Statistical Shape Modeling

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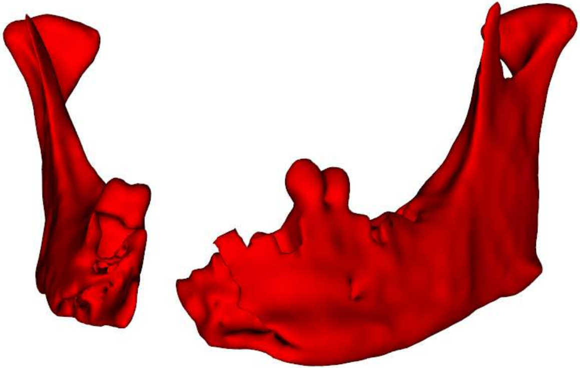
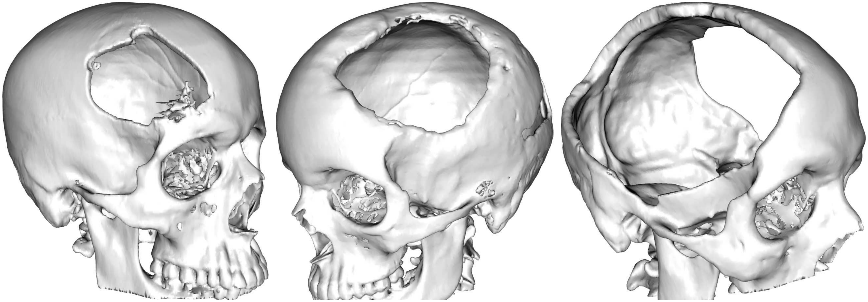
Friday Harbor, 02/21/2020

Problem I



Problem II

Cranial Trauma



Solution

- Use a statistical model to infer missing data based on the given information.
- A statistical shape model (SSM) can be a sensible choice

Background

Statistical shape models are widely used tools in computer vision and medical image analysis. They model the variability of a class of shapes by means of a normal distribution, that has been extrapolated from training data.
They are successfully being used as a prior for different algorithms, such as segmentation [13] or registration [2].

Applications of Shape models

- Object recognition
- Surgery planning
- Implant design
- Registration tasks
- Prediction of missing structures

Statistical shape model

Definition

Statistical shape models as used in the following slides are simply PCA-spaces calculated from superimposed shapes and an additional representer (e.g. a mesh) that tells us how to assemble the data from the latent space into e.g. triangular meshes and back.

Creating and modifying those models can be performed by R-package `RvtkStatismo` (*https://github.com/zarquon42b/RvtkStatismo/*), an implementation of the C++ library statismo (*https://github.com/statismo/statismo*).

Introduction

The idea behind PCA Models is to learn a (linear) model for an object class from a set of typical examples of this class. Let $O = O_1, ..., O_n$ be a set of training examples. In our cases these can be corresponding landmark configurations or meshes, where the vertices are in correspondence (i.e. pseudo-landmarks).

Shape Models

Our general goal is to estimate a (multivariate normal) probability distribution from a finite set of training data:

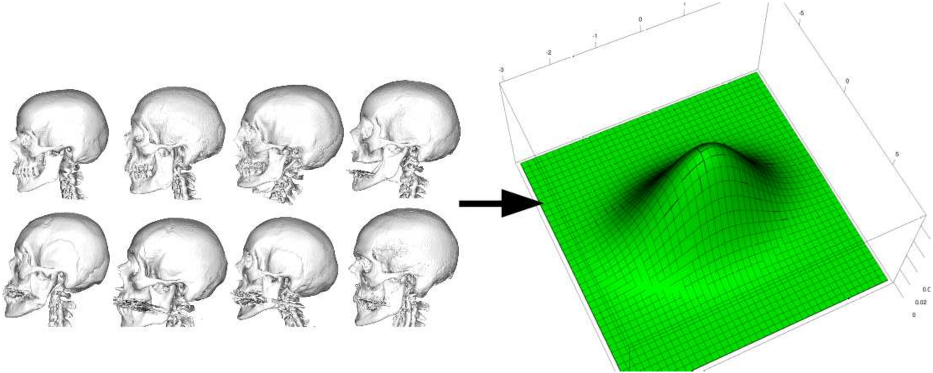


Figure: From shapes to distributions

Prerequisites

Prerequisites for generating an SSM:

- Specimens are parametrized as finite sets of 3D-pointclouds (optionally connected to form a triangular surface mesh)
- A representer that defines how the data that is transformed into vector form and back
- The positions of each point (vertex of a mesh) need to be pseudo-homologous throughout a given sample
- Differences in spatial position (and optionally size) are removed by Procrustes registration

