Exercise 3

SC Circuit

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1 Objective

The main target of this laboratory exercise is to make certain that we understand the way that switching capacitors works,better understand them and know how to solve exercises about them.Also we will work on switching between Z and S domain and see differences between them.

2 Exercise 1: Low pass filter and its SC circuit analysis

Design a low pass passive first order filter with f c =1kHz and write a transfer function H (s).Same structure must be realized in discrete z-domain (resistor must be replaced with SC circuit) where transfer function H(z) must be plotted and compared with continuous time H (s) transfer function. Draw the SC schematic. Use freqs() and freqz functions.

For the design of the low pass filter we will use a RC circuit like this:

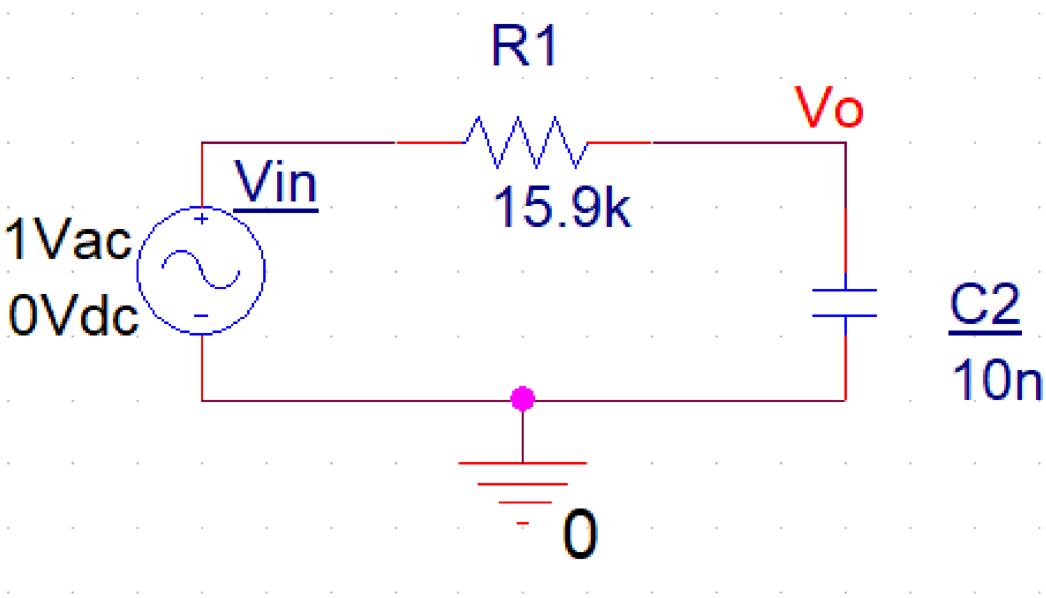

Figure 1: Schematic of the low pass filter

We need a fc of 1kHz,so we will calculate the components of such filter and confirm that we have a LPF. We know that a LPF transfer function looks like this:

$$H(s) = \frac{w_c}{s+w_c} = \frac{1}{1+s/w_c}$$

So now,we will get the transfer function of our schematic and obtain the value of the components.

$$Vo(s) = Vin(s) \cdot \frac{1/C_2 s}{1/C_2 s+R}$$

$$H(s) = \frac{Vo(s)}{Vin(s)} = \frac{1/C_2 s}{1/C_2 s+R} = \frac{1}{1+RC_2 s}$$

Comparing the obtained transfer function with the theoretical one and fixing a value to the capacitor(we will choose 10pF) we will get the value of the resistor

$$H(s) = \frac{w_c}{s+w_c} = \frac{1}{1+s/w_c} = \frac{1}{1+RC_2 s}$$

$$RC = \frac{1}{w_c} \rightarrow R = \frac{1}{2\cdot\pi\cdot f_c\cdot 10pF} = 15.9k\Omega$$

We will simulate the schematic doing and AC sweep from 10Hz to 500kHz in OrCAD to see the result:

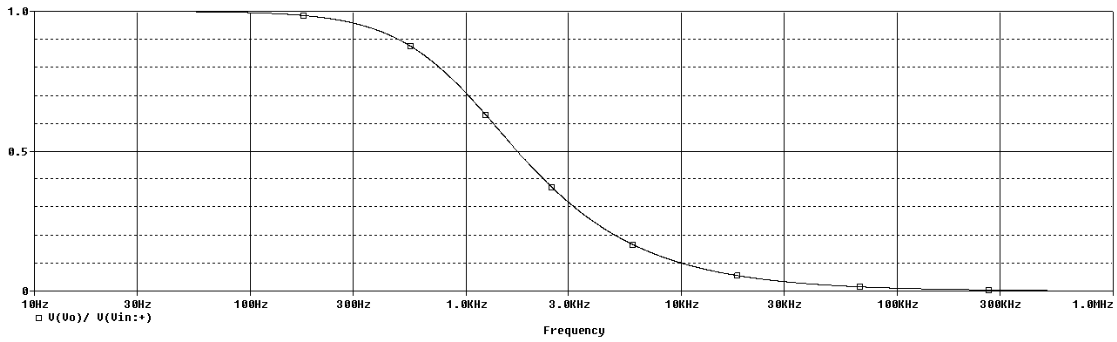

Figure 2: Bode diagram of the designed schematic

Now we will draw the corresponding SC schematic that will look like this:

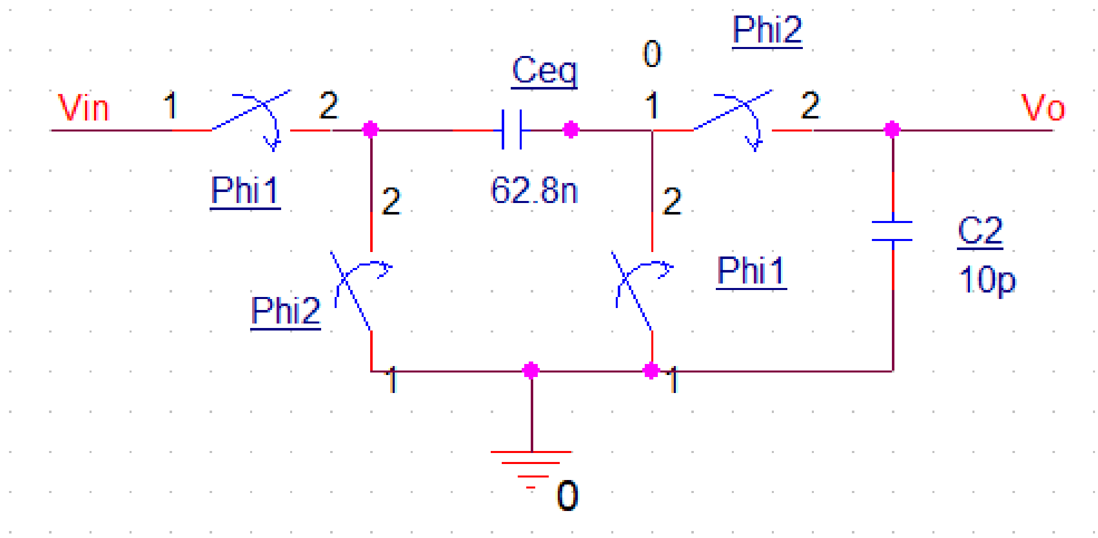

Figure 3: SC shematic

Where $Ceq = \frac{1}{R \cdot f_s}$.In order to study the SC schematic we will use the balance of charge theorem:

In (n-1) ϕ_1 will be closed and ϕ_2 open so we will have:

$$Vin(n-1) \cdot Ceq + Vo(n-1) \cdot C_2$$

In (n) ϕ_1 will be open and ϕ_2 closed so we will have:

$$Vo(n) \cdot (C_2 + Ceq)$$

Since the charge has to remain the same,we will equal the two equations and work in the Z domain to obtain the transfer function

$$Vin(n-1) \cdot Ceq + Vo(n-1) \cdot C_2 = Vo(n) \cdot (C_2 + Ceq)$$

$$Vin \cdot z^{-1} \cdot Ceq + Vo \cdot z^{-1} \cdot C_2 = Vo \cdot z \cdot (C_2 + Ceq)$$

$$Vo[-z^{-1} + (C_2 + Ceq)] = Vin \cdot z^{-1} \cdot Ceq$$

$$H(z) = \frac{Vo}{Vin} = \frac{z^{-1} \cdot Ceq}{-z^{-1} + (C_2 + Ceq)}$$

To conclude exercise 1 we will use MatLab in order to compare results and we will plot magnitude and phase diagrams of Bode both in Z and S domain using the functions $freqs$ and $freqz$.The code will be explained in the appendix via comments:

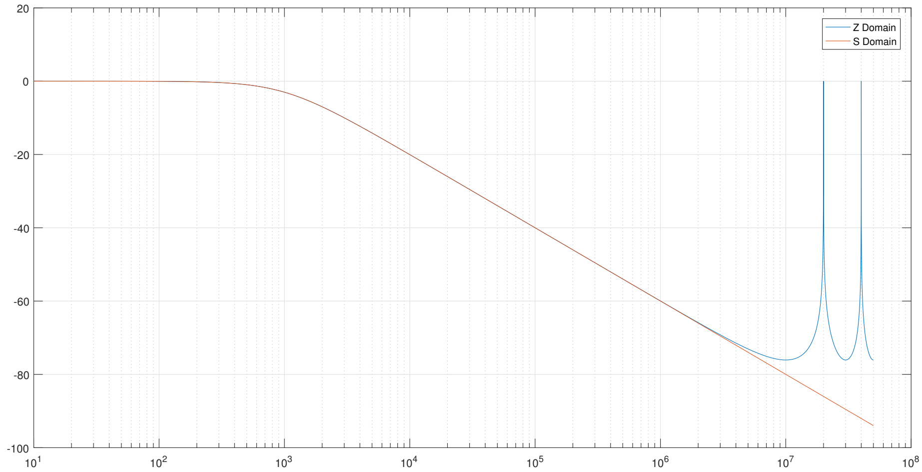

Figure 4: Magnitude diagram in S and Z Domain

Looking at the magnitude diagram we see how it decreases -20dB per decade,it is a first order filter, and we can also appreciate that the pole frequency is the expected one

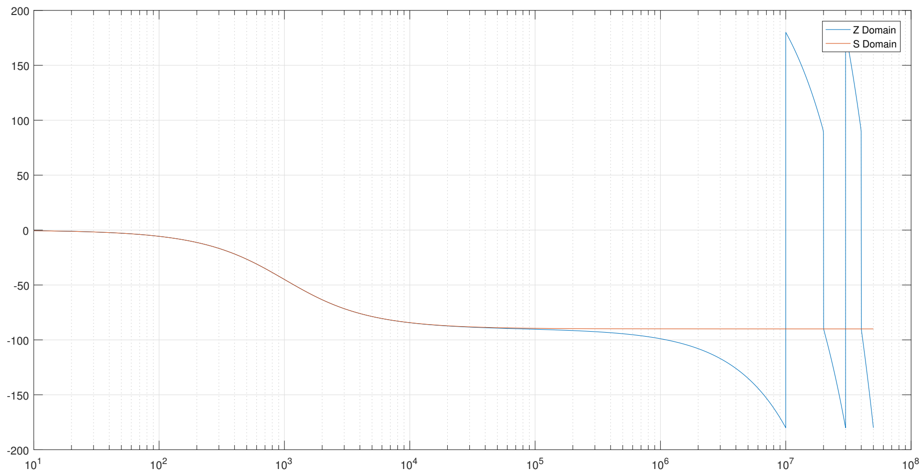

Figure 5: Phase diagram in S and Z Domain

The phase diagram is also the expected one,it decreases 90º ($\frac{\pi}{2}$ rad) and the pole frequency is 1kHz

The spikes that we are getting in both graphics are due to the digital filter ,but since we are using a high sampling frequency they are out of the filter and its not a problem.(If we would use lower Fs they may be in the middle of the filter)

3 Exercise 2: Analysis of inverting and non-inverting SC amplifier

Determine transfer function of the circuits in the Figure 6 and 7. Capacitors must be calculated from the switching frequency Fs=1 MHz and its value should not exceed 1 pF. The gain at low frequencies is 40 dB. The characteristics of the opamp and the switches are ideal.

