CLASSMATE

Context Free Grammar (CFG) and Languages

Regular Language is one class of language similarly context free Language (CFL) is also a class of language

Context-Free Language is described by context-free grammar.

Notion of Grammar :-

Let us consider English sentences of the form.

1] Mohan eats 2] Soham plays 3] Ram reads.

Any english language sentence is a collection of words but any collection of words we can not say a meaningful english language sentence because to be english language sentence some grammar rules need to be follow.

The first word of in the above sentences is a noun and the second word is a verb. A sentence of the above form can be written as

<sentence> → <noun><Verb>.

noun can be-

<noun> → Mohan | Soham | Ram.

<Verb> → eats | plays | reads.

similarly, we can construct rules for any language

ex. $L \subseteq \{0,1\}^*$

$S \rightarrow 0S1$ $\quad$ $S \rightarrow 0S1$

$S \rightarrow \epsilon$ $\quad$ $0\boxed{0S1}1$

$00\boxed{0S1}11$ $\quad$ $S \rightarrow \epsilon$

000111

Scanned by CamScanner

so string generated are $\{0^n 1^n \mid n \geq 0\}$

Definition : Context Free Grammar

A context Free Grammar G is a quadruple

$$G = (V, \Sigma, P, S) \quad \text{where.}$$

V: set of non-terminal.

Σ: set of terminal.

P: set of rewritting rules OR production rule.

S: The start symbol.

G is a context free grammar if each element of P is of the form

$$X \rightarrow \alpha \quad \text{where } X \in V \text{ and } \alpha \in (V \cup \Sigma)^*$$

Notation used :-

1] Capital letters $(A, B, C \cdots X, Y, Z)$ denote Non-terminals.

2] Small case letter $(a, b, c \cdots)$ denote terminals.

3] $\alpha, \beta, \gamma \in (V \cup \Sigma)^*$.

4] $x, y, z \ldots, u, v, w \in \Sigma^*$
(last Few letters)

for above example

$V = \{S\}$, $\Sigma = \{0, 1\}$, $P = \{S \rightarrow 0S1, S \rightarrow \epsilon\}$, $S = S$

imp $V \cap \Sigma = \phi$.

1) To get the string we start with start symbol S.
2) Use production whose left hand side is S & replace it by righ side string.
3] Above process (step 2) continue until we get desire string.

Definition : Context Free Language:

Context free grammar $G = (V, \Sigma, P, S)$ associated with language $L(G) \subseteq \Sigma^*$ as follow.

$$L(G) = \{x \in \Sigma^* \mid S \stackrel{*}{\Rightarrow} x\}$$

$L(G)$: Language generated by grammar G. represent set of all terminals strings which can be derived from start symbol of G.

A language L is context free language if there is a context free grammar G such that $L = L(G)$

ex 2. $G = (\{S\}, \{a, b\}, \{S \to aSa, S \to bSb, S \to \epsilon\}, S)$

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabaSabaa$
$\Rightarrow aabaabaa$

set of palindrome string whose length is even.

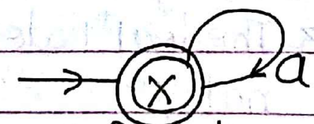
ex 3. $S \to aB \mid bA$
$A \to aS \mid bAA \mid a$
$B \to bS \mid aBB \mid b$

$S \Rightarrow aB$
$\Rightarrow a\boxed{aBB}$
$\Rightarrow aaB\boxed{bS}$
$\Rightarrow aaBb\boxed{bA}$
$\Rightarrow aaBbb\boxed{a}$
$\Rightarrow aabbba$

corresponding left most derivation

$S \Rightarrow aB$
$\Rightarrow a\boxed{aBB}$
$\Rightarrow aa\boxed{bS}B$
$\Rightarrow aabaBB$
$\Rightarrow aababB$
$\Rightarrow aababb.$

ex 4. DFA is



Grammar G for above DFA is $X \to aX$
$X \to \epsilon$

Scanned by CamScanner

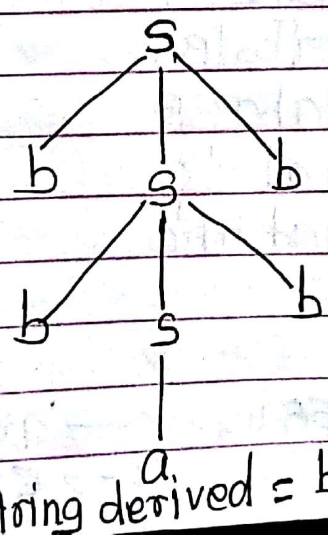
Grammar can be used to represent both.
1. Regular languages 2] Non-regular languages.

Sentential forms: (Derivation)
A string α derived from start symbol of G can be written as $S \xrightarrow{*} \alpha$ $(\alpha \in (V \cup \Sigma)^*$ which is called as sentential form.

A string can be derived in many ways, but we restrict ourselves to
1. Leftmost derivation OR sentential form
   - leftmost symbol is picked up for expansion. (Non-Terminal)
2. Rightmost derivation
   - rightmost symbol is picked up for expansion. (Non-terminal)

Parse Tree: Graphical way of seeing how a particular string is derived.

ex 4. $S \rightarrow bSb \mid a \mid b$ is a production rule. The S is start symbol.



string derived = bbabb

Properties of parse tree
1. The root node is always a node indicating start symbol.
2. The derivation is read from ~~left~~ Top to ~~right~~ bottom & from Left to right.
3. The leaf nodes are always terminals nodes.
4] The interior nodes are always the non terminal nodes.

ex 5. consider the grammar

$S \rightarrow S+S \mid S*S \mid 4$.

Derive string $x = 4+4*4$

Using left most derivation

$S \Rightarrow S*S \Rightarrow S+S*S \Rightarrow 4+S*S \Rightarrow 4+4*S \Rightarrow 4+4*4$

Using rightmost derivation

$S \Rightarrow S+S$

$\Rightarrow S+S*S \Rightarrow S+S*4 \Rightarrow S+4*4$

$\Rightarrow 4+4*4$.

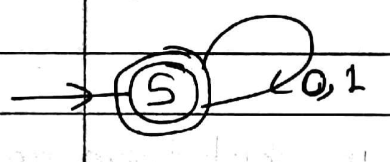
ex 6. Try to recognize the language L for given CFG.

$G = (\{S\}, \{a,b\}, P, S)$

where $P = \begin{Bmatrix} S \rightarrow aSb \\ S \rightarrow ab \end{Bmatrix}$

$\rightarrow$ $L = \{a^n b^n \mid n \geq 1\}$.

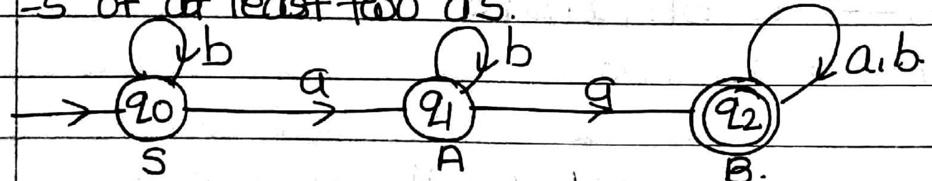
ex 7. Construct the CFG for the regular expression $(0+1)^*$



$S \rightarrow 0S \mid 1S$

$S \rightarrow \epsilon$

ex 8. construct a grammar for the language containing strings of at least two a's.



$S \rightarrow Aa\,Aa\,A$

$A \rightarrow aA \mid bA \mid \epsilon$

OR

$S \rightarrow aA \mid bS$

$A \rightarrow bA \mid aB \mid a$

$B \rightarrow aB \mid bB \mid \epsilon$.

Scanned by CamScanner

ex9. construct a grammar generating
$L = wcw^T$ where $w \in \{a,b\}^*$

→ $S \rightarrow aSa \mid bSb \mid c$

ex10. Construct CFG for the language L which has all the strings which are all palindrome over $\Sigma = \{a,b\}$

→

$S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow a$

$S \rightarrow b$

$S \rightarrow \epsilon$

ex11. construct CFG which consists of all the strings having at least one occurrence of 000

r.e $(0+1)^* \, 000 \, (0+1)^*$

$S \rightarrow ATA$

$A \rightarrow 0A \mid 1A \mid \epsilon$

$T \rightarrow 000$

ex12. construct CFG for the language in which there are no consecutive b's, the strings may or may not have consecutive a's.

→ $S \rightarrow aS \mid bA \mid a \mid b \mid \epsilon$

$A \rightarrow aS \mid a \mid \epsilon$

ex13. Recognize the CFL for the given CFG

$$\left.\begin{array}{l} S \rightarrow aB \mid bA \\ A \rightarrow a \mid aS \mid bAA \\ B \rightarrow b \mid bS \mid aBB \end{array}\right\} \text{equal no of } a \,\&\, b$$

Scanned by CamScanner

ex14. construct CFG for the language containing at least one occurrence of aa

$S \rightarrow BAB$

$A \rightarrow aa$

$B \rightarrow aB \mid bB \mid \epsilon$

ex15. construct CFG for the language containing all the strings of different first & last symbols over $\Sigma = \{0,1\}$

→ $S \rightarrow 0A1 \mid 1A0$

$A \rightarrow 0A \mid 1A \mid \epsilon$

ex 16. construct the production rules for defining a language $L = \{a^x b^x \mid x \neq y\}$

→ $S \rightarrow aSb \mid R1 \mid R2$

$R1 \rightarrow aR1 \mid a$

$R2 \rightarrow bR2 \mid b$.

ex 17. Find the CFG for the regular expression $(110+11)^*(10)^*$

→ $\underbrace{(110+11)^*}_{A} \; \underbrace{(10)^*}_{B}$

$S \rightarrow AB$

$A \rightarrow 110A \mid 11A \mid \epsilon$

$B \rightarrow 10B \mid \epsilon$

ex 18. Build a CFG for generating the integer

→ $S \rightarrow GI$

$G \rightarrow + \mid -$

$I \rightarrow DI \mid D$

$D \rightarrow 0 \mid 1 \mid 2 \mid \dots \mid 9$.

