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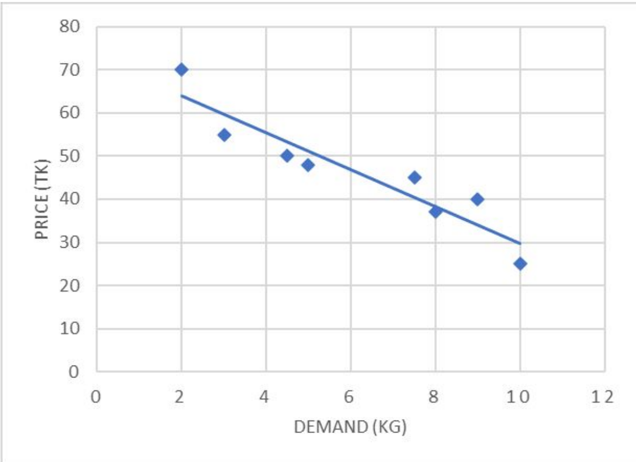
STA201 Assignment 2 Solution

Correlation & Regression

1. The demand and price of a specific product is given in the following table:

Price (Tk.)	25	37	40	45	48	50	55	70
Demand (kg)	10	8	9	7.5	5	4.5	3	2

a. Find the relationship between demand and price using a scatter diagram and comment (interpret).



b. Find correlation coefficient and comment.

$$\bar{x} = \frac{\sum x}{n} = \frac{370}{8} = 46.25 \; ; \; \bar{y} = \frac{\sum y}{n} = \frac{49}{8} = 6.125$$

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{\left(\sum x^2 - n\bar{x}^2\right)\left(\sum y^2 - n\bar{y}^2\right)}} = \frac{2013.5 - 8\times 46.25\times 6.125}{\sqrt{(18948 - 8\times(46.25)^2)(359.5 - 8\times(6.125)^2)}} = -0.933$$

Negative and strong.

c. Fit a least square regression equation (line) of demand on price and comment.

$$\hat{\beta} = \frac{\sum xy - n\overline{xy}}{\sum x^2 - n\bar{x}^2} = \frac{2013.5 - 8\times 46.25\times 6.125}{18348 - 8\times(46.25)^2} = -0.204$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 6.125+0.204*46.25 = 15.56$$

$$\hat{y} = 15.56 - 0.204x$$

d. What will be the demand when the price is Tk. 52 and Tk. 10?

If x=52

$\hat{y} = 15.56 - 0.204 \times 52 = 4.952$

If x=10

$\hat{y} = 15.56 - 0.204 \times 10 = 13.52$

e. Find coefficient of determination (how well the regression line is fitted).

Coefficient of Determination,

$R^2 = r^2 = (-0.933)^2 = 0.871$

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2. Daily studying (in hours) and marks obtained (out of 15) in a quiz of 10 students were as follows:

Studying	6	5	4	2	1	7	10	0	8	5
Marks	14	12	10	8	6	12	13	12	15	9

a. Fit the regression equation of Marks on Studying.

Dependent variable x = hours

Independent variable y = marks

$\bar{x} = \frac{48}{10} = 4.8$

$\bar{y} = \frac{111}{10} = 11.1$

$\hat{b} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2} = \frac{585-10\times4.8\times11.1}{320-10\times4.8^2} = 0.582589 \approx 0.58$

$\hat{a} = \bar{y} - \hat{b}\bar{x} = 11.1 - 0.58 \times 4.8 = 8.316 \approx 8.3$

$\bar{y} = \hat{a} + \hat{b}\bar{x} = 8.3 + 0.58x$

b. What will be the predicted marks for studying 3 hours daily?