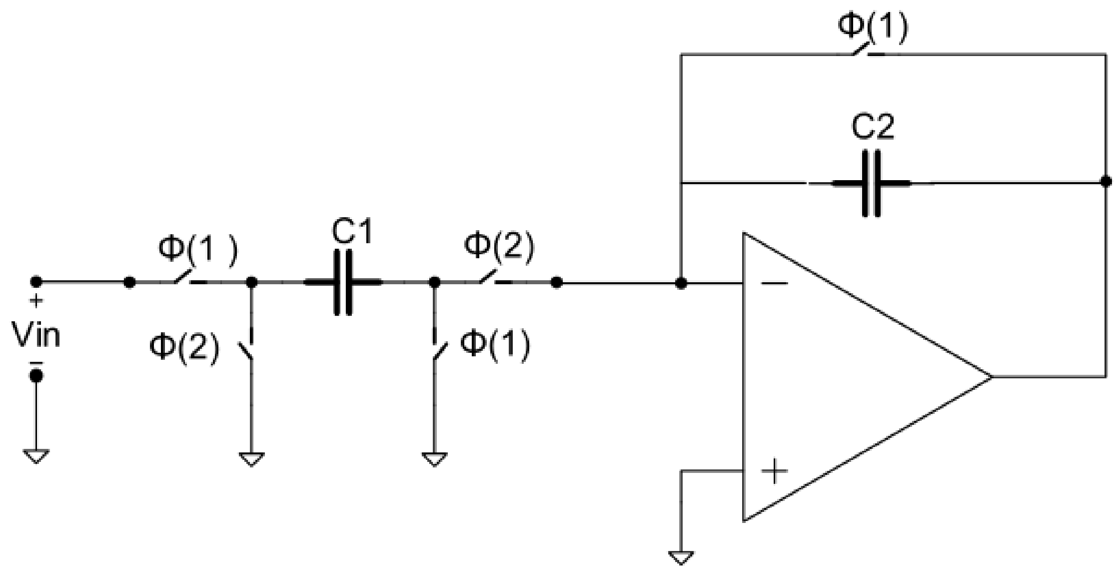

Figure 6: Non-inverting SC amplifier.

We will now obtain the transfer function of the first schematic:

In (n-1) ϕ_1 will be closed and ϕ_2 open so we will have:

$$Vin(n-1) \cdot C_1$$

In (n) ϕ_1 will be open and ϕ_2 closed so we will have:

$$Vo(n) \cdot C_2$$

We will transfer the equations to Z domain and equal them:

$$Vin(n-1) \cdot C_1 = Vo(n) \cdot C_2$$

$$Vin \cdot z^{-1} \cdot C_1 = Vo \cdot C_2$$

$$H(z) = \frac{Vo}{Vin} = \frac{C_1}{C_2} z^{-1}$$

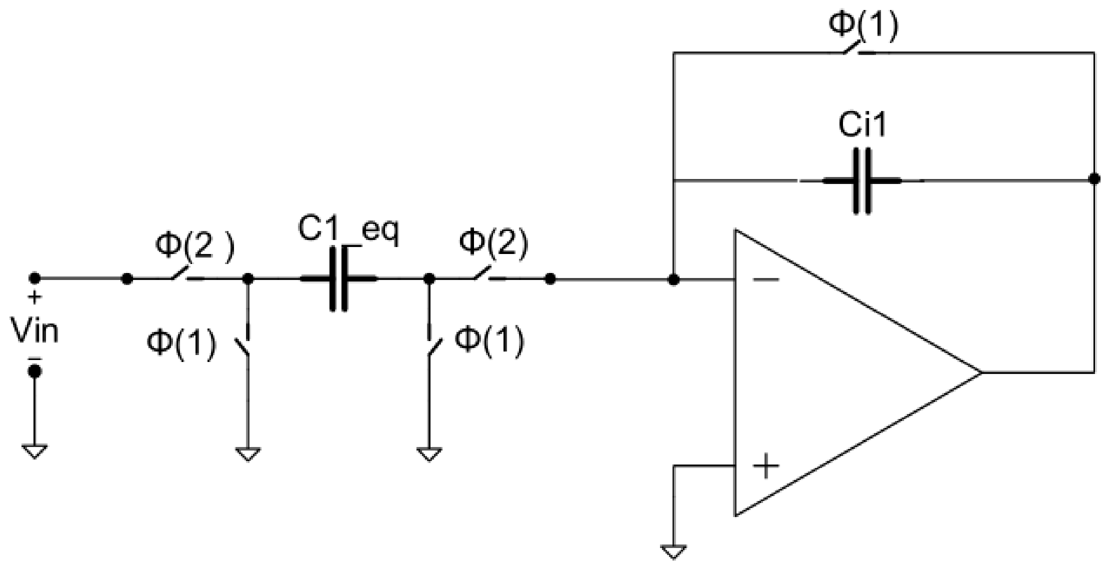

Figure 7: Inverting SC amplifier.