Scanned by CamScanner

classmate

ex 19. Build a CFG for the language $L = \{0^i 1^j 2^k \mid j > i+k\}$

$$L = 0^i \underbrace{1^i 1^k 1^p}_{} 2^k.$$

↳ This will be greater than $i+k$

We can again rewrite L for simplification as –

$$L = \underbrace{0^i 1^i}_{A} \underbrace{1^p}_{B} \underbrace{1^k 2^k}_{C}$$

$A \rightarrow 0A1 \mid \epsilon$

$B \rightarrow 1B \mid 1$

$C \rightarrow 1C2 \mid \epsilon$

ex 20. Build a CFG for the language $L = \{0^i 1^j 2^k \mid i = j\}$

$S \rightarrow AB$

$A \rightarrow 0A1 \mid \epsilon$

$B \rightarrow 2B \mid \epsilon$

ex 21. obtain CFG for the language $L = \{0^i 1^j 2^k \mid j \leq k\}$

→ Here $i \leq k$. That means the language L has two variable

$L1 = 0^i 1^j 2^j$

$L2 = 0^i 1^j 2^k$ where $k > j$

$\therefore S \rightarrow AB$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 1B2 \mid C$

$C \rightarrow 2C \mid \epsilon$

ex 22. obtain grammar to generate the language $L = \{0^n 1^{2n} \mid n \geq 0\}$

→ $S \rightarrow 0S11 \mid \epsilon$

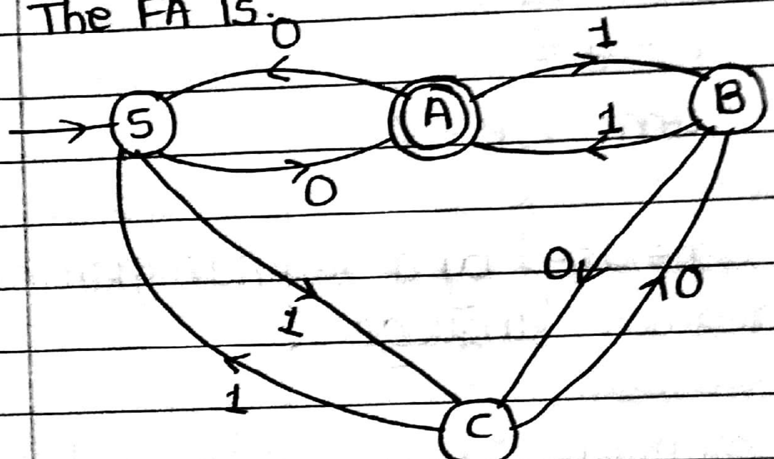
Scanned by CamScanner

ex 22 - Find the languages defined by the following grammar

i) $S \rightarrow 0A \mid 1C \mid 0$
$A \rightarrow 0S \mid 1B \mid \epsilon$
$B \rightarrow 1A \mid 0C \mid 1$
$C \rightarrow 0B \mid 1S$

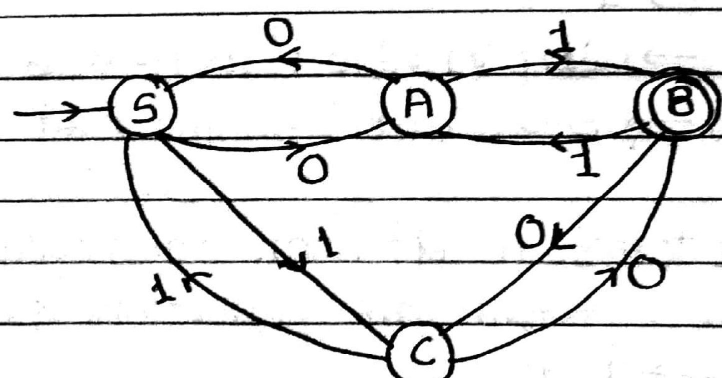
ii) $S \rightarrow 0A \mid 1C$
$A \rightarrow 0S \mid 1B \mid 1$
$B \rightarrow 0C \mid 1A \mid \epsilon$
$C \rightarrow 0B \mid 1S$ May-3 Marks-10

→ We will draw FA using the given grammar. Here S is a start state & as $A \rightarrow \epsilon$ is the rule state A is a final state.
The FA is.



string derived = {0, 011, 110, 000, 11110, 101, ...}
language containing odd number of zero's & even number of one's.

ii)



L = {01, 0001, 1101, ...} This denote a language containing odd number of one's.

Tech-Max-TCS-PU-Examples From 4-12 to 4-36

Scanned by CamScanner

Regular Grammar :-

The language accepted by finite automata can be described using a set of productions known as regular grammar. The productions of a regular grammar are of the following form:

$$A \rightarrow a$$
$$A \rightarrow aB$$
$$A \rightarrow Ba$$
$$A \rightarrow \epsilon$$

Where, $a \in T(\Sigma)$ and $A, B \in V$

A language generated by a regular grammar is known as regular language.

A regular grammar can be written in two forms.

1. Right linear form - A right linear regular grammar will have production of the given form

$$\left.\begin{array}{l} A \rightarrow a \\ A \rightarrow aB \\ A \rightarrow \epsilon \end{array}\right\}$$ Variable B in $A \rightarrow aB$ is the second symbol on the right.

2. Left-linear form - A left linear regular grammar will have production of the following form

$$\left.\begin{array}{l} A \rightarrow a \\ A \rightarrow Ba \\ A \rightarrow \epsilon \end{array}\right\}$$ Variable B in $A \rightarrow Ba$ is the first symbol on the right.

Scanned by CamScanner

classmate

DFA to right Linear Regular Grammar:-

Every DFA can be described using a set of production using the following steps:

Let the DFA, $M = (Q, \Sigma, \delta, q_0, F)$

Let the corresponding right-Linear grammar

$G = (V, \Sigma, P, S)$

1. Rename $q_0 \in Q$ as $S \in V$, relating start state of M with starting symbol of G.

2. Rename states of Q as A, B, C, D, ... where A, B, C, D, ... $\in V$

3. Creating a set of production P.

   (a) If $q_0 \in F$ then add a production $S \rightarrow \epsilon$ to P.

   (b) For every transition of the form.

   $$(B) \xrightarrow{a} (C)$$

   add a production $B \rightarrow aC$, where C is a non-accepting state.

   (c) For every transition of the form.

   $$(B) \xrightarrow{a} ((C))$$

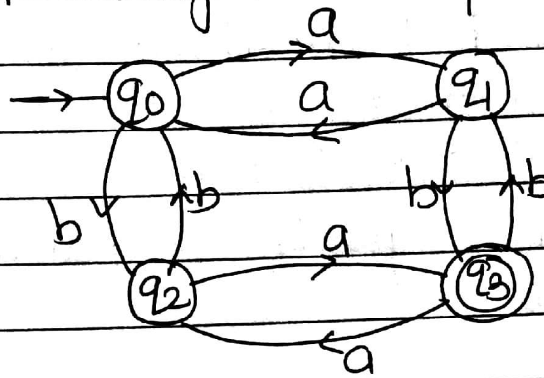
   This is optional ⇓

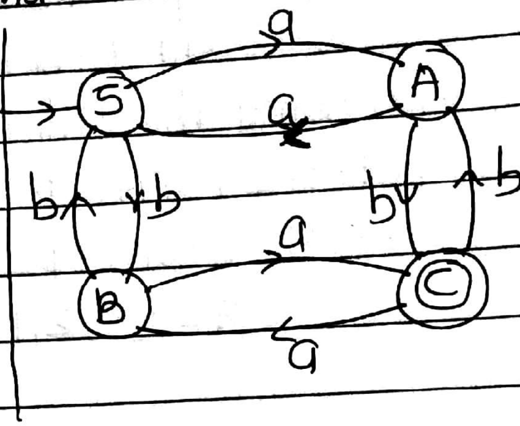
   add two production $B \rightarrow aC$, $\boxed{B \rightarrow a}$ because C is final state which produce ϵ

   where C is an accepting state.

Scanned by CamScanner

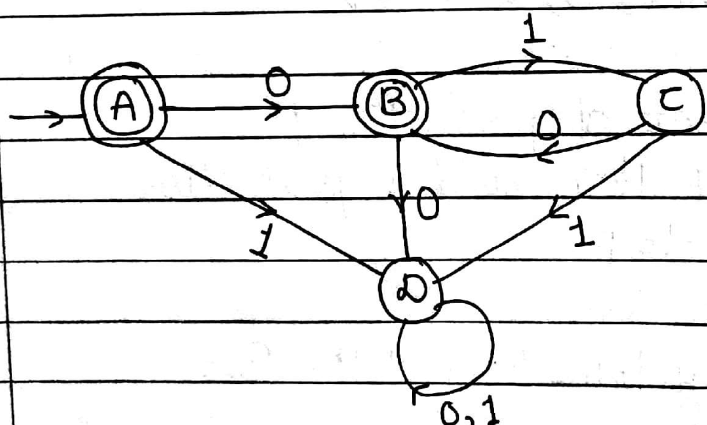
ex. Give a right-linear grammar for the DFA.



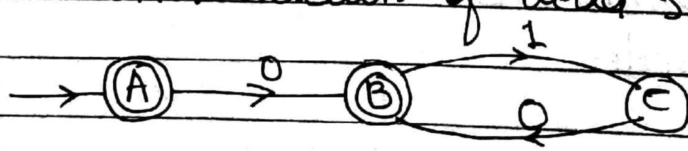


$$P = \begin{cases} S \rightarrow aA \mid bB \\ A \rightarrow aS \mid bC \mid b \\ B \rightarrow aC \mid bS \mid a \\ C \rightarrow aB \mid bA \end{cases}$$

ex 2.



state D is a dead state & it can be removed, The FA after deletion of dead state D is



$A \rightarrow \epsilon$

$A \rightarrow 0B \mid 0$

$B \rightarrow 1C \mid \epsilon$

$C \rightarrow 0B \mid 0$

Scanned by CamScanner

Right Linear Grammar to DFA :-

ex 1. convert the following right linear grammar to an equiva DFA.

q.

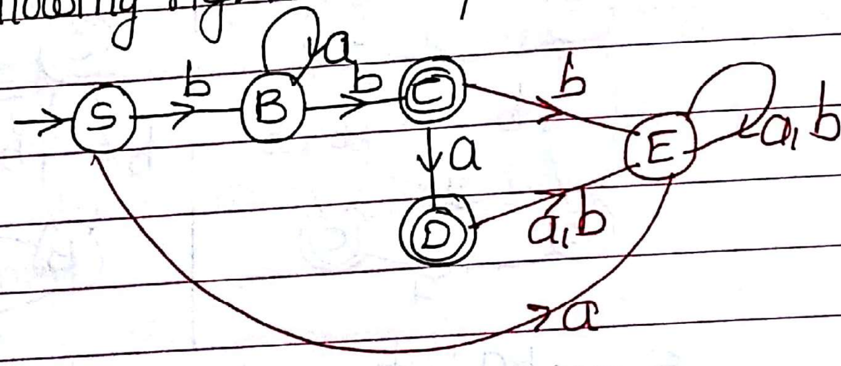
S → bB.

B → bc

B → aB

C → a

B → b



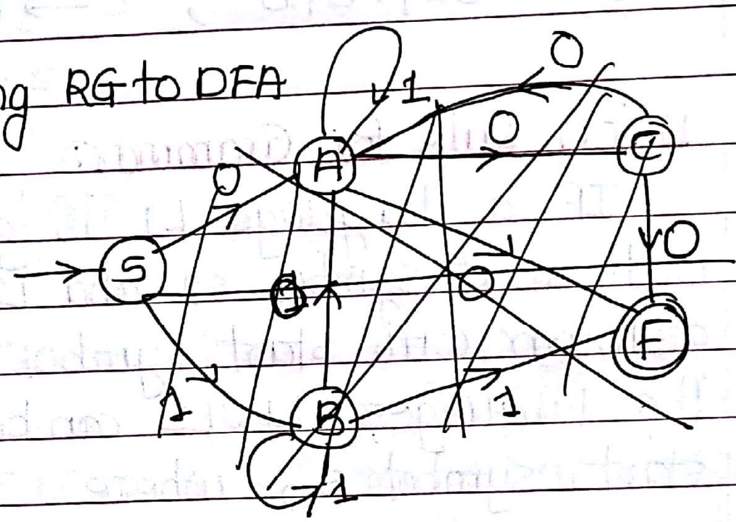
ex 2. convert following RG to DFA

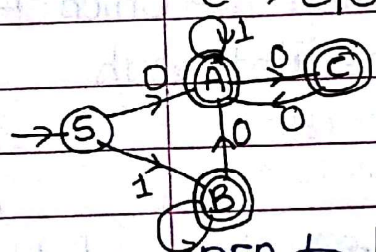
S → 0A | 1B

A → 0C | 1A | 0

B → 1B | 0A | 1

C → 0 | 0A





DFA to Left-linear Grammar :-

Following steps are required to write a left linear gramm corresponding to a DFA

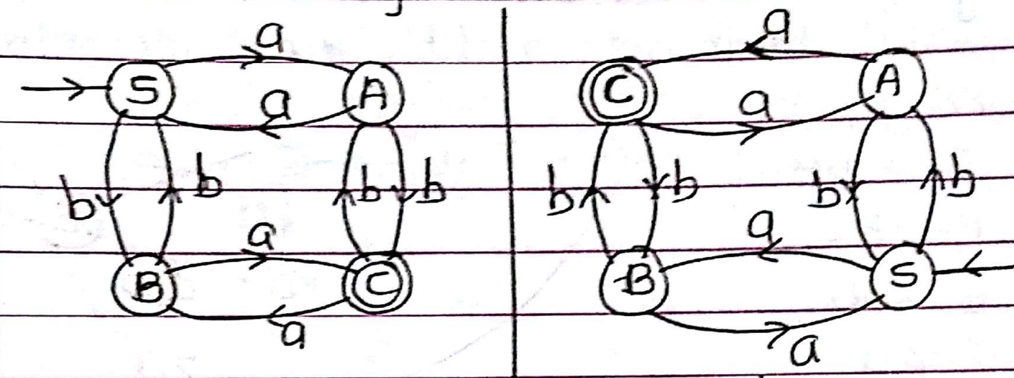
1. Interchange starting state & the final state.

