Black Hole Solutions Surrounded by Perfect Fluid in Rastall Theory

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Abstract

In this work, we obtain uncharged\charged Kiselev-like black holes as a new class of black hole solutions surrounded by perfect fluid in the context of Rastall theory. Then, we study the specific cases of the uncharged\charged black holes surrounded by regular matter like dust and radiation, or exotic matter like quintessence, cosmological constant and phantom fields. By comparing the Kiselev-like black hole solutions in Rastall theory with the Kiselev black hole solutions in GR, we find an effective perfect fluid behaviour for the black hole's surrounding field. It is shown that the corresponding effective perfect fluid has interesting characteristic features depending on the different ranges of the parameters in Rastall theory. For instance, Kiselev-like black holes surrounded by regular matter in Rastall theory may be considered as Kiselev black holes surrounded by exotic matter in GR, or Kiselev-like black holes surrounded by regular matter in GR.

Keywords: Rastall theory; Kiselev black holes.

1 Introduction

One of the basic elements of Einstein's general theory of relativity (GR) is the so-called covariant conservation of the energy-momentum tensor which via the Noether symmetry theorem leads to the conservation of some globally defined physical quantities. These conserved quantities appear as the integrals of the components of the energy-momentum tensor over appropriate space-like surfaces. These space-like surfaces admit at least one of the Killing vectors of the background spacetime as their normal. By this way, the total rest energy/mass of a physical system is conserved in the context of GR. On the other hand, some GR based new modified theories have been proposed that relax the condition of covariant energy-momentum conservation. One of these possible modification of the general theory of relativity was introduced by P. Rastall in 1972 [1, 2]. In this theory, the usual conservation law expressed by the null divergence of the energy-momentum tensor, i.e $T^{\mu\nu}_{;\mu} = 0$, is questioned. Then, a non-minimal coupling of matter fields to geometry is considered where the divergence of $T_{\mu\nu}$ is proportional to the gradient of the Ricci scalar, i.e $T^{\mu\nu}_{;\mu} \propto R^{,\nu}$, such that the usual conservation law is recovered in the flat spacetime. This can be understood as a direct accomplishment of the Mach principle representing that the inertia of a mass distribution is dependent on the mass and energy content of the external spacetime [3]. The main argument in favor of such a proposal is that the usual conservation law on $T_{\mu\nu}$ is tested only in the flat Minkowski space-time or specifically in a gravitational weak field limit. Indeed, this theory reproduces a phenomenological way for distinguishing features of quantum effects in gravitational systems, i.e the violation of the classical conservation laws [4, 5, 6], which is also reported in the f(R,T) [7] and $f(R,\mathcal{L}_m)$ [8] theories, where R,T and \mathcal{L}_m are the Ricci scalar, trace of the energy-momentum tensor and the Lagrangian of the matter sector, respectively. Also, the condition $T^{\mu\nu}_{;\mu} \neq 0$ is phenomenologically confirmed by the particle creation process in cosmology [9, 10, 11, 12, 13, 14, 15, 16]. In this regard, the Rastall theory can be considered as a good candidate

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for classical formulation of the particle creation through its non-minimal coupling [12, 17]. Moreover, some astrophysical analysis including the evolution of the neutron stars and cosmological data do not reject this modified theory [18, 19, 20]. Specially, in [18] it is shown that the restrictions on the Rastall geometric parameters are of the order of $\leq 1\%$ with respect to the corresponding value of the Einstein GR. In other words, the results in [18] confirm that the Rastall theory is a viable theory in the sense that the deviation of any extended theory of gravity from the standard GR must be weak, to pass the solar system tests. Some studies on the various aspects of this theory in the context of current accelerated expansion phase of the universe as well other cosmological problems can be found in [12, 21, 22, 23, 24, 25, 26, 27, 28]. Also, some research works are dedicated to incorporate this theory with the Brans-Dicke and scalar-tensor theories of gravity [30, 31, 32]. A modified Brans-Dicke theory incorporating Rastall's assumption, namely a nonzero divergence of the energy-momentum tensor, is introduced in [33, 34] which results in a class of viable theories with consistent field equations and gauge conditions. The implications of Rastall assumption in Kaluza-Klein theory and in inflationary cosmologies have been investigated in [35, 36]. It is also shown that this theory regenerates some loop quantum cosmological features of the universe expansion [37]. Apart from the cosmological solutions, any modified theory must also provide the solutions associated to the stellar and black hole configurations. In this line, some neutron star, black hole and wormholes solutions in the context of Rastall theory are obtained in [18, 38, 39, 40, 41, 42]. Also, a generalized version of Rastall theory is recently proposed which shows an agreement with the cosmic accelerating expansion [43]. In this regard, a dynamical factor for the proportionality of the energy-momentum tensor divergence and Ricci scalar divergence is considered. It is shown that this consideration leads to a transition from the matter dominated era to the current accelerating phase of the universe representing an agreement with some previous observations [44, 45, 46]. Finally, it should be mentioned that although Smalley first tried to get a Lagrangian for a prototype Rastall theory of gravity, with a variable gravitational constant [47], but this theory has been suffered from the lack of a consistent Lagrangian structure. This fact is known as the major drawback of this theory. But, recently a Lagrangian formulation for this theory is provided which may motivate the people to consider this theory more serious than before [48]. Besides, this theory possesses a rich structure that may be connected with some fundamental aspects of a complete theory of gravity and there are some points in favor of this theory. First of all, as mentioned before, the usual energy-momentum conservation law of Einstein's special relativity (SR) can be generalized to the curved spacetime in some different ways, including the appropriate geometric terms. Indeed, GR theory is one of the possible extension of SR to the curved spacetime by simply replacing the standard derivative with a covariant derivative, as the minimal generalization. Moreover, the classical form of the energy-momentum tensor must be modified by introducing quantities related to the curvature of the spacetime when the quantum effects are taking into account [4]. Also, due to the chirality of the quantum modes, the propagation of quantum fields in the spacetimes possessing horizons may lead to the violation of the classical conservation law which result in the so-called gravitational anomaly effect [49]. In this regard, Rastall theory can be a good phenomenological candidate in order to take into account the effects of quantum fields in curved spacetime in a covariant approach. Although, there is no action leading to the Rastall equations by implementing the variational principle, but it is possible to obtain such an action by introducing an external field in the Einstein-Hilbert action through a Lagrange multiplier. There are other geometrical models such as the well known Weyl geometry which may result in the field equations similar to the Rastall's field equations [47, 50].

