Prediction of Earthquake Magnitude

Sharmin Akhter

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1. Introduction

This project mainly purposed to predict earthquake magnitude based on earthquake depth by using Simple Linear Regression. Later I predict magnitude based on other variables. The data set give the locations of 1000 seismic events of MB > 4.0. The events occurred in a cube near Fiji since 1964. This is one of the Harvard PRIM-H project data sets. They in turn obtained it from Dr. John Woodhouse, Dept. of Geophysics, Harvard University.

A data frame with 1000 observations on 5 variables.

- **Mag**: predictor: Richer Magnitude
- **lat**: Numeric: Latitude of event
- **long**: Numeric: Longitude
- **depth**: Numeric: depth(km)
- **Stations** Numeric: Number of stations reporting

Simple linear Regression assumption.

The Linear Regression Model is based on several assumptions which are listed below:-

- Linear relationship
- Multivariate normality
- No or little multicollinearity
- No auto-correlation
- Homoscedasticity

2. Preliminary Analysis - Data Structure, Summary and Exploratory Analysis

Import libraries

```
library(tidyverse)
library(ggthemes)
library(ggrepel)
library(dplyr)
library(corrplot)
library(MASS)
library(olsrr)
```

Importing Data

```
data_quakes<- read.csv("dataset-45892.csv")
head(data_quakes)
```

```
##      lat   long depth mag stations
## 1 -20.42 181.62   562 4.8       41
## 2 -20.62 181.03   650 4.2       15
## 3 -26.00 184.10    42 5.4       43
## 4 -17.97 181.66   626 4.1       19
## 5 -20.42 181.96   649 4.0       11
## 6 -19.68 184.31   195 4.0       12
```

```
attach(data_quakes)
```

Relocate Predictor

```
data_quakes <- data_quakes %>% relocate(mag,.after= stations)
head(data_quakes)
```

```
##      lat   long depth stations mag
## 1 -20.42 181.62   562       41 4.8
## 2 -20.62 181.03   650       15 4.2
## 3 -26.00 184.10    42       43 5.4
## 4 -17.97 181.66   626       19 4.1
## 5 -20.42 181.96   649       11 4.0
## 6 -19.68 184.31   195       12 4.0
```

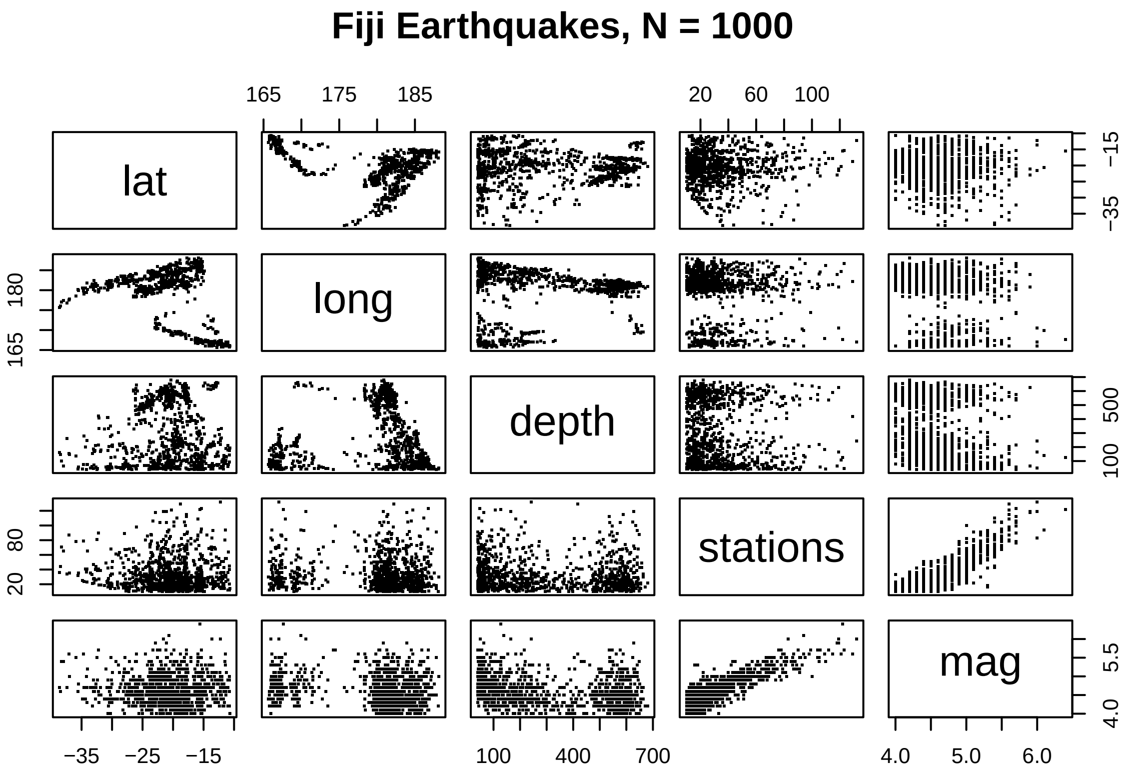
Dimension of Data

```
dim(data_quakes)
```

```
## [1] 1000    5
```

Scatterplot of the datasets

```
pairs(data_quakes, main = "Fiji Earthquakes, N = 1000", cex.main = 1.2, pch = ".")
```



Structure of the data

```r
#View(data_quakes)
str(data_quakes)
```

```
## 'data.frame':    1000 obs. of  5 variables:
##  $ lat     : num  -20.4 -20.6 -26 -18 -20.4 ...
##  $ long    : num  182 181 184 182 182 ...
##  $ depth   : int  562 650 42 626 649 195 82 194 211 622 ...
##  $ stations: int  41 15 43 19 11 12 43 15 35 19 ...
##  $ mag     : num  4.8 4.2 5.4 4.1 4 4 4.8 4.4 4.7 4.3 ...
```

```r
sum(is.na(data_quakes))
```

```
## [1] 0
```

From the structure we can see that the all variables values are numeric.

Now to get the inside idea we will look summary of the data

```r
summary(data_quakes)
```

```
##       lat             long           depth          stations     
##  Min.   :-38.59   Min.   :165.7   Min.   : 40.0   Min.   : 10.00  
##  1st Qu.:-23.47   1st Qu.:179.6   1st Qu.: 99.0   1st Qu.: 18.00  
##  Median :-20.30   Median :181.4   Median :247.0   Median : 27.00  
##  Mean   :-20.64   Mean   :179.5   Mean   :311.4   Mean   : 33.42  
##  3rd Qu.:-17.64   3rd Qu.:183.2   3rd Qu.:543.0   3rd Qu.: 42.00  
##  Max.   :-10.72   Max.   :188.1   Max.   :680.0   Max.   :132.00  
```

```
##        mag
##   Min.   :4.00
##   1st Qu.:4.30
##   Median :4.60
##   Mean   :4.62
##   3rd Qu.:4.90
##   Max.   :6.40
```

Check for relationship between mag and depth

```r
mean(data_quakes$mag[data_quakes$depth>median(data_quakes$depth)])
```

```
## [1] 4.5232
```

