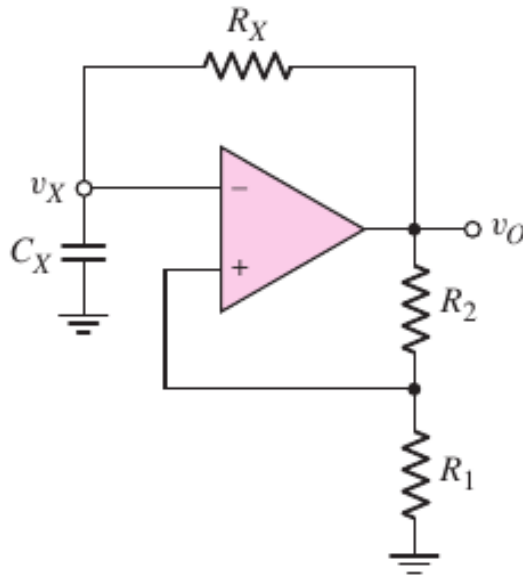


**BRAC UNIVERSITY**  
**Department of Computer Science & Engineering**  
**CSE350: Practice Problem sheet (Week6)**

Question 1



The Dual Slope ADC in week 5 problem 4 uses a clock signal of **1 MHz**. Suppose, we want to supply this clock signal from a square-wave generator circuit. The square-wave generator circuit we studied in week 6 is a schmitt-trigger oscillator, as shown in the above figure.

Assume, the saturation output voltages of the om-amp are symmetric (i.e. equal in magnitude). Now, follow the steps stated below to design the circuit.

(a)	Calculate the duty cycle of the circuit, considering symmetric output voltages.
(b)	Derive the expression of <b>Time Period, T</b> in terms of the circuit parameters.
(c)	Choose standard resistor and capacitor values to obtain the desired time period (or frequency.)
(d)	Calculate the deviation (in percentage) in frequency for your designed circuit.
(e)	How can we design a schmitt trigger oscillator circuit with a specific duty cycle (like 30%)?

## Solution:

Given specifications in the question:

1. Frequency of the square-wave,  $f = 1 \text{ MHz}$
2. Saturation output voltages of the op-amp are equal in magnitude.

$$\text{so, } |V_H| = |V_L|$$

But we know that  $V_H$  &  $V_L$  are opposite in polarity, so

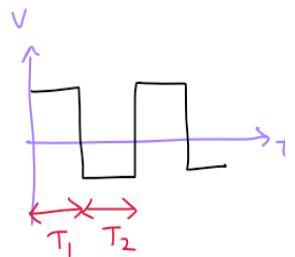
$$V_H = -V_L$$

(a)	Calculate the duty cycle of the circuit, considering symmetric output voltages.
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Duty cycle is the percentage time a signal is HIGH.

For the signal on the right,

$$\text{Duty cycle, } D = \frac{T_1}{T_1 + T_2} = \frac{T_1}{T} \quad \left( T \text{ is the time period of the signal} \right)$$



Now, we need to calculate  $T_1$  and  $T_2$ .

We have shown that,

$$T_1 = \tau \ln \left| \frac{V_H - V_L}{V_H - V_{TH}} \right| \quad \text{and} \quad T_2 = \tau \ln \left| \frac{V_L - V_{TH}}{V_L - V_{TL}} \right| ,$$

where  $\tau = R_x C_x$  and  $V_{TH} = \frac{R_1}{R_1 + R_2} V_H$  and  $V_{TL} = \frac{R_1}{R_1 + R_2} V_L$

Now, if  $V_H = -V_L$

then, it is easy to see that,  $V_{TH} = -V_{TL}$

Also, if  $V_H = -V_L$  and  $V_{TH} = -V_{TL}$

then,  $T_2$  can be written as:

$$T_2 = \tau \ln \left| \frac{V_L - V_{TH}}{V_L - V_{TL}} \right| = \tau \ln \left| \frac{-V_H + V_{TL}}{-V_H + V_{TH}} \right|$$

$$= \tau \ln \left| \frac{V_H - V_{TL}}{V_H - V_{TH}} \right| = T_2$$

so,  $T_1 = T_2$

and,  $D = \frac{T_1}{T_1 + T_2} = 0.5 \quad (50\%)$

(b) Derive the expression of Time Period, T in terms of the circuit parameters.

