ASSIGNMENT-4

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Section: 4

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Answer to the question no 1

$$P(X=x)=\begin{cases} 2kx & x=2,4,6 \\ k(x+2) & x=8 \\ 0 & \text{Otherwise} \end{cases}$$

(a)

$2(2)k+2(4)k+2(6)k+k(8+2)=1$

$\Rightarrow 4k+8k+12k+10k=1$

$\Rightarrow k=\frac{1}{34}$ [Showed]

(b)

$P(4<x\leq 8)=2(6)k+k(8+2)=\frac{12}{34}+\frac{10}{34}=\frac{11}{17}$ (Ans)

(c)

$P(2<x<4)=8$ (Ans)

(d)

$E(x)=(2\times 4k)+(4\times 8k)+(6\times 12k)+(8\times 10k)$

$=\frac{8}{34}+\frac{32}{34}+\frac{72}{34}+\frac{80}{34}=5.647$ (Ans)

20101402

(e)

$E(x^2) = (2^2 \times 4k) + (4^2 \times 8k) + (6^2 \times 12k) + (8^2 \times 10k)$

$= \frac{8}{17} + \frac{64}{17} + \frac{216}{17} + \frac{320}{17} = \frac{608}{17}$

$Var(X) = \frac{608}{17} - \left(\frac{96}{17}\right)^2 = 3.875$ (Ans)

(f)

$Var(5-3x) = Var(5) + Var(3x)$

$= 3^2 \times Var(X) = 9 \times 3.875$ [from e)

$= 34.88$ (Ans)

Answer to question 2

(a)

$P(Y \leq 8) = \int_0^8 R(Y)\,dy = \int_0^5 R(Y).dy + \int_5^8 R(Y).dy$

$= \int_0^5 \frac{y}{25}.dy + \int_5^8 \left(\frac{2}{5} - \frac{y}{25}\right).dy$

$= \left[\frac{y^2}{50}\right]_0^5 + \left[\frac{2}{5}y - \frac{y^2}{50}\right]_5^8$

$= \frac{1}{2} + \frac{21}{50} = \frac{23}{25}$ (Ans)

(b)

$$P(Y<2) = \int_0^2 R(Y)\cdot dY = \int_0^2 \frac{Y}{25}\cdot dY = \left[\frac{Y^2}{50}\right]_0^2$$

$$= \frac{2}{25}$$

$$P(Y>6) = \int_6^{10} f(Y)\cdot dY = \int_6^{10} \left(\frac{2}{5} - \frac{Y}{25}\right) dY$$

$$= \left[\frac{2}{5}Y - \frac{Y^2}{50}\right]_6^{10}$$

$$= \frac{8}{25}$$

$$\therefore P\{(Y<2) \text{ or } (Y>6)\} = \frac{2}{25} + \frac{8}{25} = \frac{2}{5} \text{ (Ans)}$$

(c)

$$E(Y) = \int_0^5 Y\left(\frac{Y}{25}\right)\cdot dY + \int_0^{10} Y\left(\frac{2}{5} - \frac{Y}{25}\right) dY$$

$$= \frac{1}{25}\int_0^5 Y^2\cdot dY + \int_5^{10}\left(\frac{2}{5}Y - \frac{Y^2}{25}\right)\cdot dY$$

$$= \frac{1}{25}\left[\frac{Y^3}{3}\right]_0^5 + \left[\frac{Y^2}{5} - \frac{Y^3}{75}\right]_5^{10}$$

$$= \frac{5}{3} + \left(\frac{20}{3} - \frac{10}{3}\right) = 5 \text{ (Ans)}$$

(d)

$$E(Y^2) = \int_0^5 \frac{y^3}{25}\,dy + \int_5^{10}\left(\frac{2}{5}y^2 - \frac{y^3}{25}\right)dy$$

$$= \frac{1}{25}\left[\frac{y^4}{4}\right]_0^5 + \left[\frac{2y^3}{15} - \frac{y^4}{100}\right]_5^{10}$$

$$= \frac{25}{4} + \left(\frac{100}{3} - \frac{125}{12}\right) = \frac{175}{6}$$

$$Var(Y) = \frac{175}{6} - 5^2 = \frac{25}{6}$$

$$SD : \sqrt{\frac{25}{6}} = 2.041 \text{ (Ans)}$$

Answer to the question no 3

(a)

$$P(A=B) = 0.08 + 0.15 + 0.1 + 0.07 = 0.4 \text{ (Ans)}$$

(b)

$$P(A+B=4) = 0.04 + 0.1 + 0.03 = 0.17$$

$$P(A+B \geq 4) = 0.14 + 0.06 + 0.04 + 0.01 + 0.07 + 0.05 + 0.06$$
$$= 0.46 \text{ (Ans)}$$

(c)

A	0	1	2	3	4
$P_A(A)$	0.19	0.30	0.25	0.14	0.12

B	0	1	2	3
$P_B(B)$	0.19	0.30	0.23	0.23

$E(A) = (0\times0.19) + (1\times0.3) + (2\times0.25) + (3\times0.14) + (4\times0.12)$

$= 1.7$ (Ans)

(d)

$P(B=21)(A=3) = \frac{0.04}{0.14} = \frac{2}{7}$ (Ans)

(e)

$P(A) = 0.12$, $P(B=0) = 0.19$

$P(A=4, B(0=0)) = 0$

$P(A=4 \cap B=0) = 0.12\times0.19 = 0.0228$

Since $P(A,B)(4,0) \neq P_{A,B}(A=4 \cap B=0)$

$\therefore$ A and B are no independent random variables. (Ans)

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Answer to the question 4

(a)

$P(R) = \frac{4}{12}$

$P(\text{Red ball in the 5th term}) = \left(\frac{8}{12}\right)^4 \times \frac{4}{12} = \frac{16}{243}$ (Ans)

(b)

$P(\text{non-white}) = \frac{10}{12}$

10 non white balls are expected in 12 terms

					"	"	$\frac{12}{10}$ "
∴ 1	"	"	"				$= 6/5$ turns (Ans)

(c)

$P(B) = \frac{6}{12} = \frac{1}{2}$; 1 Blue ball expected in 2 turn each.

x = number of turns required to get one Blue ball.