2. Reverse the direction of all the transitions.

3. Write the grammar from the transition graph in left - linear form.

Scanned by CamScanner

ex Give a left linear grammar for the DFA shown in bellow



$S \to Ba \mid Ab$, $A \to Sb \mid Ca \mid a$

$B \to Sa \mid Cb \mid b$, $C \to Bb \mid Aa$

Union Rule for Grammar:

If a language L1 is generated by a grammar with start symbol S1 and L2 is generated by a grammar with start symbol S2 then the union of the languages L1 U L2 can be generated with start symbol S, where

$$S \to S1 \mid S2$$

ex Let the language L1 and L2 are given as below

$L1 = \{a^n \mid n > 0\}$

$L2 = \{b^n \mid n > 0\}$

production for L1 are

$S1 \to aS \mid a$

production for L2 are.

$S2 \to bS \mid b$

Then the productions for $L = L1 \cup L2$ can be written

$S \to S1 \mid S2$

$S1 \to aS \mid a$

$S2 \to bS \mid b$

Concatenation Rule for Grammar:

If a language L1 is generated by a grammar with start symbol S1 and L2 is generated by a grammar with start symbol S2 then the concatenation of the language L1.L2 can be generated with start symbol S, where.

$$S \rightarrow S1S2$$

by considering previous example.

$L = L1.L2$

$S \rightarrow S1S2$

$S1 \rightarrow aS|a$

$S2 \rightarrow bS|b$

Ambiguous Grammar:-

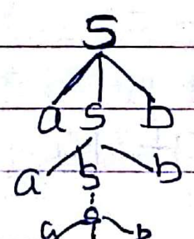
Let $G = (V, \Sigma, P, S)$ be a CFG

A string $w \in L(G)$ is said to be ambiguously derivable if there exists more than one derivation tree OR Leftmost derivation for w. in G.

A CFG G is said to be ambiguous if there is at-least one word or string in L(G) which is ambiguously derivable. otherwise it is said to be unambiguous.

example of unambiguous grammar

$S \rightarrow aSb | ab$ $\quad L = \{a^n b^n \; n > 0\}$

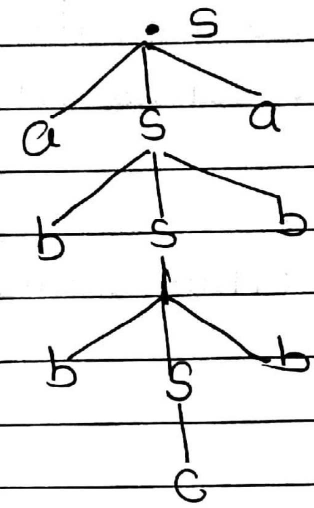


Scanned by CamScanner

2) $S \rightarrow aSa$

$S \rightarrow bSb$ $\quad L = \{ wcw^R \mid w \in \{a, b\}^* \}$

$S \rightarrow c$



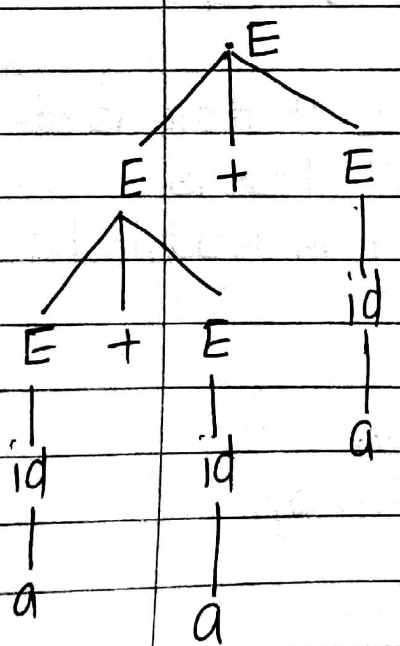
ex3. $E \rightarrow E+E$

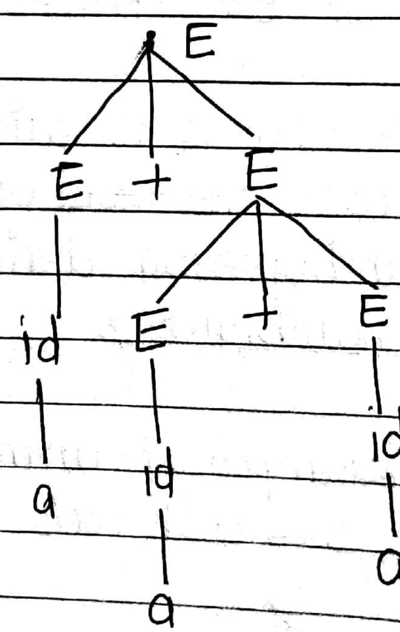
$E \rightarrow E*E$

$E \rightarrow id$

$id \rightarrow a|b|c$

string $a+a+a$



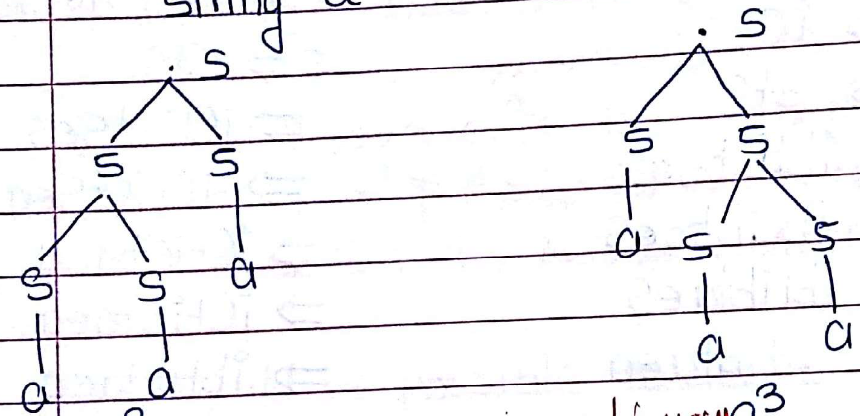


a+a+a above is ambiguous grammar a+a+a

Scanned by CamScanner

ex $S \rightarrow SS$ $L = \{a^n \mid n > 0\}$
$S \rightarrow a$

string a^3



a^3 given grammar is ambiguous a^3

A CFL L can be generated by many grammars G1, G2, G3...... L is said to be unambiguous. If there is an unambiguous grammar generating it.

If all the grammars generting L are ambiguous, i.e There is no unambiguous grammar generating L then L is said to be an inherently ambiguous CFG. There are inherently ambiguous CFL.

It is undecidable to find whether a CFG is ambiguous or not.

ex Consider the following grammar
$S \rightarrow iCtS \mid iCtSeS \mid a$
$C \rightarrow b$

For the ~~following~~ string ibtibtaea find the following

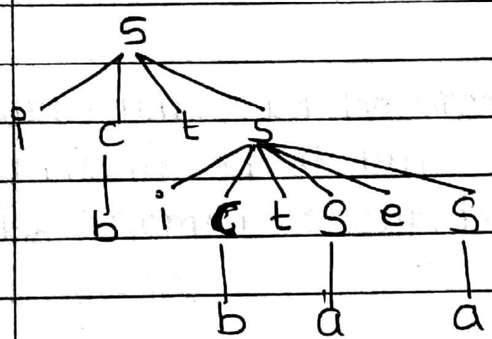
i) Leftmost derivation 2) Rightmost derivation.
3) Parse tree 4) check if the above grammar is ambigu[ous]

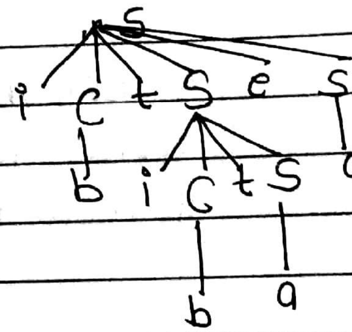
Leftmost derivation.

S ⇒ iCtS
⇒ ibtS
⇒ ibtiCtSeS
⇒ ibtibtSeS
⇒ ibtibtaeS
⇒ ibtibtaea

Rightmost derivation.

S ⇒ iCtS
⇒ iCtiCtSeS
⇒ iCtiCtSea
⇒ iCtiCtaea
⇒ iCtibtaea
⇒ ibtibtaea.





Examples Technical - 5-21 to 5-28

Page No

classmate

Simplification of CFG

Removal of useless symbols:- Let $G=(V,\Sigma,P,S)$ be a CFG. A non-terminal symbol X is useful if there is a derivation $S \rightarrow \alpha X \beta \stackrel{*}{\Rightarrow} w$ for some α, β & w where $\alpha, \beta \in (V \cup \Sigma)^*$, $w \in \Sigma^*$. Otherwise X is useless.

i.e
1] It should be possible to derive a terminal string from X.
2] It should be possible to derive a sentential form containing X from S. (X must be reachable from S)
terminal / Non-terminal

ex $G=(V,\Sigma,P,S)$ where $V=\{S,T,X\}$ $\Sigma=\{0,1\}$

$S \rightarrow 0T \mid 1T \mid X \mid 0 \mid 1$

$T \rightarrow 00$

$S \rightarrow X$

$X \rightarrow ?$ therefore X is useless symbol.

so remove it

Final P is

$S \rightarrow 0T \mid 1T \mid 0 \mid 1$

$S \rightarrow 00$

(Non-generating)

Lemma 1: Given a CFG $G=(V,\Sigma,P,S)$ with $L(G) \neq \phi$ we can effectively find an equivalent CFG $G'=(V',\Sigma,P',S)$ such that for each A in V' there is some $w \in \Sigma^*$ for which $A \stackrel{*}{\Rightarrow} w$

Scanned by CamScanner

ex $G = (V, \Sigma, P, S)$, $V = \{S, A, B, C, E\}$ $\Sigma = \{b, c, d\}$

$S \rightarrow AB$ $S \rightarrow BC$

$S \rightarrow AC$ $C \rightarrow cEd$

$B \rightarrow b$ $E \rightarrow cd$

$E \rightarrow cEd$

→ construct V' & P'

Initially V' consist of all terminals that give ~~termi~~-nal string.

