Autonomous NNR based Non-Periodic Oscillator

Shruti Murarka
Electronics and Communication Engineering Department
Visvesvaraya National Institute of Technology
Nagpur, India
shrutim@students.vnit.ac.in

***Abstract*—In this paper, an autonomous circuit that behaves chaotically has been presented. The analysis of Chua's circuit offers an understanding of how to design an electronic circuit to exhibit chaotic behaviour, which can then be used to create applications based on the same and techniques to prevent chaos in a circuit. Ngspice simulation toolbox is used to model the circuit.**

***Index Terms*—Chua's circuit, Nonlinear characteristic, Double Scroll**

I. Introduction

The theory of the Linear system has been thoroughly developed, and mathematical tools are available to analyze such systems. Many observed observations are dismissed by many scientists and experimentalists because linear structure theory cannot describe them. In the last decade, there has been a surge of interest in studying systems of unusually complex waveforms, also known as strange attractors. These attractors have become extremely popular in a number of nonlinear deterministic systems.

As a result, it is important that today's students are introduced to these dynamic chaos phenomena. In terms of education, students must learn not only how to control and avoid chaos but also how to design chaotic circuits and create applications that explore these phenomena. Understanding the educational importance of exposing undergraduate students to the phenomenon of chaos, Lonngren [2] outlines a fascinating circuitry experiment to demonstrate the nature of chaos. The stated laboratory experiment, along with the corresponding theory, is a good starting point for the student to comprehend and understand chaos. As a continuation of this aim and to improve students' understanding of chaos, Hamill [3] proposed a series of ten chaotic circuits simulated using PSpice. These circuits, some of which are very basic, show how chaos can be created.

This paper can be viewed as a brief study of Chua's Circuit using operational amplifiers and BJT transistors. The circuit's simplicity has made it a popular real-world example of a chaotic system, has become a universal paradigm for chaos. It is a result of attempts to enhance students' knowledge of not just how chaos is simulated but also how to execute it.

II. Chua's Circuit

A. History

Leon Chua grasped the mechanism that caused chaos in both Lorenz' and Rössler's equation models. Following his analysis of the unsuccessful effort to replicate Lorenz' equations electronically, Chua chose to construct a simplified circuit with the same chaos generating characteristics.

Chua used a systematic non-linear circuit synthesis technique to construct a circuit. After a two-year battle, Matsumoto successfully programmed the circuit equations on a computer. Chaos was confirmed by the appearance of a strange attractor [1].

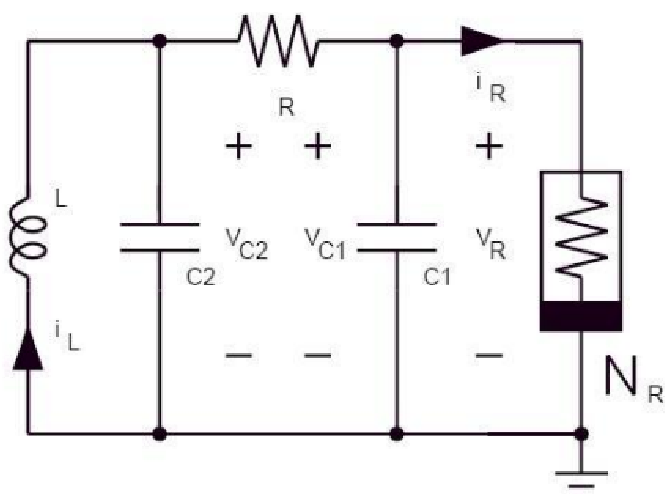


Fig. 1: Chua's Circuit.

B. Chaotic criteria

Autonomous circuit must contain:

1) one or more nonlinear elements,
2) one or more locally active resistors,
3) three or more energy-storage elements.

To display chaotic behaviour. Chua's Circuit is the most fundamental electronic circuit that meets these specifications. Fig 1 shows Chua's circuit, which includes two capacitors, a resistor, an inductor and a nonlinear resistor NR. The two capacitors and inductor acts as energy storing element. A "locally active resistor" is an active device with negative resistance that provides the power to produce the oscillating current. The unit NR, also known as "Chua's diode", incorporates the locally active resistor and nonlinearity.