Capturing morphological variability

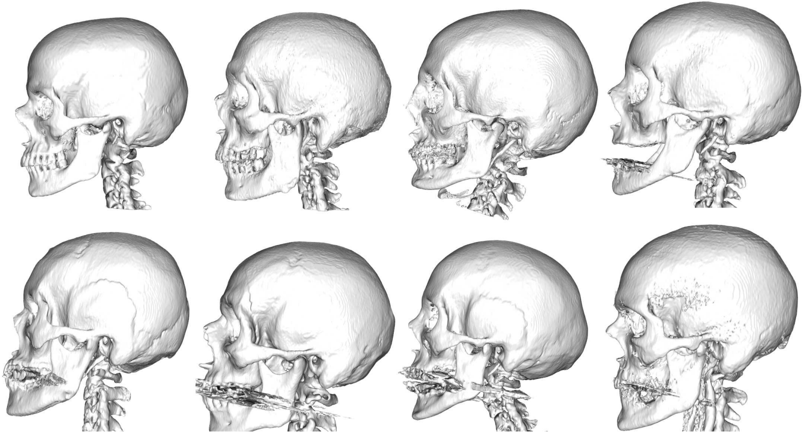


Figure: Variability of the cranial vault

Establishing Correspondences

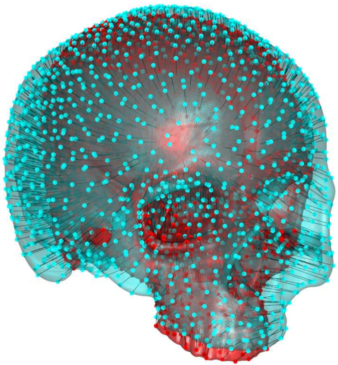


Figure: Point-to-point correspondences between two registered and aligned human crania.

PCA-models - Details

Each shape s can be parameterized as a linear combination of the eigenvectors of the sample's covariance matrix.

$$\Sigma = \frac{1}{n-1} \sum_{i=1} (s_i - \mu)(s_i - \mu)^T$$

with its eigendecomposition

$$\Sigma = UD^2U^T$$

Then each shape within the model can be represented as

$$s = s(\alpha) = \mu + UD\alpha =: \mu + Q\alpha$$

with the coefficient vectors being distributed according to $\mathcal{N}(0, I_n)$ and the shapes according to $\mathcal{N}(\mu, QQ^T) = \mathcal{N}(\mu, \Sigma)$.

PCA-models - Sampling

Knowing that the coefficients are distributed according to $\mathscr{N}(0, I_n)$, we can then use any suitable random number generator to draw instances from our distribution.

SSM

Sampling from the distribution

Model applications

A model like that can now be used e.g. to find the best shapes given a specific problem:

- A registration task can now be posed as an optimization problem

$$(\alpha^*) = \underset{\alpha}{\operatorname{argmin}}\, Distance\,(R, S(\alpha))$$

- A free-form registration can be regularized by an SSM
- In clinical cases the deviation between the registration result and the affected shape can be interpreted in terms of pathology

Model applications

Trauma

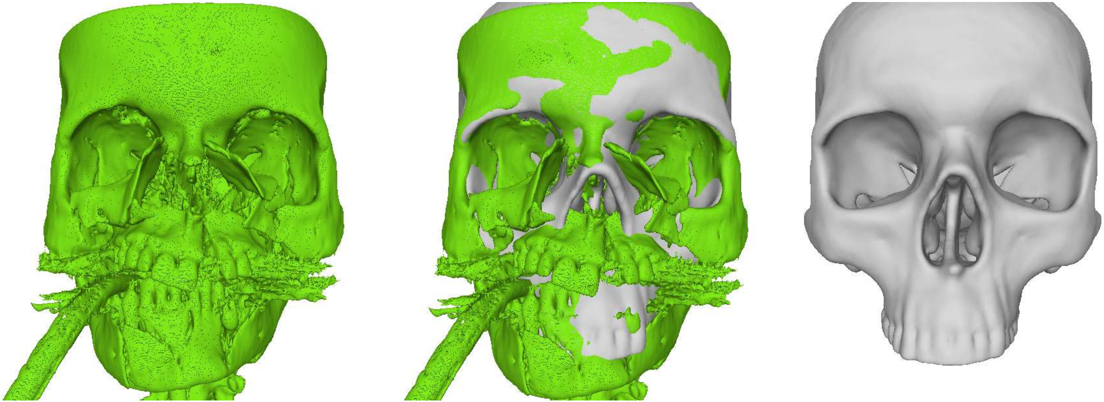


Figure: Estimations for midfacial trauma repositioning

Introduction

Given the multivariate normality of our distribution, we can use Bayesian reasoning to not only create a single prediction (like in a standard linear regression model), but we can compute a posterior distribution, given some (partial) prior knowledge.

Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Example I

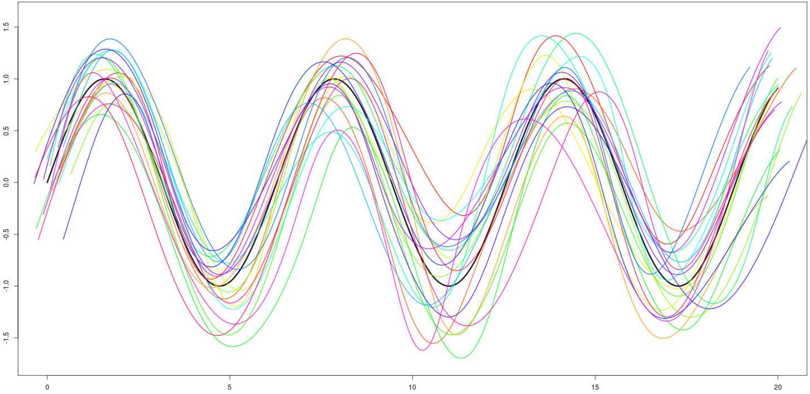


Figure: Distribution of shapes

Example I

Constrained by a single point

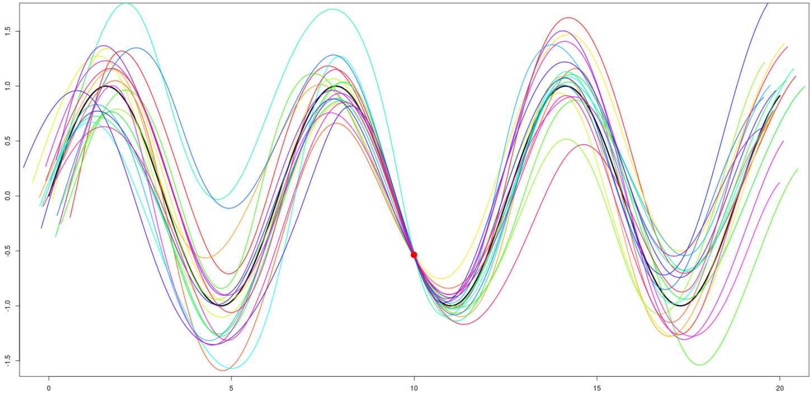


Figure: Constrained by a single point

Example I

Constrained by three points

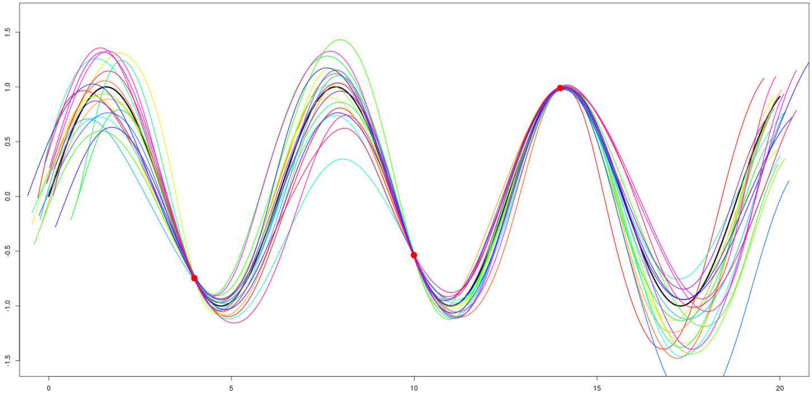


Figure: Constrained by three points

Example I

Constrained by 9 points

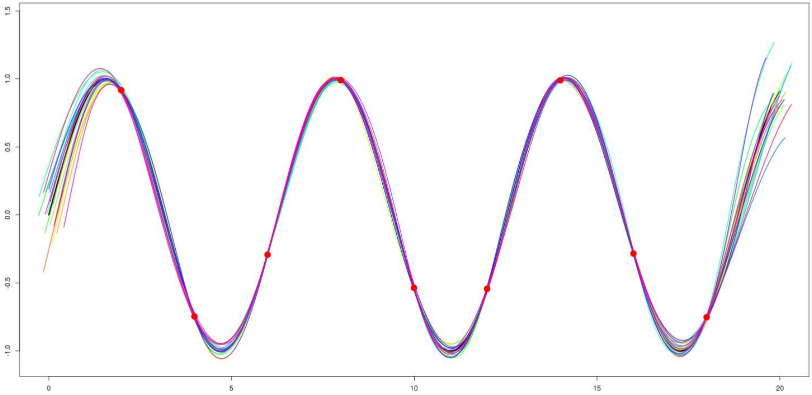


Figure: Constrained by 9 points

Example I

Constrained by 15 points

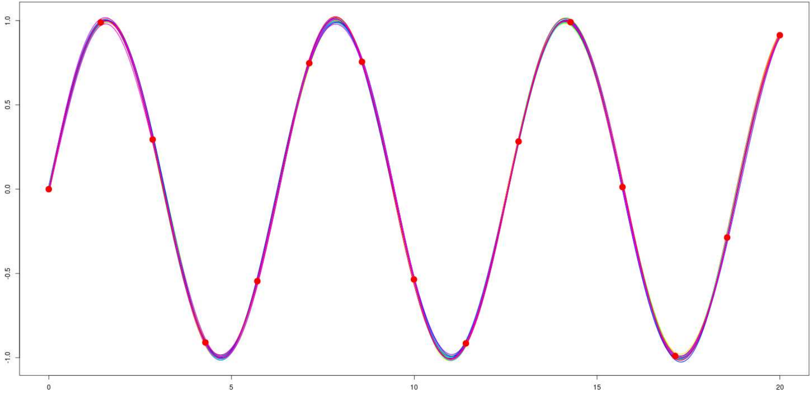


Figure: Constrained by 15 points

Posterior SSM

Following notation from above, each shape s can be written as:

$$s = s(\alpha) = \mu + UD\alpha =: \mu + Q\alpha$$

Exploiting the Gaussian distribution, we can calculate a posterior shape model, given some prior known information (e.g. the position of a subset of coordinates):
Having q known entries in the shape vector, then $\mu_g \in \mathbb{R}^q$ denotes the subvector of the mean shape and $Q_g \in \mathbb{R}^q$ the according submatrix of Q.

