(21) $y' + 4y = e^{-4t}$

$y(0) = 2$

The laplace tranform of the differential equation is,

$$s\mathcal{L}(y) - y(0) + 4\mathcal{L}(y) = \frac{1}{s+4}$$

Now,

$$\mathcal{L}(y) = \frac{1}{(s+4)^2} + \frac{2}{s+4}$$

$$\therefore y = te^{-4t} + 2e^{-4t}$$

(23) Given,

$y'' + 2y' + y = 0$

$y(0) = 1$

$y'(0) = 1$

$$\therefore s^2\mathcal{L}(y) - sy(0) - y'(0) + 2[s\mathcal{L}(y) - y(0)] + \mathcal{L}(y) = 0$$

Now,

$$\mathcal{L}(y) = \frac{s+3}{(s+1)^2} = \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

$$\therefore y = e^{-t} + 2te^{-t}$$

(26) $y'' - 4y' + 4y = t^3$

$y(0) = 1$

$y'(0) = 0$

$$\mathcal{L}(y) = \frac{s^5 - 4s^4 + 6}{s^4(s-2)^2}$$

$$= \frac{3}{4}\frac{1}{s} + \frac{9}{3}\frac{1}{s^2} + \frac{3}{4}\frac{2}{s^3} + \frac{1}{4}\frac{3!}{s^4} + \frac{1}{4}\frac{1}{s-2} - \frac{13}{8}\frac{1}{(s-2)^2}$$

Thus,

$$y = \frac{3}{4} + \frac{9}{8}t + \frac{3}{4}t^2 + \frac{1}{4}t^3 + \frac{1}{4}e^t - \frac{13}{8}e^{2t}$$

(29) Given,

$y'' - y' = e^t \cos t$

$y(0) = 0$

$y'(0) = 0$

$$\therefore s^2\mathcal{L}\{y\} - sy(0) - y'(0) - s\mathcal{L}\{y\}\cdot y(0) = \frac{s-1}{(s-1)^2+1}$$

Now,

$$\mathcal{L}\{y\} = \frac{1}{s(s^2-2s+2)}$$

$$= \frac{1}{2}\frac{1}{s} - \frac{1}{2}\frac{s-1}{(s-1)^2+1} + \frac{1}{2}\frac{1}{(s-1)^2+1}$$

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Thus,

$$y = \frac{1}{2} - \frac{1}{2} e^{t} \cos t + \frac{1}{2} e^{t} \sin t$$

(30) $y'' - 2y' + 5y = 1 + t$

$y(0) = 0$

$y'(0) = 4$

$\therefore s^2 \mathcal{L}\{y\} - s y(0) - y'(0) - 2[s\mathcal{L}\{y\} - y(0)] + 5\mathcal{L}\{y\} = \frac{1}{s} + \frac{1}{s^2}$

Now,

$$\mathcal{L}\{y\} = \frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)}$$

$$= \frac{7}{25} \cdot \frac{1}{s} + \frac{1}{5} \frac{1}{s^2} + \frac{-75/25 - 109/25}{s^2 - 2s + 5}$$

$$= \frac{7}{25} \frac{1}{s} + \frac{1}{5} \cdot \frac{1}{s^2} - \frac{7}{25} \cdot \frac{s-1}{(s-1)^2 + 2^2} + \frac{51}{25} \cdot \frac{2}{(s-1)^2 + 2^2}$$

Thus,

$$y = \frac{7}{25} + \frac{1}{5} t - \frac{7}{25} e^{t} \cos 2t + \frac{5t}{25} e^{t} \sin 2t.$$