ID : 21301163 (1)

প্র 33: $y' + 6y = e^{4t}$, $y(0) = 2$

Using Laplace transform,

$$L(y') + L(6y) = L(e^{4t})$$

$$sY(s) - y(0) + 6Y(s) = \frac{1}{s-4}$$

$$sY(s) - 2 + 6Y(s) = \frac{1}{s-4}$$

$$Y(s)(s+6) = \frac{1}{s-4} + 2$$

$$Y(s)(s+6) = \frac{1+2s+8}{s-4}$$

$$Y(s) = \frac{2s-7}{(s-4)(s+6)}$$

Using partial fraction, $\frac{2s-7}{(s-4)(s+6)} = \frac{A}{s-4} + \frac{B}{s+6}$

$$\Rightarrow \frac{2s-7}{(s-4)(s+6)} = \frac{As + 6A + Bs - 4B}{(s-4)(s+6)}$$

$$\Rightarrow \frac{2s-7}{(s-4)(s+6)} = \frac{s(A+B) + 6A - 4B}{(s-4)(s+6)}$$

$$\Rightarrow 2s - 7 = s(A+B) + 6A - 4B$$

Equiting Like-terms, $A + B = 2$ $\Rightarrow 6A - 4B = -7$

Solving these equations, $A = \frac{1}{10}$ & $B = \frac{19}{10}$

$$\frac{2s-7}{(s-4)(s+6)} = \frac{1}{10(s-4)} + \frac{19}{10(s+6)}$$

$$Y(s) = \frac{1}{10(s-4)} + \frac{19}{10(s+6)}$$

Using inverse Laplace transform,

$$\boxed{y(t) = \frac{1}{10} e^{4t} + \frac{19}{10} e^{-6t}}$$ Ans

36: $y'' - 4y' = 6e^{3t} - 3e^{-t}$; $y(0) = 1$, $y'(0) = -1$.

We know, $L(F') = sL(F) - F(0)$ & $L(F'') = s^2 L(F) - sF(0) - F'(0)$

So, the given differential equation is equated to,

$L(y'') - 4L(y') = L(6e^{3t} - 3e^{-t})$ where, $F(0) = y(0) = 1$
$F'(0) = y'(0) = -1$ — (1)

$\Rightarrow s^2 L(y(t)) - sF(0) - F'(0) - 4(sL(y(t)) - F(0))$
$= L(6e^{3t}) - L(3e^{-t})$ [Using linearity property of laplace transformation & using the initial value we get from (1)]

$\Rightarrow s^2 L(y(t)) - s\times 1 + 1 - 4sL(y(t)) + 4 = \frac{6}{s-3} - \frac{3}{s+1}$

$\therefore L(e^{at}) = \frac{1}{s-a}$

$\Rightarrow \cancel{s^2 L(y} (s^2 - 4s) L(y(t)) - 5 - s = \frac{3s+15}{s^2 - 2s - 3}$

$\Rightarrow L(y(t)) = \dfrac{\frac{3s+15}{s^2-2s-3} - (5-3)}{s^2 - 4s}$

$\Rightarrow L(y(t)) = \dfrac{s^3 - 7s^2 + 10s + 30}{s^4 - 6s^3 + 5s^2 + 12s}$

$= \frac{-2}{s-3} + \frac{5}{2s} - \frac{3}{5(s+1)} + \frac{11}{10(s-4)}$

Taking the inverse laplace transformation,

$y(t) = L^{-1}\left\{\frac{-2}{s-3} + \frac{5}{2s} - \frac{3}{5(s+1)} + \frac{11}{10(s-4)}\right\}$

Using the linearity property of inverse L.T,

$y(t) = L^{-1}\left(\frac{2}{s-3}\right) + L^{-1}\left(\frac{5}{2s}\right) - L^{-1}\left(\frac{3}{5(s+1)}\right) + L^{-1}\left(\frac{11}{10(s-4)}\right)$

$$\boxed{\therefore y(t) = -2e^{3t} - \frac{3}{5}e^{-t} + \frac{11}{10}e^{4t} + \frac{5}{2}}$$

Ans

Q39: $2y''' + 3y'' - 3y' - 2y = e^{-t}$, $y(0) = 0$, $y'(0) = 0$ & $y''(0) = 1$.