On the other hand, the direct local impacts of cosmic backgrounds upon the known black hole solutions have been paid attention recently. It is shown by Babichev et al [51] that for a universe filled by phantom field, the black hole mass diminishes due to the accreting particles of the phantom scalar field into the central black hole. But this is a global impact indeed. The local changes in the spacetime geometry next to the central black hole can be obtained by a modified metric including the surrounding space time of the black hole. In this regard, an analytical static spherically symmetric solution to Einstein filed equations has been obtained by Kiselev [52]. This solution is characterized by the equation of state parameters of the black hole surrounding fields which generally can be dust, radiation or a dark energy component [52, 53]. In [53], a Reissner-Nordström black hole surrounded by radiation and dust and a Schwarzschild black hole surrounded by quintessence, as the special cases of the Kiselev general solution, their phase transitions as well as their thermodynamical properties are investigated. The dynamics of a neutral and a charged particle around the Schwarzschild black surrounded by a quintessence matter have been discussed in [54]. The rotating Kiselev solution and Kerr-Newman Kiselev solution have been also obtained in [55, 56, 57, 58]. Phase transition,

quasinormal modes and Hawking radiation of Schwarzschild black hole in the quintessence field are studied in [59, 60, 61]. Also, One may refer to [62, 63, 64, 65, 66, 67] for more detail in thermodynamical analysis of the Schwarzschild, Reissner-Nordström and Reissner-Nordström-AdS black holes in a quintessence background.

The essence of the Rastall theory is associated to the high curvature environments and consequently the black holes physics can provide an appropriate ground in order to investigate this theory in more details. Therefore, in this paper, our aim is to obtain the surrounded Kiselev-like black hole solutions as a new class of non-vacuum black hole solutions of this theory. The organization of the paper is as follows. In section 2, the general analytical static spherical symmetric surrounded black hole solutions in Rastall theory is obtained. Then, in the next five subsections 2.1 - 2.5, the special cases of the surrounded uncharged\charged black holes by the dust, radiation, quintessence, cosmological constant and phantom fields are addressed. Finally, in section 3, some concluding remarks are represented.

2 Surrounded Black Hole Solutions in Rastall Theory

In this section, we are looking for the general non-vacuum spherically symmetric static uncharged\charged black hole solutions in the context of the Rastall theory of gravity. Based on the Rastall's hypothesis [1, 2], for a spacetime with Ricci scalar R filled by an energy-momentum source of $T_{\mu\nu}$, we have

$$T^{\mu\nu}{}_{;\mu} = \lambda R^{;\nu},\tag{1}$$

where λ is the Rastall parameter, a measure for deviation from the standard GR conservation law. Then, the Rastall field equations can be written as

$$G_{\mu\nu} + \kappa \lambda g_{\mu\nu} R = \kappa T_{\mu\nu},\tag{2}$$

where κ is the Rastall gravitational coupling constant. This field equations reduce to GR field equations in the limit of $\lambda \to 0$ and $\kappa = 8\pi G_N$ where G_N is the Newton gravitational coupling constant.

In order to obtain black hole solutions, we consider the general spherical symmetric spacetime metric in the standard Schwarzschild coordinates as

$$ds^{2} = -f(r)dt^{2} + \frac{dr^{2}}{f(r)} + r^{2}d\Omega^{2},$$
(3)

where f(r) is a generic metric function depending on the radial coordinate r and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ is the two dimensional unit sphere. Using this metric, we obtain nonvanishing components of the Rastall tensor defined as $H_{\mu\nu} = G_{\mu\nu} + \kappa\lambda g_{\mu\nu}R$ as

$$H^{0}{}_{0} = G^{0}{}_{0} + \kappa \lambda R = -\frac{1}{f}G_{00} + \kappa \lambda R = \frac{1}{r^{2}}(f'r - 1 + f) + \kappa \lambda R,$$

$$H^{1}{}_{1} = G^{1}{}_{1} + \kappa \lambda R = fG_{11} + \kappa \lambda R = \frac{1}{r^{2}}(f'r - 1 + f) + \kappa \lambda R,$$

$$H^{2}{}_{2} = G^{2}{}_{2} + \kappa \lambda R = \frac{1}{r^{2}}G_{22} + \kappa \lambda R = \frac{1}{r^{2}}\left(rf' + \frac{1}{2}r^{2}f''\right) + \kappa \lambda R,$$

$$H^{3}{}_{3} = G^{3}{}_{3} + \kappa \lambda R = \frac{1}{r^{2}sin^{2}\theta}G_{33} + \kappa \lambda R = \frac{1}{r^{2}}\left(rf' + \frac{1}{2}r^{2}f''\right) + \kappa \lambda R,$$

$$(4)$$

where the Ricci scalar reads as

$$R = -\frac{1}{r^2} \left(r^2 f'' + 4r f' - 2 + 2f \right), \tag{5}$$

in which the prime sign represents the derivative with respect to the radial coordinate r. Then, regarding the nonvanishing components of the Rastall tensor H^{μ}_{ν} , the total energy-momentum tensor supporting this spacetime should have the following diagonal form

$$T^{\mu}_{\ \nu} = \begin{pmatrix} T^{0}_{0} & 0 & 0 & 0\\ 0 & T^{1}_{1} & 0 & 0\\ 0 & 0 & T^{2}_{2} & 0\\ 0 & 0 & 0 & T^{3}_{3} \end{pmatrix},\tag{6}$$

which must also obey the symmetry properties of the Rastall tensor H^{μ}_{ν} . Regarding the equations in (4), the equalities $H^0_0 = H^1_1$ and $H^2_2 = H^3_3$ require $T^0_0 = T^1_1$ and $T^2_2 = T^3_3$, respectively. Then, one can construct a general total energy-momentum tensor T^{μ}_{ν} possessing these symmetry properties in the following form

$$T^{\mu}{}_{\nu} = E^{\mu}{}_{\nu} + \mathcal{T}^{\mu}{}_{\nu},$$
 (7)

where $E^{\mu}_{\ \nu}$ is the trace-free Maxwell tensor given by

$$E_{\mu\nu} = \frac{2}{\kappa} \left(F_{\mu\alpha} F_{\nu}{}^{\alpha} - \frac{1}{4} g_{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right), \tag{8}$$

so that $F_{\mu\nu}$ is the antisymmetric Faraday tensor satisfying the following vacuum Maxwell equations

$$F^{\mu\nu}{}_{;\mu} = 0,$$

$$\partial_{[\sigma} F_{\mu\nu]} = 0.$$
 (9)