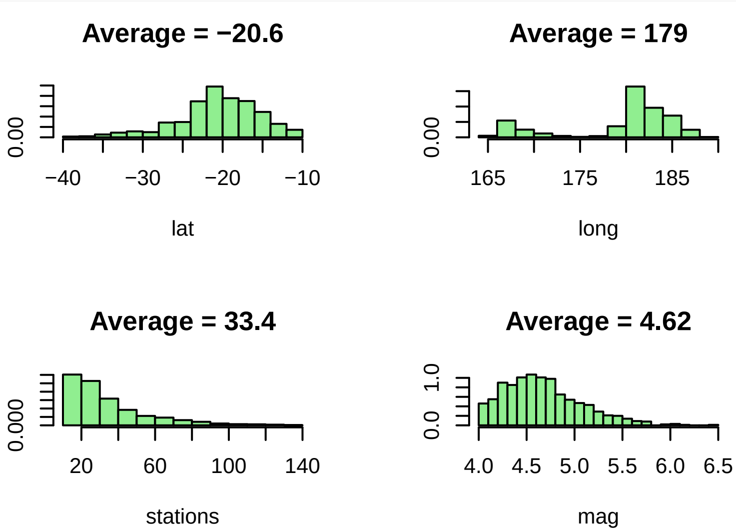
```r
mean(data_quakes$mag[data_quakes$depth<median(data_quakes$depth)])
```

```
## [1] 4.7176
```

From the above mean we can see the there is an inverse relationship between an earthquake's depth and its magnitude.

Now we look at the variables distribution:

```r
pred = par(mfrow = c(3,3))
for ( i in 1:5 ) {
  truehist(data_quakes[[i]], xlab = names(data_quakes)[i], col = 'lightgreen', main = paste("Average ="
}
#pred
```

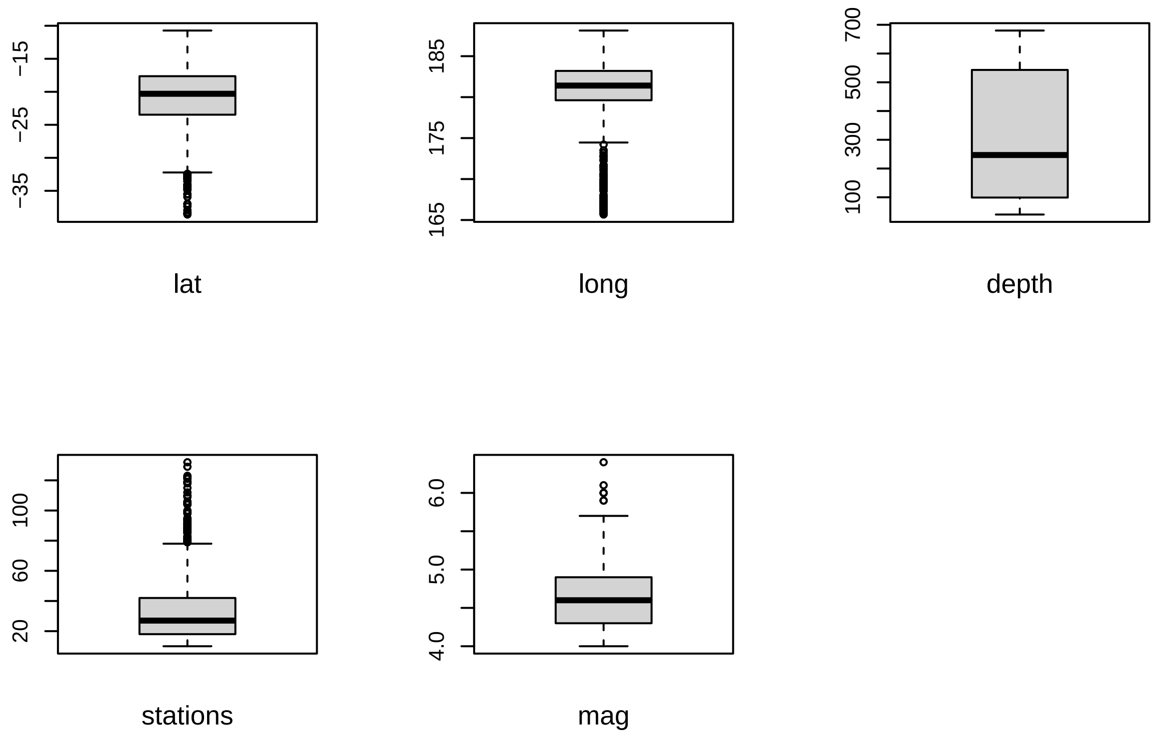


From the above distribution we can say that variable are asymmetric.

Next I will produce boxplot for each of the variables to see the outliers

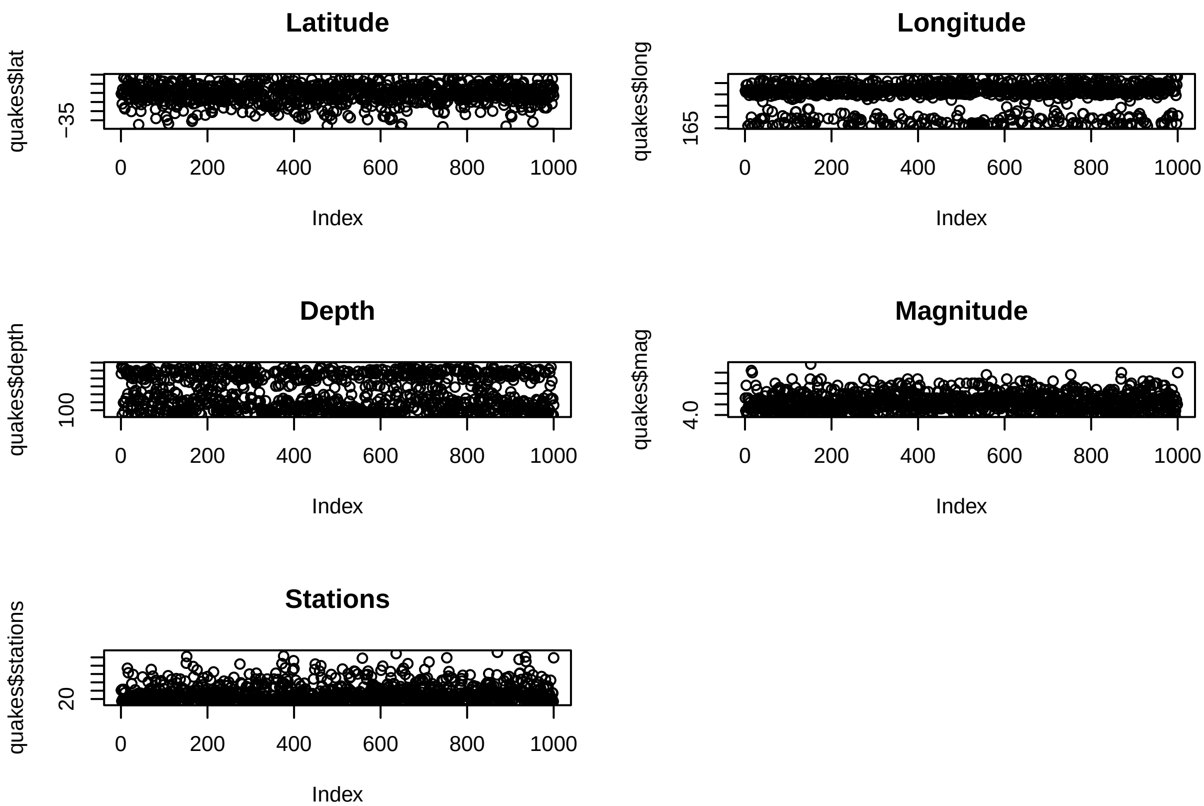
For each variables, we consider observations that lie outside 1.5 * IQR as outliers.

```
par(mfrow = c(2,3))
for ( i in 1:5) {
  boxplot(data_quakes[[i]])
  mtext(names(data_quakes)[i], cex = 0.8, side = 1, line = 2)
}
```



