Platforms & Tools

D2L (DEN): Syllabus
Lecture Notes
Lecture Videos
HW Assignments
HW Submissions
Any other reference material
Exams

Piazza: Discussion Board

Roles & Responsibilities

- Instructor
- TAs
- Graders
- Course Producers
- CS Dept. Advisors
- DEN Support

Textbooks

- Algorithms Design by Jon Kleinberg & Eva Tardos

- Supplemental Textbook:
  Introduction to Algorithms, 3rd edition, by Cormen et al.

Your Responsibilities

- Attending lectures and discussions
- Completing reading assignments
- Doing HW problems
- Doing as many other practice problems as possible

Grading Scale

90 - 100	A	60 - 64.99	C^+
85 - 89.99	A^-	55 - 59.99	C
75 - 84.99	B^+	50 - 54.99	C^-
70 - 74.99	B	45 - 49.99	D
65 - 69.99	B^-	Below 45	F

- Grade boundaries will be reduced by the difference between class average grade and 75 if class average falls below 75
- At least 20% of the class will receive an A grade.
- At least 10% of the class will receive an A^- grade

Prerequisites

- Discrete Math - Mathematical Induction
- Sorting methods
- Basic data structures: Arrays, stacks, queues, linked lists
- Basics of graphs: Trees, cycles, DAG, adjacency list/matrix, etc.
- Graph search algorithms: BFS, DFS

High level Syllabus

- Introduction
- Review of some prereq's + asymptotic notation
- Major algorithmic techniques
  - Greedy
  - Divide & Conquer
  - Dynamic Programming
- Network Flow
- Computational Complexity Theory
- Approximation Algorithms
- Linear Programming

Corrections

1- An algorithm is a set of instructions in machine language.

2- ... Algorithmic science advanced on Wall Street ...

3- ... Invite 6 million algorithms for a listen ...

When studying a problem, we go through the following steps:

1- Come up with a concise problem statement

2- Present a solution

3- Prove correctness

4- Perform complexity analysis

Stable Matching

Stable Matching Example

Problem: We are interested in matching $\underline{n}$ men with $\underline{n}$ women so that they could stay happily married ever after.

Step 1: Come up with a concise problem statement.

We have a set of $\underline{n}$ men, $M = \{m_1, \ldots m_n\}$

We have a set of $\underline{n}$ women, $W = \{w_1, \ldots w_n\}$

Def. A <u>Matching</u> S is a set of ordered pairs.

Def. A <u>perfect matching</u> S' is a matching with the property that each member of M and each member of W appear in exactly one pair in S'.

Add notion of preferences

Each man $m \in M$ ranks all women

- $\underline{m}$ prefers $\underline{w}$ to $\underline{w'}$ if $\underline{m}$ ranks $\underline{w}$ higher than w'.

- Ordered ranking of $\underline{m}$ is his preference list

$$P_{m_i} = \{ w_{i_1}, w_{i_2}, \ldots w_{i_n} \}$$

Same for women, i.e. each woman $w \in W$ ranks all men...

<u>Def.</u> Matching S is stable if

1- It is perfect

2- There are no instabilities WRT S

<u>Step 1:</u> Input: Preference lists for a set of $\underline{n}$ men & $\underline{n}$ women.

Output: Set of $\underline{n}$ marriages w/ no instabilities

<u>Step 2:</u> Gale-Shapley Alg.

Step 3 Proof of Correctness

① From the woman's perspective, she starts single, and once she gets engaged

② From the man's perspective, he starts single, gets engaged,

③

Proof by Contradiction

Assume an instability exists in our solution involving two pairs (m,w), (m',w')

Q: Did $\underline{m}$ propose to w' at some point in the execution?

If no, then $\underline{w}$ must be higher than $\underline{w'}$ on his list $\Rightarrow$ contradiction!

If yes, he must have been rejected in favor of m'' and due to (1) either $m'' = m'$ or m' is better than m''

$\Rightarrow$ contradiction!

Step 4 Complexity Analysis

1- Identify a free man

2- For a man $\underline{m}$, identify the highest ranked woman to whom he has not yet proposed.