We can write,

$$2s^3 Y(s) - 2s^2 \cancel{y(0)} - 2s\cancel{y'(0)} - 2y''(0) + 3s^2 Y(s) - 3s\cancel{y(0)} - 3\cancel{y'(0)}$$
$$- 3sY(s) + 3\cancel{y(0)} - 2Y(s) = \frac{1}{s+1}$$

$$\Rightarrow (2s^3 + 3s^2 - 3s - 2) \cdot Y(s) = \frac{1}{s+1} + 2 = \frac{1+2(s+1)}{s+1}$$

$$Y(s) = \frac{2s+3}{(s+1)(2s^3+3s^2-3s-2)}$$

$$= \frac{2s+3}{(s+1)(s-1)(2s^2+5s+2)}$$

$$= \frac{2s+3}{(s+1)(s-1)(s+2)(s+\frac{1}{2})}$$

Now,
$$\frac{2s+3}{(s+1)(s-1)(s+2)(s+\frac{1}{2})} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2} + \frac{D}{s+\frac{1}{2}} \quad \text{---(1)}$$

So,
$$2s+3 = A(s-1)(s+2)(s+\tfrac{1}{2}) + B(s+1)(s+2)(s+\tfrac{1}{2}) + C(s+1)(s-1)(s+\tfrac{1}{2}) + D(s+1)(s-1)(s+2)$$

If, $s = -1$ then $1 = A$

" , $s = 1$ then $5 = 9B \Rightarrow B = \frac{5}{9}$

" , $s = -2$ then $-1 = \frac{3}{2}C \Rightarrow C = \frac{2}{9}$

" , $s = -\frac{1}{2}$ then $2 = \frac{-9}{8} \Rightarrow D = -\frac{16}{9}$

putting the values in (1),

$$y(s) = \frac{1}{s+1} + \frac{5/9}{s-1} + \frac{2/9}{s+2} + \frac{-16/9}{s+\frac{1}{2}}$$

$$y(t) = e^{-t} + \frac{5}{9}e^{t} + \frac{2}{9}e^{-2t} - \frac{16}{9}e^{-1/2 t}$$

$$\boxed{\therefore y(t) = -\frac{8}{9}e^{-1/2} + 1/9\, e^{-2t} + \frac{5}{18}e^{t} + \frac{1}{2}e^{-t}.}$$

Ans

ID: 2130163

40: $y''' + 2y'' - y' - 2y = \sin 3t$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$.

We can write,

$$s^3Y(s) - s^2 y(0) - \cancel{s y'(0)} - y''(0) + 2s^2Y(s) - \cancel{2sy(0)} - 2y(0)$$
$$- sY(s) + y(0) - 2Y(s) = \frac{3}{s^2+9}$$

Now,

$$(s^3 + 2s^2 - s - 2)\cdot Y(s) = \frac{3}{s^2+9} + 1 = \frac{3 + s^2 + 9}{s^2+9}$$

$$Y(s) = \frac{s^2+12}{(s^2+9)(s^3+2s^2-s-2)}$$

$$\Rightarrow Y(s) = \frac{s^2+12}{(s^2+9)[s^2(s+2)-(s+2)]}$$

$$\Rightarrow Y(s) = \frac{s^2+12}{(s^2+9)(s+2)(s+1)(s-1)}$$

Now,

$$\frac{s^2+12}{(s^2+9)(s+2)(s+1)(s-1)} = \frac{As+B}{s^2+9} + \frac{C}{s+2} + \frac{D}{s+1} + \frac{E}{s-1} \quad (1)$$

So,

$$s^2 + 12 = (As+B)(s+2)(s+1)(s-1) + C(s^2+9)(s+1)(s-1) + D(s^2+9)(s+2)(s-1) + E(s^2+9)(s+2)(s+1)$$

When $s = -2$ then, $39C = 16 \Rightarrow C = \frac{16}{39}$

If, $s = -1$ then, $13 = -20D \Rightarrow D = \frac{-13}{20}$

" , $s = 1$ then, $13 = 60E \Rightarrow E = \frac{13}{60}$

And, when, $s \to \infty$ then, $0 = A + C + D + E$ so, $A = \frac{3}{130}$

" $s = 0$, $12 = -2B - 9C - 18D + 18E$. So, $B = \frac{-3}{65}$

Putting values in (1),

$$Y(s) = \frac{3/130}{s^2+9} + \frac{-3/65}{s^2+9} + \frac{16/39}{s+2} + \frac{-13/20}{s+1} + \frac{13/60}{s-1}$$

$$\therefore \boxed{y(t) = \frac{3}{130}\cos 3t - \frac{1}{65}\sin 3t + \frac{16}{39}e^{-2t} - \frac{13}{20}e^{-t} + \frac{13}{60}e^{t}}$$

Ans