Considering the spherical symmetry existing in the spacetime metric (3) imposes the only non-vanishing components of the Faraday tensor $F^{\mu\nu}$ to be $F^{01}=-F^{10}$. Then, from the equations in (9), one obtains

$$F^{01} = \frac{Q}{r^2},\tag{10}$$

where Q is an integration constant playing the role of a electrostatic charge. Thus, the equations (3), (8) and (10) give the only non-vanishing components of the Maxwell tensor E^{μ}_{ν} as

$$E^{\mu}{}_{\nu} = \frac{Q^2}{\kappa r^4} \ diag(-1, -1, 1, 1), \tag{11}$$

representing an electrostatic field and clearly possesses the symmetries in H^{μ}_{ν} tensor. On the other hand, \mathcal{T}^{μ}_{ν} describes the energy-momentum tensor of the surrounding field defined as [52]

$$\mathcal{T}^{0}{}_{0} = -\rho_{s}(r),$$

$$\mathcal{T}^{i}{}_{j} = -\rho_{s}(r)\alpha \left[-(1+3\beta)\frac{r_{i}r^{j}}{r_{n}r^{n}} + \beta\delta^{i}{}_{j} \right].$$
(12)

This form of \mathcal{T}^{μ}_{ν} indicates that the spatial sector is proportional to the time sector, denoting the energy density ρ_s , with the arbitrary parameters α and β related to the internal structure of the black hole surrounding field. Here, we used the subscript "s" for denoting the surrounding field which generally can be a dust, radiation, quintessence, cosmological constant, phantom field or even any combination of them. By taking the isotropic average over the angles we have [52]

$$\langle \mathcal{T}^i{}_j \rangle = \frac{\alpha}{3} \rho_s \delta^i{}_j = p_s \delta^i{}_j,$$
 (13)

since it is supposed that $\langle r^i r_j \rangle = \frac{1}{3} \delta^i{}_j r_n r^n$. Thus, one has the barotropic equation of state for the surrounding field

$$p_s = \omega_s \rho_s, \quad \omega_s = \frac{1}{3}\alpha,$$
 (14)

where p_s and ω_s are the pressure and equation of state parameter, respectively. Thus, the field equations (4) with respect to the total energy-momentum tensor in (7), (11) and (12) exactly provide the principle of additivity and linearity condition supposed in the reference [52] which was proposed to determine the free parameter β of the energy momentum-tensor of the surrounding field as

$$\beta = -\frac{1 + 3\omega_s}{6\omega_s}.\tag{15}$$

Then, the non-vanishing components of the T^{μ}_{ν} tensor can be obtained in the following form

$$\mathcal{T}^{0}{}_{0} = \mathcal{T}^{1}{}_{1} = -\rho_{s},$$

$$\mathcal{T}^{2}{}_{2} = \mathcal{T}^{3}{}_{3} = \frac{1}{2}(1+3\omega_{s})\rho_{s},$$
(16)

which also possess the same symmetries in the Rastall tensor H^{μ}_{ν} . Consequently, our total constructed energy-momentum tensor in (7) admits all of the symmetry properties of H^{μ}_{ν} . One may just consider the \mathcal{T}^{μ}_{ν} as the only supporting energy-momentum tensor of the Rastall field equations. In this way, the obtained solutions will describe the surrounded uncharged black hole solutions in the context of the Rastall theory which differ from the ones in GR, as we see later. Including the Maxwell tensor E^{μ}_{ν} in T^{μ}_{ν} provides the possibility of obtaining most general class of the static surrounded charged black hole solutions in the framework of this theory. In the following, we solve the field equations and obtain its general solution. Then, we address both of the uncharged charged solutions.

The $H^0_0 = T^0_0$ and $H^1_1 = T^1_1$ components of the Rastall field equations give the following differential equation

$$\frac{1}{r^2}(rf'-1+f) - \frac{\kappa\lambda}{r^2}(r^2f''+4rf'-2+2f) = -\kappa\rho_s - \frac{Q^2}{r^4},\tag{17}$$

and $H^2_{\ 2}=T^2_{\ 2}$ and $H^3_{\ 3}=T^3_{\ 3}$ components read as

$$\frac{1}{r^2} \left(rf' + \frac{1}{2}r^2 f'' \right) - \frac{\kappa \lambda}{r^2} \left(r^2 f'' + 4rf' - 2 + 2f \right) = \frac{1}{2} (1 + 3\omega_s) \kappa \rho_s + \frac{Q^2}{r^4}.$$
 (18)

Thus, we have two unknown functions f(r) and $\rho_s(r)$ which can be determined analytically by the above two differential equations. Now, by solving the set of differential equations (17) and (18)¹, one obtains the following general solution for the metric function

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{N_s}{r^{\frac{1+3\omega_s - 6\kappa\lambda(1+\omega_s)}{1-3\kappa\lambda(1+\omega_s)}}},$$
(19)

with the energy density in the form of

$$\rho_s(r) = -\frac{3W_s N_s}{\frac{3(1+\omega_s)-12\kappa\lambda(1+\omega_s)}{1-3\kappa\lambda(1+\omega_s)}},$$
(20)

where M and N_s are two integration constants representing the black hole mass and surrounding field structure parameter, respectively in which

$$W_s = -\frac{(1 - 4\kappa\lambda)\left(\kappa\lambda\left(1 + \omega_s\right) - \omega_s\right)}{\left(1 - 3\kappa\lambda(1 + \omega)\right)^2},\tag{21}$$

is a geometric constant depending on the Rastall geometric parameters κ and λ as well as the equation of state parameter ω_s of the black hole surrounding field. Note that the integration constant N_s represents the characteristic features of the surrounding field. For $\lambda=0$, i.e in the GR limit, we have $\rho_s(r)=-\frac{3}{\kappa}\mathcal{W}_sN_sr^{-3(1+\omega_s)}$ where $\mathcal{W}_s=\omega_s$ as in [52]. Note that in [52], the author used the units of $4\pi G_N=1$ with a metric possessing a negative signature.

Regarding the weak energy condition representing the positivity of any kind of energy density of the surrounding field, i.e $\rho_s \geq 0$, imposes the following condition on the geometric parameters of the theory

$$W_s N_s \le 0. (22)$$

This condition implies that for the surrounding field with geometric parameter $W_s > 0$, we need $N_s < 0$ and conversely for $W_s < 0$, we need $N_s > 0$. Then, considering that W_s is given by (21), the sign of the metric parameter N_s depends on the Rastall geometric parameters κ , λ and the equation of state parameter ω_s of the surrounding field. In this regard, any set of κ , λ and ω_s parameters may admit a different positive or negative N_s values.

