University of California, Santa Barbara

CMPSC 138 SUMMER 2018

Homework II: Due Wednesday, August 15, during the discussion session.

1. Suppose Σ is an alphabet with k letters and $n > 0$ is an integer. How many words are there in the language $L = \{w \in \Sigma^* \mid |w| = n\}$?

2. Define the languages

$$L_1 = \{u \in \Sigma^* \mid u^R = u\}, \quad L_2 = \{uv \in \Sigma^* \mid u = v^R\}.$$

   (a) Prove that $L_2 \subseteq L_1$. (Hint: Section 1.2, Problems 3 and 4).

   (b) Prove that L_2 is a proper subset of L_1.

3. Consider the definitions of the *star-closure* L^* and the *positive closure* L^+ of a language L given on p. 20. When do we have

   (a) $\lambda \in L^*$?

   (b) $\lambda \in L^+$?

   (c) $L^* = L^+$?

4. Prove that $(\mathcal{L}_1^* \mathcal{L}_2^*)^* = (\mathcal{L}_1 \cup \mathcal{L}_2)^*$.

5. Determine the following languages:

   (a) $(ab)^*/b$,

   (b) $(ab)^*/a*$,

   (c) $(ab)^*/a^+$.

6. Prove or disprove the following claims:

   (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2.

   (b) $(L^R)^* = (L^*)^R$ for all languages L.

7. Find grammars for $\Sigma = \{a, b\}$ that generate the sets of

   (a) all strings with exactly one a.

   (b) all strings with at least one a.

   (c) all strings with no more tha three a's.

   (d) all strings with at least three a's.

8. Let $\Sigma = \{a, b\}$. Prove that if $w \in a\,\Sigma^*\,b$ then the number of occurrences of ab in w plus the number of occurrences of ba in w is always an odd number.

9. Let Σ be an alphabet and $w \in \Sigma^*$. Show that $x = w^2$ is the *unique solution* to the equation

$$x^2 = w\,x\,w$$

   over Σ^* (Note that this is an equation in which the unknown x is a word over Σ).

10. Design a transducer (serial adder) which takes as input two ternary (base 3) n-digit numbers and and calculates the n digits of their sum as in Example 1.17, pp. 33-34 of the text. You can assume a similar left-to-right representation of the digits.