Mathematical Methods in Engineering and Applied Science
Problem Set 7

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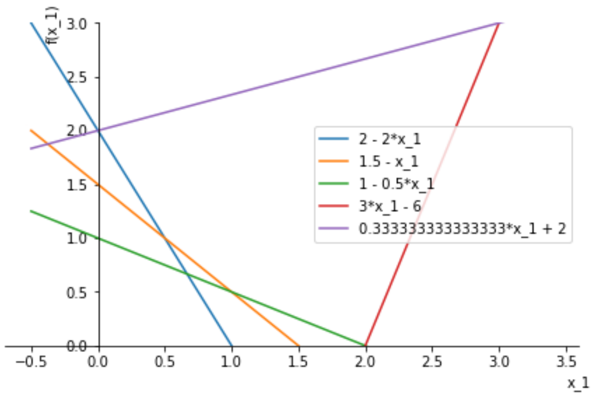
(1). Solve the following LP problem both in Matlab using linprog (or elsewhere using an analog of linprog) and directly by plotting the required regions:
minimize $f(x) = x_1 + x_2$ subject to:
$2x_1 + x_2 \geq 2$
$2x_1 + 2x_2 \geq 3$
$x_1 + 2x_2 \geq 2$
$3x_1 - x_2 \leq 6$
$3x_2 - x_1 \leq 6$
After expression x_2 as a $f(x_1)$ plot the lines:
Our solution lays in the largest inner triangle.
Min function $f(x) = x_1 + x_2 => x_2 = f(x) - x_1$ - line with 45^0 between itsalfe and x_1
The solution is $x_2 = 1.5 - x_1$, that line the same as yellow.=> $f(x) = 1.5$



$$x_1 \in \left[\frac{1}{2} : 1\right], \qquad x_2 = \in \left[\frac{1}{2} : 1\right]$$

Programming solution through the scipy.optimize.linprog :
fun: 1.5000000000042775, x=[0.5, 1]
$f(x) \approx 1.5$

(2). Consider the data:
$x = 1 : 24$
$y = [75,77,76,73,69,68,63,59,57,55,54,52,50,50,49,49,49,50,54,56,59,63,67,72]$.
with the cubic, $y = Ax^3 + Bx^2 + Cx + D$, chosen to fit them.

(a) Find the best fit by using **fminsearch** starting with the initial condition (1,1,1,60).
I tried to minimise the error:
$Error = |f(x) - y|\_2$
Error value = 6.123091
Iterations: 451
A = 9.18552584e-03, B = -1.59522492e-01, C = -1.74893782e+00, D = 8.02351658e+01

(b) Set up the normal equation and find the least-squares fit by solving it.
$Au = y$, where $u^T = [A, B, C, D]$, $A = [x^3, x^2, x, 1]$, $x = [1 : 24]$,
$Au = y => u = (A^T A)^{-1} A^T y = A^+ y$
A = 9.18550049e-03, B = -1.59521638e-01, C = -1.74894429e+00, D = 8.02351779e+01
Error = 6.123091284927192

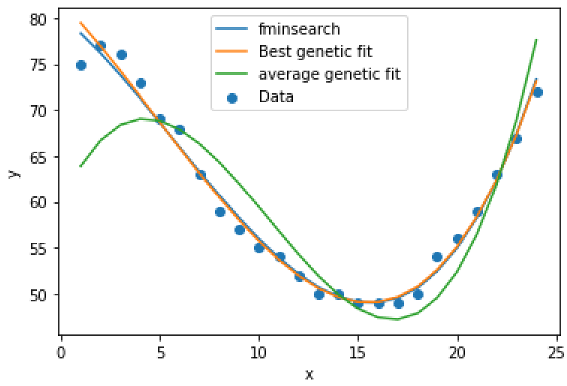
(c) Find the fit by using the genetic algorithm starting with the same initial condition as in (a). If there is no convergence, or it is too slow, try other initial conditions.
Iterations: 500
Initial: as in (a)
Error = 22.14773768147146
A = 2.22958123e-02 B = -7.02423277e-01 C= 4.75604243e+00 D = 5.98320401e+01

(d) Compare the computation times by the genetic algorithm with the fminsearch method, for example using tic and toc commands in Matlab. The minima in both cases have to agree with each other within 1%. You should use the same initial conditions in both methods.
Best rersult from genetic alg:
Error = 6.33551427690039

A = 8.30041793e-03 B = -1.21700964e-01 C= -2.22105010e+00 D = 8.17972981e+01
Relative error = 3.46 %
Time = 8.9379 sec
Initial are as in (a)
relative error > than 1 % but I tried a lot with a lot of iterations, it is the best fit that I've gove.

