Consistency of Matrix

Seat No.: 2644 Enrolment No. 102CTBMCS2122038

NATIONAL FORENSIC SCIENCES UNIVERSITY

B.Tech-M.Tech computer Science and Engineering (Cyber Security)

Semester - I - March-2022

Subject Code: CTBTCSE SI P2 **Date:** 21/03/2022

Subject Name: Engineering Mathematics

Time: **Total Marks: 100**

Instructions:

1. **Write down each question on separate page.**
2. **Attempt all questions.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

[(9+6)+(6+6) (6+4)·(4+6)

Marks

Q.1 (a) If $A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$, Find the value of $A^2-6A+8I$, Where I is second order Unit Matrix. **05**

(b) If $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 0 & 1 & 2\\ 3 & 1 & 0 & 5\end{bmatrix}_{4\times3}$ and $B=\begin{bmatrix}2 & 1 & 0\\ 3 & 2 & 1\\ 1 & 0 & 1\end{bmatrix}_{3\times3}$ Find AB or BA, whichever exist. **05**

(4x3 3x3) 3x3 x 4x3

(c) Find $\begin{bmatrix}x & y & z\end{bmatrix}_{1\times3}\times\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}_{3\times3}\times\begin{bmatrix}x\\ y\\ z\end{bmatrix}_{1\times3}$. **07**

A·B:

or

(c) Find the inverse of $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ $R_1\to R_1-R_2$ $\begin{bmatrix}1 & 3 & 0\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ **07**

Q.2 (a) Find the rank of the matrix $A=\begin{bmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ **05**

(b) If Z_1 and Z_2 be two complex numbers, show that $(Z_1+Z_2)^2+(Z_1-Z_2)^2=2\left[|Z_1|^2+|Z_2|^2\right]$ **05**

$(a+b)^2+(a-b)^2=2|(a)|^2+|(b)|^2$ $2\left[|z_1|^2+|z_2|^2\right]$

(c) Evaluate $\left(1+i\sqrt{3}\right)^{90}+\left(1-i\sqrt{3}\right)^{90}$ **07**

or

15/3 3 $a^2xy\,x2+xy.$

(c) If $\sin A+\sin B+\sin C=\cos A+\cos B+\cos C=0$ — 07

Prove that $\sin(A+B)+\sin(B+C)+\sin(C+A)=0$

Q.3 (a) If $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=\frac{\pi}{3}$, then find the complex number Z. — 05

(b) Simplify $\dfrac{(\cos 2\theta+i\sin 2\theta)^{\frac{2}{3}}(\cos\theta-i\sin\theta)^2}{(\cos 3\theta-i\sin 3\theta)^2(\cos 5\theta-i\sin 5\theta)^{\frac{1}{3}}}$ — 05

$\theta =$

$\tan^{-1} \theta y/x = \pi/3$

(c) Use De Moivre's theorem to solve $Z^4-Z^3+Z^2-Z+1=0$ — 07

or

(c) If $1+2i$ is a root of the equation $Z^4-3Z^3+8Z^2-7Z+5=0$, then find all other roots. — 07

$2k i$

Q.4 (a) $Find\ y_n$ if $y=x^2e^{ax}$ — 05

(b) $If\ y=e^{a\sin^{-1}x}$ prove that $(1-x^2)y_2-xy_1=a^2y$ — 05

$uv' = u'v + uv'$

(c) $If\ y=\sin\log(x^2+2x+1)$,

prove that $(x+1)^2y_{n+2}+(2n+1)(x+1)y_{n+1}+(n^2+4)y_n=0$ — 07

or

$\pi/6 + \pi/3$ $\quad \frac{\pi-2\pi}{6} = \pi/6$ $\quad (f\cdot g)' = f'g + fg'$

(c) $Find\ y_n$ if $y=\dfrac{x^4}{(x-1)(x-2)}$ — 07

Q.5 (a) Define Maclaurian's formula and Expand $x^4-11x^3+43x^2-60x+14$ in power of $x-3$. — 05

(b) Evaluate the followings: — 05

$(i)\lim\limits_{x\to\frac{\pi}{2}}(\cos x)^{\frac{\pi}{2}-x}$ $\quad (ii)\lim\limits_{x\to 0}\dfrac{xe^x-\log(1+x)}{x^2}$

$x = 3$

(c) Define Euler's modified statement — 06

and find $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$, where $u=\log\left(\dfrac{x^3+y^3}{3x+4y}\right)$