Posterior SSM

The given data can then be modeled as

$$s_g = \mu_g + Q_g \alpha$$

or, more general:

$$s_g = \mu_g + Q_g \alpha + \varepsilon$$

to account for the distance from s_g to the model space. Including the noise term, with the distribution of shapes being

$$p(s_g) = \mathcal{N}(\mu_g, Q_g Q_g^T + \sigma^2 I_q)$$

Posterior SSM

From

$$p(\alpha) = \mathcal{N}(0, I_n)$$

and

$$p(s_g|\alpha) = \mathcal{N}(\mu_g + Q_g\alpha, \sigma^2 I_q)$$

we can now compute the conditional distribution

$$p(\alpha|s_g)$$

and finally

$$p(s|s_g)$$

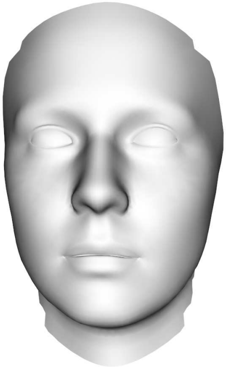
again a multivariate normal distribution, using Bayes' theorem (for details, see [4, 1]).

Conditional SSM

Bayesian reasoning can also be used to condition an SSM to surrogate variables (a.k.a covariates) such as age, sex or BMI to obtain an appropriate shape model, given the additional information [3].

Examples: Faces for different Ages

Low age vs. high age



What do do, when we have no sample?

You might be wondering:

- If SSM helps with a registration task but
- also it needs a set of registered data

What do do, when we have no sample?

We magically create one!

Gaussian Process Morphable Models

Recall: Each shape can be modeled as

$$s = \mu + UD\alpha = \mu + \sum \alpha_i d_i u_i$$

where d is the square root of the eigenvalue and and u the corresponding eigenvector.

- This can be viewed as a variation of shapes but also as
- A distribution of deformations around a mean shape μ

We can model a distribution of smooth deformations around the template using a so-called kernel function. A suitable kernel function would be a *Gaussian kernel:*

$$k_g(x,y) = exp(-\|x-y\|^2/\sigma^2)$$

where x and y are two coordinates on our template shape and then model the respective $3x3$ entry of that specific part of our artificial “covariance”-matrix as

Example

Example

Assuming the distance between two coordinates *x* and *y* is *2* mm and we use a scale factor of *1*, we set $\sigma = 10$, then $k_g(x,y) = e^{(-\frac{4}{100})} = 0.96$ and then the corresponding entry in the covariance matrix becomes

$$\begin{matrix} \vdots & \cdots & \cdots & \cdots & \vdots \\ 0.96 & 0 & 0 & \cdots & \vdots \\ 0 & 0.96 & 0 & \cdots & \vdots \\ 0 & 0 & 0.96 & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \vdots \end{matrix}$$

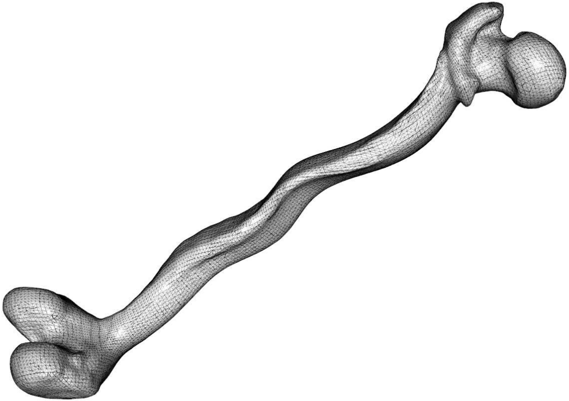
In the following example we use a Gaussian kernel with $\sigma = 50$ and a scaling factor $\lambda = 50$.
The “covariance” matrix between two points is now estimated as

$$cov(x,y) = \lambda \cdot diag(e^{\frac{-(x-y)(x-y)^t}{\sigma^2}})$$

Kernel Combinators

The application of these kernels can also be combined summation and or multiplication of the covariance functions.
We can, for example increase the variability of a small sample by adding a set of kernels to the empirical kernel.

Gaussian Deformation model



Example: Femur

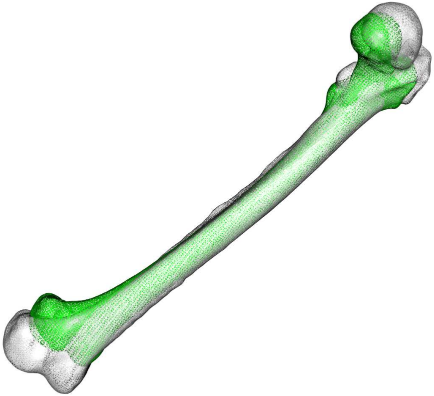


Figure: Reference (white) and target shape (green)

Prior Knowledge

Three landmarks

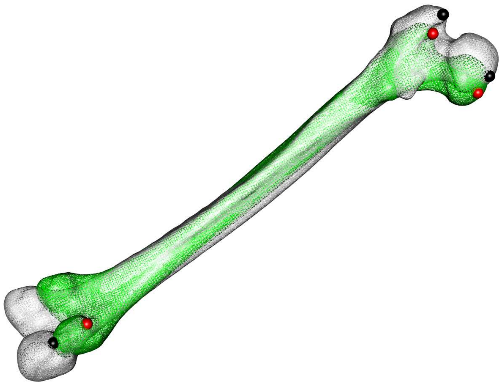


Figure: Reference (white) and target shape (green)

