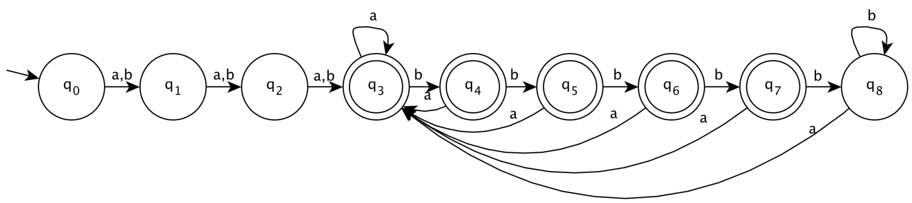
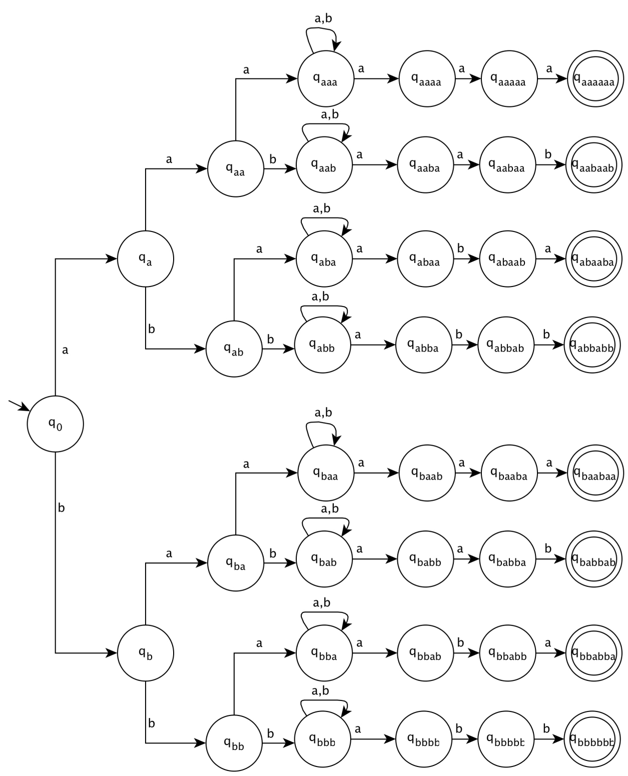
CS 138: Formal languages and Automota
Homework 3 solutions

1. .



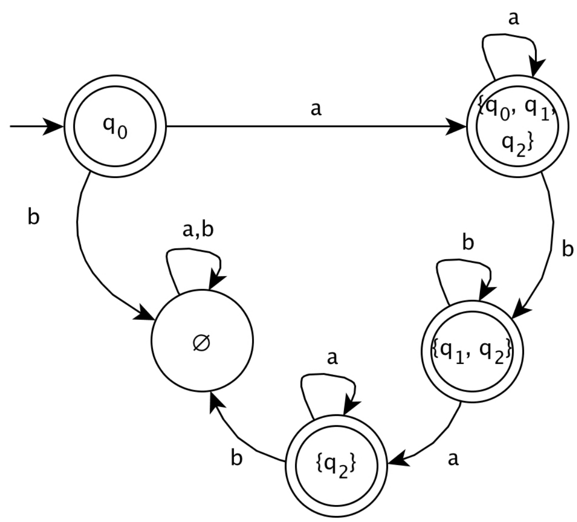
2. .



3. (a) $\delta^*(q_0, 1011) = \{q_2\}$

   (b) $\delta^*(q_1, 01) = \{q_1\}$

4. .



5. Since L is a regular language, we can construct a corresponding dfa, M, such that $L(M) = L$. By definition, L^R consists of all strings in language L in reverse order. We will construct an nfa, M_R, representing L^R such that $L(M_R) = L^R$. M_R will contain an additional start state with λ-transitions to the final states of M. The direction of every transition in M is reversed. Also, the start state of M will be the final state of M_R. The construction of the nfa M_R is as follows:

   Let $M = (Q, \Sigma, \delta, q_L, F)$

   $M_R = (Q \cup \{q_0\}, \Sigma_r, \delta_r, q_0, \{q_L\})$ and $q_0 \notin Q$

   $p \in \delta(q, a) \Longleftrightarrow q \in \delta_r(p, a)$ for $a \in \Sigma$

   Now we need to show that $w \in L$ iff $w^R \in L^R$

6. Let L be a regular language. This implies that there exists a DFA, $M = (Q, \Sigma, \delta, q_0, F)$ which accepts L. We construct a NFA M' which accepts chopLeft(L). M' has the same set of states, we only need to make the transitions from the initial state to other states as λ transition. Note that if the initial state q_0 has a loop back transition, we don't change that transition. Clearly M' accepts chopLeft(L). Hence chopLeft(L) is regular.

   $\forall \delta(q_0, a_i) = q_j$: if $q_j \neq q_i \Longrightarrow \delta(q_0, \lambda) = q_j$

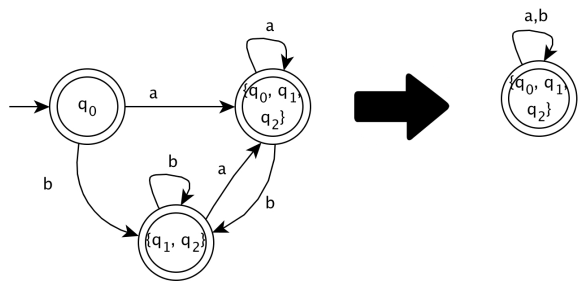
7. (a) $b^*ab^*ab^*$

   (b) $b^*(a + \lambda)b^*(a + \lambda)b^*(a + \lambda)b^*$

   (c) $(a+b+c)^*a(a+b+c)^*b(a+b+c)^*c(a+b+c)^* + (a+b+c)^*a(a+b+c)^*c(a+b+c)^*b(a+b+c)^* + (a+b+c)^*b(a+b+c)^*a(a+b+c)^*c(a+b+c)^* + (a+b+c)^*b(a+$

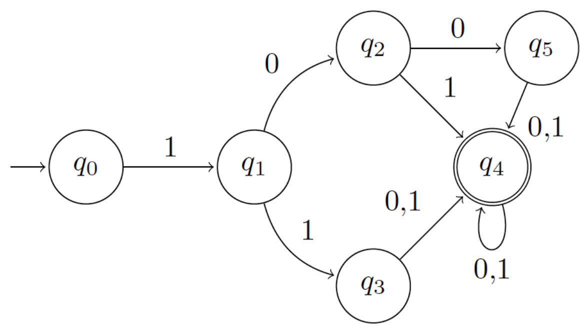
$b+c)^*c(a+b+c)^*a(a+b+c)^* + (a+b+c)^*c(a+b+c)^*a(a+b+c)^*b(a+b+c)^*$
$+ (a+b+c)^*c(a+b+c)^*b(a+b+c)^*a(a+b+c)^*$

8. .

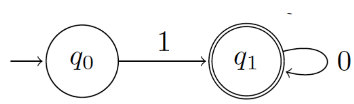


9. (a) $(1+0)^*(10)$
   (b) $\lambda + 0 + 1 + (0+1)^*(00+01+11)$
   (c) $1^*0(1+01^*0)^*$

10. a)



b)



c)