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Assignment No. 06

Q.1 Write short Note on

a) Recursive & recursively enumerable language

⟹

* Recursive language:-

A language L is Turing decidable if there exist a halting Tm m such that

$$L = L(M)$$

For every I/p m/c has to halt.

M halts on w if either M accepted w or M reject w M is said to be a halting TM if $\forall w \in \Sigma^*$ M halts on w Decidable languages are subset of Turing recognizable.

* Recursively enumerable language:-

A language L is Turing recognizable, if there exists a TM M such that

$$L = L(M)$$

It means; TM for input

1) if $w \in L$, then Tm accept w & halt.
2) if $w \notin L$ then Tm either reject the w & halt

OR

it may goes into infinite loop.

∴ If $w \notin L$ there is no guarantee of halting the TM. language of these catagory called Recursively Enumerable languages

Recursively enumerable language for which it is not sure that on which input the TM will ever halt.

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b) Halting Problem :-

i) The halting problem is a special kind of a problem wherein a Machine is proved to be undecidable in its behaviour at one point where its composition is changed.

ii) For this we assigne a universal machine for this example & a couple of similar machines.

iii) We consider two simple machines, one (A) which performs addition of two numbers & another (B) that converts a number from decimal to binary

iv) Machine A cannot execute the output of Machine B neither can machine B execute machine A's output.

v) Now, we have a Machine D which executes any machine's (here A or B) output talking that Machine's blueprint and that Machine's individual input set as the input

vi) So, machine D will execute machine A's output on gaining Machine's A's blueprint & so it is the case for Machine B.

vii) Now, we construct a universal Machine V that has Machine D (to identify the Machine to be executed) & a negator machine N (to reserve the output).

viii) If we give machine B's blueprint & machine A's input set to V, we should be getting an error, which doesn't happen as the negator reserve the false output to true.

ix) If we give machine B's blueprint & machine B's input set to V, we should be getting the correct, which doesn't happen as the negator reserves the 'true' output to 'false'

x) This problem of undecidability is often what is termed as the Halting problem.

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c) Rice's Theorem :-

⇒ i) Rice's theorem helps to explain one aspect of the pervasiveness of undecidability. Here is the theorem & its proof,
following the needed definition.

ii) A property of languages is a predicate $p : P(\Sigma^*) \Rightarrow \{false, true\}$ for some alphabet Σ. That is the input of p is a language and the output is a truth value.

iii) The value $P(L)$ = true means L has property p.

iv) The value $p(L)$ = False means L does not have property p.

v) Example properties are : is finite, is infinite, is r.e. etc.

vi) A non-trivial property of r.e. languages is a property of languages push that $P(L_0)$ for some r.e. language L_0 and $P(L_1)$ for some r.e. language L_1.

Proof:

We assume that ϕ does not have a property $p : P(\phi)$ = False we show that $A_{TM} \leq M_{LP}$. For this we must exhibit a Turing computable function for which $m' = f(M, \omega)$ is a machine accepting a language with property p if m accepts ω.

Let the behaviour of m' on input x to be :-

i) Run m on ω.

ii) If Rejects, reject

iii) Run m_1 on x, where M_1 is fixed machine for which $P((m_1)) = 1$. We know that such a M_1 exists because P is a non-trivial solution of r.e. languages.

iv) If m accepts, accept; if m_1 rejects, reject.

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d) Post Correspondence Problem :-

⇒ i) The post correspondence problem (PCP) consists of two lists of strings over some alphabet Σ, the two lists must be of equal length.

ii) If there exists a solution to PCP, there exists infinitely many solutions

	LIST A	LIST B
i	w_i	x_i
1	110	110110
2	0011	00
3	0110	110

- A & B are defined above & let $\Sigma = \{0, 1\}$

- for instance let $m = 3$, $i_1 = 2$, $i_2 = 3$ & $i_3 = 1$
- If solution exists to the above PCP, it should verify the following conditions i.e. $w_2, w_3, w_1 = x_2 x_3 x_1$
i.e. 0011010110 = 0011011 0110. since the above condition verified, therefore the solution is the list 2, 3, 1.
- It is not necessary this solution is unique, i.e. there can exists more than one solution.