Set - B

Date: 6^{th} November '23

BRAC UNIVERSITY

Department of Computer Science Engineering

Midterm Examination

CSE 481: Quantum Computing 1

Total Marks: 40 **Time: 60 minutes**

Please write your name and/or ID on every page of your answer script. Answer all the questions.

Question 1:[total marks 20]

Let x be the second last digit of your ID number. Let y be the last digit of your ID number.

Consider the following quantum states:

$$|\psi_a\rangle = \frac{2}{3}|0\rangle + \frac{1-2i}{3}|1\rangle$$

$$|\psi_b\rangle = \frac{x}{y}|0\rangle + \frac{x-yi}{4}|1\rangle$$

(i) Examine the quantum states $|\psi_a\rangle$ & $|\psi_b\rangle$ to determine whether they are normalized. If not, provide the necessary steps to normalize them and write down their normalized forms. **[2]**

(ii) $|\psi_a\rangle$ is measured in the basis:

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, \qquad |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Find the probability of getting a $|+\rangle$. **[8]**

(iii) If another measurement is performed using the same basis as in question (ii), what is the probability of obtaining outcome $|-\rangle$ both the first and the second time ? **[1]**

(iv) Determine the inner product $\langle 1|\psi_b\rangle$. Additionally, state the probability of getting a $|1\rangle$ when we measure $|\psi_b\rangle$. **[3]**

(v) Express the relationship between the inner product and the probability found in question (iv) in one sentence. **[1]**

(vi) Compute the inner product $\langle\psi_b|\psi_a\rangle$. Provide a brief explanation of what the result signifies. **[5]**

Question 2: [total marks 20 + bonus 2]

Consider the following quantum states:

$$|\psi_a\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle$$

$$|\psi_b\rangle = \frac{x}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

(a) Find a value of x so that the two states are orthogonal. **[4]**
(b) Find a value of x so that $|\psi_a\rangle$ is normalized. **[4]**
(c) For what value(s) of x are the two states orthonormal? **[2]**

Consider the quantum state:

$$|\psi\rangle = \frac{1}{2}|0\rangle - \frac{\sqrt{3}}{2}|1\rangle$$

(d) Sheldon and Howard place a bet. They will measure $|\psi\rangle$ in the basis of $\{|\psi_a\rangle, |\psi_b\rangle\}$ and if it collapses to $|\psi_a\rangle$, Sheldon wins \$20 from Howard and if it collapses to $|\psi_b\rangle$, Sheldon loses \$20 dollars to Howard.
Find the expectation value of profit/loss for Sheldon from the bet.

(Hint: if an event has a probability of 1/3 to show 1 and 2/3 to show 2, the expectation value will be $expectation = \frac{1}{3} * 1 + \frac{2}{3} * 2$*)* **[5]**

(e) Sheldon hates losing and decides to cheat. So he passses the $|\psi\rangle$ through a unitary gate $U = \begin{pmatrix} p & q \\ 0 & -1 \end{pmatrix}$ to create a new state, $|\psi_{new}\rangle$, such that he will ***always wins*** the bet.

Write down the state of $|\psi_{new}\rangle$ in $|0\rangle$ and $|1\rangle$ basis. Then write down the matrix equation applying the gate U to $|\psi\rangle$. Hence find the values of p and q. **[5]**

Bonus: Write down the matrix of the gate that Howard can apply to $|\psi_{new}\rangle$ to make the game as it was before Sheldon cheated. **[2]**