$\cos 90^\circ$ $\quad (0)^0$

Q.6 (a) $If\ u=x+y$ and $v=2x-y$, find $\dfrac{\partial(x,y)}{\partial(u,v)}$ — 05

(b) $If\ u=4x+xy-y^2\ ; x=\cos 3t, y=\sin 3t$, find $\dfrac{du}{dt}$ at $t=0$ — 05

(c) Find Maxima and Minima of the function — 06

$f(x,y)=x^3+y^3-3x-12y+20$

$u = x+y$
$x = y-u$
$y = x-u$

$v = 2x-y$
$y = 2x - v$
$x = \frac{y-v}{2}$

Seat No.: ________ Enrolment No.________

NATIONAL FORENSIC SCIENCES UNIVERSITY

B.Tech-M.Tech computer Science and Engineering – Semester (ATKT Exam) – I - July-2022

Subject Code: CTBTCSE SI P2

Date: 18 /07/2022

Subject Name: Engineering Mathematics - I

Time: 11:00 Am to 2:00 pm

Total Marks: 100

Instructions:

1. **Write down each question on separate page.**
2. **Attempt all questions.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

Marks

Q.1 (a) Find the eigenvalues and eigenvectors for $A=\begin{bmatrix}11 & -4 & -7\\ 7 & -2 & -5\\ 10 & -4 & -6\end{bmatrix}$ 05

(b) If $A=\begin{bmatrix}3 & -5 & 4\\ 4 & 6 & 1\\ 4 & 2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1 & 5\\ 6 & 7 & 3\\ 2 & 4 & 1\end{bmatrix}$ Find $2A+3B$. 05

(c) If $A=\begin{bmatrix}-2 & 4\\ 3 & 1\end{bmatrix}$, Find the value of $A^2+A-14I$, Where I is second order Unit Matrix. 07

or

(c) Find the eigenvalues and eigenvectors for $D=\begin{bmatrix}2 & -2 & 2\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$ 07

Q.2 (a) Verify Cayley Hamilton theorem for $A=\begin{bmatrix}3 & -1 & 4\\ 1 & 1 & 2\\ 4 & 5 & 7\end{bmatrix}$ 05

(b) If $Z_1=2+3i$ and $Z_2=-1+2i$ be two complex numbers, then find $\frac{Z_1}{Z_2}$, Z_1Z_2, Z_1+Z_2, Z_1-Z_2 05

(c) Evaluate $(1+i)^{20}+(1-i)^{20}$ 07

or

(c) Solve $Z^4+i=0$ 07

1

$11-\lambda(+(-2-\lambda)+(-6-\lambda)$

Q.3 (a) Solve the following system of linear equations:

$$2x+y-z=2,\ \ x-3y+z=1,\ \ 2x+y-z=6$$ 05

(b) Simplify $\dfrac{(\cos 5\theta+i\sin 5\theta)^{\frac{3}{5}}(\cos 7\theta-i\sin 7\theta)^{2}}{(\cos 3\theta-i\sin 3\theta)^{3}(\cos 8\theta-i\sin 8\theta)^{\frac{1}{8}}}$ 05

(c) Find the polar and exponential form of the equation $\sqrt{3}-i$ 07

or

(c) Find all first and second order partial derivatives w.r.t x and y for $u=3x^4-5xy^3-x^2y^2+y^3$ 07

Q.4 (a) $if\ \sqrt{x+y}-\sqrt{y-x}=c$ prove that $y_2=\dfrac{2}{c^2}$ 05

(b) $if\ y=e^{a\sin^{-1}x}$ prove that $x^2y_2+xy_1+y=o$ 05

(c) Find y_n for $y=\sin^2 x\cos^2 x$ 07

or

(c) Find y_n for $y=\dfrac{x^2+2x-1}{(x+2)(x-1)(x-4)}$ 07

Q.5 (a) Define Maclaurian's formula and find expansion of log (1+x). 05

(b) Evaluate the followings: 05

$$(i)\lim_{x\to 0}\frac{e^x-1-x}{x^2}\qquad (ii)\lim_{x\to y}\frac{x^y-y^x}{x^x-y^y}$$

(c) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at point $(-1,4)$ 06

for the function $f(x,y)=2x^4-3x^2y^3+3xy-5y^3$

Q.6 (a) If $f=\log\left(\dfrac{x^2+y^2}{x+y}\right)$ then find $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}$ 05

(b) If $f=r^2+s^2$ and $r=\sin 3t,\ s=\cos 2t$ find $\dfrac{df}{dt}$ 05

(c) Find local minimum, maximum and saddle point for $f(x,y)=x^3+y^3-3x-12y+20$ 06