University of California, Santa Barbara

CMPSC 138 SUMMER 2018

Homework V: Due Wednesday, September 5, during the discussion session.

1. Consider the CFG $G = (\{S, A\}, \{a, b\}, P, S)$ where P consists of five S-productions and four A-productions as given below

$$\begin{aligned} S &\to aaS \mid bbS \mid abA \mid baA \mid \lambda \\ A &\to aaA \mid bbA \mid abS \mid baS \ . \end{aligned}$$

   What is $\mathcal{L}(G)$? Prove your answer.

2. Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are languages generated by the two grammars whose productions are

$$\begin{array}{ll} S \to XY \mid YY \mid a & \qquad S \to SX \mid b \\ X \to SY \mid b \mid c & \qquad X \to YS \mid XS \mid c \\ Y \to XY \mid SX \mid a & \qquad Y \to SY \mid a \end{array}$$

   respectively. Construct a grammar G which generates the language $\mathcal{L}_1\mathcal{L}_2 \cup \mathcal{L}_1^+$.

3. Consider the language $\mathcal{L}$ over the alphabet $\{a, b, c\}$ consisting of all strings of the form $a^i b^j c^k$, where $i, j, k > 0$ and either $i = j$ or $j = k$. Is $\mathcal{L}$ regular? Is it context-free? Justify your answers.

4. For *each* word below, determine whether or not it is generated by *each* CFG given on the right (so that there are 16 parts to the problem), and draw a derivation tree for it if it is:

$$\begin{array}{ll} aabb & \qquad CFG\ 1.\ \ S \to aSb \mid ab \\ abaa & \qquad CFG\ 2.\ \ S \to aS \mid bS \mid a \\ abba & \qquad CFG\ 3.\ \ S \to aS \mid aSb \mid X\ ;\ \ X \to aXa \mid a \\ aaaa & \qquad CFG\ 4.\ \ S \to aAS \mid a\ ;\ \ A \to SbA \mid SSba \end{array}$$

5. Construct a reduced grammar (i.e. a grammar without useless symbols) equivalent to the CFG $G = (\{S, A, B, C\}, \{a, b, c\}, P, S)$, where P is the set of productions

$$\begin{aligned} S &\to aA \mid bC \\ A &\to aSA \mid bAC \\ B &\to ABc \mid bSC \mid b \\ C &\to aAC \mid bc \end{aligned}$$

6. Consider the CFG G

$$\begin{aligned} S &\to aSA \mid \lambda \\ A &\to aA \mid a \end{aligned}$$

   (a) Show that G is ambiguous.
   (b) Find an unambiguous CFG equivalent to G.
   (c) Find an unambiguous CFG that generates $\mathcal{L}(G) \setminus \lambda$.

7. Consider the CFG G whose productions are as below:

$$\begin{aligned} S &\to AbB \mid B \\ A &\to CD \mid a \\ B &\to S \mid b \\ C &\to BbS \mid \lambda \\ D &\to \lambda \end{aligned}$$

(a) Eliminate λ−productions from G and write down the resulting grammar.

(b) Eliminate the unit productions from the grammar you have obtained in part (a) and write down the resulting grammar.

(c) Eliminate useless symbols from the grammar you have obtained in part (b) and write down the resulting grammar.

8. Construct a grammar in Chomsky Normal Form that generates $\mathcal{L}(M)$ where M is the NFA $M = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_4\})$ and δ is given by

δ	a	b	λ
q_1	$\{q_2\}$	$\{q_1\}$	ϕ
q_2	ϕ	$\{q_2\}$	$\{q_3\}$
q_3	$\{q_1\}$	ϕ	$\{q_4\}$
q_4	$\{q_2\}$	$\{q_4\}$	ϕ

9. Construct a Chomsky Normal Form grammar for $\{a^m ww^R b^m \mid w \in \{a, b\}^*, \;\; m \geq 1\}$.

10. (This topic will be discussed in lecture on Tuesday, September 4) Consider the grammar G whose productions are

$$\begin{aligned} S &\rightarrow XZ \mid ZY \\ X &\rightarrow ZX \mid a \\ Y &\rightarrow XZ \mid a \\ Z &\rightarrow YY \mid b \end{aligned}$$

(a) Use the CYK algorithm to show that $bababa \in \mathcal{L}(G)$.

(b) Use part (a) to construct a derivation tree for $bababa$.