Create a histogram to get better information which boxplot cann't provide

```
par(mfrow = c(3, 2))
plot(quakes$lat,main="Latitude")
plot(quakes$long, main="Longitude")
plot(quakes$depth, main="Depth")
plot(quakes$mag, main="Magnitude")
plot(quakes$stations, main="Stations")
```

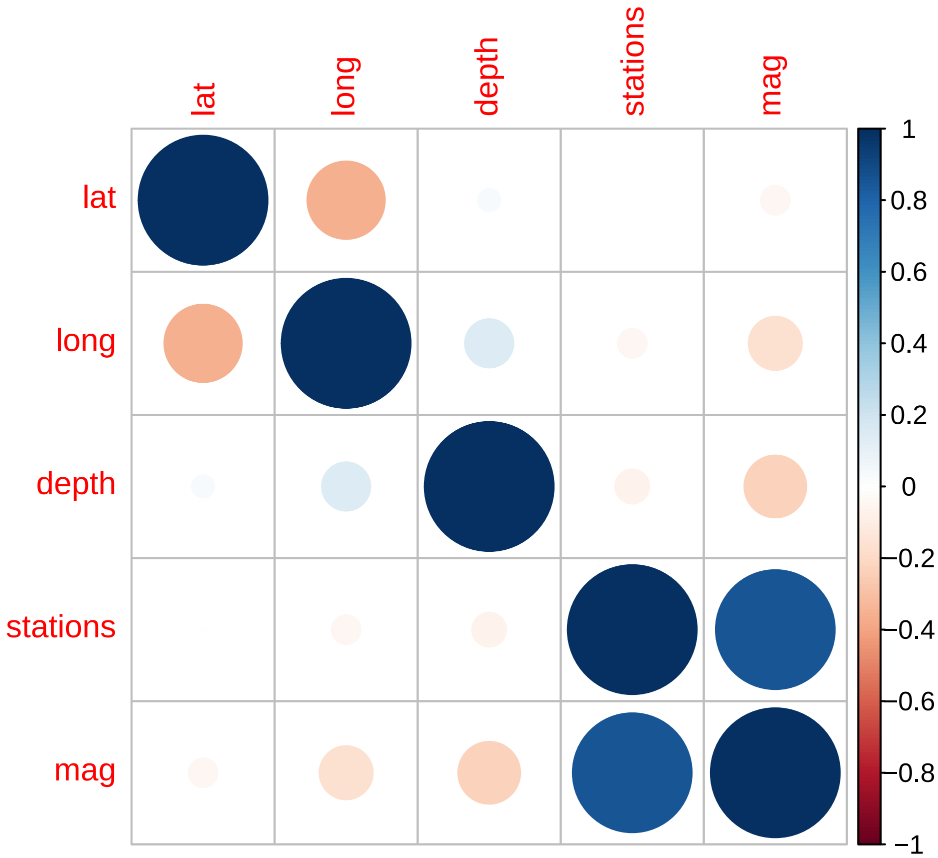


Correlation matrix

```
cormat<- round(cor(data_quakes),2)
cormat
```

```
##             lat  long depth stations   mag
## lat        1.00 -0.36  0.03     0.00 -0.05
## long      -0.36  1.00  0.14    -0.05 -0.17
## depth      0.03  0.14  1.00    -0.07 -0.23
## stations   0.00 -0.05 -0.07     1.00  0.85
## mag       -0.05 -0.17 -0.23     0.85  1.00
```

```
corrplot(cormat)
```



From the above correlation matrix we can say that mag has negative correlation with lattitude and depth, it has positive correlation with longitude and no correlation with stations.

Outlier detection

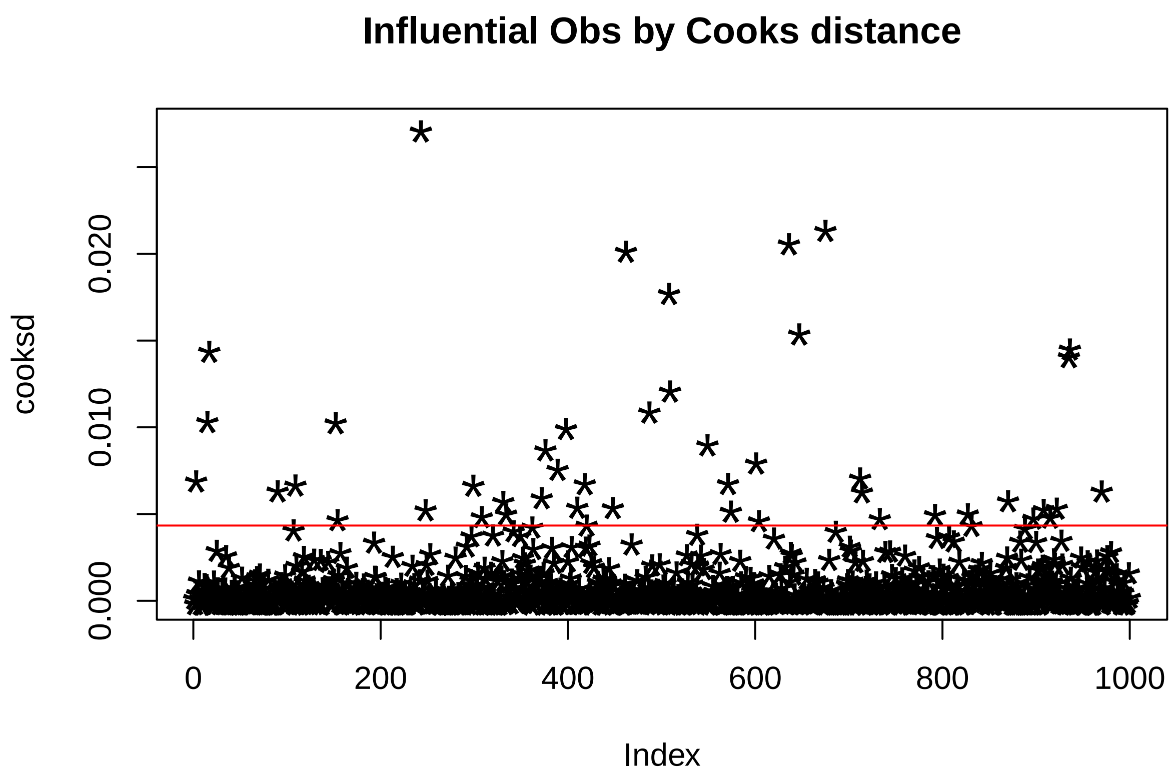
```
outliers = c()
for ( i in 1:5 ) {
  stats = boxplot.stats(data_quakes[[i]])$stats
  bottom_outlier_rows = which(data_quakes[[i]] < stats[1])
  top_outlier_rows = which(data_quakes[[i]] > stats[5])
 outliers = c(outliers , top_outlier_rows[ !top_outlier_rows %in% outliers] )
  outliers = c(outliers , bottom_outlier_rows[ !bottom_outlier_rows %in% outliers] )
}
outliers
```

```
##   [1]  41  81 104 107 110 164 165 166 310 410 418 419 425 426 476
##  [16] 477 484 487 525 530 570 606 610 621 622 627 647 649 744 890
##  [31] 903 952   7  12  15  17  22  27  32  37  40  45  48  53  63
##  [46]  64  73  78  87  91  92  94  99 108 117 118 119 120 121 126
##  [61] 133 136 141 143 145 148 152 154 155 157 159 160 163 170 192
##  [76] 205 222 226 230 239 243 250 251 252 254 258 263 267 268 292
##  [91] 300 301 305 311 312 318 320 321 325 328 330 334 352 357 360
## [106] 365 381 382 384 389 400 402 408 413 416 417 429 437 441 443
## [121] 453 456 467 474 490 492 496 504 507 508 509 517 524 527 528
## [136] 531 532 534 536 538 539 541 542 543 544 545 546 547 552 553
## [151] 560 571 581 583 587 593 594 596 597 612 613 618 620 625 629
```

```
## [166]  638  642  653  655  656  672  675  681  686  699  701  712  714  716  721
## [181]  725  726  735  754  756  759  765  766  769  779  781  782  787  797  804
## [196]  813  825  827  837  840  844  852  853  857  860  865  866  869  870  872
## [211]  873  883  884  887  888  891  893  908  909  912  915  916  921  927  930
## [226]  962  963  969  974  980  982  986  987  988  997 1000   28   70  151  167
## [241]  176  214  275  297  313  358  372  376  380  397  399  448  449  459  462
## [256]  486  512  558  601  605  623  636  651  657  663  702  753  758  850  920
## [271]  922  935  936  944
```

