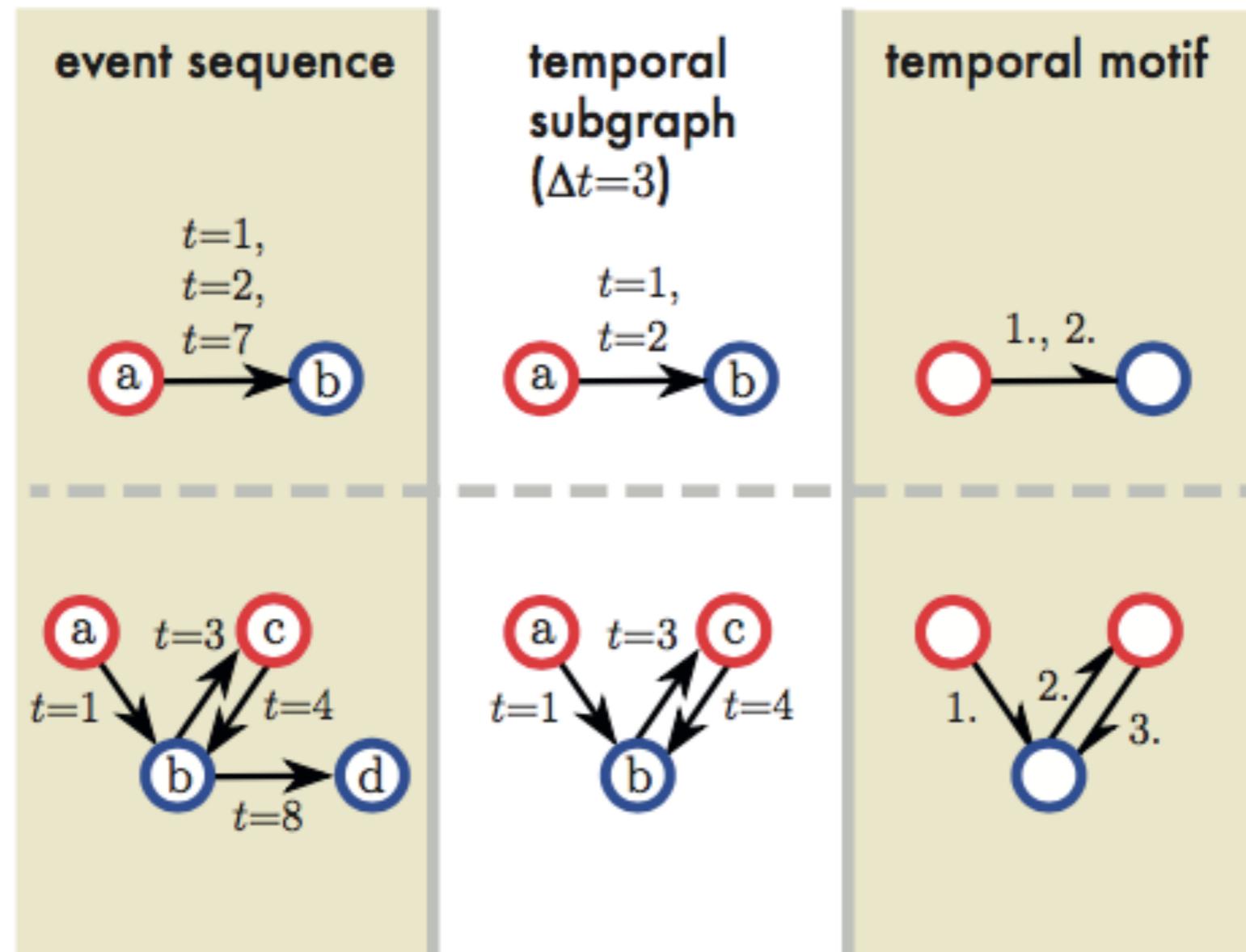
Equivalence classes:  
forget identities of nodes  
and exact timing



1

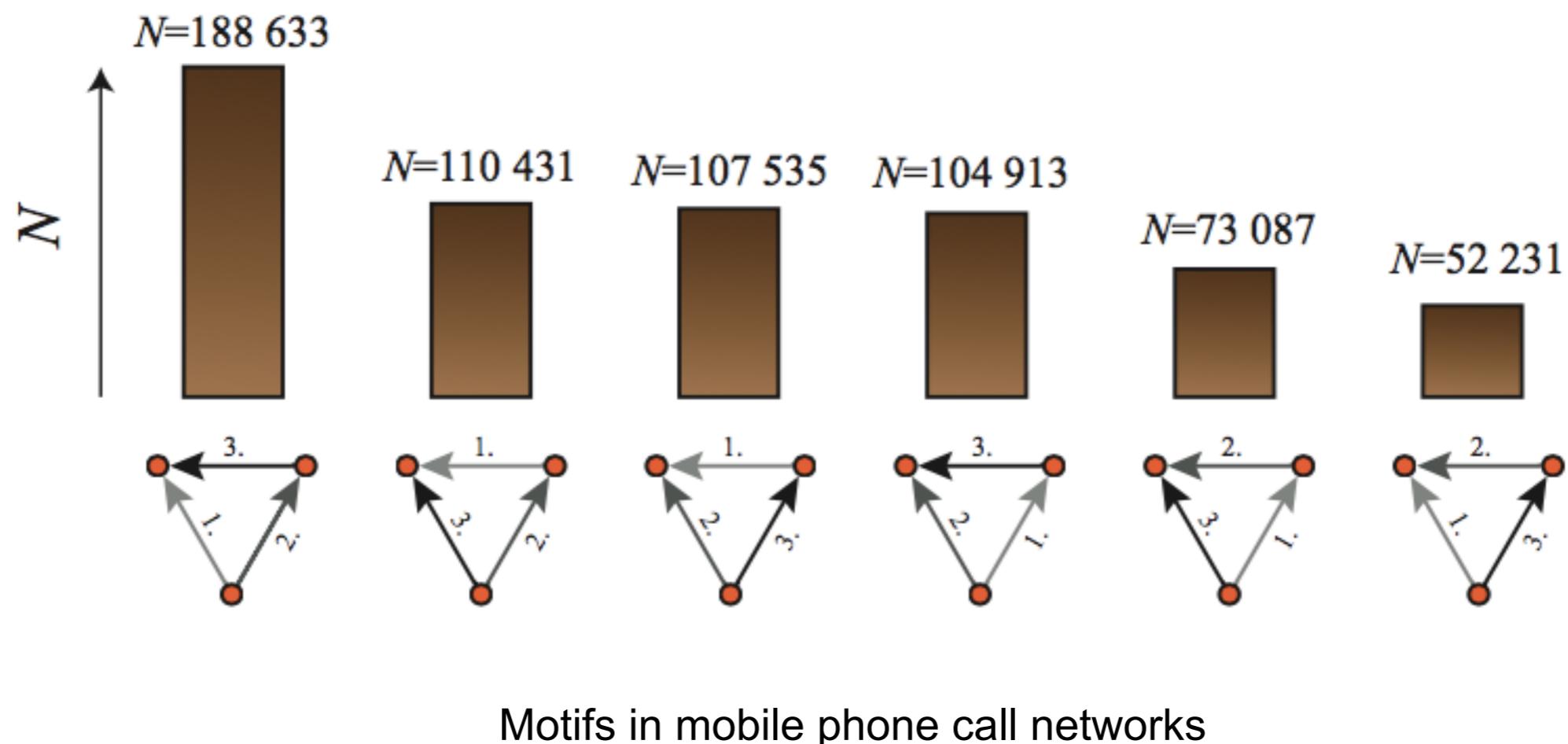
2

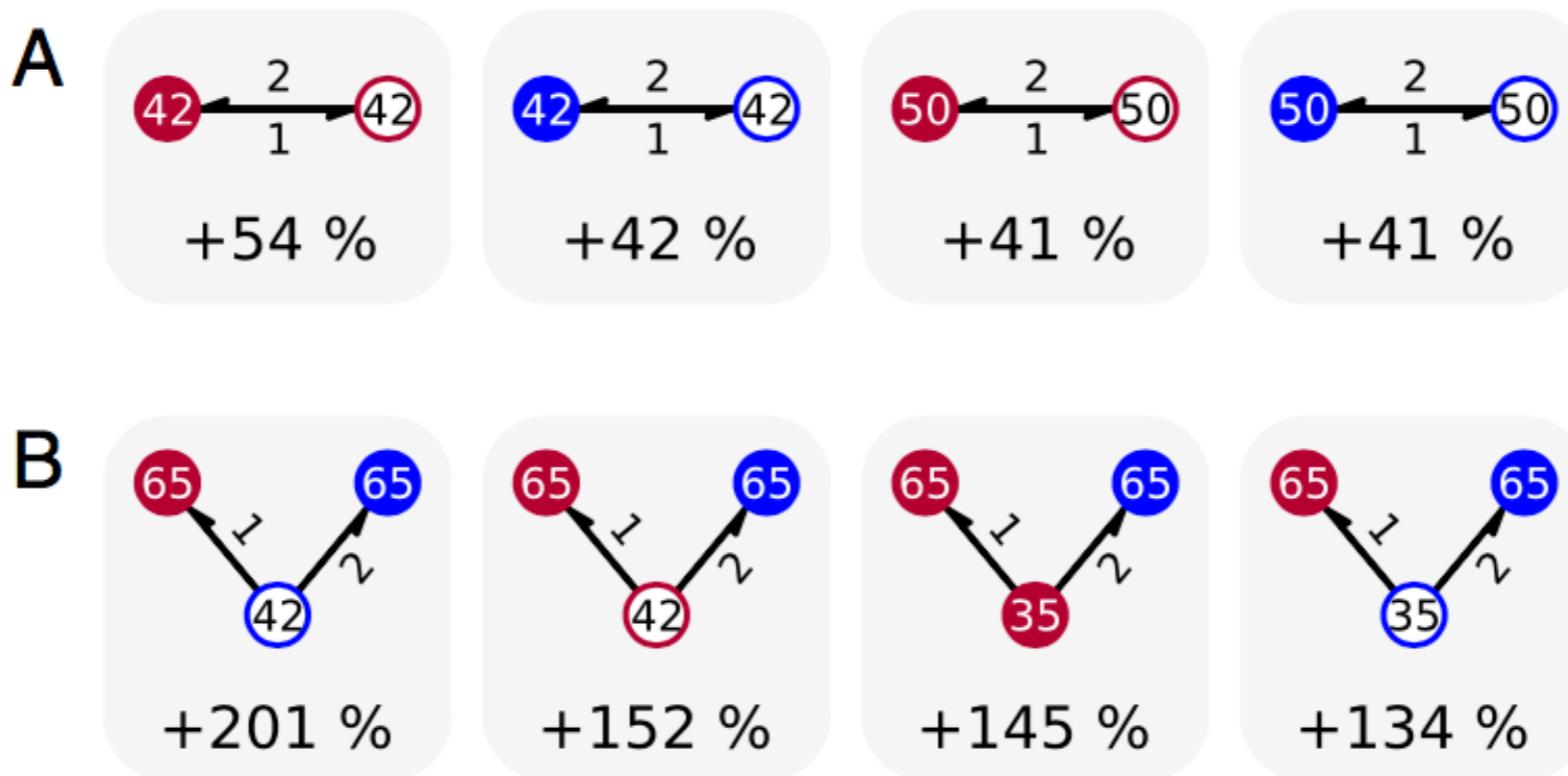
3



which motifs are most frequent?  
 metadata on nodes, compare with null models, ...

# Structure: temporal motifs





**Fig. 4.** The most common temporal motifs exhibit shared properties. (A) The four most common returned-call motifs. The numbers inside the nodes denote the age group (18–26, 27–32, 33–38, 39–45, 46–55, or 56–80; the value shown is the weighted average rounded to closest integer). The open nodes denote postpaid and filled prepaid customers; red denotes female, and blue, male. The arrows denote events, and the numbers next to them show their temporal order. In all four cases, the first call takes place from the prepaid (filled node) to the postpaid (open node) customer. The number below each motif shows the relative occurrence compared with the null model. (B) The four most common out-star motifs. In all four cases, the two receivers have the same age, a pattern that is typical for the most common out-stars.

Temporal motifs reveal homophily, gender-specific patterns, and group talk in call sequences  
L. Kovanen et al., PNAS (2013)

> Finding Structures:  
decomposing a temporal network

# Detection of structures: decomposition of temporal network

Data: temporal network with discrete time intervals



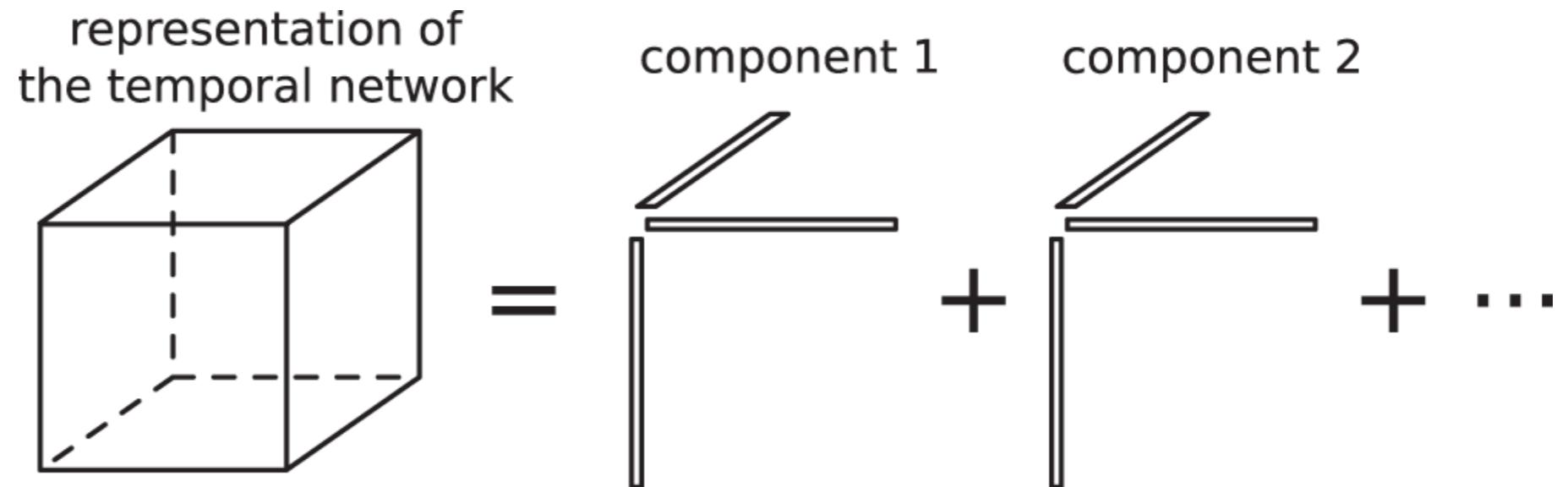
Time-dependent adjacency matrix  $A(i,j,t)$



Three-way tensor  $T$

# Detection of structures: decomposition of temporal network

## Kruskal decomposition



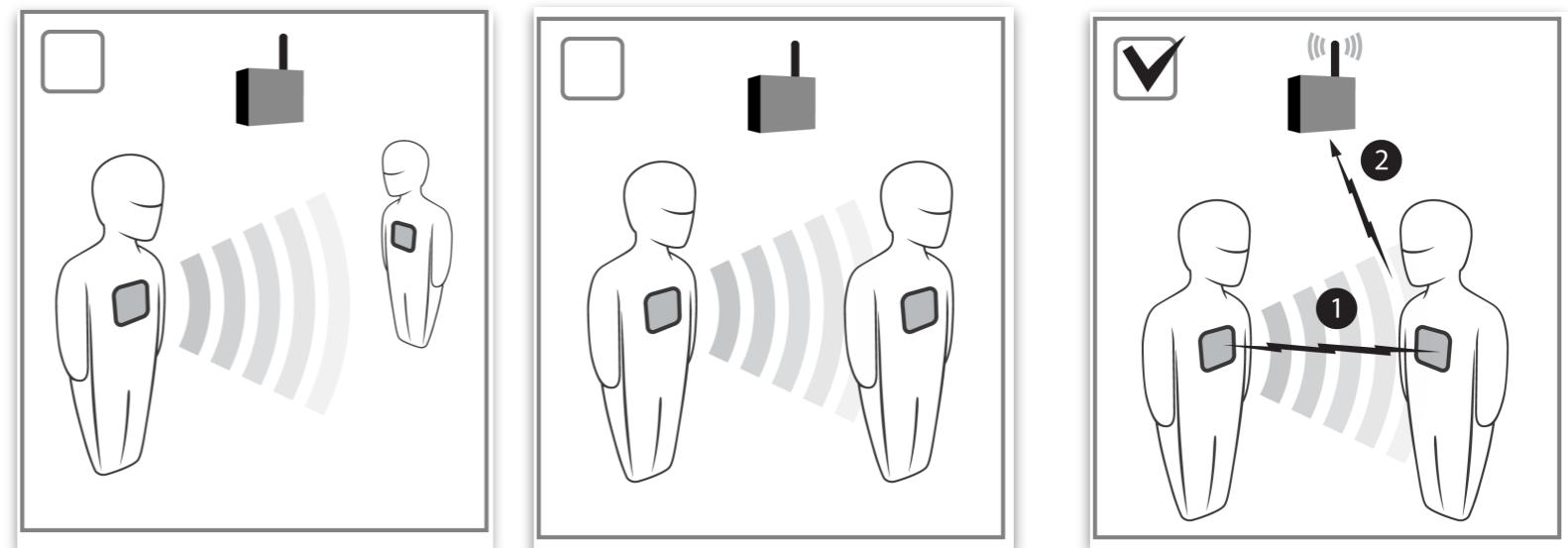
$$T \approx \tilde{\mathcal{T}} = \sum_{r=1}^R \mathcal{S}_r = \sum_{r=1}^R \mathbf{a_r} \circ \mathbf{b_r} \circ \mathbf{c_r}$$

$$t_{ijk} \approx \sum_{r=1}^R a_{ir} b_{jr} c_{kr}$$

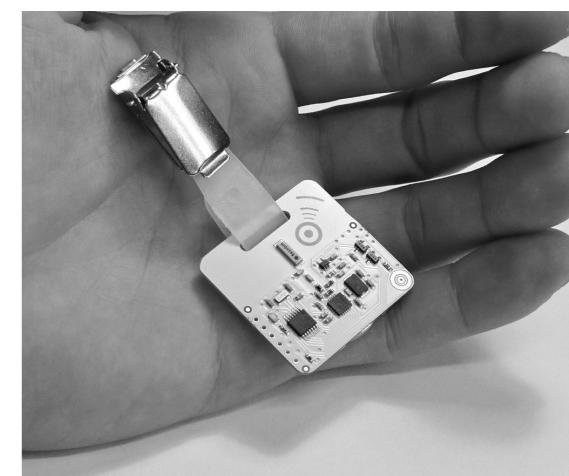
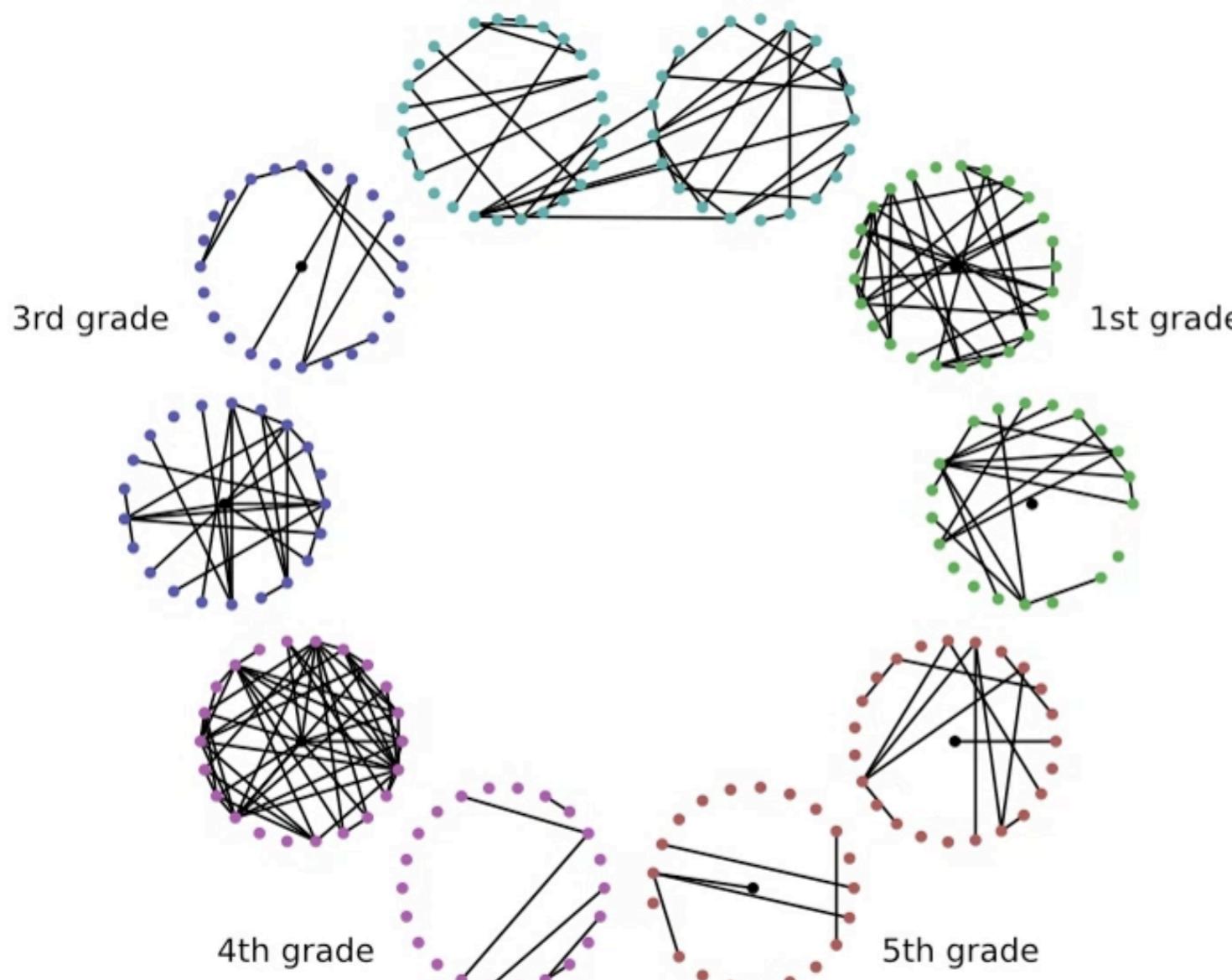
undirected network:  $a=b$

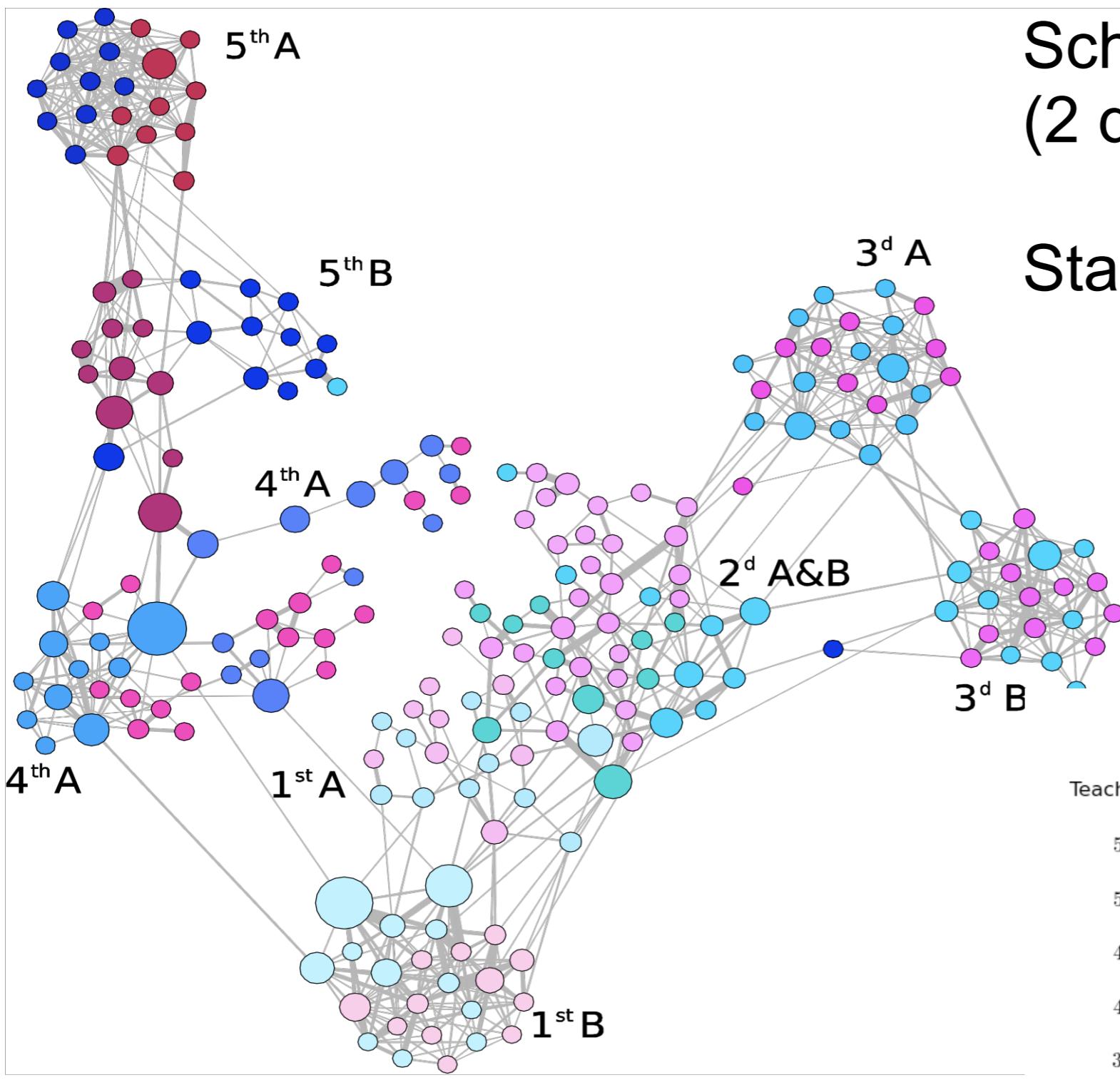
$a_{ir}$  = membership of node  $i$  to component  $r$   
 $c_{kr}$  = timeline of component  $r$