For fminsearch:

Time = 0.02535 sec
Tnitial are as in (a)
Error = 6.123091
Relative error ≈ 0
A= 9.18552584e-03 B = -1.59522492e-01 C = -1.74893782e+00 D = 8.02351658e+01



(3). Find the Fourier cosine series of $sin\ x$ and sine series of $cos\ x$ on $[0, \pi]$. Which one does a better job of representing its function and why? Make plots of 2-term as well as 10-term approximations together with the original functions.

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos(kx) + b_k \sin(kx)$$

where

$$a_k = \frac{1}{L}\int_{-L}^{L} f(x)\cos(kx)\,dx$$

$$b_k = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx$$

For cosine series of $sin(x)$:

$$L = \frac{\pi}{2}$$

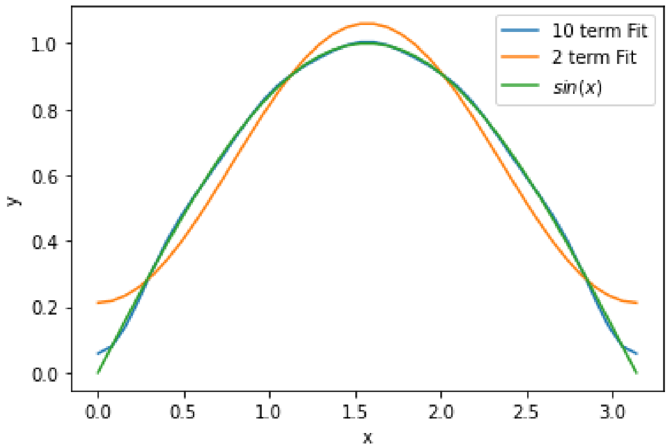
$$a_k = \frac{2}{\pi}\int \sin(x)\cos(kx)\,dx = \frac{1}{\pi}\left\{\begin{matrix} -\frac{\cos^2(x)}{2} & for\ k = \pm 1 \\ -\frac{k sin(x)\sin(kx)}{k-1} - \frac{\cos(x)\cos(kx)}{k^2-1} & otherwise \end{matrix}\right\}$$

$$a_k = \frac{2}{\pi}\int_0^{\pi} \sin(x)\cos(kx)\,dx = \frac{1}{\pi}\left\{\begin{matrix} 0 & for\ k = \pm 1 \\ -\frac{\cos(k\pi)}{k^2-1} - \frac{1}{k^2-1} & otherwise \end{matrix}\right\}$$

$$a_0 = \frac{2}{\pi}\left(-\frac{\cos(0*\pi)}{k^2-1} - \frac{1}{k^2-1}\right) = \frac{4}{\pi}$$

As result

$$\sin(x) \approx \frac{2}{\pi} + \sum_{k=2}^{n} \frac{2}{\pi}\left(-\frac{\cos(k\pi)}{k^2-1} - \frac{1}{k^2-1}\right)\cos(kx)$$

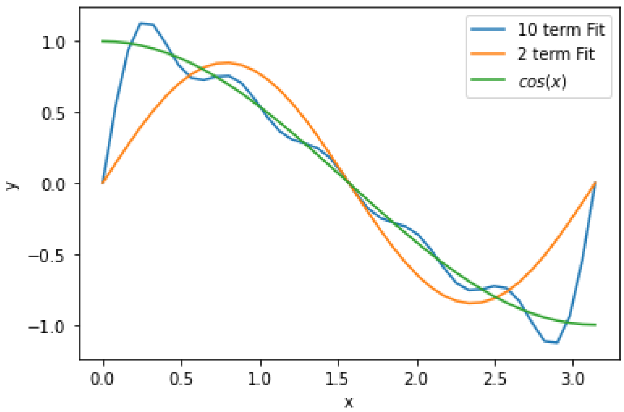


For sine series of $cos(x)$:

$$b_k = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx$$

$$b_k = \frac{2}{\pi}\int_{0}^{\pi} \cos(x)\sin(kx)\,dx = \frac{2}{\pi}\left(\frac{k\cos(\pi k)}{k^2-1} + \frac{k}{k^2-1}\right) for\ k \geq 2$$

$$\cos(x) \approx \sum_{k=1}^{n} \frac{2}{\pi}\left(\frac{k\cos(\pi k)}{k^2-1} + \frac{k}{k^2-1}\right)\sin(kx)$$