If $x = 3$

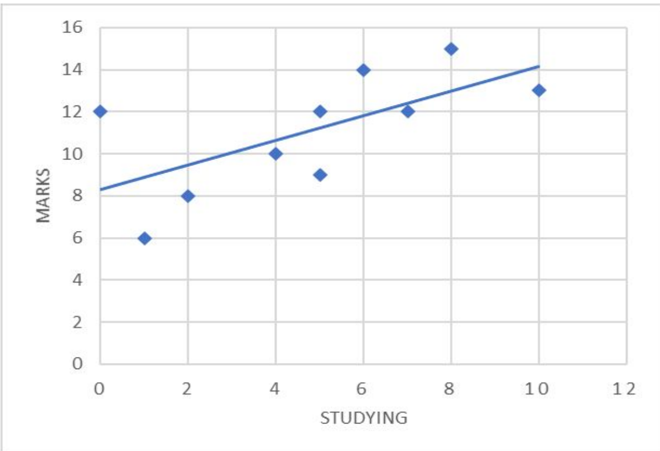
$\bar{y} = \hat{a} + \hat{b}\bar{x} = 8.3 + 0.58 \times 3 = 10.4$

c. Can you verify the relationship between marks and daily studying? Try all the methods.

Correlation coefficient,

$$r = \frac{\sum xy - n\bar{x}\bar{y}}{\sqrt{(\sum x^2 - n\bar{x}^2)((\sum y^2 - n\bar{y}^2)}} = \frac{585-10\times4.8\times11.1}{\sqrt{(320-10\times4.8^2)(1303-10\times11.1^2)}} = 0.6549277 \approx 0.65$$

Strong and positive correlation.



Probability

3. In a simultaneous throw of a pair of fair 6-sided dice, find the probability of getting:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a. 8 as the sum
   For this event, E = { (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }
   So, the probability of getting 8 as the sum, P(E) = $\frac{5}{36}$

b. a doublet (two dice landing on the same value)
   For this event, E = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) }
   So, the probability of getting a double, P(E) = $\frac{6}{36}$

c. a doublet of prime numbers
   For this event, E = {(2, 2), (3, 3), (5, 5)}
   So, the probability of getting a double of prime numbers, P(E) = $\frac{3}{36}$ = $\frac{1}{12}$

d. a doublet of odd numbers
   For this event, E = {(1, 1), (3, 3), (5, 5)}
   So, the probability of getting a double of odd numbers, P(E) = $\frac{3}{36}$ = $\frac{1}{12}$

e. a sum greater than 9
   For this event, E = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}
   So, the probability of getting a sum greater than 9, P(E) = $\frac{6}{36}$ = $\frac{1}{6}$

f. an even number on first
   For this event,
   E = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
   So, the probability of getting an even number on first, P(E) = $\frac{18}{36}$ = $\frac{1}{2}$

g. an even number on one and a multiple of 3 on the other
   For this event, E = {(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (3, 2), (6, 2), (6, 6), (3, 4), (6, 4), (3, 6)}
   So, the probability of getting an even number on one and a multiple of 3 on the other
   P(E) = $\frac{11}{36}$

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4. A bag contains 25 balls numbered 1 through 25. Suppose an odd number is considered a 'Success'. Two balls are drawn from the bag with replacement. Find the probability of getting:

a. Two successes

There are 13 odd numbers from 1 to 25 and so the probability of selecting a ball having odd number in the first draw is, A = $\frac{13}{25}$.
Since two balls are drawn with replacement and again, for the second selection there are 13 odd numbers and so the probability of selecting a ball having odd number in the second draw is, A = $\frac{13}{25}$.
Here, the events A & B are independent.
Thus, the probability of getting two successes = $\frac{13}{25} * \frac{13}{25}$ = 0.27

b. exactly one success

One odd numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting exactly one success is, P(E) = ($\frac{13}{25} * \frac{12}{25}$) + ($\frac{12}{25} * \frac{13}{25}$) = 0.49

c. at least one success

It can happen during the draw that two odd numbered balls are selected and again, one odd numbered ball can be selected either in the first draw or in the second draw. Thus, the probability of getting at least one success,
P(E) = ($\frac{13}{25} * \frac{13}{25}$) + ($\frac{13}{25} * \frac{12}{25}$) + ($\frac{12}{25} * \frac{13}{25}$) = 0.76

d. no successes

Even, no odd numbered can be selected during the drawn and so, the probability of getting no successes is, P(E) = $\frac{12}{25} * \frac{12}{25}$ = 0.23

5. An elementary school is offering two optional language classes, one in French and the other in Spanish. These classes are open to any of the 120 upper grade students in the school. Suppose there are 32 students in the French class, 36 in the Spanish class, a total of 8 who are in both classes. If an upper grade student is randomly chosen, what is the probability that this student is not enrolled in any one of these classes?