# Example: contacts in a primary school



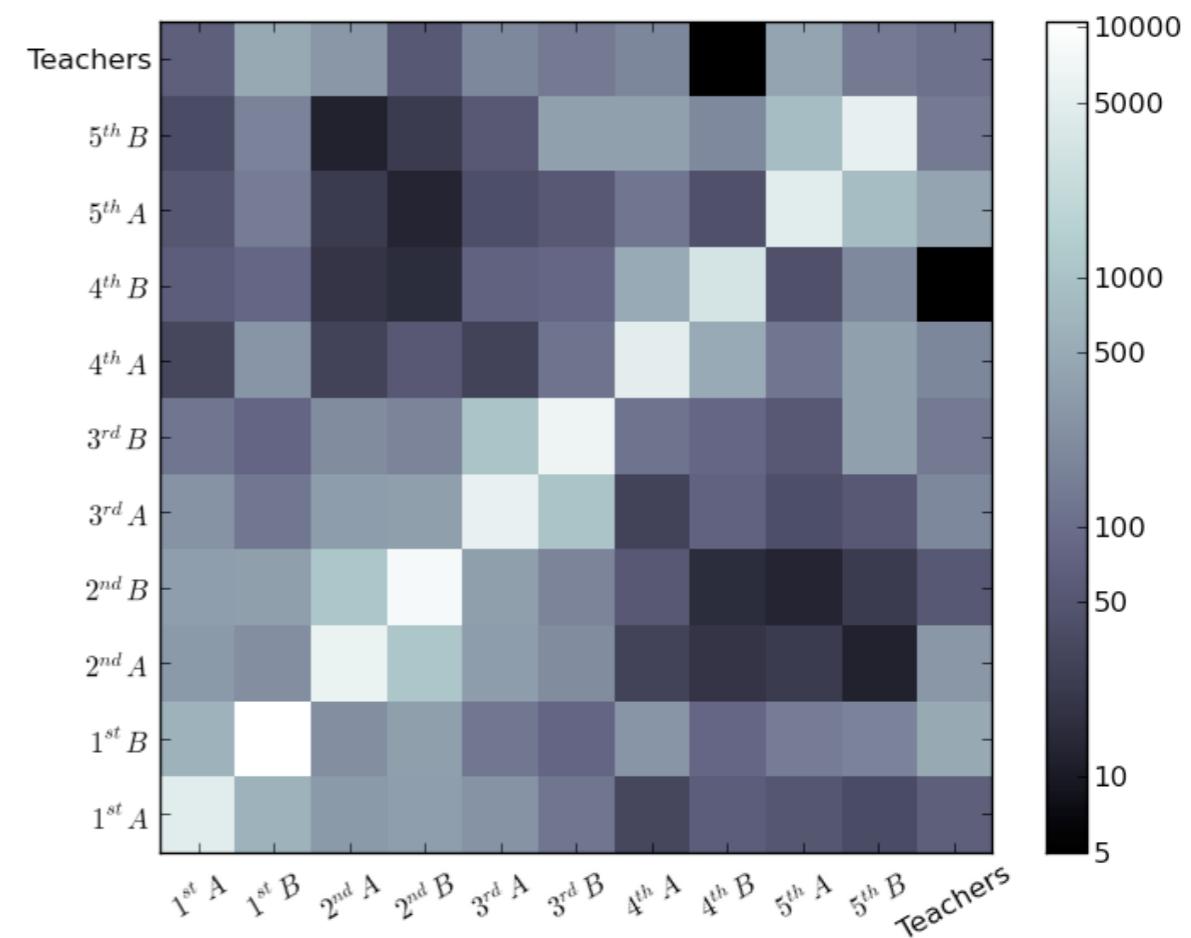
2nd grade

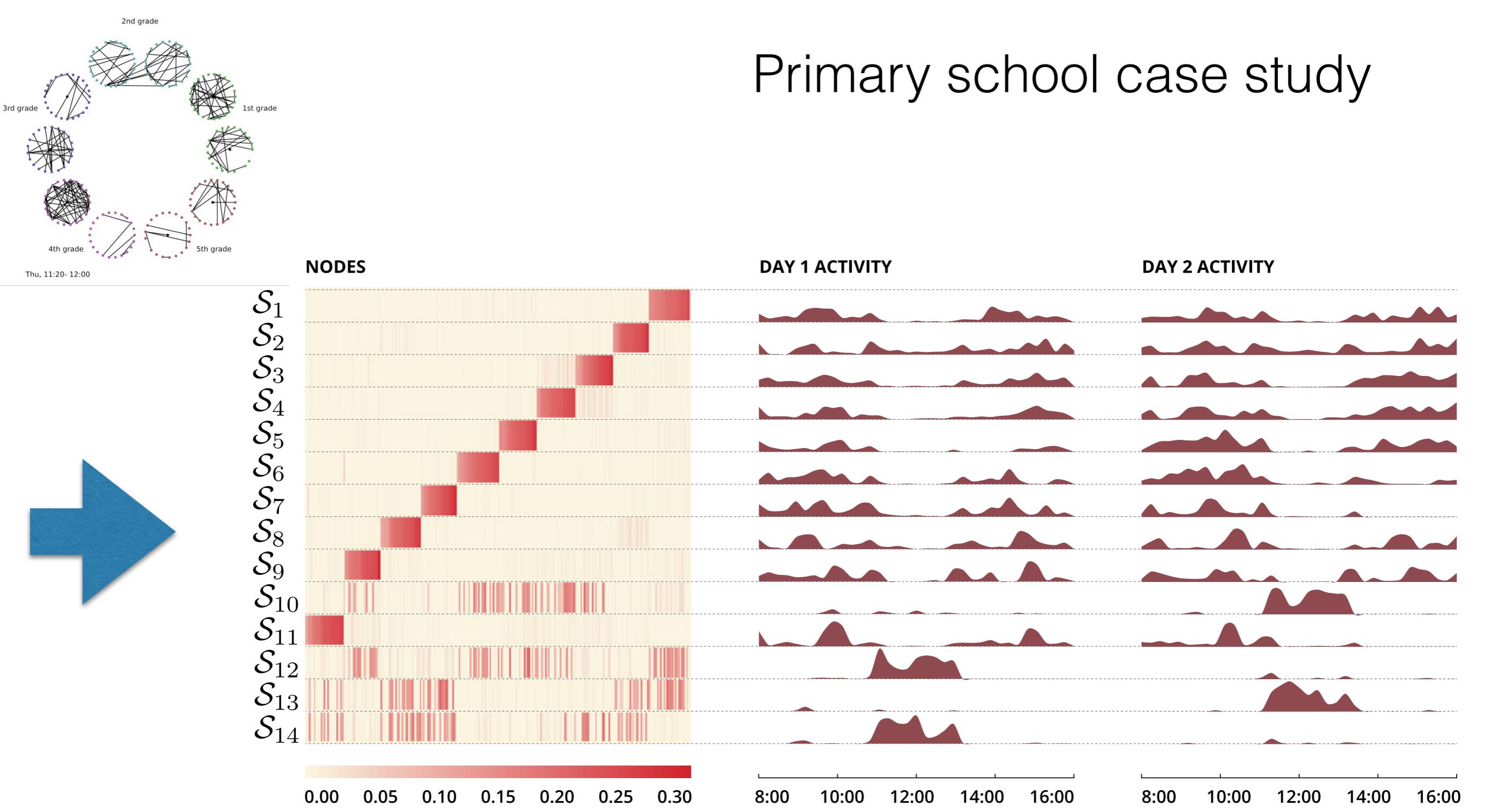




School cumulative f2f network  
(2 days, 2 min threshold)

Statically: 10 classes





10 classes (also revealed by static analysis)

4 mixed-membership components (links with synchronised activities) = breaks

# Relevance for spreading processes

arXiv.org > physics > arXiv:1501.02758

Search...

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Physics > Physics and Society

[Submitted on 12 Jan 2015]

## Revealing latent factors of temporal networks for mesoscale intervention in epidemic spread

Laetitia Gauvin, André Panisson, Alain Barrat, Ciro Cattuto

The customary perspective to reason about epidemic mitigation in temporal networks hinges on the identification of nodes with specific features or network roles. The ensuing individual-based control strategies, however, are difficult to carry out in practice and ignore important correlations between topological and temporal patterns. Here we adopt a mesoscopic perspective and present a principled framework to identify collective features at multiple scales and rank their importance for epidemic spread. We use tensor decomposition techniques to build an additive representation of a temporal network in terms of mesostructures, such as cohesive clusters and temporally-localized mixing patterns. This representation allows to determine the impact of individual mesostructures on epidemic spread and to assess the effect of targeted interventions that remove chosen structures. We illustrate this approach using high-resolution social network data on face-to-face interactions in a school and show that our method affords the design of effective mesoscale interventions.

### Effect of removing a component:

NB:

- components 12, 14 correspond to less total weight than “class”-components
- actionable interventions

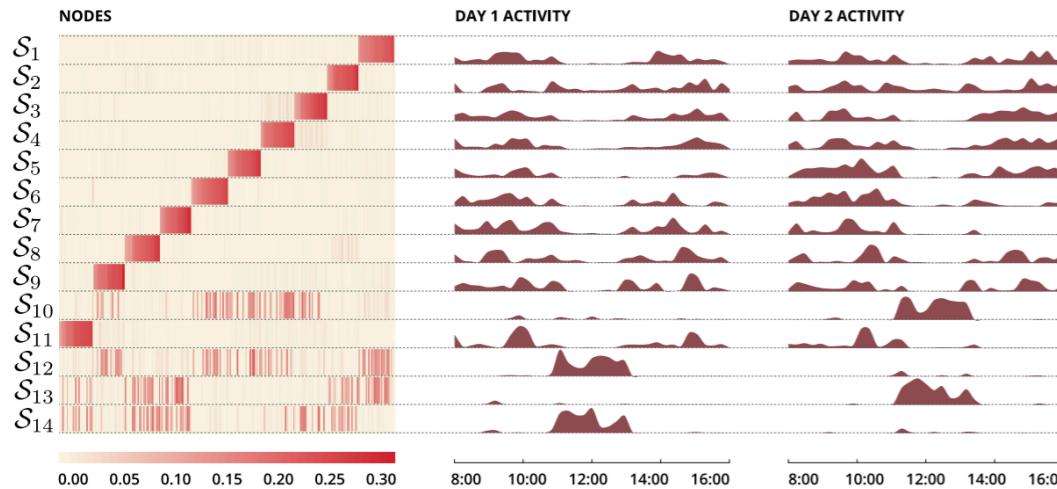
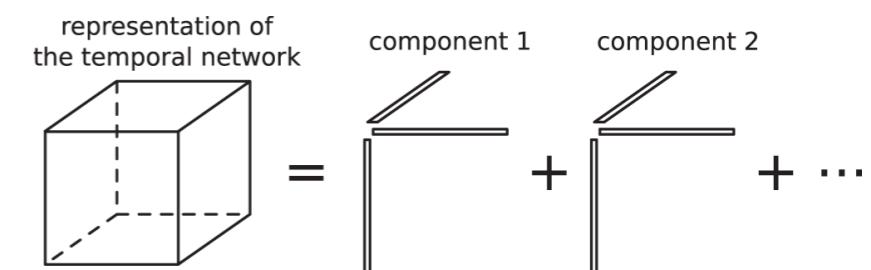


Figure 1: Components obtained by tensor factorization of the school temporal network.



$$T \approx \tilde{T} = \sum_{r=1}^R \mathcal{S}_r = \sum_{r=1}^R \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$$

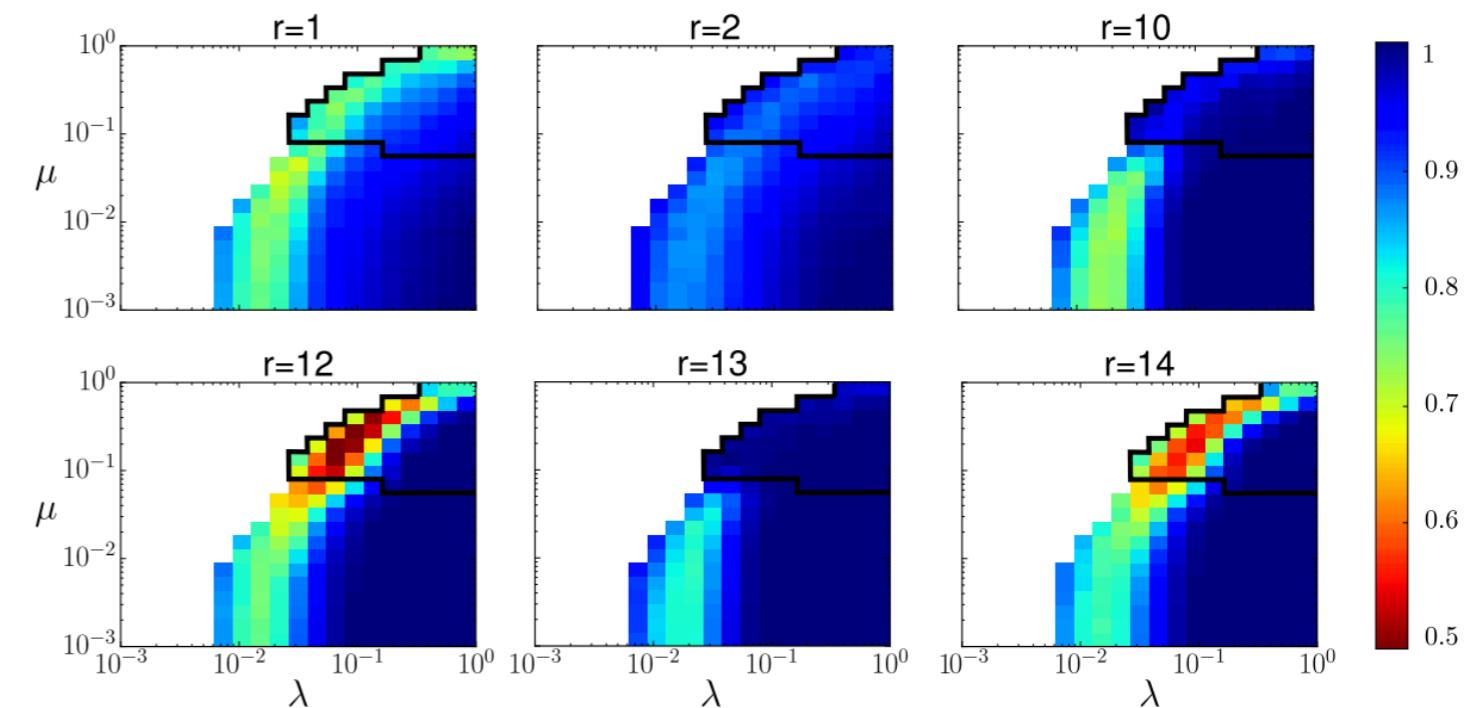


Figure 3: Epidemic size ratio as a function of SIR model parameters. Each heat map corresponds to a targeted intervention that selectively removes one component. For each removed component

# >Finding structures: Backbones - significant ties



Article | OPEN | Published: 15 January 2019

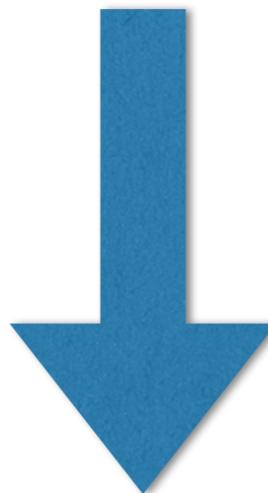
The structured backbone of temporal  
social ties

Teruyoshi Kobayashi, Taro Takaguchi & Alain Barrat

*Nature Communications* **10**, Article number: 220 (2019) | Download Citation

# Filtering methods - static networks

Large scale data: mix of “important” and “random/noisy” links

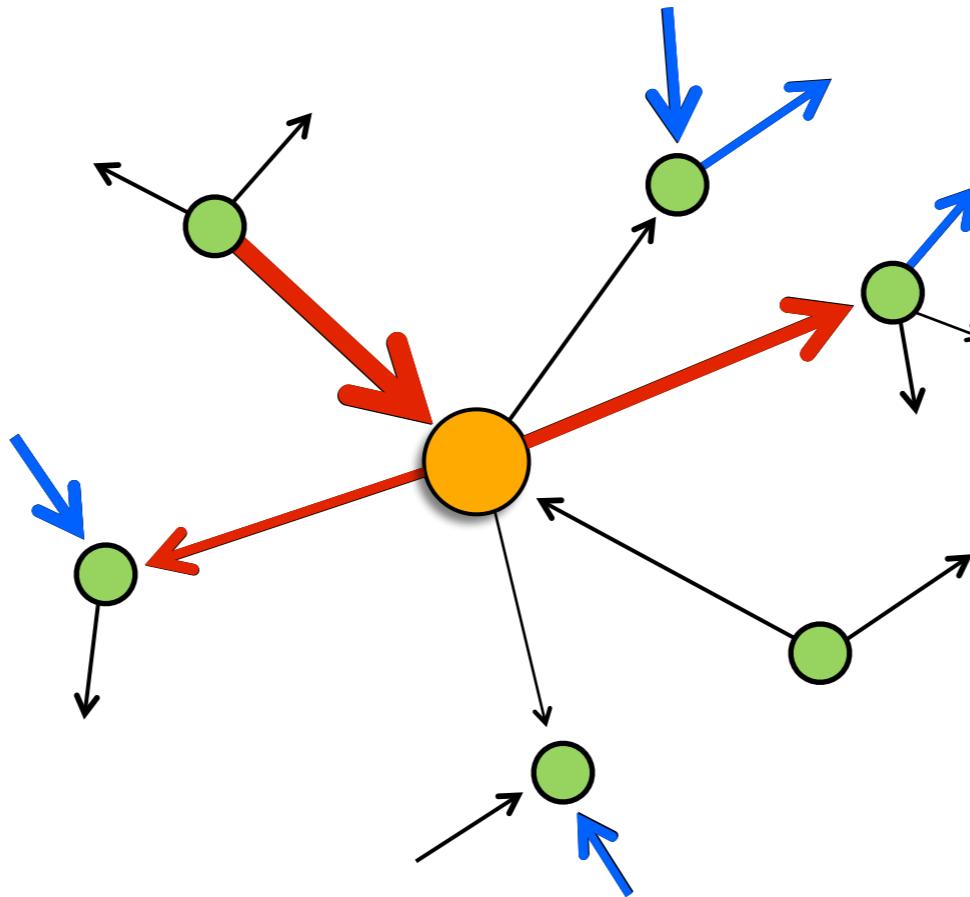


How to extract the most relevant ties?

# Filtering methods - static networks

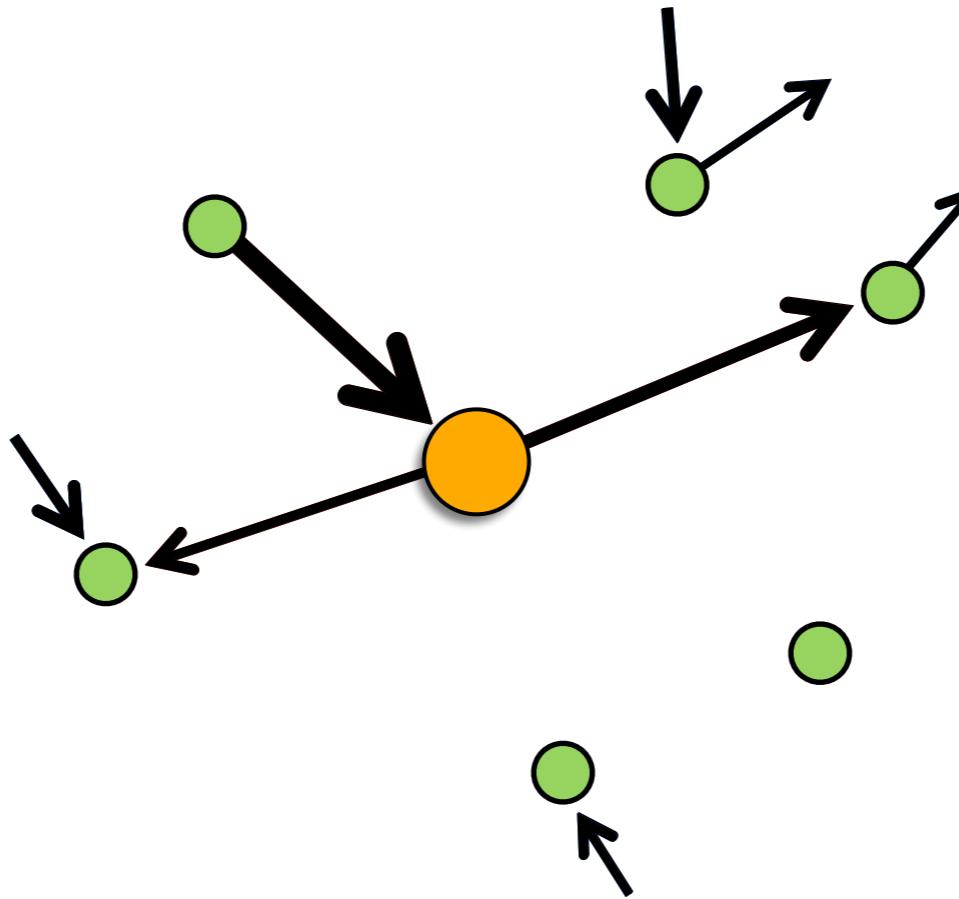
- Retaining only large weights: links with weights larger than a (tunable) threshold => **global filter**
- Filters **based on a null model**: retain only weights larger than expected in a null model
  - **Disparity filter**: multiscale backbone (Serrano et al., PNAS 2009): for each node, retain edges that are “important for that node” => local filter

# Disparity filter



The disparity filter selects the statistically significant links when the weights are heterogeneously distributed.

# Disparity filter



The disparity filter selects the statistically significant links when the weights are heterogeneously distributed.

# Filtering methods - static networks

- Retaining only large weights: links with weights larger than a (tunable) threshold => **global filter**
- Filters **based on a null model**: retain only weights larger than expected in a null model
  - **Disparity filter**: multiscale backbone (Serrano et al., PNAS 2009): for each node, retain edges that are “important for that node” => local filter
  - **Enhanced configuration model** (Gemmetto et al., arXiv: 1706.00230) = max entropy ensemble of networks with fixed degree and strength sequences (**null model**) => **p-value of observing a weight  $w_{ij}$  between i and j larger than or equal to the real weight** => retain only links with small p-value

# Filtering method - temporal network

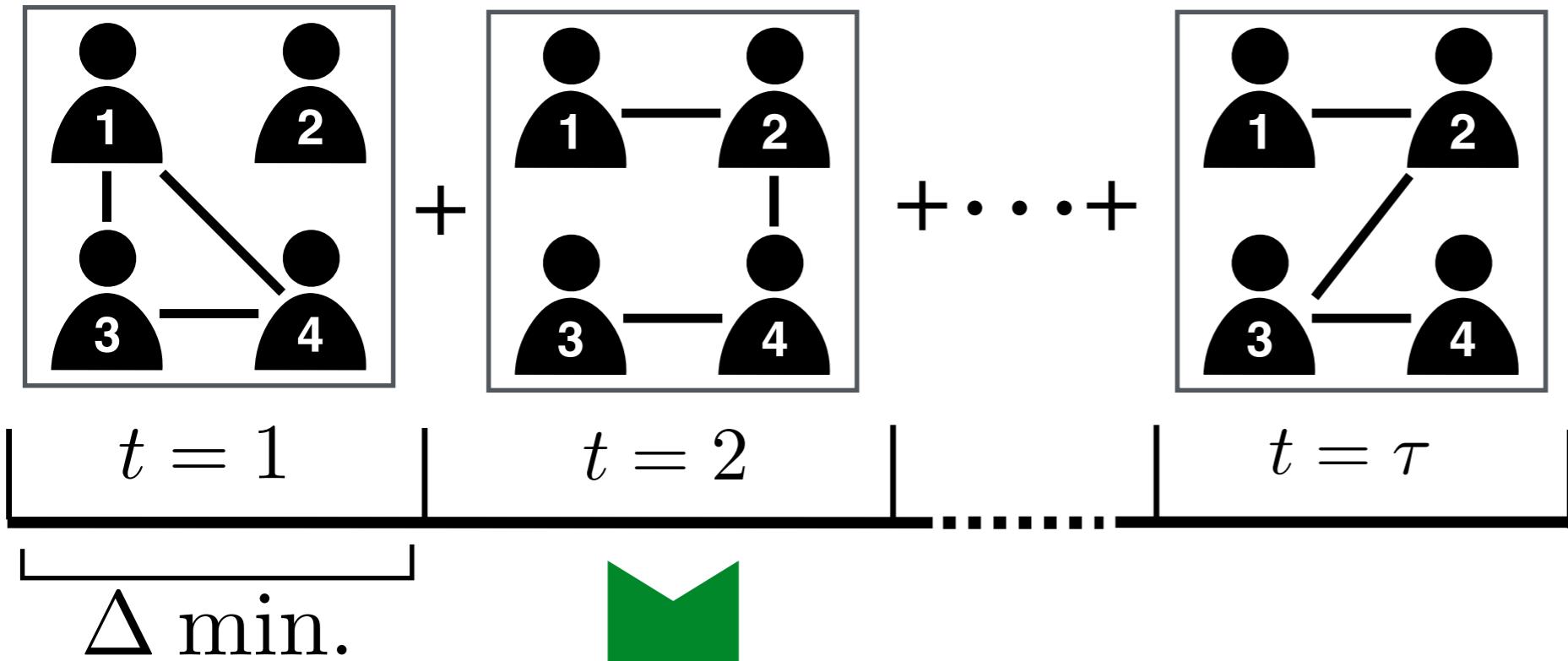
Use a **temporal null model**:

- define the temporal fitness model
  - node i, activity  $a_i$
  - probability of interaction at any time:  $u(a_i, a_j) = a_i a_j$
- evaluate activities  $a_i^*$  of nodes (max likelihood)
- compute the probability distribution of the number of interactions

$$g(m_{ij} | a_i^*, a_j^*) = \binom{\tau}{m_{ij}} u(a_i^*, a_j^*)^{m_{ij}} (1 - u(a_i^*, a_j^*))^{\tau - m_{ij}}.$$

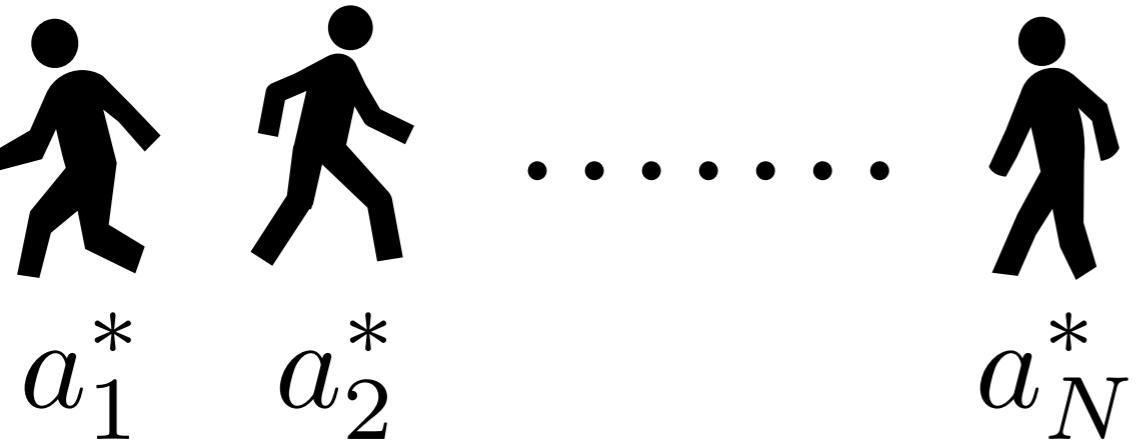
- ij is a significant tie if the observed number has a small p-value w.r.t.  $g$