3- For a woman $\underline{w}$, decide if $\underline{w}$ is engaged, and if so to whom

4- For a woman $\underline{w}$ and two men $\underline{m}$ & $\underline{m'}$, decide which man is preferred by $\underline{w}$

5- Place a man back in the list of free men.

1. Identify a free man

2. Identify the highest ranked woman to whom <u>m</u> has not yet proposed.

Men's preference list: ManPref[1..n, 1..n],
where
ManPref[m, i] denotes the ith
woman on man <u>m</u>'s preference list.

To find next woman <u>w</u> to whom <u>m</u>
will be proposing to:

$w =$

3_ Determine woman $\underline{w}$'s status

4_ Determine which man is preferred by $\underline{w}$.

Preparation before entering GS iterations

Create a Ranking array where Ranking [w,m] contains the rank of man m based on w's preference

w_{i_1}

w_{i_2}

m_i w_{i_3}

$\vdots$

w_{i_n}

Def. Woman $\underline{w}$ is a valid partner of a man $\underline{m}$ if there is a stable matching that contains the pair (m, w)

Def. m's best valid partner is

Claim: Every execution of the G-S algorithm (When men propose) results in the same stable matching regardless of the order in which men propose.

Plan: to prove this, we will show that when men propose, they always end up with their best valid partner.

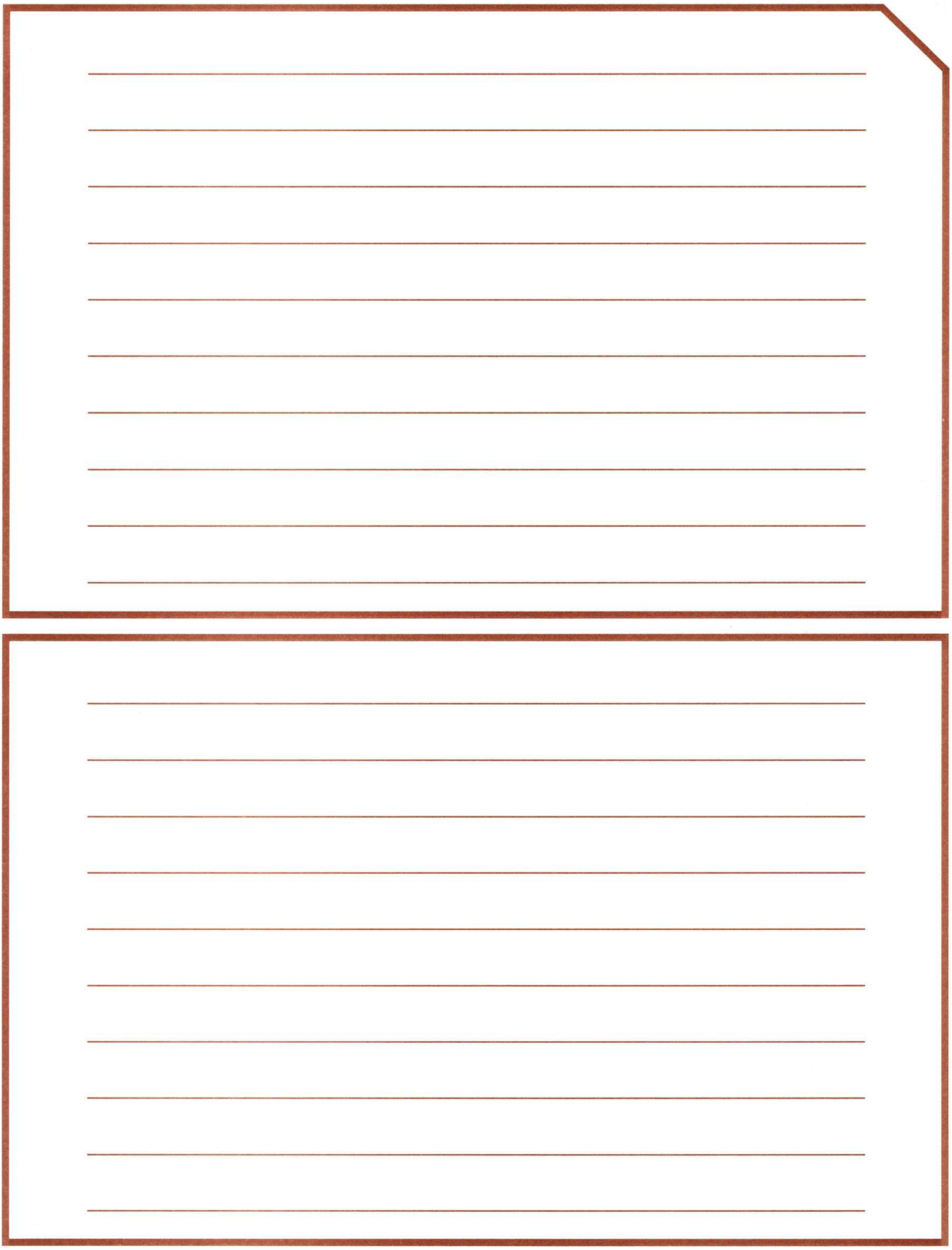
Proof by contradiction:

Say $\underline{m}$ is the first man rejected by a valid partner w.

Claim: When men propose, women end up with their worst valid partners

Proof: By contradiction . . . .

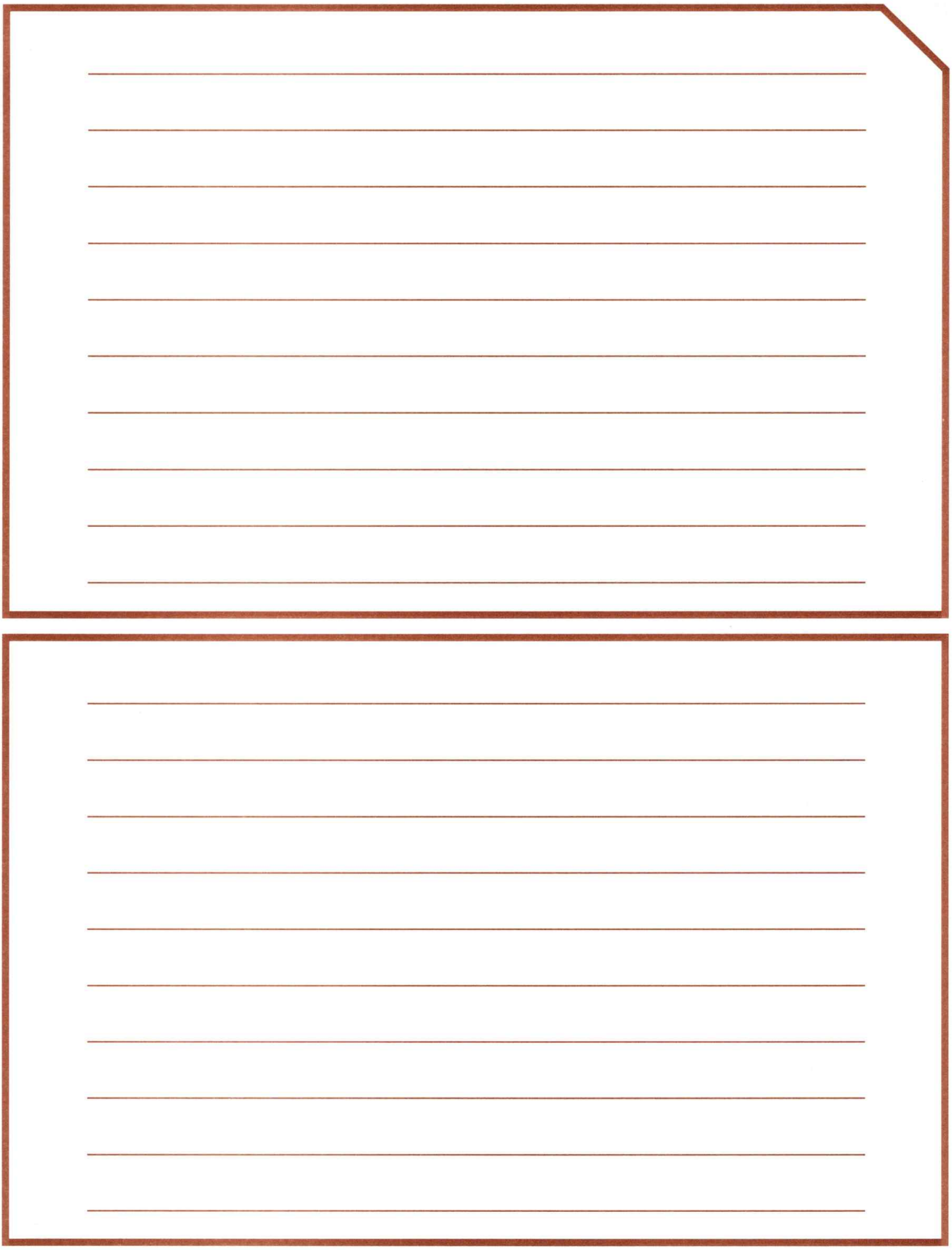
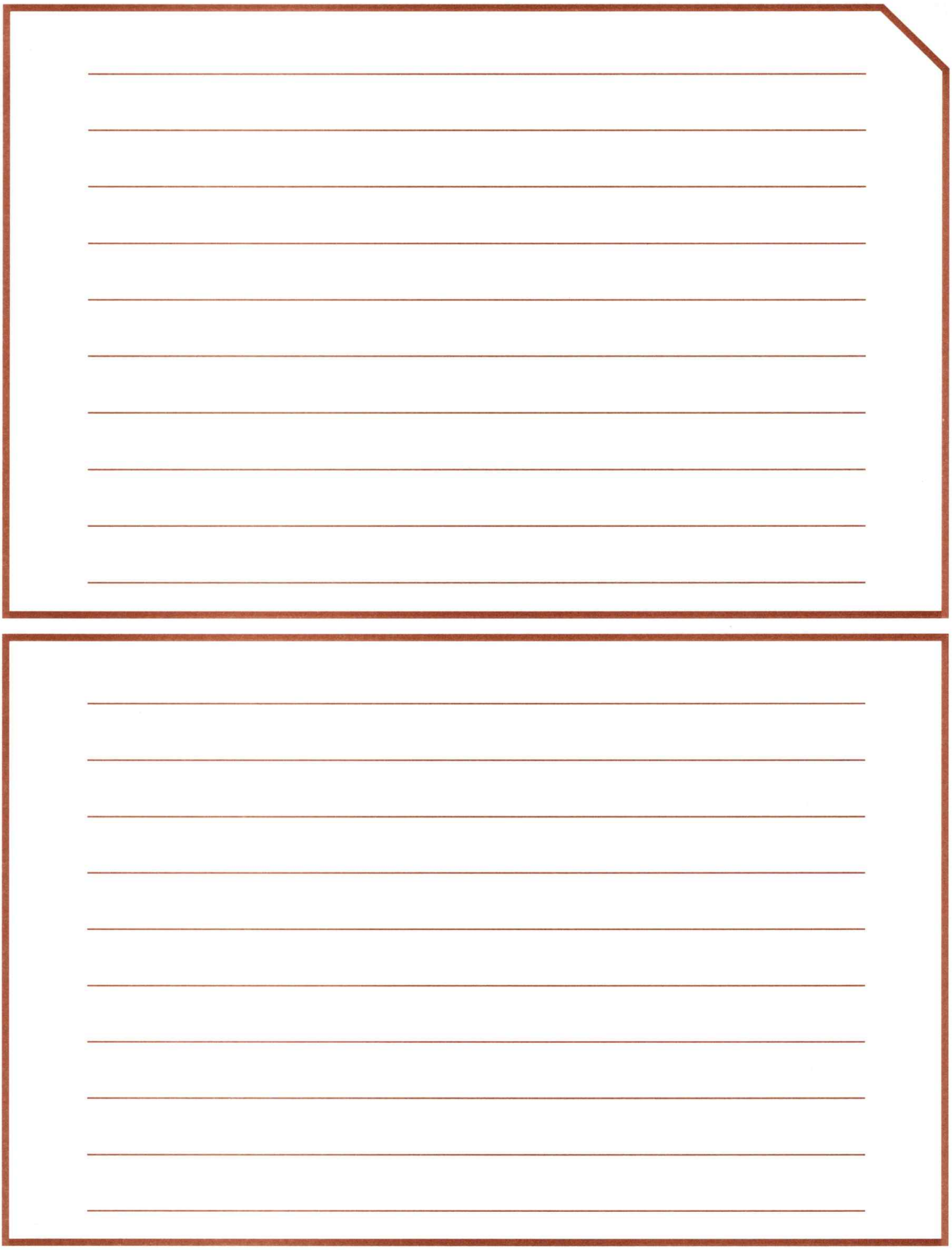
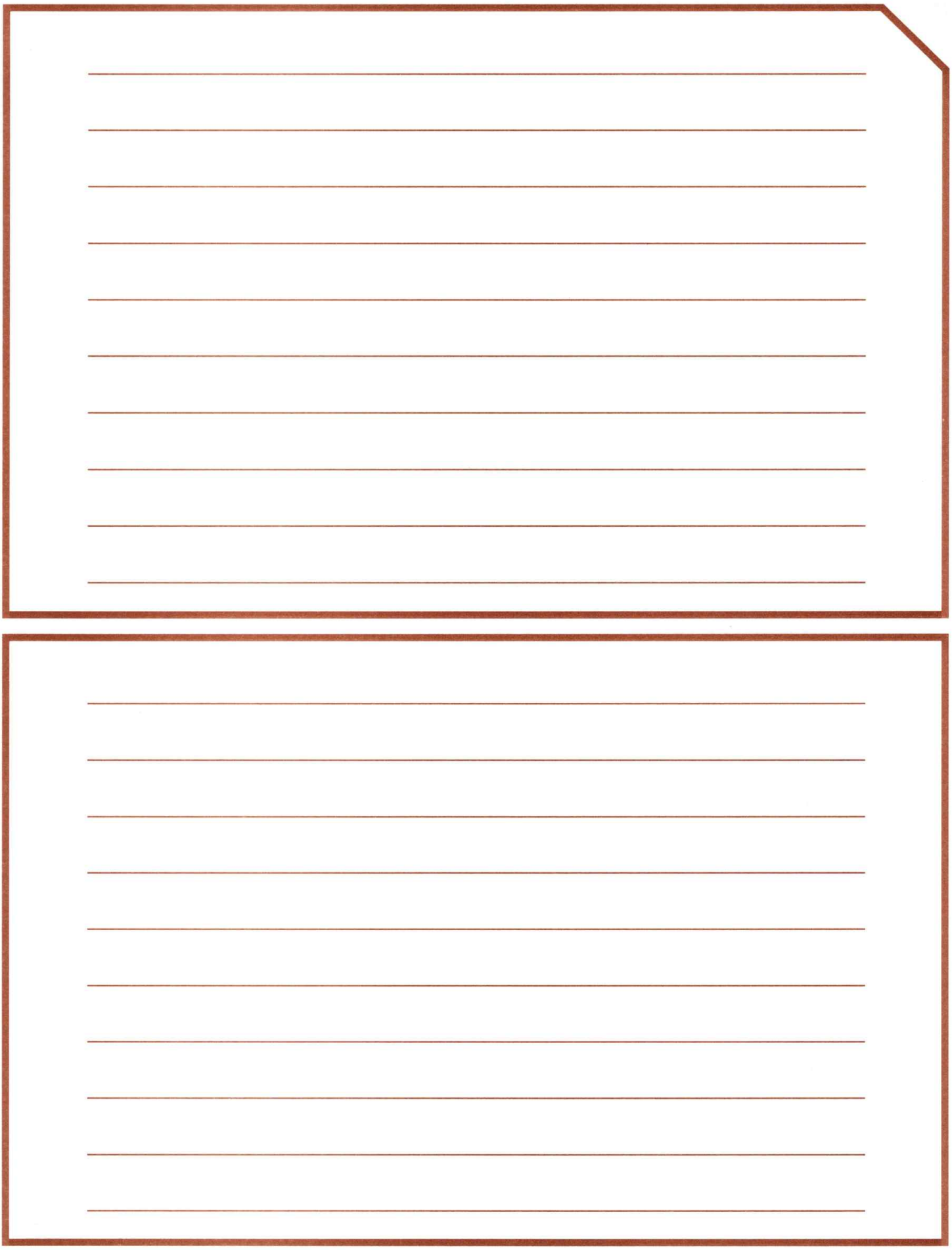
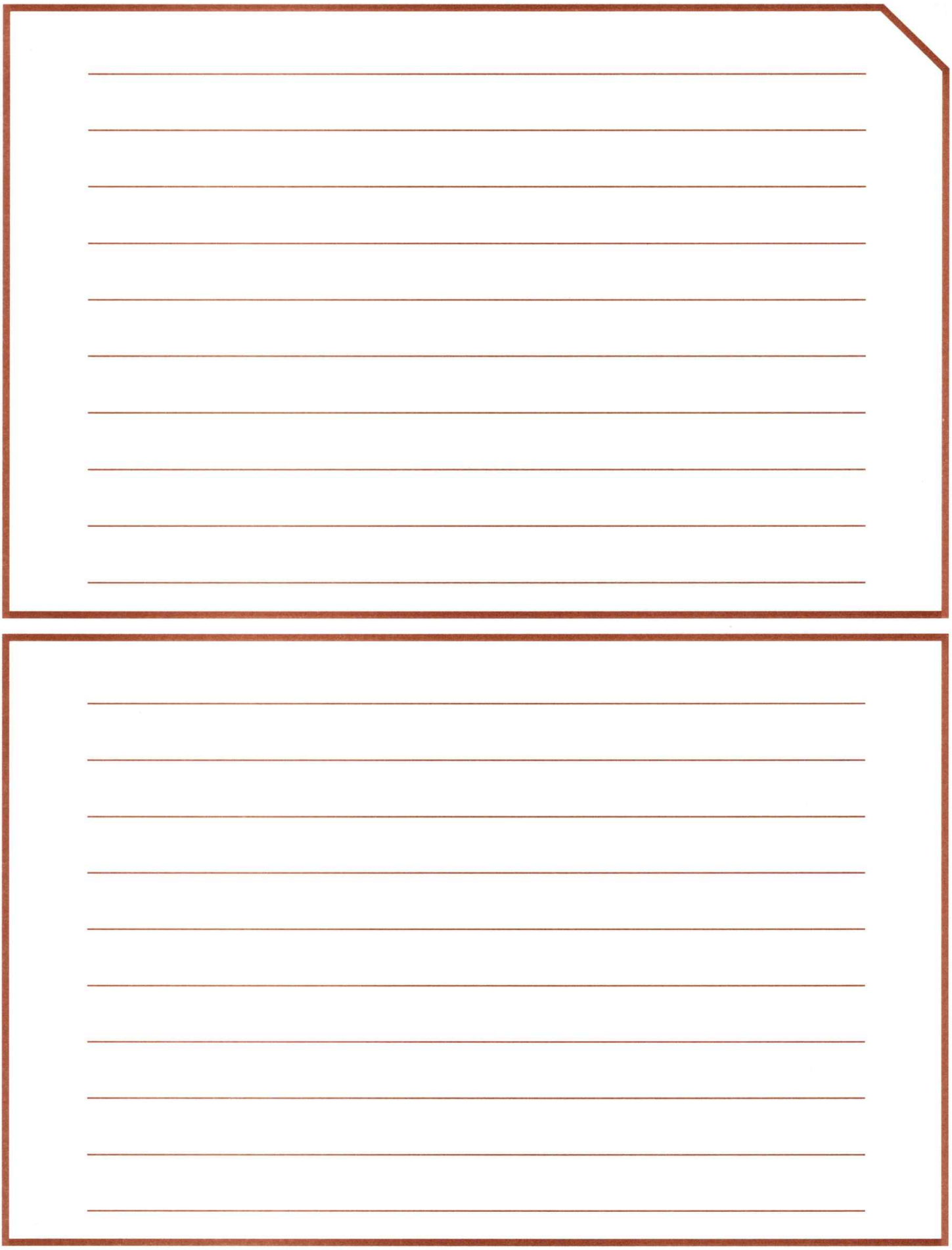
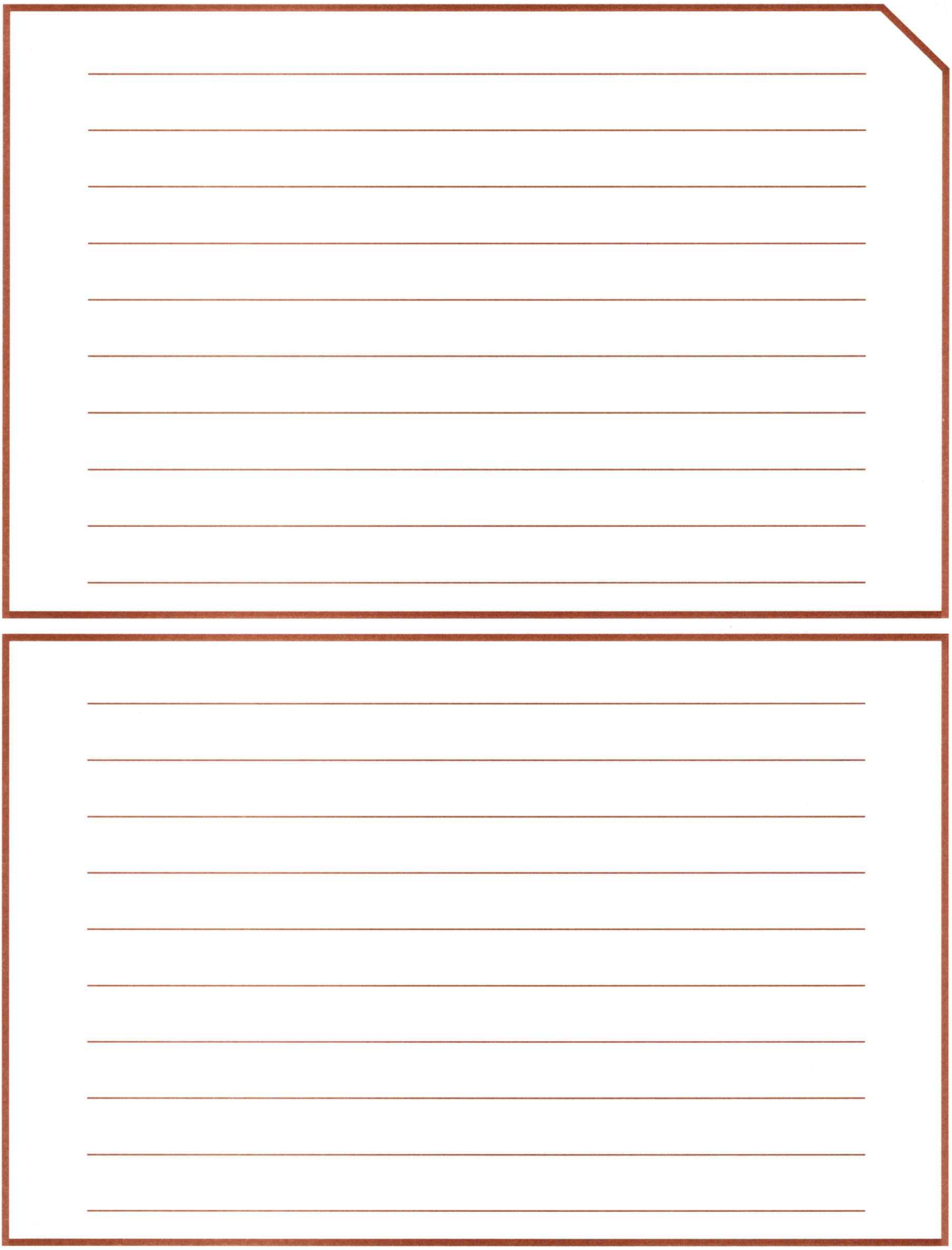