We use the Cook's distance to detect influential observations.

```
mod = lm(mag ~ ., data = data_quakes)
cooksd = cooks.distance(mod)
plot(cooksd, pch = "*", cex = 2, main = "Influential Obs by Cooks distance")
abline(h = 4*mean(cooksd, na.rm = T), col = "red")
```



Clean outliers

```
clean_outliers = as.numeric(rownames(data_quakes[cooksd > 4 * mean(cooksd, na.rm=T), ]))
outliers = c(outliers , clean_outliers[ !clean_outliers %in% outliers ] )

clean_Data = data_quakes[-outliers, ]
summary(clean_Data)
```

```
##       lat              long           depth          stations
##  Min.   :-32.20   Min.   :177.8   Min.   : 40.0   Min.   :10.00
##  1st Qu.:-23.74   1st Qu.:181.0   1st Qu.:143.0   1st Qu.:17.00
##  Median :-20.88   Median :182.0   Median :407.5   Median :23.50
```

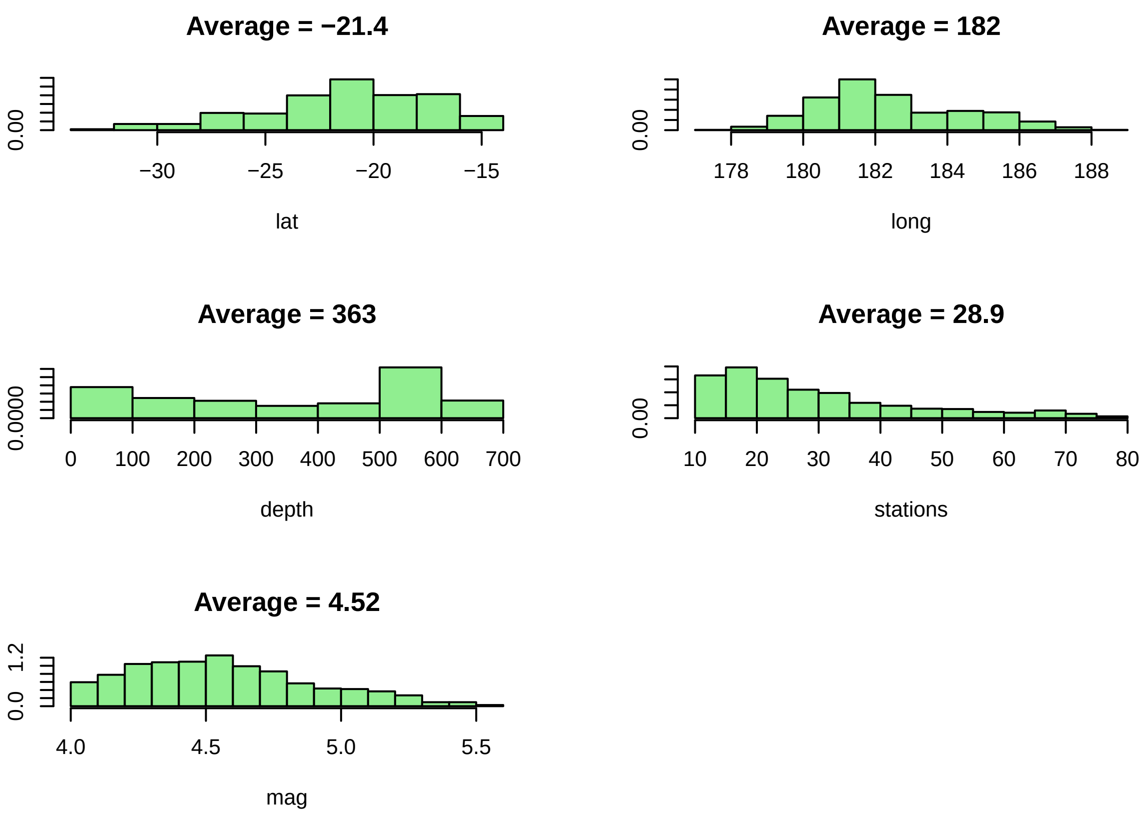
```
## Mean   :-21.36   Mean   :182.4   Mean   :362.7   Mean   :28.93
## 3rd Qu.:-18.12   3rd Qu.:183.9   3rd Qu.:562.0   3rd Qu.:36.00
## Max.   :-14.85   Max.   :188.1   Max.   :680.0   Max.   :78.00
##      mag
## Min.   :4.000
## 1st Qu.:4.300
## Median :4.500
## Mean   :4.518
## 3rd Qu.:4.700
## Max.   :5.500
```

```r
str(clean_Data)
```

```
## 'data.frame':    708 obs. of  5 variables:
##  $ lat     : num  -20.4 -20.6 -18 -20.4 -19.7 ...
##  $ long    : num  182 181 182 182 184 ...
##  $ depth   : int  562 650 626 649 195 194 211 622 583 554 ...
##  $ stations: int  41 15 19 11 12 15 35 19 13 19 ...
##  $ mag     : num  4.8 4.2 4.1 4 4 4.4 4.7 4.3 4.4 4.4 ...
```

Histogram plot after remove outliers

```r
par(mfrow=c(3,2))
for ( i in 1:5) {
  truehist(clean_Data[[i]], xlab = names(clean_Data)[i], col = 'lightgreen', main = paste("Average =", s
}
```



By removing the outliers, the dataset size reduced to 708 observations of 5 variables. Now, the variables are approximatly normaly distributed. By comparing with the previous histogram that contains high influency outliers we can see that the skewness is reduced in the new histogram.

