HW2

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(1). Problem

Calculate Fermi energy (in eV), mobility, and drift velocity of free electrons (external s-electrons) in Zinc at normal temperature and pressure (NTP). An applied electric field is 2 V/cm, resistivity:
$\rho = 5.9 \cdot 10^{-8}\, Ohm \cdot m$, density: $D = 7130\, kg/m^3$.

1. Fermi Energy

   Zinc atomic weight: $M = 65{,}38$
   Number of free electrons: = 2
   Electron concentration:
   $$n = \frac{num\ of\ free\ electrons * N_A * D}{M} = 1.3 * 10^{29}\ atom/m^3$$
   Fermi Energy:
   $$E_F = \frac{h^2}{8m}\left[3 * \frac{n}{\pi}\right]^{\frac{2}{3}} = 1.5 * 10^{-18} J = 9.4 eV$$

2. Mobility

   Relaxation time $\tau = \frac{m}{ne^2p}$

   Electric field $E = 2\frac{V}{cm} = 2 * 10^2\ V/m$

   Drift velocity $v_d = \frac{eE}{m} * \tau$

   Mobility $\mu = \frac{v_d}{E} = \frac{e}{m} * \frac{\tau}{1} = \frac{e}{1}\frac{1}{ne^2p} = \frac{1}{nep} = 8 * 10^{-4}\frac{m^2}{Vs}$

3. drift velocity

   Drift velocity $v_d = \frac{eE}{m} * \tau = \frac{E}{1} * \frac{1}{nep} = E * \mu = 0.16\frac{m}{s}$

(2). Problem

Calculate a ratio of the Fermi velocity of the free electrons in metal to the average velocity of free electrons at T = 300 K. The concentration of free electrons in metal is 1.81·10^29 m-3.

$$n = 1.81 * 10^{29}\frac{1}{m^3}$$

Fermi energy T =0: $E_{F0} = \frac{h^2}{2m}\left(\frac{3}{8\pi}\right)^{\frac{2}{3}} n^{\frac{2}{3}} = 1.85 * 10^{-18} J$

Fermi energy T>0: $E_F = E_{F0}\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right) = 1.85 * 10^{-18} J \approx E_{F0}$

$$< E_F > = \frac{3}{5}E_{F0}$$

$$v_1 = \sqrt{2\frac{< E_F >}{m}}, v2 = \sqrt{2\frac{E_F}{m}}$$

$$\frac{v2}{v1} = \sqrt{\frac{E_F}{< E_F >}} = \sqrt{\frac{5}{3}} = 1.29$$

(3). Problem

Derive Drude-Zommerfeld permittivity formula for noble metals.

Free electron contribution:

$$m_e\ddot{r} + m_e\mathrm{T}\dot{r} = eE_0\exp(-i\omega t), \qquad T = \frac{v_F}{l}, \qquad r(t) = r_0\exp(-i\omega t)$$

$$m_e[-\omega^2 r_0\exp(-i\omega t) + \mathrm{T}(-\mathrm{i}\omega)r_0\exp(-i\omega t)] = eE_0\exp(-i\omega t)$$

$$r_0[-\omega^2 - \mathrm{i}\omega\mathrm{T}] = \frac{eE_0}{m_e}$$

$$r_0 = \frac{\frac{eE_0}{m_e}}{-\omega^2 - \mathrm{i}\omega\mathrm{T}}$$

Such as $P(\omega) = ner(\omega)$

$$P(\omega) = ne\frac{\frac{eE_0}{m_e}}{-\omega^2 - \mathrm{i}\omega\mathrm{T}}\exp(-i\omega t) = -\frac{\frac{ne^2}{m_e}}{\omega^2 + \mathrm{i}\omega\mathrm{T}}E_0\exp(-i\omega t)$$

$$= -\frac{\frac{ne^2}{\varepsilon_0 m_e}}{\omega^2 + \mathrm{i}\omega\mathrm{T}}\varepsilon_0 E(\omega) = -\frac{\omega_p^2}{\omega^2 + \mathrm{i}\omega\mathrm{T}}\varepsilon_0 E(\omega),$$