Sine series of cosine looks worse, because $\sin(0) = \sin(\pi) = 0$, while $\cos(0) = 1$

(4). Find the complex Fourier series $u = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ of $u = sgn(x)$ on $[-\pi, \pi]$. Plot the magnitude of the coefficients, $|c_n|^2$, as a function of n. What is $\sum_{n=-\infty}^{\infty} |c_n|^2$?

$$sgn(x) = \begin{matrix} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{matrix}$$

$$u = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$

$$c_n = \frac{1}{2L}\int_{-L}^{L} u(x)e^{-inx}dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} u(x)e^{-inx}dx =$$

$$= \frac{i}{2\pi n}\left(e^{i\pi n} - 2 + e^{-i\pi n}\right) = \frac{i}{2\pi n}\left(e^{i\pi n} - 2 + e^{-i\pi n}\right) =$$

$$= \frac{i}{2\pi n}(\cos(\pi n) + i\sin(\pi n) - 2 + \cos(\pi n) - i\sin(\pi n)) =$$

$$= \frac{i}{\pi n}(\cos(\pi n) - 1)$$

$$u = \sum_{n=-\infty}^{\infty} \frac{i}{\pi n}(\cos(\pi n) - 1)e^{inx}$$

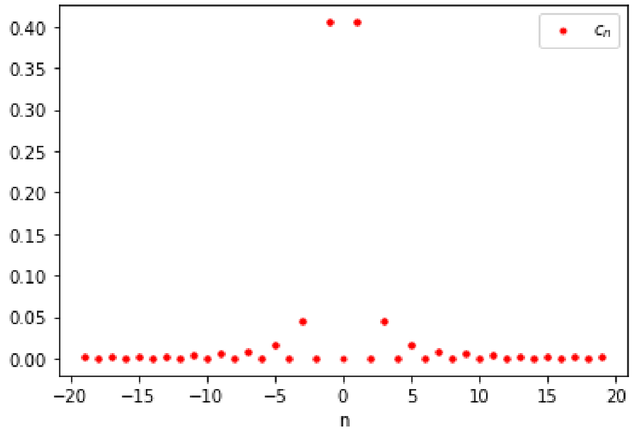
Let's find magnitude of the coefficients, $|c_n|^2$

Problem in $n \to 0$

$$c_0 = \lim_{n\to 0} \frac{i(\cos(\pi n)-1)}{\pi n} = \lim_{n\to 0} \frac{i(\cos(\pi n)-1)'}{(\pi n)'} = \lim_{n\to 0} \frac{i\pi(-\sin(\pi n))}{\pi} = 0$$

$|c_0| = 0$

Other $c_n = \frac{i}{\pi n}(\cos(\pi n) - 1)$



$\sum_{n=-\infty}^{\infty}|c_n|^2$ – is the sum of all "energies", I think $\sum_{n=-\infty}^{\infty}|c_n|^2 \to 1$ because, analogically to vectors: square of vector is the sum of squares of orthogonal components of this vector: $|sign(x)|^2 = 1$, and I think $\sum_{n=-\infty}^{\infty}|c_n|^2$converge to 1.

(5). A harmonic oscillator is hit with the force $f(t) = (-1)^k$, $t \in [k\pi, (k+1)\pi)$, $k = 0, 1, 2...$ The equation of motion is $\ddot{x} + \omega_0^2 x = f(t)$ and the initial conditions are $x(0) = \dot{x}(0) = 0$.

