University of California, Santa Barbara

CMPSC 138 SUMMER 2018

Homework VI: Due Wednesday, September 12, during the discussion session.

1. Do Problem 1, Section 7.1 of the text.
2. Do Problem 6 (c), (h), (k), Section 7.1 of the text.
3. Do Problem 7, Section 7.2 of the text.
4. Do Problem 3, Section 7.3 of the text.
5. Do Problem 18, Section 8.2 of the text (Hint: Theorem 8.5).
6. Suppose G is a CFG and $w \in \mathcal{L}(G)$ with $w \neq \lambda$. How long is a derivation of w in G in terms of $|w|$ if
   (a) G is in Chomsky normal form,
   (b) G is in Greibach normal form.

   Prove your answers.
7. Consider the grammar

$$\begin{aligned} S &\rightarrow bX \\ X &\rightarrow bXYZ \mid aY \mid b \\ Y &\rightarrow a \\ Z &\rightarrow c \end{aligned}$$

   (a) Give (formally) the transition rules of a PDA M with three states q_0, q_1, q_f that accepts the language generated by this grammar.
   (b) Give the sequence of moves M makes in processing the word $w = b^3ac$, using the " $\vdash$ " notation.
   (c) Construct a derivation for w that corresponds to the above sequence of moves M makes.
   (d) Construct the derivation tree corresponding to the derivation in part (c) above.
8. Suppose in a PDA M, $\delta(q_1, a, b) = \{(q_1, \alpha), (q_2, \lambda), (q_3, \beta)\}$. Construct the set of all instantaneous descriptions (IDs) $\mathcal{I}$ such that $(q_1, aw, bx) \vdash \mathcal{I}$.
9. (a) Is $\mathcal{L}_1 = \{xcy \mid x, y \in \{a,b\}^*, \ \ |x| = |y| \ \}$ a deterministic CFL? Give reasons.
   (b) Is $\mathcal{L}_2 = \{xy \mid x, y \in \{a,b\}^*, \ \ |x| = |y| \ \}$ a deterministic CFL? Give reasons.
   (c) Is $\mathcal{L}_3 = \{xx^R \mid x \in \{a,b\}^* \ \}$ a deterministic CFL? Give reasons.
10. Consider the PDA $M = (\{q_0, q_1, q_2\}, \{a,b\}, \{a,b,z\}, \delta, q_0, z, \{q_2\})$ where δ is given by

$$\begin{array}{ll} \delta(q_0, a, a) = \{(q_0, aa)\} & \delta(q_0, \lambda, a) = \{(q_1, a)\} \\ \delta(q_0, b, a) = \{(q_0, ba)\} & \delta(q_0, \lambda, b) = \{(q_1, b)\} \\ \delta(q_0, a, b) = \{(q_0, ab)\} & \delta(q_1, a, a) = \{(q_1, \lambda)\} \\ \delta(q_0, b, b) = \{(q_0, bb)\} & \delta(q_1, b, b) = \{(q_1, \lambda)\} \\ \delta(q_0, a, z) = \{(q_0, az)\} & \delta(q_1, \lambda, z) = \{(q_2, z)\} \\ \delta(q_0, b, z) = \{(q_0, bz)\} & \end{array}$$

   (a) Starting with

$$(q_0, abba, z) \vdash (q_0, bba, az) \vdash (q_0, ba, baz) \vdash \cdots$$

   give the remaining sequence of moves the machine makes to accept $abba$.
   (b) Is M deterministic? Why?