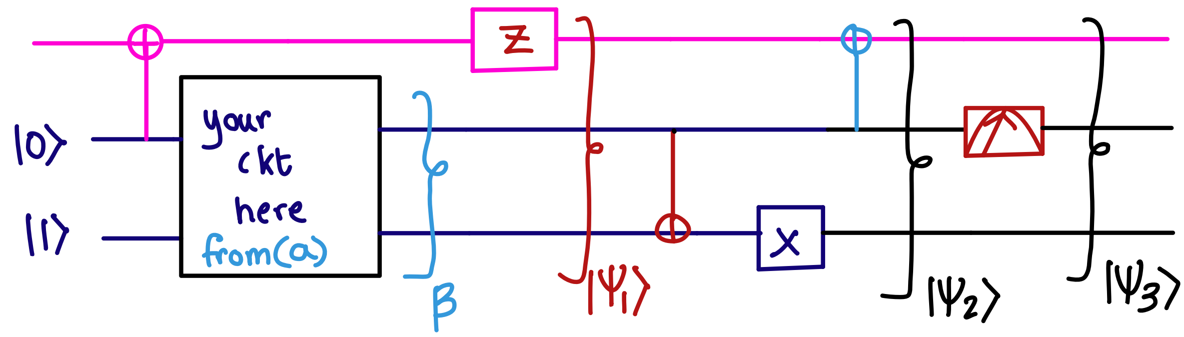
Questions

1.



(a) Design a quantum network such that $\beta = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$.

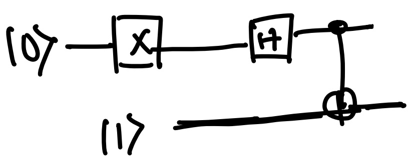
(b) Find the composite state $|\psi_1\rangle$

(c) Find $|\psi_2\rangle$

(d) The second qubit is measured to be $|1\rangle$. Find $|\psi_3\rangle$.

(e) what is the probability of measuring $|1\rangle$ on third (bottom-most) qubit now.

ⓐ



$$CNOT_{12}\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle \right)$$

$$= \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

ⓑ $|\psi_1\rangle = ?$

$$CNOT_{21}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \right) \Rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = (\text{unchanged})$$

$|\psi_1'\rangle$ (before H gate) $= \frac{1}{\sqrt{2}}|00\rangle + |10\rangle$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

$$= \frac{1}{2}\left(|001\rangle + |101\rangle - |010\rangle - |110\rangle\right)$$

$$|\psi_1\rangle = \frac{1}{2}\left(|001\rangle - |101\rangle - |010\rangle + |110\rangle\right)$$

(c) $|\psi_2\rangle = CNOT_{23}\left(\frac{1}{2}\ |001\rangle - |101\rangle - |010\rangle + |110\rangle\right)$

$$= \frac{1}{2}\left(|001\rangle - |101\rangle - |011\rangle + |111\rangle\right)$$

(d) $-\frac{1}{2}|011\rangle + \frac{1}{2}|111\rangle$

normalize $\quad \frac{-1}{\sqrt{2}}|011\rangle + \frac{1}{\sqrt{2}}|111\rangle$

$\beta_{00} = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

Q.2

$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is an observable.

given $|\psi\rangle = \frac{x-2i}{\sqrt{3}}|0\rangle + \frac{4}{5}|1\rangle$ $\quad x =$ last digit of ID

(a) Normalize $|\psi\rangle$

(b) Find the possible values upon measuring S_z and their corresponding collapsed states.

(c) Find the expectation value of observing S_x on $|\psi\rangle$.

(a) $$|\psi\rangle = \frac{\frac{3-2i}{\sqrt{3}}|0\rangle + \frac{4}{5}|1\rangle}{\sqrt{\left(\frac{3-2i}{\sqrt{3}}\right)\left(\frac{3+2i}{\sqrt{3}}\right) + \left(\frac{4}{5}\right)^2}} = \underline{\qquad\qquad}$$

(b) $\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ eigenvalue = observed value.