Finally, regarding (19), our metric (3) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{s}}{r^{\frac{1+3\omega_{s} - 6\kappa\lambda(1+\omega_{s})}{1-3\kappa\lambda(1+\omega_{s})}}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{s}}{r^{\frac{1+3\omega_{s} - 6\kappa\lambda(1+\omega_{s})}{1-3\kappa\lambda(1+\omega_{s})}}}} + r^{2}d\Omega^{2}.$$
 (23)

¹Substituting $\kappa \rho_s(r)$ from differential equation (17) into (18) gives a differential equation for f(r) leading to the solution (19). Then, by substituting the obtained f(r) into the differential equations (17) or (18), one obtains the appropriate form of $\rho_s(r)$ as given by (20) and (21).

In the limit of $\lambda \to 0$ and $\kappa = 8\pi G_N$, we recover the Reissner-Nordström black hole surrounded by a surrounding field in GR which was firstly found by Kiselev [52] as

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{s}}{r^{3\omega_{s}+1}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{s}}{r^{3\omega_{s}+1}}} + r^{2}d\Omega^{2}.$$
 (24)

Our obtained static solution (23) is new and possesses some interesting features. By comparing the metric (23) with the Kiselev metric (24) in GR, we may obtain an effective equation of state parameter ω_{eff} for the modification term resulting from the geometry of the Rastall theory.

The notion of "effective equation of state" in Rastall theory has already been studied in the cosmological context, where a solution for the entropy and age problems of the Standard Cosmological Model were provided [?] by considering Brans-Dicke and Rastall theories of gravity and performing a perturbative analysis. It was shown that by introducing an "effective equation of state", the Rastall theory exhibits satisfactory properties at perturbative level in comparison to the Brans-Dicke theory.

In the next subsections, the surrounded black hole by the dust, radiation, quintessence, cosmological constant and phantom fields, as the subclasses of the general solution (23), as well as their interesting features are studied in detail. At last, we recall that the cases $\kappa\lambda = \frac{1}{4}$ and $\kappa\lambda = \frac{1}{6}$ are generally excluded due to the divergence of the Rastall gravitational coupling constant, as discussed in [1, 40].

2.1 The Black Hole Surrounded by the Dust Field

For the dust surrounding field, we set $\omega_d = 0$ [52, 68]. Then, the metric (23) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{d}}{r^{\frac{1-6\kappa\lambda}{1-3\kappa\lambda}}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{d}}{r^{\frac{1-6\kappa\lambda}{1-3\kappa\lambda}}}} + r^{2}d\Omega^{2}.$$
 (25)

This metric differs from the metric of the surrounded charged black hole by a dust field in GR [52]. One can realize that in GR, i.e in the limit of $\lambda \to 0$ and $\kappa = 8\pi G_N$, the black hole in the dust background appears as a charged black hole with an effective mass $M_{eff} = 2M + N_d$. Thus, we see that for $\kappa\lambda \neq 0$, the geometric parameters κ and λ of the Rastall theory can play an important role leading to distinct solutions relative to GR. Setting Q = 0 or E^{μ}_{ν} in the total energy-momentum tensor in (7), one arrives at uncharged Kiselev-like black hole solutions in the dust background. One can realize that for $\kappa\lambda \neq 0$ the Rastall correction term never behaves as the mass or charge terms, and introduces a new character to the black hole, not comparable to the mass and charge terms. The presence of such nontrivial character can drastically change the thermodynamics, causal structure and Penrose diagrams, due to the Rastall geometric parameters, with respect to those of GR.

In this case, the geometric parameter W_d given by the relation (21) reads as

$$W_d = -\frac{\kappa \lambda \left(1 - 4\kappa \lambda\right)}{\left(1 - 3\kappa \lambda\right)^2}.$$
 (26)

Then, regarding the weak energy condition represented by the relation (22), for $0 \le \kappa\lambda < \frac{1}{4}$ it is required that $N_d > 0$, while for $\kappa\lambda < 0 \cup \kappa\lambda > \frac{1}{4}$, we need $N_d < 0$ for the field structure constant. In this case, \mathcal{W}_d and consequently ρ_d are effectively different from their GR counterparts such that $\rho_d = \frac{3\lambda(1-4\kappa\lambda)N_d}{(1-3\kappa\lambda)^2} r^{-\frac{3-12\kappa\lambda}{1-3\kappa\lambda}}$.

By comparing this metric with the Kiselev metric (24) in GR, we may obtain an effective equation of state parameter ω_{eff} for the modification term resulting from the geometry of the Rastall theory as

$$\omega_{eff} = \frac{1}{3} \left(-1 + \frac{1 - 6\kappa\lambda}{1 - 3\kappa\lambda} \right). \tag{27}$$

One may realize that ω_{eff} can never be zero (representing a background dust matter) except for the $\kappa\lambda = 0$ corresponding to GR limit. Then, the solutions of this theory are effectively different from those of GR. Regarding (27), two interesting classes are distinguishable as

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_d value	N_d value
$\frac{2}{10}$	$-\frac{1}{2}$	violated	$-\frac{5}{4}$	positive
$\frac{2}{9}$	$-\frac{2}{3}$	violated	-1	positive
$\frac{3}{10}$	-3	violated	20	negative

Table 1: Some $\kappa\lambda$ values in the range $\frac{1}{6} < \kappa\lambda < \frac{1}{3}$ and their corresponding effective equation of state ω_{eff} parameters accompanied by the geometric parameters \mathcal{W}_d and N_d .

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_d value	N_d value
$\frac{1}{8}$	$-\frac{1}{5}$	respected	$-\frac{32}{25}$	positive
$\frac{1}{9}$	$-\frac{1}{6}$	respected	$-\frac{5}{4}$	positive
$\frac{1}{10}$	$-\frac{1}{7}$	respected	$-\frac{60}{49}$	positive
$\frac{4}{10}$	2	respected	15	negative
$\frac{1}{2}$	1	respected	4	negative
1	$\frac{1}{2}$	respected	$\frac{3}{4}$	negative

Table 2: Some $\kappa\lambda$ values in the range $\kappa\lambda < \frac{1}{6} \cup \kappa\lambda > \frac{1}{3}$ and their corresponding effective equation of state ω_{eff} parameters accompanied by the geometric parameters \mathcal{W}_d and N_d .

• $\frac{1}{6} < \kappa\lambda < \frac{1}{3}$ which leads to $\omega_{eff} \le -\frac{1}{3}$. In this case, we have an effective surrounding fluid with an effective equation of state parameter ω_{eff} , playing the role of dark energy, which leads to an effective repulsive gravitational effect. Then, regarding this range for $\kappa\lambda$, such black holes may contribute to the accelerating expansion of the universe in the Rastall theory of gravity. In the language of Raychaudhuri equation, such an effective surrounding fluid violating the strong energy condition can account for the accelerating expansion of the universe. Some $\kappa\lambda$ values in the range $\frac{1}{6} < \kappa\lambda < \frac{1}{3}$ and their corresponding effective equation of state ω_{eff} parameters accompanied by the geometric parameters \mathcal{W}_d and N_d are listed in Table 1.