$B \rightarrow b$

$E \rightarrow cd$.

$V' = \{B, E\}$

Search B & E or both BE on right hand side of production rule.

$C \rightarrow c\underline{E}d$.

Include C in V'

$V' = \{B, E, C\}$

Search B, E and C or all three on right side of production rule.

$S \rightarrow BC$

include S in V'

$V' = \{B, E, C, S\}$

Finally there is no production rule where rightside contains B, E, C or S.

Therefore A is useless symbol.

remove A, from all production rules involving A either on left or right side.

$$P' \left\{ \begin{array}{l} S \rightarrow BC \\ B \rightarrow b \\ E \rightarrow cEd \\ C \rightarrow cEd \quad E \rightarrow cd \end{array} \right\}$$

Page No

Scanned by CamScanner

classmate

(non-reaching)

lemma 2 :- Given a CFG $G = (V, \Sigma, P, S)$ we can effective find an equivalent CFG $G' = (V', \Sigma', P', S)$ such that for each X in $V' \cup \Sigma'$ there exist α & $\beta \in (V' \cup \Sigma')^*$ for which $S \rightarrow \alpha X \beta$.

ex →

$S \rightarrow AB$ $\qquad$ $B \rightarrow bB$

$A \rightarrow aAb$ $\qquad$ $B \rightarrow b$

$A \rightarrow ab$ $\qquad$ ~~$S \rightarrow cCd$~~

~~$C \rightarrow ed$~~.

$V = \{S, A, B, C\}$ $\qquad$ $\Sigma = \{a, b, c, d\}$

$V' = \{S, A, B\}$ $\qquad$ $\Sigma' = \{a, b\}$

From start symbol S we are unable to reach at C therefore C & d are also not contributing anything to the L(G).

Theorem:- Every nonempty CFL is generated by a CFG with no useless symbols.

steps to remove useless symbols

1] Apply lemma 1

2] Apply lemma 2.

Scanned by CamScanner

ex Find CFG with no useless symbols equivalent to

$S \rightarrow AB \mid CA$ $\quad B \rightarrow BC \mid AB$

$A \rightarrow a$ $\quad C \rightarrow aB \mid b$.

$\longrightarrow$

$S \rightarrow AB$ $\quad V = \{S, A, B, C\}$

$S \rightarrow CA$ $\quad \Sigma = \{a, b\}$

$B \rightarrow BC$

$B \rightarrow AB$ $\quad V' = \{A, C, S\}$

$A \rightarrow a$ $\quad \Sigma = \{a, b\}$

$C \rightarrow aB$

$C \rightarrow b$.

$G' = (V', \Sigma, P', S)$

$V' = \{A, C, S\}$, $\Sigma = \{a, b\}$

$P' = \{S \rightarrow CA, A \rightarrow a, C \rightarrow b\}$

$V'' = \{S, A, C\}$

$\Sigma'' = \{a, b\}$

$P'' = \{S \rightarrow CA, A \rightarrow a, C \rightarrow b\}$.

ex Remove useless symbols from the following grammar

$S \rightarrow aA$

$S \rightarrow bB$ $\quad V' = \{A, D, E, S\}$

$A \rightarrow aA$ $\quad P' = S \rightarrow aA$

$A \rightarrow a$ $\quad A \rightarrow aA$

$B \rightarrow bB$ $\quad A \rightarrow a$

$D \rightarrow ab$ $\quad D \rightarrow ab$

$D \rightarrow Ea$ $\quad D \rightarrow Ea$

$E \rightarrow aC$ $\quad E \rightarrow d$

$E \rightarrow d$ $\quad \Sigma = \{a, b, d\}$

Scanned by CamScanner

$V'' = \{S, A$

$\Sigma'' = \{a\}$

$P'' = S \rightarrow aA$

$A \rightarrow aA$

$A \rightarrow a.$

ex 3. Eliminate the useless symbols from the following grammar

$S \rightarrow aS$ $\quad G = (V, \Sigma, P, S)$

$S \rightarrow A$ $\quad V = \{S, A, C, B\}$

$S \rightarrow C$ $\quad \Sigma = \{a, b\}$

$A \rightarrow a$ $\quad G' = (V', \Sigma, P', S)$

$B \rightarrow aa$ $\quad V' = \{A, B, S\}$

$C \rightarrow aCb$ $\quad \Sigma = \{a, b\}$

$P' = S \rightarrow aS$

$S \rightarrow A$

$A \rightarrow a$

$B \rightarrow aa$

$G'' = (V'', \Sigma'', P'', S)$

$V'' = \{S, A\}$

$\Sigma'' = \{a\}$

$P'' = S \rightarrow aS$

$S \rightarrow A$

$A \rightarrow a$

Scanned by CamScanner

Removal of ϵ-production:-

$A \rightarrow \epsilon$ is called an ϵ-production.

Theorem: If $L = L(G)$ for some CFG $G = (V, \Sigma, P, S)$, then $L - \{\epsilon\}$ is $L(G')$ for a CFG G' with no useless symbols or ϵ productions.

A non terminal A is nullable if $A \overset{*}{\Rightarrow} \epsilon$

ex 1. $S \rightarrow 0S \mid 1S \mid \epsilon$

$\rightarrow$ $S \rightarrow 0S \mid 1S \mid 0 \mid 1$

ex 2. $S \rightarrow SaSbS$

$S \rightarrow \epsilon$

$S \rightarrow SaSbS \mid SaSb \mid aSbS \mid SabS \mid aSb \mid Sab \mid abS \mid$
ab

ex 3. $S \rightarrow XY\cancel{Z}$

$X \rightarrow 0X \mid \epsilon$

$Y \rightarrow 1Y \mid \epsilon$

$\rightarrow$

$S \rightarrow XYX \mid XY \mid YX \mid XX \mid X \mid Y.$

$X \rightarrow 0X \mid 0$

$Y \rightarrow 1Y \mid 1$

ex

$S \rightarrow AB$	
$A \rightarrow aAA \mid \epsilon$	$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow bBB \mid \epsilon$	$B \rightarrow bBB \mid bB \mid b$
$\rightarrow$ $S \rightarrow AB \mid A \mid B$	

Scanned by CamScanner

Removal of Unit production - A production of the form $A \rightarrow B$ is known as a unit production. where $A, B \in V$

The Technique is based on expansion of unit production until it disappears. This technique works in most of the cases. This technique does not work if there is a cycle of unit productions such as,

$A1 \rightarrow A2$, $A2 \rightarrow A3$, $A3 \rightarrow A4$ & $A4 \rightarrow A1$

ex 1. $S \rightarrow ABA \mid BA \mid AA \mid AB \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$.

$\rightarrow$ $S \rightarrow ABA \mid BA \mid AA \mid AB \mid \underline{aA \mid a \mid bB \mid b}$.

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$.

ex 2. $E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid I$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$\rightarrow$

$E \rightarrow E+T \mid T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$T \rightarrow T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$F \rightarrow (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$

Scanned by CamScanner

classmate

steps for simplification of CFG
1] Remove ϵ production.
2] Remove unit production.
3] Lemma 1
4] Lemma 2.

ex. Simplify the following grammar

$S \rightarrow ASB \mid \epsilon$
$A \rightarrow aAS \mid a$
$B \rightarrow SbS \mid A \mid bb$

→ 1) Remove ϵ production.

$S \rightarrow ASB \mid AB$
$A \rightarrow aAS \mid a \mid aA$
$B \rightarrow SbS \mid bS \mid Sb \mid b \mid A \mid bb$

2) Remove unit production.

$S \rightarrow ASB \mid AB$
$A \rightarrow aAS \mid a \mid aA$
$B \rightarrow SbS \mid bS \mid Sb \mid b \mid aAS \mid a \mid aA \mid bb$

3) Lemma 1.

$G' = (V', \Sigma, P', S)$
$V' = \{A, B, S\}$

4) Lemma 2

$G'' = (V'', \Sigma'', P'', S)$
$V'' = \{A, S, B\}$
$\Sigma'' = \{a, b, bb\}$
$P'' = S \rightarrow ASB \mid AB$
$A \rightarrow aAS \mid a \mid aA$
$B \rightarrow SbS \mid bS \mid Sb \mid b \mid bb \mid aAS \mid aA \mid b$

Scanned by CamScanner

ex 2. Simplify the following grammar.

$S \to 0A0 \mid 1B1 \mid BB$.

$A \to C$.

$B \to S \mid A$

$C \to S \mid \epsilon$

At the time of removing unit production, there is a chain in production

$\therefore S \to B, B \to A, A \to C, C \to S$

$\therefore$ remove unit production & write the production of S

→ Remove ϵ production.

$S \to$ ~~0A0+~~ $0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B$

$B \to S \mid A$, $A \to C$

$C \to S$

Remove unit production.

~~S → 0A0 | 00 | 1B1 | 11 | BB |~~

$\boxed{S \to B,\ B \to S.}$, $B \to A$, $A \to C$, $C \to S$

$S \to B \to A \to C$ $\qquad$ $S \to B \to S$

$B \to A \to C \to S \to B$

The Grammar without unit productions

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$C \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

The symbol C is not reachable & hence it can be deleted.

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$.

Scanned by CamScanner

ex Simiplify the following grammar:

$S \to Ab$

$A \to a$

$B \to C|b$

$C \to D$

$D \to E$

$E \to a$

→ 1) Remove ϵ production. (does not contain ϵ production)

2) Remove unit production.

$S \to Ab$ $A \to a$

$B \to a|b$ $C \to a$

$D \to a$ $E \to a$

3) Lemma 1.

$G' = (V', \Sigma, P', S)$

$V' = \{A, B, C, D, E, S\}$

$S \to Ab$ $A \to a$

$B \to a|b$ $C \to a$

$D \to a$ $E \to a$

4) $G'' = (V'', \Sigma'', P'', S)$

$V'' = \{S, A\}$

$\Sigma'' = \{a\}$

$P'' = \begin{cases} S \to Ab \\ A \to a \end{cases}$

ex: $S \to aC|SB$

$A \to bSCa$

$B \to aSB|bBC$

$C \to aBC|ad$

Page No

→ There is no ϵ or unit production.

$G'' = (V', \Sigma, P', S)$

$V' = \{C, S, A\}$

$$P' = \left\{\begin{array}{l} S \to aC \\ A \to bSCa \\ C \to ad. \end{array}\right\}$$

$G'' = (V'', \Sigma'', P'', S)$

$V'' = \{S, C\}$

$\Sigma'' = \{ad\}$

$$P'' = \left\{\begin{array}{l} S \to aC \\ C \to ad. \end{array}\right\}$$

ex Remove the unit productions

$S \to 0A \mid 1B \mid C$

$A \to 0S \mid 00$

$B \to 1 \mid A$

$C \to 01$

→ $S \to 0A \mid 1B \mid 01$

$A \to 0S \mid 00$

$\cancel{B \to 1 \mid 0S}$ $\cancel{A \to 0S \mid 00}$ $B \to 1 \mid 0S \mid 00$

$C \to 01$

ex Remove ϵ production & unit production.

$S \to ABC \mid BaB$

$A \to aA \mid BaC \mid aaa$

$B \to bBb \mid a \mid D$

$C \to CA \mid AC$

$D \to \epsilon$

Scanned by CamScanner

classmate

→ 1) ϵ production.