2.1 Model building

Now we fit a simple linear regression model to predict earthquake magnitude based on earthquake depth.

$$mag = \beta 0 + \beta 1.depth$$

```
lmmod<- lm(mag~depth, data = clean_Data)
summary(lmmod)
```

```
##
## Call:
## lm(formula = mag ~ depth, data = clean_Data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.59315 -0.26308 -0.05362  0.19998  0.93990
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.614e+00  2.474e-02 186.477  < 2e-16 ***
## depth       -2.645e-04  5.896e-05  -4.486 8.48e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3311 on 706 degrees of freedom
## Multiple R-squared:  0.02771,    Adjusted R-squared:  0.02634
## F-statistic: 20.12 on 1 and 706 DF,  p-value: 8.476e-06
```

```
anova(lmmod)
```

```
## Analysis of Variance Table
##
## Response: mag
##            Df Sum Sq Mean Sq F value    Pr(>F)
## depth       1  2.206 2.20597  20.123 8.476e-06 ***
## Residuals 706 77.395 0.10963
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Adjusted R^2= 0.02634 , RSE= 0.3311 so the model is not good fit.

2.1 To check for non constant variance

ANOVA for reduced model

```
red<- resid(lmmod)
rs<- red^2
red.lm<- lm(rs~depth, data = clean_Data[1:708,])
summary(red.lm)
```

```
##
```

```
## Call:
## lm(formula = rs ~ depth, data = clean_Data[1:708, ])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.11005 -0.09598 -0.05240  0.03625  0.77461
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.102e-01  1.071e-02  10.285   <2e-16 ***
## depth       -2.332e-06  2.552e-05  -0.091    0.927
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1433 on 706 degrees of freedom
## Multiple R-squared:  1.183e-05,  Adjusted R-squared:  -0.001405
## F-statistic: 0.008351 on 1 and 706 DF,  p-value: 0.9272
```

```
anova(red.lm)
```

```
## Analysis of Variance Table
##
## Response: rs
##            Df  Sum Sq   Mean Sq F value Pr(>F)
## depth       1  0.0002 0.0001715  0.0084 0.9272
## Residuals 706 14.5005 0.0205390
```

Breush-Pagan test for constancy of error variance

$$\chi_0 = \frac{n^2}{2} * \frac{SSR^*}{SSE^2}$$

From above anova table and summary we have n = 708, SSR = 14.500, SSE* = 77.395

```
chi0 <-((708^2)/2) * (14.500/(77.395^2))
chi0
```

```
## [1] 606.7066
```

```
chi_crit <- qchisq(0.95,706)
chi_crit
```

```
## [1] 768.924
```

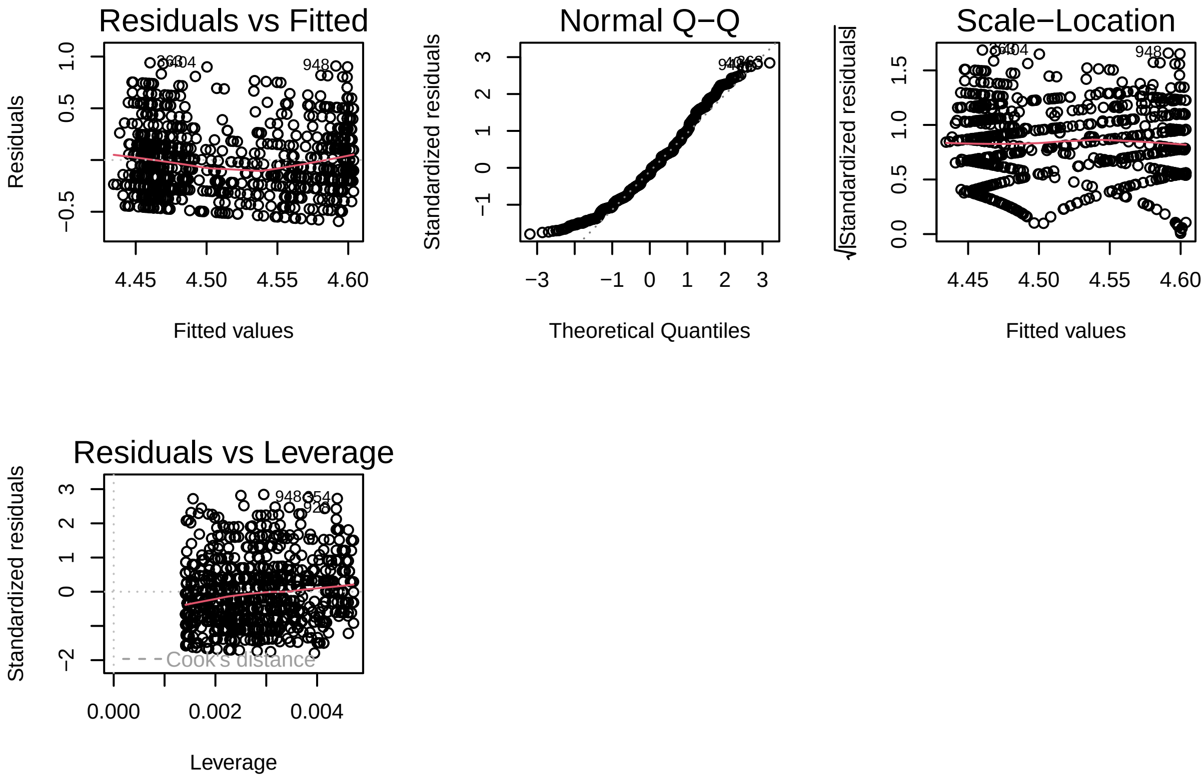
Hypothesis(null and alternative):

H0:beta1=0 and H1:beta1!=0

Since the chi0 = 606.7066<chi_crit=768.924 so we can reject null hypothesis.

Plot for full model without outliers in the dataset to predict magnitude based on depth

