Enrolment No. 102CT BMCS2122028

NATIONAL FORENSIC SCIENCES UNIVERSITY, DELHI CAMPUS

B-Tech, 2nd Sem.

Mid-Semester Examination

Date: 12.05.2022

Subject Name: Engineering Mathematics
Time: 90 minutes

Total Marks: 50

Instructions: All questions are mandatory.

1. Prove that: **[10]**

$${}_0\int^{\pi/2} \sin^{2m-1}x \,.\, \cos^{2n-1}x \; dx = \Gamma(m)\,\Gamma(n) \,/\, 2\,\Gamma(m+n) \quad ; \; m>0 \;\&\; n>0$$

2. (a)

Evaluate: $I = {}_0\int^{2} x^2 / \sqrt{(2-x)} \; . dx$ **[5]**

$\frac{4}{3} + \frac{\sqrt{8}}{3}$

(b)

Evaluate: $I = {}_0\int^{\infty} dx \,/\, (1+x^4)$ **[5]**

$\frac{4+\sqrt{8}}{3} = \frac{4+\sqrt{8}}{3}$

3. Using reduction method prove that:

$$\int sin^n(x)dx = \frac{-Sin^{n-1}(x)Cos(x)}{n} + \frac{n-1}{n} Sin^{n-2}(x)dx$$ **[10]**

4.

Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ by changing the order of integration. **[10]**

5.

Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{1}{(1+x^2+y^2)} dydx$ **[10]**

$Cx \cdot -Sin$

$\Rightarrow 0 - 1 \cdot 1 \quad 2 \cdot 2a = \frac{4a}{3} - \frac{4a^3}{4a}$

$\frac{2\sqrt{4a^2}}{3} - \frac{4a^3}{4a} \Rightarrow \frac{16a - 12a^3}{12a}$

$\frac{8a\sqrt{16a^3}}{12a} - \frac{12a^3}{4a} \Rightarrow \frac{16a - 12a^3}{12a}$

$a^3/4$

$\beta(m,n) \int_0^1 x^{m-1} + (1-x)^{n-1} / dx$

$\Rightarrow \left(\frac{1}{1+x^2} + \frac{(\sqrt{1+x^2})^3}{3} \right) dx$

$\frac{(\sqrt{1+x^2})^3}{3} \Rightarrow 1 + x^4 \cdot \sqrt{1+x^2}$

$\Rightarrow (1+x^2)$

$\Rightarrow \frac{1}{1+x^2 + \cancel{4a}}$ $\quad \frac{1}{1} + \frac{1/2}{3} \quad \frac{3+1/2}{1} \quad \frac{6+1}{2} = 7/2$

$\left((1+x^2)\left[1 + \left(\frac{\sqrt{1+x^2}}{3}\right)\right]\right]_0^1$

$\Rightarrow \frac{3+1/2}{3} = \frac{7/2}{3/1} \quad \frac{6}{7}$

$1+1\left[1 + \frac{\sqrt{1+1^2}}{3}\right] = 2\left[1 + \frac{\sqrt{2}}{3}\right]$ $\quad 1 \quad \frac{3+1}{3} = \frac{4}{3} + \frac{1}{2}$

$= 2 \cdot \left(\frac{1}{1} + \frac{1/2}{3}\right) \Rightarrow$ $\quad \frac{1}{1} + \frac{1}{3} = \frac{3+1}{3} = 4/3 \quad \frac{8+3}{6} = \frac{11}{6} \quad 4\sqrt{8}/3$

Seat No.: 2819

Enrolment No. 102CTBMCS2122038

NATIONAL FORENSIC SCIENCES UNIVERSITY

B.Tech-M.Tech computer Science and Engineering - Semester - II - August-2022

Subject Code: CTBTCSE SII P2 **Date: 08 /08 /2022**

Subject Name: Engineering Mathematics 2

Time: 11:00AM to 2:00PM **Total Marks: 100**

Instructions:

1. **Write down each question on separate page.**
2. **Attempt all questions.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}\,dx$	**05**
	(b)	Define Reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$	**05**
	(c)	Evaluate $\int_0^{\pi} (1-\cos\theta)^3\,d\theta$	**07**
		or	
	(c)	Evaluate $\int_0^5 \sqrt{x(5-x)^3}\,dx$	**07**
Q.2	**(a)**	Define Gamma function and prove $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$	**05**
	(b)	Using Gamma function prove $\Gamma(n)=2\int_0^{\infty} e^{-x^2} x^{2n-1}\,dx$	**05**
	(c)	Evaluate $\int_0^{\infty} \frac{x^5(1-x^4)}{(1+x)^{18}}\,dx$	**07**
		or	
	(c)	Find the length of the arc of the curve $x=a\cos\theta$ and $y=a\sin\theta$.	**07**
Q.3	**(a)**	Evaluate $\int_0^1 \int_0^x (xy^2+1)\,dy\,dx$	**05**
	(b)	Evaluate $\int_0^2 \int_{y^2}^4 (x^2+y^2)\,dx\,dy$ by changing the order of integration.	**05**

(P.T.O...

(c) Evaluate $\iint_R (x^2+y^2)\,dA$, R is the annular region between the two circles $x^2+y^2=1$ and $x^2+y^2=5$ by changing the polar coordinates. 07

or

(c) Find the volume of the region bounded by the surface $x^2+y^2=4$ between the planes $y+z=3$ and $z=0$. 07

Q.4 (a) Solve $x\sqrt{1+y^2}\,dx+y\sqrt{1+x^2}\,dy=0$ 05

(b) Solve $\left(x^2y^2+xy+1\right)ydx+\left(x^2y^2-xy+1\right)xdy=0$ 05

(c) Define Leibnitz linear equation and solve $\dfrac{dy}{dx}+\dfrac{1}{x^2}y=6e^{\frac{1}{x}}$ 07

or

(c) Solve $\left(2x^2+3y^2-7\right)xdx-\left(3x^2+2y^2-8\right)ydy=0$ 07

Q.5 (a) Discuss the converges of $(i)\sum_{n=1}^{\infty}\left(\dfrac{n}{n+1}\right)^2$ $(ii)\sum_{n=1}^{\infty}\dfrac{6^n+7^n}{8^n}$ 05

(b) Discuss the converges of $\sum\left(\sqrt[3]{n^3+1}-n\right)$ 05

(c) Find the radius of the converges and interval of converges of the series 06

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x+2)^n}{n}$$

Q.6 (a) Test the converges of the series $\dfrac{1}{1!}+\dfrac{3}{2!}+\dfrac{5}{3!}+\dfrac{7}{4!}+\ldots.$ 05

(b) Evaluate $\int_1^2\int_2^3\int_0^1 xyz\;dzdxdy$ 05

(c) Find the area of surface generated by revolving curve $y=2\sqrt{x},\ 1\le x\le 2$, about $x-axis$. 06