(a) Expand $x(t)$ in the Fourier series in t, plug it into the equation and determine the solution. Assume that ω_0 is not an integer. Plot the solution $x(t)$ over $t \in [0, 10\pi]$ at $\omega_0 = \sqrt{2}$ including sufficient number of terms in the series.

$$x(t) = \sum_{\omega=-\infty}^{\infty} e^{\frac{i\omega\pi t}{L}} \hat{x}(\omega)$$

$$\ddot{x}(t) = \sum_{\omega=-\infty}^{\infty} -\left(\frac{\omega\pi}{L}\right)^2 e^{\frac{i\omega\pi t}{L}} \hat{x}(\omega) =$$

$$\ddot{x} + \omega_0^2 x = \sum_{\omega=-\infty}^{\infty} \left(\omega_0^2 - \left(\frac{\omega\pi}{L}\right)^2\right) e^{\frac{i\omega\pi t}{L}} \hat{x}(\omega) = f(t)$$

multyply by $e^{-\frac{ip t\pi}{L}}$and integrate over$[0,2\pi]$ by t.

$$\sum_{\omega=-\infty}^{\infty} \left(\omega_0^2 - \left(\frac{\omega\pi}{L}\right)^2\right)\hat{x}(\omega) \int_0^{2\pi} e^{\frac{i(\omega-p)\pi t}{L}} dt = \int_0^{2\pi} f(t) e^{-\frac{ip\pi t}{L}} dt$$

Where $L = \pi$

$$\int_0^{2\pi} e^{i(\omega-p)t} dt = \frac{e^{i(\omega-p)2\pi} - e^0}{i(\omega-p)} = 0 \; if \; p \neq \omega$$

For $p = \omega$ we have:

$$\int_0^{2\pi} e^{i(\omega-p)t} dt = \int_0^{2\pi} e^{i(0)t} dt = 2\pi \; if \; p = \omega$$

Past this in our equation.

$$\sum_{\omega=-\infty}^{\infty} \left(\omega_0^2 - \left(\frac{\omega\pi}{L}\right)^2\right)\hat{x}(\omega) \int_0^{2\pi} e^{\frac{i(\omega-p)\pi t}{L}} dt = \sum_{\omega=-\infty}^{\infty} \left(\omega_0^2 - \left(\frac{\omega\pi}{\pi}\right)^2\right)\hat{x}(\omega) 2\pi\delta_{p\omega} =$$

$$= \left(\omega_0^2 - \left(\frac{p\pi}{\pi}\right)^2\right) 2\pi\hat{x}(p) = (\omega_0^2 - p^2)2\pi\hat{x}(p) = \int_0^{2\pi} f(t) e^{ipt} dt$$

Now consider right part:

$$\int_0^{2\pi} f(t)e^{-ipt}dt = \int_0^{\pi} e^{-ipt}dt + \int_{\pi}^{2\pi} (-1)e^{-ipt}dt = \frac{1}{-ip}(e^{-ip\pi} - e^0 - e^{-ip2\pi} + e^{-ip\pi})$$
$$= \frac{-4i}{p} \ \ if \ \ p = 2n+1,$$

else if $p = 2n$:

$$\int_0^{2\pi} f(t)e^{-ipt}dt = 0$$

else if $p = 0$

$$\int_0^{\pi} dt - \int_{\pi}^{2\pi} dt = \pi - 2\pi + \pi = 0$$

Let's connect left and right parts:

If $p = 0$

$$(\omega_0^2 - 0)2\pi\hat{x}(0) = 0$$

Suppose $\omega_0 \neq 0$ than we have $\hat{x}(0) = 0$

If (p = 2n) then

$$(\omega_0^2 - (2n)^2)2\pi\hat{x}(2n) = 0$$

Suppose $\hat{x}(2n) \neq 0 => (\omega_0^2 - (2n)^2) = 0 => n = \pm\frac{\omega_0}{2}, \hat{x}(2n)$ is any

If (p = 2n+1) then

$$(\omega_0^2 - (2n+1)^2)2\pi\hat{x}(2n+1) = \frac{-4i}{2n+1}$$
$$\hat{x}(2n+1) = \frac{-4i}{(2n+1)(\omega_0^2 - (2n+1)^2)2\pi}$$