## Interaction snapshots (binary)



## Aggregate adjacency

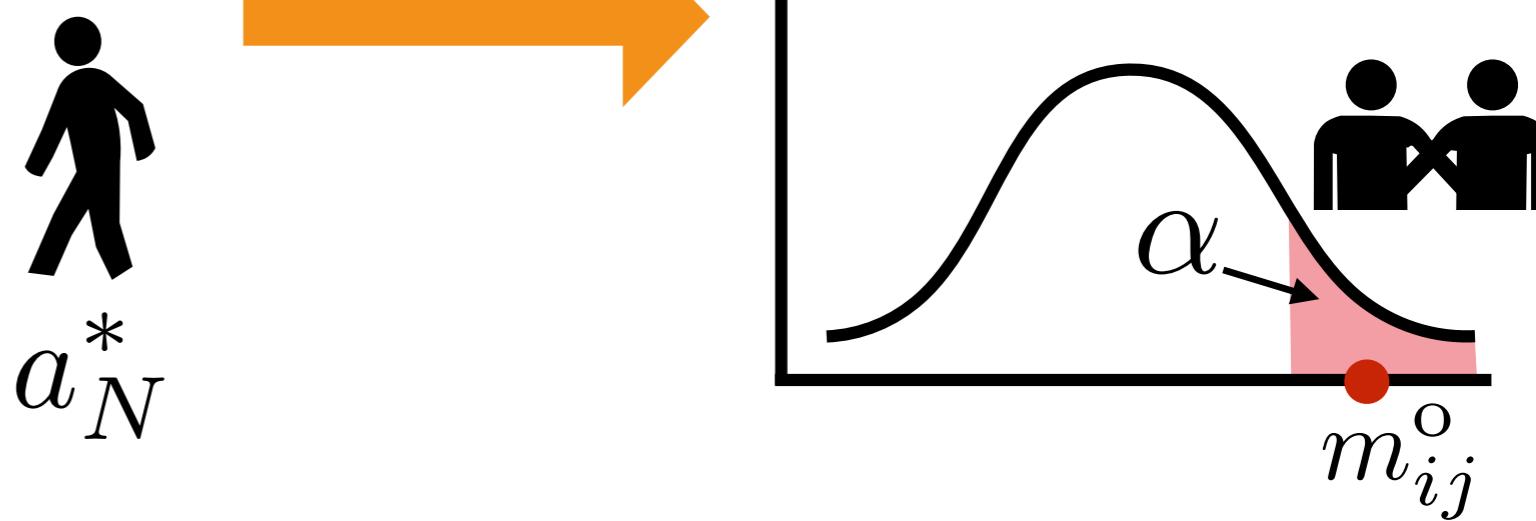
### Activity estimation



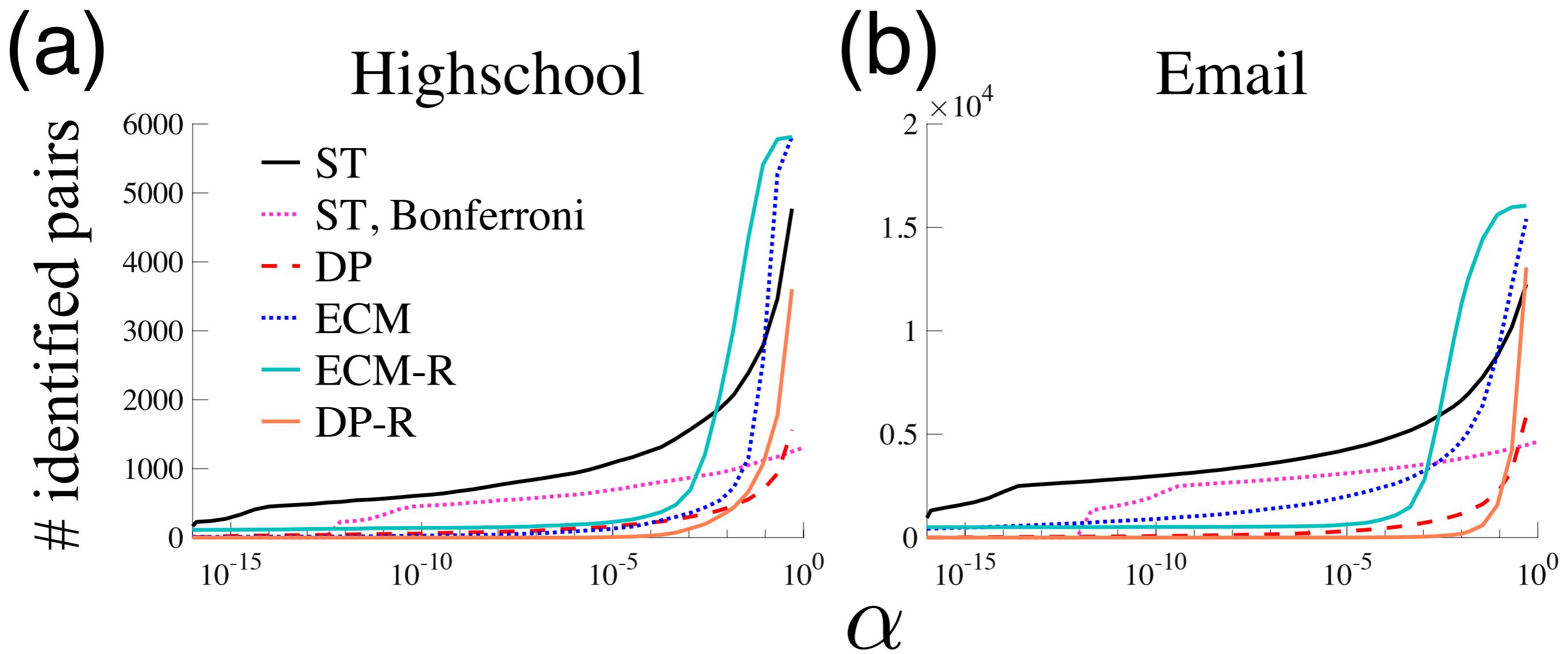
$$\{a_i^*\}$$

### Null distribution

$$m_{ij} \sim B(\tau, u(a_i^*, a_j^*))$$

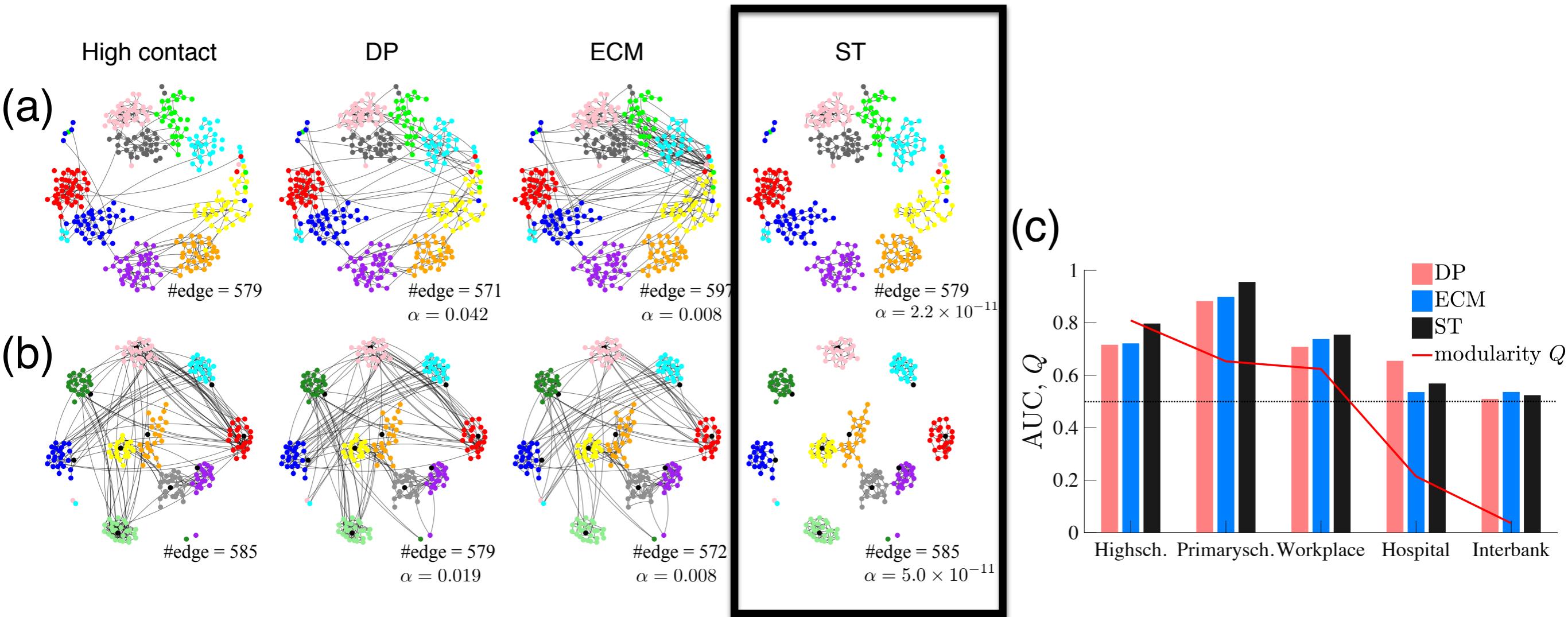


# Examples



- More ties detected than with other methods
- Range of significance values with **stable** backbone
-

# Example: SocioPatterns datasets



- More ties detected than with other methods
- Range of significance values with stable backbone
- Significant ties seem to be mostly within groups, when a community structure is present

# Beyond significant ties: significant structures

Probability that i,j,k form a triangle of **simultaneous** interactions at a given time

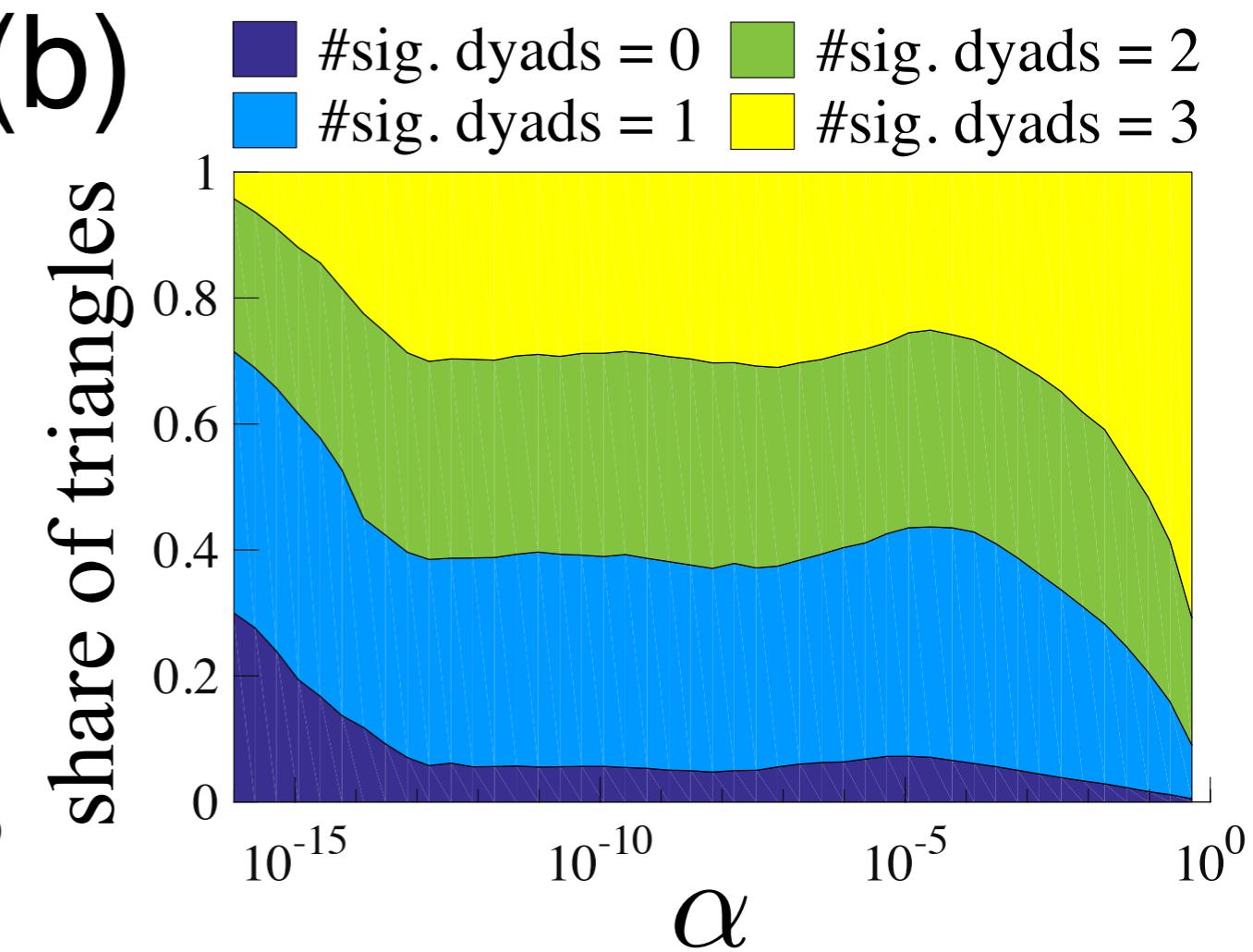
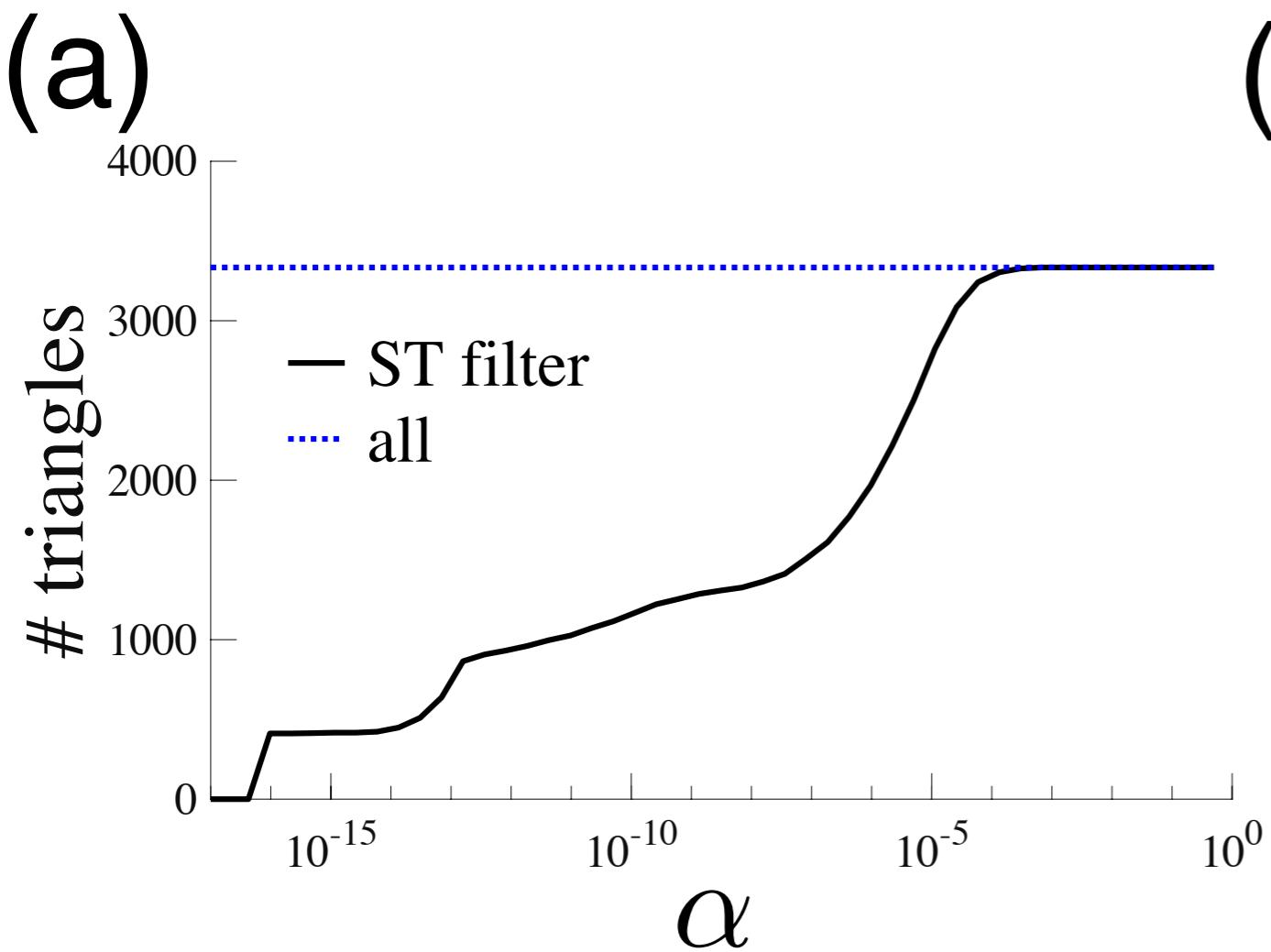
$$v(i, j, k) = u(a_i^*, a_j^*) \cdot u(a_j^*, a_k^*) \cdot u(a_k^*, a_i^*),$$

Probability that the triangle i,j,k is observed  $r_{ijk}$  times in the null model:

$$h(r_{ijk} | a_i^*, a_j^*, a_k^*) = \binom{\tau}{r_{ijk}} v(i, j, k)^{r_{ijk}} (1 - v(i, j, k))^{\tau - r_{ijk}}$$

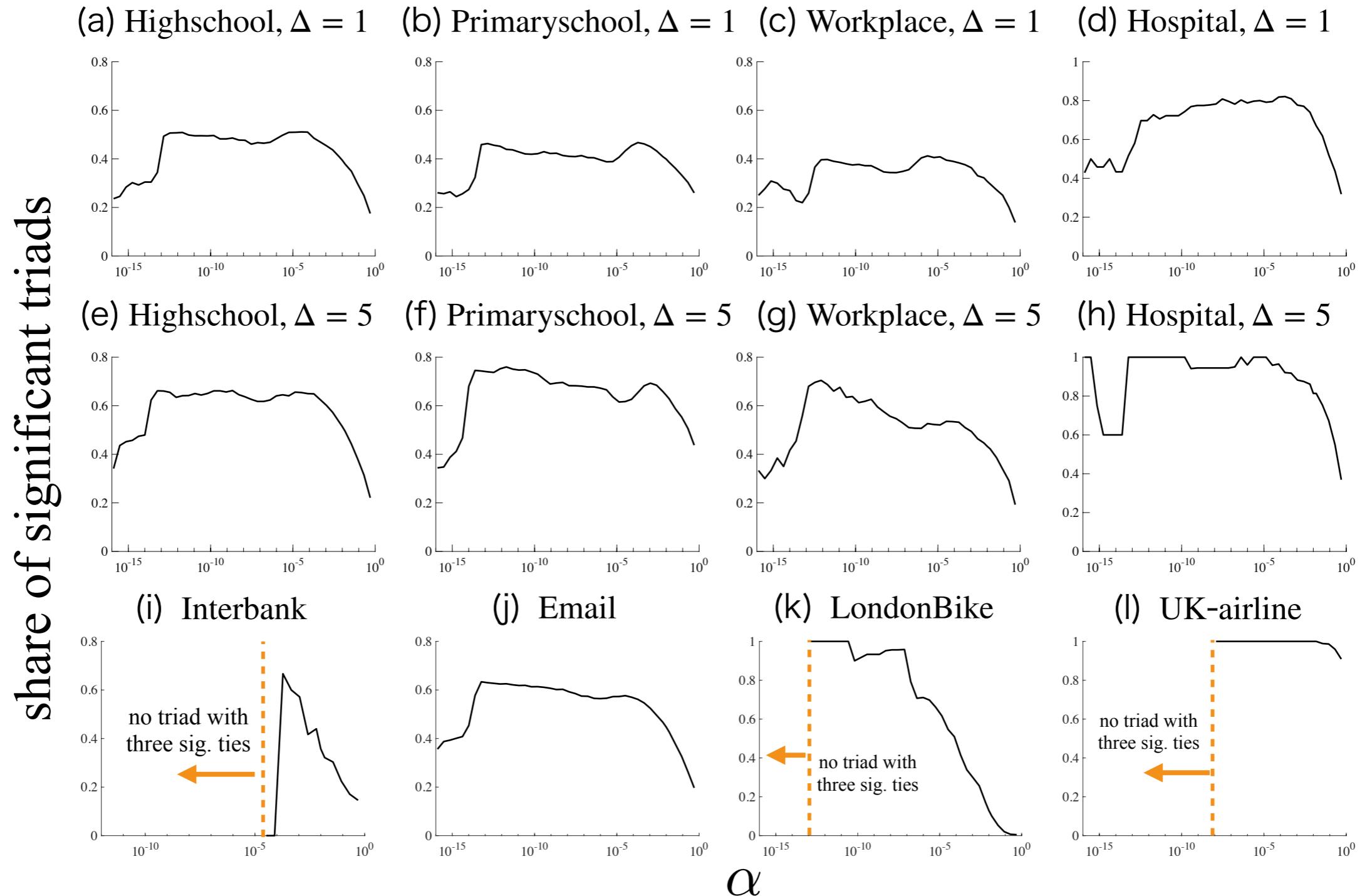
=> possibility to **assign a significance to a triad of simultaneous interactions** or to other temporal motifs/structures  
(not possible with static filters)

# Beyond significant ties: significant structures



Significant structures are not only sets of significant ties

# Not all triangles of 3 significant dyads are significant triads



Share of significant triads among the triangles composed by three significant ties  
vs. the filtering level  $\alpha$ , for various data sets and temporal resolutions.

# Extension: taking into account groups

Modified **temporal null model**:

- node  $i$ , activity  $a_i$ , group  $c_i$
- probability of interaction at any (discrete) time:

$$u(a_i, a_j) \equiv a_i a_j \times (\delta_{c_i, c_j} + p(1 - \delta_{c_i, c_j}))$$

evaluate activities  $a_i^*$  of nodes and parameter  $p^*$  (max likelihood)

compute the probability distribution of the number of interactions  $m_{ij}$  during  $\tau$  in the null model

$$g(m_{ij} | a_i^*, a_j^*) = \binom{\tau}{m_{ij}} u(a_i^*, a_j^*)^{m_{ij}} (1 - u(a_i^*, a_j^*))^{\tau - m_{ij}}.$$

$ij$  is a **Significant Tie** if the observed number of interactions  $m_{ij}^0$  has a small p-value w.r.t.  $g$

# Using the backbone

Q1: does the backbone contain *enough* information?  
if not, what additional information should we keep?

Q2: to do what?

A2: to use in data-driven numerical simulations of dynamical processes

A1: Main idea: original network  $\approx$  backbone + noise

- create surrogate data by adding “noise” (null model) to the backbone
- use the surrogate data in numerical simulations



**nature  
communications**

Open Access | Published: 13 November 2015

**Compensating for population sampling in  
simulations of epidemic spread on  
temporal contact networks**

Mathieu Génois, Christian L. Vestergaard, Ciro Cattuto & Alain Barrat

# Creating surrogate data

- Input:
  - backbone  $\mathcal{B}_f$  (fraction  $f$  of original data)
  - number  $W$  of temporal edges in original data
  - (fitted) statistics of (inter-)contact times
- Procedure:
  - compute activities  $a_i$  of nodes in backbone
  - $\forall i, j$  not in  $\mathcal{B}_f$  extract  $m_{ij}$  from binomial distribution  $g(m_{ij} | a_i, a_j)$
  - if  $\sum m_{ij} = W^* < W$ , use  $u(a_i, a_j) = a_i a_j W / W^*$
  - create timelines on each link  $i, j$ , using statistics of (inter-)contact times



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Compensating for population sampling in simulations of epidemic spread on temporal contact networks

NB: case of groups

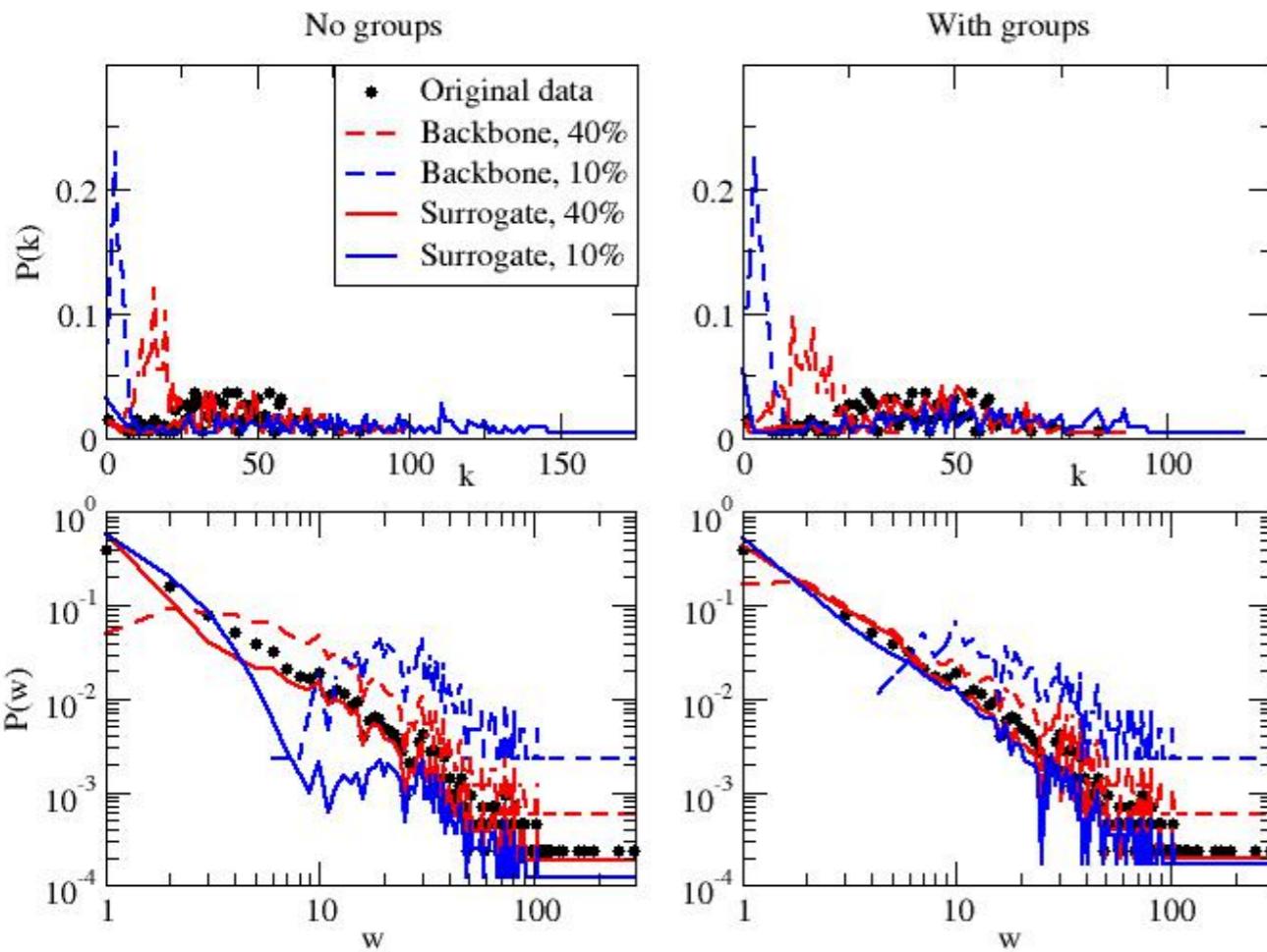
additional input: group membership,  $W_{inter}$ ,  $W_{intra}$