Here, the probability that the student is enrolled in the French class is, P(F) = $\frac{32}{120}$
And, the probability that the student is enrolled in the Spanish class is, P(S) = $\frac{36}{120}$
Again, the probability that the student is enrolled in both classes is, P(E ∩ S) = $\frac{8}{120}$
So, if an upper grade student is randomly chosen, the probability that this student is enrolled in any one of these classes, P(E U S) = $\frac{32}{120} + \frac{36}{120} - \frac{8}{120}$
Thus, if an upper grade student is randomly chosen, the probability that this student is not enrolled in any one of these classes, $P(E\ U\ S)^c$ = 1 – [$\frac{32}{120} + \frac{36}{120} - \frac{8}{120}$] = 0.5

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6. Assume that the chances of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drugs reduces its chances by 25%. At a time, a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Let us define the events,
A_1: Person is treated with meditation & yoga
A_2: Person is treated with drug
B: Person has heart attack

We have to find the probability that the patient followed a course of meditation and yoga given that the patient selected at random suffered from a heart attack.

$$P(A_1|B) = \frac{P(B|A_1)*P(A_1)}{P(B|A_1)*P(A_1)+P(B|A_2)*P(A_2)}$$

It is given that, meditation & yoga and drug has equal probabilities,

$$P(A_1) = 0.5,\ P(A_2) = 0.5$$

We know the chances of having a heart attack without any treatment is 40%

Meditation reduces the risk by 30% so the risk becomes 70% of the original.
Therefore, the probability of having heart attack if is treated with meditation,
$P(B|A_1) = 0.40 \times 0.70 = 0.28$

The drug reduces the risk by 25% so the risk becomes 75% of the original.
Therefore, the probability of having heart attack, if he treated with drugs,
$P(B|A_2) = 0.40 \times 0.75 = 0.30$

Therefore,

$$P(A_1|B) = \frac{P(B|A_1)*P(A_1)}{P(B|A_1)*P(A_1)+P(B|A_2)*P(A_2)} = \frac{(0.28*0.5)}{(0.28*0.5)+(0.30*0.5)} = \frac{0.28}{0.28+0.30} = \frac{0.28}{0.58} = \frac{14}{29}$$

7. Bag A contains 3 red and 4 black balls and Bag B contains 4 red and 5 black balls. One ball is transferred from Bag A to Bag B and then a ball is drawn from Bag B. The ball so drawn is found to be red in color. Find the probability that the transferred ball is black.

Let us define the events as follows:
A_1: ball transferred from Bag I to Bag II is red
A_2: ball transferred from Bag I to Bag II is black
B: ball drawn from Bag II is red

Probability of selecting red ball from Bag I, $P(A_1) = \frac{3}{7}$
Probability of selecting black ball from Bag I, $P(A_2) = \frac{4}{7}$

When a red ball is added to Bag II, the total number of balls in Bag II, is 10 and the number of red balls is 5.
So, $P(B|A_1) = \frac{5}{10} = \frac{1}{2}$

When black ball is added to Bag II, the total number of balls in Bag II, is 10 and the number of red balls is 4.
So, $P(B|A_2) = \frac{4}{10} = \frac{2}{5}$

We have to find,
$P(A_2|B) = \frac{P(B|A_2)*P(A_2)}{P(B|A_1)*P(A_1)+P(B|A_2)*P(A_2)} = \frac{\frac{4}{7}*\frac{2}{5}}{\frac{3}{7}*\frac{1}{2}+\frac{4}{7}*\frac{2}{5}} = \frac{16}{31}$