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ASSIGNMENT NO. 02

Q.1 Define
a) Regular Expression
b) Regular language

→ a) Regular Expression :-

R.E. is used for specifying the strings of regular language & is defined as
(1) 'ϕ' is R.E. for specifying {} (null set)
(2) 'ϵ' is R.E. for specifying {ϵ} (Epsilon)
(3) 'a' is R.E. for specifying {a}
(4) Let R & S be two R.E. for specifying LR & LS respectively
i) (R)/(S) is R.E. for specifying LR U LS
(ii) (R)(S) is R.E. for specifying LR.LS
(iii) (R)* is R.E. for specifying LR*

(b) Regular language :-

It is a language that can be expressed with a regular expression or a deterministic or non-deterministic finite Automata or state machine A language. A language is a set of strings which are made up of characters from a specified alphabet, or set of symbols. Regular languages are a subset of the set of all strings.

Closure properties of regular ~~expression~~ languages

The class of regular languages is closed under
i) Union ii) Intersection
iii) Complimentation (iv) set difference
v) Concatination
vi) Kleen Closure
vii) Reversal

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Q.2 Write short Note on:

a) Arden's Theorem :-

i) The Arden's theorem is used in regular languages to determine whether a given expression has a unique solution

ii) It states that for two regular expressions out of which one does not contain e as its input, there exist a unique solution.

iii) Let P & Q be two regular expressions so if P doesn't have ϵ as its input, $R = Q + RP$ will have a unique solution represented by $R = QP^*$

Proof :-

We need to consider the fact that a regular expression r can be represented as $\epsilon + r^2 + r^3 + \cdots$

So, we begin the proof using

$R = Q + RP$

$R = Q + (Q + RP)P$

$= Q + QP + RP^2$

Again replacing $R = Q + RP$ in RHS gives as

$R = Q + QP + QP^2 + RP^3$

$R = Q(\epsilon + P + P^2 + \cdots)$

But $(\epsilon + P + P^2)$ can be replaced by P^*

Hence, $R = QP^*$ a unique solution hence exists.

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b) Pumping lemma for RL :-

statement :-

- Let L be a regular language & let z be a word of L such that $|z| \geq n$, where n is the minimum number of states required for recognizing L.
- Then as per pumping lemma, we can write z as $z = uvw$, where $|uv| \geq n$ & $1 \leq |v| \leq n$ such that all the strings of the form uv^iw where $i \geq 0$ would belong to L.

Applications of Pumping lemma :-

1) Select a string z in the language L.
2) Breaks the ~~strings~~ string z into x, y & z accordance with the above conditions imposed by the pumping lemma.
3) Now, check if there is any contradiction to the pumping lemma for any value i.

c) ~~Closu~~ Closure Properties of RL :-

i) Closure & Union :-

If L & m are regular languages so in LUM

proof :- let L & M be the languages of regular expression R & S respectively

Then R+S is a regular expression whose language is LUM

ii) Concatenation :-

Let R & S be two regular expressions specifying regular languages L(R) & L(S) resp. Now if we perform contbatination operation on languages L(R) & L(S). Then resulting language after

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the operation is performed will also be a regular language which be specified by a regular expression

R-S is the regular expression for specifying L(R).L(S)

iii> **Intersection :-**

Regular languages are closed under intersection i.e. if we perform intersection between two regular languages then the resulting language will be also a regular language

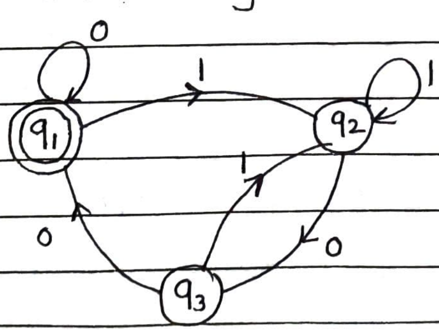
iv> **Difference :-**

Regular languages are closed under set difference operator i.e. if we perform set difference operator between / on the two regular languages then the resulting language will be also a regular language

v> **Reversal :-**

Regular languages are closed under reversal i.e. if we take a reversal of a regular language then the resulting language will be regular

Q.3 Convert the following DFA to RE



$\Rightarrow q_1 = 0q_1 + 1q_2 + E$

$q_2 = 0q_3 + 1q_2$

$q_3 = 0q_1 + 1q_2$

$q_2 = 0q_3 + 1q_2$

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$= 1q_2 + 0q_3$

$q_2 = 1^* 0q_3$ ..... using Arden's Theorem.

$q_3 = 0q_1 + 1q_2$
$= 0q_1 + 11^* q_3$
$= 11^* 0q_3 + 0q_1$
$= (11^*0)^* 0q_1$ .... Using Arden's Theorem

$q_1 = 0q_1 + 1q_2 + \epsilon$
$= 0q_1 + 11^* 0q_3 + \epsilon$
$= 0q_1 + 11^* 0(11^*0)^* 0q_1 + \epsilon$
$= (0 + 11^* 0(11^*0)^* 0) q_1 + \epsilon$
$= [(0 + 11^* 0(11^*0)^* 0)]^* \epsilon$

$\boxed{q_1 = (0 + 11^* 0(11^*0)^* 0)^*}$

Q.4 Prove : $L = \{a^p \mid p \text{ is prime}\}$ is not regular

→ Let's assume L is a regular & p is a prime number
prime numbers - 2, 3, 5, 7 ......
- aa
aaa
aaaaa......

Now consider $x = \underset{u\,v\,w}{aaaaa}$, $|x|$ is prime say p

$i = 2$
$\underset{u}{a}\,\underset{v}{a}\,\underset{v}{a}\,\underset{w}{aaa}$

but p+1 is not a prime number. Hence whatever we have assumed becomes contradictory.
Hence L is not regular

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Q.5 Show that $L = \{ww \mid w \in \{0,1\}^*\}$ is not regular.

→ Let us assume L is regular, consider k is pumping lemma constant, k represent length of w
consider

$$x = \underbrace{\underline{0101}}_{k} \underbrace{0101}_{k}$$

$$x = 0101 \; 0101$$

According to pumping lemma there exist uvw

$\therefore |uv| \leq k \quad , |v| \geq 0$

$i = 2$

$uvvw$

$$\underbrace{01101}_{k+1} \; \underbrace{0101}_{k}$$

$\therefore uvvw \notin L$

∴ We have a contradiction therefore our assumption is wrong.

Hence it is not regular.