# CS 138: Formal languages and Automota
# Homework 2 solutions

1. $k^n$

2. (a) For any $uv$ in $L_2$ we need to show that $(uv)^R = uv$:
   $(uv)^R = v^R u^R = v^R (v^R)^R = v^R v = uv$

   (b) Find a word $w$ such that $w$ is in $L_1$ but not in $L_2$ for example $w = aba$

3. (a) Always

   (b) if $\lambda \in L$

   (c) if $\lambda \in L$

4. Prove it in 2 steps: first, show that if $x \in (L_1^* L_2^*)^*$ then $x \in (L_1 \cup L_2)^*$. Second show that if $x \in (L_1 \cup L_2)^*$ then $x \in (L_1^* L_2^*)^*$. To prove each part you can use induction. For example for the first part, what we have to do is:

   Basis: show that $(L_1^* L_2^*)^0 = \lambda \in (L_1 \cup L_2)^*$

   Inductive hypothesis: assume that $(L_1^* L_2^*)^k \in (L_1 \cup L_2)^*$ for $k = 1, 2, ..., n$

   Inductive Step: show that $(L_1^* L_2^*)^{n+1} \in (L_1 \cup L_2)^*$

5. (a) $(ab)^* a$

   (b) $(ab)^*$

   (c) $\lambda$

6. (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$
   If $x \in (L_1 \cup L_2)^R \Leftrightarrow x^R \in ((L_1 \cup L_2)^R)^R \Leftrightarrow x^R \in L_1 \cup L_2 \Leftrightarrow x^R \in L_1$ or $x^R \in L_2 \Leftrightarrow x \in L_1^R \cup L_2^R$

   (b) proof by induction or similar to the previous step

7. (a) $S \to AaA$
   $A \to bA \mid \lambda$

   (b) $S \to AaA$
   $A \to aA \mid bA \mid \lambda$

(c) $S \to BABABAB$

$A \to a \mid \lambda$

$B \to bB \mid \lambda$

(d) $S \to BaBaBaB$

$B \to bB \mid aB \mid \lambda$

8. We use induction to prove this. Consider $x$ as the number of occurrences of $ab$ in the $w$.

   Induction basis: for the first word $ab$, $x = 1$ which is an odd number.

   Inductive assumption: for word $w$ of length $n$, $x$ is an odd number.

   Inductive step: to increase the length of $w$ to $n + 1$ we need to add a symbol. This symbol can be added in any of the following positions:

   between $a$ and $a$, between $a$ and $b$, between $b$ and $a$, or between $b$ and $b$.

   Since we have two symbols in our alphabet, we will have eight different combinations. It is easy to see that $x$ is odd in each case.

9. We first show that the length of $x$ is even. We use contradiction. Suppose that $|x|$ is an odd number, so:

   $x^2 = wxw \implies 2|x| = |w| + |x| + |w| \implies 2|x| = 2|w| + |x|$

   It is clear that $2|x|$ and $2|w|$ are both even so $|x|$ can not be odd.

   Next, we split $x$ in half such that $x = yz$ and $|y| = |z|$. So we will have $yzyz = wyzw$ Also we know that $|y| = |z|$ , and $|yz| = |wy| = |zw|$ so it is easy to show that $|y| = |z| = |w|$ and finally $y = z = w$, so: $x = yz = ww = w^2$

10. The answer is shown below: