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## Tutorial: 01

**BRAC University**

Semster: Fall 2023
Course No: CSE481
Course Title: QUANTUM COMPUTING
Section: 01
Faculty: JMD, RMT

Lecture: 1
Week: 1
Time: 30 minutes
Date: October 4, 2023

## Question 1

In physics, we often use vectors. Does this definition of vector spaces fit what we call vectors in physics? For example, Force is a vector quantity and it can be described using 3 co-ordinates $(x, y, z)$. Does the set of all the forces form a vector space?

$$\vec{F} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \quad \text{where } a_x, a_y, a_z \in \mathbb{R}$$

## Question 2

Does the set of all 3 × 1 matrix containing complex numbers form a vector space?

$$S = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \text{where } a, b, c \in \mathbb{C}$$

## Question 3

Does the set of all 3 × 1 matrix whose sum of elements is zero, form a vector space?

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : \text{where, } a + b + c = 0 \ ; \ a, b, c \in \mathbb{C} \right\}$$

## Question 4

Does the set of all 3 × 1 matrix whose sum of elements is one, form a vector space?

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : \text{where, } a + b + c = 1 \ ; \ a, b, c \in \mathbb{C} \right\}$$

## Question 5

Are these vectors independent? And what's the dimension of vector space they form?

$$u = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}, w = \begin{bmatrix} 5 \\ -10 \\ -8 \end{bmatrix}$$

## Question 6

$\langle u|v\rangle$ $\langle v|u\rangle$

Calculate the inner product between the following vectors: (u,v) and (v,u). Can you find a relation between these values? If so, can you justify that?

$$|u\rangle = \begin{bmatrix} 0.5 - 3i \\ 4 + i \\ 2 + 0.2i \end{bmatrix} \text{ and } |v\rangle = \begin{bmatrix} -3i \\ 2 - i \\ 2.5 \end{bmatrix}$$

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## Question 7

What's the norm of this vector?

$$v = \begin{bmatrix} -3i \\ 2-i \\ 2.5 \end{bmatrix}$$

## Question 8

Check if the following vectors orthogonal:

- $\begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4i \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4i \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1.5i \\ -1.5i \end{bmatrix}$

[1]

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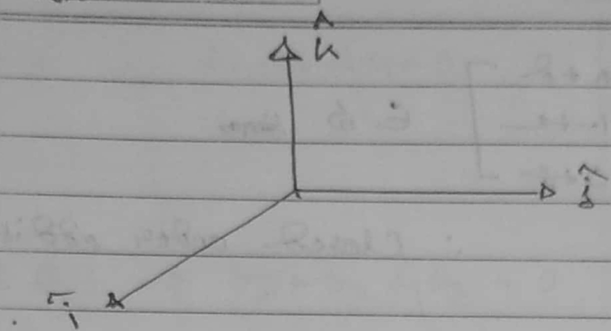
1)




$\vec{F} = ax\hat{i} + ay\hat{j} + az\hat{k}$ ; $a_x, a_y, a_z \in \mathbb{R}$

Let,

$\vec{F_1} = ax\hat{i} + ay\hat{j} + az\hat{k}$

$+ \vec{F_2} = bx\hat{i} + by\hat{j} + bz\hat{k}$

$\vec{F_1} + \vec{F_2} = \underbrace{(a+b)}_{\mathbb{R}}x\,\hat{i} + \underbrace{(a+b)}_{\mathbb{R}}y\,\hat{j} + \underbrace{(a+b)}_{\mathbb{R}}z\,\hat{k} \in \vec{F}$

$\vec{F_1} = ax\hat{i} + \cancel{b}ay\hat{j} + az\hat{k}$

$+ \vec{F_2} = bx\hat{i} + by\hat{j} - az\hat{k}$

$\vec{F_1} + \vec{F_2} = \underbrace{(a+b)}_{\mathbb{R}}x\,\hat{i} + \underbrace{(a+b)}_{\mathbb{R}}y\,\hat{j} + \underbrace{0}_{\mathbb{R}}z\,\hat{k} \in \vec{F}$

$\therefore$ Closed under addition.

$c \cdot \vec{F} = \underbrace{cax\hat{i}}_{\mathbb{R}} + \underbrace{cay\hat{j}}_{\mathbb{R}} + \underbrace{caz\hat{j}}_{\mathbb{R}} \in \vec{F}$ where $c \in \mathbb{R}$

$\mathbb{R} * \mathbb{R} = \mathbb{R}$

Thus closed under scalar multiplication.

$\therefore$ This is a Real Vector Space.

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2) $\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} \in S$ ~~since~~

since $a+d \in \mathbb{C}$ $\therefore$ Closed under addition
$b+e \in \mathbb{C}$
$c+f \in \mathbb{C}$

$c \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c \cdot a \\ c \cdot b \\ c \cdot c \end{bmatrix} \in S$

since,
$ca \in \mathbb{C}$
$cb \in \mathbb{C}$ $\therefore$ closed under scalar Multiplication
$cc \in \mathbb{C}$ $c \in \mathbb{R}/\mathbb{C}$

Thus, $S$ forms a vector space.

3) Let,
$U = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$ $V = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

$U+V$
$\Rightarrow \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 5 \\ -1 \\ -4 \end{bmatrix} \in S$ since $5+(-1)+(-4)=0$ and
$5, -1, -4 \in \mathbb{C}$

$\therefore$ closed under addition.

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$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \in S \Rightarrow a_1 + a_2 + a_3 = 0.$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S \Rightarrow b_1 + b_2 + b_3 = 0$$

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1+b_1 \\ a_2+b_2 \\ a_3+b_3 \end{bmatrix} \in S$$

$(a_1 + b_1)$

$$c \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} c \cdot a \\ c \cdot b \\ c \cdot c \end{bmatrix} \in S$$

$$\underbrace{c \cdot a}_{C} + \underbrace{c \cdot b}_{C} + \underbrace{c \cdot c}_{C} = 0$$

$\Rightarrow c(a + b + c) = 0$

$\Rightarrow c(0) = 0$

$\Rightarrow 0 = 0$

$\therefore$ Closed under scalar Multiplication.

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \in S \Rightarrow a_1 + b_1 + c_1 = 0 \qquad \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \in S \Rightarrow a_2 + b_2 + c_2 = 0$$

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2 \\ c_1+c_2 \end{bmatrix} \in S$$

$$\underbrace{(a_1 + a_2)}_{C} + \underbrace{(b_1 + b_2)}_{C} + \underbrace{(c_1 + c_2)}_{C} = 0$$

$$\Rightarrow \underbrace{(a_1 + b_1 + c_1)}_{C} + \underbrace{(a_2 + b_2 + c_2)}_{C} = 0.$$

$\Rightarrow 0 + 0 = 0$

$\Rightarrow 0 = 0$

$\therefore$ Closed under addition.

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$$5\begin{bmatrix}3\\1\\-4\end{bmatrix} = \begin{bmatrix}15\\5\\-20\end{bmatrix} \in S$$

$15+5+(-20)$
$= 0$

$\therefore$ closed under scalar Multiplication.

Thus $S$ forms a vector space.

4) $U = \begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix} \in S \Rightarrow a_1+b_1+c_1=1$ | $V = \begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix} \in S \Rightarrow a_2+b_2+c_2 = 1$

$U+V$

$$\Rightarrow \begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix} + \begin{bmatrix}a_2\\b_2\\c_2\end{bmatrix} = \begin{bmatrix}a_1+a_2\\b_1+b_2\\c_1+c_2\end{bmatrix} \in S$$

$\Rightarrow (a_1+a_2)+(b_1+b_2)+(c_1+c_2) = 1$
$\Rightarrow (a_1+b_1+c_1)+(a_2+b_2+c_2) - 1$
$\Rightarrow 1+1 \neq 1$
$\Rightarrow 2 \neq 1$

$\therefore$ NOT closed under addition.

$$c.U = c\begin{bmatrix}a_1\\b_1\\c_1\end{bmatrix} = \begin{bmatrix}c.a_1\\c.b_1\\c.c_1\end{bmatrix} \in S \Rightarrow c.a_1 + c.b_1 + c.c_1 = 1$$

$\Rightarrow c(a_1+b_1+c_1) - 1$
$\Rightarrow c(1) \neq 1$
$\Rightarrow c \neq 1$

$\therefore$ NOT closed under scalar multiplication.

Thus $S$ is not a vector space.