C. Dynamics

Applying Kirchhoff's Current Law & Kirchhoff's Voltage Law, we get

$$
\begin{aligned}
C_1\frac{dV_1}{dt} &= \frac{(V_1 - V_2)}{R} - f(V_1) \\
C_2\frac{dV_2}{dt} &= \frac{(V_1 - V_2)}{R} + i_3 \\
L\frac{di_3}{dt} &= -V_2
\end{aligned} \tag{1}
$$

Equation (1) can also be written as

$$
\begin{aligned}
\frac{dV_1}{dt} &= \frac{1}{C_1}[G(V_1 - V_2) - f(V_1)] \\
\frac{dV_2}{dt} &= \frac{1}{C_2}[G(V_1 - V_2) + i_3] \\
\frac{di_3}{dt} &= -\frac{V_2}{L}
\end{aligned} \tag{2}
$$

Dimensionless version of (2) is

$$
\begin{aligned}
\frac{dx}{dt} &= \alpha[y - x - f(x)] \\
\frac{dy}{dt} &= x - y + z \\
\frac{dz}{dt} &= -\beta y
\end{aligned} \tag{3}
$$

Here, x(t) represents voltage around capacitor C_1,
y(t) represents voltage around capacitor C_2,
z(t) represents current in conductor L_1,
& f(x) defines the electrical response of the nonlinear resistor.

D. Negative Nonlinear Resistor (NNR)

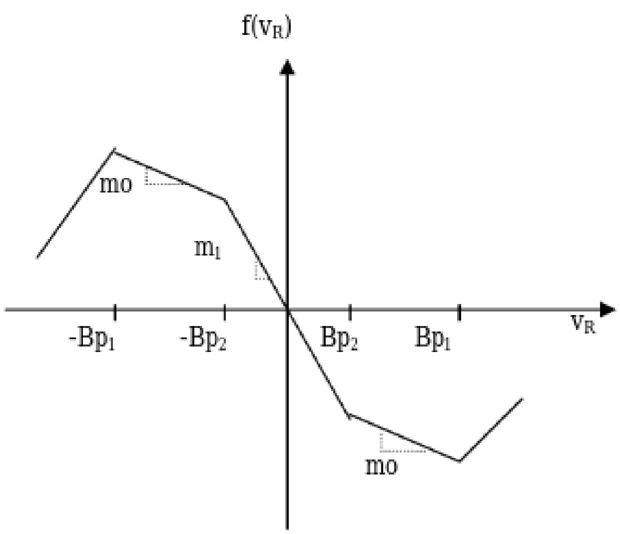


Fig. 2: I vs V Graph for Chua's Diode(Nr).

An ordinary resistor's I vs V graph is a positive sloped straight line. These resistors, in addition to their other functions, absorb electrical energy from the voltage or current sources in a circuit and generate heat. A negative resistor, on the other hand, provides energy to the circuit and has a negative slope on the I vs V graph.

The nonlinear Chua's function is described as
$f(V_r) = m_0V_r + \frac{1}{2}(m_1 - m_0)[|V_r + B_p| - |V_r - B_p|]$

III. Circuit Design

Chua's Diode is not commercially available, but it is implemented in different forms by active circuits.

A. Using Op-Amp [4]

Figure 3 shows implementation of Chua's Circuit. The nonlinear resistor is implemented by the combination of two operational amplifiers and six linear resistors. The desired V-I characteristics is produced by connecting two op-amp voltage-controlled negative resistance convertors Nr_1 and Nr_2 in parallel.

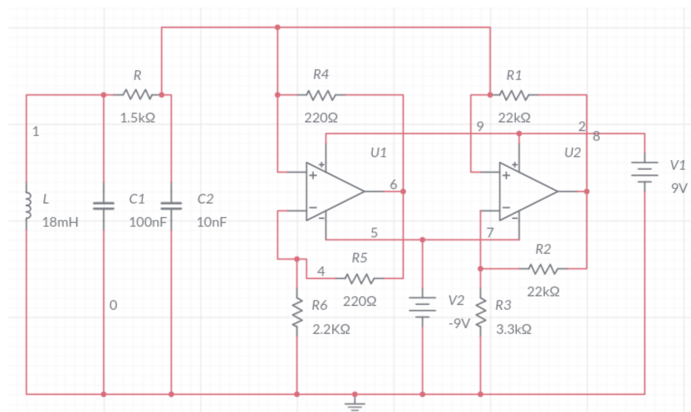


Fig. 3: Realization of Chua's Circuit using 2 opamps & 6 linear resistors to implement Nr.