```
par(mfrow=c(2,3))
plot(lmmod)
```



Based on the above graphs, we observe the following -

- Residual vs fitted: There is curvature in the plot indicating that there is non linear relationship in the datasets.
- The normal Q-Q plot shows a fairly straight line, indicating the errors are more-or-less normally distributed.

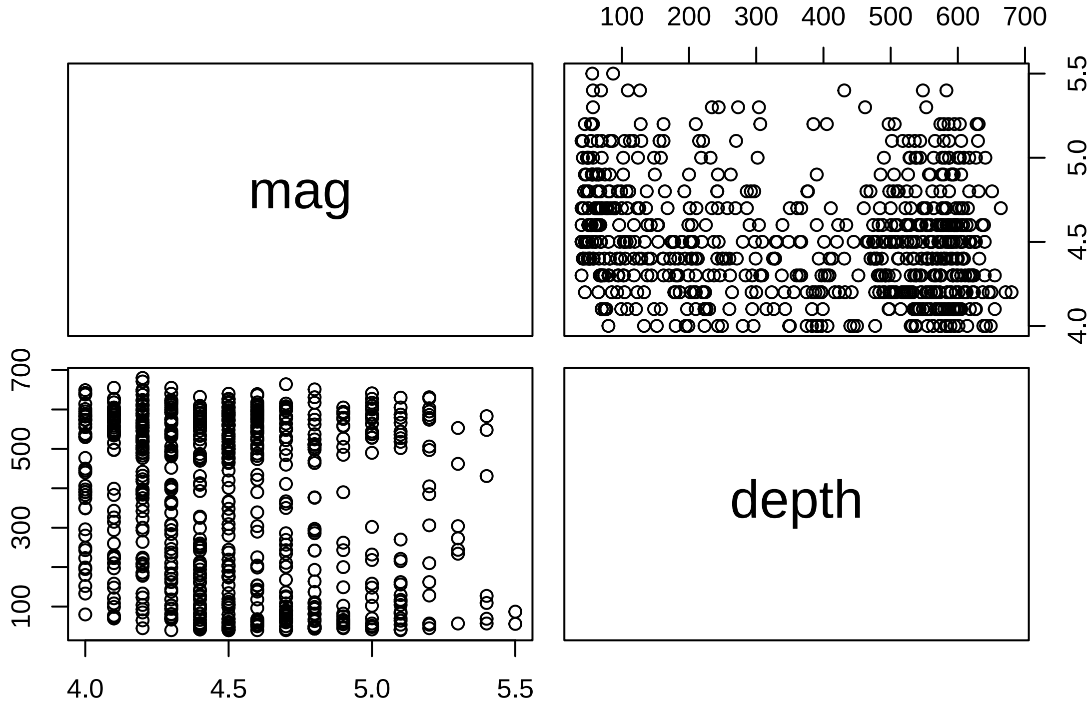
Based on the above summary of the fitted model we make the following observations:

- The multiple R- squared of the full and reduced model is 0.02771 and 1.183e-05. Adjusted R-square value is 0.02634 and -0.001405 respectively for full and reduced model.
- Since the errors seem to follow normal distribution based on Q-Q plot so taking level of significance 0.01.

Identify multicolinearity of with.

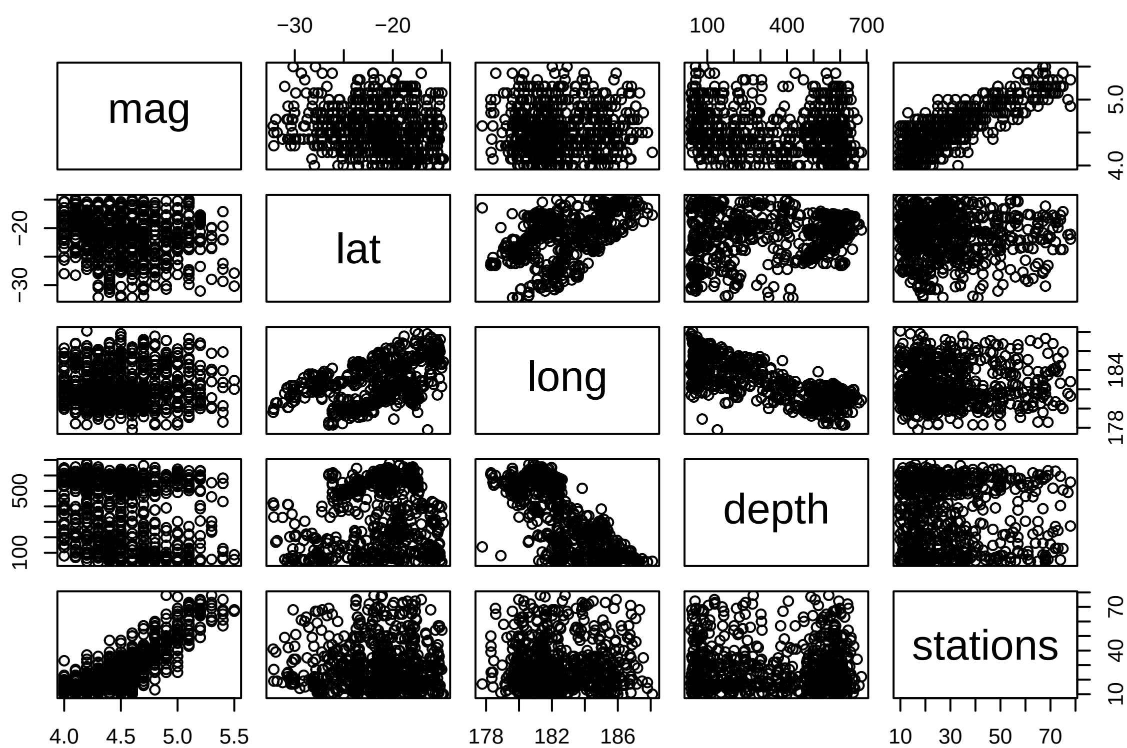
Now we look for the deeper analysis of the data

```
pairs(mag~depth,data = clean_Data[1:708,])
```



With all variables

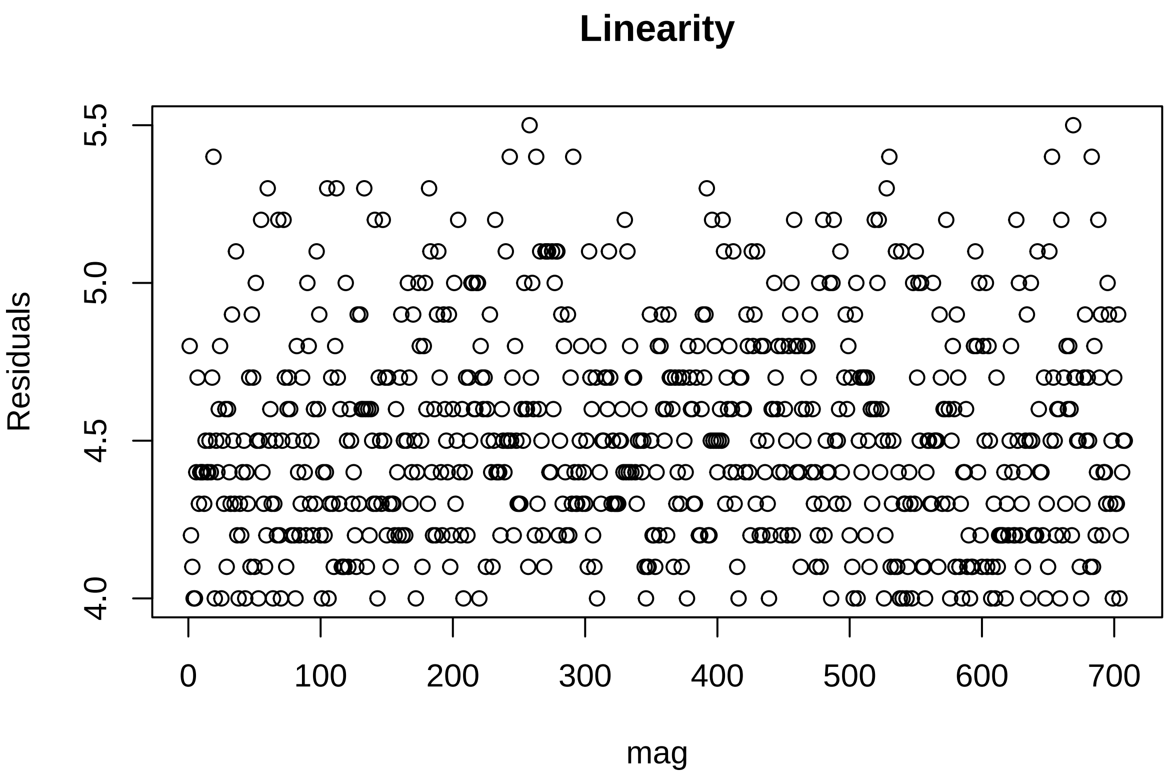
```
pairs(mag~.,data = clean_Data[1:708,])
```



Multicollinearity occurs when the features (or independent variables) are highly correlated

Linearity

```
plot(clean_Data$mag,clean_Data$residuals,xlab="mag",ylab="Residuals",main="Linearity")
```



from the scatterplot we can see that the relationship between response and feature variables is linear.

Further analysis

Multiple Linear Regression

Now, we fit a multiple linear regression model with mag as the response and all other variables as regressors. We plot the basic summary plots based on the fitted model, lmmod1, say, to get more idea about the data

```
lmmod1<-lm(mag~.,data=clean_Data[1:708,])
summary(lmmod1)
```

```
##
## Call:
## lm(formula = mag ~ ., data = clean_Data[1:708, ])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.51312 -0.12390 -0.00318  0.12307  0.45420
##
```