$$A v = \lambda v$$

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\det\left[\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right] = \det\begin{pmatrix} -\lambda & \hbar/2 \\ \hbar/2 & \lambda \end{pmatrix} = 0$$

$$\lambda^2 - \frac{\hbar^2}{4} = 0$$

$$\lambda = \pm \frac{\hbar}{2}$$

$+\frac{\hbar}{2}$

$$\left[\begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} - \begin{pmatrix} \hbar/2 & 0 \\ 0 & \hbar/2 \end{pmatrix}\right]\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -\hbar/2 & \hbar/2 \\ \hbar/2 & -\hbar/2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$-\frac{\hbar}{2}x + \frac{\hbar}{2}y = 0$$

$$\frac{\hbar}{2}x - \frac{\hbar}{2}y = 0$$

$$x = y \qquad \text{if } x = k$$

$$\begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

similarly do for the other eigen value

$$\text{state} = 1|0\rangle + 1|1\rangle$$

$$= \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

(c) $\langle \psi | S_x | \psi \rangle$

$$= \left(\frac{1}{\left(\frac{3-2i}{\sqrt{3}}\right)\left(\frac{3+2i}{\sqrt{3}}\right) + \left(\frac{4}{5}\right)^2} \right) \begin{bmatrix} \frac{3-2i}{\sqrt{3}} & \frac{4}{5} \end{bmatrix} \begin{pmatrix} 0 & \hbar/2 \\ \hbar/2 & 0 \end{pmatrix} \begin{bmatrix} \frac{3-2i}{\sqrt{3}} \\ \frac{4}{5} \end{bmatrix}$$

$$\downarrow$$

$$= k' \begin{bmatrix} \frac{3-2i}{\sqrt{3}} & \frac{4}{5} \end{bmatrix} \begin{bmatrix} \frac{4}{5} \cdot \frac{\hbar}{2} \\ \frac{3-2i}{\sqrt{3}} \frac{\hbar}{2} \end{bmatrix}$$

$$= k' \left(\frac{3-2i}{\sqrt{3}} \cdot \frac{4}{5} \cdot \frac{\hbar}{2} + \frac{4}{5} \cdot \frac{3-2i}{5} \cdot \frac{\hbar}{2} \right)$$

Q3

$$U|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \qquad U|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$$

(a) Write U as a matrix.

(b) Find $U(|+\rangle)$

(c) Is U a quantum gate?

(a) $U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$

(b) $U|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} =$ ____________

(c) if $U^\dagger U = I$ then U is unitary and thus a valid Q-gate.

$$U^\dagger = (U^T)^*$$

$$U^\dagger U = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & +i \\ +i & 1 \end{pmatrix}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1+1 & \\ & 1+1 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = I$$

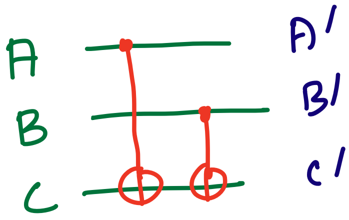
valid Q-gate

From Wong

Problems

Exercise: 3.20, 3.21, 4.3, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.19, 4.20

Ex: 4.16 (Wong)



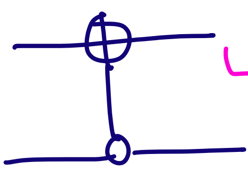
A	B	C	A'	B'	C'	
0	0	0	0	0	0	
0	0	1	0	0	1	
0	1	0	0	1	1	flip
0	1	1	0	1	0	flip
1	0	0	1	0	1	flip
1	1	1	1	1	1	flip x 2

no change

Ex 4.17 (Wong)

anti-controlled-NOT = U

LHS



flips upper qbit when it is 0 (opposite of CNOT)

$U|00\rangle = |10\rangle$

$U|01\rangle = |01\rangle$

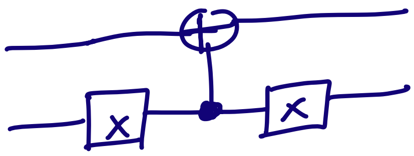
$U|10\rangle = |00\rangle$

$U|11\rangle = |11\rangle$

$\therefore$ matrix = $\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

RHS

U'



$U'|00\rangle = |10\rangle$

$U'|01\rangle = |01\rangle$

$U'|10\rangle = |00\rangle$

$U'|11\rangle = |11\rangle$

$$U' = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\therefore \; U = U'$ ckts equivalent

similarly Do Exercise 4.15, 4.18.

Final Syllabus

- All basics from the mid.

Wong

- 3.3.3, 3.3.4, 3.3.5, 3.4.1, 3.4.2
- 4.2.1, 4.2.2, 4.2.3, 4.3.1, 4.3.2, 4.4.1, 4.4.2
- Postulates of QM and examples done in class
- Bell states, Entanglement
- No cloning theorem
- Quantum teleportation
- Quantum parallelism