We already know that,  $T_1 = T_2$

$$\text{so, } T = T_1 + T_2 = 2T_1 = 2\tau \ln \left| \frac{V_H - V_{TL}}{V_H - V_{TH}} \right|$$

Now, we have to express this in terms of circuit parameters  $R_x$ ,  $C_x$ ,  $R_1$ ,  $R_2$ . For this, just follow the steps below:

$$T = 2R_x C_x \ln \left| \frac{V_H - \frac{R_1}{R_1 + R_2} V_L}{V_H - \frac{R_1}{R_1 + R_2} V_H} \right| = 2R_x C_x \ln \left| \frac{V_H + \frac{R_1}{R_1 + R_2} V_H}{V_H - \frac{R_1}{R_1 + R_2} V_H} \right|$$

$$= 2R_x C_x \ln \left| \frac{1 + \frac{R_1}{R_1 + R_2}}{1 - \frac{R_1}{R_1 + R_2}} \right| = 2R_x C_x \ln \left| \frac{2R_1 + R_2}{R_2} \right|$$

$$\text{so, } \boxed{T = 2R_x C_x \ln \left( 1 + 2 \frac{R_1}{R_2} \right)}$$

(c) Choose standard resistor and capacitor values to obtain the desired time period (or frequency.)

Desired frequency,  $f = 1 \text{ MHz} = 10^6 \text{ Hz}$ .

$$\text{so, } T = \frac{1}{f} = \frac{1}{10^6} \text{ sec} = 10^{-6} \text{ sec} = 1 \mu\text{sec}.$$

It is actually a 'design problem'. so, we have some freedom.

$$\text{We need } T = 1 \mu\text{sec}; \text{ also } T = 2R_x C_x \ln \left( 1 + 2 \frac{R_1}{R_2} \right)$$

↓

Let's choose  $R_1 = R_L = 1 \text{ k}\Omega$

$$\text{so, } T = 2 R_x C_x \ln(3) = 2.197 R_x C_x = 1 \text{ } \mu\text{sec.}$$

$$\Rightarrow R_x C_x = 0.455 \text{ } \mu\text{sec.}$$

I will now choose standard resistor & capacitor values to get  $R_x C_x$  close to  $0.455 \text{ } \mu\text{sec.}$

choosing  $R_x = 1 \text{ k}\Omega$ ,  $C_x = 0.47 \text{ nF}$   
gives  $R_x C_x = 0.47 \text{ } \mu\text{sec.}$

(d)	Calculate the deviation (in percentage) in frequency for your designed circuit.
-----	---------------------------------------------------------------------------------

We chose,  $R_1 = R_L = 1 \text{ k}\Omega$

$$R_x = 1 \text{ k}\Omega, C_x = 0.47 \text{ nF.}$$

$$\text{so, } T' = 2 R_x C_x \ln\left(1 + 2 \frac{R_1}{R_L}\right) = 2 \times 0.47 \ln(3) = 1.0327 \text{ } \mu\text{sec.}$$

$$\therefore f' = \frac{1}{T'} = 0.9683 \text{ MHz.}$$

$$\begin{aligned} \text{Percentage deviation (between desired frequency and achieved frequency)} &= \frac{f' - f}{f} \times 100\% \\ &= \frac{0.9683 - 1}{1} \times 100\% \end{aligned}$$

$$\approx -3.477$$

(e) How can we design a schmitt trigger oscillator circuit with a specific duty cycle (like 30%)?

