CS 138: Formal languages and Automota
Homework 5 solutions

1. The language Even-even consisting of all strings that contain both an even number of a's and an even number of b's.

   $L = \{w \in \{a, b\}^* \mid n(a) = 2k \text{ and } n(b) = 2k' \text{ and } k, k' \geq 0\}$

   To prove your answer you need to show that every word with even number of a and b is generated by the grammar and also show that every word generated by this grammar has even numbers of a and b. You can use induction.

2. $$\begin{aligned}
&S \to S_1S_2|S_3\\
&S_3 \to S_1S_3|S1\\
&S_1 \to X_1Y_1|Y_1Y_1|a\\
&X_1 \to S_1Y_1|b|c\\
&Y_1 \to X_1Y_1|S_1X_1|a\\
&S_2 \to S_2X_2|b\\
&X_2 \to Y_2S_2|X_2S_2|c\\
&Y_2 \to S_2Y_2|a
\end{aligned}$$

3. L is not regular, you can use pumping lemma to show that or consider the intersection of L with $L' = a^+b^+$. If L is regular then since L' is also regular, $L \cap L' = a^nb^n$ has to be regular which is not! Contradiction! so, L is not regular.

   L is context-free.

   $$\begin{aligned}
&S \to S_1|S_2\\
&S_1 \to aAbC|\lambda\\
&A \to aAb|\lambda\\
&C \to cC|\lambda\\
&S_2 \to DbBc|\lambda\\
&D \to aD|\lambda\\
&B \to bBc|\lambda
\end{aligned}$$

4. (a) CFG 1 generates only $aabb$: $S \Rightarrow aSb \Rightarrow aabb$

   (b) CFG 2 generates $abaa$, $abba$, and $aaaa$ as follow

   $S \Rightarrow aS \Rightarrow abS \Rightarrow abaS \Rightarrow abaa$

$$S \Rightarrow aS \Rightarrow abS \Rightarrow abbS \Rightarrow abba$$
$$S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaS \Rightarrow aaaa$$

(c) CFG 3 generates only $aaaa$: $S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaS \Rightarrow aaaX \Rightarrow aaaa$

(d) CFG 4 does not generate any of the words.

5.

$$\begin{aligned} S &\to bC \\ C &\to bc \end{aligned}$$

6. (a)

$$S \Rightarrow aSA \Rightarrow aA \Rightarrow aaA \Rightarrow aaaA \Rightarrow aaaa$$
$$S \Rightarrow aSA \Rightarrow aaSAA \Rightarrow aaAA \Rightarrow aaaA \Rightarrow aaaa$$

(b)

$$\begin{aligned} S &\to aA|\lambda \\ A &\to aA|a \end{aligned}$$

(c)

$$\begin{aligned} S &\to aA \\ A &\to aA|a \end{aligned}$$

7. (a)

$$\begin{aligned} S &\to AbB|B|bB \\ A &\to C|a \\ B &\to S|b \\ C &\to BbS \end{aligned}$$

(b)

$$\begin{aligned} S &\to AbB|b|bB \\ A &\to BbS|a \\ B &\to AbB|b|bB \\ C &\to BbS \end{aligned}$$

(c)

$$\begin{aligned} S &\to AbB|b|bB \\ A &\to BbS|a \\ B &\to AbB|b|bB \end{aligned}$$

8.

$$\begin{aligned} q_1 &\to aq_2|bq_1 \\ q_2 &\to bq_2|q_3|\lambda \\ q_3 &\to aq_1|q_4|\lambda \\ q_4 &\to aq_2|bq_4|\lambda \end{aligned}$$

- Eliminate λ-productions

$$\begin{aligned}
q_1 &\to aq_2|bq_1|a \\
q_2 &\to bq_2|q_3|b \\
q_3 &\to aq_1|q_4 \\
q_4 &\to aq_2|bq_4|a|b
\end{aligned}$$

- Eliminate the unit productions

$$\begin{aligned}
q_1 &\to aq_2|bq_1|a \\
q_2 &\to bq_2|aq_1|aq_2|bq_4|a|b \\
q_3 &\to aq_1|aq_2|bq_4|a|b \\
q_4 &\to aq_2|bq_4|a|b
\end{aligned}$$

- Eliminate useless symbols

$$\begin{aligned}
q_1 &\to aq_2|bq_1|a \\
q2 &\to bq_2|aq_1|aq_2|bq_4|a|b \\
q_4 &\to aq_2|bq_4|a|b
\end{aligned}$$

- Chomsky Normal Form

$$\begin{aligned}
q_1 &\to q_aq_2|q_bq_1|a \\
q_2 &\to q_bq_2|q_aq_1|q_aq_2|q_bq_4|a|b \\
q_4 &\to q_aq_2|q_bq_4|a|b \\
q_a &\to a \\
q_b &\to b
\end{aligned}$$

9.

$$\begin{aligned}
S &\to aSb|aCb|ab \\
C &\to aCa|bCb|aa|bb
\end{aligned}$$

- Chomsky Normal Form

$$\begin{aligned}
S &\to DB|EB|AB \\
C &\to EA|FB|AA|BB \\
A &\to a \\
B &\to b \\
D &\to AS \\
E &\to AC \\
F &\to BC
\end{aligned}$$

10. (a)

$$
\begin{aligned}
&V_{11} = V_{33} = V_{55} = \{Z\} \\
&V_{22} = V_{44} = V_{66} = \{X, Y\} \\
&V_{12} = V_{34} = V_{56} = \{S, X\} \\
&V_{23} = V_{45} = \{S, Y\} \\
&V_{13} = V_{35} = \{S, Y\} \\
&V_{24} = V_{46} = \{Z\} \\
&V_{14} = V_{25} = V_{36} = \{Z\} \\
&V_{15} = \{Z\} \\
&V_{26} = \{S, X, Y\} \\
&V_{16} = \{S, X, Y\}
\end{aligned}
$$

Since $S \in V_{16}$, $bababa \in L(G)$.

(b) $S \Rightarrow XZ \Rightarrow ZXZ \Rightarrow ZXYY \Rightarrow ZXXZY \Rightarrow ZXZXZY$ (and then perform only transitions to terminal symbols to get $bababa$)