Constrained Model

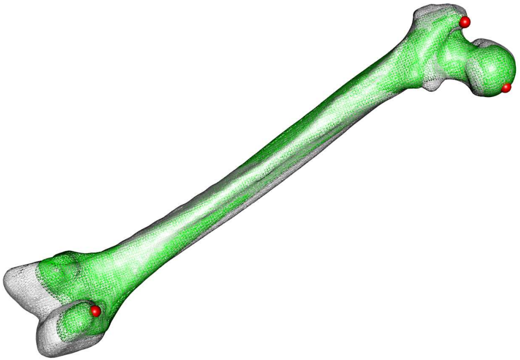
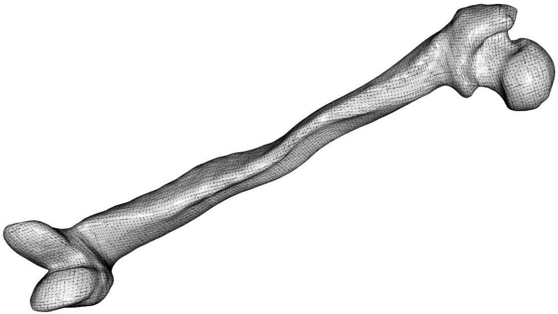


Figure: Posterior mean

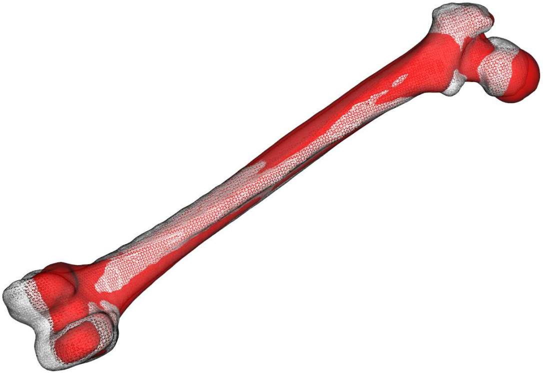
Constrained model

Model constrained by the landmark (with an assumed placement error variance of 2mm)

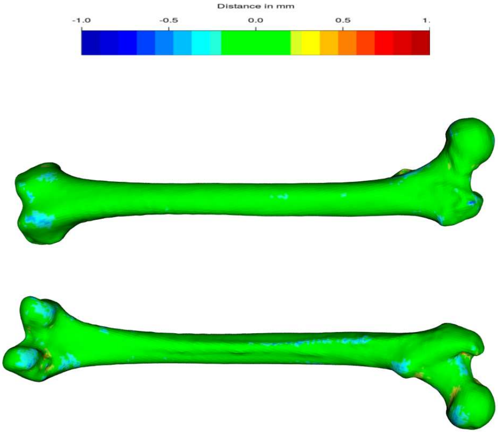


Surface Matching based on the Gaussian deformation model

The deformation model is now used to dampen deformations and to make them more "probable".



Matching Error



Distortion

Minimal mesh distortion

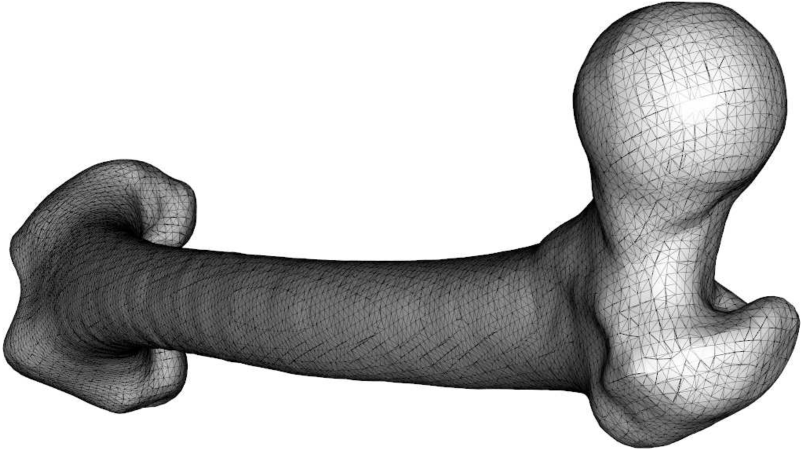


Figure: Looking at the structure generated by the marching cubes, it can be seen that the mesh's internal

Introduction and Motivation

Quantifying phenotypic variability in humans is one of the central interests in biological anthropology. The aim hereby is usually to find causes and effects of this variability.
In the study presented here, however, we intended to exploit explicit knowledge about the shape variability of the ***cranial vault*** in a healthy sample in order to estimate and restore pathological defects. This allows to objectify and simplify surgery planning.

Approach

- Find SSM instance most similar to a given cranium with defects
- Detect the defect/missing area
- Find a deformation that maps the SSM instance seamlessly to the defect cranium, reconstructing the defect.

Material I

SSM generation

An SSM was created based on registered[1] CT-scans:

- 131 pathologically unaffected individuals, taken in the course of medical treatment at the University Clinics Freiburg.
- 61 females and 70 males
- Average age = 53.2 years
- Slice thickness between 1 and 2 mm

[1] Registration was performed using the software ANTsR[2]

Material II

Result Validation

- Our proposed method was evaluated on the CT-scans of 31 pathologically unaffected individuals, not included in the SSM to avoid statistical self-inference.