If($p = 0$)

$$(\omega_0^2 - 0)2\pi\hat{x}(0) = 0 => \hat{x}(0) = 0$$

Finally:

$$x(t) = \sum_{\omega=-\infty}^{\infty} e^{\frac{i\omega\pi t}{L}}\hat{x}(\omega) = \sum_{\omega=-\infty}^{\infty} e^{\frac{i2\omega\pi t}{\pi}}\hat{x}(2\omega) + \sum_{\omega=-\infty}^{\infty} e^{\frac{i(2\omega+1)\pi t}{\pi}}\hat{x}(2\omega+1)$$
$$+ \sum_{\omega=-\infty}^{\infty} e^{\frac{i(0)\pi t}{L}}\hat{x}(0) = e^{\frac{+i\omega_0\pi t}{\pi}}\hat{x}_1(2\omega) + e^{\frac{-i\omega_0\pi t}{\pi}}\hat{x}_2(2\omega) +$$
$$+ \sum_{\omega=-\infty}^{\infty} e^{\frac{i(2\omega+1)\pi t}{\pi}}\frac{-4i}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)2\pi} = Ae^{+i\omega_0 t} + Be^{-i\omega_0 t} +$$
$$+ \sum_{\omega=-\infty}^{\infty} \big(\cos\big((2\omega+1)t\big) + i\ \sin\big((2\omega+1)t\big)\big)\frac{-4i}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)2\pi} =$$
$$= Ae^{+i\omega_0 t} + Be^{-i\omega_0 t} + \sum_{\omega=-\infty}^{\infty} \frac{2\sin\big((2\omega+1)t\big)}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)\pi}$$

cosine is droped out because: if $g(\omega,t) = \cos\big((2\omega+1)t\big) * \frac{-4i}{(2\omega+1)\left(\omega_0^2-(2\omega+1)^2\right)2\pi}$, then $g(-\omega,t) = g(\omega,t)$, so sum $\sum_{\omega=-\infty}^{\infty} g(\omega,t)$ is 0.

Initial x(0) =0 => $Ae^0 + Be^0 = 0 => B = -A$

Initial $\dot{x}(0) = i\omega_0 Ae^{+i\omega_0 0} + i\omega_0 Ae^{-i\omega_0 0} + \sum_{\omega=-\infty}^{\infty} \frac{2\cos\left((2\omega+1)0\right)}{(\omega_0^2-(2\omega+1)^2)\pi} = 0$

$$i\omega_0 A + i\omega_0 A + \sum_{\omega=-\infty}^{\infty} \frac{2}{(\omega_0^2 - (2\omega+1)^2)\pi} = 0$$
$$A = \sum_{\omega=-\infty}^{\infty} -\frac{2}{(\omega_0^2 - (2\omega+1)^2)\pi(2i\omega_0)} = \sum_{\omega=-\infty}^{\infty} \frac{i}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0}$$

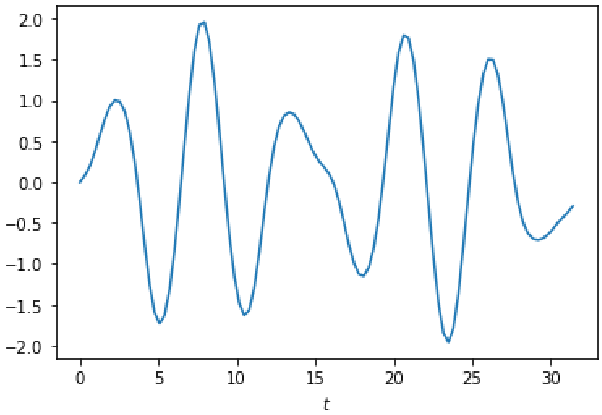
$$Ae^{+i\omega_0 t} - Ae^{-i\omega_0 t} = \sum_{\omega=-\infty}^{\infty} \frac{i}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0}\left(e^{+i\omega_0 t} - e^{-i\omega_0 t}\right)$$

$$= \sum_{\omega=-\infty}^{\infty} \frac{i}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0}\left(\cos(\omega_0 t) + isin(\omega_0 t) - \cos(\omega_0 t) + isin(\omega_0 t)\right) =$$

$$= \sum_{\omega=-\infty}^{\infty} \frac{i}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0}\left(2isin(\omega_0 t)\right) = \sum_{\omega=-\infty}^{\infty} \frac{-2sin(\omega_0 t)}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0}$$

$$x(t) = \sum_{\omega=-\infty}^{\infty} \frac{-2sin(\omega_0 t)}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0} + \sum_{\omega=-\infty}^{\infty} \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)\pi}$$

Plot:



$$x(t) = \sum_{\omega=-\infty}^{\infty} \frac{-2sin(\omega_0 t)}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0} + \sum_{\omega=-\infty}^{\infty} \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)\pi}$$

