Contents

1. Theory of relativity

1.1. Introduction

The two most successful physical theories of the 19th century, (classical) mechanics and electromagnetism, stood the test of time and work almost seamlessly hand in hand. There is however one troublesome aspect that, when taken into account, has the two theories completely at odds: light.

Maxwell's Equations for electromagnetism, in accord to the experimental results, predict that light should travel with speed:

$$
\begin{aligned}
c &= \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.854 \times 10^{-12} F/m)(1.256 \times 10^{-6} N/A^2)}} = \\
&= \frac{1}{\sqrt{11.120 \times 10^{-18} (\cancel{s^4} \cdot \cancel{A^2} \cdot \cancel{kg} \cdot \cancel{m} \cdot \cancel{s^{-2}} / \cancel{kg} \cdot m^2 \cdot \cancel{m} \cdot \cancel{A^2})}} = \\
&= \frac{1}{\sqrt{11.120 \times 10^{-18} s^2/m^2}} = \frac{1}{3.334 \times 10^{-9} s/m} \approx 3.000 \times 10^8 m/s
\end{aligned}
$$

That is, light apparently moves at speed c in any circumstance. This is problematic, because mechanics postulates that there is no such thing as an "absolute velocity": all velocities depend on the reference frame from which they are observed, as the principle of Galilean relativity states.

Consider two inertial reference frames, S and S', where S' is moving at constant speed v (along the x axis) with respect to S. Suppose that a beam of light were to be travelling at speed c (again, along the x axis) with respect to S; then, applying Galilean relativity, one would expect the light beam to travel at velocity $c - v$ with respect to S'. However, when measuring the speed of the light beam with respect to S' one still finds that it moves with speed c.

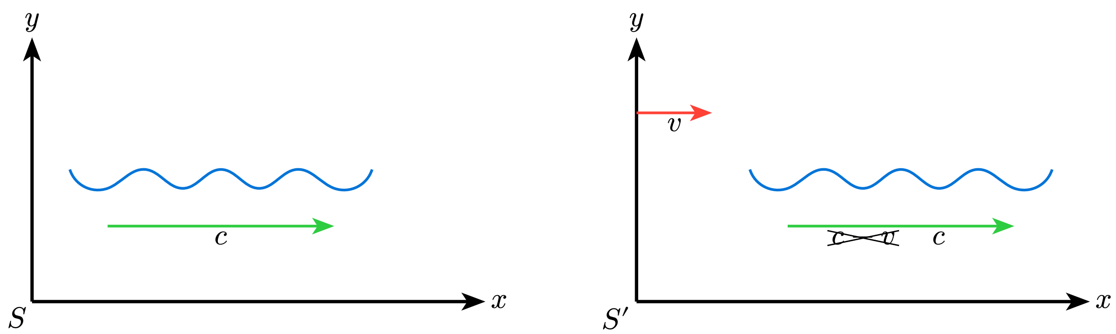

Figure 1: With respect to an inertial frame of reference S, a light beam (in blue) moves with velocity c. With respect to another inertial frame of reference S', that is moving with respect to S at constant velocity v, it would be assumed that the light beam would travel with velocity $v - c$ with respect to S'. However, this is not the case: the light beam moves at velocity c both with respect to S and with respect to S'.

One hypothesis that was advanced early on to reconcile the two theories was to postulate that the entire Universe is permeated by a peculiar substance called **luminiferous ether**, or just **ether**, and that the speed of light was constant when measured with respect to the "special" frame of reference of the ether. Stated otherwise, the value of c arising from Maxwell equations does not refer to the speed of light "per se", but the value measured with respect to this priviledged frame of reference.

If this were to be true, the contradiction would be solved, since the Galilean principle of relativity now works again. Given a frame of reference S, if it were to be possible to compute the velocity v with which S moves with respect to the frame of reference of the ether, the speed of light measured from S would now be $c - v$.