$D \to \epsilon$

$S \to ABC \mid AC \mid BaB \mid Ba \mid aB \mid a$

$A \to aA \mid BaC \mid aC \mid aaa$

$B \to bBb \mid bb \mid a \mid$

$C \to CA \mid AC$

2) No Unit production.

3)

$G' = (V', \Sigma, P', S)$

$V' = \{S, A, B\}$

$S \to BaB$ $\quad S \to a$ $\quad B \to bBb$

$S \to Ba$ $\quad A \to aA$ $\quad B \to bb$

$S \to aB$ $\quad A \to aaa$ $\quad B \to a$

4) $G'' = (V'', \Sigma'', P'', S)$

$V'' = \{S, B\}$

$\Sigma'' = \{a, bb\}$

$S \to BaB$ $\quad S \to a$ $\quad B \to a$

$S \to Ba$ $\quad B \to bBb$

$S \to aB$ $\quad B \to bb$

Page No

Scanned by CamScanner

Normal Forms :

1] Chomsky's Normal Form (CNF)

Every CFL without ϵ can be generated by a CFG with rules of the form

$$A \rightarrow BC$$

$$A \rightarrow a$$

where $A, B, C \in V$ and $a \in \Sigma$

Steps to convert CFG to CNF :-

1] For every terminal symbol Introduce a new non-terminal

2] If $A \rightarrow B1B2 \cdots Bm$ is a rule in CFG then the rule in CNF

$A \rightarrow B1D1$

$D1 \rightarrow B2D2$ $\qquad$ $D_{n-3} \rightarrow B_{n-2}D_{n-2}$

$D2 \rightarrow B3D3$ $\qquad$ $D_{n-2} \rightarrow B_{n-1}B_n$

⋮

ex $S \rightarrow aSb$

$S \rightarrow ab$

→ CNF - step 1 :

$S \rightarrow ASB$

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

step 2 -

$S \rightarrow AD1$

$D1 \rightarrow SB$

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

Page No

ex 2. Convert CFG to CNF.

$S \rightarrow aSa$ $\quad S \rightarrow c$ $\quad \Big\} L = \{ wcw^R \; w \in \{a, b\}^* \}$

$S \rightarrow bSb$

→ $S \rightarrow ASA$ $\quad S \rightarrow c$ $\quad B \rightarrow b$

$S \rightarrow BSB$ $\quad A \rightarrow a$

step 2: $S \rightarrow AD$ $\quad S \rightarrow BE$

$D \rightarrow SA$ $\quad E \rightarrow SB$

$S \rightarrow c$ $\quad B \rightarrow b$

$A \rightarrow a$

ex 3. Convert the given CFG to CNF

$S \rightarrow aSa \mid bSb \mid a \mid b.$

→ step 1: $S \rightarrow ASB \mid a$

$S \rightarrow BSB \mid b$

$A \rightarrow a$

$B \rightarrow b$

Step 2: $S \rightarrow AD$ $\quad S \rightarrow BE$ $\quad A \rightarrow a, S \rightarrow a$

$D \rightarrow SB$ $\quad E \rightarrow SB$ $\quad B \rightarrow b, S \rightarrow b$

ex 4 Convert the given CFG to CNF $G = (V, \Sigma, P, S)$

strickly follow the algo.

$V = \{S, A, B\}$, $T = \{a, b\}$ p consist of

$S \rightarrow aB$ $\quad A \rightarrow a$ $\quad A \rightarrow bAA$ $\quad B \rightarrow aS$

$S \rightarrow bA$ $\quad A \rightarrow aS$ $\quad B \rightarrow b$ $\quad B \rightarrow aBB$

→ step 1: $S \rightarrow AB$ $\quad A \rightarrow a$ $\quad A \rightarrow BAA$

$S \rightarrow BA$ $\quad A \rightarrow AS$ $\quad B \rightarrow b$

Ans written at the end of this Notebook

$B \rightarrow AS$ $\quad B \rightarrow ABB$

step 2

$S \rightarrow AB$ $\quad A \rightarrow BD$ $\quad B \rightarrow AE$

$S \rightarrow BA$ $\quad D \rightarrow AA$ $\quad E \rightarrow BB$

$A \rightarrow a$ $\quad B \rightarrow b$

$A \rightarrow AS$ $\quad B \rightarrow AS$

Scanned by CamScanner

ex5. Convert the following grammar to chomsky's normal form.

$S \to \sim S \mid [S] \mid p \mid q.$

$\Sigma = \{\sim, [,], p, q\}$

→ $A \to \sim$ $D \to p$

$B \to [$ $E \to q$

$C \to]$

step 1: $S \to AS$

$S \to BSC$

step2: $S \to AS$ $A \to \sim$ $D \to p$

$S \to BF$ $B \to [$ $E \to q$.

$F \to SC$ $C \to]$

ex6. Convert CFG to CNF

$S \to ASB \mid \epsilon$

$B \to SbS \mid A \mid bb$

$A \to aAS \mid a$

→ First simplify the CFG

$S \to ASB \mid AB$

$B \to SbS \mid bS \mid Sb \mid b \mid A \mid bb$

$A \to aAS \mid aA \mid a$

Remove ε production

$S \to ASB \mid AB$

$B \to SbS \mid bS \mid Sb \mid b \mid aAS \mid aA \mid a \mid bb$

$A \to aAS \mid aA \mid a$

Remove Unit production.

$S \to AD$

$D \to SB$

$\boxed{S \to ASB} \mid AB$

$\boxed{B \to SBS}$ $B \to SE$

$E \to BS$

$B \to BS$

$B \to SB$

$B \to b$

$B \to AF$

$F \to AS$

$\boxed{B \to AAS}$

$B \to AA$

$B \to a$

$B \to BB$

$\boxed{A \to AAS}$ — $A \to AG$

$G \to AS$

$A \to AA$

$A \to a$

S → ASB | AB

B → SbS | Sb | bS | b | aAS | aA | a | bb

A → aAS | aA | a

→ Step 1

Q → a P → b

S → ASB	S → AX , X → SB
S → AB	S → AB
B → SPS	B → SY , Y → PS
B → SP	B → SP
B → PS	B → PS
B → b	B → b
B → QAS	B → QZ , Z → AS
B → QA	B → QA
B → a	B → a
B → PP	B → PP
A → QAS	A → QD , D → AS
A → QA	A → QA
A → a	A → a

ex Conver the following grammar to CNF form

S → ABA

A → aA | bA | ∈

B → bB | aA | ∈

→ Remove ∈ production.

S → ABA | AB | BA | AA | A | B

A → aA | a | bA | b

B → bB | b | aA | a

~~S → AR1~~ ~~S → AB~~ ~~S → AA~~

~~R1 → BA~~ ~~S → BA~~

Page No

Remove unit production.

$S \to ABA \mid AB \mid BA \mid AA \mid aA \mid a \mid bA \mid b \mid bB \mid b \mid aA \mid a$

? $A \to aA \mid a \mid bA \mid b$

$B \to bB \mid b \mid aB \mid a$.

1]	$S \to ABA$	1] $S \to AR1$, $R1 \to BA$
2]	$S \to AB \mid BA \mid AA$	
3]	$S \to aA$	$S \to R2A$, $R2 \to a$
4]	$S \to bA$	$S \to R3A$, $R3 \to b$
5]	$S \to a \mid b$	
6]	$S \to bB$	$S \to R3B$.
7]	$S \to aA$	$S \to R2A$
8]	$A \to aA$	$A \to R2A$
9]	$A \to a \mid b$.	
10]	$A \to bA$	$A \to R3A$
11]	$B \to bB$	$B \to R3B$
12]	$B \to aB$	$B \to R2B$.
13]	$B \to a \mid b$.	

ex Find the CNF equivalent to $S \to aAbB$, $A \to aA$, $B \to bB \mid b$.

→ Terlmals are $\{a, b\}$

$R1 \to a$ $\quad$ $A \to R1A$

$R2 \to b$ $\quad$ $B \to R2B \mid b$.

$S \to R1AR2B$

→ $S \to R1X$ $\quad$ $R2 \to b$.

$X \to AX1$ $\quad$ $A \to R1A$

$X1 \to R2B$ $\quad$ $B \to R2B$

$R1 \to a$ $\quad$ $B \to b$.

classmate

~~Greibach Normal form (GNF)~~

ex Convert the grammar given below to its equivalent G[NF]

$S \rightarrow PQP$ $Q \rightarrow 1Q \mid \epsilon$ PU-DEC-2009

$P \rightarrow 0P \mid \epsilon$

$\rightarrow$ $S \rightarrow PQP \mid PP \mid PQ \mid QP \mid P \mid Q$

$P \rightarrow 0P \mid 0$

$Q \rightarrow 1Q \mid 1$.

$S \rightarrow PQP \mid PP \mid PQ \mid QP \mid 0P \mid 0 \mid 1Q \mid 1$

? $P \rightarrow 0P \mid 0$

$Q \rightarrow 1Q \mid 1$

Scanned by CamScanner

classmate

ex. Find equivalent CNF.

$S \to bA \mid aB$

$A \to bAA \mid aS \mid a$

$B \to aBB \mid bS \mid b$

MU-2010.

$\to$ $\Sigma = \{a, b\}$

$C_a \to a$	$C_a \to a$, $C_b \to b$
$C_b \to b$	
$S \to C_bA \mid C_aB$	$S \to C_bA \mid C_aB$
$A \to C_bAA \mid C_aS \mid a$	$A \to C_bC_1$, $C_1 \to AA$
$B \to C_aBB \mid C_bS \mid b.$	$A \to C_aS$, $A \to a.$
	$B \to C_aC_2$, $C_2 \to BB$
	$B \to C_bS$, $B \to b.$

ex Convert the following grammar to CNF

$S \to Aba$, $S \to aab$, $B \to AC$

$\to$ $G' = (V', \Sigma, P, S)$

$V' = \{S\}$

$S \to aab.$	$S \to C_aC_1$
$C_a \to a$	$C_1 \to C_bC_b$
$C_b \to b.$	$C_a \to a$
$S \to C_aC_bC_b$	$C_b \to b$

0

GNF Example

ex $S \to aSb$

$S \to ab.$

1] $S \to ASB$	2] $S = A1$	$A1 \to A2\,A1\,A3$
$S \to AB$	$A = A2$	$A1 \to A2A3$
$A \to a$	$B = A3$	$A2 \to a$
$B \to b$		$A3 \to b$

Page No

Scanned by CamScanner

3] $A1 \to A2A1A3$
$A1 \to A2A3$
$A2 \to a$
$A3 \to b$

$A_i \to A_j \gamma$, $j > i$

4] $A3 \to b$
$A1 \to aA3$

$A2 \to a$
$A1 \to aA1A3$

$\Rightarrow$ $S \to aB \mid aSB$
$A \to a$
$B \to b$

ex 2 $S \to SS$, $S \to aSb$, $S \to ab$

$\to$ 1] $S \to SS$
$S \to ASB$
$S \to AB$
$A \to a$
$B \to b$

2] $S = A1$
$A = A2$
$B = A3$

$A1 \to A1A1$
$A1 \to A2A1A3$
$A1 \to A2A3$
$A2 \to a$
$A3 \to b$

3] Use lemma 2
$A1 \to A1A1 \mid A2A1A3 \mid A2A3$
$a = A1$, $\beta 1 = A2A1A3$, $\beta 2 = A2A3$
$A1 \to A2A1A3 \mid A2A3$
$A1 \to A2A1A3Z \mid A2A3Z$
$Z \to A1$
$Z \to A1Z$
$A2 \to a$
$A3 \to b$

4] $A1 \to aA1A3$
$A1 \to aA3$
$A1 \to aA1A3Z$
$A1 \to aA3Z$
$Z \to A1$
$Z \to A1Z$
$A2 \to a$
$A3 \to b$

5] $A1 \to aA1A3$
$A1 \to aA3$
$A1 \to aA1A3Z$
$A1 \to aA3Z$
$A2 \to a$
$A3 \to b$
$Z \to aA1A3$
$Z \to aA3$

$Z \to aA1A3Z$
$Z \to aA3Z$
$Z \to aA1A3Z$
$Z \to aA3Z$
$Z \to aA1A3ZZ$
$Z \to aA3ZZ$

① ②

Above two rules are repeated so write once only.

ex3 Convert the given CFG to GNF

$S \rightarrow ABA$, $A \rightarrow aA \mid \epsilon$, $B \rightarrow bB \mid \epsilon$

→ First simplify given grammar

i) Remove ϵ production.