The slopes m_0, m_1 & breakpoints B_{p1}, B_{p2} can be calculated as:

$$
\begin{aligned}
m_1 &= -\frac{R_2}{R_1R_3} - \frac{R_5}{R_4R_6} \\
m_0 &= -\frac{R_2}{R_1R_3} + \frac{1}{R_4} \\
B_{p1} &= \frac{R_3}{R_2 + R_3}E_{sat} \\
B_{p2} &= \frac{R_6}{R_5 + R_6}E_{sat}
\end{aligned} \tag{4}
$$

where E_{sat} is saturation voltage of the operational amplifier.

B. Using BJT Transistors

Figure 4 shows implementation of Chua's circuit. The nonlinear resistor is implemented by the combination of two diodes, six linear resistors and four npn and two pnp transistors.

The op-amps are replaced by the combination of a differential amplifier, level shifter and output stage. A differential amplifier is made up of 2 transistors and outputs the amplified version of the difference between the two input signal. Dual input balanced output is used. A level shifter is used to shift dc output voltage to zero. At the output stage, voltage is amplified. Figure 4 shows the implementation of the circuit by replacing the Chua's Diode using transistors.

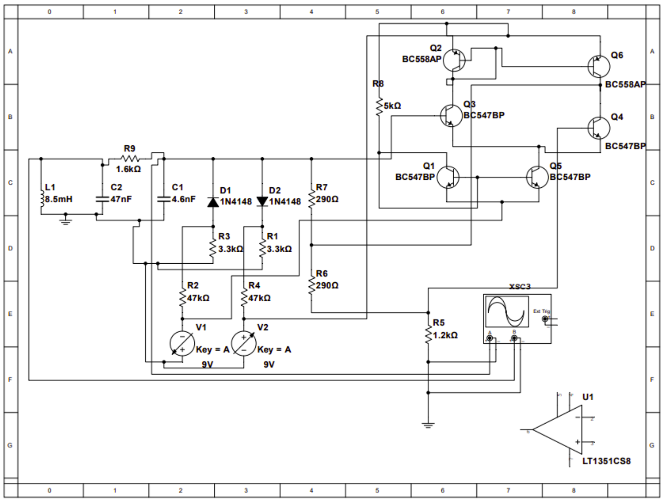


Fig. 4: Realization of Chua's Circuit using BJT Transistor to implement Nr.

IV. Simulation Results using Ngspice

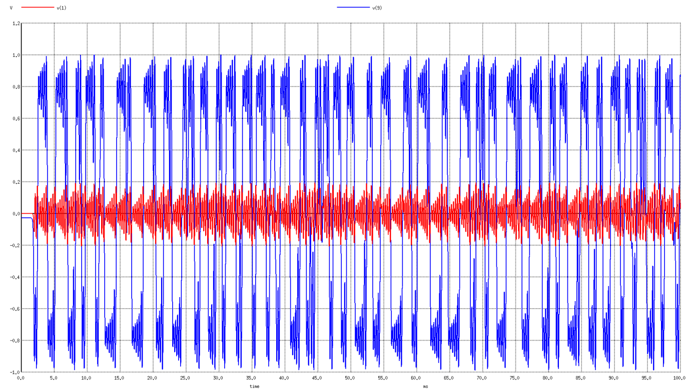
E_{sat} is usually given in the datasheet of OpAmp. [4]

- We choose R_1 so that it will not significantly load the amplifier.
- $R_2 = R_1 = 22K\Omega$
- $R_3 = \frac{E_{sat}}{(B_{p2}-E_{sat})m_0 - B_{p2}m_1} = 3.3K\Omega$
- $R_4 = \frac{E_{sat}}{B_{p2}m_0 - m_1}$
- $R_5 = R_4 = 220\Omega$
- $R_6 = \frac{E_{sat}}{(E_{sat}-B_{p2})(m_0 - m_1)} = 2.2K\Omega$

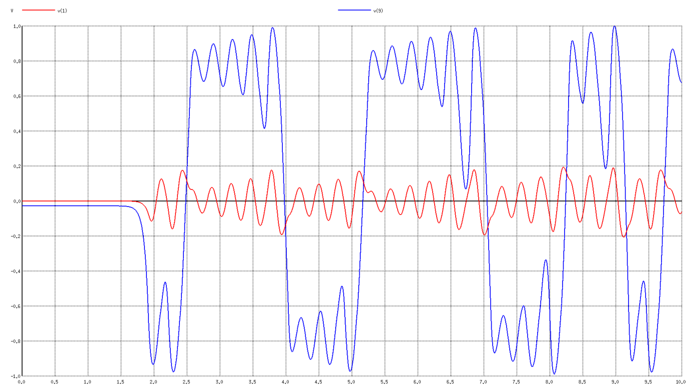
The circuits are powered by two 9V sources, one positive and one negative. Output are observed across resistor R,(nodes having capacitors C_1 and C_2).