Interestingly, for $\kappa\lambda=\frac{2}{10}$ and $\frac{2}{9}$, the effective equation of state ω_{eff} lies in the quintessence range while for $\kappa\lambda=\frac{3}{10}$, it lies in the strong phantom range. This represents the fact that for a given κ , the more large values of λ , namely the more strong coupling $g_{\mu\nu}R$ in Rastall theory, the more strong acceleration phase.

• $\kappa\lambda < \frac{1}{6} \cup \kappa\lambda > \frac{1}{3}$ which leads to $\omega_{eff} \geq -\frac{1}{3}$. In this case, we have an effective surrounding fluid with an equation of state parameter respecting to the strong energy condition possessing the usual attractive gravitational effect. This may contribute to the decelerating expansion or even the contraction of universe depending on the value of the effective equation of state parameter ω_{eff} . In the language of Raychaudhuri equation, such a regular effective matter which respects to the strong energy condition, can justify the deceleration phase. Some $\kappa\lambda$ values in the range $\kappa\lambda < \frac{1}{6} \cup \kappa\lambda > \frac{1}{3}$ and their corresponding effective equation of state ω_{eff} parameters accompanied by the geometric parameters \mathcal{W}_d and N_d are listed in Table 2.

Interestingly, for $\kappa\lambda = \frac{1}{2}$, the effective equation of state $\omega_{eff} = 1$ belongs to the stiff matter possessing very strong attractive gravitational effect.

2.2 The Black Hole Surrounded by the Radiation Field

For the radiation surrounding field, we set $\omega_r = \frac{1}{3}$ [52, 68]. Then, the metric (23) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2} - N_{r}}{r^{2}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2} - N_{r}}{r^{2}}} + r^{2}d\Omega^{2}.$$
 (28)

It is interesting that this case is the same as in GR and the geometric effects of the Rastall parameters do not appear for a black hole surrounded by the radiation field [52]. Also, the geometric parameter W_r given by the relation (21) reads as

$$W_r = \frac{1}{3},\tag{29}$$

and consequently with regard to the weak energy condition for this case, represented by the relation (22), it is required that $N_r < 0$ for the radiation field structure parameter. Then, by defining the positive structure parameter $\mathcal{N}_r = -N_r$, we have

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2} + \mathcal{N}_{r}}{r^{2}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2} + \mathcal{N}_{r}}{r^{2}}} + r^{2}d\Omega^{2},\tag{30}$$

which is the metric of the well known Reissner-Nordström black hole with an effective charge $Q_{eff} = \sqrt{Q^2 + \mathcal{N}_r}$. This result is interpreted as the positive contribution of the characteristic feature of the surrounding radiation field to the effective charge of the black hole. The appearance of effective charge in the black hole solution cannot change the causal structure and Penrose diagrams of this black hole solution, in comparison to the Reissner-Nordström black hole.

Setting Q=0 or switching off the electrostatic energy-momentum tensor E^{μ}_{ν} in the total energy-momentum tensor in (7), one arrives at the Kiselev black hole solutions in the radiation background. In that case, the resulting metric will be the Reissner-Nordström black hole with the charge term \mathcal{N}_r . Also, note that for a radiation background, not only the metric and the geometric parameter W_r are the same as in GR but also the energy density ρ_r of the background radiation has the same form in comparison to the GR's as $\rho_r = \frac{\mathcal{N}_r}{\kappa r^4}$. It is seen that the value of radiation energy density ρ_r of the background depends not only on the characteristic feature of the surrounding radiation field \mathcal{N}_r , but also it depends on the gravitational constant of the Rastall theory κ . In general, Rastall's gravitational constant may differs from the Newton gravitational constant. However, if one sets $\kappa = 8\pi G_N$ as in GR, the corresponding energy densities in both of these theories will be the same. Such a situation occurs also in the cosmological context of the Rastall theory [69]. In the cosmological setup, the metric solution, i.e the scale factor, of the universe filled by the radiation fluid is exactly the same as in GR. Then, the evolutions of the universe during the radiation dominated era are the same for both of the GR and Rastall theories. This fact can be understood by inspecting the original field equations of the Rastall theory such that for a radiation fluid, we have T=0 and R=0 indicating that everything should be the same as in GR theory.

2.3 The Black Hole Surrounded by the Quintessence Field

For the quintessence surrounding field, we set $\omega_q = -\frac{2}{3}$ [52, 68]. Then, the metric (23) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{q}}{r^{\frac{-1-2\kappa\lambda}{1-\kappa\lambda}}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{q}}{\frac{-1-2\kappa\lambda}{1-\kappa\lambda}}} + r^{2}d\Omega^{2}.$$
 (31)

This metric differs from the metric of the surrounded charged black hole by a quintessence field in GR [52]. Here, it is seen that for $\kappa\lambda \neq 0$, the geometric parameters κ and λ of the Rastall theory can play an important role leading to distinct solutions, in comparison to GR. In this case, setting Q=0 or $E^{\mu}_{\nu}=0$ in the total energy-momentum tensor in (7), one arrives at uncharged Kiselev-like black hole solutions in the quintessence background. Due to the appearance of nontrivial N_q term, the causal structure and Penrose diagram will be different from those of Reissner-Nordström black hole in GR.

In this case, the geometric parameter W_q given by the relation (21) reads as

$$W_q = -\frac{(1 - 4\kappa\lambda)(2 + \kappa\lambda)}{3(1 - \kappa\lambda)^2}.$$
(32)

Then, considering the weak energy condition given by the relation (22), we require $N_q > 0$ for $0 \le \kappa \lambda < \frac{1}{4}$ and $N_q < 0$ for $\kappa \lambda > \frac{1}{4}$. The equation (32) shows that \mathcal{W}_q and consequently the corresponding energy density ρ_q effectively differ from their GR counterparts such that $\rho_q = \frac{(1-4\kappa\lambda)(2+\kappa\lambda)N_q}{\kappa} r^{-\frac{1-4\kappa\lambda}{1-\kappa\lambda}}$.

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_q value	N_q value
$-\frac{1}{2}$	$-\frac{1}{3}$	violated	$-\frac{12}{25}$	positive
$\frac{4}{10}$	$-\frac{4}{3}$	violated	15	negative
$\frac{1}{2}$	$-\frac{5}{3}$	violated	4	negative

Table 3: Some $\kappa\lambda$ values in the range $-\frac{1}{2} \leqslant \kappa\lambda < 1$ and their corresponding effective equation of state ω_{eff} parameters with their behaviors, accompanied by the geometric parameters \mathcal{W}_q and N_q .