# Preliminary results

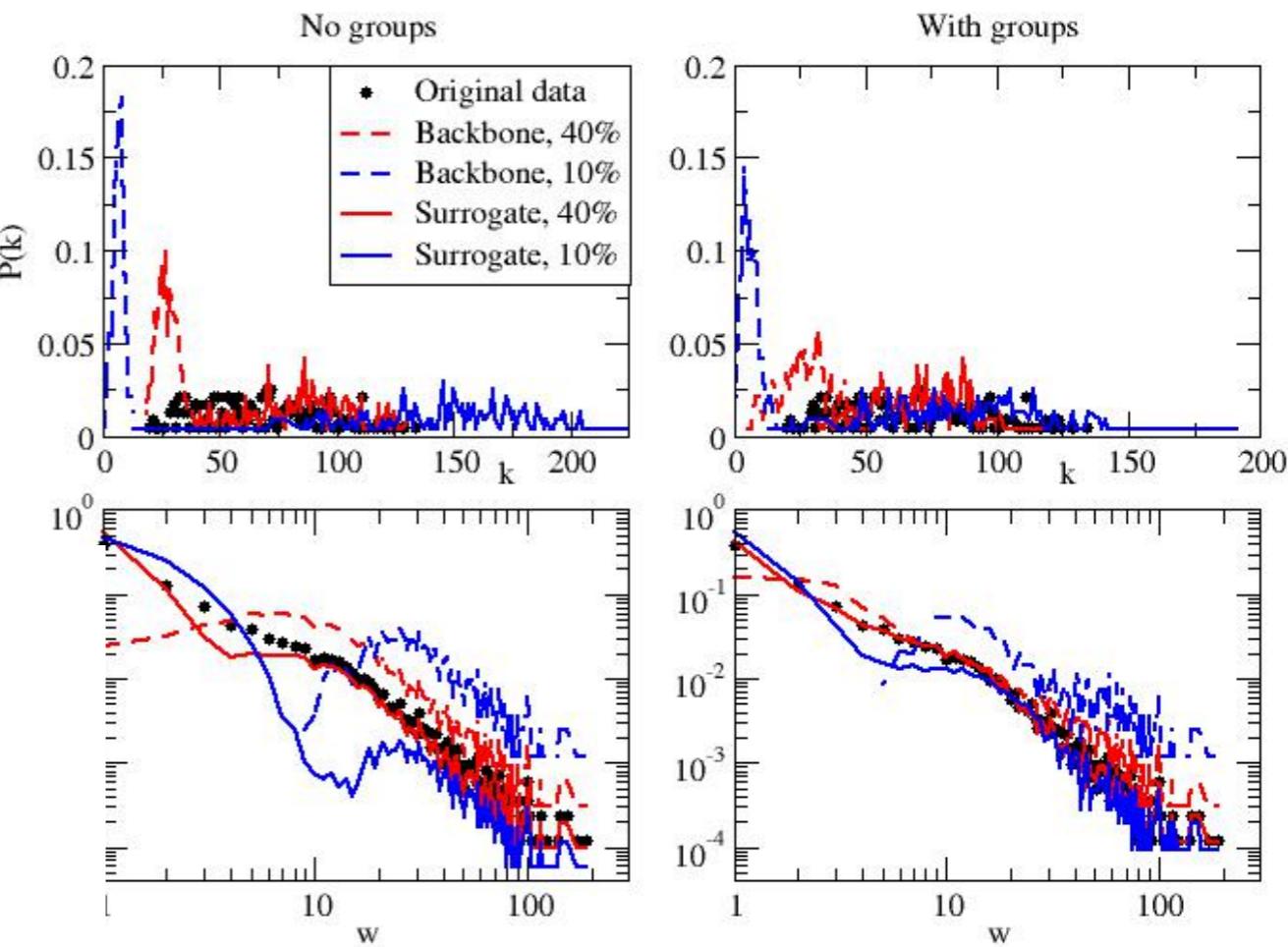
## Degrees and weights statistics

- original data
- backbone
- surrogate data

### Offices



### School



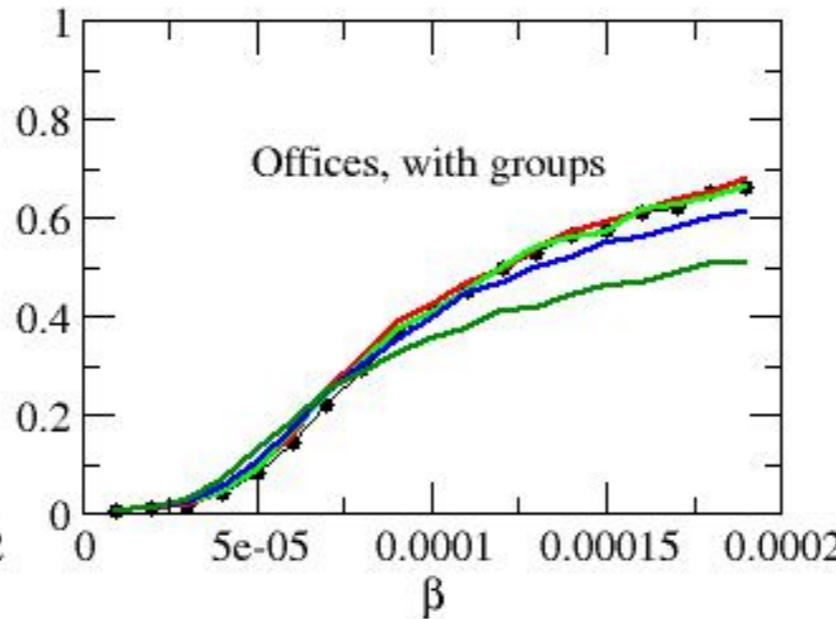
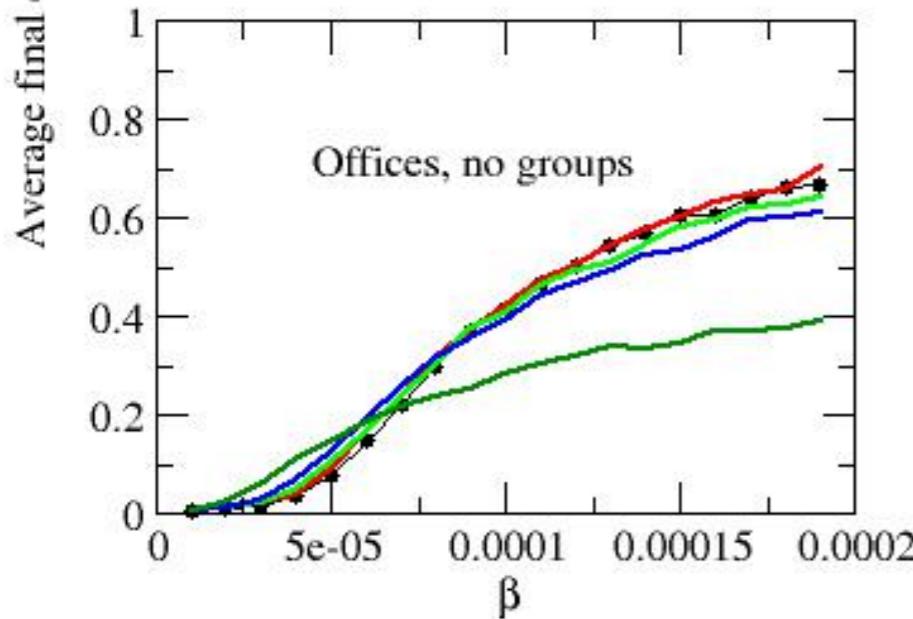
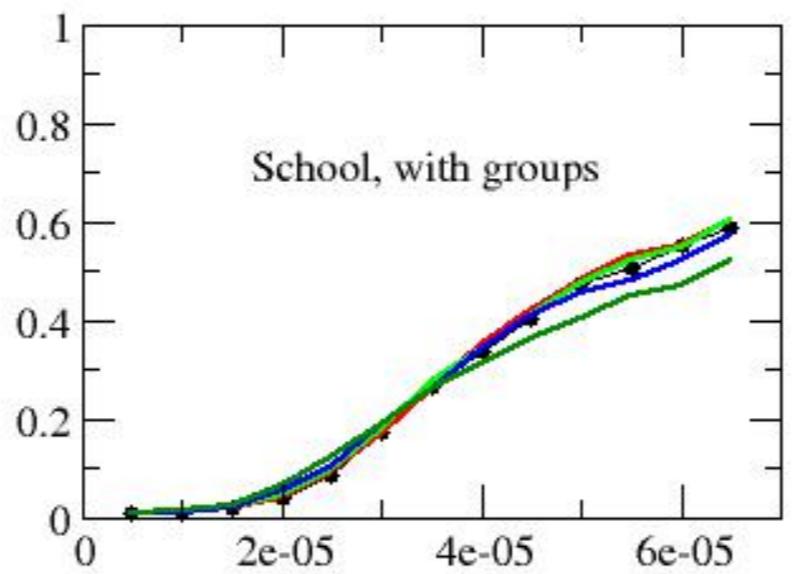
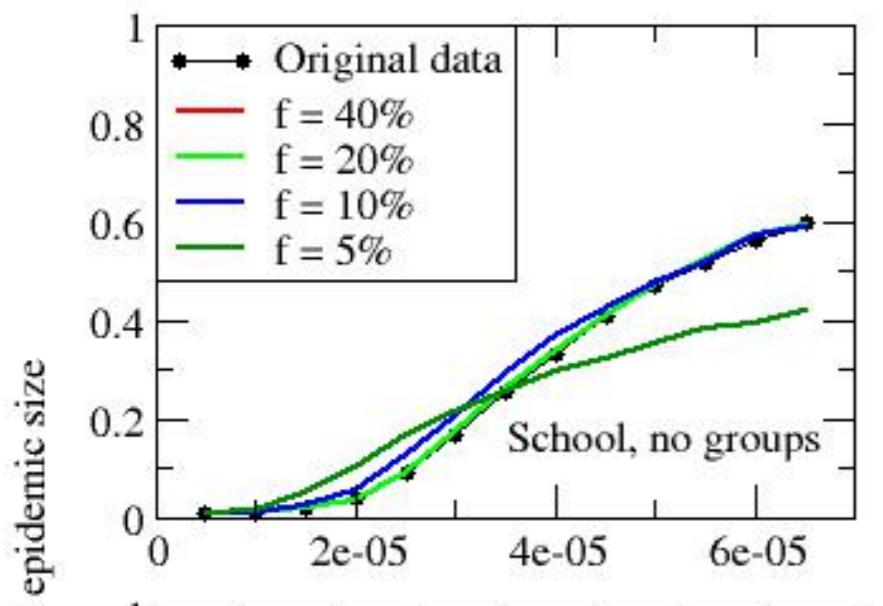
Also:  
strong correlation between strength in surrogate and strength in data  
weaker correlation between degree in surrogate and degree in data

# Preliminary results

(static) SIR process on (i) original data (ii) surrogate data



$f$  = size of backbone



School:  
groups (classes) of same size  
strong community structure

Offices:  
groups of different sizes  
weak community structure

## Future work:

Other reconstruction methods

SIR on temporal surrogate (in progress, with C. Présigny, P. Holme)

Other dynamical processes

Backbone of events (in progress, with M. Karsai)

# >Finding structures: Span cores



## Mining (maximal) Span-cores from Temporal Networks

Full Text: [PDF](#) [Get this Article](#)

Authors: [Edoardo Galimberti](#) [ISI Foundation & University of Turin, Turin, Italy](#)  
[Alain Barrat](#) [Aix Marseille Univ, CNRS, CPT, & ISI Foundation, Marseille, France](#)  
[Francesco Bonchi](#) [ISI Foundation & Eurecat, Turin, Italy](#)  
[Ciro Cattuto](#) [ISI Foundation, Turin, Italy](#)  
[Francesco Gullo](#) [UniCredit, Rome, Italy](#)

Published in:



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[ACM](#) New York, NY, USA ©2018  
[table of contents](#) ISBN: 978-1-4503-6014-2 doi:>[10.1145/3269206.3271767](#)



2018 Article



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## SCIENTIFIC REPORTS

Article | [Open Access](#) | Published: 27 July 2020

## Relevance of temporal cores for epidemic spread in temporal networks

[Martino Ciaperoni](#), [Edoardo Galimberti](#), [Francesco Bonchi](#), [Ciro Cattuto](#), [Francesco Gullo](#) & [Alain Barrat](#)

[Scientific Reports](#) **10**, Article number: 12529 (2020) | [Cite this article](#)

# Reminder: k-core decomposition for static networks

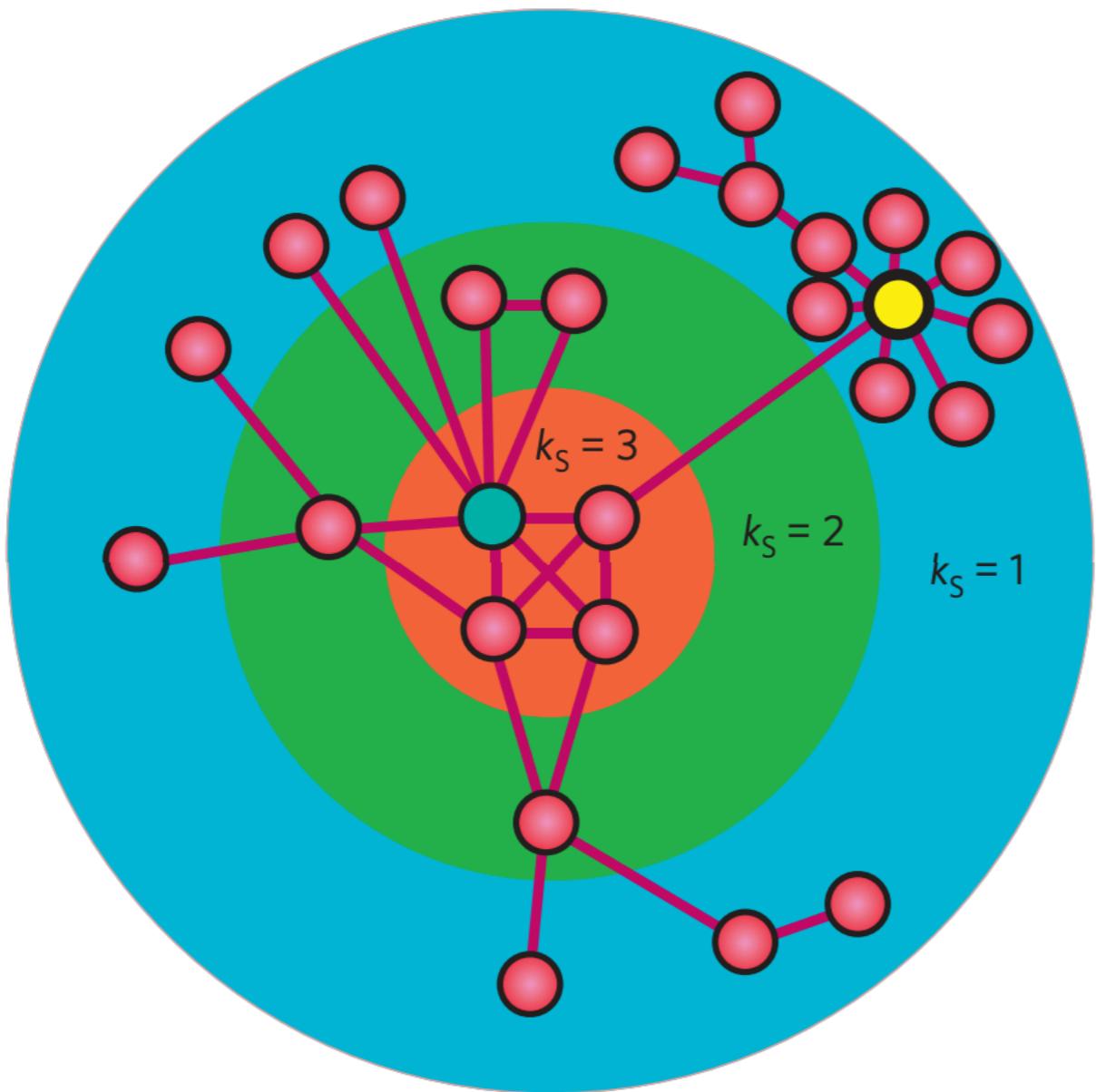
graph G=(V,E)

–**k-core** of graph G: **maximal subgraph** such that for all vertices in this subgraph have degree **at least k**

–vertex i has **shell index** k iff it belongs to the k-core but not to the (k+1)-core

–**k-shell**: ensemble of all nodes of shell index k

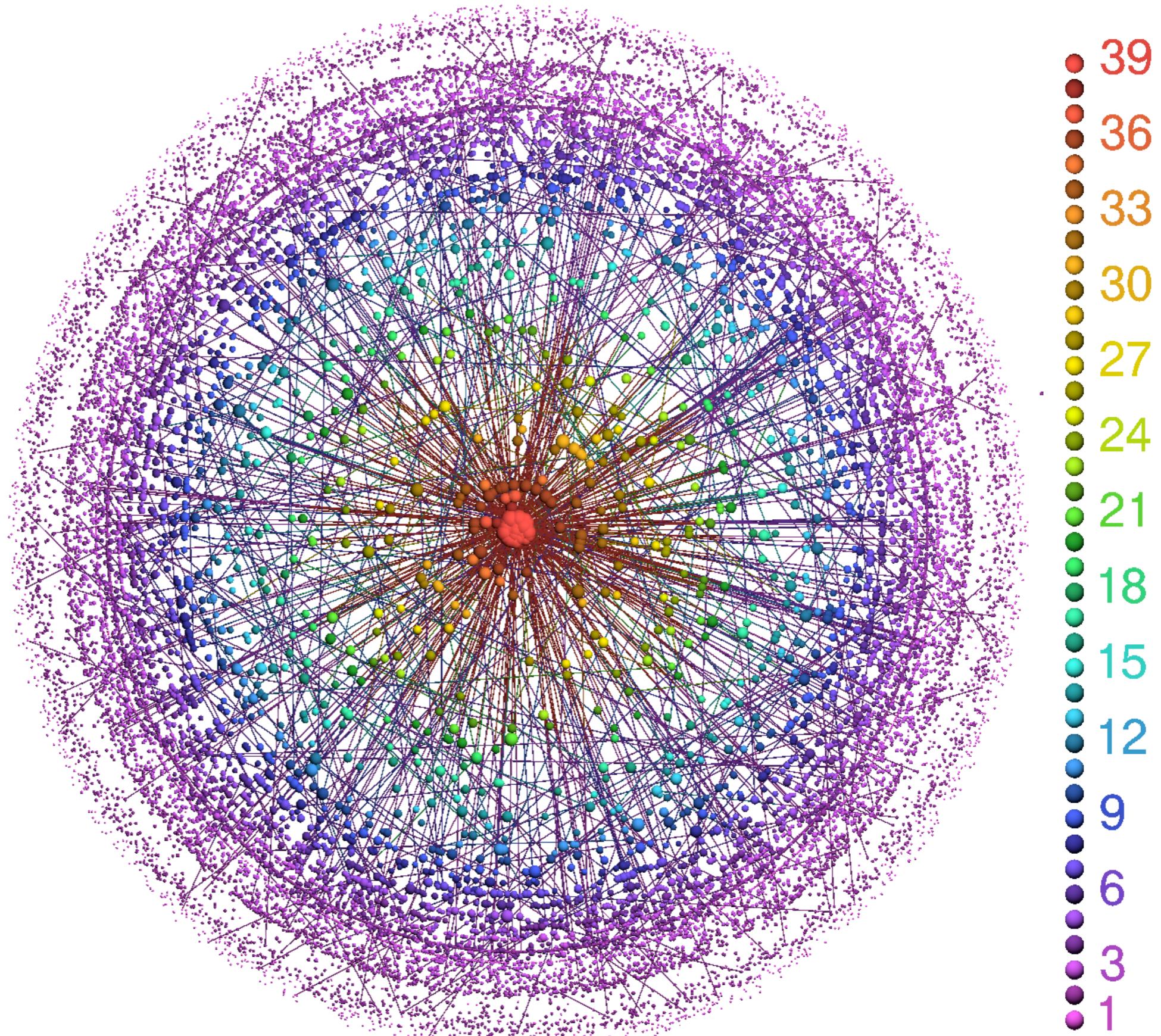
# Reminder: k-core decomposition for static networks



(picture from Kitsak et al., Nat Phys 2010)

<http://lanet-vi.fi.uba.ar/>

- 12
- 47
- 187
- 747
- 2986





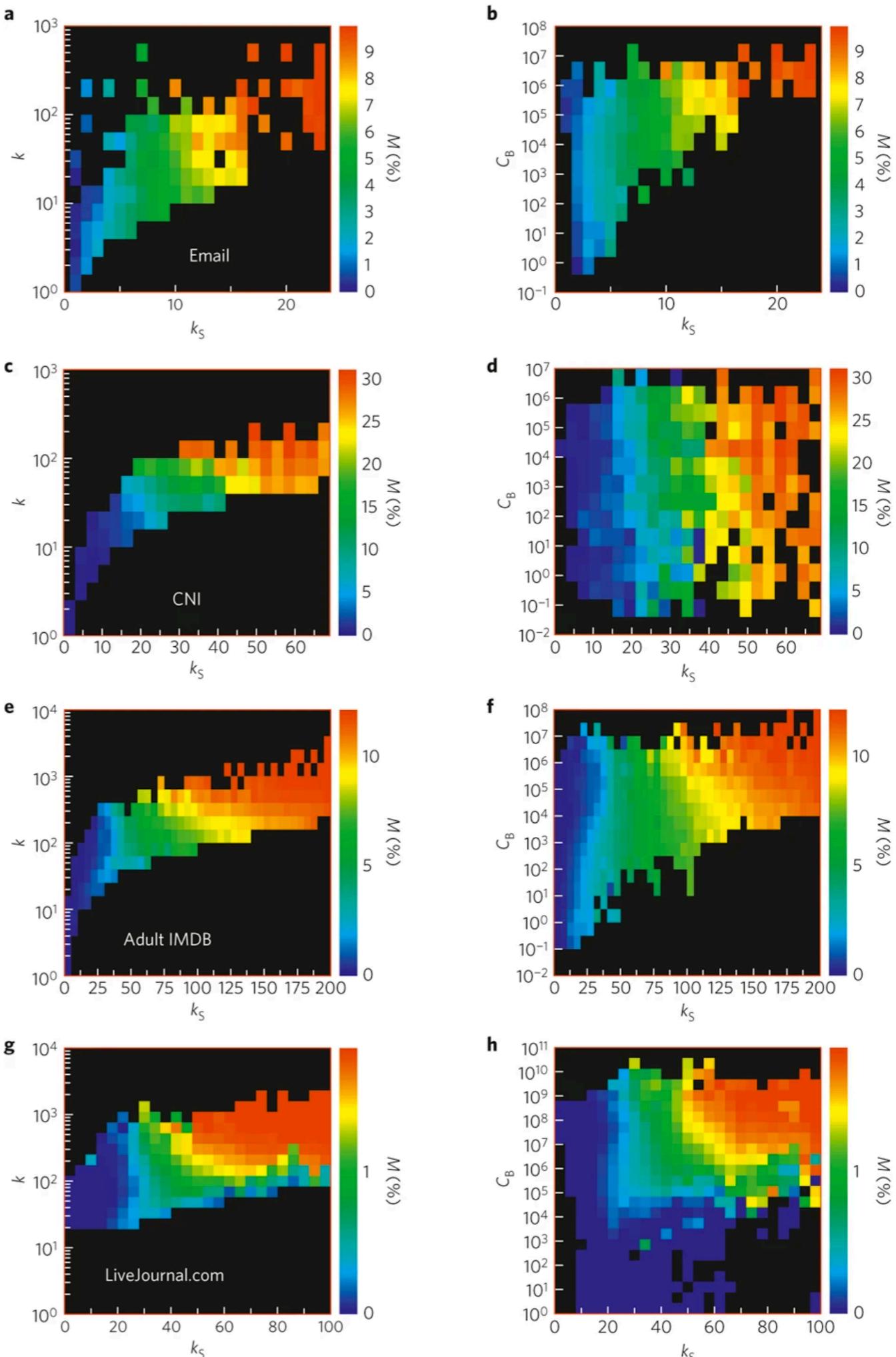
Published: 29 August 2010

# Identification of influential spreaders in complex networks

Maksim Kitsak, Lazaros K. Gallos, Shlomo Havlin, Fredrik Liljeros, Lev Muchnik, H. Eugene Stanley & Hernán A. Makse

Nature Physics 6, 888–893(2010) | Cite this article

## Size of an outbreak as a function of the seed's properties



# Span-core: definition

Temporal network G, set of vertices V,  
temporal interval  $T = [0, 1, \dots, t_{max}]$

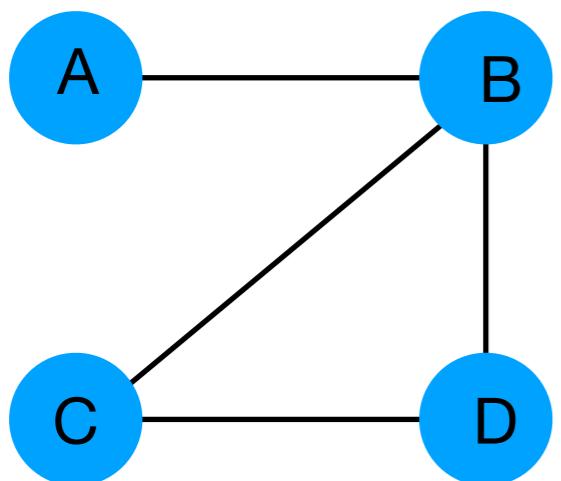
Set of edges at time t:  $E_t$

Set of edges active **at all times** t of an **interval**  $\Delta : E_\Delta = \bigcap_{t \in \Delta} E_t$

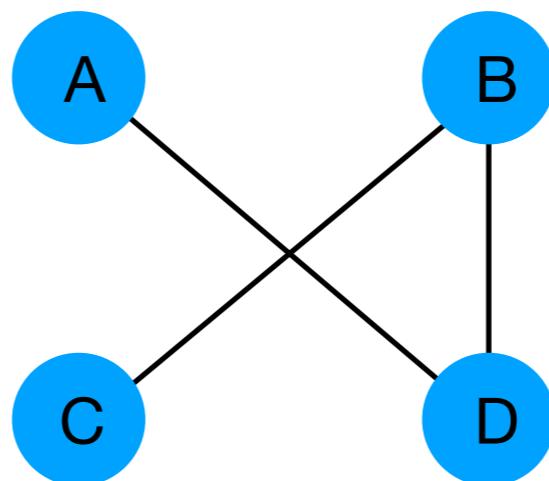
**Temporal degree** of a node within a subgraph S during  $\Delta$ :

$$d_\Delta(S, u) = |\{v \in S \mid (u, v) \in E_\Delta[S]\}|$$

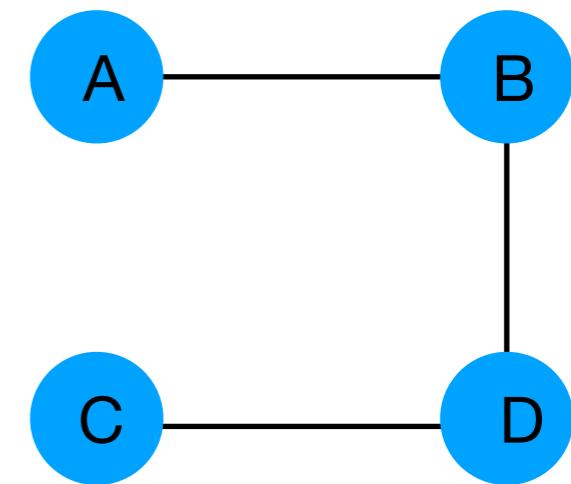
=number of nodes **in S** to which u is linked **at all times** during  $\Delta$



$t$



$t+1$



$t+2$

Temporal degrees on  $[t, t+1]$ :

$$d(A, [t, t+2]) = 0$$

$$d(B, [t, t+2]) = 1$$

$$d(C, [t, t+2]) = 0$$

$$d(D, [t, t+2]) = 1$$

# Span-core: definition

DEFINITION 2 (( $k, \Delta$ )-CORE). *The  $(k, \Delta)$ -core of a temporal graph  $G = (V, T, \tau)$  is (when it exists) a maximal and non-empty set of vertices  $\emptyset \neq C_{k, \Delta} \subseteq V$ , such that  $\forall u \in C_{k, \Delta} : d_\Delta(C_{k, \Delta}, u) \geq k$ , where  $\Delta \subseteq T$  is a temporal interval and  $k \in \mathbb{N}^+$ .*

i.e., all nodes of the core have **degree at least  $k$  within the core** during the temporal interval,  
and have **at least  $k$  “constant” neighbors in the core** during that interval

DEFINITION 3 (MAXIMAL SPAN-CORE). *A span-core  $C_{k, \Delta}$  of a temporal graph  $G$  is said maximal if there does not exist any other span-core  $C_{k', \Delta'}$  of  $G$  such that  $k \leq k'$  and  $\Delta \subseteq \Delta'$ .*

# Extracting the span-cores

[https://github.com/egalimberti/span\\_cores](https://github.com/egalimberti/span_cores)

## Naive algorithms

- All span cores:
  - generate all temporal intervals,
  - construct graphs of permanent links for each interval
  - build the k-core decomposition for each graph
- Maximal span cores:
  - for each interval, consider the cores of highest order
  - keep only the maximal ones

# Extracting the span-cores

More efficient algorithms

[https://github.com/egalimberti/span\\_cores](https://github.com/egalimberti/span_cores)

- Exploit the **containment property**:

**PROPOSITION 1 (SPAN-CORE CONTAINMENT).** *For any two span-cores  $C_{k,\Delta}, C_{k',\Delta'}$  of a temporal graph  $G$  it holds that*

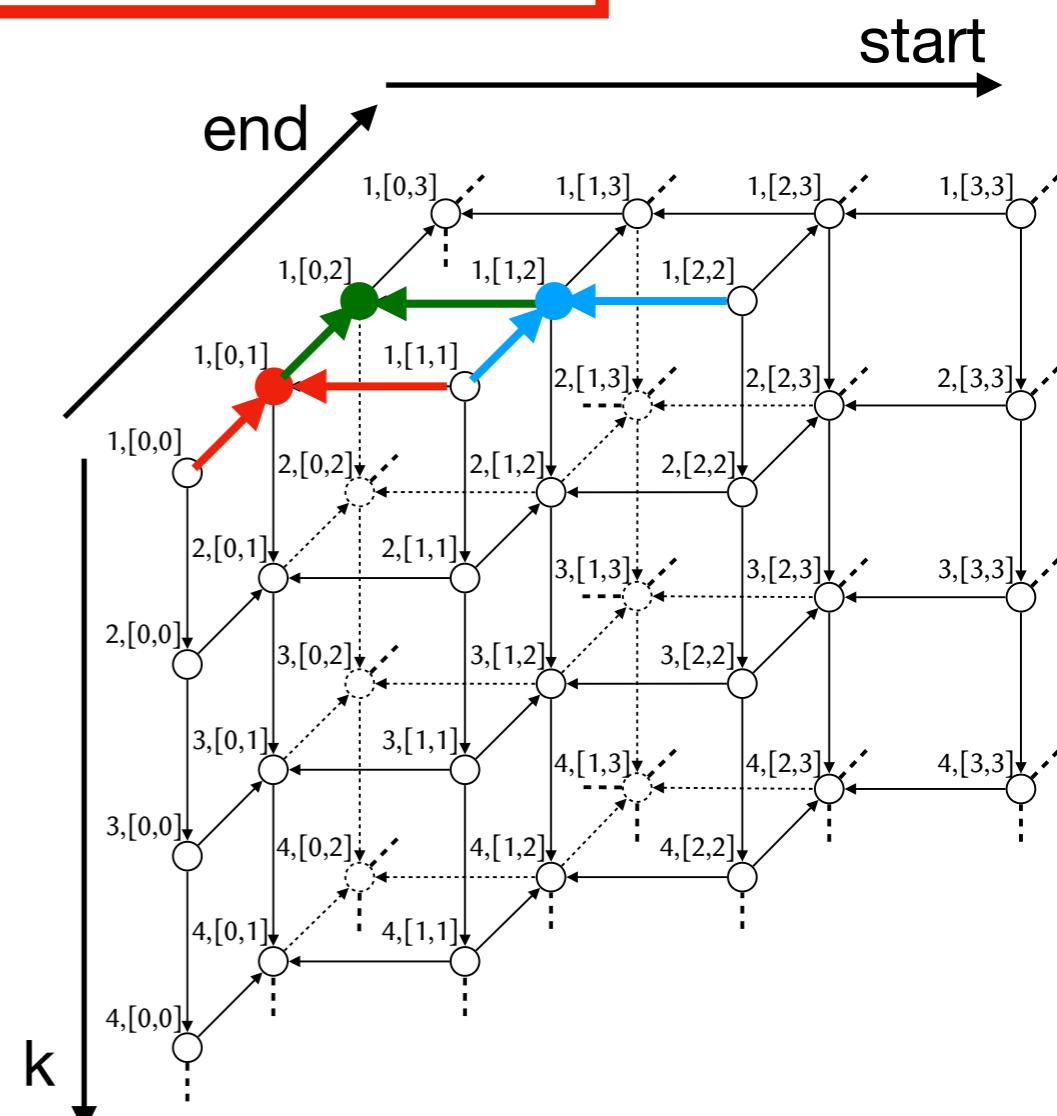
$$k' \leq k \wedge \Delta' \sqsubseteq \Delta \Rightarrow C_{k,\Delta} \subseteq C_{k',\Delta'}.$$