Methods I

Registration of the SSM to the target cranium

- Place 6 anatomical landmarks to establish an initial correspondence between SSM and target shape
- Constrain the SSM to those landmarks (including an error margin due to placement errors)
- Fit SSM to target shape minimizing a symmetric mesh-mesh distance

Landmark placement

Landmarks

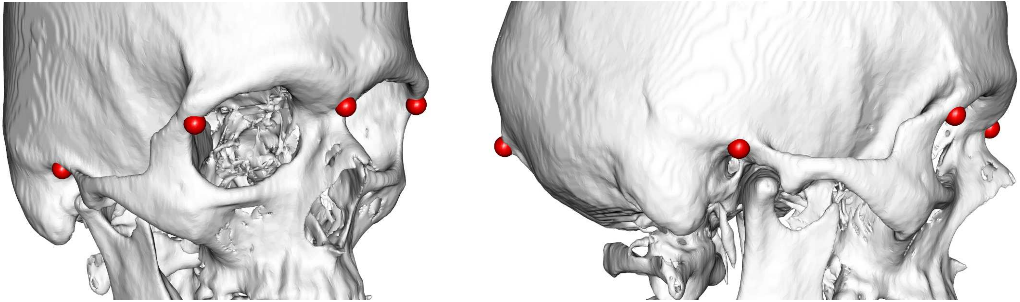


Figure: Landmark positions.

SSM Registration

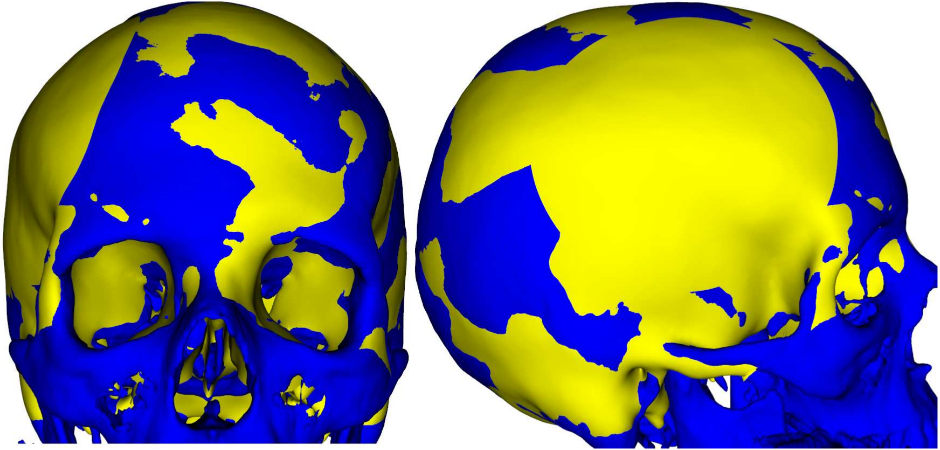


Figure: SSM instance (yellow) after fitting to the target (blue)

Methods II

Detection of defect areas

- 3000 semi-landmarks are sampled on the cranial vault of the SSM
- Semi-landmarks are projected onto the target surface.
- Distances to the target mesh and mesh topology are compared to properties on SSM instance.
- Discarding coordinates deemed unsuitable, the SSM instance is deformed on the target using a Thin-plate spline deformation.

Identification of defects

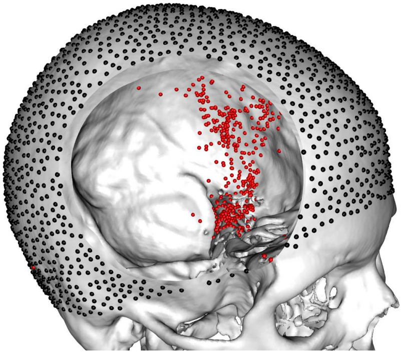


Figure: Identification of (potentially) unusable correspondences. The red spheres represent the projection of the semi-landmarks from the fitted SSM to the target mesh that are excluded from the warping. The black spheres are those projections that are used in the final deformation.

Final deformation

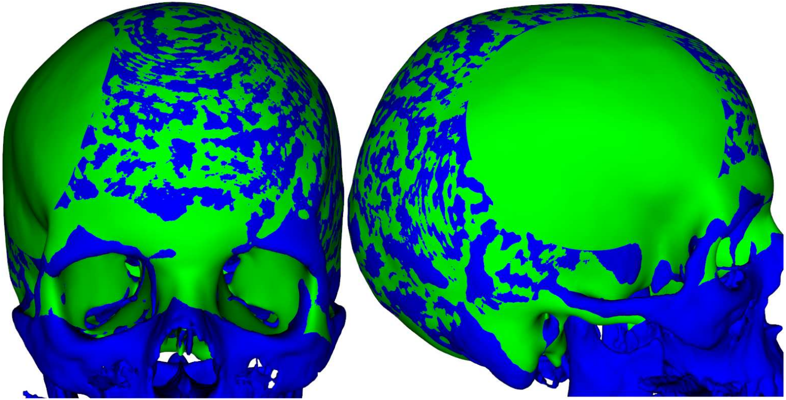


Figure: Green surface: The result from SSM fitting warped to the target surface (blue) using a TPS based on semi-landmarks.

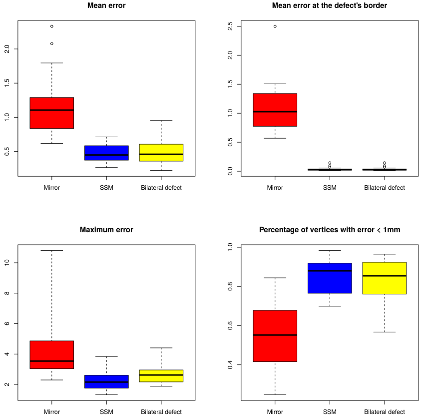
Methods III

Evaluation

- Both unilateral and bilateral defects were created virtually on the cranial vault.
- Displacement at the entire defect as well as the borders was evaluated by measuring the distances between the reconstructed surface to the removed parts
- Additionally, the results for the unilateral case were compared to the current gold standard of mirroring, performed by a trained surgeon.