In this case, by comparing the metric (31) with the original Kiselev metric (24) in GR, one can obtain an effective equation of state parameter ω_{eff} for the modification term resulting by the geometry of the Rastall theory as

$$\omega_{eff} = \frac{1}{3} \left(-1 - \frac{1 + 2\kappa\lambda}{1 - \kappa\lambda} \right). \tag{33}$$

One may realize that ω_{eff} can never be $-\frac{2}{3}$ (the background quintessence filed), except for the $\kappa\lambda = 0$ which corresponds to the GR limit. Then, the solutions of this theory are effectively different from GR's. Regarding (33), two interesting classes are distinguishable as

• $-\frac{1}{2} \leqslant \kappa \lambda < 1$ which leads to $\omega_{eff} \le -\frac{1}{3}$. In this case, we have an effective surrounding fluid with an equation of state parameter violating the strong energy condition which leads to a repulsive gravitational effect like as the background quintessence field but with a different repulsive strength. This may contribute to the accelerating expansion of the universe. Regarding the appropriate range for $\kappa\lambda$, such black holes may contribute to the accelerating expansion of the universe in the Rastall theory. Using the Raychaudhuri equation, such an effective surrounding quintessence field violating the strong energy condition can justify the acceleration expansion of the universe. Some $\kappa\lambda$ values in the range $-\frac{1}{2} \leqslant \kappa\lambda < 1$ and their corresponding effective equations of state ω_{eff} parameter with its behavior, accompanied by the geometric parameters \mathcal{W}_q and N_q are given in Table 3.

Interestingly, the case of $\kappa\lambda=-\frac{1}{2}$ leads to $\omega_{eff}=-\frac{1}{3}$ representing an effective surrounding quintessence field weaker than the one with $\omega_q=-\frac{2}{3}$. In the cosmological setup and through the second Friedmann equation, the acceleration equation, $\omega_{eff}=-\frac{1}{3}$ corresponds to a universe with a uniform expanding velocity, i.e $\ddot{a}=0$ where a is the scale factor of the ambient FRW universe filled by an effective field with $\omega_{eff}=-\frac{1}{3}$. For, $\kappa\lambda=\frac{4}{10}$ and $\kappa\lambda=\frac{1}{2}$, it is seen that the effective surrounding field possesses a repulsive character stronger than the quintessence with $\omega_{eff}=-\frac{4}{3}$ and $\omega_{eff}=-\frac{5}{3}$ which eventually lie in the phantom regime.

• $\kappa\lambda\leqslant-\frac{1}{2}$ \cup $\kappa\lambda>1$ which leads to $\omega_{eff}\geq-\frac{1}{3}$. In this case, we have an effective surrounding fluid with an equation of state parameter respecting to the strong energy condition possessing an attractive gravitational effect. This may contribute to the decelerating expansion or even in contraction of the universe. In this case, although the black hole is surrounded by the quintessence field with $\omega_q=-\frac{2}{3}$, however the effective equation of state ω_{eff} regarding the appropriate range for $\kappa\lambda$ does not belong to the quintessence range. For such a regular effective matter which respects to the strong energy condition, the Raychaudhuri equation can justify the deceleration phase or even the contraction of the universe. Some $\kappa\lambda$ values in the range $\kappa\lambda\leqslant-\frac{1}{2}$ \cup $\kappa\lambda>1$ and their corresponding effective equations of state ω_{eff} parameters with their behaviors, accompanied by the geometric parameters \mathcal{W}_q and N_q are given in Table 4.

In this case, the Rastall's correction term in metric (31) can never behave as the charge term, i.e as $\frac{1}{r^2}$, to increases or decrease the charge's effect. But interestingly for $\kappa\lambda=-2$, which leads to the effective equation of state $\omega_{eff}=0$ representing an effective dust matter, it exactly behaves like the mass term, i.e $\frac{1}{r}$. The sign of metric parameter N_q for $\kappa\lambda=-2$ is positive and consequently, the correction term contributes to increase the effect of Schwarzschild mass term. A similar but reverse effect is reported in [51] in which for a universe filled by a phantom field, the black hole mass smoothly decreases due to the accreting particles of the phantom scalar field into the central black hole. This

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_q value	N_q value
-1	$-\frac{1}{6}$	respected	$-\frac{5}{16}$	positive
$-\frac{3}{2}$	$-\frac{1}{15}$	respected	$-\frac{28}{121}$	positive
-2	0	respected	$-\frac{9}{49}$	positive
$\frac{3}{2}$	$\frac{7}{3}$	respected	$\frac{20}{49}$	negative
2	$\frac{4}{3}$	respected	$\frac{7}{25}$	negative
$\frac{5}{2}$	1	respected	36 169	negative

Table 4: Some $\kappa\lambda$ values in the range $\kappa\lambda \leqslant -\frac{1}{2} \cup \kappa\lambda > 1$ and their corresponding effective equation of state ω_{eff} parameters with their behaviors, accompanied by the geometric parameters W_q and N_q .

fact can be investigated for the case of a universe filled by a quintessence filed, which is out of the scope of the present paper. Also, $\kappa\lambda=\frac{5}{2}$ leads to the equation of state parameter $\omega_{eff}=1$ denoting a stiff matter. In conclusion, it is seen that although the surrounding field is an essentially quintessence but the effective field is not the quintessence like filed, possessing a negative equation of state parameter, rather it can behave effectively as dust or even stiff matter possessing a zero or positive equation of state parameters, respectively.

2.4 The Black Hole Surrounded by the Cosmological Constant Field

For the cosmological constant surrounding field, we set $\omega_c = -1$ [52, 68]. Then, the metric (23) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - N_{c}r^{2}\right)du^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - N_{c}r^{2}} + r^{2}d\Omega^{2}.$$
 (34)

It is interesting that this case is the same as what was already obtained in GR by Kiselev [52]. Then, the Rastall and Einstein theories behave the same in the cosmological constant background. Here, setting Q=0 or switching off E^{μ}_{ν} in the total energy-momentum tensor in (7), one arrives at uncharged Kiselev-like black hole solutions in the de Sitter or anti-de Sitter background.

