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Memo: Futures Hedging

I. Introduction

There are times when your carefully chosen portfolio gets hammered by inevitable price change by the Fed, Congress, etc. Future contracts are one of the most common derivatives used to protect a security from such risks. The main reason that investors use future contracts is because future positions can offset the gain or loss of their portfolios. One might wonder, "Why don't investors just temporarily sell all the securities if they want to protect their securities?" This method can be a bad strategy because investors cannot gain benefit from dividends and frequent trading costs them transaction fees.

For this project, we used future contracts S&P 500 E-mini and Russell 2000 E-mini to hedge each security. For our portfolio we have 2,000,000 shares of each security that comprise of two large caps namely Goldman Sachs Group Inc. (GS) and Home Depot Inc. (HD) and one small cap ADTRAN, Inc (ADTN). In this case, we worry about the short term loss from the market but also want to keep our holding because we believe that their long run performance is bright. The overall goal for this project is to mitigate the equity exposure risk by shorting some stock-market futures. Transaction fees, taxes and all other fees are ignored in this project to simplify the computation.

II. Findings

After excluding the unsystematic risks by diversification of a portfolio, each asset should be a considerable reflection of its index. An asset's sensitivity to the market return is referred to as beta coefficients and is calculated by the covariance of an asset return and the benchmark return divided by the variance of the benchmark return.

Choosing the appropriate benchmark requires a process that considers various factors:

1. **Risk profile**: The first step is to determine the risk profile, which can include many factors such as how long the funds will be and other financial resources such as cash reserves.
2. **Asset Allocation**: Most portfolios consist of multiple assets including stocks, bonds, commodities and cash. Each asset is weighted against the overall portfolio to determine the percentage of the portfolio that asset is made of. Overall asset model can have multiple benchmarks for each security.

3. **Risk**: Each asset has its own risk assessment and it varies widely amongst companies. One way to measure risk it using the Sharpe Ratio. A higher Sharpe ratio means a superior overall risk-adjusted return.
4. **Building a Benchmark**: Building a custom benchmark requires some kind of software. You can do it for free using software tools offered by ETF companies.

Once a benchmark is chosen you may discover that you are taking too much or too little risk. In this project, however, the securities in our portfolio are given and futures are limited to S&P 500 E-Mini Futures and Russell 2000 E-Mini Futures. Since our portfolio consists of two large caps and one small cap we will be using the return on S&P 500 as a benchmark for the large caps and the return on Russell 2000 as a benchmark for the small cap. The extent to which the stock prices mirror its index values will be evaluated by its beta coefficient.

The rolling window for beta coefficient gives more reliable results by reducing the market idiosyncrasies over a given period of time. These idiosyncrasies might include market crash, rise in foreign exchange pricing, etc. These unexpected events highly influence the calculations for the maximum return of the portfolio. For example, 30-day rolling windows uses 30-day return instead of daily return in beta calculation. As this method is likely to eliminate some unusual market behaviors, rolling window beta is more reliable for hedging in the long run.

The final result did not show any advantages of using dynamic hedging. However, for longer hedging period, we can infer that dynamic hedging performs better as it requires constant re-balancing of the futures positions.

III. Discussion

III. a. Methods

Static and Dynamic Hedging

A static hedging is the one that does not require a constant re-balancing of the portfolio given other characteristic change such as volatility of the stock. Static hedging is not a definite term but it is understood as a way of reducing the frequency of the purchase or sale of the underlying asset. In practice hedging must be done at discrete times and it is costly. Theoretically speaking, the model for the underlying assets in the portfolio is not perfect because we do not know the parameters value accurately.

Dynamic hedging is a technique that seeks to limit an investments exposure to delta and gamma by adjusting the hedge as the underlying security changes. The strategy is frequently used by financial professionals working with derivatives. Derivatives dealers often find that they hold large numbers of short options positions on a risky security which they want to offset by purchasing long options, but that they cannot find long options because these types of options are not as available.

Data Collection and Manipulation

Two different sets of data were collected: historical and current. Historical data of each stock and index were collected through the website *Yahoo! Finance* taking adjusted close prices for daily and monthly data (*Daily Raw Data (Q1)* and *Monthly Raw Data (Q1)* tabs). Current stock and futures price were recorded on every Friday at 3pm for four weeks from practice trading accounts that are assigned for this project. The current prices are used to conduct hedging whose period is four weeks, from March 23, 2018 to April 20, 2018.