For duty cycle to be different than 50%, we need

$$|V_H| \neq |V_L|$$

Follow homework problem 6.4

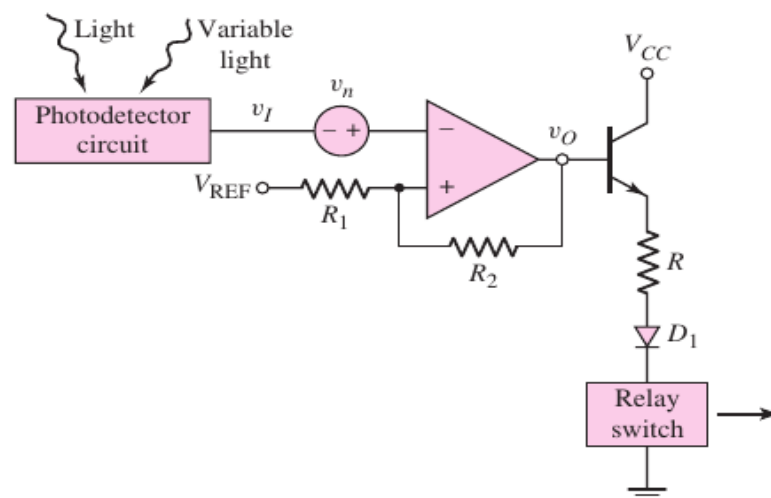
## Question 2

Previously, we designed a street light control circuit using a simple op-amp comparator. But it damaged some of the street lights.

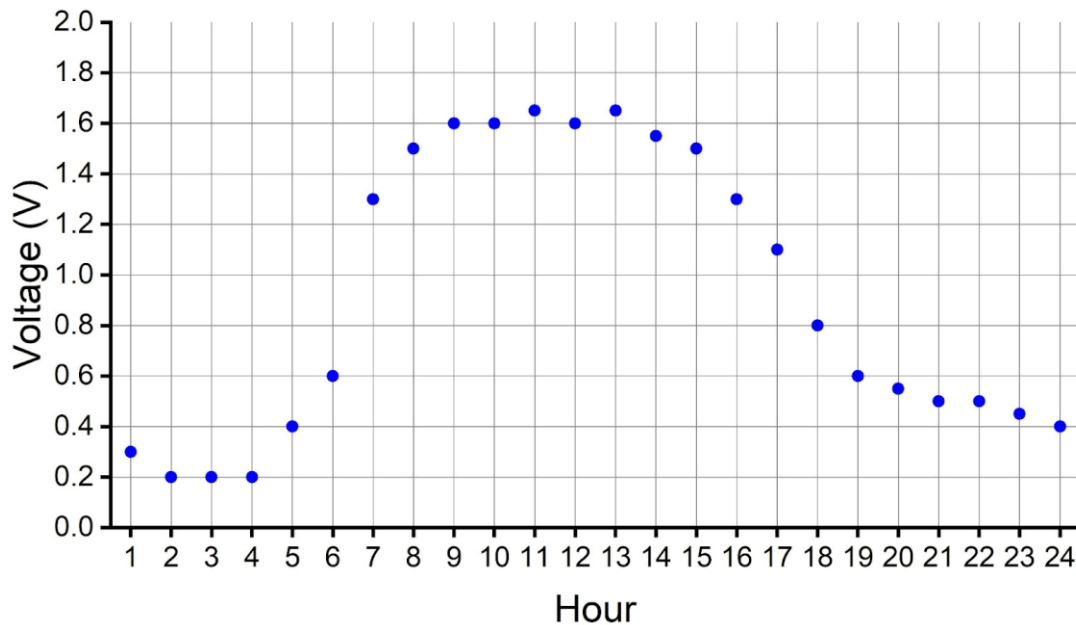
Now, we want to design a street light control circuit using Schmitt trigger, as shown below. This design employing Schmitt trigger circuit is robust to environmental noise and photodetector's shot-noise, so hopefully it will not damage the street lights this time.

We want to install our street light control system in Mohammadpur, Dhaka. So, we went there last week to collect photodetector circuit data. Our collected data over 24 hours is shown in the plot below. Note that, the output voltage of a photodetector circuit is directly proportional to the amount of light incident on it and we use this output voltage as an input voltage of the schmitt trigger circuit.

Now, help us in selecting the appropriate values of the circuit parameters.



(a)	Choose an appropriate value of the switching voltage $V_s$ .
(b)	We found that the combined noise voltage has a peak-to-peak voltage of around 0.1 V. Considering this, choose an appropriate value of the hysteresis width of the schmitt trigger circuit.
(c)	For your chosen switching voltage and hysteresis width, select the values of $R_1$ , $R_2$ , $V_{REF}$ .



## Solution:

(a)	Choose an appropriate value of the switching voltage $V_s$ .
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In the plot, we can see that photodetector's output voltage rises steeply between 6 am - 7 am. Also, the output voltage decreases sharply between 4 pm - 6 pm. So, these are the time windows when we have to turn on/off the street lights.