In this case, the geometric parameter W_c given by the relation (21) reads as

$$W_c = -(1 - 4\kappa\lambda). \tag{35}$$

Then, considering the weak energy condition given by the relation (32), we require $N_c > 0$ for $0 \le \kappa \lambda < \frac{1}{4}$, and $N_c < 0$ for $\kappa \lambda > 1/4$, corresponding to de Sitter or anti-de Sitter backgrounds, respectively. This shows that the sign of cosmological constant in the Rastall theory depends on its geometric parameters κ and λ . Although the form of metric (34) in this theory is the same as in GR for cosmological constant background, but the energy density of the cosmological constant differs from the GR due to the geometric features of the Rastall theory through the equations (20) and (35). In this case, the energy density of the cosmological constant is given by $\rho_c = \frac{3(1-4\kappa\lambda)N_c}{\kappa}$. A similar situation occurs in the cosmological context of the Rastall theory where the metric solution of the field equations, i.e the scale factor, for the universe dominated by the cosmological constant has a similar form as in GR, i.e it has an exponential form. In this case, by comparing the obtained result in [69] as $H \propto \sqrt{1-\frac{2}{3}(\frac{3-2\lambda}{2\lambda-1})\rho}$ with the GR's as $H \propto \sqrt{\Lambda}$, we see that although the solutions have the same form but the geometric properties of the Rastall theory may affect the energy density of the background cosmological constant.²

² One should note to a little different notation for the field equations in our work and [69], where the field equations are defined as $R_{\mu\nu} - \frac{\lambda}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$ and $T^{\mu\nu}_{;\mu} = \frac{1-\lambda}{2\kappa} T^{;\nu}$.

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_p value	N_p value
$-\frac{1}{2}$	-3	violated	$-\frac{12}{25}$	positive
$\frac{1}{2}$	$-\frac{7}{9}$	violated	4	negative
1	$-\frac{1}{2}$	violated	$\frac{3}{4}$	negative

Table 5: Some $\kappa\lambda$ values in the range $-1 < \kappa\lambda < \frac{3}{2}$ and their associated effective equation of state parameters ω_{eff} parameters with their behaviors, accompanied by the geometric parameters W_q and N_q .

2.5 The Black Hole Surrounded by the Phantom Field

For the phantom surrounding field, we set $\omega_p = -\frac{4}{3}$ [68]. Then, the metric (23) takes the following form

$$ds^{2} = -\left(1 - \frac{2M}{r} + \frac{Q^{2}}{r^{2}} - \frac{N_{p}}{r^{\frac{-3+2\kappa\lambda}{1+\kappa\lambda}}}\right)dt^{2} + \frac{dr^{2}}{1 - \frac{2M}{r} - \frac{N_{p}}{r^{\frac{-3+2\kappa\lambda}{1+\kappa\lambda}}}} + r^{2}d\Omega^{2}.$$
 (36)

This metric differs from the metric of the surrounded charged black hole by a phantom field in GR [52]. For $\kappa\lambda\neq 0$, the geometric parameters κ and λ of the Rastall theory plays an important role leading to distinct solutions in comparison to GR. Also, setting Q=0 or switching of E^{μ}_{ν} in the total energy-momentum tensor in (7), one arrives at uncharged Kiselev-like black hole solutions in the phantom background. Due to the appearance of nontrivial N_p term, the causal structure and Penrose diagram will be different from those of Reissner-Nordström black hole in GR.

In this case, the geometric parameter W_p given by the relation (21) reads as

$$W_p = -\frac{1}{3} \frac{(1 - 4\kappa\lambda)(4 - \kappa\lambda)}{(1 + \kappa\lambda)^2}.$$
 (37)

Then, considering the weak energy condition given by the relation (22), we require $N_p > 0$ for $0 \le \kappa \lambda < \frac{1}{4} \cup \kappa \lambda > 4$ and $N_p < 0$ for $\frac{1}{4} < \kappa \lambda < 4$. The equation (37) shows that \mathcal{W}_p and consequently the corresponding phantom energy density ρ_p effectively differs from their GR counterparts such that $\rho_p = \frac{(1-4\kappa\lambda)(4-\kappa\lambda)}{\kappa(1+\kappa\lambda)^2} N_p r^{\frac{1-4\kappa\lambda}{1+\kappa\lambda}}$.

 $\frac{(1-4\kappa\lambda)(4-\kappa\lambda)}{\kappa(1+\kappa\lambda)^2}N_p\ r^{\frac{1-4\kappa\lambda}{1+\kappa\lambda}}.$ By comparing this metric with the Kiselev metric (24), we may obtain an effective equation of state parameter ω_{eff} for the modification term resulting from the geometry of the Rastall theory as

$$\omega_{eff} = \frac{1}{3} \left(-1 - \frac{3 - 2\kappa\lambda}{1 + \kappa\lambda} \right). \tag{38}$$

One may realize that ω_{eff} never can be $-\frac{4}{3}$ (the background phantom field), except for the $\kappa\lambda = 0$ corresponding to GR limit. Then, two interesting classes are distinguishable as

- $-1 < \kappa \lambda < \frac{3}{2}$ leading to $\omega_{eff} \le -\frac{1}{3}$. Then, we have a surrounding fluid with an effective equation of state parameter ω_{eff} which violates the strong energy condition resulting in a repulsive gravitational force. Then, in this range of $\kappa\lambda$, these black holes may contribute to the accelerating expansion of the universe in the Rastall theory. For such an effective surrounding quintessence field violating the strong energy condition, the Raychaudhuri equation can account for the acceleration expansion of the universe. In Table 5, some $\kappa\lambda$ values in the range $-1 < \kappa\lambda < \frac{3}{2}$ and their associated effective equations of state parameters ω_{eff} parameter with their behaviors, accompanied by the geometric parameters W_q and N_q are given. Interestingly, for $\kappa\lambda = -\frac{1}{2}$, we have $\omega_{eff} = -3$ which has a repulsive character stronger than the background phantom field with $\omega_p = -\frac{4}{3}$ while for $\kappa\lambda = \frac{1}{2}$ and $\kappa\lambda = 1$, we have an effective field with a repulsive character weaker than the background phantom field with $\omega_p = -\frac{4}{3}$ but still lying in the quintessence range.
- $\kappa\lambda < -1 \cup \kappa\lambda \ge \frac{3}{2}$ which leads to $\omega_{eff} \ge -\frac{1}{3}$. In this case, we have an effective surrounding fluid with an equation of state parameter respecting to the strong energy condition which leads to a attractive

$\kappa\lambda$ value	ω_{eff} value	SEC	\mathcal{W}_p value	N_p value
$-\frac{3}{2}$	$\frac{11}{3}$	respected	$-\frac{28}{121}$	positive
-2	2	respected	$-\frac{9}{49}$	positive
$-\frac{5}{2}$	13 9	respected	$-\frac{44}{289}$	positive
2	$-\frac{2}{9}$	respected	$\frac{7}{25}$	negative
$\frac{5}{2}$	$-\frac{1}{7}$	respected	$\frac{36}{169}$	negative
3	$-\frac{1}{12}$	respected	$\frac{11}{64}$	negative
4	0	respected	$\frac{15}{121}$	negative