$S \rightarrow ABA \mid AB \mid BA \mid AA \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$.

2) Remove unit production.

$S \rightarrow ABA \mid AB \mid BA \mid AA \mid aA \mid bB \mid a \mid b$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$.

3) Remove useless symbols.

$G' = (V', \Sigma, P', S)$

$V' = \{A, B, S\}$

4] $G'' = (V'', \Sigma'', P'', S)$

$V'' = \{A, B, S\}$

$\Sigma'' = \{a, b\}$

GNF form:

1] Let $P \rightarrow a$

$Q \rightarrow b$

$S \rightarrow ABA \mid AB \mid BA \mid AA \mid PA \mid QB \mid a \mid b$.

$A \rightarrow PA \mid a$

$B \rightarrow QB \mid b$

2] $S = A1$, $A = A2$, $B = A3$, $P = A4$, $Q = A5$

A1

Scanned by CamScanner

2] Greibach Normal form (GNF)

Every CFL without ϵ can be generated by a CFG with rules of the form

$$A \rightarrow aB_1 \ldots\ldots B_m$$
$$A \rightarrow a$$

Where $A, B \ldots\ldots B_m \in V$ & $a \in \Sigma$

steps to convert CFG to GNF:

1] For every terminal symbol introduce a new non-terminal symbol.

2] Introduce an order among ~~ther~~ non-terminal by renaming them. (No need of step 2)

Lemma 1 : Let $G = (V, \Sigma, P, S)$ be a given CFG & if there is a production $A \rightarrow Ba$ and $B \rightarrow \beta_1 | \beta_2 | \ldots | \beta_n$

Then we can convert A rule to GNF as.

$$A \rightarrow \beta_1 a | \beta_2 a | \beta_3 a | \ldots | \beta_n a$$

For example : $S \rightarrow Aa$

$$A \rightarrow aA | bA | aAS | b$$

then we can convert S rule in GNF as

$$S \rightarrow aAa | bAa | aASa | ba$$
$$A \rightarrow aA | bA | aAS | b$$

Note that both the rules are in GNF.

Scanned by CamScanner

Lemma 2: Let $G = (V, \Sigma, P, S)$ be a given CFG & if there is a production

$A \rightarrow A\alpha_1 | A\alpha_2 | A\alpha_3 \ldots | A\alpha_n | \beta_1 | \beta_2 | \ldots | \beta_n$

such that β_i do not start with A then equivalent grammar in GNF can be

$A \rightarrow \beta_1 | \beta_2 | \ldots | \beta_n$.

$A \rightarrow \beta_1 Z | \beta_2 Z | \beta_3 Z | \ldots | \beta_n Z$

$Z \rightarrow \alpha_1 | \alpha_2 | \alpha_3 | \ldots | \alpha_n$

$Z \rightarrow \alpha_1 Z | \alpha_2 Z | \ldots | \alpha_n Z$

For example: $A \rightarrow A1 | 0B | 2$

Here $\beta_1 \Rightarrow 0B$, $\beta_2 = 2$, $\alpha_1 = 1$

$A \rightarrow 0B | 2$

$A \rightarrow 0BZ | 2Z$

$Z \rightarrow 1$

$Z \rightarrow 1Z$

3] Use lemma 1 & lemma 2 & convert the rules in such a way that at the end of this steps, the rules are in GNF or of the form $A_i \rightarrow A_j \gamma$, $j > i$ you may also have some Z rules. $[A_1 \rightarrow A_n]$

4] Convert the A_i rules into GNF $[A_n \rightarrow A_i]$

5] Convert the Z rules into GNF.

Scanned by CamScanner

3] $A1 \to A2A3A2$ $\quad$ $A2 \to A4A2$
$A1 \to A2A3$ $\quad$ $A2 \to a$
$A1 \to A3A2$ $\quad$ $A3 \to A5A3$
$A1 \to A2A2$ $\quad$ $A3 \to b$.
$A1 \to A4A2$ $\quad$ $A4 \to a$
$A1 \to A5A3$ $\quad$ $A5 \to b$
$A1 \to a$
$A1 \to b$

4]
$A1 \to a\cancel{A3A2} \mid \cancel{aA3} \mid \cancel{bA2} \mid \cancel{aA2} \mid \cancel{aA2} \mid bA3 \mid a \mid b$
$A2 \to aA2 \mid a$ $\quad$ $A1 \to A4A2A3A2$
$A3 \to bA3 \mid b$ $\quad$ $A1 \to A4A2A3$
$A4 \to a$
$A5 \to b$

$S \to aABA \mid aBA$	$S \to aAA \mid aA$
$S \to aAB \mid aB$	$S \to aA \mid bB \mid a \mid b$
$S \to bBA \mid bA$	$A \to aA \mid a$ $\quad$ $P \to a$
	$B \to bB \mid b$ $\quad$ $Q \to b$

ex 2. Convert given CFG to GNF where $V = \{S, A\}$, $T = \{0, 1\}$ and P is

$S \to AA \mid 0$
$A \to SS \mid 1$

steps-1] Not necessory.
2] Consider $S = A1$, $A = A2$, ∓
$A1 \to A2A2 \mid 0$ —— 1
$A2 \to A1A1 \mid 1$ —— 2

3] Rule 2 is not in GNF form so
$A2 \to A2A2A1 \mid 0A1 \mid 1$ $\quad$ $[A1 \to A2A2 \mid 0]$
using lemma 2
$\beta1 = 0A1$ $\quad$ $\beta2 = 1$ $\quad$ $\alpha1 = A2A1$.
$\therefore A2 \to 0A1 \mid 1$
$A2 \to 0A1Z \mid 1Z$

Scanned by CamScanner

$Z \rightarrow A2A1$ $A1 \rightarrow A2A2 | 0$

$Z \rightarrow A2A1Z$

4]

~~A1 → 1A2 | 0~~

~~A1 → 0A1 | 1A2 | 0 | 0A1ZA~~

$A1 \rightarrow A2A2 | 0$

$A2 \rightarrow 0A1 | 1 | 0A1Z | 1Z | \cancel{A2A1} | \cancel{A2A1Z}$

$Z \rightarrow A2A1 | A2A1Z$

$A1 \rightarrow 0A1A2 | 1A2 | 0A1ZA2 | 1ZA2 | 0$

$A2 \rightarrow 0A1 | 1 | 0A1Z | 1Z$

$Z \rightarrow A2A1 | A2A1Z$.

5] $A1 \rightarrow 0A1A2 | 1A2 | 0A1ZA2 | 1ZA2 | 0$

$A2 \rightarrow 0A1 | 1 | 0A1Z | 1Z$

$Z \rightarrow 0A1A1 | 1A1 | 0A1ZA1 | 1ZA1$.

$Z \rightarrow 0A1A1Z | 1A1Z | 0A1ZA1Z | 1ZA1Z$.

Now rewrite the rules by converting back $A1=S, A2=A$

$S \rightarrow 0SA | 1A | 0SZA | 1ZA | 0$

$A \rightarrow 0S | 1 | 0SZ | 1Z$

$Z \rightarrow 0SS | 1S | 0SZS | 1ZS$

$Z \rightarrow 0SSZ | 1SZ | 0SZSZ | 1ZSZ$

Ex 3. Convert the grammar $S \rightarrow AB$, $A \rightarrow BS | b$, $B \rightarrow SA | a$ into GNF.

→ Given Grammar

$S \rightarrow AB$

$A \rightarrow BS | b$

$B \rightarrow SA | a$

Scanned by CamScanner

→ Step 1 : Not required.

Step 2. Let $S = A1$, $A = A2$, $B = A3$

$A1 \rightarrow A2A3$

$A2 \rightarrow A3A1 \mid b$

$A3 \rightarrow A1A2 \mid a$

Step 3.

$A3 \rightarrow A1A2 \mid a$ Not in GNF

$\therefore A3 \rightarrow A2A3A2 \mid a$ $[A1 \rightarrow A2A3]$

$\therefore A3 \rightarrow A3A1A3A2 \mid bA3A2 \mid a$

Use lemma 2.

$\alpha_1 = A1A3A2$ $\beta 1 = bA3A2$ $\beta 2 = a$

$A3 \rightarrow bA3A2 \mid a$

$A3 \rightarrow bA3A2Z \mid aZ$

$Z \rightarrow A1A3A2$

$Z \rightarrow A1A3A2Z$

Step 4 : $A1 \rightarrow A3A1A3 \mid bA3$

$\therefore A1 \rightarrow bA3A2A1A3 \mid aA1A3 \mid bA3A2ZA1A3 \mid$
$aZA1A3 \mid bA^3$

$A2 \rightarrow bA3A2A1 \mid aA1 \mid bA3A2ZA1 \mid aZA1 \mid b$

$A3 \rightarrow bA3A2 \mid a \mid bA3A2Z \mid aZ$

$Z \rightarrow A1A3A2 \mid A1A3A2Z$

Step 5 : Above production rules are same, There is difference in

$Z \rightarrow bA3A2A1A3A3A2 \mid aA1A3A3A2 \mid bA3A2ZA1A3A3A2 \mid$
$aZA1A3A3A2 \mid bA3A3A2$

$Z \rightarrow bA3A2A1A3A3A2Z \mid aA1A3A3A2Z \mid bA3A2ZA1A3A3$
$aZA1A3A3A2Z \mid bA3A3A2Z$

ex4. Convert the following grammar into Greibach Normal form

$S \rightarrow AB1 | 0$

$A \rightarrow 00A | B$

$B \rightarrow 1A1.$

May-12, Marks-10

First Remove unit production.

$S \rightarrow AB1 | 0$

$A \rightarrow 00A | 1A1.$ $B \rightarrow 1A1$

$S \rightarrow$ ~~A1A11B1~~ $| 0 | A1A11$

$A \rightarrow 00A | 1A1$

step1: Let $P \rightarrow 0$, $Q \rightarrow 1$

$S \rightarrow$ ~~AQAQBQ~~ $| 0 | AQAQQ$

$A \rightarrow PPA | QAQ$, $A3 \rightarrow 0$ $A4 \rightarrow 1$

step2:- $S = A1$, $A = A2$, $P = A3$, $Q = A4$.