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5) $c_1 u + c_2 v + c_3 w = 0$

$$\Rightarrow c_1 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix} + c_3 \begin{bmatrix} 5 \\ -10 \\ -8 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2c_1 + 3c_2 + 5c_3 \\ -c_1 - 4c_2 - 10c_3 \\ c_1 - 2c_2 - 8c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ -1 & -4 & -10 & 0 \\ 1 & -2 & -8 & 0 \end{array}\right] \xrightarrow{R_2 = R_2 - (-1/2) R_1} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -\frac{15}{2} & 0 \\ 1 & -2 & -8 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -\frac{15}{2} & 0 \\ 1 & -2 & -8 & 0 \end{array}\right] \xrightarrow{R_3 = R_3 - (1/2) R_1} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -\frac{15}{2} & 0 \\ 0 & -\frac{3}{4} & -\frac{17}{4} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -15/2 & 0 \\ 0 & -3/4 & -17/4 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 - \left(\frac{-3/4}{-5/2}\right) R_2} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -15/2 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$\cancel{-2c_3 = 0}$ | $\cancel{-5/2\, c_2 - 15/2\, c_3 = 0}$ | $\cancel{2c_1 + 0 + 0 = 0}$

$\cancel{\therefore c_3 = 0}$ | $\cancel{\Rightarrow -5/2\, c_2 - 0 = 0}$ | $\cancel{\therefore c_1 = 0}$

$\cancel{\Rightarrow c_2 = 0}$

$$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -15/2 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3/-2} \left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & -5/2 & -15/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-\frac{5}{2} c_2 - \frac{15}{2} c_3 = 0$

$\Rightarrow \frac{-5c_2 - 15c_3}{2} = 0$

$\Rightarrow -c_2 - 3c_3 = 0$

$\Rightarrow c_2 + 3c_3 = 0$

$2c_1 + 3c_2 + 5c_3 = 0$

$\cancel{\Rightarrow 2c_1 + 3c_2 = 0}$

$\Rightarrow 2c_1 + 3(3c_3) + 5c_3 = 0$

$\Rightarrow 2c_1 + 14c_3 = 0$

$2c_1 + 3c_2 + 5c_3 = 0$

$\Rightarrow 2c_1 + 3c_2 + 5(-1/3\, c_2)$

$\Rightarrow 2c_1 + 4/3\, c_2 = 0$

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$\therefore$ Linearly Dependent.

Dimension = 2

Line → 1D
Sheet → 2D
Point → None

Calculating Determinant.

$$\begin{bmatrix} 2 & 3 & 5 \\ -1 & -4 & -10 \\ 1 & -2 & -8 \end{bmatrix} = 2(32-20)-3(8+10)+5(2+4)$$
$$= 0$$

Can't be Basis. Thus linearly dependent

6) $U^T, V$

$\Rightarrow [0.5+3i \quad 4\bar{+}i \quad 2+0.2i]^T \cdot \begin{bmatrix} -3i \\ 2-i \\ 2.5 \end{bmatrix}$

$\Rightarrow (0.5-3i)(-3i) + (4+i)(2-i) + (2+0.2i)(2.5)$

$\Rightarrow -1.5i - 9 + 8 + 2i - 4i + 1 + 5 + 0.5i$

$\Rightarrow 5$

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6) $$\begin{bmatrix} 0.5+3i & 4-i & 2-0.2i \end{bmatrix} \cdot \begin{array}{c} |v\rangle \\ \begin{bmatrix} -3i \\ 2-i \\ 2.5 \end{bmatrix} \end{array}$$

$\Rightarrow (0.5+3i)(-3i) + (4-i)(2-i) + (2-0.2i)(2.5)$

$\Rightarrow -1.5i + 9 + 8 - 2i - 4i - 1 + 5 - 0.5i$

$\Rightarrow 21 - 8i$

$$\begin{array}{c} \langle v| = |v\rangle^{\dagger} \\ \begin{bmatrix} 3i & 2+i & 2.5 \end{bmatrix} \end{array} \begin{array}{c} |v\rangle \\ \begin{bmatrix} 0.5-3i \\ 4+i \\ 2+0.2i \end{bmatrix} \end{array}$$

$\Rightarrow (3i)(0.5-3i) + (2+i)(4+i) + (2.5)(2+0.2i)$

$\Rightarrow 1.5i + 9 + 8 + 4i + 2i - 1 + 5 + 0.5i$

$\Rightarrow 21 + 8i$

7) $|v| = \sqrt{(-3i \cdot 3i) + (2-i)(2+i) + (2.5+0i)(2.5-0i)}$

$= \sqrt{9 + 5 + 6.25}$

$= 4.5$

8) $\begin{bmatrix} 3 \\ 0 \end{bmatrix} \begin{bmatrix} 4i \\ 0 \end{bmatrix} = \begin{bmatrix} -12i \\ 0 \end{bmatrix} = 12i + 0 = 12i$

$\begin{bmatrix} 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 4i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$ orthogonal

$\begin{bmatrix} -2 \\ 2 \end{bmatrix} \begin{bmatrix} 1.5i \\ -1.5i \end{bmatrix} = \cancel{\begin{bmatrix} 3 \\ 3 \end{bmatrix}} = 3 - 3 = 0$ $\therefore$ orthogonal