R can be adjusted from 0 to 2kΩ to fine-tune the circuit. Chaos exists somewhere between the two extremes. I found 1.6kΩ a good value. Transient solution of 100ms gives the waveforms V_{C1} and V_{C2} shown in fig 5a. Voltage V_2 seems to be bouncing around two different DC levels. A closer view shows details are shown in fig 5b. When I change the x-axis to be V2 and the y-axis to V1, we get the beautiful double scroll plot shown in fig 5c. I also plotted the frequency domain plot, and we can see audible range frequencies are included as shown in fig 5d.

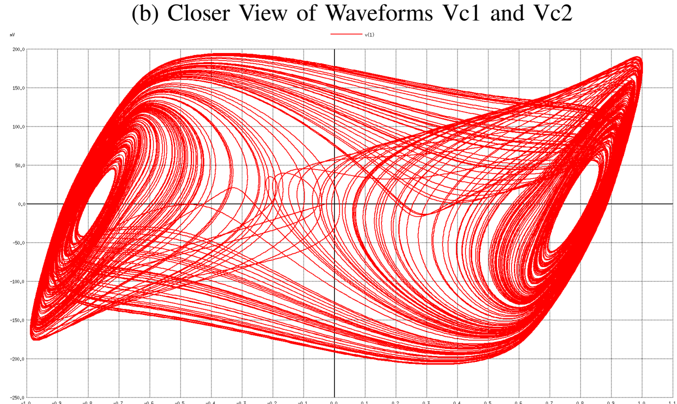
Similar analysis for transistor circuit is shown in 6. I found $R_9 = 1.6K\Omega$ tuned value for chaos. The transient solution of 100ms gives the waveforms V_{C1} and V_{C2} shown in fig 6a. For details, closer view is shown in fig 6b.



(a) Waveforms for Vc1 and Vc2



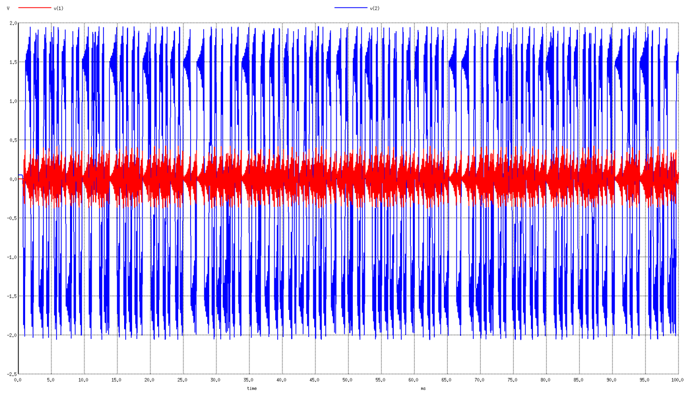
(b) Closer View of Waveforms Vc1 and Vc2



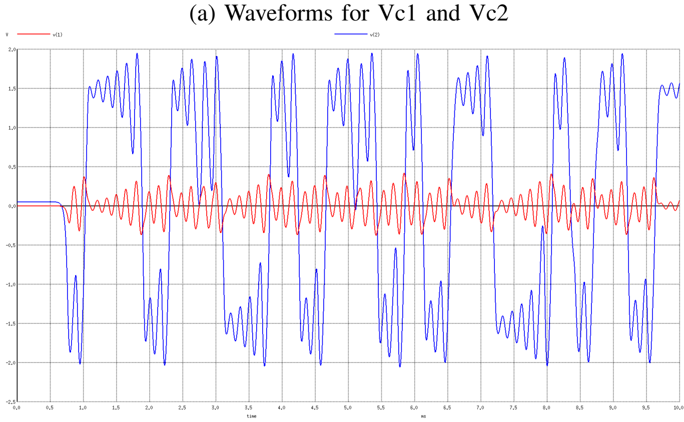
(c) Double Scroll Plot

(d) FFT Plot (0-20KHz)