- generate temporal intervals of increasing size
- start the decomposition from intersection of previously found cores

Examples:

**core k=1 on interval [0,1]** is obtained by starting from the intersection of **core 1 on interval [0,0] and core 1 on interval [1,1]**

**core k=1 on interval [0,2]** is obtained by starting from the intersection of **core 1 on interval [0,1] and core 1 on interval [1,2]**



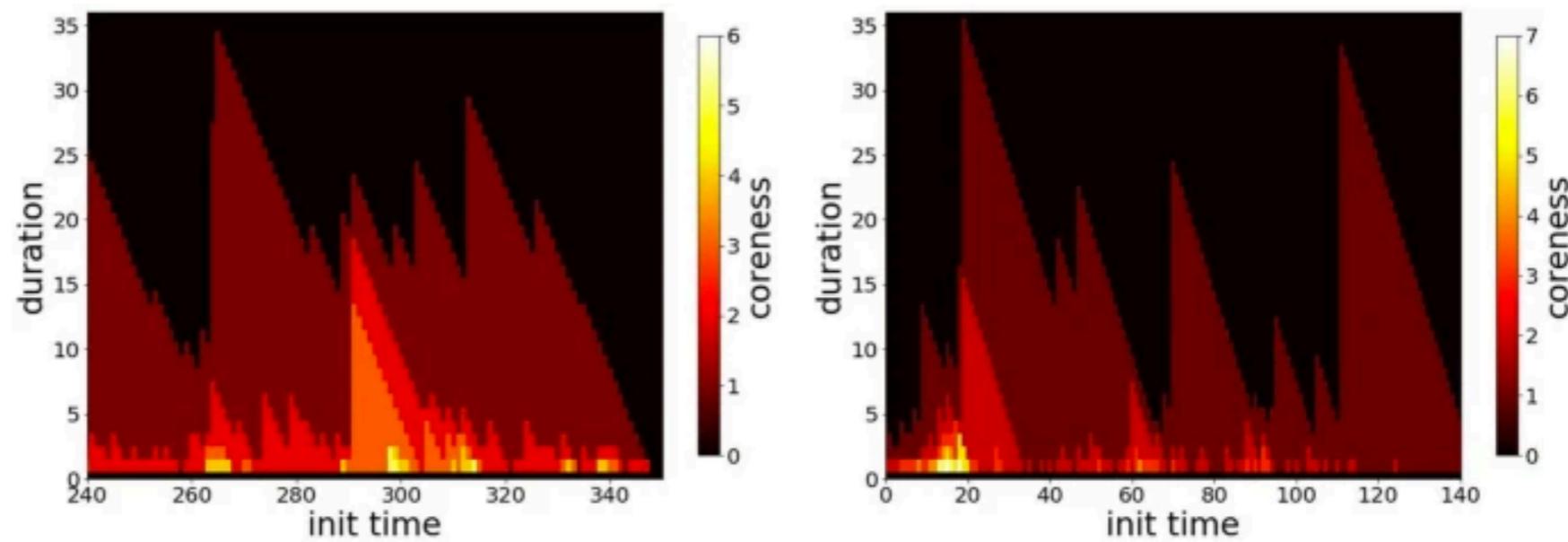
# Span-core: examples

Face-to-face contact networks

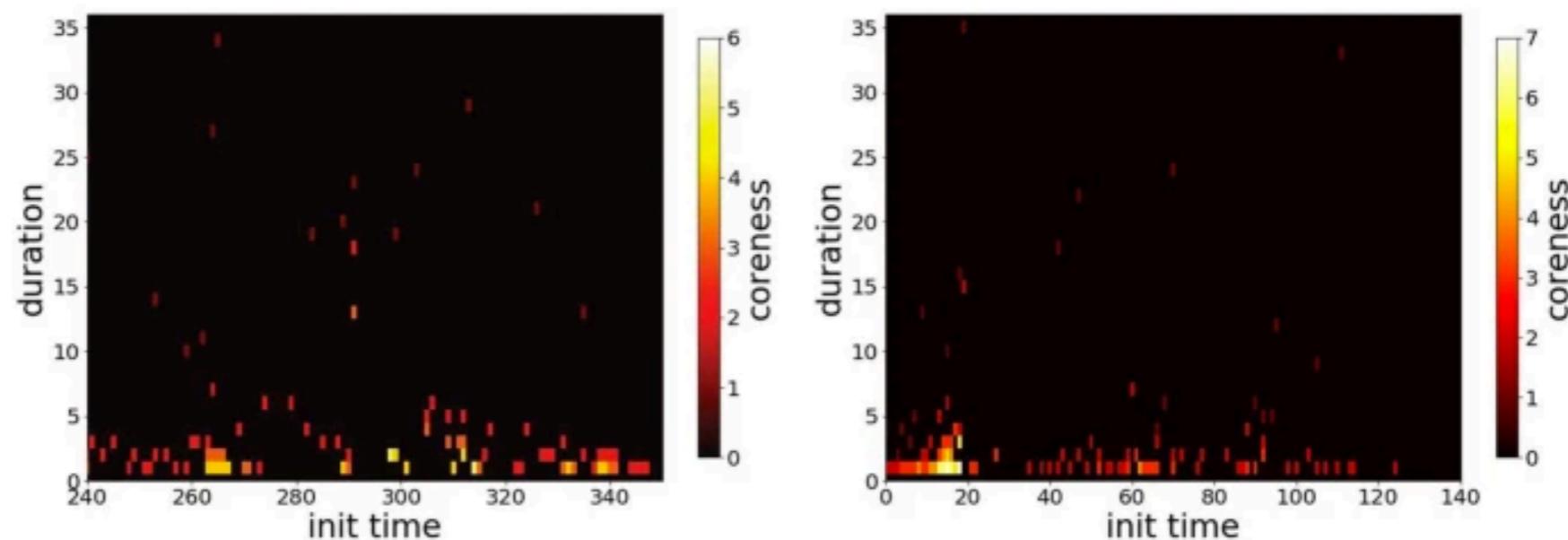
Hightschool (left) and Workplace (right) data:

Order of the span-cores as a function of their starting time and of the temporal span length

All span cores



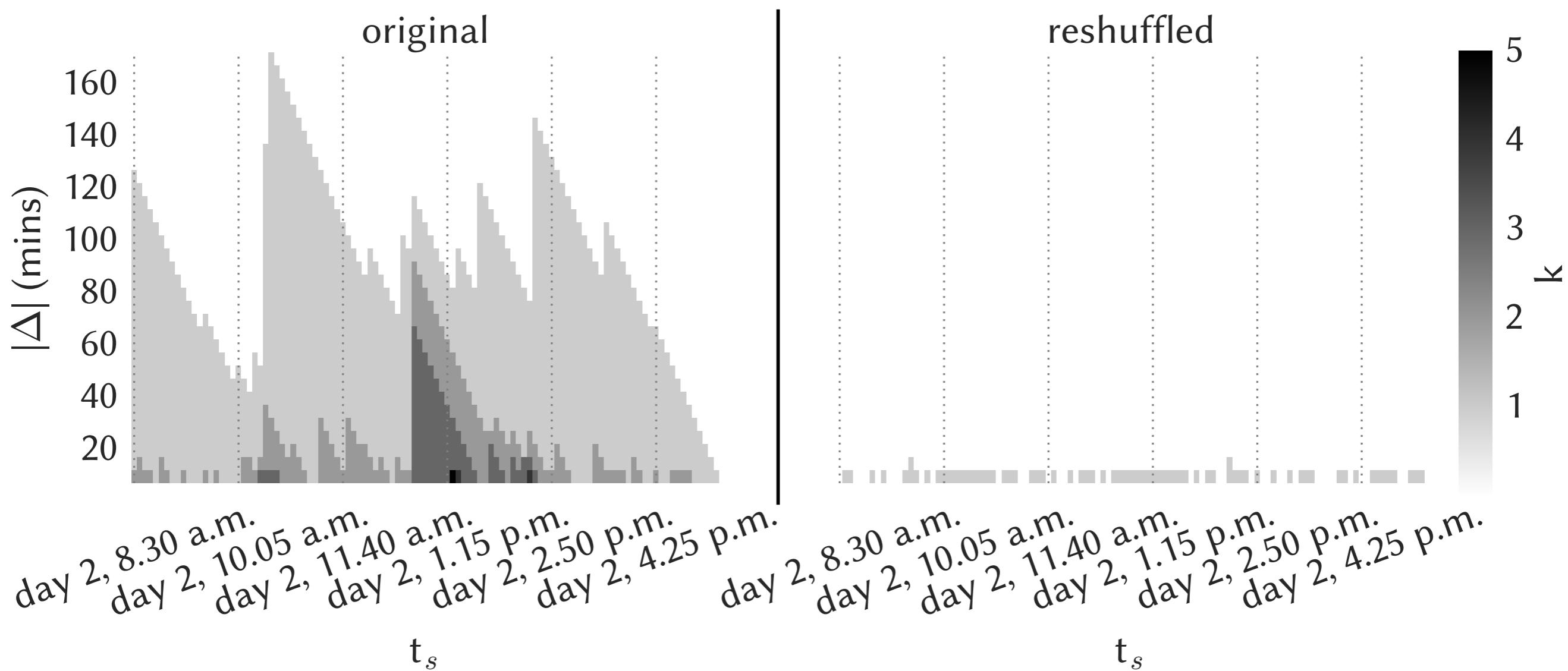
Maximal span cores



# Span-core: examples

Primary school data:

Order of the span-cores as a function of their starting time and of the temporal span length

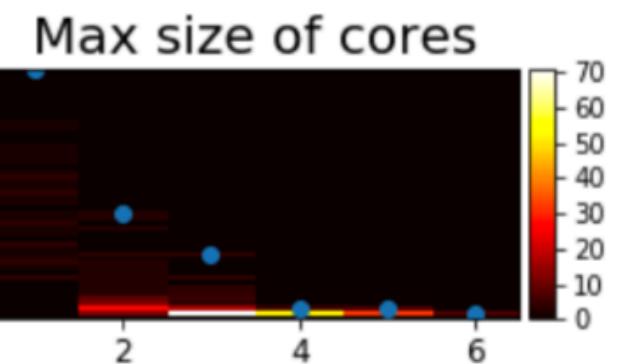
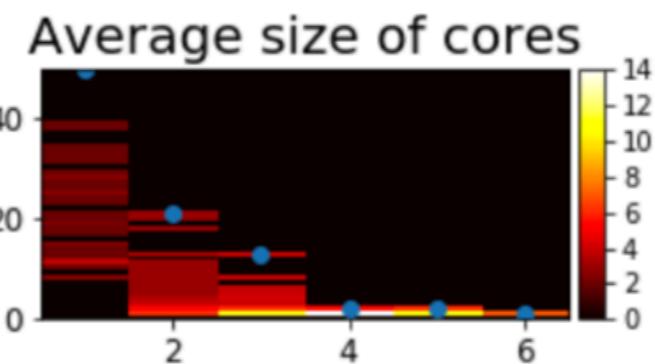
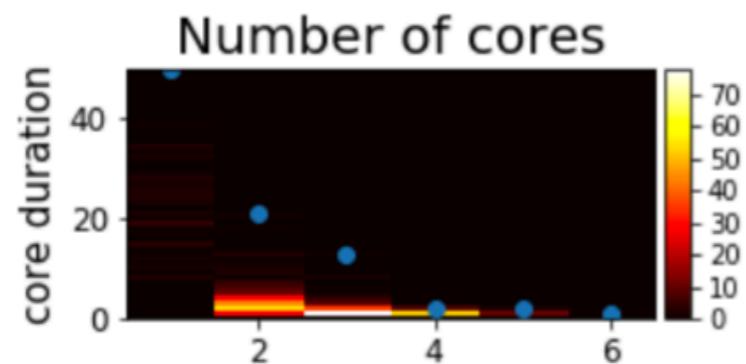


# Span-core: examples

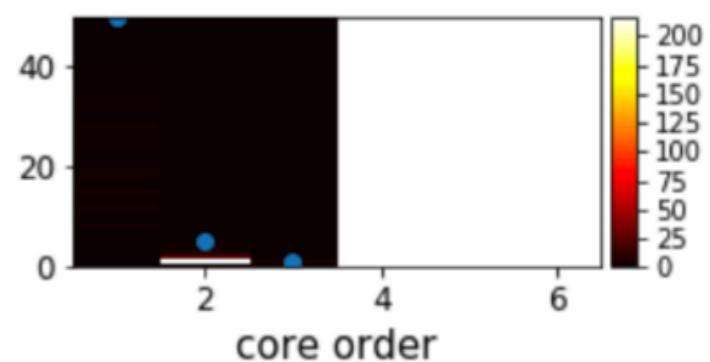
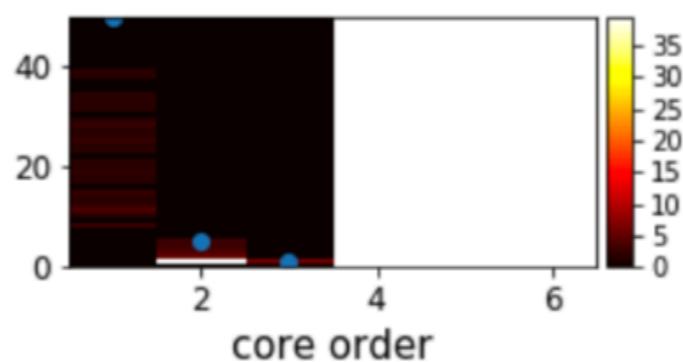
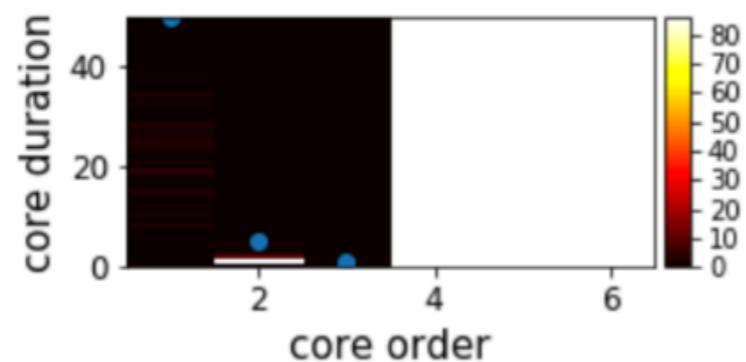
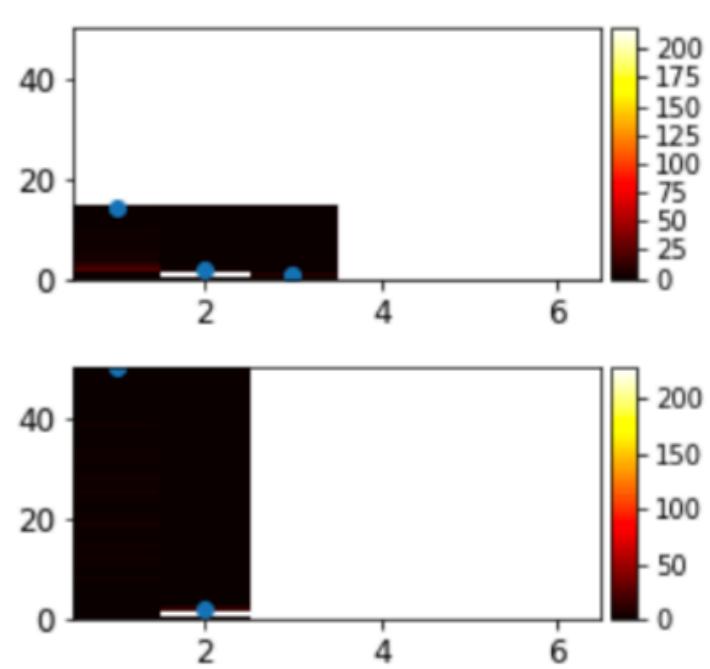
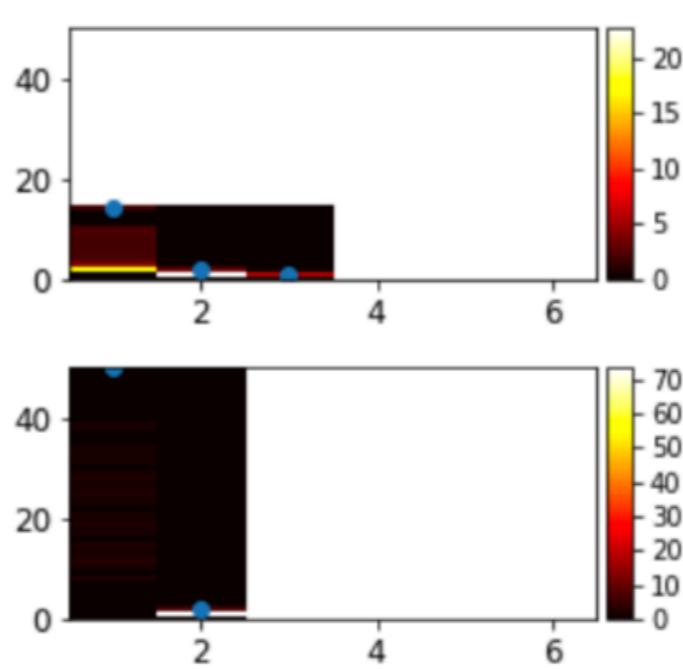
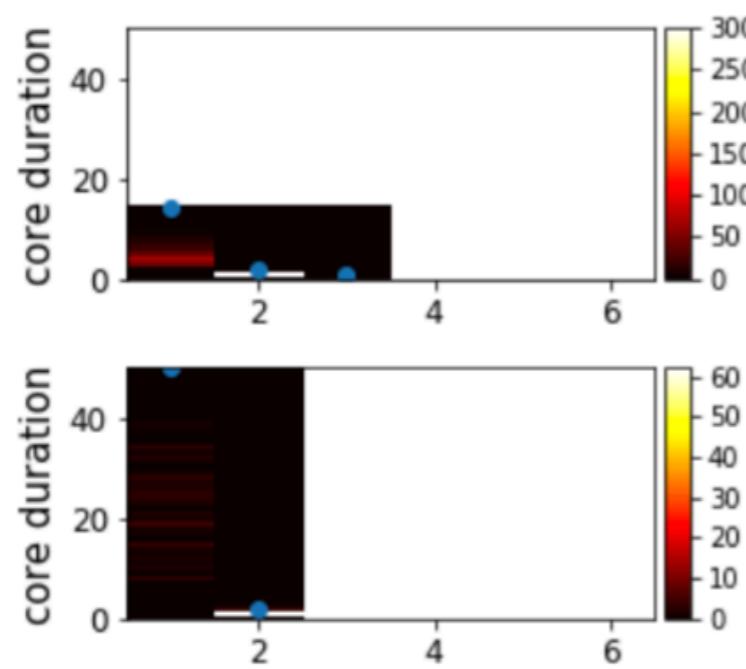
Face-to-face contact networks

Hghschool data, maximal span cores

Original data



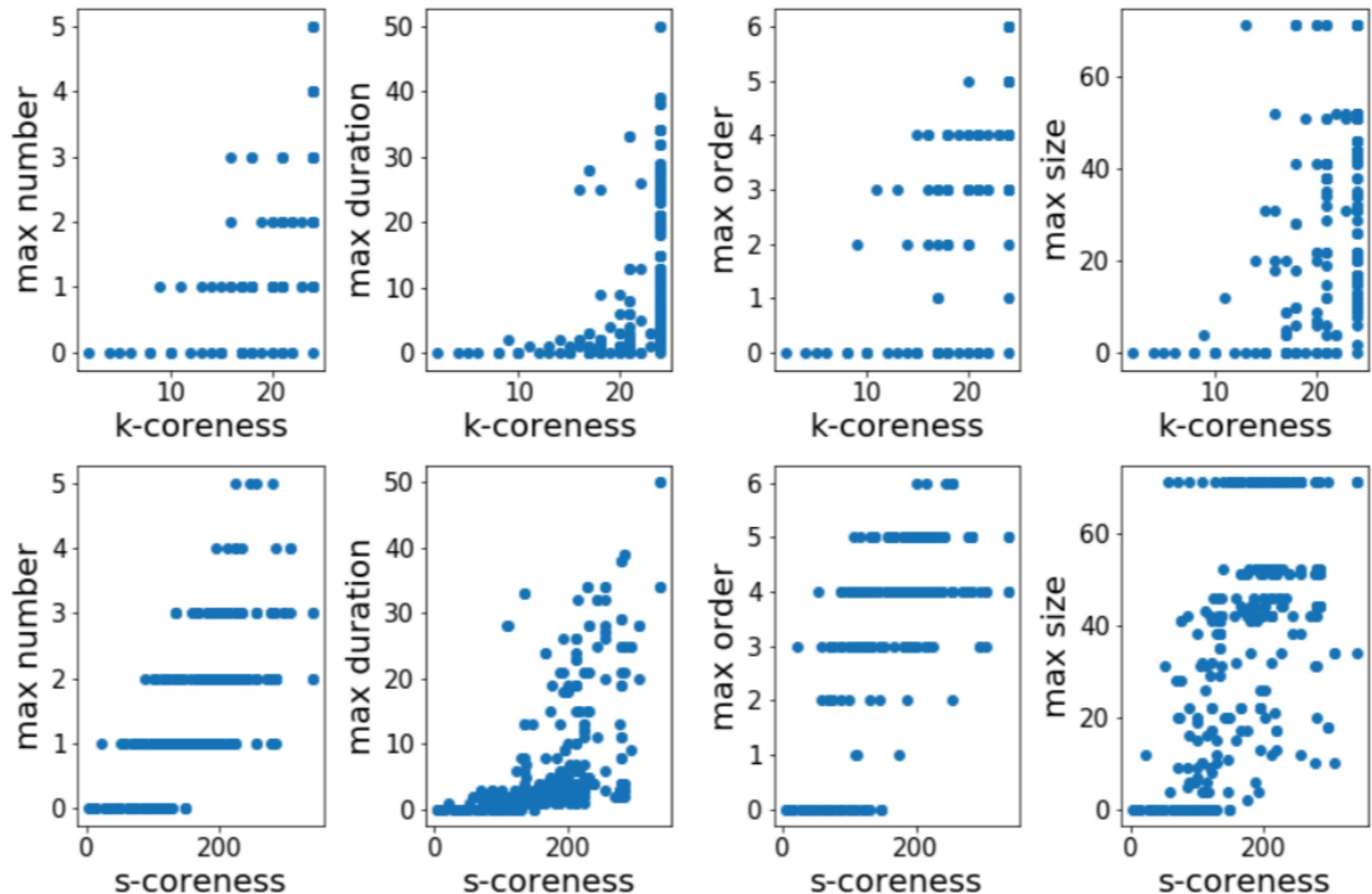
Reshuffled data



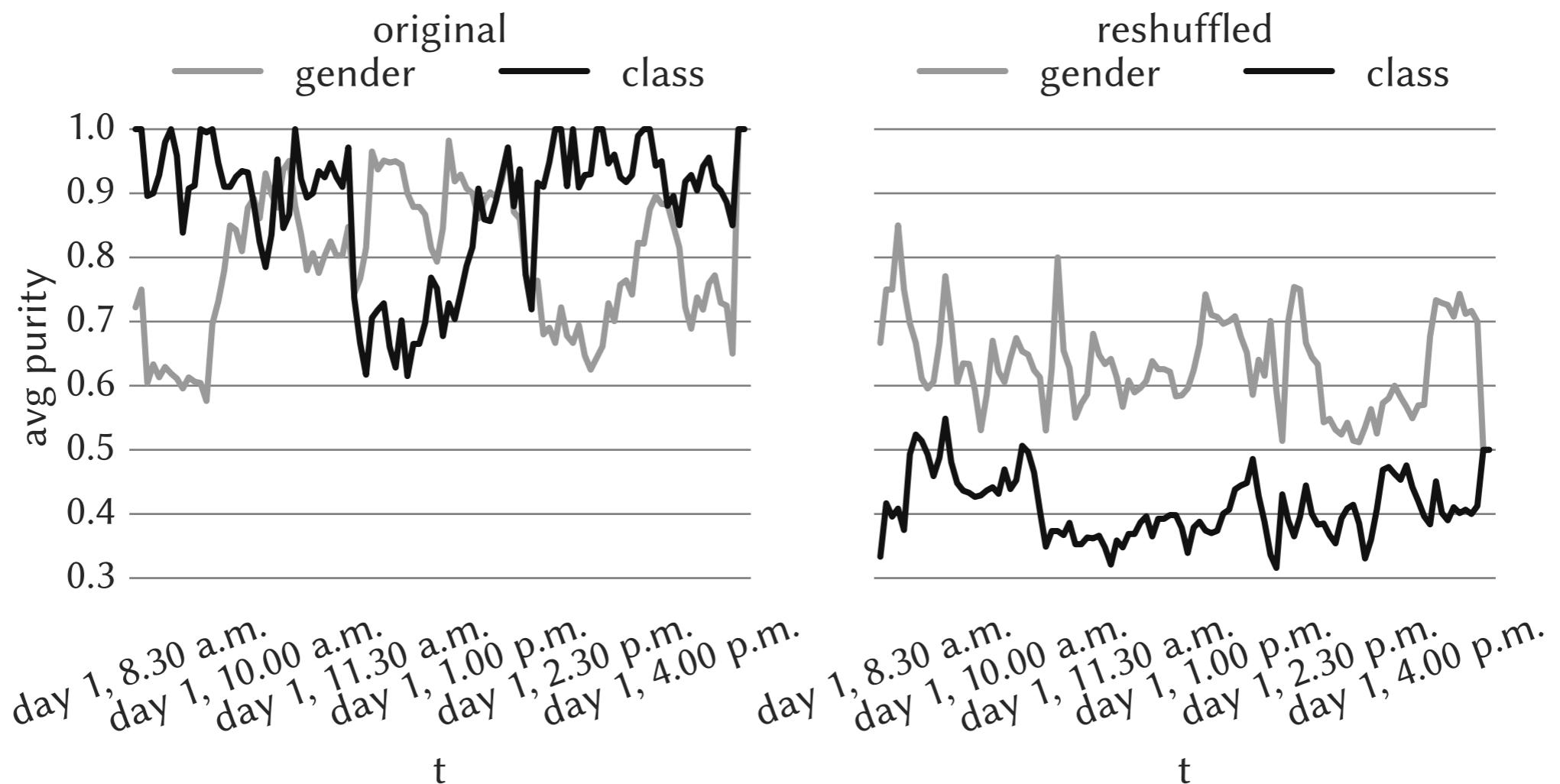
# Span-core: examples

Face-to-face contact networks

Hightschool data, maximal span cores properties vs static (aggregated) coreness measures