$A1 \rightarrow$ ~~A2A4A2A4A~~ $A2A4A2A4A4.$

$A2 \rightarrow A3A3A2 | A4A2A4.$

step3: $Ai \rightarrow AjY$, $j > i$

$A1 \rightarrow A2A4A2A4A4$

$A2 \rightarrow A3A3A2 | A4A2A4$, $A3 \rightarrow 0$, $A4 \rightarrow 1$

Step 4:

$A1 \rightarrow A3A3A2A4A2A4A4 | A4A2A4A4A2A4A4.$

$A2 \rightarrow 0A3A2 | 1A2A4.$

$A1 \rightarrow 0A3A2A4A2A4A4 | 1A2A4A4A2A4A4$

$A3 \rightarrow 0$

$A4 \rightarrow 1$

After simplification we are getting

$S \rightarrow 0$

Which are in GNF

Above Answer is not correct. Please do not refer it.

Page No

ex5. $S \rightarrow AB \mid AC$

$A \rightarrow aA \mid bAa \mid a$

$B \rightarrow bbA \mid aB \mid AB$

$C \rightarrow aCa \mid aD$

$D \rightarrow aD \mid bC$

→ Given grammar is not simplified.

lemma1: $V' = \{a, A, B, S\}$

$P' =$ $S \rightarrow AB$

$A \rightarrow aA \mid bAa \mid a$

$B \rightarrow bbA \mid aB \mid AB$

lemma2: $V'' = \{S, A, B\}$

$\Sigma'' = \{a, b\}$

$P'' =$ $S \rightarrow AB$

$A \rightarrow aA \mid bAa \mid a$

$B \rightarrow bbA \mid aB \mid AB$

Step 1. Let $X \rightarrow a$ & $Y \rightarrow b$.

$S \rightarrow AB$

$A \rightarrow XA \mid \cancel{b}YAX \mid a$

$B \rightarrow \cancel{bb}YYA \mid XB \mid AB$

step2: Let $S = A1$, $A = A2$, $B = A3$,

$X = A4$, $Y = A5$.

$A1 \rightarrow A2A3$

$A2 \rightarrow A4A2 \mid A5A2A4 \mid a$

$A3 \rightarrow A5A5A2 \mid A4A3 \mid A2A3$.

$A4 \rightarrow a$

$A5 \rightarrow b$.

step3: $A1 \rightarrow \cancel{A2A3}\ A4A2A3 \mid A5A2A4A3 \mid aA3$

$A2 \rightarrow A4A2 \mid A5A2A4 \mid a$

$A3 \rightarrow A5A5A2 \mid A4A3 \mid A4A2A3 \mid A5A2A4A3$

$\mid aA3$

$A4 \to a$ $A5 \to b$

step 4: $A1 \to aA2A3 \mid bA2A4A3 \mid aA3$

$A2 \to aA2 \mid bA2A4 \mid a$

$A3 \to bA5A2 \mid aA3 \mid aA2A3 \mid bA2A4A3 \mid aA3$

$A4 \to a$, $A5 \to b$.

put $A1 = S$, $A2 = A$, $A3 = B$, $A4 = X$, $A5 = Y$.

$S \to aAB \mid bAXB \mid aB$

$A \to aA \mid bAX \mid a$

$B \to bYA \mid aB \mid aAB \mid bAXB \mid aB$

$X \to a$

$Y \to b$

ex6: $S \to ASB \mid SS \mid AB$

$A \to 1$

$B \to 1$

step 1: No required.

step 2: Let $S = A1$, $A = A2$, $B = A3$.

$A1 \to A2A1A3 \mid A1A1 \mid A2A3$

$A2 \to 1$

$A3 \to 1$

step 3: $A1 \to A1A1$ Not in GNF.

Apply lemma 2.

$A1 \to A2A1A3 \mid A1A1 \mid A2A3$

$\beta 1 = A2A1A3$

$\beta 2 = A2A3$

$\alpha_1 = A1$

$A1 \to A2A1A3 \mid A2A3$

$A1 \to A2A1A3Z \mid A2A3Z$

$Z \to A1$

$Z \to A1Z$

$A2 \to 1$

$A3 \to 1$

Scanned by CamScanner

step4. $A1 \rightarrow 1A1A3 \mid 1A3$

$A1 \rightarrow 1A1A3Z \mid 1A3Z$

$Z \rightarrow A1$

$Z \rightarrow A1Z$

$A2 \rightarrow 1$

$A3 \rightarrow 1$

step5. $A1 \rightarrow 1A1A3 \mid 1A3$

$A1 \rightarrow 1A1A3Z \mid 1A3Z$

$Z \rightarrow 1A1A3 \mid 1A3 \mid 1A1A3Z \mid 1A3Z$

$Z \rightarrow 1A1A3Z \mid 1A3Z \mid 1A1A3ZZ \mid 1A3ZZ$

$A2 \rightarrow 1$

$A3 \rightarrow 1$

ex 7: Convert the following CFG to GNF.

$S \rightarrow AB \mid BC$

$A \rightarrow AB \mid a$

$B \rightarrow AA \mid CB \mid b$

$C \rightarrow a \mid b$

→ step 1 – Not required.

step2: Let $S = A1$, $A = A2$, $B = A3$, $C = A4$.

$A1 \rightarrow A2A3 \mid A3A4$

$A2 \rightarrow A2A3 \mid a$

$A3 \rightarrow A2A2 \mid A4A3 \mid b$

$A4 \rightarrow a \mid b$.

steps: $\boxed{A1 \rightarrow A2A3 \mid A3A4.}$

$A2 \rightarrow A2A3 \mid a$ Not in GNF

$\beta 1 = a$ $\quad \alpha_1 = A3$

$\therefore$ $\boxed{\begin{array}{l} A2 \rightarrow a \\ A2 \rightarrow aZ \\ Z \rightarrow A3 \\ Z \rightarrow A3Z \end{array}}$

Scanned by CamScanner

$A3 \rightarrow aA2 \mid aZA2 \mid A3A2 \mid A3ZA2 \mid A4A3 \mid b.$

$A3 \rightarrow A3A2 \mid A3ZA2 \mid aA2 \mid aZA2 \mid A4A3 \mid b$

$\beta 1 = aA2, \ \beta 2 = aZA2, \ \beta 3 = A4A3 \ \ \beta 4 = b.$

$\alpha_1 = A2, \ \ \alpha 2 = ZA3.$

$A3 \rightarrow aA2 \mid aZA2 \mid A4A3 \mid b.$

$A3 \rightarrow aA2Z \mid aZA2Z \mid aA3Z \mid bZ \mid bA3Z.$

$Z \rightarrow A2 \mid ZA3$

$Z \rightarrow A2Z \mid ZA3Z$

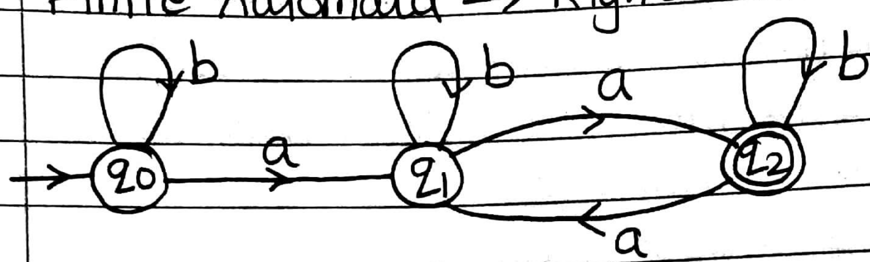
$\boxed{A4 \rightarrow a \mid b}$

Step 4:

Scanned by CamScanner

classmate

Finite Automata $\rightarrow$ Right Linear Grammar.



$q_0 = S$, $q_1 = A$, $q_2 = B$

$S \rightarrow bS \mid aA$

$A \rightarrow bA \mid aB$

$B \rightarrow bB \mid aA \mid \epsilon$.

Scanned by CamScanner

classmate

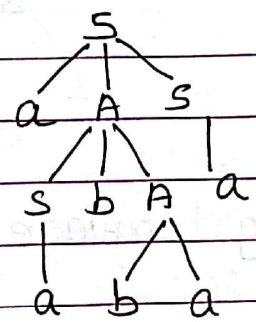
Example on CFG

1] Let G = (V, T, P, S) be the CFG having following set of productions. Derive the string aabbaa using leftmost derivation & rightmost derivation. [MU-May 2009]

$S \rightarrow aAS \mid a,$

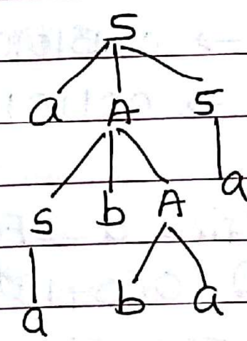
$A \rightarrow SbA \mid SS \mid ba$

Leftmost derivation :



$S \rightarrow aAS$
$\rightarrow aSbAS$
$\rightarrow aabAS$
$\rightarrow aabbaS$
$\rightarrow aabbaa$

Rightmost derivation



$S \rightarrow aAS$
$\rightarrow aAa$
$\rightarrow aSbAa$
$\rightarrow aSbbaa$
$\rightarrow aabbaa$

2] Consider the grammar

$S \rightarrow 0B \mid 1A$

$A \rightarrow 0 \mid 0S \mid 1AA$

$B \rightarrow 1 \mid 1S \mid 0BB$

For the string 00110101 find the following [MU-Dec 2011]

Leftmost derivation →

$S \rightarrow 0B$
$\rightarrow 00BB$
$\rightarrow 001B$
$\rightarrow 0011S$
$\rightarrow 00110B$
$\rightarrow 001101S$
$\rightarrow 0011010B$
$\rightarrow 00110101$

Page No

Rightmost derivation

$S \rightarrow 0B$

$\rightarrow 00BB$

$\rightarrow 00B1S$

$\rightarrow 00B10B$

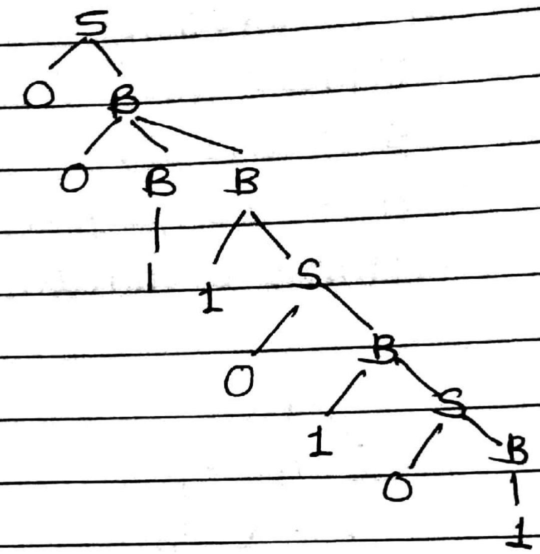
$\rightarrow 00B101S$

$\rightarrow 00B1010B$

$\rightarrow 00B10101$

$\rightarrow 00110101$

Parse Tree.



3] Give a CFG for the following Language.

