To: Dr. Zhixin Wu
From: Shuto Araki, Muhammad Shahraiz Niazi, Syed Mehdi
Subject: **Financial Engineering Project 1**
Date: March 11, 2018

Memo: Optimal Portfolio Construction and the Efficient Frontier

I. Introduction

The finance and investing world was transformed when the Modern Portfolio Theory was introduced by Harry Markowitz in the middle of the 20th century. The theory suggests that investors can construct an optimal portfolio for a given amount of risk, which will allow them to maximize their return. With the help of this theory, investors have different combinations of portfolios that form the "efficient frontier." Investors can choose to invest at any point on this frontier for maximum returns depending on how risk-averse they are.

The theory also emphasizes a fundamental part of investing: diversification. Investing in a single stock is dangerous, as the stock may fall victim to a variety of factors such as internal company issues, industry-related issues, and/or economy wide issues. Unsystematic risks can be reduced by holding a variety of stocks, which allows for the portfolio to not be affected by just one company/industry. The Modern Portfolio Theory helps us understand how diversifying our portfolio helps us reduce the overall portfolio risk (which will be lower than the risk of an individual asset) and how to properly allocate weightage to these assets to construct the efficient frontier.

This project looks to use Markowitz's theory and apply it to our portfolio in order to find the efficient frontier. More specifically, the efficient frontier can be obtained by calculating the minimal risk for any given level of return.

Our portfolio contains two large cap stocks (Goldman Sachs Group Inc. (GS) and Home Depot Inc. (HD)), one small cap stock (ADTRAN(ADTRAN)), a corporate bond (iShares IBoxx $ Invest Grade Corp Bd Fd(LQD)), and a treasury (iShares Barclays 20+ Yr Treas.Bond (ETF)(TLT)).

II. Findings

As stated above, the two major takeaways from the Modern Portfolio Theory are diversification and maximizing returns for a specific amount of risk for an investor. The unconstrained efficient frontier maximizes the portfolio return by changing the weights given to the 5 assets. Our results show us that Home Depot Inc.,ADTRAN and the corporate bond are sometimes given negative weightages (short selling) in order to construct the unconstrained efficient frontier. This frontier gives us a range of expected risk, ranging from approximately 0.580% to 2.088%. In comparison, the constrained efficient frontier has a lower range of expected risk, falling between approximately 0.732% to 2.226%. This difference would have been much more observable had it not been for the last data point in the constraint frontier, as it tries to optimize the return by taking a steeper risk. This shows that the portfolio's expected risk can be reduced by diversification because the constrained frontier is forced to take at least 5% of each asset and not more than 50%.

The 5, 10, 20, and 30 year indices also give us valuable information. When we look at the optimum weights used for the indices, we find that as a higher return is required, our portfolio gives a higher weightage to stock indices than it does to bond indices. The S&P 500, which looks at large cap stocks, is usually weighed

higher than the Russell 2000, which looks at large and small cap stocks. Bond indices are usually given minimum weight as higher returns are desired but they help keep the risk down since they are safer assets.

Lastly, we can see that our portfolio is weighed similarly as the indices. Most of the weightage is given to large cap stocks, since they are usually financially well-off companies and perform to a high caliber. Weightage given to corporate bonds and US treasurys in both portfolios generally follow the same trend; the indices are higher when the expected return is lower and falls to the minimum weightage as we require a higher return. Trends in small cap stocks is less clear, as it fluctuates but ends up going higher for the indices but stays on the minimum weightage for our small cap stock (ADTRAN). However, this can be attributed to the consistent negative returns for our small cap stock due to poor stock performance in the observed 2 year period.

III. Discussion

III. a. Methods

Data Collection:

For the 5 asset portfolio, weekly adjusted closing price data in the recent 2 years for aforementioned stocks and bonds were collected from Yahoo Finance. Adjusted closing price was used in order to take dividends and splits into account to better represent the price movement in history.