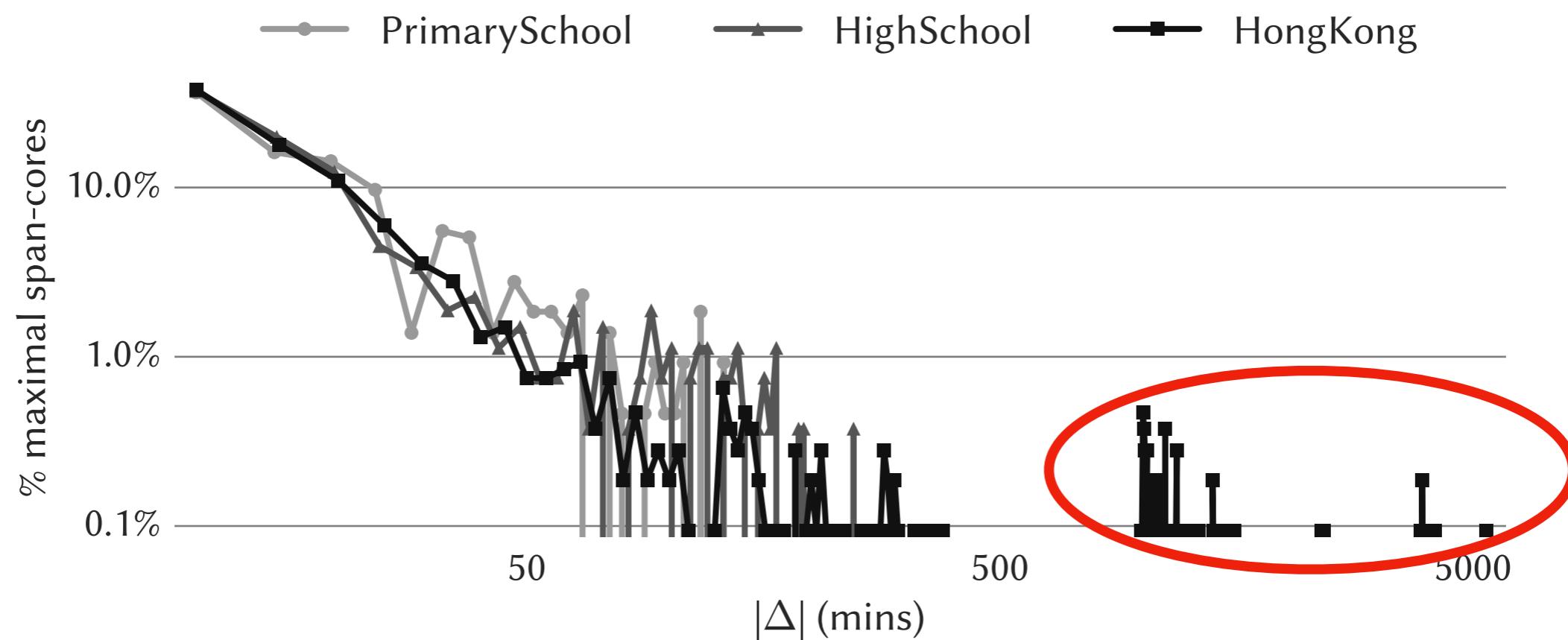
# Span-core: examples



Primary school:

Average gender and class purity of the maximal span-cores

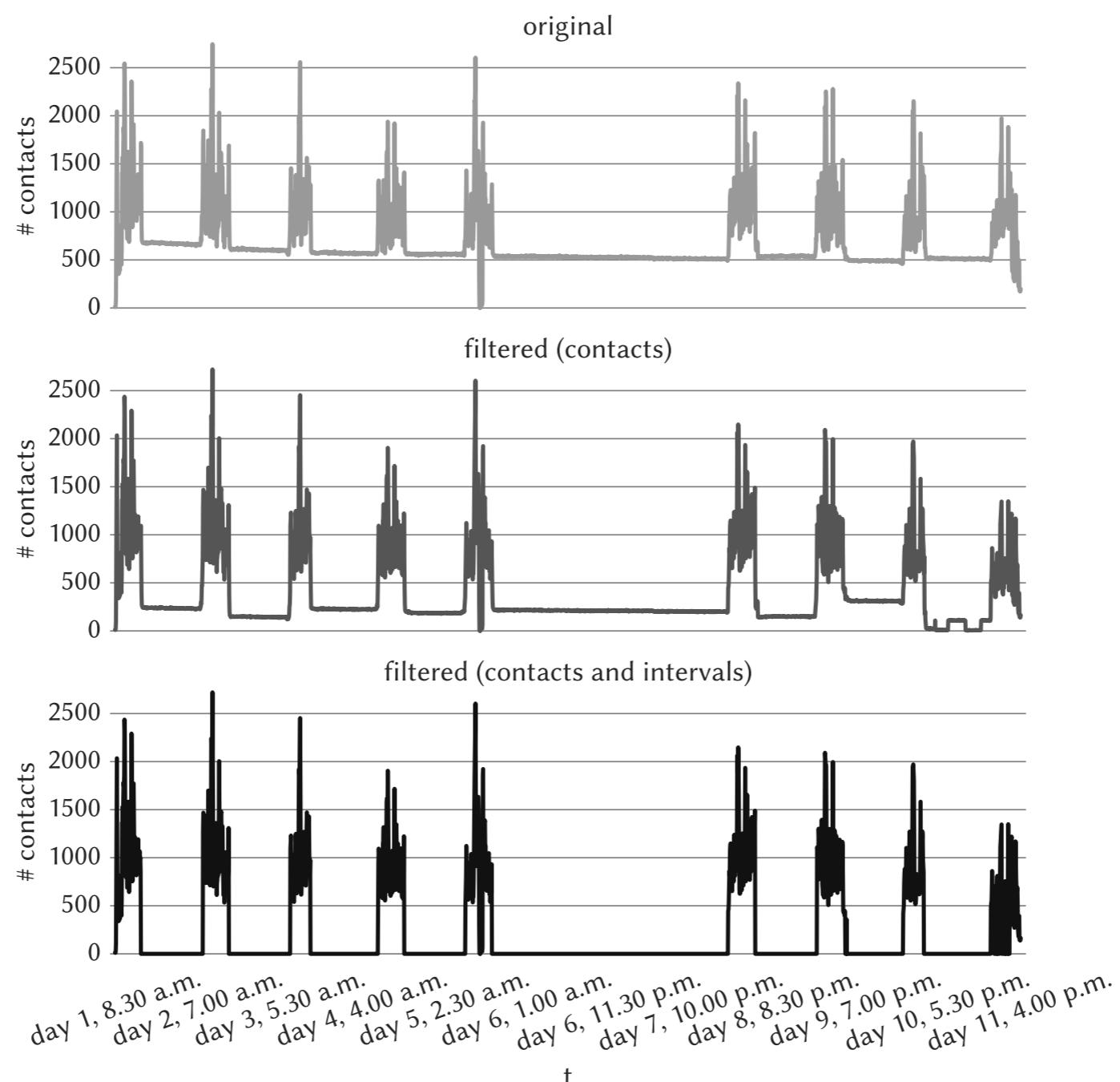
# Span-core: examples



Distributions of maximal span-cores lengths

# Span-core: anomaly detection

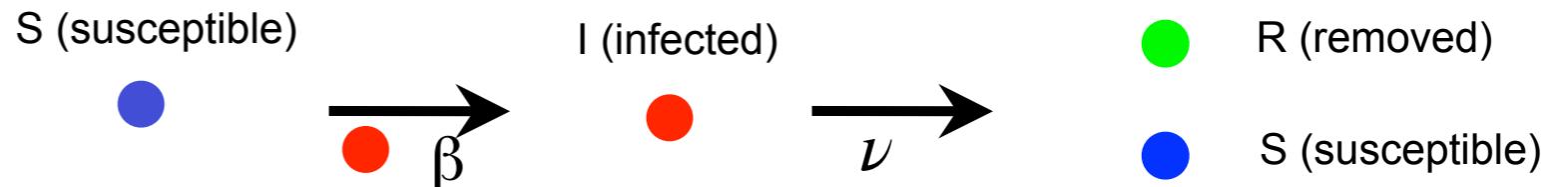
Hong-Kong primary school: filtering anomalous contacts



# Span cores and spreading processes

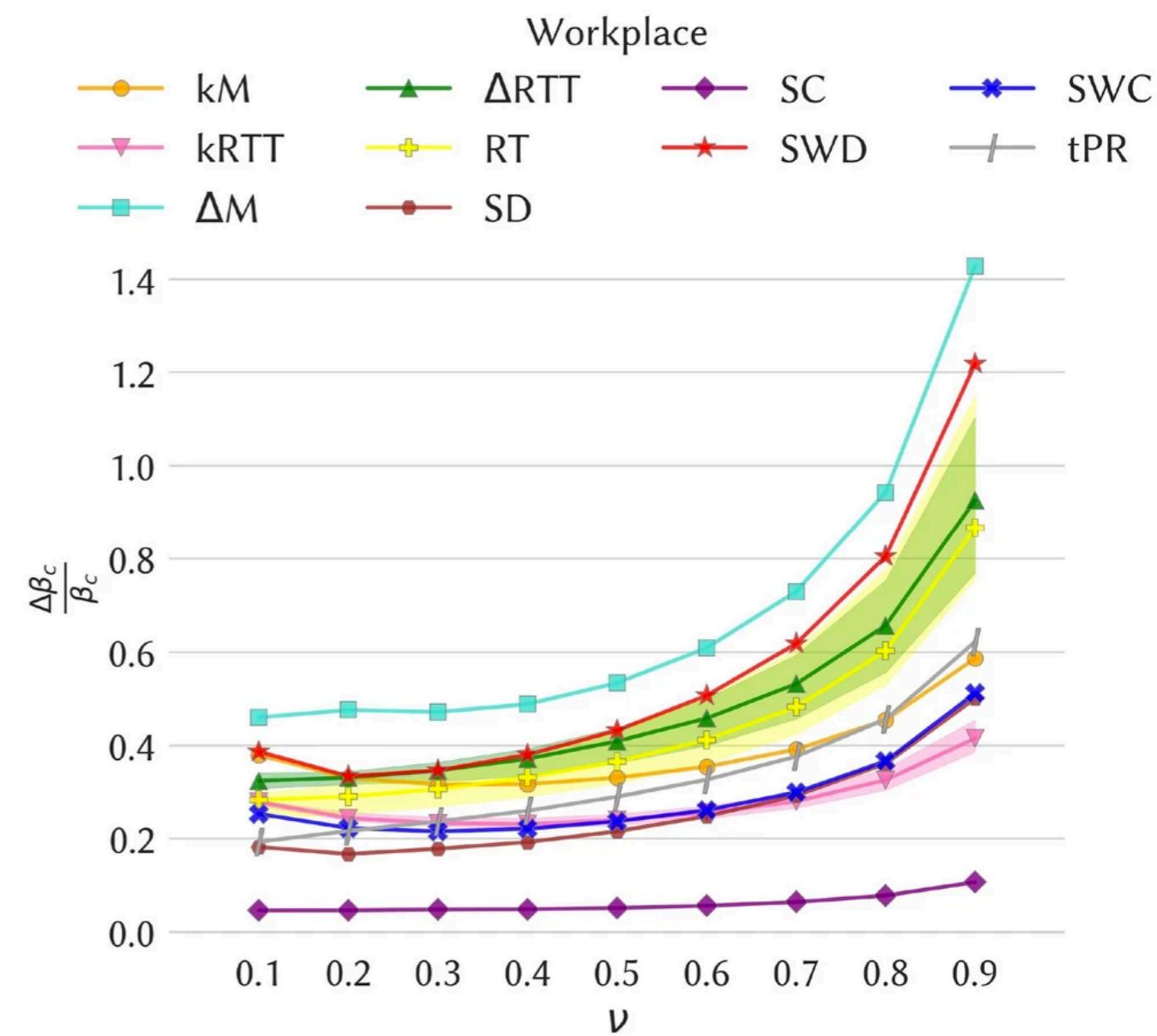
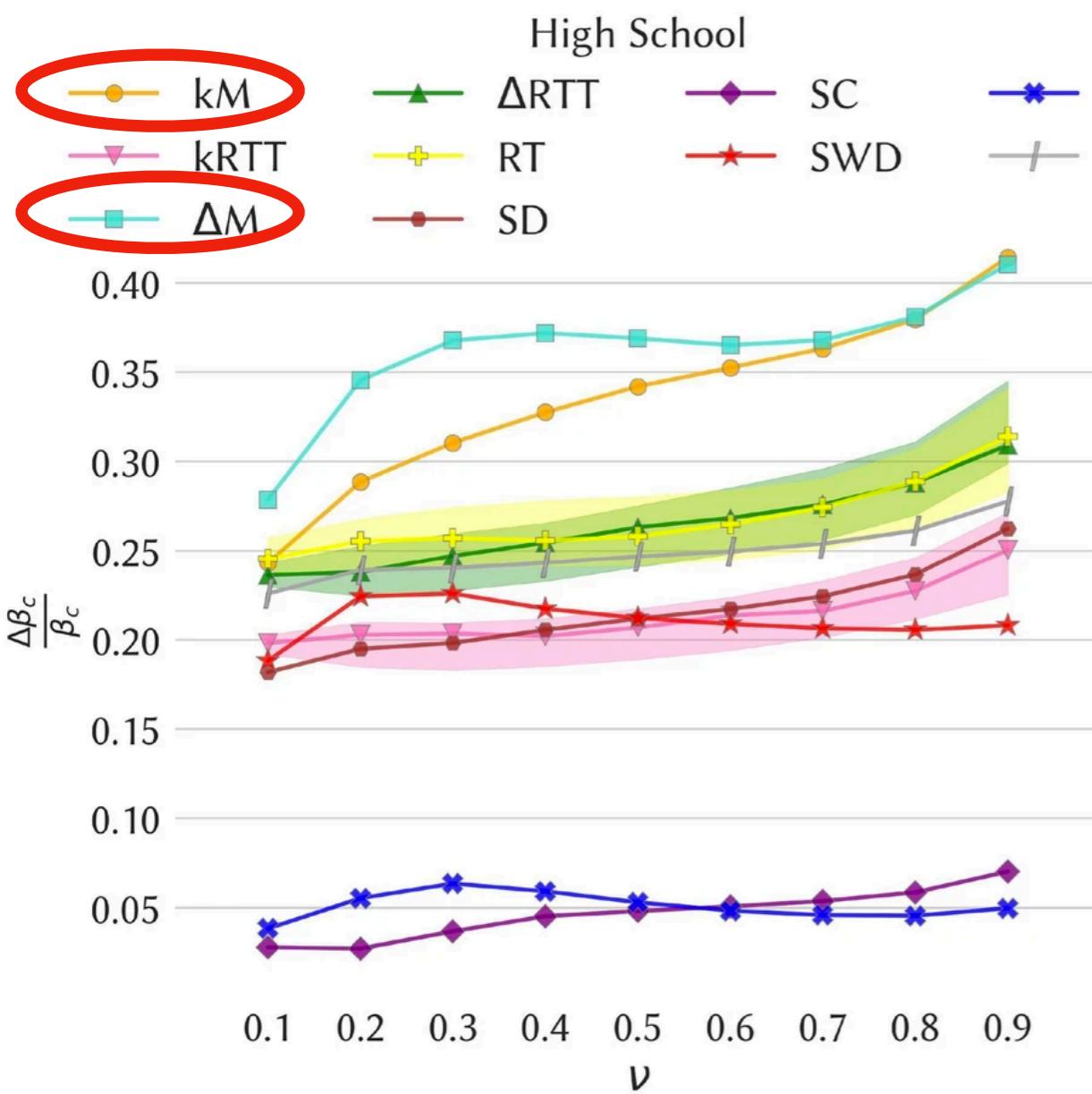
## Procedure

1. remove temporal edges from maximal span-cores with large order/duration
2. observe impact on outcome of spreading processes, e.g. SIS, SIR
3. compare with removal of same number of temporal edges according to other strategies



# Span cores and spreading processes

Relative change in SIS epidemic threshold when **removing temporal edges** with targeted strategies based either on span-cores or on aggregated measures or on temporal PageRank



# Span cores and spreading processes

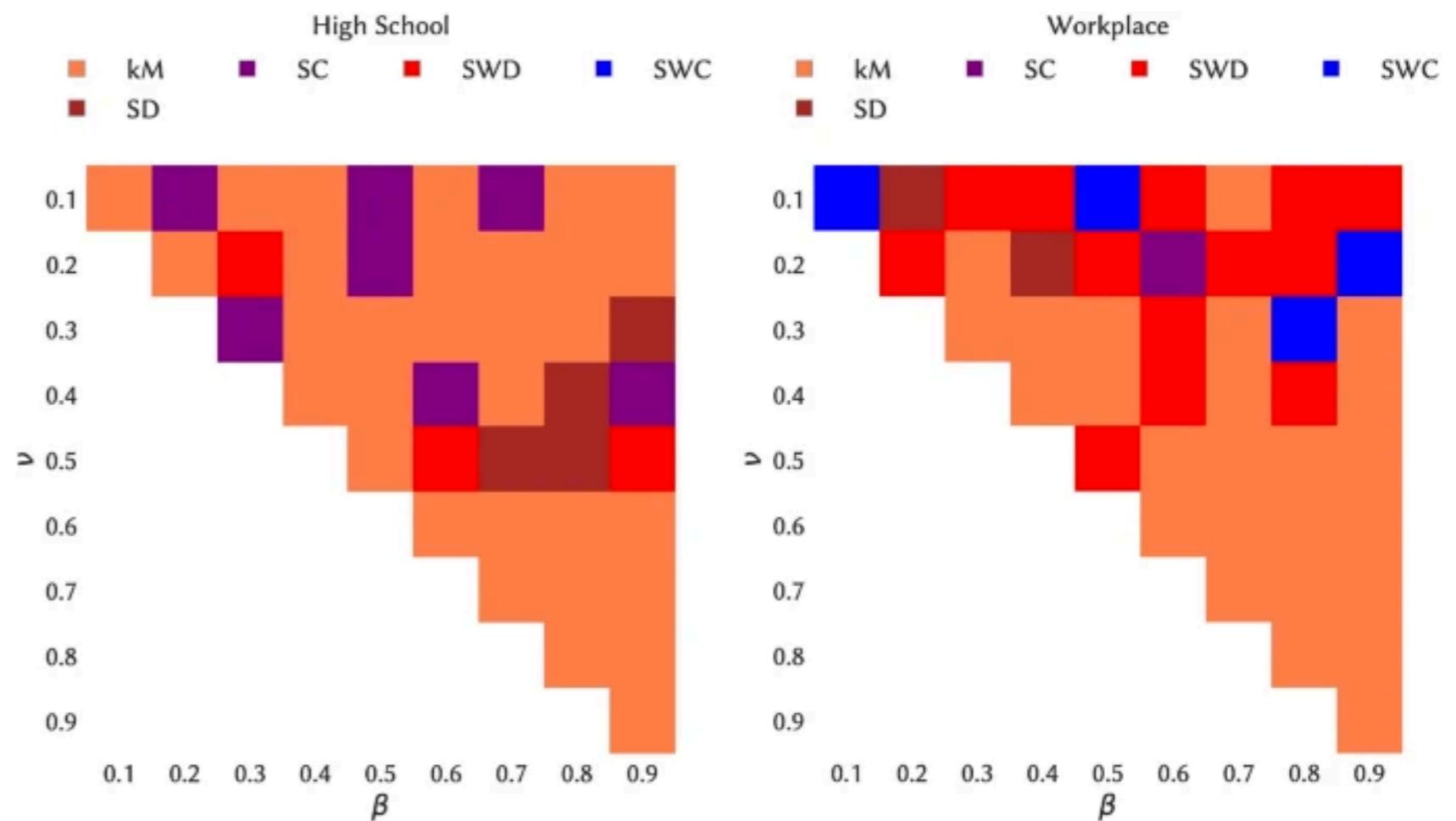
## Procedure

1. consider an SIR process with seed in maximal span cores
2. compare outbreak size with random seed
3. compare with other seeding strategies