(a) $0(0+1)^*01(0+1)^*1$

$\rightarrow$ CFG for $(0+1)^*$

$S1 \rightarrow 0S1 \mid 1S1 \mid \epsilon$

$\therefore$ $S \rightarrow 0S101S11$

$S_1 \rightarrow 0S_1 \mid 1S_1 \mid \epsilon$

(b) $R = (0+1)1^*(1+(01)^*)$

CFG for $(0+1)$

$S_1 \rightarrow 0 \mid 1$

CFG for 1^*

$S_2 \rightarrow 1S_2 \mid \epsilon$

CFG for $(1+(01)^*)$

$S_3 \rightarrow 1 \mid S_4$

$S_4 \rightarrow 01S_4 \mid \epsilon$

$\therefore P = \{$ $S \rightarrow S_1S_2S_3$

$S_1 \rightarrow 0 \mid 1$

$S_2 \rightarrow 1S_2 \mid \epsilon$

$S_3 \rightarrow 1 \mid S_4$

$S_4 \rightarrow 01S_4 \mid \epsilon$ $\}$

Page No

Scanned by CamScanner

4] Consider the grammar

S → 0B | 1A

A → 0 | 0S | 1AA

B → 1 | 1S | 0BB

For the string 00110101 find the following

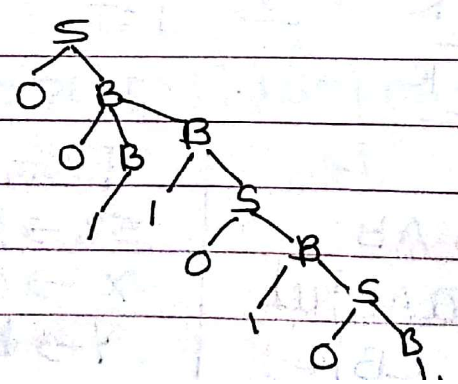
(i) Leftmost derivation

S → 0B	[S → 0B]
→ 00BB	[B → 0BB]
→ 001B	[B → 1]
→ 0011S	[B → 1S]
→ 00110B	[S → 0B]
→ 001101S	[B → 1S]
→ 0011010B	[S → 0B]
→ 00110101	[B → 1]

Rightmost derivation :-

S → 0B	[S → 0B]
→ 00BB	[B → 0BB]
→ 00B1S	[B → 1S]
→ 00B10B	[S → 0B]
→ 00B101S	[B → 1S]
→ 00B1010B	[S → 0B]
→ 00B10101	[B → 1]
→ 00110101	[B → 1]

Parse Tree



Scanned by CamScanner

5] Give CFG for

(a) $L = \{a^i b^j c^q \;;\; i+j=q;\ i,j \geq 1\}$

$a^i \underline{b^j c^j} c^i$

$S \rightarrow aSc \mid aXc$

$X \rightarrow bXc \mid bc$

(b) $L = \{a^i b^j c^k \mid i = j+k\}$

$a^k \underline{a^j b^j} c^k$

$S \rightarrow aSc \mid aXc$

$X \rightarrow aXb \mid ab$

(c) $L = \{a^i b^j c^k \mid j = i+k\}$

$\underline{a^i b^i}\ \underline{b^k c^k}$

$S \rightarrow AB$

$A \rightarrow aAb \mid ab$

$B \rightarrow bBc \mid bc$

(d) $L = \{a^i b^j c^k \mid i=j \text{ or } j=k\}$

$\underline{a^i b^i} c^k$ — $i=j$ — (1)

$a^i \underline{b^j c^k}$ ... $j=k$. — (2)

From (1)	From (2)	
$S1 \rightarrow AB$	$S2 \rightarrow XY$.	$S \rightarrow S1 \mid S2$
$A \rightarrow aAb \mid ab$	$X \rightarrow aX \mid a$	
$B \rightarrow cB \mid c$	$Y \rightarrow bYc \mid bc$	

Scanned by CamScanner

(e) $0^i 1^{i+k} 0^k$ where $i, k \geq 0$

→ $\underline{0^i 1^i}\ \underline{1^k 0^k}$

$S \rightarrow XY$

$X \rightarrow 0X\cancel{1} \mid \epsilon$

$Y \rightarrow 1Y0 \mid \epsilon$

(f) $L = \{0^i 1^j 0^k \mid j > i+k\}$

$$\underbrace{0^i 1^i}_{X}\ \underbrace{1^p}_{Y}\ \underbrace{1^k 0^k}_{Z}$$

$S \rightarrow XYZ$

$X \rightarrow 0X1 \mid 01$

$Y \rightarrow 1Y \mid 1$

$Z \rightarrow 1Z0 \mid 10$

(g) CFG for matching parenthesis

→ $S \rightarrow SS \mid (S) \mid \epsilon$

(h) strings with at least two 0's $\Sigma = \{0,1\}$

RE $= 1^* 0 1^* 0 (0+1)^*$

$S \rightarrow X0X0Y$

$X \rightarrow 1X \mid \epsilon$

$Y \rightarrow 0Y \mid 1Y \mid \epsilon$

(i) odd length strings in $\{0,1\}^*$ with middle symbol 1

→ $S \rightarrow 0S0 \mid 1S1 \mid 1S0 \mid 0S1 \mid 1$

(j) $L = \{0^m 1^n 0^{m+n} \mid m,n >= 0\}$

$0^m \underline{1^n 0^n} 0^m$

$S \rightarrow XYZ$

$X \rightarrow 0X \mid \epsilon$

$Y \rightarrow 1Y0 \mid \epsilon$

Scanned by CamScanner

(k) $L = \{0^a 1^b 2^c \mid a - c = b\}$

$0^c \, \underline{0^b 1^b} \, 2^c$

$S \to 0S2 \mid 0X2$

$X \to 0X1 \mid 01$

(6) Find CFG

(i) string containing alternate sequence of 0's & 1's

$S \to 0S \mid 1S \mid \epsilon$

(ii) The string containing no consecutive b's but a's can be consecutive

$S \to aS \mid bX \mid b \mid \epsilon$

$X \to aS \mid a$

(iii) The set of all strings over $\{a, b\}$ with exactly twice as many a's than b's

$S \to aSaSbS \mid aSbSaS \mid bSaSaS \mid \epsilon$

(iv) Language having number of a's greater than number of b's

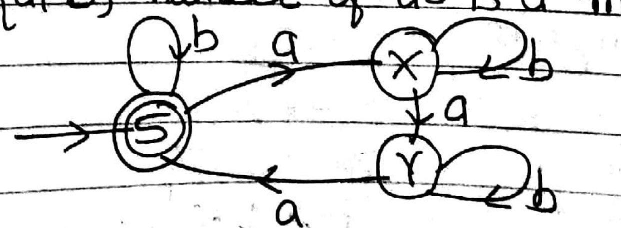
$S \to SaS \mid aSS \mid SSa \mid a \mid aX \mid Xa$

$X \to aB \mid bA$

$A \to aX \mid bAA \mid a$

$B \to bX \mid aBB \mid b$

(7) $\Sigma = \{a, b\}$ number of a's is a multiple of 3



$S \to bS \mid aX \mid \epsilon$

$X \to aY \mid bX$

$Y \to aS \mid bY$

Scanned by CamScanner

CLASSMATE

(8) Let G be the grammar MU-209, 2012, 13

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

Find (a) Leftmost derivation (2) rightmost derivation (3) parse tree (4) Is the grammar unambiguous? for given strings (A) aaabbabbba (B) bbaaabbaaba (C) 00110101

(9) Test whether the following grammars are ambiguous

(1) $S \rightarrow 0S1S \mid 1S0S \mid \epsilon$

(2) $S \rightarrow AA$, $A \rightarrow aAb \mid bAa \mid \epsilon$ MU-May-10

CNF example NO-04

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid bAA \mid aS$

$B \rightarrow aS \mid b \mid aBB$

→ step 1

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow XB \mid YA$

$A \rightarrow a \mid YAA \mid XS$

$B \rightarrow XS \mid b \mid XBB$

step 2:

$S \rightarrow XB \mid YA$

$A \rightarrow a \mid YD1 \mid XS$

$D1 \rightarrow AA$

$B \rightarrow XS \mid b \mid XD2$

$D2 \rightarrow BB$, $X \rightarrow a$, $Y \rightarrow b$

Page No

Scanned by CamScanner

classmate

ex Convert the following grammar to CNF

$S \rightarrow AACD$

$A \rightarrow aAb \mid \epsilon$

$C \rightarrow aC \mid a$

$A \rightarrow aDa \mid bDb \mid \epsilon$

→ Simplification of G

1] Remove ϵ - production

$S \rightarrow AACD \mid ACD \mid CD$

$A \rightarrow aAb \mid ab$

$C \rightarrow aC \mid a$

$A \rightarrow aDa \mid bDb$

2] No unit production

3] Remove useless symbol.

ⓐ Non-generating symbol.

A, C,

$A \rightarrow aAb \mid ab$

$C \rightarrow aC \mid a$

since, the starting symbol itself is non-geneeting it is an invalid grammar.

ex Convert the following grammars to CNF

$A \rightarrow aBb \mid bBa$

$B \rightarrow aB \mid bB \mid \epsilon$

→ Remove ϵ production

$A \rightarrow aBb \mid ab \mid bBa \mid ba$.

$B \rightarrow aB \mid a \mid bB \mid b$

CNF - $C_a \rightarrow a$ (Rule 1)

$C_b \rightarrow b$

$A \rightarrow C_aBC_b \mid C_aC_b \mid C_bBC_a \mid C_bC_a$

$B \rightarrow C_aB \mid a \mid C_bB \mid b$

Page No

Scanned by CamScanner

$A \rightarrow C_aC_1 \mid C_aC_b \mid C_bC_2 \mid C_bC_a$ (Rule 2)

$C_1 \rightarrow BC_b$

$C_2 \rightarrow BC_a$

$B \rightarrow C_aB \mid a \mid C_bB \mid b$

$C_a \rightarrow a$

$C_b \rightarrow b$

ex. Convert the following grammar to CNF.

$S \rightarrow ABC \mid BaB$

$A \rightarrow aA \mid BaC \mid aaa$

$B \rightarrow bBb \mid a \mid D$

$C \rightarrow CA \mid AC$

$D \rightarrow \epsilon$ MU - 2012.

→ Remove ϵ production.

$B \rightarrow bBb \mid a \mid D \mid \epsilon$

$D \rightarrow \epsilon$

$S \rightarrow ABC \mid AC \mid BaB \mid Ba \mid aB \mid a$

$A \rightarrow aA \mid BaC \mid aC \mid aaa$

$B \rightarrow bBb \mid bb \mid a \mid D$

$C \rightarrow CA \mid AC$

Remove unit production.

$B \rightarrow D$ but $D \rightarrow \epsilon$

$\therefore$ remove $B \rightarrow D$

$S \rightarrow ABC \mid AC \mid BaB \mid Ba \mid aB \mid a$

$A \rightarrow aA \mid BaC \mid aC \mid aaa$

$B \rightarrow bBb \mid bb \mid a$

$C \rightarrow CA \mid AC$

Scanned by CamScanner

Remove useless symbol.

Non-generating variable.

A, B, S i.e C is non-generating Symbol.

$\therefore S \rightarrow BaB \mid aB \mid Ba \mid a$

$A \rightarrow aA \mid aaa$

$B \rightarrow bBb \mid bb \mid a$

Symbol A is not rechable from S.

$\therefore S \rightarrow BaB \mid aB \mid Ba \mid a$

$B \rightarrow bBb \mid bb \mid a.$

CNF Rule1 -

$X \rightarrow a \quad , \quad Y \rightarrow b$

$\therefore S \rightarrow BXB \mid XB \mid BX \mid a$

$B \rightarrow YBY \mid YY \mid a$

Rule 2

$S \rightarrow BA1 \mid XB \mid BX \mid a$

$A1 \rightarrow XB$

$B \rightarrow YA2 \mid YY \mid a.$

$A2 \rightarrow BY$

$X \rightarrow a$

$Y \rightarrow b.$

Scanned by CamScanner