(b) (Optional, for extra credit). Redo part (a) for $\omega_0 = 1$.

$$x(t) = \sum_{\omega=-\infty}^{\infty} \frac{-2sin(\omega_0 t)}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0} + \sum_{\omega=-\infty}^{\infty} \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)\pi}$$

Consider $(2\omega + 1) \to 1, \omega_0 = 1$

$$\lim_{(2\omega+1)\to\pm1} \frac{-2sin(\omega_0 t)}{(\omega_0^2 - (2\omega+1)^2)\pi\omega_0} + \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(\omega_0^2 - (2\omega+1)^2)\pi} =$$

$$\lim_{(2\omega+1)\to\pm1} \frac{-2sin(t)}{(1 - (2\omega+1)^2)\pi} + \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(1 - (2\omega+1)^2)\pi}$$

$$= \lim_{(2\omega+1)\to\pm1} \frac{-2sin(t)(2\omega+1) + 2\sin\left((2\omega+1)t\right)}{(1 - (2\omega+1)^2)\pi}$$

$$= \lim_{p\to\pm1} \frac{-2sin(t)\mathrm{p} + 2\sin(\mathrm{p}t)}{(1 - \mathrm{p}^2)\pi} = \lim_{p\to\pm1} \frac{-2sin(t) + 2\,\mathrm{t} * \cos(\mathrm{p}t)}{-2\mathrm{p}\pi} = \pm\frac{sin(t) - \mathrm{t}\cos(t)}{\pi}$$

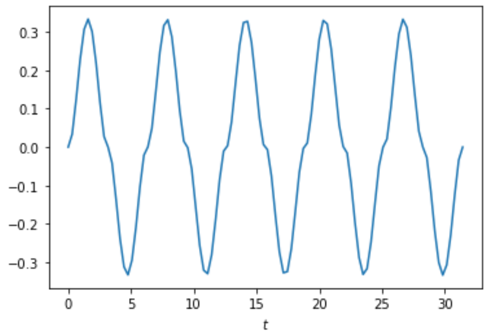
For $\omega = 0$ $lim = \frac{sin(t) - tcos(t)}{\pi}$ for $\omega = -1$ $lim = -\frac{sin(t) - tcos(t)}{\pi}$

$$x_1(t) = \sum_{\omega=-\infty}^{\infty} \frac{-2sin(t)}{(1 - (2\omega+1)^2)\pi} + \sum_{\omega=-\infty}^{\infty} \frac{2\sin\left((2\omega+1)t\right)}{(2\omega+1)(1 - (2\omega+1)^2)\pi} for\ all\ \omega \neq 0, -1$$

$$x_2(t) = \frac{sin(t) - \mathrm{t}\cos(t)}{\pi} for\ \omega = 0$$

$$x_3(t) = -\frac{sin(t) - \mathrm{t}\cos(t)}{\pi} = -x_2(t)\ for\ \omega = -1$$

$$x(t) = x_1(t) + x_2(t) + x_3(t) = x_1(t)$$



(6). Write down the Fourier matrix F8 and decompose it into three factors containing F4. Let f = zeros(8, 1) be a zero vector.

$$F_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 & \omega^4 & \omega^5 & \omega^6 & \omega^7 \\ 1 & \omega^2 & \omega^4 & \omega^6 & \omega^8 & \omega^{10} & \omega^{12} & \omega^{14} \\ 1 & \omega^3 & \omega^6 & & & & & \\ 1 & \omega^4 & \omega^8 & .. & .. & .. & .. & .. \\ 1 & \omega^5 & \omega^{10} & & & & & \\ 1 & \omega^6 & \omega^{12} & & & & & \\ 1 & \omega^7 & \omega^{14} & & & & & \omega^{49} \end{bmatrix}$$

Where $\omega = e^{-i\frac{2\pi}{8}} = e^{-i\frac{\pi}{4}}$

$$F_8 = \begin{bmatrix} I_4 & D_4 \\ I_4 & -D_4 \end{bmatrix} \begin{bmatrix} F_4 & 0 \\ 0 & F_4 \end{bmatrix} P$$

$$D_4 = \begin{bmatrix} 1 & & & \\ & \omega & & \\ & & \omega^2 & \\ & & & \omega^3 \end{bmatrix}; F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{bmatrix};$$

P= even-odd permutation

(a) Modify f so that f (1) = 1, find the Fourier transform $\hat{f}$ of f and plot $|\hat{f}|^2$.