```
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.507e+00  1.660e+00   2.716 0.006772 **
## lat         -1.175e-02  3.117e-03  -3.769 0.000178 ***
## long        -3.602e-03  8.657e-03  -0.416 0.677453
## depth       -2.124e-04  7.662e-05  -2.772 0.005717 **
## stations     1.708e-02  4.044e-04  42.235  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1737 on 703 degrees of freedom
## Multiple R-squared:  0.7335, Adjusted R-squared:  0.732
## F-statistic: 483.7 on 4 and 703 DF,  p-value: < 2.2e-16
```

```
anova(lmmod1)
```

```
## Analysis of Variance Table
##
## Response: mag
##            Df Sum Sq Mean Sq   F value    Pr(>F)
## lat         1  2.971   2.971   98.4350 < 2.2e-16 ***
## long        1  1.538   1.538   50.9706 2.344e-12 ***
## depth       1  0.047   0.047    1.5494    0.2136
## stations    1 53.831  53.831 1783.8104 < 2.2e-16 ***
## Residuals 703 21.215   0.030
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Adjusted R^2 = 0.732, RSE = 0.1737

2.2 Check for non constant variance

ANOVA for reduced model

```
red<- resid(lmmod1)
rs<- red^2
red.lm<- lm(rs~ lat+long+depth+stations, data = clean_Data[1:708,])
summary(red.lm)
```

```
##
## Call:
## lm(formula = rs ~ lat + long + depth + stations, data = clean_Data[1:708,
##     ])
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.03721 -0.02607 -0.01411  0.01220  0.23350
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.623e-01  3.664e-01   0.716    0.474
## lat          1.502e-04  6.881e-04   0.218    0.827
## long        -1.228e-03  1.911e-03  -0.643    0.521
## depth       -1.121e-05  1.692e-05  -0.663    0.508
## stations    -3.449e-05  8.928e-05  -0.386    0.699
```

```
##
## Residual standard error: 0.03835 on 703 degrees of freedom
## Multiple R-squared:  0.001375,   Adjusted R-squared:  -0.004307
## F-statistic: 0.242 on 4 and 703 DF,  p-value: 0.9145
```

```
anova(red.lm)
```

```
## Analysis of Variance Table
##
## Response: rs
##            Df  Sum Sq    Mean Sq F value Pr(>F)
## lat         1 0.00051 0.00051145  0.3477 0.5556
## long        1 0.00002 0.00001873  0.0127 0.9102
## depth       1 0.00067 0.00067415  0.4584 0.4986
## stations    1 0.00022 0.00021943  0.1492 0.6994
## Residuals 703 1.03394 0.00147075
```

Breush-Pagan test for constancy of error variance

$$\chi_0 = \frac{n^2}{2} * \frac{SSR^*}{SSE^2}$$

From above anova table and summary we have n = 708, SSR = 1.03394, SSE* = 21.215

```
chi0 <-((708^2)/2) * (1.03394/(21.215^2))
chi0
```

```
## [1] 575.7657
```

```
chi_crit <- qchisq(0.95,703)
chi_crit
```

```
## [1] 765.7925
```

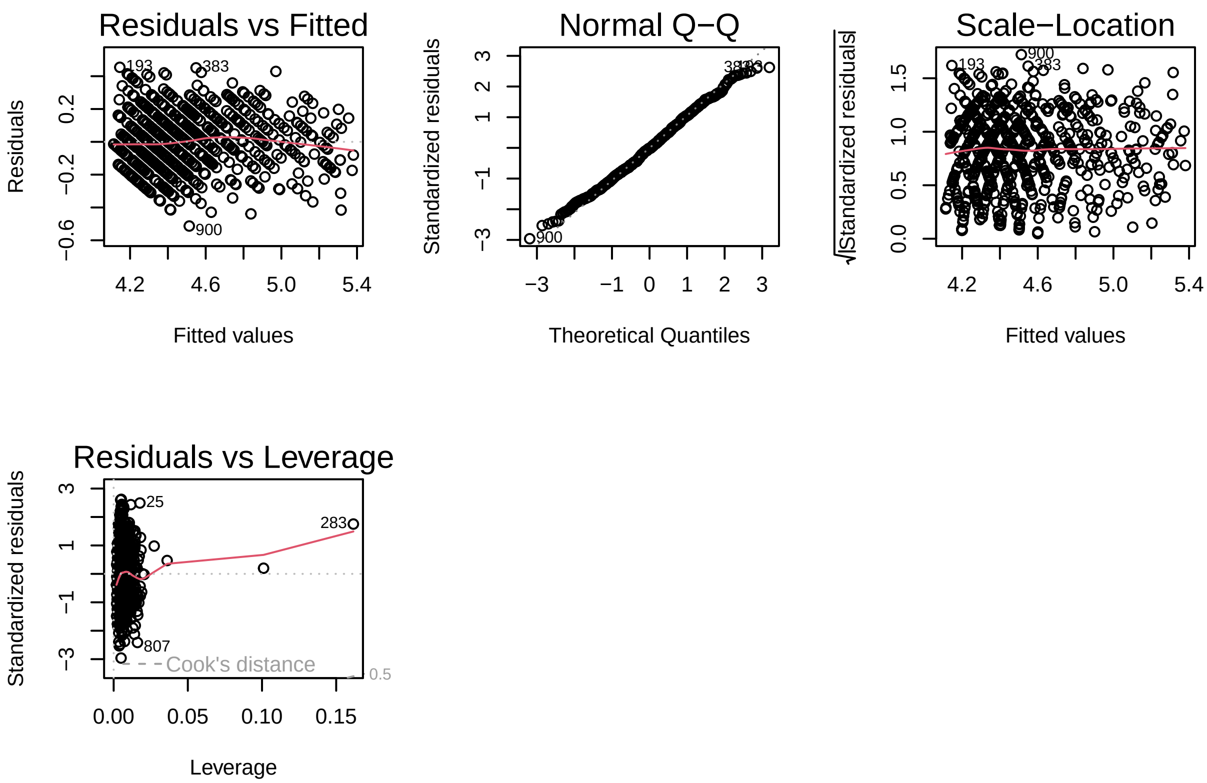
Hypothesis(null and alternative):

H0:beta1=0 and H1:beta1!=0

Since the chi0 = 575.7657<chi_crit=765.7925 so we can reject null hypothesis.

Plot for full model without outliers

```
par(mfrow=c(2,3))
plot(lmmod1)
```



Based on the above graphs, we observe the following -

- Residual vs fitted: There is curvature in the plot indicating that there is non linear relationship in the datasets.
- The normal Q-Q plot shows a fairly straight line, indicating the errors are more-or-less normally distributed.

Based on the above summary of the fitted model we make the following observations:

- The multiple R- squared of the full and reduced model is 0.7335 and 0.001375. Adjusted R-square value is 0.732 and -0.004307 respectively for full and reduced model.
- Since the errors seem to follow normal distribution based on Q-Q plot so taking level of significance 0.01.

