University of California, Santa Barbara

CMPSC 138 SUMMER 2018

Homework IV: Due Wednesday, August 29, during the discussion session.

1. Do Problem 19, Section 4.3 of the text.
2. Do Problem 20, Section 4.3 of the text.
3. Decide whether each statement below is true or false. If it is true, prove it. If not, give a counterexample. The alphabet is $\{a, b\}$.
   (a) If $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and $\mathcal{L}_1$ is not regular, then $\mathcal{L}_2$ is not regular.
   (b) If $\mathcal{L}_1 \subseteq \mathcal{L}_2$ and $\mathcal{L}_2$ is not regular, then $\mathcal{L}_1$ is not regular.
   (c) If $\mathcal{L}_1$ and $\mathcal{L}_2$ are nonregular, then $\mathcal{L}_1 \cup \mathcal{L}_2$ is nonregular.
   (d) If $\mathcal{L}_1$ and $\mathcal{L}_2$ are nonregular, then $\mathcal{L}_1 \cap \mathcal{L}_2$ is nonregular.
   (e) If $\mathcal{L}_1$ is regular and $\mathcal{L}_2$ is nonregular, then $\mathcal{L}_1 \cap \mathcal{L}_2$ is nonregular.
   (f) If $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \ldots$ are all regular, then $\cup_{n=1}^{\infty} \mathcal{L}_n$ is regular.
   (g) If $\mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_1\mathcal{L}_2$ is regular.
   (h) If $\mathcal{L}_1 \cup \mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_2$ is regular.
   (i) If $\mathcal{L}_1\mathcal{L}_2$ is regular and $\mathcal{L}_1$ is finite, then $\mathcal{L}_2$ is regular.
4. Is the language $\{a^i b^j \mid i + j \text{ is even }\}$ regular? If so, construct the transition diagram of a DFA accepting it. If not, use the pumping lemma for regular languages to show that it is not regular.
5. Suppose all the productions of a CFG G are of the form
   $$A \to a\,B \ , \quad A \to B\,a \ , \quad A \to a \ .$$
   Does it follow that $\mathcal{L}(G)$ is regular? Prove or give a counterexample.
6. Let $\mathcal{L}$ be the language denoted by the regular expression a^*b.
   (a) Find a right-linear grammar for $\mathcal{L}$.
   (b) Find a left-linear grammar for $\mathcal{L}$.
7. Let $\mathcal{L} = \{a^m b \mid m \geq 0\} \cup \{b^m a \mid m \geq 1\}$.
   (a) Find the 9-th word in $\mathcal{L}$ in lexicographic order
   (b) Find the 9-th word in $\mathcal{L}^R$ in canonical order.
   (c) Find a regular expression for $\mathcal{L}$.
   (d) Construct the minimum state DFA M_{min} for $\mathcal{L}$.
8. Describe a decision algorithm to answer the following question:
   Given a DFA M accepting $\mathcal{L}$, and a string w, is w a substring of an element of $\mathcal{L}$?
9. Use the pumping lemma to show that the language $\mathcal{L} = \{a^m b^k \mid m > k\} \cup \{a^m b^k \mid m \neq k - 1\}$ is nonregular.
10. Consider the grammar $G = (\{S, A\}, \{a, b\}, S, P)$ where P is the set of productions
    $$\begin{array}{lcl} S & \to & aA \\ A & \to & abS \mid b \end{array}$$
    (a) Construct a DFA that accepts $\mathcal{L} = \mathcal{L}(G)$.
    (b) Construct a regular expression denoting $\mathcal{L}$.
    (c) Construct a left-linear grammar for $\mathcal{L}$.