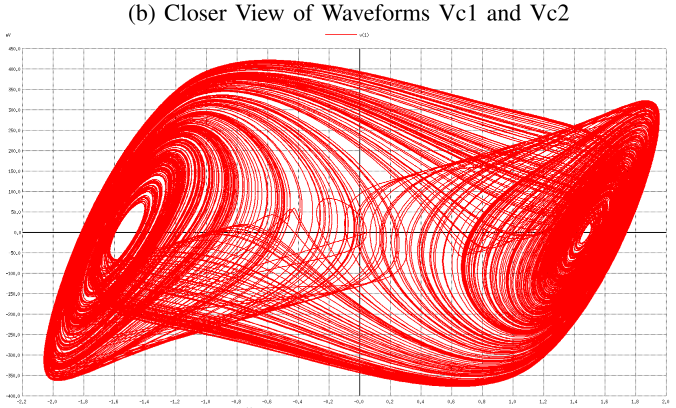
Fig. 5: Output for Chua's OpAmp based Model



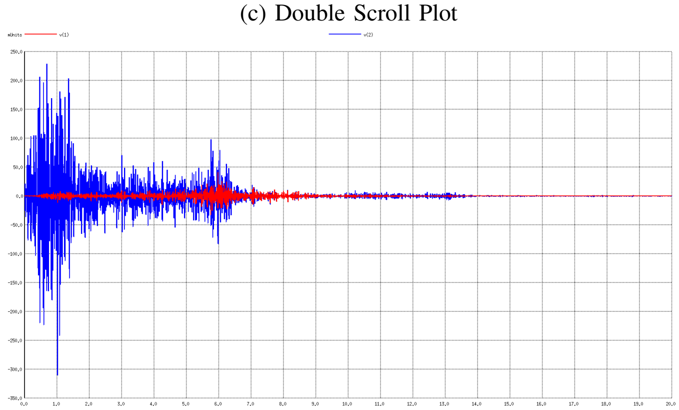
(a) Waveforms for Vc1 and Vc2



(b) Closer View of Waveforms Vc1 and Vc2



(c) Double Scroll Plot



(d) FFT Plot (0-20KHz)

Fig. 6: Output for Chua's Transistor based Model

Double scroll Plot (V_{C1} vs V_{C2}) is shown in fig 6c. It is also observed from frequency domain plot shown in fig 6d, circuit has audible frequencies.

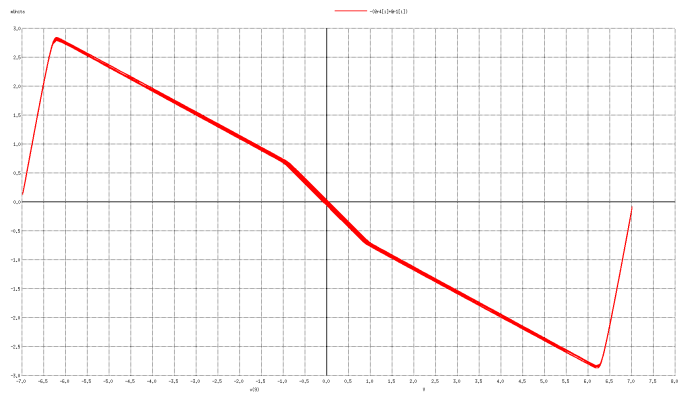


Fig. 7: IV Characteristics of Chua's Diode (Nr).

The VI characteristics across Chua's Diode (Nr) is shown in fig 7. It resembles characteristics of negative nonlinear resistance shown in fig 2.

V. Conclusion

An implementation of a Chua's Circuit has been presented in this paper. Chua's Circuit generates time-varying output without requiring a time-varying input, making it an autonomous circuit. I have gone over the circuit theory concepts that underpin the design of negative resistors. I have defined a design technique for synthesizing a Chua Diode by linking two op-amps in parallel and also using transistors.

From ngspice simulations of Chua's Circuit models, realizations of Chua's Diode using transistors and op-amps exhibit similar dynamical behaviours. Both emulates the non-periodic oscillator's dynamical actions, such as limit cycle oscillations, unstable/chaotic double scrolls, and boundary crises [7]. Both of Chua's generalized circuits, which are optimal for IC development, are designed to produce robustly stable chaotic signals.

References

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