CS 138: Formal languages and Automota
Homework 4 solutions

1. if L is regular, so $\bar{L}$. Since intersection of two regular languages is regular, therefore $L_1 = \bar{L} \cap L(w_1cw_2) = a^ncb^n$ has to be regular. however it is easy to show that L_1 is not regular using Pumping Lemma.

2. if L is regular, so L^R. We can write L as $\{w : w \in L_1, w \in L_2^R\} = L_1 \cap L_2^R$.

3. (a) It's false. We can give $L_1 = \{a^nb^n \mid n \geq 0\}$ and $L_2 = \Sigma^*$ as counterexamples. Here $L_1 \subseteq L_2$ and L_1 is not regular. But L_2 is regular.

   (b) It's false. We can give $L1 = \{ab\}$ and $L_1 = \{a^nb^n \mid n \geq 0\}$ as counterexamples. Here $L_1 \subseteq L_2$ and L_2 is not regular. But L_1 is regular.

   (c) It's false. We can give $L_1 = \{a^nb^n \mid n \geq 0\}$ and $L_2 = \overline{L_1} = \Sigma^* - L_1$ as counterexamples. Here L_1 and L_2 are not regular. But $L_1 \cup L_2 = \Sigma^*$ is regular.

   (d) It's false. We can give $L_1 = \{a^nb^n \mid n \geq 0\}$ and $L_2 = \{b^na^n \mid n \geq 0\}$ as counterexamples. Here L_1 and L_2 are not regular. But $L_1 \cap L_2 = \lambda$ is regular.

   (e) It's false. We can give $L_1 = \{ab\}$ and $L_2 = \{a^nb^n \mid n \geq 0\}$ as counterexamples. Here L_1 is regular and L_2 is not regular. But $L1 \cap L2 = \{ab\}$ is regular.

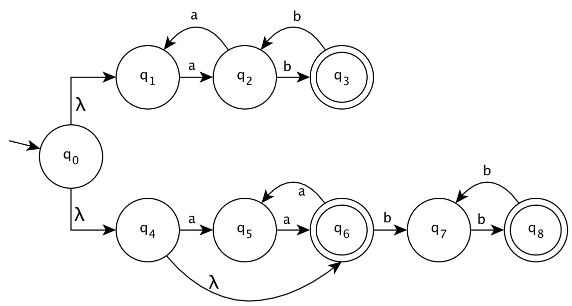
   (f) It's false. We can give $L_=\{ab\}$, $L_2 = \{a^2b^2\}$, ..., $L_n = \{a^nb^n\}$ as counterexamples. Here L_1, L_2, ..., L_n are all regular. But $\bigcup_{n=1}^{\infty} L_n = \{a^nb^n \mid n > 0\}$ is nonregular.

   (g) It's true. If L_1 is finite then L_1 is regular, so both L_1 and L_2 is a regular language and regular languages are closed under concatenation.

   (h) It's true. We can write $(L_1 \cup L_2) - L_1 = L_2 - L_1$ and $L_2 = (L_1 \cap L_2) \cup (L_2 - L_1)$. Both L_1 and $L_1 \cup L_2$ are regular, therefore $L_2 - L_1$ is regular because set difference is closed under regular languages. $L_1 \cap L_2$ is regular because since L_1 is finite, the intersection is finite, therefore regular. L_2 is union of two regular languages, therefore it is regular.

   (i) It's false. We can give $L_1 = \{\lambda, a\}$, $L_2 = \{a^n \mid n \text{ is a composite number}\}$, and $L_1L_2 = \{a^n \mid n \geq 4\}$ as counterexamples. In this case, L_1L_2 is regular, $L1$ is finite but L_2 is nonregular.

4. .



5. No, here is a grammer that follows the fiven rules, but generates the non-regular language $a^n b^n$.

   $S \to aA|\lambda$

   $A \to Sb$

6. (a) $S \to aS|b$

   (b) $S \to Ab$

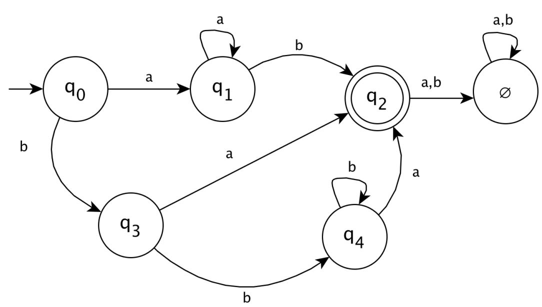
   $A \to Aa|\lambda$

7. (a) $a^n b$

   (b) ba^4

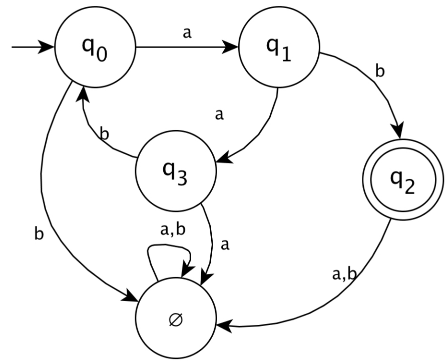
   (c) $a^*b + b^+a$

   (d) .



8. Take the DFA M, add λ transitions from initial state to all other states and λ transitions from all non-final states to the final states, the resulting NFA accepts all substrings of L including λ.

9. Choose $\omega = a^{n!}b^{(n+1)!}$, clearly because $|xy| \le n$, $y = a^t$, $0 < t \le n$, so $xy^kz = a^{n!+t(k-1)}b^{(n+1)!}$ which means if $k = \frac{n*n!-1}{t} + 1$ then $xy^kz = a^{(n+1)!-1}b^{(n+1)!}$



10. (a)

    (b) $a(aba)^*b$

    (c) $S \to Ab$

    $A \to Aaba|a$