The ether hypothesis was short-lived, however, since it became clear that no such substance exists.

2. Quantum mechanics

2.1. Introduction

Quantum mechanics was a new paradigm developed in stages to answer questions that classical physics was unable to answer. In particular, it is a framework that is necessary to model reality at very small scales (atoms and molecules).

The first staple point of quantum mechanics is the idea that energy is not a *continuous* quantity, but is instead a *discrete* quantity, that is, an integer multiple of a fixed elementary value.

The fundamental physical constant that regulates the size of this fundamental energy bit is called the **Planck constant**, denoted as h:

$$h = 6.626 \times 10^{-34} J \cdot s$$

Since this value is very small, on large scales energy appears continuous, because the "steps" between different energy values are infinitesimal and become relevant only on small scales.

The second staple point of quantum mechanics is that the difference between particles and waves becomes blurred. In this sense, it is possible for matter particles to exhibit wave-like properties and it is possible for waves to exhibit particle-like properties.

2.1.1. Black body radiation

The first problem that quantum mechanics aided in solving is the description of the emitted radiation of a **black body**. A black body is an idealized physical body that is capable of absorbing any electromagnetic radiation, regardless of its frequency or angle of incidence, and that therefore emits back energy only and exclusively because of this absorbtion.

The spectrum of all the frequencies of electromagnetic radiations emitted from a black body, also referred to as **emission spectrum**, is given by:

$$I(f) = \frac{d^4 E}{d\theta dA \cos(\theta) dt df} = \frac{d^4 \Phi}{d\theta dA \cos(\theta) df}$$

Where E is the energy, A is the surface area, θ is the angle of emission, t is the timem f is the frequency and Φ is the flux. Since A and θ are effectively chosen by the experimenter, and are therefore always known, the only variable at play is the frequency (or the wavelength, which is just its reciprocal).

Experimentally, the emission spectrum forms a curved shape with a peak at a certain wavelenght λ_{max}. It is possible to explicitly define the relationship between these two quantities as follows:

$$I(f, T) = \frac{2\pi h f^3}{c^2} \frac{1}{\exp\left(\frac{hf}{k_B T}\right) - 1}$$

Where T is the temperature, h is the Planck constant and k_B is the **Boltzmann constant**:

$$k_B = 1.38 \times 10^{-23} \frac{J}{\text{K}}$$

The total energy emitted by the black body is given by:

$$E_{\text{tot}} = \sigma T^4$$

Where σ is the **Stefan-Boltzmann constant**:

$$\sigma 5.67 \times 10^{-8} \frac{W}{m^2 \mathrm{K}^4}$$

The formula suggests an intuitive result, mainly that the amount of energy emitted increases as the temperature increases, as vice versa.

It can be shown that the value of $\lambda_{\max}$ is inversely proportional to the temperature.

It is to be expected that the aforementioned equation for $I(f, T)$ were to be derived from Maxwell equations. Interestingly, there is no way to do so. To derive the expression for $I(f, T)$ it is necessary to assume that each particle acts as an harmonic oscillator that emits energy is chunks, not as a continuous stream.

Given a certain frequency, a single chunk of energy δE is given by:

$$\delta E = hf$$

2.1.2. Photoelectric effect

The second phenomena that quantum mechanics aided in explaining was the **photoelectric effect**. This phenomena is the expulsion of electrons, called *photoelectrons*, from a piece of metal when hit by light. This happens because the energy given to the plate by the light is sufficient to break the bond that links electrons to their nucleus, and are thus ejected.

It is possible to experimentally determine the number of electrons that are ejected from the plate and their energy with respect to the light frequency and intensity, and then derive from Maxwell equations the corresponding equations and see if the results match.

In the framework of classical mechanics, the energy transfer from light to electrons is no different than heating an object. This process requires time, so it is expected that when the plate is illuminated there should be some delay before the plate starts emitting electrons. Whan happens instead is that the electrons are emitted immediately, as soon as the plate is hit.