3. Model Selection

In this section we will develop a best subset model for predicting the Earthquakes.

```
lm.1<- lm(mag~.,clean_Data)
lm.1
```

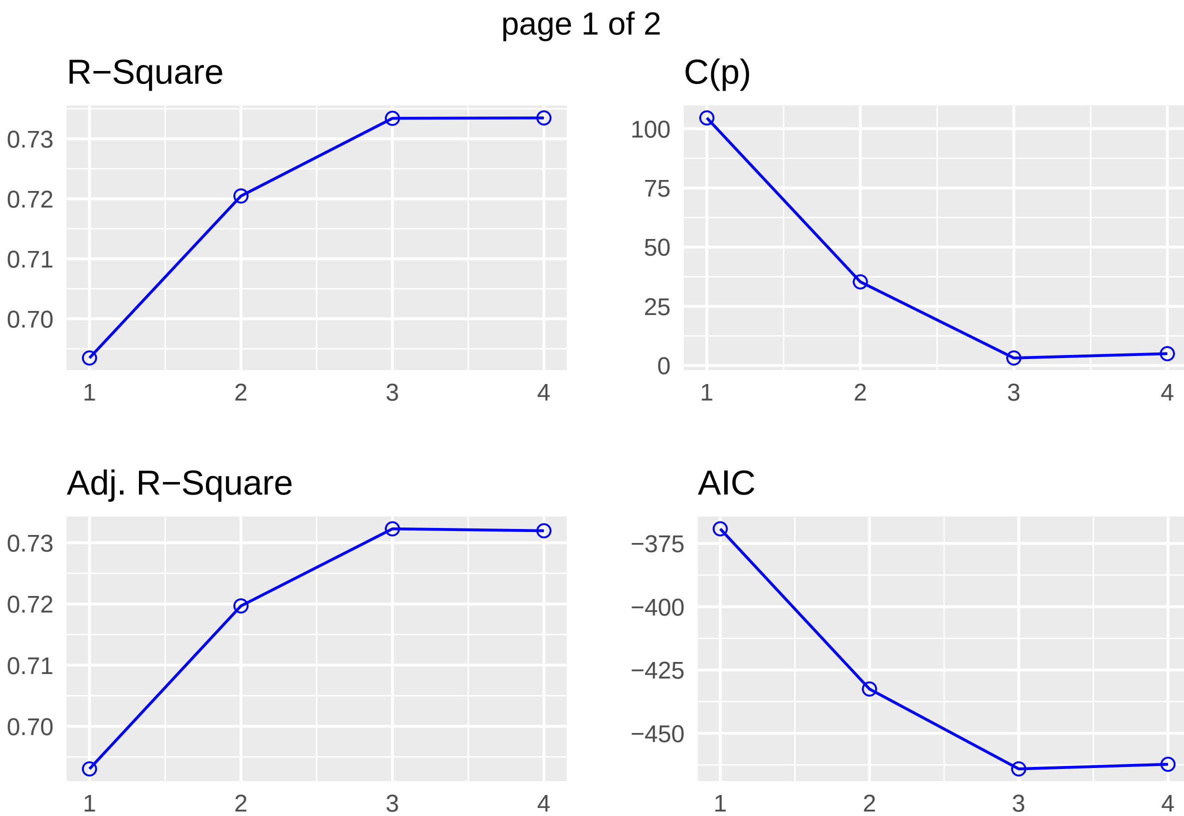
```
##
## Call:
## lm(formula = mag ~ ., data = clean_Data)
##
```

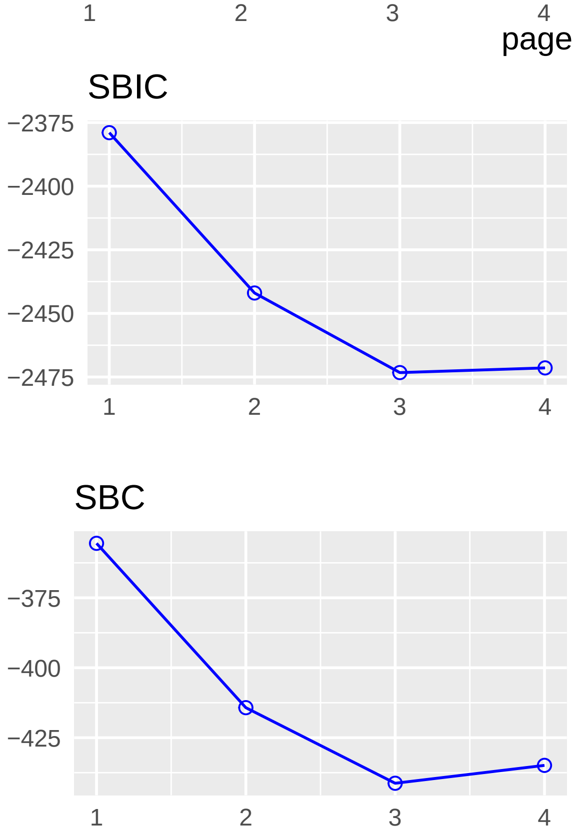
```
## Coefficients:
## (Intercept)          lat         long        depth     stations
##   4.5073439   -0.0117481   -0.0036022   -0.0002124    0.0170808
```

```
plt<- ols_step_best_subset(lm.1)
plt
```

```
##       Best Subsets Regression
## --------------------------------------
## Model Index    Predictors
## --------------------------------------
##      1         stations
##      2         lat stations
##      3         lat depth stations
##      4         lat long depth stations
## --------------------------------------
##
##                                                       Subsets Regression Summary
## -----------------------------------------------------------------------------------------------------
##                        Adj.        Pred
## Model    R-Square    R-Square    R-Square      C(p)         AIC           SBIC           SBC         MSE
## -----------------------------------------------------------------------------------------------------
##   1        0.6935      0.6930      0.6918    104.5580    -369.2199    -2378.9653    -355.5326    24.4
##   2        0.7205      0.7197      0.7181     35.3204    -432.5187    -2441.9774    -414.2689    22.3
##   3        0.7334      0.7323      0.7304      3.1732    -464.0916    -2473.2537    -441.2793    21.3
##   4        0.7335      0.7320      0.7295      5.0000    -462.2659    -2471.4119    -434.8913    21.3
## -----------------------------------------------------------------------------------------------------
## AIC: Akaike Information Criteria
##  SBIC: Sawa's Bayesian Information Criteria
##  SBC: Schwarz Bayesian Criteria
##  MSEP: Estimated error of prediction, assuming multivariate normality
##  FPE: Final Prediction Error
##  HSP: Hocking's Sp
##  APC: Amemiya Prediction Criteria
```

```
plot(plt)
```





We can see that model 4 with lat, long, depth, stations as predictor variables is selected based on R^2 adjusted criterion with highest R^2 adjusted value. C(p) value leads to model 4 as for this model which is small. This 4 predictor variable model is also selected by the AIC. We can see SBC criterion which leads to model 4.

Based on the 4 criteria model 4 turned out to be the best model.

4. Summary

In this project I have predicted earthquake magnitude based on earthquake depth. For this I have collected data with 1000 observations of 5 variables and all the values in the data sets are numeric.

In the preliminary analysis part of the project I analyzed data with various method. A few approaches were taken to addressed the analysis of the data. Box plot shows the outliers of the data and then for better analysis I plotted histrogram. It shows there more than 200 outliers on the datasets. I checked for influence of the outliers by cooks distance and clean the outliers.

After that I fitted a model with magnitude and depth which is a simple linear regression model. By the assumptions of linear regression i checked for multicolinearity, linearity of the variables. From the scatter plot of multicolinearity and linearity its visible that the data in linear and normally distributed.

For the further analysis I was fitted a model with earthquake magnitude based on other variables.

The diagnostic plots show an improvement over the base one. However, the performance of the model decreases as showcased by smaller R-squared and RMSE values for both methods.

In the last section the best models shows that with lat, long, depth and stations