For the second schematic we will do pretty much the same:

In (n-1) ϕ_1 will be closed and ϕ_2 open so we wont have any equation

In (n) ϕ_1 will be open and ϕ_2 closed so we will have:

$$Vi(n) \cdot C1_{eq} + Vo(n) \cdot Ci_1$$

We will transfer the equations to Z domain and equal them:

$$0 = Vi(n) \cdot C1_{eq} + Vo(n) \cdot Ci_1$$

$$Vi \cdot C1_{eq} + Vo \cdot Ci_1 = 0$$

$$H(z) = \frac{Vo}{Vin} = -\frac{C1_{eq}}{Ci_1}$$

In both we have the same scenario,the gain is the division of both and since we want a gain of 40dB (100 in linear) we will need C1 and $C1_{eq}$ 100 times higher than C2 and Ci_1

.Since they need to be lower than 1pF We will take C1 and $C1_{eq}$ 1pF and C2 and Ci_1 will be 100 times lower, that means 10fF.

4 Exercise 3: Comparison of CT and SC integrators

Calculate transfer function of the integrator in Figure 3. Determine the value of R1, Rf and Cf in a way, to achieve low frequency gain of A DC =10 with corner frequency f c =1 kHz. Realize SC integrator and replace R1 and R f with circuit shown in the Figure 4. For proper inverting operation, it is essential to pay attention to the phases on each switch driven with 1 MHz. Prepare .m file to analyse and compare CT and SC transfer functions. Use freqs() and freqz() functions.

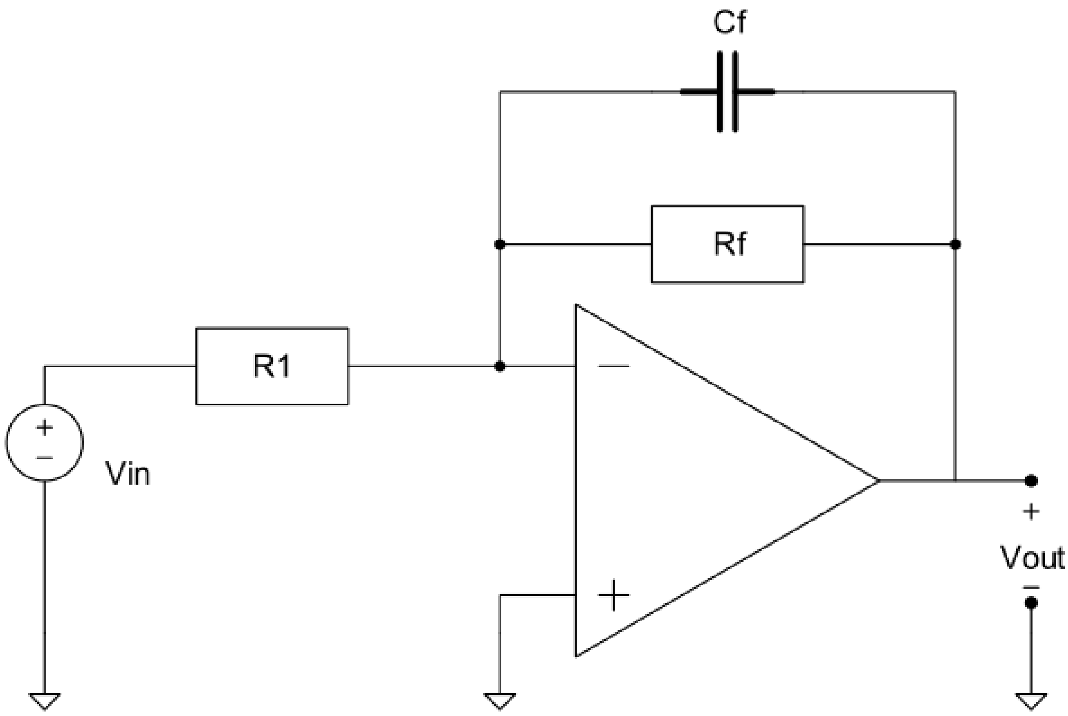

Figure 8: CT integrator.