For the indices portfolio, monthly historical price data in the recent 30 years were given for this project: S&P 500 index for large cap company stocks, Russell 2000 for small cap company stocks, U.S. Corporate Investment Grade for company bonds, and U.S. Treasury for the U.S. government bond (zero default-risk). S&P 500 and Russell 2000 are dividends and splits adjusted. The indices portfolio is analyzed in order to provide the economic overview in comparison to the five assets under different timeframes: recent 5, 10, 20, and 30 years.

Construction of Efficient Frontiers:

The Excel Solver was utilized to write macros to find unconstrained and constrained efficient frontiers. The objective function was set to minimize the risk. The Solver changes the weights (except for one that is adjusted so that the total weight becomes one) to calculate the expected return and the expected risk. Since the risk (standard deviation of the portfolio price) is a quadratic function of the weights, the Solver can always find the global minimum risk (Capiński, 73).

For the unconstrained efficient frontier, since the possible portfolio return and risk are infinite, the limits were set from minimum return of an individual security (TLT in this case) to maximum return of an individual security (GS in this case). Thus, a weight range from -100% to +100%. However, it is important to note that the unconstrained efficient frontier is impractical. Heavily investing all the assets into one or two risky assets does not diversify the portfolio.

For the constrained efficient frontier, the weight constraints are added so that all the weights will be between 5% and 50%. This constraint ensures the diversification of the investment and reduce the future risk. Since the historical data cannot exactly indicate the future risk and return, it is essential to make sure the investments are diversified.

III. b. Analysis

a. 5 Asset Portfolio Analysis

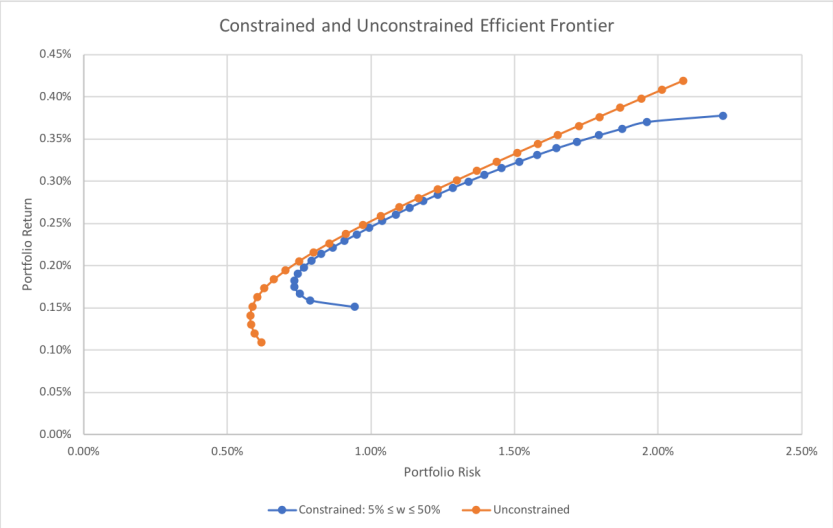

Figure 1: Constrained and Unconstrained Efficient Frontier

As we can see from figure 1, the unconstrained frontier has a higher range of return (0.109% to 0.419%) than the constrained frontier (0.151% to 0.378%). This higher return can be associated with the fact that the unconstrained portfolio can give more weightage to assets that are doing better in the 2 year historical prices we are looking. However, theories in finance state that a higher return is achievable only through high risk. This can be seen in figure 1, where the unconstrained efficient frontier has a higher range of risk (0.618% to 2.089%) compared to the constrained frontier (0.733% to 2.23%). Note that rational investors only invest on points after the “hook”(as they can earn higher return for same risk) seen in the efficient frontiers. So, while the unconstrained efficient frontier lets us get a higher return, it is riskier than our constrained frontier.