It should also be reasonable that an higher light intensity would correspond to an higher energy of the photoelectrons. Instead, the intensity of the light has no influence on the energy of the photoelectrons, which is instead proportional to the frequency of the light.

The third puzzling observation is that the photoelectric effect only happens when the incoming light has a frequency equal or above a certain threshold, specific for each metal. In classical mechanics this has no explanation, since the photoelectric effect should happen, albeit with different degree of intensity, when employing light of any frequency.

The quantum explanation is instead to assume that light is composed of elementary massless particles called **photons**, whose energy is given by the black body harmonic oscillator model. If this is the case, all three issues are solved, because:

1. If each electron is hit by photons one by one, there's no need to wait for the body to absorb energy, since energy absorbtion is "one-shot". Therefore, the electron explusion is instantaneous;
2. Since $E = hf$, energy is indeed dependent on the frequency;
3. If the absorbed energy is insufficient, the electron is immediately recaptured by the charge of the nucleus. Being the energy dependent on the frequency, this explains the existence of a frequency threshold.

2.1.3. Gasses and radiation

When a gas is traversed by light, it is expected that the overall resulting frequency of the light is lowered, but preserved. In a similar fashion, when light is induced to emit light (by a sparkle), it is expected that every frequency is emitted. What is observed instead is that, for each gas, only specific frequencies are emitted/preserved.

The nuclear model of the atom was tested in an experiment by Rutherford. Of course, it is not possible to study atoms simply with a microscope, because the scale is too small even for the greatest magnifier. An alternative approach is to employ electric charges, since both electrons and protons are charged.

An extremely thin plate of gold is used as a probe. Gold is used because it is both very dens and very soft, and is therefore possible to craft extremely thin plates. A radioactive source emits alpha particles (which are just helium nuclei). A detector entirely surrounds the plate, so that it is possible to observe where (and if) alpha particles are deflected. What happens is that some particles, albeit in small number, are indeed deflected, sometimes with great angle, whereas in a non-nuclei model of the atom all particles would have stroke through.

The proper model of the atom was worked out by Rutherford, who imagined the atom as an incredibly dense nucleus of positive charges with electrons orbiting around it, so that the electromagnetic force between the two acts as a centripetal force. The deflection of the alpha particles are then caused by the electromagnetic repulsion of the nuclei when alpha particles get too close to them.

The problem with the model is that the electron orbiting around the nucleus, in order to be able to maintain its orbit, would emit radiation, which in turn means it would gradually loose angular momentum until it would spiral onto the nucleus. Even assuming this to be true, the time for this to happen would be too narrow for matter to exist.

A solution was to assume that the electron does indeed orbit around the nucleus, but can only find itself at very specific distances from it. For this to be possible, it is necessary to assume that the angular momentum of the electron is quantized, according to the formula:

$$|p| = mvr = n\frac{h}{2\pi} \text{ with } n \in \mathbb{N}$$

In particular, the energy of the electron can be found by employing Coulomb's Law and the expression for the electric potential:

$$E = K + U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}$$

Where e is the electric charge of the electron and r is its distance from the nucleus.

Being the energy quantized, the only non constant member of the equation, r, must also be quantized. This means that r cannot be lower than a certain threshold, which is determined as $0.53 \times 10^{-10}m$.

Each possible value of energy that an electron can possess is called an **energy level**. Each time an electron exchanges energy with the environment, it hops from one energy level to another. In particular, by releasing energy it goes down one level, by absorbing energy it goes up one level.

Let E_0 be the lowest possible energy level (the one associated to ($r = 0.53 \times 10^{-10}m$). It is possible to relate the energy of a generic level n with respect to E_0:

$$E_n = \frac{ke^2}{2n^2E_0}$$

Since $E = hf$, this explains why gasses can absorb/release only certain frequencies, because they are the ones that match the (fixed) energy amounts needed for electrons to move.