First we will his transfer function on S domain.We will start obtaining the parallel between Rf and Cf:

$$Z_f = \frac{R_f C_f}{Rf+C_f} = \frac{\frac{R_f}{sC_f}}{R_f+\frac{1}{sC_f}}$$

Transfer function $H(s) = \frac{Vo}{Vi} = -\frac{Z_f}{R_1} = -\frac{R_f}{R1}\frac{1}{1+\frac{s}{wc}}$

We can see by the transfer function that it will behave like a low pass filter with a gain of $-\frac{R_f}{R_1}$ and since we want a A_{DC} gain of 10 R_f will be 10 times higher than R_1, we will take $R_1 = 1k\omega$ and $R_f = 10k\Omega$, we can also $C_f = \frac{1}{2\cdot\pi\cdot R_f\cdot f_c = 15.915pF}$

So the equivalent circuit in SC will be like this:

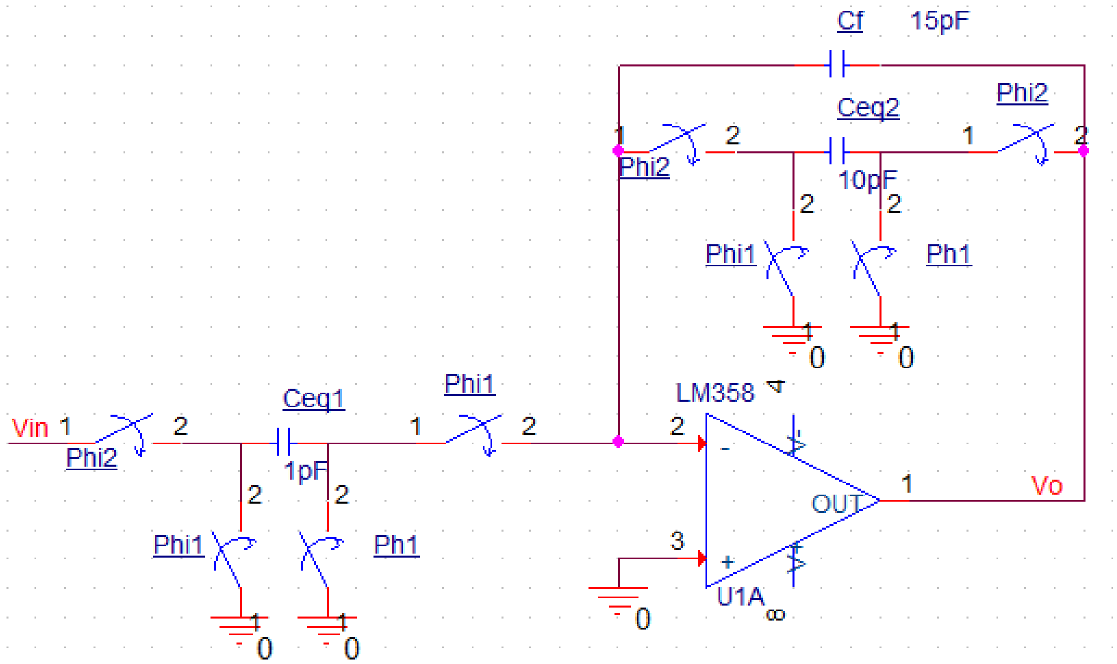

Figure 9: Equivalent SC schematic

In (n-1) ϕ_1 will be closed and ϕ_2 open so we will have:

$$Vo(n-1)\cdot C_f$$

In (n) ϕ_1 will be open and ϕ_2 closed so we will have:

$$Vin(n)\cdot C_{eq1} + Vo(n)(C_{eq2} + C_f)$$

We will transfer the equations to Z domain and equal them:

$$Vo(n-1)\cdot C_f = Vin(n)\cdot C_{eq1} + Vo(n)(C_{eq2}+C_f)$$

$$Vo\cdot z^{-1}\cdot C_f = Vin\cdot C_{eq1} + Vo(C_{eq2}+C_f)$$

$$H(z) = \frac{Vo}{Vin} = \frac{\frac{-C_{eq1}}{C_f}}{1+\frac{C_{eq2}}{C_f}}\cdot\frac{1}{1-\frac{1}{1+\frac{C_{eq2}}{C_f}}\cdot z^{-1}}$$

Finally we will prepare the MatLab code in order to plot the transfer functions.The code explanation will be in the appendix