With the historical data, daily, monthly, 30-day, and 24-month returns are calculated respectively. The 30-day and 24-month returns are calculated in order to reduce the idiosyncrasies in the price data. This is, as mentioned above, rolling windows. Various beta coefficients were calculated for different time periods (summarized in *Betas (Q3 and Q4)* tab). Different betas tell us the risk contribution of well diversified portfolio to an asset in different time periods. For instance, ADTN has considerably small beta against either of the indices. It means that ADTN carries smaller systematic risk that cannot be diversified away.

Choosing Beta

Since rolling window reduces the effect of unexpected events that should not be accounted for, we concluded that rolling window betas are better for hedging in the end. Under the Capital Asset Pricing Model, the following equation is assumed: $portfolioReturn = r_f + \beta(r_m - r_f)$ where r_f is the return on a risk-free investment and r_m is the return on the market, which is approximated as the return on S&P 500 or Russell 2000. In order to hedge the portfolio well, beta has to be (close to) one because the portfolio return equals to the return on the market. The better the stock price mirrors the market, the less risk the portfolio bears.

As shown in Table 1., it is hard to tell which rolling window (30-Day or 24-Month) is better with the given information. Thus, we decided experiment on both patterns. Since our 30-Day rolling window betas are generally small (especially ADTN is way smaller than 1), we predicted that the 24-Month rolling window betas preserves the initial portfolio value better. However, it is not conclusive because GS occupies about 57% of the portfolio value consistently and beta of 1.37 might be too high

Table 1.

Betas with Rolling Windows			
	Goldman Sachs Group Inc. (GS)	Home Depot Inc. (HD)	ADTRAN, Inc. (ADTN)
30-Day Rolling Window			
S&P 500	0.7603457	1.09273675	-0.369694837
Russell 2000	0.597621114	0.713526581	0.406740803
24-Month Rolling Window			
S&P 500	1.373243928	0.651005877	0.991384794
Russell 2000	0.974528147	0.335411099	1.187882357

III. b. Results

The initial portfolio value was $865,360,000.00.

The number of future contract to short was calculated by the following formula: $N = \beta \frac{V_A}{V_F}$ where V_A denotes the current value of the portfolio and V_F is the current value of one futures contract.

Static Hedging Portfolio

The final portfolio value hedged by 30-Day rolling window beta was $874,790,696.50.

The final portfolio value hedged by 24-Month rolling window beta was $871,762,846.50.

Dynamic Hedging Portfolio

The final portfolio value hedged by 30-Day rolling window betas was $874,798,086.40.

The final portfolio value hedged by 24-Month rolling window betas was $871,717,846.90.

In both cases, the 24-Month rolling window is closer to the initial portfolio value by about 47%. Thus, it is significantly better to use the 24-Month rolling window in this hedging. Comparing the dynamic hedging with the static hedging, we can see that the dynamic hedging is slightly better, but the difference is so slight that it is probably not appropriate to conclude that the dynamic hedging is better. However, the hedging period here is only four weeks. Longer hedging period would bear different results. Moreover, changing window frames might result in better hedging. There are various ways to calculate and explain beta coefficients and hard to find the definitive answer to this problem.

III. c. Limitations

Although dynamic hedging reduces a portfolio exposure to the underlying asset, it has limitations. One limitation is that the position still has the exposure even if the position is delta-neutral. Dynamic hedging needs to be constantly adjusted with movements of the underlying asset. Expanding on this our underlying asset prices did not significantly change and the results of dynamic hedging; hence shorting the futures did not give a reasonable change to make analysis.

Another big limitation was finding the appropriate benchmark. We used the return of S&P 500 as a benchmark for our large caps and return on Russell 2000 as a benchmark for our small cap stocks. Benchmarking simply helps you spot the areas which needs improvement. It does not contribute to help to solving the issues in hand. Benchmark simply compares numbers, it does not take into account the micro and macro factors that led to the industry fail or succeed.

The portfolio also has some limitations because there are only three stocks: two large cap stocks and one small cap stock. It is not well diversified. Thus, the portfolio is exposed to unsystematic risks that are unique to each security such as company policy changes and news that are specific to some companies. The unsystematic risks cannot be taken into account by beta hedging because it assumes that the portfolio is well diversified.

Capital Asset Pricing Model (CAPM) also has several weaknesses because it is based on some unrealistic assumptions such as: Existence of risk free assets, all assets being perfectly divisible and marketable, existence of homogeneous expectations about the expected returns, assets returns are normally distributed, etc. CAPM is a single period model - it looks at the end of the year return, CAPM cannot be empirically tested because we cannot test investors expectations, and the CAPM assumes that a security's required rate of return is based on only one factor (the stock market- beta).

IV. References

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