where $\omega_p^2 = \frac{ne^2}{\varepsilon_0 m_e}$

$$D = \varepsilon_0\varepsilon(\omega)E(\omega) = \varepsilon_0 E(\omega) + P(\omega) = \varepsilon_0 E(\omega)\left(1 - \frac{\omega_p^2}{\omega^2 + \mathrm{i}\omega\mathrm{T}}\right) \Rightarrow$$

$$\varepsilon(\omega) = \left(1 - \frac{\omega_p^2}{\omega^2 + \mathrm{i}\omega\mathrm{T}}\right)$$

(4). Problem

Using classical theory, estimate maximal linear velocity of the electron surface during its rotation, which is necessary to provide the magnetic moment due to the spin of the electron, equal to:

Pms = 9.3·10-24 Am2.

Compare the result with the speed of light c. Hints: refer to the textbook Ch 7 and the ball moment of inertia.

Electron radius: $r = 2.8 * 10^{-15}m$

$$\mu = \gamma\left(\frac{e}{2m_e}\right)S$$

For electron $\gamma = -2$

$$S = Jw_{max} = \frac{2}{5} m_e R^2 v_{max}/R \ = \frac{2}{5} m_e R v_{max}$$

$$\mu = \gamma \left(\frac{e}{2m_e}\right) \frac{2}{5} m_e R v_{max} = \gamma \left(\frac{e}{5}\right) R v_{max} => v = \mu \frac{5}{\gamma e R} = 5 * 10^{10} \frac{m}{s}$$

(5). Problem

The same magnetic field H is applied, first, to a paramagnetic bulk material, and, second, to a diamagnetic bulk material. Considering an equal absolute value of magnetization in both cases, which is 0.02 %, calculate the ratio of magnetic induction B.

$$B = \mu_0 (H + M) = \mu_0 (H + xH)$$

$$|x_1| = |x_2| = x = 0.02\%, \qquad x_1 \geq 0 => x_1 = 0.02\% = x$$

$$x_2 \leq \ 0 => x_2 = -0.02\% = -x$$

$$\frac{B_1}{B_2} = \frac{1 + x_1}{1 + x_2} = \frac{1 + x}{1 - x} = 1.0004$$

(6). Problem

An electron in a hydrogen atom circulates with a radius 0.052 nm. Calculate the change in its magnetic moment if a magnetic induction (B) = 3.3 Wb/m2 acts at right angles to the plane of orbit.

$$F_{new} = F_{old} + F_{lorentz}$$

$$mr\omega^2 = mr\omega_0^2 \pm Bev = mr\omega_0^2 \pm Ber\omega$$

$$\omega^2 \pm \frac{Be\omega}{m} - \omega_0^2 = 0$$

$$(\omega_0 + \Delta\omega)^2 \pm \frac{Be}{m} (\omega_0 + \Delta\omega) - \omega_0^2 = 0$$

$$\omega_0^2 + 2\Delta\omega\omega_0 + \Delta\omega^2 \pm \frac{Be}{m} (\omega_0 + \Delta\omega) - \omega_0^2 \approx (\Delta\omega \ll \omega_0)$$

$$2\Delta\omega\omega_0 + \Delta\omega^2 \pm \frac{Be}{m} (\omega_0) \approx$$

Neglecting $\Delta\omega^2$

$$2\Delta\omega\omega_0 \pm \frac{Be}{m} (\omega_0) = 0$$

$$\Delta\omega = \pm \frac{Be}{2m}$$

From $\mu = -\frac{er^2\omega}{2} => \Delta\mu = -\frac{er^2\Delta\omega}{2}$

$$\Delta\mu = \pm \frac{er^2}{2} \frac{Be}{2m} = \pm \frac{e^2 r^2}{4} \frac{B}{m} = 6.2 * 10^{-29} A\, m^2$$