Table 6: Some $\kappa\lambda$ values in the range $\kappa\lambda < -1 \cup \kappa\lambda \ge \frac{3}{2}$ and their corresponding effective equation of state ω_{eff} parameters with their behaviors, accompanied by the geometric parameters W_p and N_p .

gravitational effect. This may contribute to the decelerating expansion or even in contraction of the universe. In this case, although the black hole is surrounded by the phantom field with $\omega_p = -\frac{4}{3}$, but the effective equation of state ω_{eff} regarding the appropriate range of $\kappa\lambda$ does not belong to the phantom range. This effect may cause the contraction of universe filled by such a black holes in the Rastall theory of gravity. For such a regular effective matter which respects to the strong energy condition, the Raychaudhuri equation can justify the deceleration phase or even the contraction of the universe. Some $\kappa\lambda$ values in the range $\kappa\lambda < -1 \cup \kappa\lambda \ge \frac{3}{2}$ and their corresponding effective equations of state ω_{eff} parameter with their behaviors, accompanied by the geometric parameters \mathcal{W}_p and N_p are given in Table 6. In this case, the Rastall's correction term in metric (36) can never behave like the charge term, i.e as $\frac{1}{r^2}$, but interestingly for $\kappa\lambda=4$, which leads to the effective equation of state $\omega_{eff} = 0$ representing an effective dust matter, it exactly behaves like the mass term, i.e. $\frac{1}{r}$. For this case, the sign of N_q for $\kappa\lambda=4$ is negative and the correction term contributes to decreases the effect of Schwarzschild mass term. Such an effect is reported in [51] in which for a universe filled by a phantom field approaching to the Big Rip, the black hole mass gradually decreases due to the accreting particles of the phantom scalar field into the central black hole. In conclusion, it is seen that although the surrounding field is an essentially phantom field but the effective surrounding field is not the phantom field, rather it can be effectively a quintessence, dust or even stiff matter.

3 Conclusion

We have obtained general uncharged\charged Kiselev-like black hole solutions surrounded by perfect fluid in the context of Rastall theory. Then, we have investigated in more detail the specific cases of the black holes surrounded by dust, radiation, quintessence, cosmological constant and phantom fields. In each case, the weak energy condition, representing a positive energy density, is applied to put constraint on the physical parameters of this modified theory. By comparing the new term in the metric, resulted from the Rastall theory, with the Kiselev solution in GR, an effective behaviour for the black hole surrounding field is realized. It is shown that the effective fluid has different characteristics through its effective equation of state parameter ω_{eff} depending on the $\kappa\lambda$ values. In the case of black hole in a dust background with $\omega_d = 0$, for $\frac{1}{6} < \kappa\lambda < \frac{1}{3}$, we have $\omega_{eff} \leq -\frac{1}{3}$ violating the strong energy (SEC) condition, while for $\kappa\lambda < \frac{1}{6} \cup \kappa\lambda > \frac{1}{3}$, we have $\omega_{eff} \geq -\frac{1}{3}$ respecting to strong energy condition. For a black hole in a quintessence background with $\omega_q = -2/3$, we have $\omega_{eff} \leq -\frac{1}{3}$ for $-\frac{1}{2} \leq \kappa\lambda < 1$ violating the strong energy condition, while $\omega_{eff} \geq -\frac{1}{3}$ for $\kappa\lambda < \frac{3}{2}$, we have $\omega_{eff} \leq -\frac{1}{3}$, while for $\kappa\lambda < -1 \cup \kappa\lambda \geq \frac{3}{2}$, we have $\omega_{eff} \geq -\frac{1}{3}$. For such an effective surrounding fluid violating/respecting the strong energy condition, the Raychaudhuri equation can account for the accelerating/decelerating expansion of the universe, respectively. For each of these special classes, some interesting $\kappa\lambda$ values and their corresponding ω_{eff} as well as the defined Rastall geometric parameters \mathcal{W}_p and \mathcal{N}_p are given in the tables 1 to 6. For example, for the black hole in dust background, for $\kappa\lambda = \frac{2}{10}$ and $\frac{2}{9}$, the effective equation of state ω_{eff} lies in the quintessence

range while for $\kappa\lambda = \frac{3}{10}$, it lies in the strong phantom regime possessing repulsive gravitational effect. For $\kappa\lambda = \frac{1}{2}$, we have $\omega_{eff} = 1$ which belongs to the stiff matter with stronger gravitational attraction than the background dust. In the case of a black hole in a quintessence background with $\omega_q = -\frac{2}{3}$, the case of $\kappa\lambda = -\frac{1}{2}$ leads to $\omega_{eff} = -\frac{1}{3}$ representing an effective surrounding quintessence field weaker than the background. For, $\kappa\lambda = \frac{4}{10}$ and $\kappa\lambda = \frac{1}{2}$, it is seen that the effective surrounding field possesses a repulsive character stronger than the quintessence with $\omega_{eff} = -\frac{4}{3}$ and $\omega_{eff} = -\frac{5}{3}$, respectively, which lie in the phantom regime. Also, for $\kappa\lambda = -2$, we have $\omega_{eff} = 0$ representing an effective dust field while $\kappa\lambda = \frac{5}{2}$ leads to the equation of state parameter $\omega_{eff} = 1$ denoting a stiff matter. In latter cases, it is seen that although the surrounding field is an essentially quintessence but the effective field is not the quintessence like filed, possessing a negative equation of state parameter, rather it can behave effectively as dust or even stiff matter possessing a zero or positive equation of state parameters, respectively. Finally, for a black hole in a phantom background with $\omega_p = -4/3$, for $\kappa \lambda = -\frac{1}{2}$, we have $\omega_{eff} = -3$ which has a repulsive character stronger than the background phantom field, while for $\kappa \lambda = \frac{1}{2}$ and $\kappa \lambda = 1$, we have effective fields with repulsive character weaker than the background phantom field, still lying in the quintessence range. For $\kappa\lambda = 4$, we have $\omega_{eff} = 0$ representing an effective dust field. Then, it is seen that for the latter cases, although the surrounding field is an essentially phantom field but the effective surrounding field is not the phantom field, rather it can be effectively a quintessence, dust or even stiff matter. It is predicted that the new terms appearing in the Kiselev-like black holes may cause for some drastic changes in their horizons, causal structures and thermodynamical aspects, in comparison to the Kiselev black holes in GR. Such study is under work by the authors and will be reported, elsewhere.

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