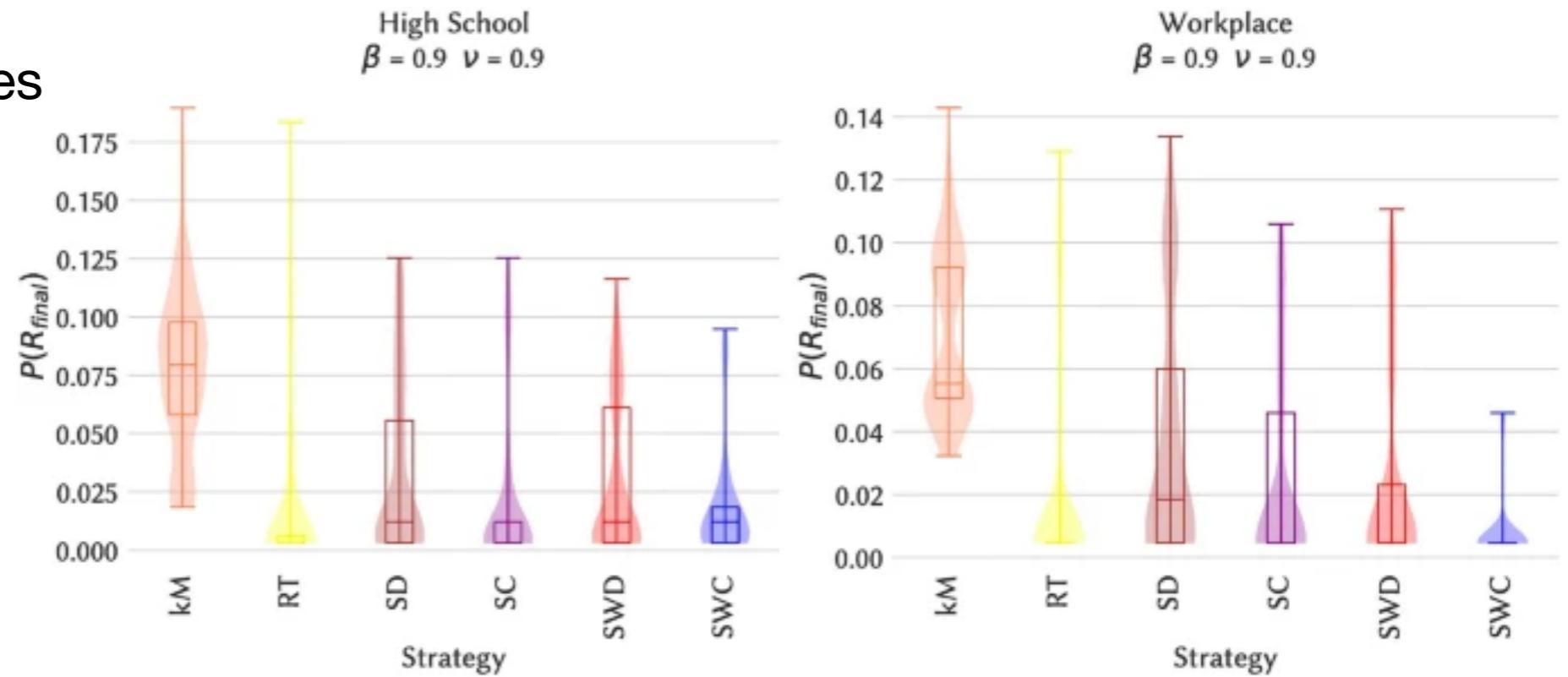
# Span cores and spreading processes

Best seeding strategy

kM: cores with largest order

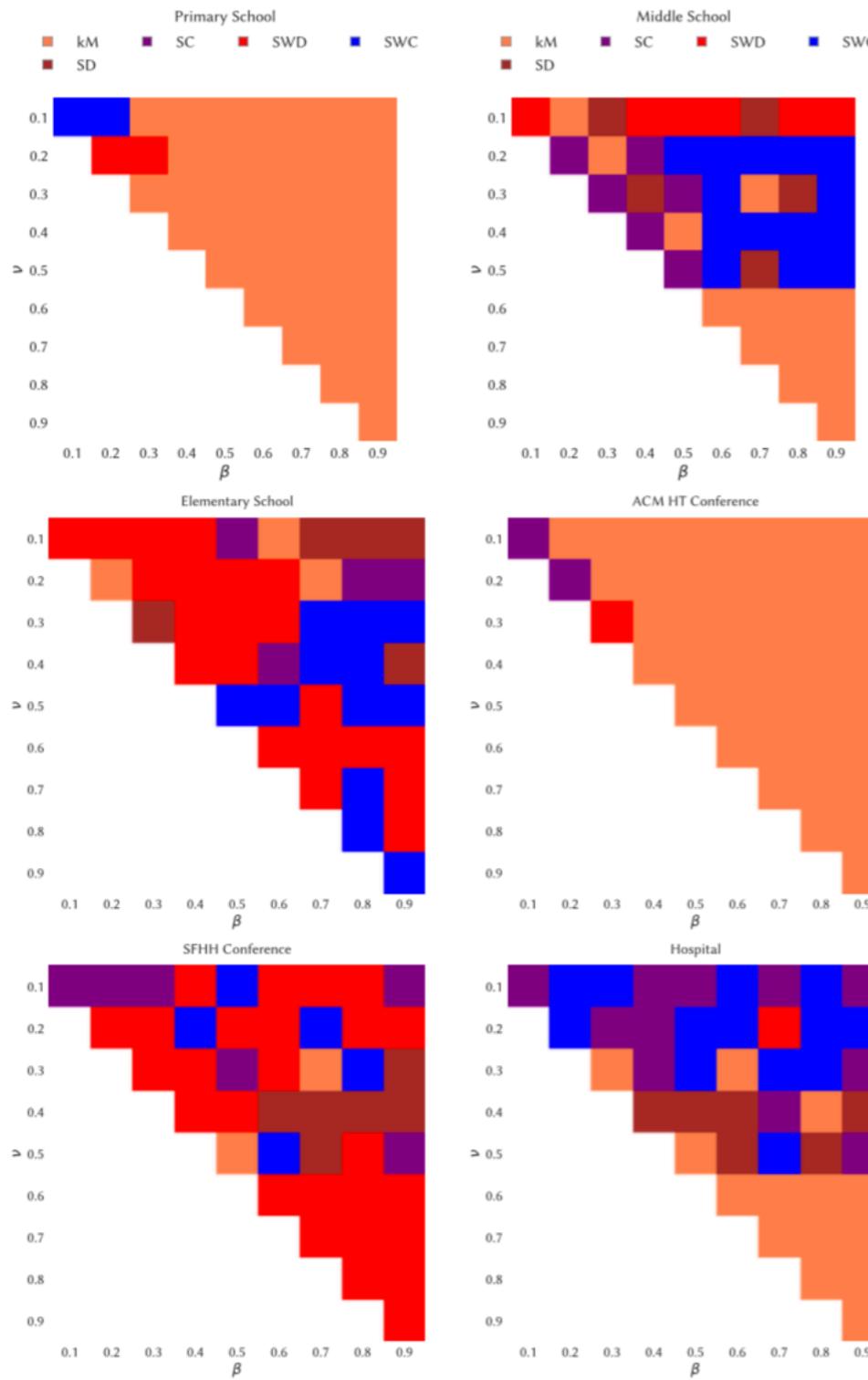


Distributions of outbreak sizes

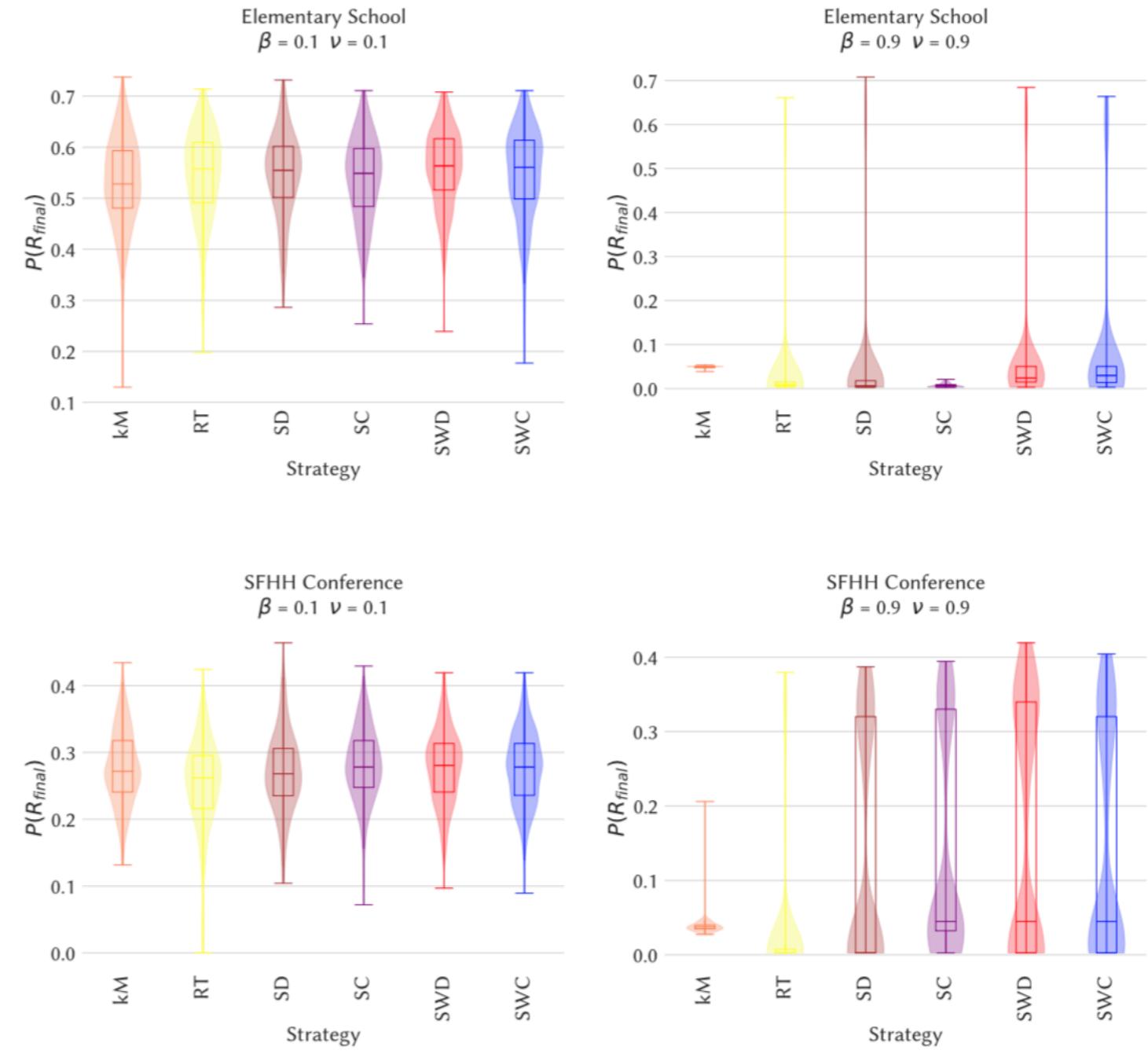


# Span cores and spreading processes

Best seeding strategy

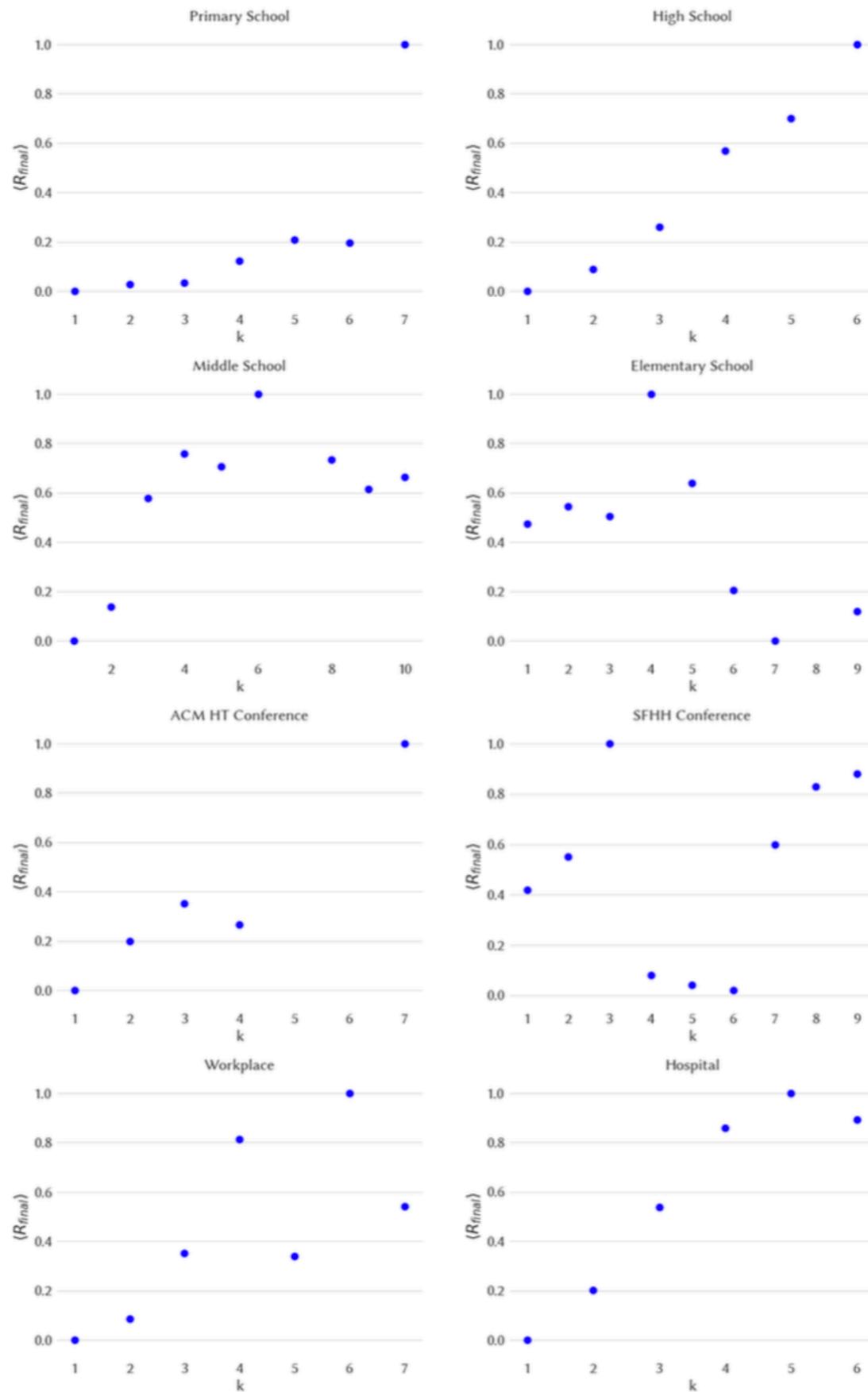


Distributions of outbreak sizes



# Span cores and spreading processes

Effect of the order of the core  
to which the seed belongs



# Span-cores

- Generalization of static core decomposition
- Effective algorithms to find (maximal) span-cores
- Uncovers strongly connected structures together with their duration/span
- Uncovers structures not trivially related to activity
- Broad distributions of durations
- Detection of anomalies in data
- Relevance in spreading processes

→ Need to take span-core structures into account in temporal network modelling

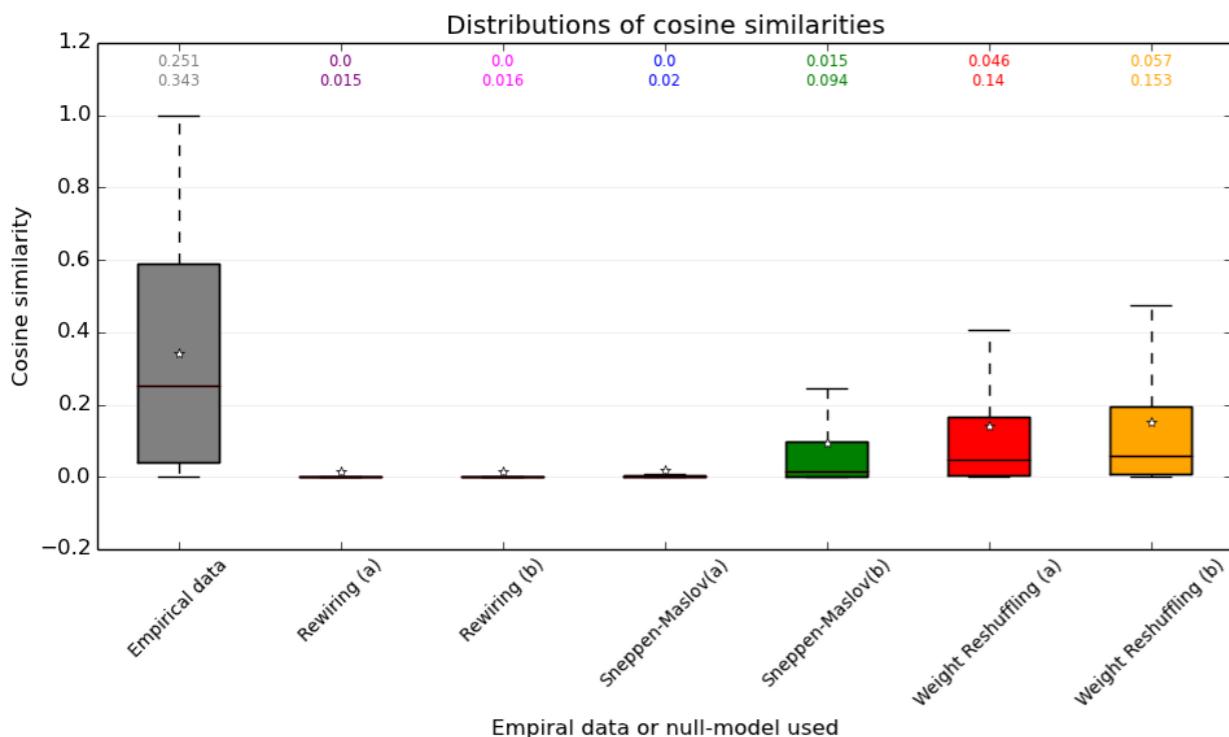
>Timescales and states

# Persistence of patterns

Compare **successive snapshots** or time-aggregated networks:

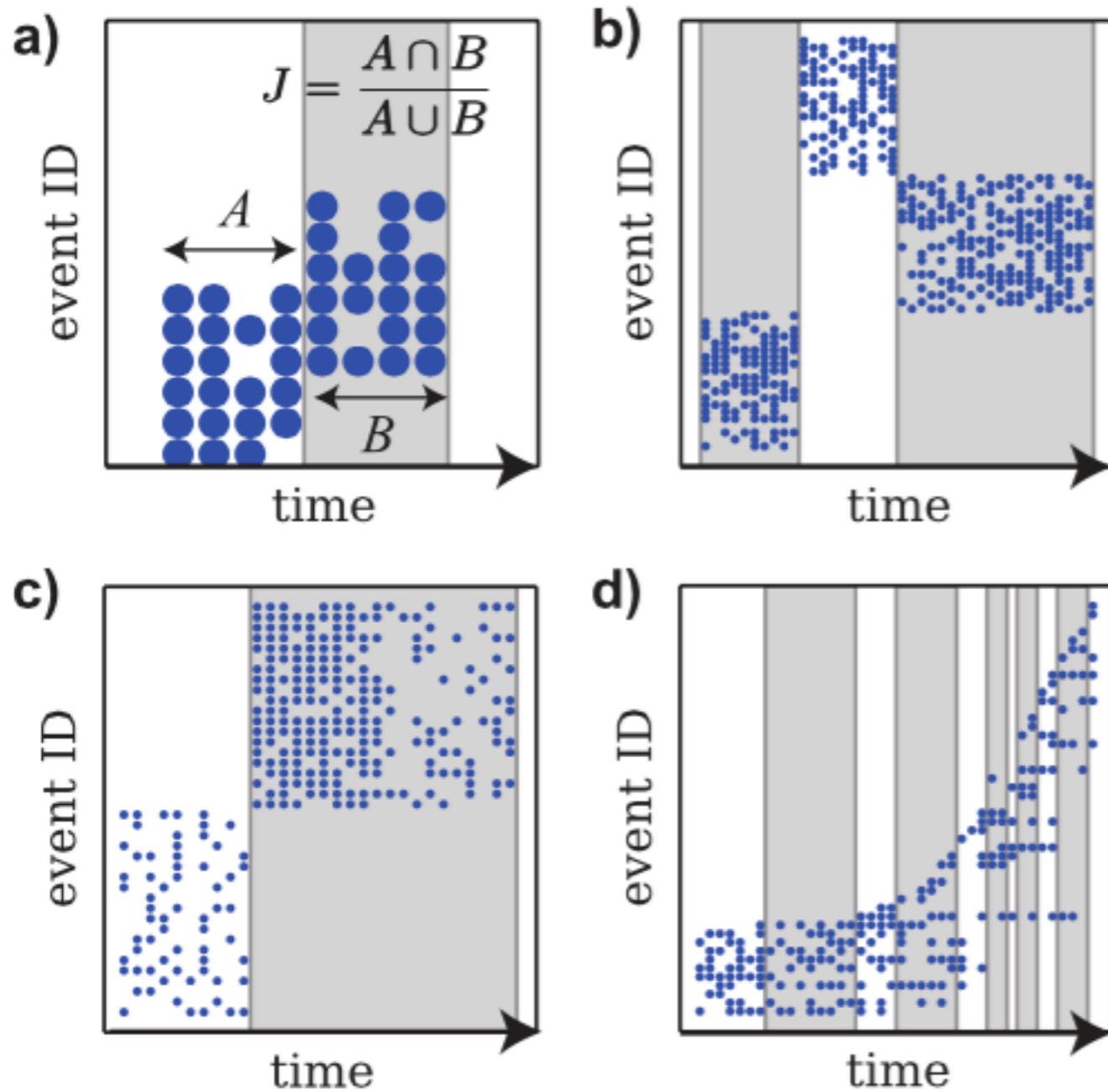
- correlation between (weighted) adjacency matrices
- correlation between contact matrices
- local similarity of neighbourhoods

$$\sigma_i = \frac{\sum_j w_{ij,(1)} w_{ij,(2)}}{\sqrt{\sum_j w_{ij,(1)}^2 \sum_{i,j} w_{ij,(2)}^2}}$$



Example: contacts in a high school,  
neighbourhood similarities between different days,  
vs different null models  
(Fournet & Barrat, PLoS ONE 2014)

# Detection of time-scales



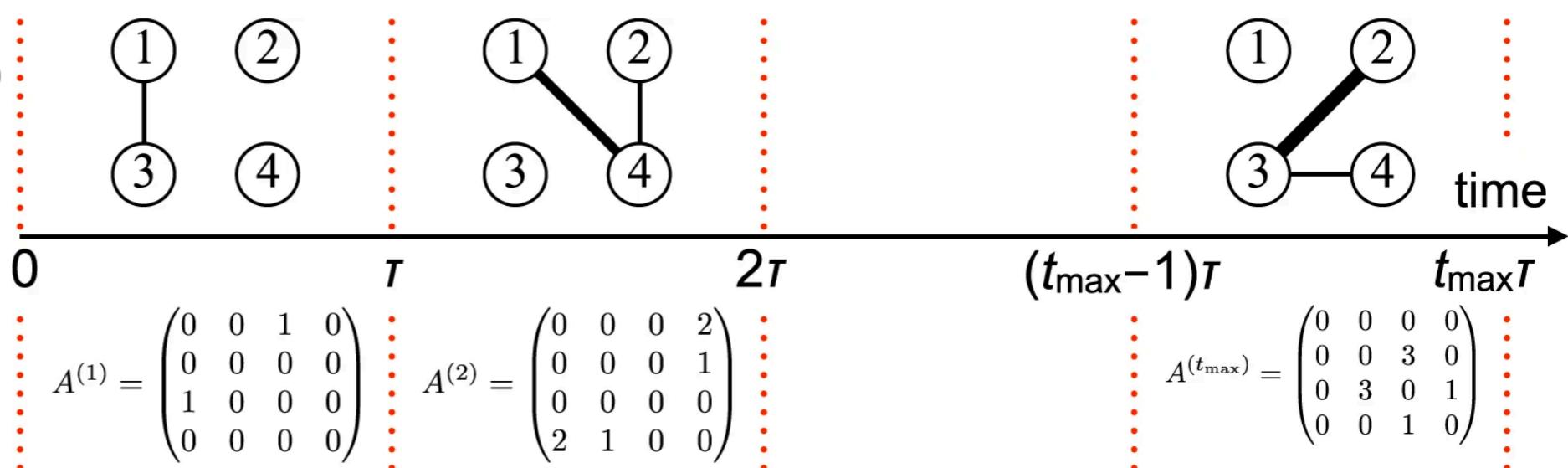
Comparison of successive time-windows of growing length  
Maximisation of overlap

Article | Open Access | Published: 28 January 2019

# Detecting sequences of system states in temporal networks

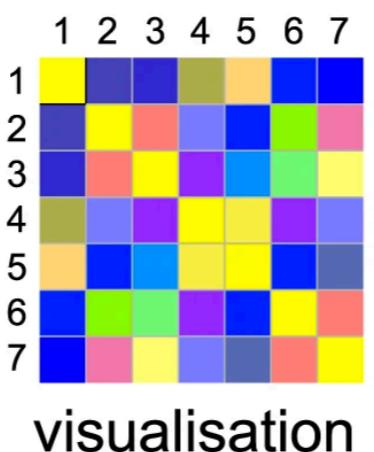
Naoki Masuda  & Petter Holme

Scientific Reports 9, Article number: 795 (2019)

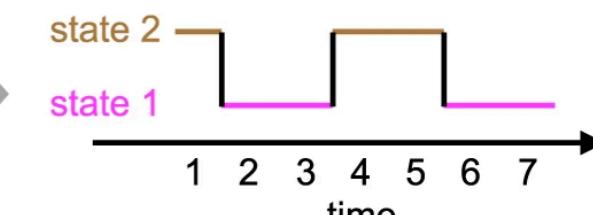
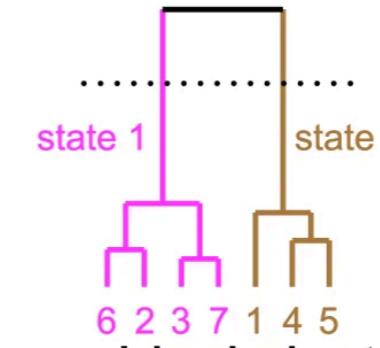


1. matrix of distance between snapshots
2. hierarchical clustering

→ states



distance matrix ( $d(i, j)$ )



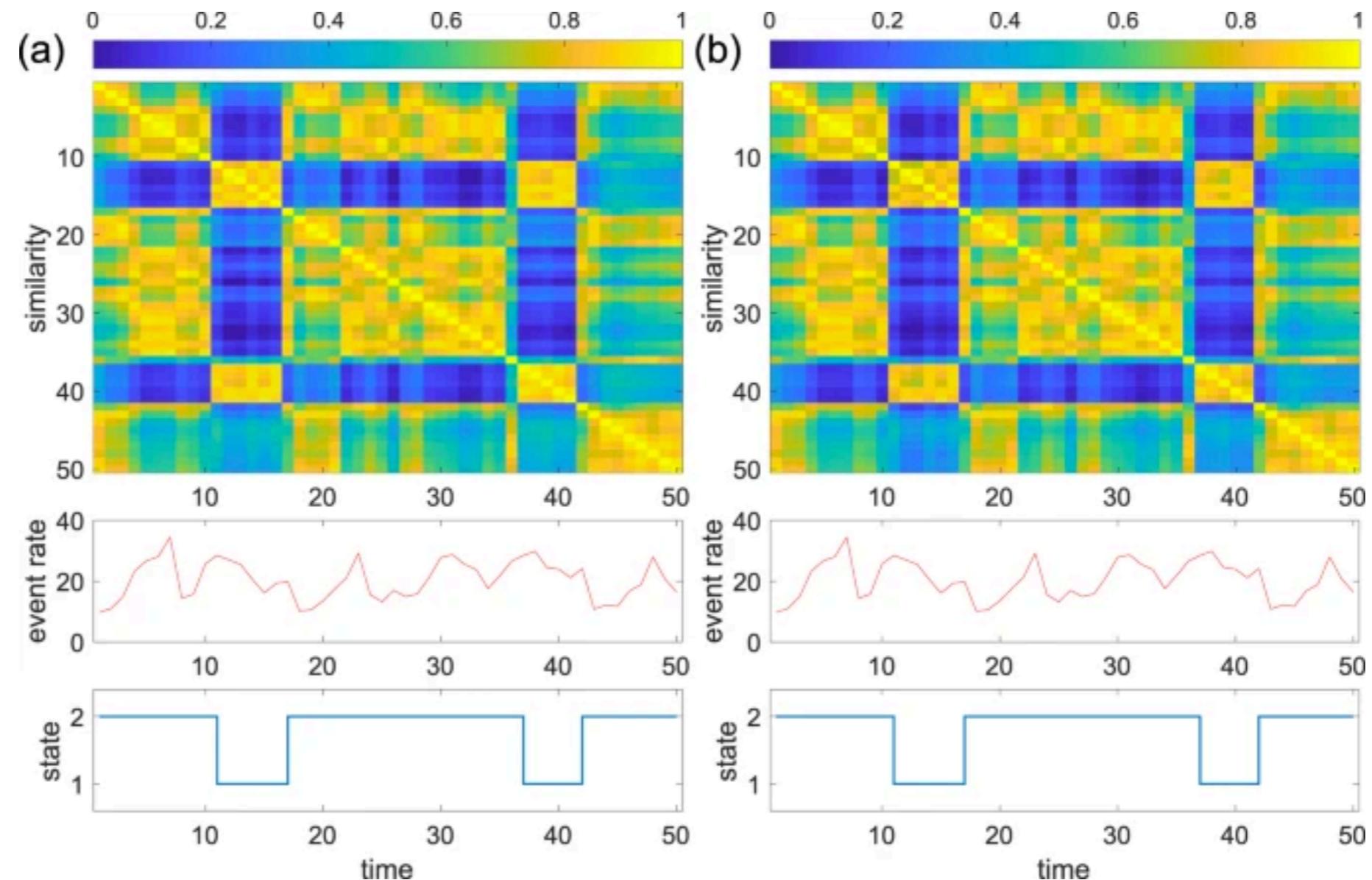
NB:  
various possible distances  
various clustering algorithms

Article | Open Access | Published: 28 January 2019

# Detecting sequences of system states in temporal networks

Naoki Masuda  & Petter Holme

Scientific Reports 9, Article number: 795 (2019) | Cite this article

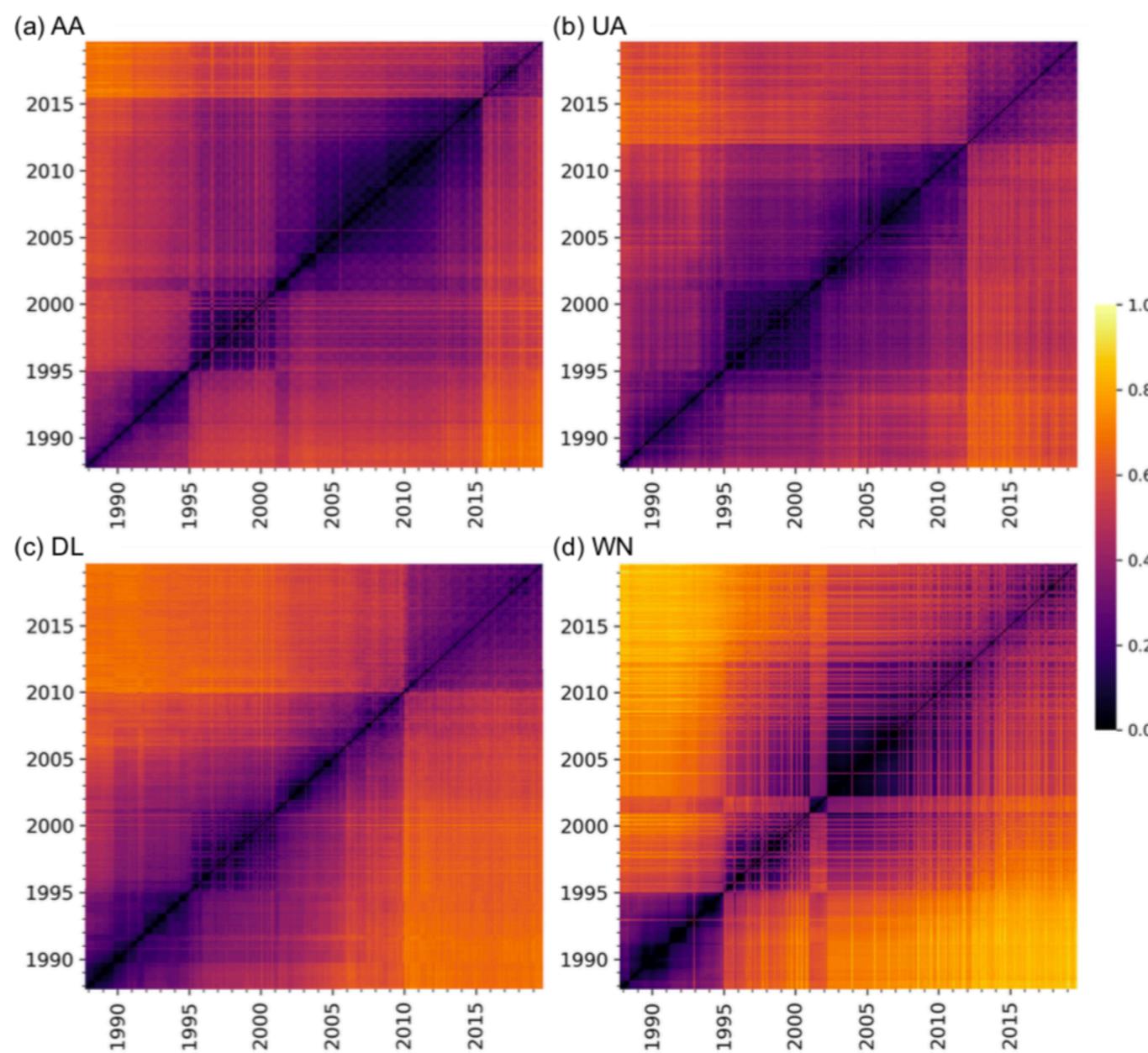


Face-to-face contacts in a primary school

[Submitted on 29 May 2020]

# Recurrence in the evolution of air transport networks

Kashin Sugishita, Naoki Masuda

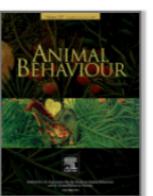




ELSEVIER

# Animal Behaviour

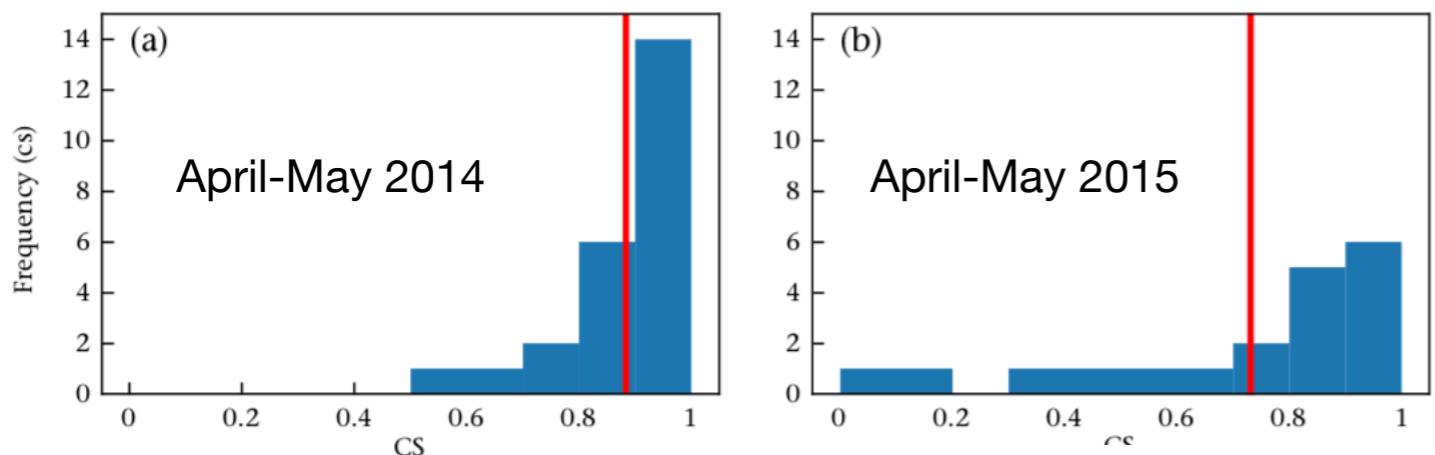
Volume 157, November 2019, Pages 239-254