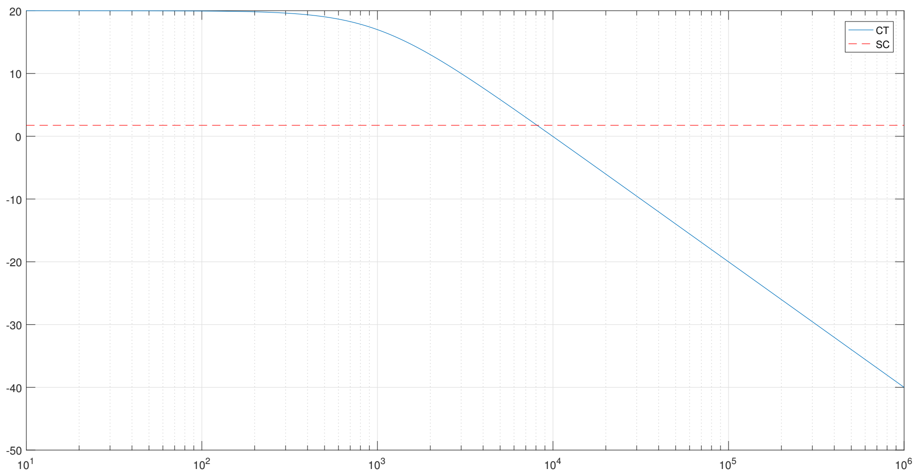

Figure 10: Comparison of CT and SC transfer function

We can see how the CT behaves as a LPF while the SC is constant through all the frequencies.

5 Appendix

5.1 Exercise 1

```
% We define the frequency vector and sampling frequency
freq= 0:10:50e6;
Fs = 20e6;
% We define the calculated elements R and C2
R=15.9e3;
C2=10e-9;

% For S Domain
% We define numerator and denominator of the transfer function
Bs=[1];
As=[R*C2 1];
% We define angular frequency
w=2*pi*freq;
% We calculate the transfer fucntion
[Hs,Ws] = freqs(Bs,As,w);
% We transform from angular freq to freq
fs=Ws/(2*pi);
% We calculate the phase and transform it to degrees
phase = angle(Hs);
degPhaseS = phase*180/(pi);


% For Z Domain
Ceq=1/(R*Fs);
Bz=[0 Ceq];
Az=[Ceq+C2 -C2];

[Hz,Wz] = freqz(Bz,Az,freq,Fs);
fz=Wz/2*pi;

```

```
phase = angle(Hz);
degPhaseZ = phase*180/(pi);

% Finally we plot the results
figure(1)
semilogx(fs,20*log10(Hz))
grid on
hold on
semilogx(fs,20*log10(Hs))
legend('Z Domain','S Domain')
% The spikes are due to the digital filter,but since we are using a higher
% sampling freq they are out of the filter and its not a problem.(If we use
% lower Fs they may be in the middle of the filter)

figure(2)
semilogx(fs,degPhaseZ)
grid on
hold on
semilogx(fs,degPhaseS)
legend('Z Domain','S Domain')
% Spikes also appear in phase but we can see that the phase is pretty much
% the same as in continuous time
```

5.2 Exercise 3

```
clear all;
close all;

% We define the data providen in the statemen
fc=1e3;             % Pole freq
Fs=1e6;             % Sampling freq
R1=1e3;
```

```
Rf=10e3;
Cf=1/(2*pi*Rf*fc)
f=0:10:1e6;             % Array of frequenciess
w=2*pi.*f;              % Array of anguar frequencies

% In S Domain
num=-Rf/R1;
den=[(Rf*Cf)  1];

% We calculate the value of Ceq1 and Ceq2
Ceq2=1/(Rf*Fs)          % We get the formula from the equivalency of
    a resistor in SC
Ceq1=1/(R1*Fs)

% In S domain
num1=-(Ceq1/Ceq2)/(1-Ceq1/Ceq2);
den1=[1  -1/(1+Ceq1/Ceq2)];

% We use both fucntion to get H and W
[h,w_plot]=freqs(num,den,w);

[h1,w_plot1]=freqz(num1,den1,w);

% We plot the results
semilogx(w_plot/(2*pi),20*log10(abs(h)));
grid;
hold on
semilogx(w_plot1/(2*pi),20*log10(abs(h1)), '--r');
```