Evaluation

The proposed method yielded very low estimation errors



Error distribution

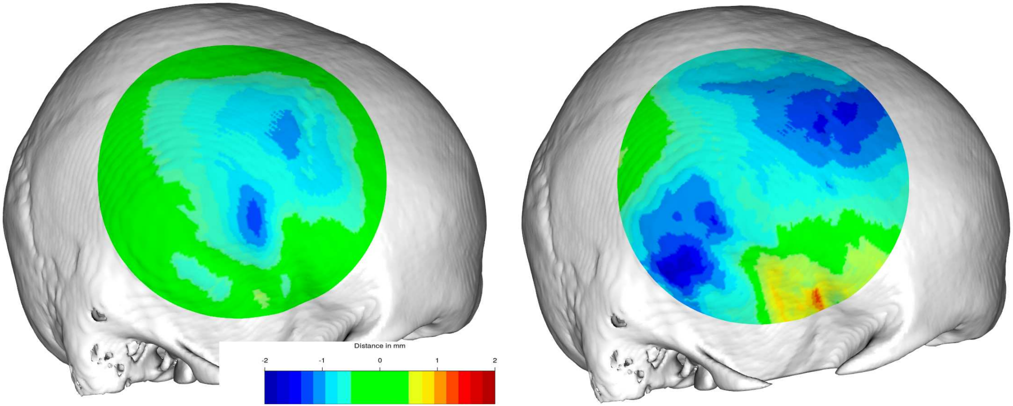


Figure: Average per-vertex estimation error for our method (left) and the gold standard using a mirroring technique (right)

Real world application

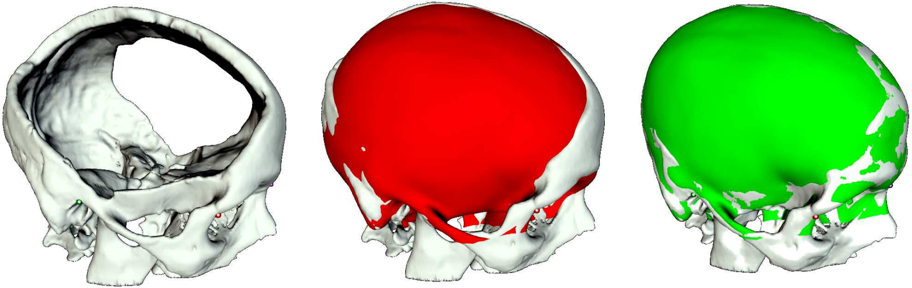
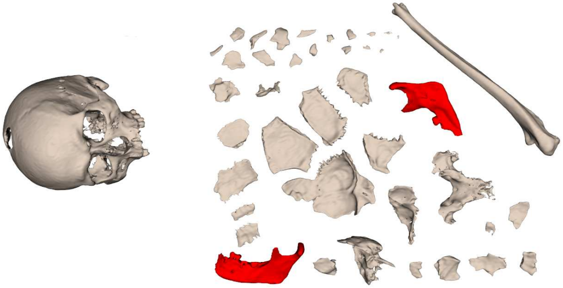
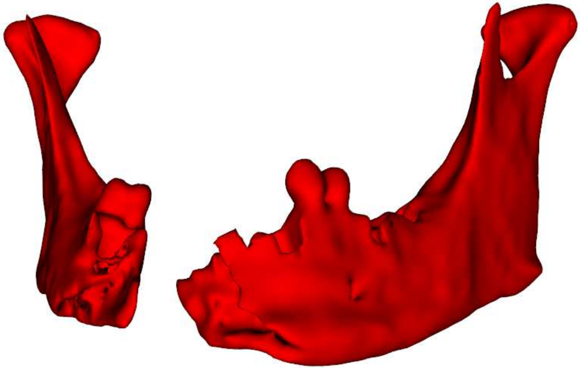


Figure: Application of the method to a cranium with severe defects. Left: target; center: SSM fitted to target; right: fitted SSM instance warped to target mesh.