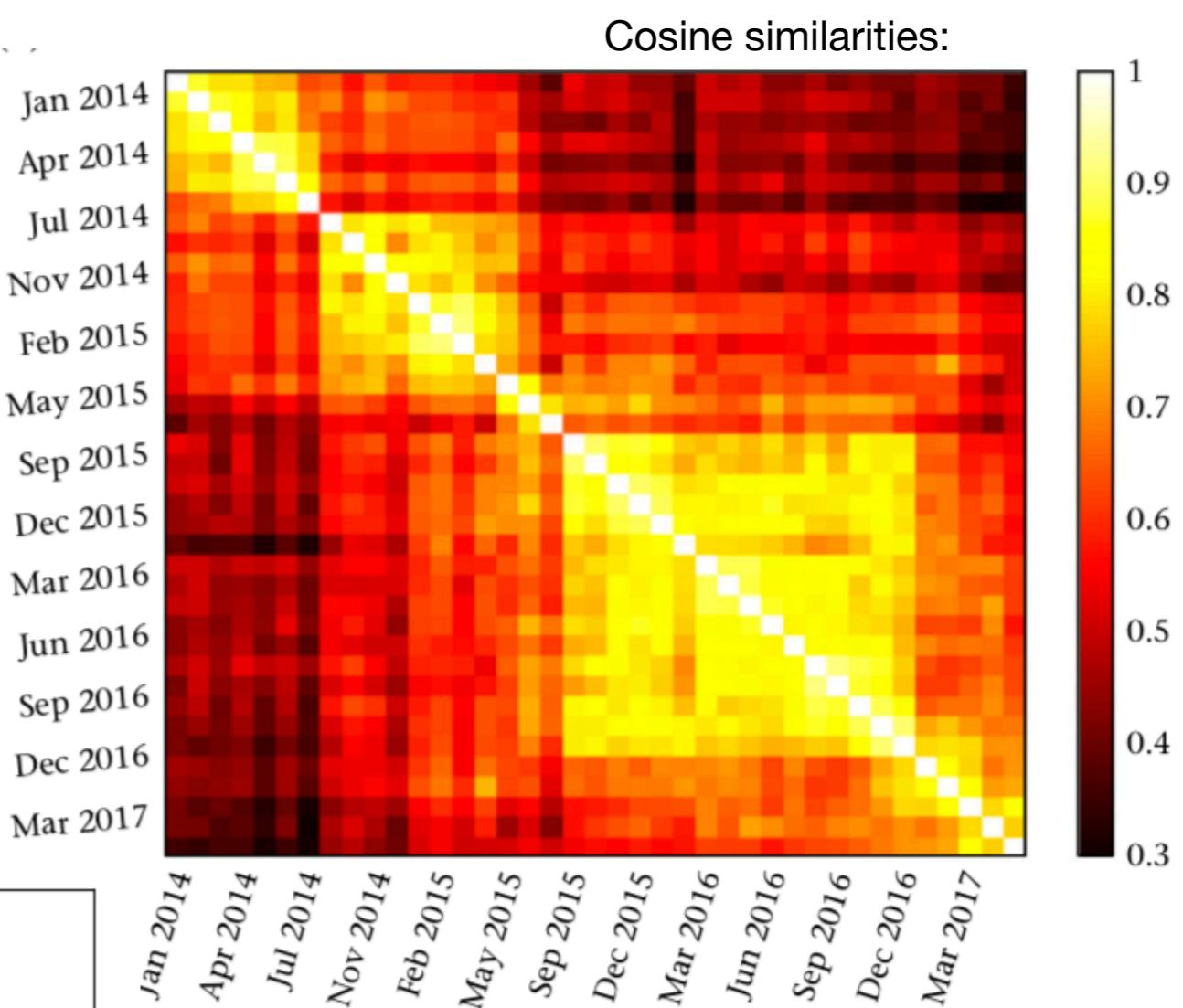
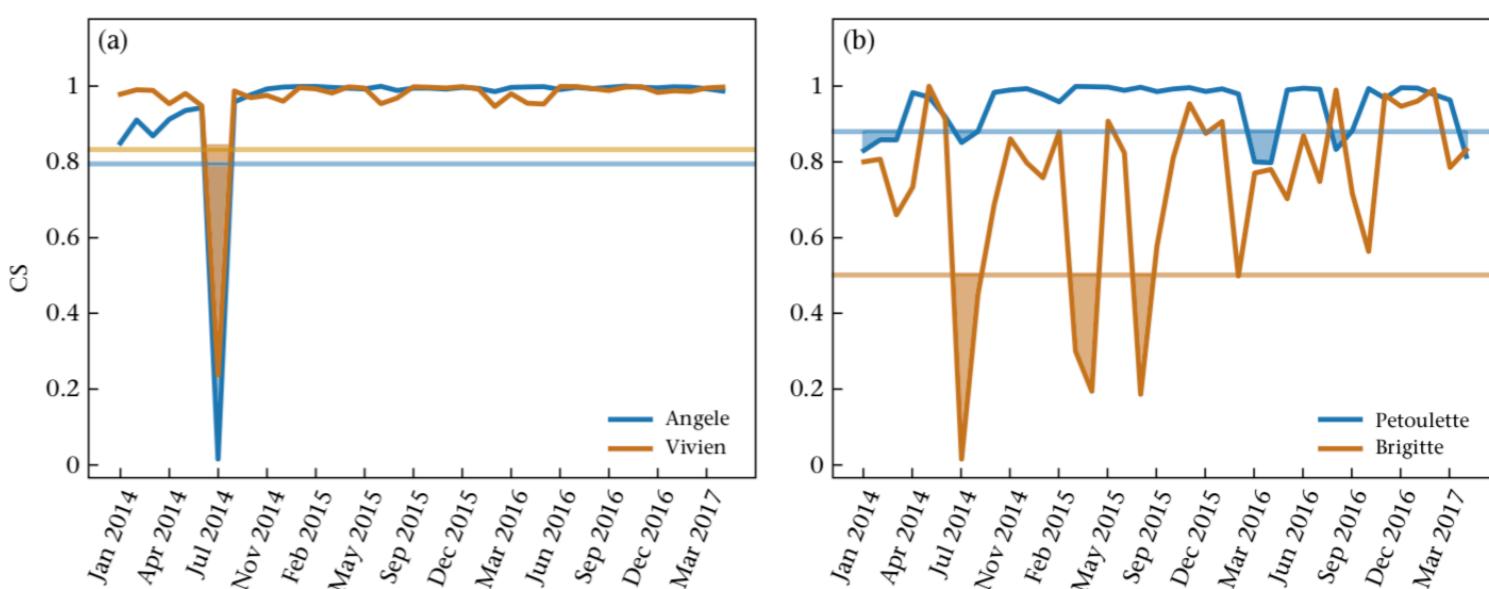
## Detecting social (in)stability in primates from their temporal co-presence network

Valeria Gelardi <sup>a</sup>, Joël Fagot <sup>b</sup>, Alain Barrat <sup>a, c</sup>, Nicolas Claidière <sup>b</sup>

Distributions of similarities  
(analysis beyond the average):



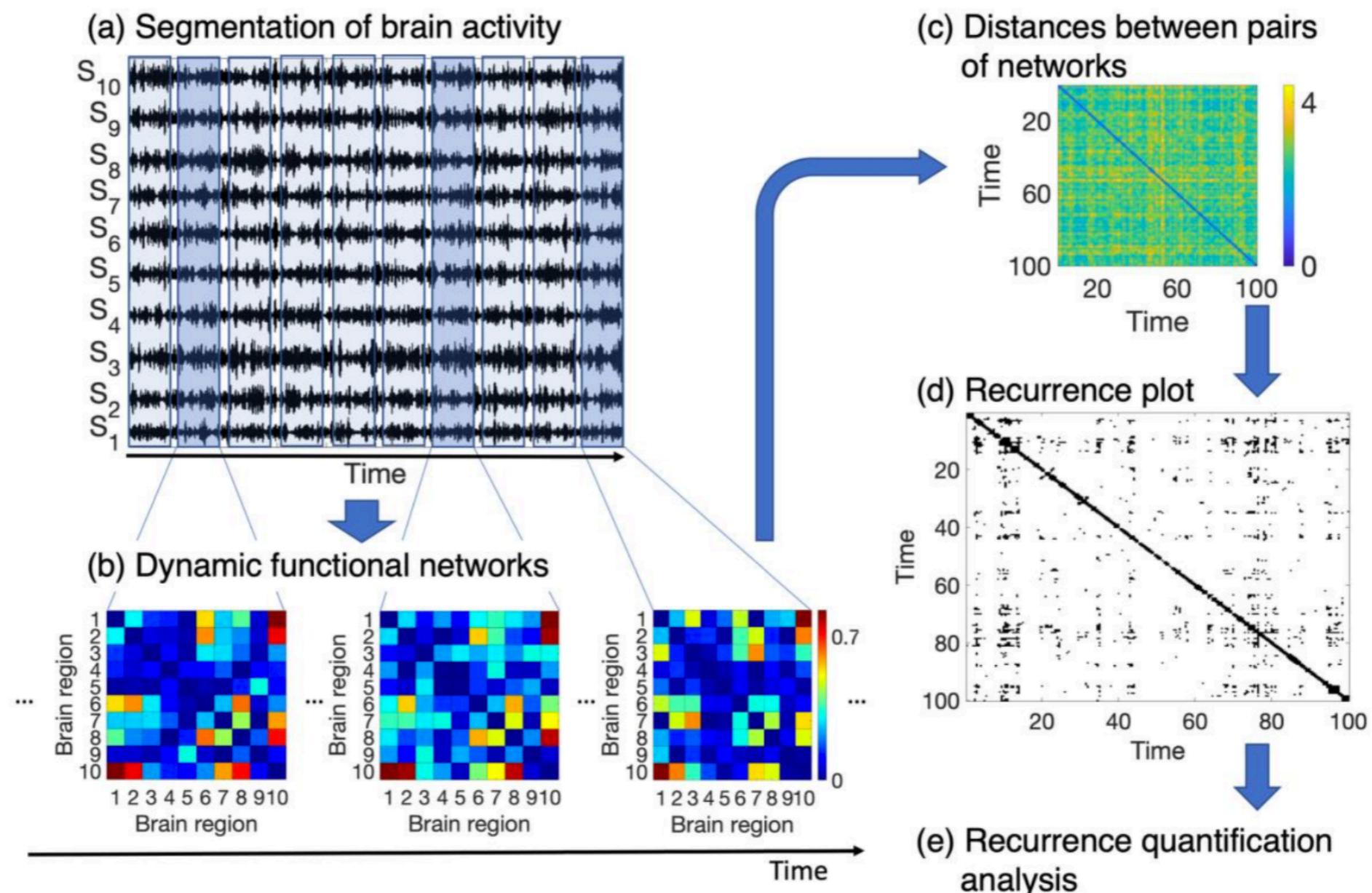
Ego networks  
CS(t,t+1):



[Submitted on 11 Jan 2020]

# Recurrence Quantification Analysis of Dynamic Brain Networks

Marinho A. Lopes, Jiaxiang Zhang, Dominik Krzemiński, Khalid Hamandi, Qi Chen, Lorenzo Livi, Naoki Masuda

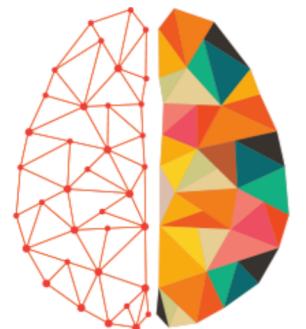


**Computer Science > Social and Information Networks***[Submitted on 24 Jul 2020 ([v1](#)), last revised 19 Aug 2020 (this version, v3)]*

# Detecting Dynamic States of Temporal Networks Using Connection Series Tensors

**Shun Cao, Hiroki Sayama**

Many temporal networks exhibit multiple system states, such as weekday and weekend patterns in social contact networks. The detection of such distinct states in temporal network data has recently been explored as it helps reveal underlying dynamical processes. A commonly used method is network aggregation over a time window, which aggregates a subsequence of multiple network snapshots into one static network. This method, however, necessarily discards temporal dynamics within the time window. Here we develop a new method for detecting dynamic states in temporal networks using information regarding the timeline of contacts between each pair of nodes. We apply a similarity measure informed by the techniques of processing time series and community detection to sequentially discompose a given temporal network into multiple dynamic states (including repeated ones). Experiments with empirical temporal network data demonstrated that our method outperformed the conventional approach using simple network aggregation in revealing interpretable system states. In addition, our method allows users to analyze hierarchical temporal structures and to uncover dynamic state at different spatial/temporal resolutions.



# NETWORK NEURO SCIENCE

Quarterly (winter, spring,  
summer, fall)

## Dynamic core-periphery structure of information sharing networks in entorhinal cortex and hippocampus

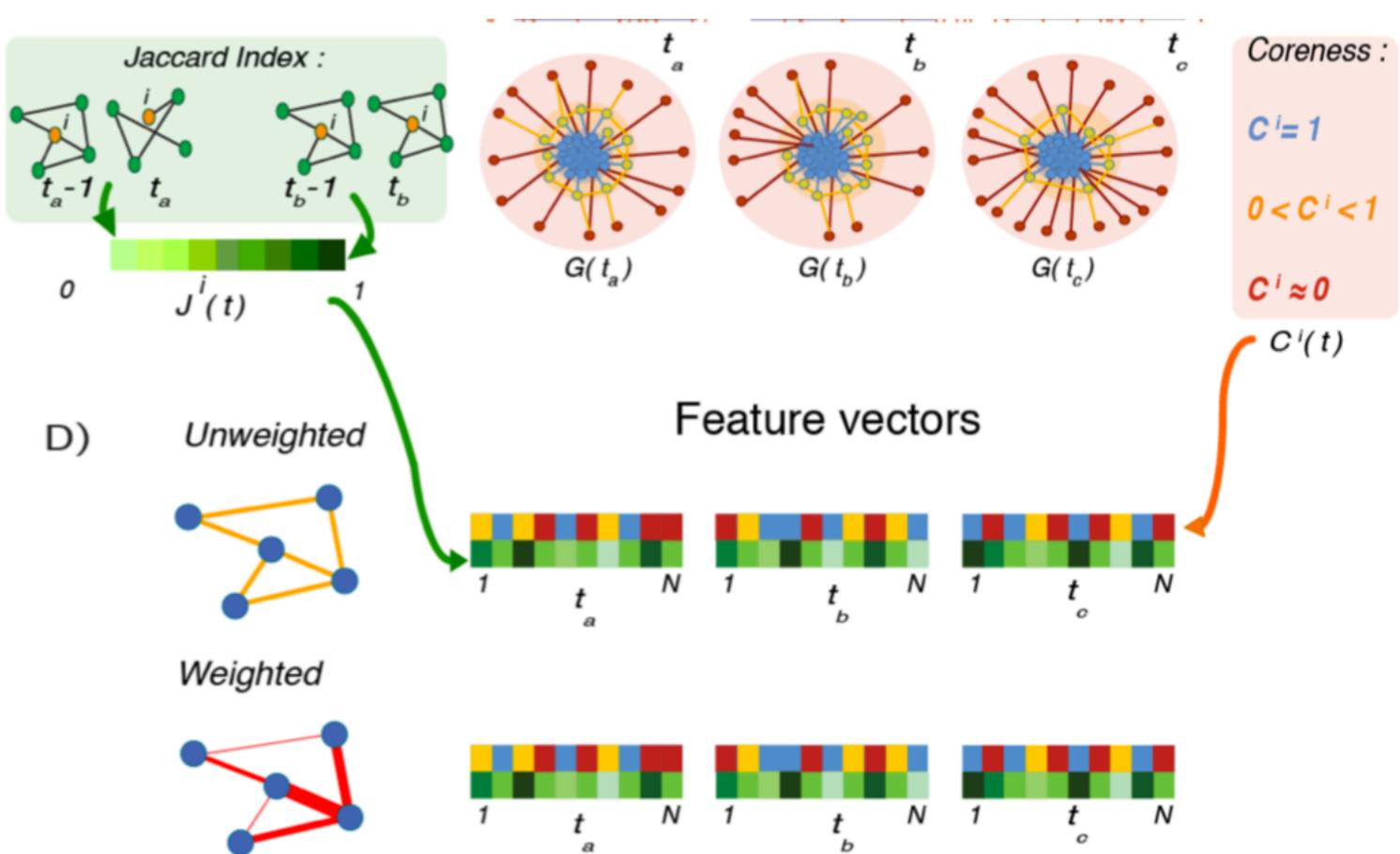
Nicola Pedreschi, Christophe Bernard, Wesley  
Clawson, Pascale Quilichini, Alain Barrat and  
Demian Battaglia

Posted Online April 28, 2020

[https://doi.org/10.1162/netn\\_a\\_00142](https://doi.org/10.1162/netn_a_00142)

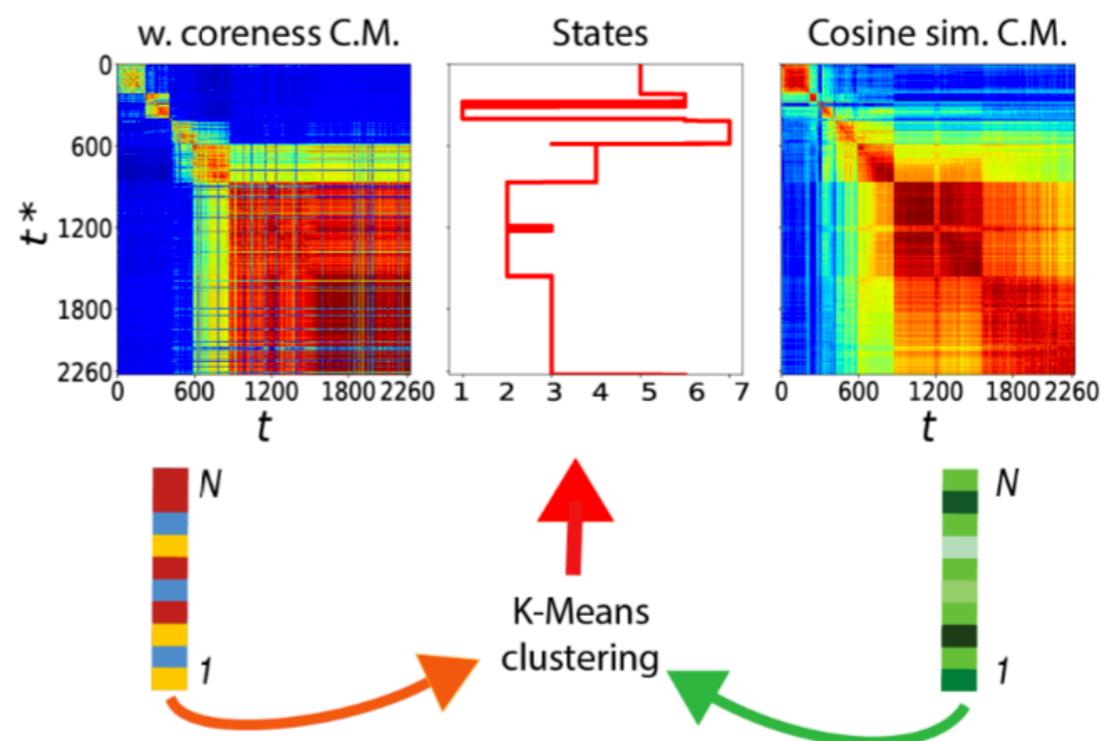
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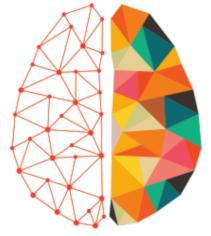
Network Neuroscience  
Early Access  
v1-20



1. Vector of features (here coreness+liquidity) at each time
2. K-means clustering of times

→ states





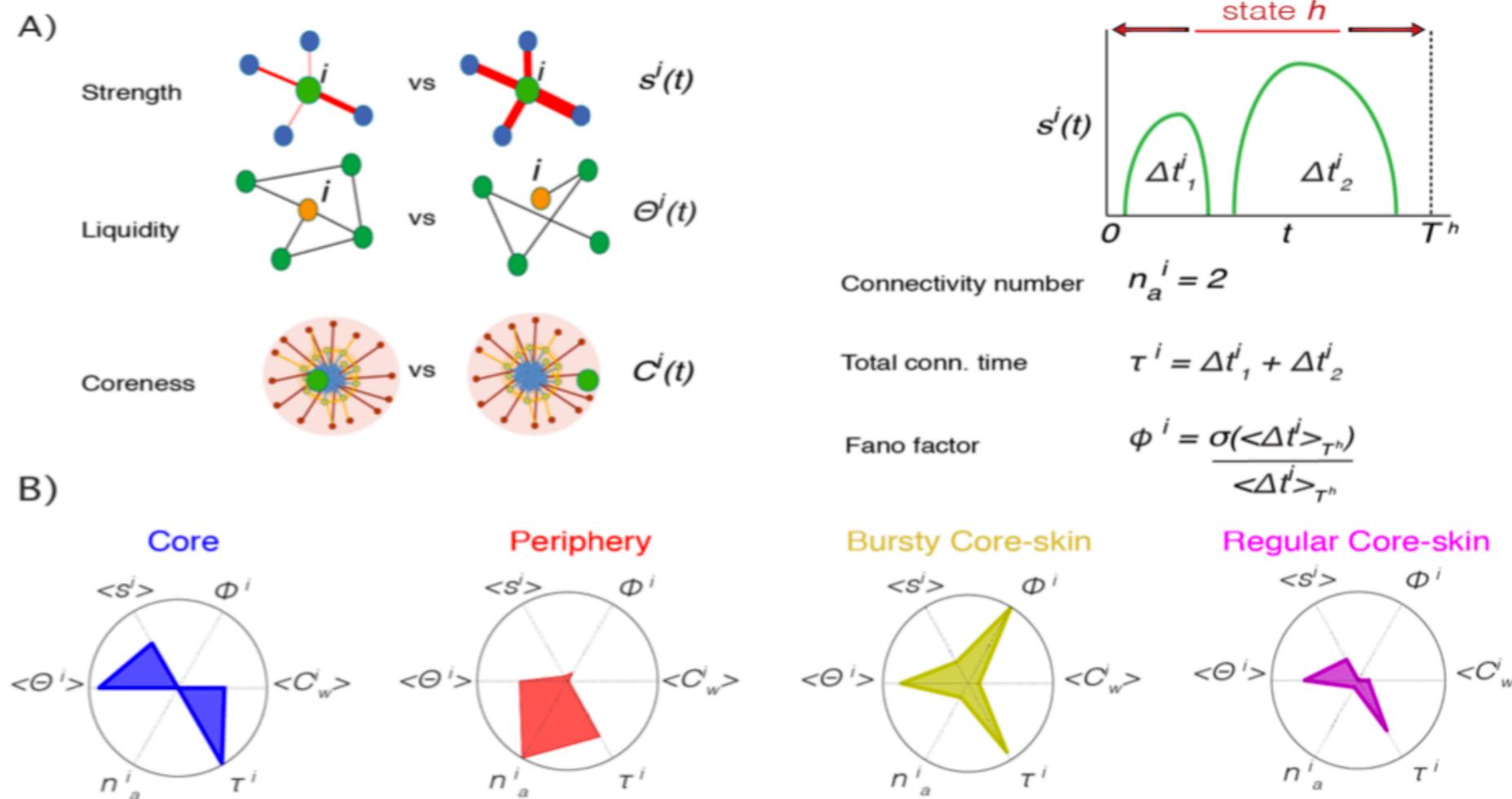
# Dynamic core-periphery structure of information sharing networks in entorhinal cortex and hippocampus

Nicola Pedreschi, Christophe Bernard, Wesley Clawson, Pascale Quilichini, Alain Barrat and Demian Battaglia

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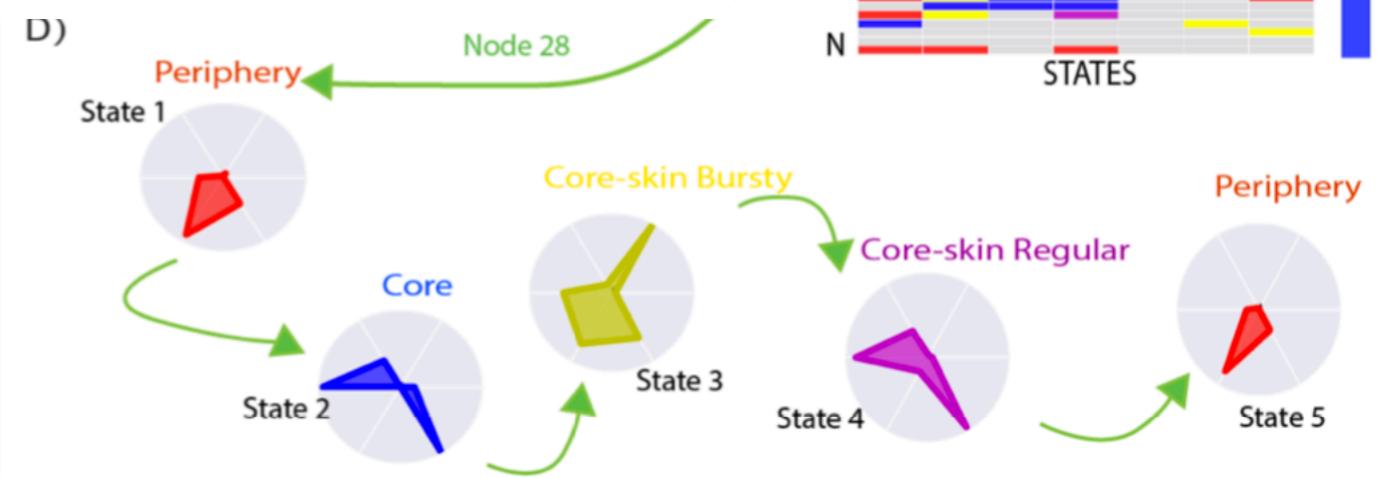
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v1.20



**NB:**  
dynamics *within each state*  
different types of dynamics for different nodes

→ different connectivity profiles



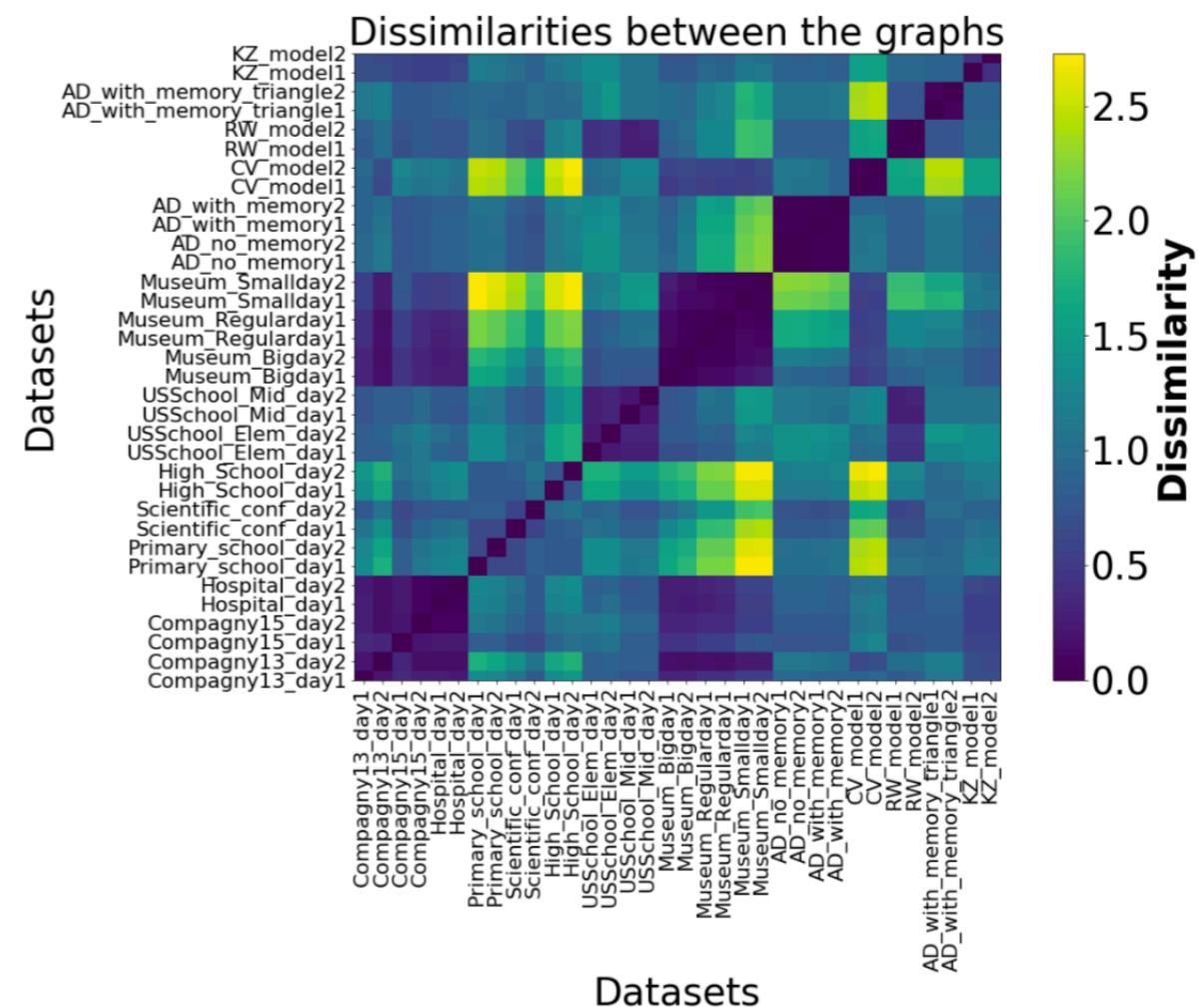
# Current work

Backbone of events (with T. Bassanetti, M. Karsai)

Beyond spreading processes

Comparison of temporal networks (with E. Andres, M. Karsai)

Temporal simplicial complexes



# Temporal networks: Still very open field!

Data

Structures in data

Incompleteness of data

Models

Processes on temporal networks

...

Datasets: [www.sociopatterns.org/datasets](http://www.sociopatterns.org/datasets)