$Var(x) = \frac{1 - 1/2}{\left(\frac{1}{2}\right)^2} = 2$ (Ans)

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Answer to the question 5

(a)

$n = 6$, $P(B) = \frac{6}{12}$

X = no of blue balls after 6 turns

$P(X=3) = {}^6C_3 \times \left(\frac{6}{12}\right)^3 \left(\frac{6}{12}\right)^3 = \frac{5}{16}$ (Ans)

(b)

P(5B after 6 turns) $= {}^6C_5 \times \left(\frac{6}{12}\right)^5 \times \left(\frac{6}{12}\right)^5 = 3/32$

P(6B after 6 turns) $= {}^6C_6 \times \left(\frac{6}{12}\right)^6 = \frac{1}{64}$

P(more than 4B after 6 turns) $= \frac{3}{32} + \frac{1}{64} = \frac{1}{64}$

(Ans)

(c)

4 red balls picked after 12 turns

$\therefore$ after 48 turns number of red balls picked

$= 4/12 \times 48 = 16$ red balls.

(Ans)

(d)

$n = 36$, $p = \frac{2}{12}$

x = number of white balls picked after 36 turns.

$Var(x) = 36 \times \frac{2}{12} \times (1 - \frac{2}{12}) = 5$

$\therefore SD = \sqrt{5} = 2.236$ (Ans)

Answer to the question 6

(a)

Every 12 moths, 6 earthquakes occur

In, 4 month $= \frac{6}{12} \times 4$

$= 2$ earthquake occur (Ans)

(b)

6 earthquakes occur in a year

In 2 years, 12 earthquakes are expected to occur, $\lambda = 12$

X = no of earthquakes occuring in 2 years.

$P(x = 7) = \frac{e^{-12} \times 12^7}{7!} = 0.0436$ (Ans)

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(c)

$\lambda = 6$

x = number of earthquakes occuring in a year

$$P(x \geq 9) = 1 - P(x \leq 8)$$

$$= 1 - \left[e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!} + \frac{6^5}{5!} + \frac{6^6}{6!} + \frac{6^7}{7!} + \frac{6^8}{8!}\right)\right]$$

$$= 1 - e^{-6}\left(\frac{1709}{5}\right) = 0.1528 \text{ (Ans)}$$

Answer to the question 7

(a)

$X \sim N[73, 22^2]$

$$P(x \geq 75) = P\left(z > \frac{75-73}{22}\right)$$

$$= P(z \geq 0.09)$$

$$= 0.5 - 0.0359 = 0.4641 \text{ (Ans)}$$

(b)

$P(x \leq 45) = P\left(z \leq \frac{45-73}{22}\right)$

$= P(z \leq 1.27)$

$= 0.5 - 0.3980 = 0.102$ (Ans)

(c)

$P(55 < x < 90) = P\left(\frac{55-73}{22} < z < \frac{90-73}{22}\right)$

$= P(0.82 < z < 0.77)$

$= 0.2794 + 0.02939 = 0.5733$ (Ans)

(d)

$P(x = 65) = 0$ (Ans)

(e)

$P(x < x) = 0.08$

$P\left(z \leq \frac{x-73}{22}\right) = 0.08$

$\Rightarrow 0.5 + P\left(z \leq \frac{x-73}{22}\right) = 008$

$\Rightarrow \frac{x-73}{22} = -1.41$

$\Rightarrow x = 41.98$ (Ans)

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(f)

In every normal distribution mean = median = mode

The mean is always taken to be 0, which is being calculated by using normal distribution.

(Ans)

Answer to the question no 8

(a)

$\frac{1}{\lambda} = 100000 \Rightarrow \lambda = 1 \times 10^{-5}$

~~P exp~~ $P(x) \sim \exp(\lambda = 1 \times 10^{-5})$

$P(x < 10000) = \int_0^{1000} 1 \times 10^{-5} e^{-10^{-5}x} \, dx$

$= 1 - e^{-10^{-5} \times 10000} = 0.0952$ (Ans)

(b)

$P(x > 120000) = \int_{120000}^{\infty} 10^{-5} \cdot e^{10^{-5}x} \, dx$

$= e^{-10^{-5} \times 120000} = 0.3012$ (Ans)

(c)

$P(60000 < x < 100000)$

$= P(x < 100000) \; P(x < 60000)$

$= -e^{-10^{-5} \times 100000} + e^{-10^{-5} \times 6 \times 10^{4}} = 0.181$ (Ans)

(d)

Considering 10% TV sets lastmore than

$P(x > t) = 1 - P(x < t)$

$\Rightarrow 0.1 = 1 - (1 - e^{-\lambda t})$

$\Rightarrow e^{-\lambda t} = 0.1$

$\Rightarrow e^{-10^{-5} t} = 0.1$

$\Rightarrow -10^{-5} t = \ln(0.1)$ [taking ln]

$\Rightarrow t = 230258.51$ hours

≈ 230259 hours (Ans)

—x—