Department of Mathematics and Natural Sciences

Semester: Fall 2023

Midterm Examination

Course Title: Mathematics for Machine Learning & Signal Processing

Course Code: MAT 215, Section: 15

Total marks: 20 Date: November 5, 2023

Times: 1 hour (9.0 am -10.0 am) Room: Online

Answer any FOUR including question 1:

Q1. [2+3]

a. Determine whether the function $u = e^{-x}(x\ sin\ y - y\cos y)$ is harmonic. For harmonic function, find the conjugate harmonic function v and express $u + iv$ as an analytic function of z.

b. Find the indicated roots and locate them graphically: $\left(-2\sqrt{3} - 2i\right)^{\frac{1}{5}}$

Q2. [2+2+1]

a. State the necessary and sufficient conditions for being a function analytic. Hence show that the function $sinh\ 4z$ is analytic.

b. Graph and express in rectangular form: $2e^{\frac{5\pi i}{4}}$.

c. If $\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are reals then find the values of x and y.

Q3. [2+3]

a. Find $\lim_{z\to 0}\left(\frac{\sin z}{z}\right)^{\frac{1}{z^2}}$.

b. Suppose A, B, C are the point set defined by $|z + i| < 3, |z| < 5, |z + 1| < 4$. Represent graphically $(A \cap B \cap C)$.

Q4. [2+3]

a. Show that $sin^{-1}z = -i\ log[iz + (1 - z^2)^{\frac{1}{2}}]$.

b. Show that $ln\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \left(\frac{4\pi}{3} + 2k\pi\right)i, k = 0, \pm 1, \pm 2, \ldots\ldots$

What is the principal value?

Q5. [2+2+1]

a. Suppose $f(z) = 3z^2 + 2z$. Prove that $\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} = 6z_0 + 2$.

b. Show that $f(z) = \frac{z^2+1}{z^2-3z+2}$ is continuous for all z outside $|z| = 2$.

c. Evaluate $tanh^{-1}\infty$.