	Expected Returns	Risk	Risk-to-Return Ratio
GS	0.41880%	3.46653%	8.277285639
HD	0.41641%	2.35456%	5.654468444
ADTN	0.21655%	3.75099%	17.32134894
LQD	0.12583%	0.68538%	5.446654805
TLT	0.10879%	1.61671%	14.86081978

Figure 2: Expected Returns, Risk, and Risk-Return ratio

The constrained efficient frontier allows for the portfolio to include all the assets (in a long position) to some degree (5% to 50%), which allows for the unsystematic risk to fall. The unconstrained frontier allows for short selling, which causes high weightage for assets that are doing well or give high returns. In figure 2, we can see the expected return, risk, and risk to return ratio for all our individual assets. As we demand a higher return on our portfolio, the efficient portfolio gives more weightage to assets with higher returns and low risk to return ratios. Our excel file shows that as we require a higher return, the weightage for Goldman Sachs, Home Depot, and the treasury bond increase, while the weightage for ADTRAN and the investment bond decreases. The intuition behind the weightages assigned to GS,HD, and ADTRAN are based on their risk to return ratio and how risk averse an investor is. However, the weightages for our corporate investment and treasury bond are less intuitive. The weightage for the treasury bond increases and weightage for the treasury decreases as we require a higher return, even though the treasury offers both a lower risk and return than the corporate bond. An explanation for this could be the high correlation (80%) between the two assets. The constrained

efficient frontier however, stays true to our analysis as it gives a higher weightage to GS and HD and lower weightage to ADTRAN, the corporate investment, and treasury as we require a higher return.

b. Long Term Indices Portfolio Analysis

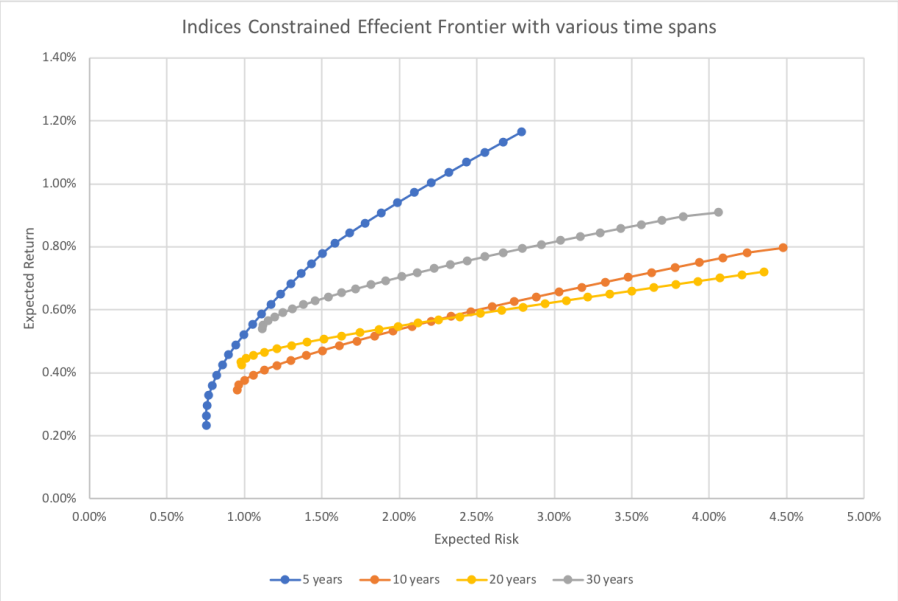

Figure 3: Efficient Frontier for long term indices

	min return	min risk	max return	max risk
5 year	0.23188%	0.75286%	2.78702%	1.16517%
10 year	0.34555%	0.95359%	0.79644%	4.47562%
20 years	0.42509%	0.97805%	0.72122%	4.35394%
30 years	0.53977%	1.11398%	0.90919%	4.05924%

Figure 4: Risk and Return for long term Indices Portfolios

Figure 3 shows the 5,10,20, and 30 year indices portfolio efficient frontiers. The 5 year indices portfolio has the lowest range of risk and the highest range of return as seen in Figure 4. This can be attributed to the strong stock market and economy, which after recovering from the 2008 recession has been doing well. With the economy doing well, more money is demanded in the economy, which has decreased bond prices (since economics tell us that the money market and bond market work against each other). Individuals are investing heavily in the stock market, with indices such as the S&P 500 and DJIA hitting all time highs too. This overall positive trend in the economy is seen by the high returns and low risks associated with the 5 year indices portfolio.