(b) Now let $f\ (1:\ 2)\ =\ 1$ leaving the other components 0, and plot $|\hat{f}|^2$.

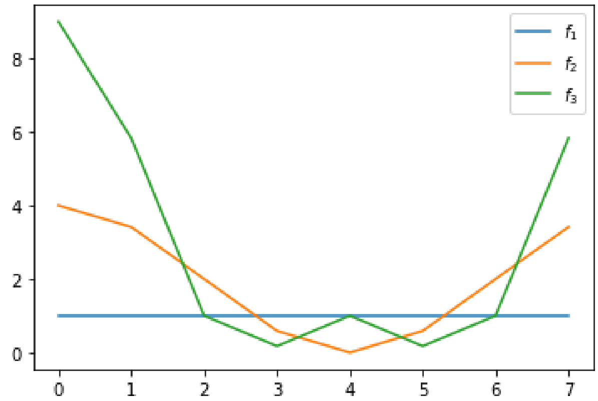
(c) Do the same with f (1 : 3) = 1. Explain your observations.

$$\hat{f} = F_8 f$$

$$f_1 = [10000000]^T, f_2 = [11000000]^T, f_3 = [11100000]^T$$

$$\hat{f}_1 = F_8 f_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} . \hat{f}_2 = F_8 f_2 = \begin{bmatrix} 2 \\ 1 + e^{-\frac{i\pi}{4}} \\ 1 - i \\ 1 + e^{-\frac{3i\pi}{4}} \\ 0 \\ 1 + e^{\frac{3i\pi}{4}} \\ 1 + i \\ 1 + e^{\frac{i\pi}{4}} \end{bmatrix}$$

$$\hat{f}_3 = F_8 f_3 = \begin{bmatrix} 3 \\ 1 - i + e^{-\frac{i\pi}{4}} \\ -i \\ 1 + e^{-\frac{3i\pi}{4}} + i \\ 1 \\ 1 - i + e^{\frac{3i\pi}{4}} \\ i \\ 1 + e^{\frac{i\pi}{4}} + i \end{bmatrix}$$



$\hat{f}_1$uses only one $\omega^0 = 1$

$\hat{f}_2$uses ω^0, ω^1

$\hat{f}_3$uses $\omega^0, \omega^1, \omega^2$

(7). (Optional, for extra credit). Given the data: x = 1 : 10, y = [0.78, 1.27, 1.33, 1.69, 1.96, 1.67, 2.07, 2.11, 1.91, 1.92], determine the least squares fit of the form y = a (1 − exp (−bx)) by setting up a nonlinear system of equations for a and b and solving it with Newton's method (that you should implement yourself)

$$y = a\,(1 - exp\,(-bx))$$

$$F = \sum \left(y_i - a\,(1 - exp\,(-bx_i))\right)^2$$

$$\frac{\partial F}{\partial a} = -2\sum \left(y_i - a\,(1 - exp\,(-bx_i))\right)(1 - exp\,(-bx_i))$$

$$\frac{\partial F}{\partial b} = -2a\sum \left(y_i - a\,(1 - exp\,(-bx_i))\right) exp\,(-bx_i)(x_i)$$

$$a_{k+1} = a_k - \frac{F(a_k, b_k)}{\frac{\partial F}{\partial a}(a_k, b_k)}$$

$$b_{k+1} = b_k - \frac{F(a_k, b_k)}{\frac{\partial F}{\partial b}(a_k, b_k)}$$

The most unstable method I've ever used. Through the attemts I came to algorithm:

For i in (1:20)

For j in (1:100)

Calculate a_{k+1} $(b_k\ is\ locked)$

Choose a_{best} such that F is min

$a_k = a_{best}$

For j in (1:100)

Calculate b_{k+1} $(a_k\ is\ locked)$

Choose b_{best} such that F is min

$b_k = b_{best}$

After that I obtained a_k, b_k, such that F is min.
Initial gues:

$$a_0 = 5, b_0 = 2$$

I tried to calculate a_{k+1} and b_{k+1} together but there was chaos.
Best Fit:
F = 0.155785135254788 a = 2.00938295431986 b = 0.460632487826948