Now, observe the voltage values during these time windows.

'1 V' seems to be a right choice for switching voltage.

- (b) We found that the combined noise voltage has a peak-to-peak voltage of around 0.1 V. Considering this, choose an appropriate value of the hysteresis width of the schmitt trigger circuit.

The main advantage of schmitt trigger over comparator circuit is the noise immunity. The hysteresis width actually determines how much immune the circuit is to noise voltage.

so choose,  $\text{Hysteresis width} = \text{peak to peak noise voltage} = 0.1 \text{ V}$

- (c) For your chosen switching voltage and hysteresis width, select the values of  $R_1$ ,  $R_2$ ,  $V_{\text{REF}}$ .

We need,  $V_S = 1 \text{ V}$

$$\text{Hysteresis width} = 0.1 \text{ V}$$

$$\text{Now, } \text{Hysteresis width} = V_{TH} - V_{TL} = \frac{R_1}{R_1 + R_2} (V_H - V_L) \dots \dots (1)$$

$$\text{Let's assume, } V_H = 5 \text{ V, } V_L = -5 \text{ V}$$

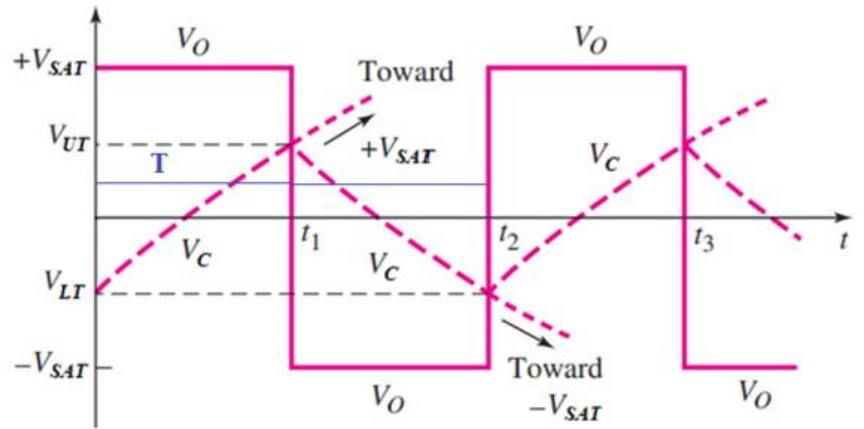
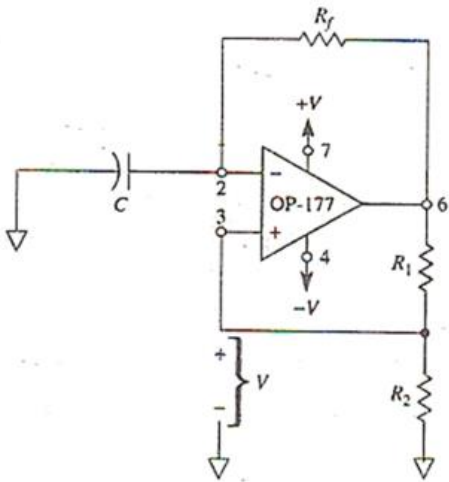
$$\text{so, } (1) \Rightarrow \frac{R_2}{R_1} = 99$$

$$\text{Choose, } R_1 = 1 \text{ K; so } R_2 = 99 \text{ K;}$$

$$\text{and } V_S = \frac{R_2}{R_1 + R_2} V_{\text{REF}} = 1 \text{ V} \Rightarrow V_{\text{REF}} = 1.01 \text{ V}$$



**Question 3 :** Assume for the square wave generator below,  $R_2 = 0.86 * R_1$  and  $+V_{sat} = -V_{sat}$  .  
Prove that  $T = 2R_1C$