The 10 year indices portfolio has a range of return lower and range of risk higher than the 5 year indices portfolio as seen in figure 4. While the 5 year indices portfolio did well because of the economy and stock market doing well, the 10 year indices portfolio suffers due to the financial crisis of 2008. The 2008 financial crisis had a significant impact on not only the stock market, but the entire U.S. (and world) economy. The crisis, which started due to subprime borrowing and mortgage backed securities, led to one of the largest bear markets in history and caused the financial market to crash. Indices such as the S&P 500 fell by nearly 50% during the crisis and even Corporation after corporation was going through bankruptcy and people were

unwilling to buy stocks or corporate bonds (due to high possibility of default risk). US treasurys, which are often seen as the safest asset, were victims of loss of trust but were highly traded. Bond prices rose significantly higher, since people didn't want to hold money, which caused yields to crash. All of these factors contributed to the higher risk and lower return we see in the 10 year indices portfolio compared to the 5 year indices portfolio.

The 20 year indices portfolio has a higher range of risk than the 5 year indices and has lowest range of expected return for the 4 long term indices as seen in figure 4.This means that investors only reap minimal return with more risk. The 20 year indices portfolio not only had to face the 2008 financial crisis, but also the dot-com bubble. The dot-com bubble, caused by the inflow of investments in to the rapid technological advancements in 1997, saw certain indices like the S&P 500 and DJIA fall by about 50%. The economy, like 2008, again was affected heavily by the financial market, going in to recession. The combined effect of the two major crisis has caused the 20 year indices portfolio to give the lowest return for all the long term indices.

The 30 year portfolio has a range of return that is higher than the 20 year indices portfolio and a range of risk lower than both the 10 year and 20 year indices. The 30 year portfolio suffers from another bear market in 1990 in addition to the 2008 and dot-com one. However, as we can see in figure 3 , the 30 year indices efficient frontier is higher than the 10 year and 20 year indices portfolio. This means it awarded a higher magnitude of return for the same amount of risk, which can be attributed to the strong market of the 90s. This is because while the U.S. economy goes through recessions and bear markets during the 30 year period, it still has substantial growth with in those 30 year periods.

III. c. Limitations

The project has several limitations that could affect the data. Firstly, the two-year data for our project does not provide a clear depiction of the respective stocks throughout their history or of the stock market in total. This may lead to misleading results for our portfolio management. Secondly, the stock market is victim to several macroeconomic issues and financial bear market that make the portfolio theory hard to perfectly implement and interpret. Thirdly, the efficient frontier and the modern portfolio theory have some assumptions that properly depict reality. For example, an asset return follows a normal distribution. Additionally, Markowitz's theory assumes that all investors are rational and avoid risks but in reality there are irrational and risk-seeking investors that can influence the economy greatly. This can cause great fluctuations in the stock prices and may produce faulty analysis of the whole market. Fourthly, the efficient frontier is a moving target. If one looks at the frontier between bonds and equities 10 –year intervals the highest return for the lowest risk ranges from 100% bonds to 100% equity. This is not very efficient.

IV. Conclusion

The 5 asset portfolio was assigned different weightages to optimize the maximum return for different levels of risk. In order to assign weightages, we created two different models: the unconstrained efficient frontier and the constrained efficient frontier. We found that the constrained efficient frontier was better because it gave us lower risk overall than the unconstrained efficient frontier. This was possible because the constrained efficient frontier allowed us to have a more diversified portfolio, which reduced unsystematic risk. We also looked at long term indices portfolios, which were used to construct efficient frontiers for the respective portfolios. Our analysis showed that our 5 asset portfolio and long term indices portfolios generally follow similar trends by assigning a higher weightage to stock assets compared to bond assets as a higher return is desired.

V. References

Capiński, Marek, and Tomasz Zastawniak. *Mathematics for Finance: An Introduction to Financial Engineering*. London: Springer, 2011. Print.

Markowitz, Harry. “Portfolio Selection.” The Journal of Finance, vol. 7, no. 1, 1952, pp. 77–91.

Investopedia."Why the current social security cost-of-living index is hurting retirees." Investopedia. 11 March. 2018. Web. 11 March. 2018.

NBCNews. “11 historic bear markets.” National Broadcasting Company News. 11 March. 2018. Web. 11 March. 2018.