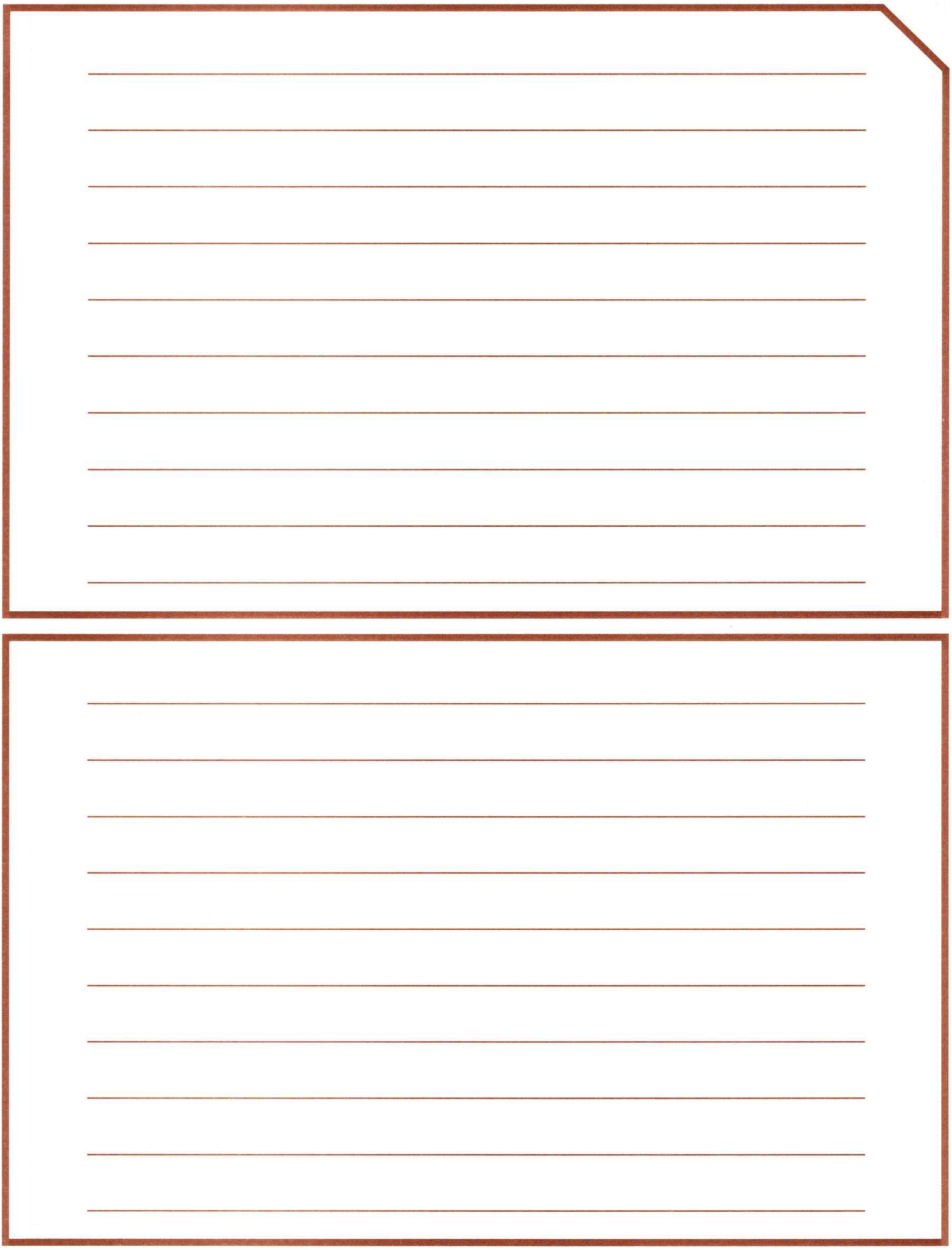
Suppose we end up with a matching S where for a pair (m,w) in S, m is <u>not</u> w's worst valid partner.
So there must be another matching S' where w is paired with a man m' whom she likes less.



Discussion 1

1. Prove that every execution of the G-S algorithm (when men are proposing) results in the same stable matching regardless of the order in which men propose.

2. Prove that when we run the G-S algorithm with men proposing, women end up with their worst valid partners.

3. True or False:
   In every stable matching that Gale–Shapley algorithm may end up with when men propose, there is a man who is matched to his highest-ranked woman.

4. In a connected bipartite graph, is the bipartition unique? Justify your answer.