Solution

Given,  $R_2 = 0.86 R_1$  and if  $+V_{sat} = -V_{sat}$ , proof that

$$T = 2 R_f C.$$

$$\text{Now } T_1 = R_f C \ln \left| \frac{+V_{sat} - V_{LT}}{+V_{sat} - V_{UT}} \right|$$

$$= R_f C \ln \left| \frac{+V_{sat} - \frac{R_2}{R_1 + R_2} (-V_{sat})}{+V_{sat} - \frac{R_2}{R_1 + R_2} (+V_{sat})} \right|$$

$$= R_f C \ln \left| \frac{+V_{sat} + \frac{R_2}{R_1 + R_2} V_{sat}}{V_{sat} - \frac{R_2}{R_1 + R_2} V_{sat}} \right|$$

$$= R_f C \ln \left| \frac{\frac{R_1 + 2R_2}{R_1 + R_2}}{\frac{R_1}{R_1 + R_2}} \right|$$

$$T_1 = R_f C \ln \left| \frac{R_1 + 2R_2}{R_1} \right| \dots \text{--- (1)}$$

Now since  $R_2 = 0.86 R_1$ , substituting in eqn (1)

$$T_1 = R_f C \ln \left| \frac{R_1 + 1.72 R_1}{R_1} \right|$$

$$= R_f C \ln | 2.72 |$$

$$S_o \quad T_1 = R + C$$

similarly, we can prove that

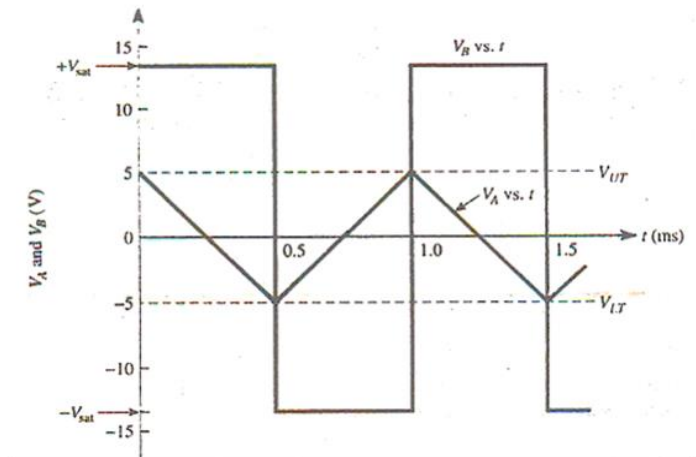
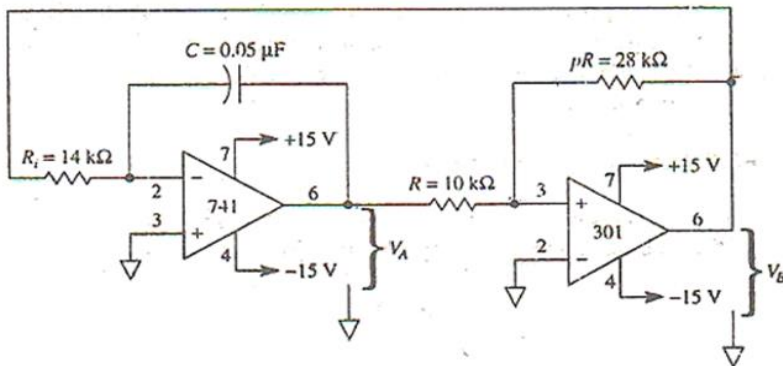
$$T_2 = R + C$$

$$S. \quad T = T_1 + T_2 = 2R + C \quad [proves]$$

**Question 4:** Assume for the triangular wave generator below,  $+V_{sat} = -V_{sat}$ .

Prove that

$$f = \frac{p}{4R_i C}$$



We know,

$$V_{UT} = \frac{V_{sat}}{P}$$

$$, \text{ so, } V_{UT} - V_{LT} = \frac{2V_{sat}}{P}$$

$$V_{LT} = \frac{-V_{sat}}{P}$$

$$V_{LT} - V_{UT} = -\frac{2V_{sat}}{P}$$

Now

$$T_1 = R_i C \left( \frac{V_{UT} - V_{LT}}{V_{sat}} \right)$$

$$= R_i C \left( \frac{2V_{sat}/P}{V_{sat}} \right)$$

$$= \frac{2R_i C}{P}$$

$$T_2 = R_i C \left( \frac{V_{LT} - V_{UT}}{-V_{sat}} \right)$$

$$= R_i C \left( \frac{-2V_{sat}/P}{-V_{sat}} \right)$$

$$= \frac{2R_i C}{P}$$

$$\therefore T = T_1 + T_2 = \frac{4R_i C}{P}$$

$$f = \frac{P}{4R_i C}$$