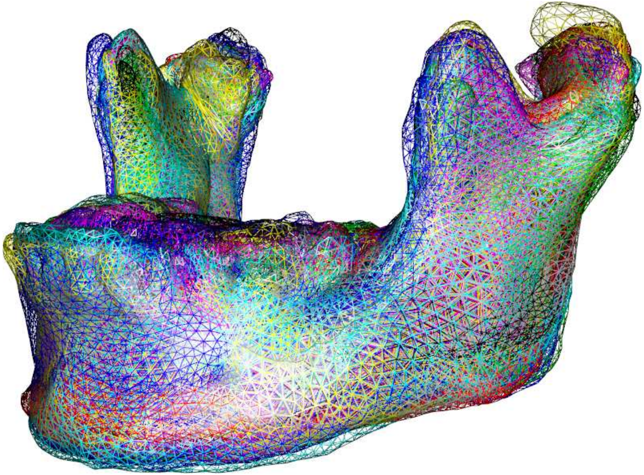
Forensic case



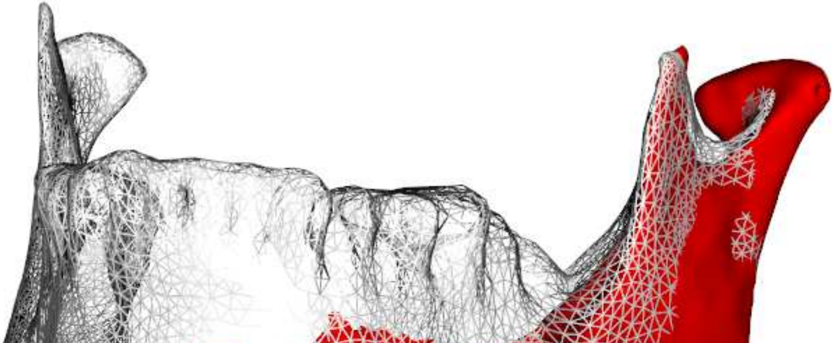
Mandible fragments



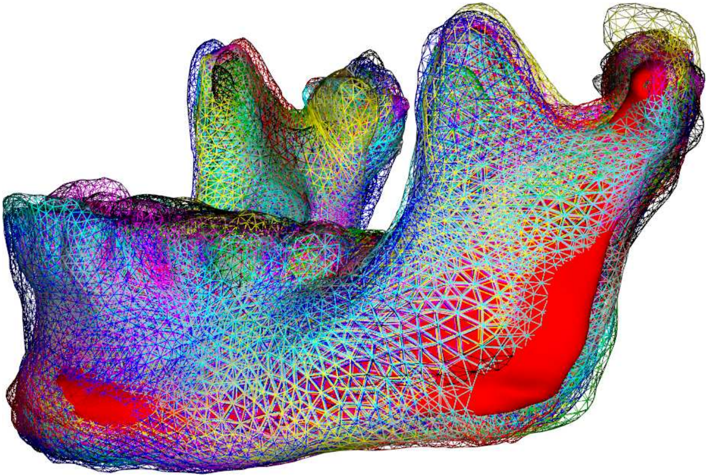
Shape model



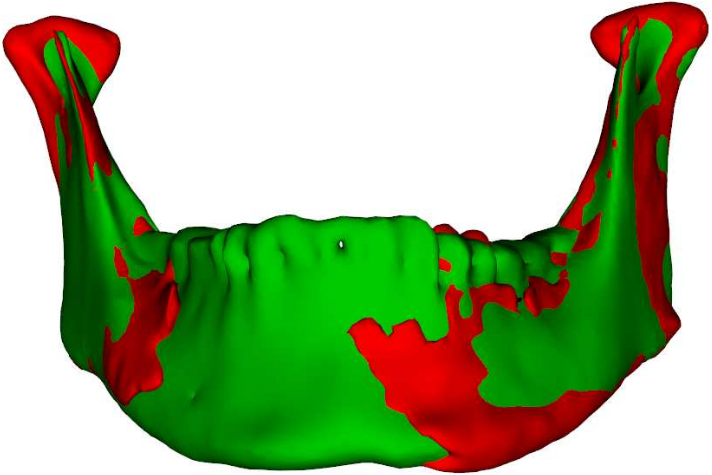
Registration



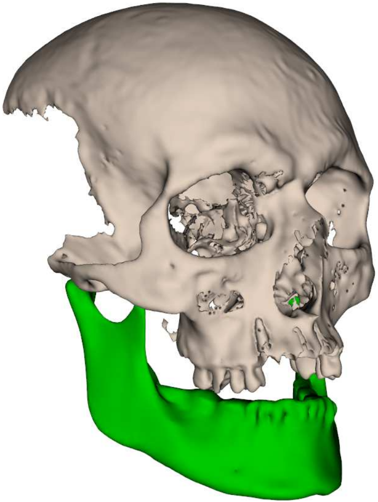
The defect mandible registered to the model



Estimation of the missing parts using a constrained model

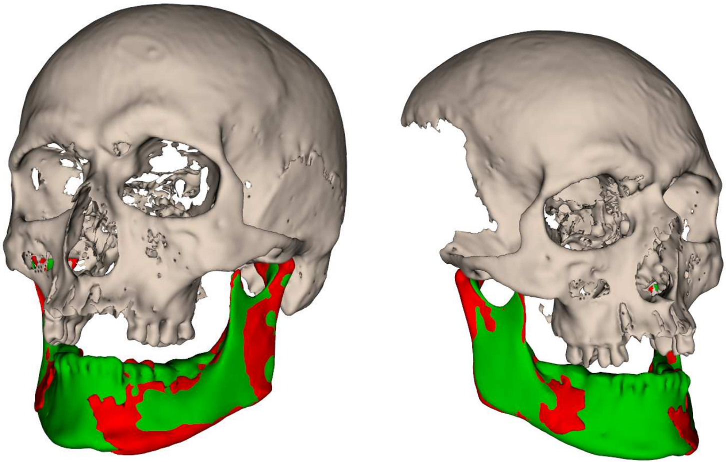


Rotated to the skull



Rotated to the skull

Original fragments are red



Statistical Shape Modeling Viewer & MOOC

And Summer School in African Winter

Statismo Viewer: https://github.com/statismo/statismo/wiki/Statismo-Viewer
MOOC: https://www.futurelearn.com/courses/statistical-shape-modelling/4
Summer School in Cape Town (SA): http://shapemodelling.cs.unibas.ch/pmm2020/

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