AI4Good: Solutions

June 2020

2. **(10 points) Neural Network Representation**
Consider the binary classification problem described in the figure below (right), with the two classes being represented by solid triangles and empty squares. Let us refer to the squares as Class 0 and the triangles as Class 1. In this problem we will consider a classifier using the network in the figure (right). All activation functions in the hidden layer being Heavyside step functions :

$$H[x] = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0, \end{cases} \tag{1}$$

Fill-in the values of the model parameters (w_{11}, w_{12}, w_{13}, w_{21}, w_{22}, w_{23}, u_1, u_2, u_3, b_1, b_2, b_3 and c) that result in the correct class prediction for the data presented in the figure.

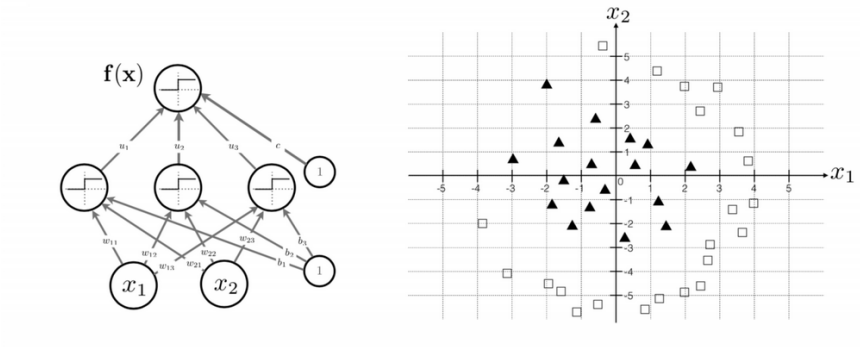

Figure 1: Question

The weights:
$u_1, u_2, u_3 \in \{0, 1\}$
$c, b, w \in \mathbb{R}$
$u_1 = \mathbb{H}(x_1 w_{11} + x_2 w_{21} + b_1)$
$u_2 = \mathbb{H}(x_1 w_{12} + x_2 w_{22} + b_2)$
$u_3 = \mathbb{H}(x_1 w_{13} + x_2 w_{23} + b_3)$
squares: 0
triangles: 1

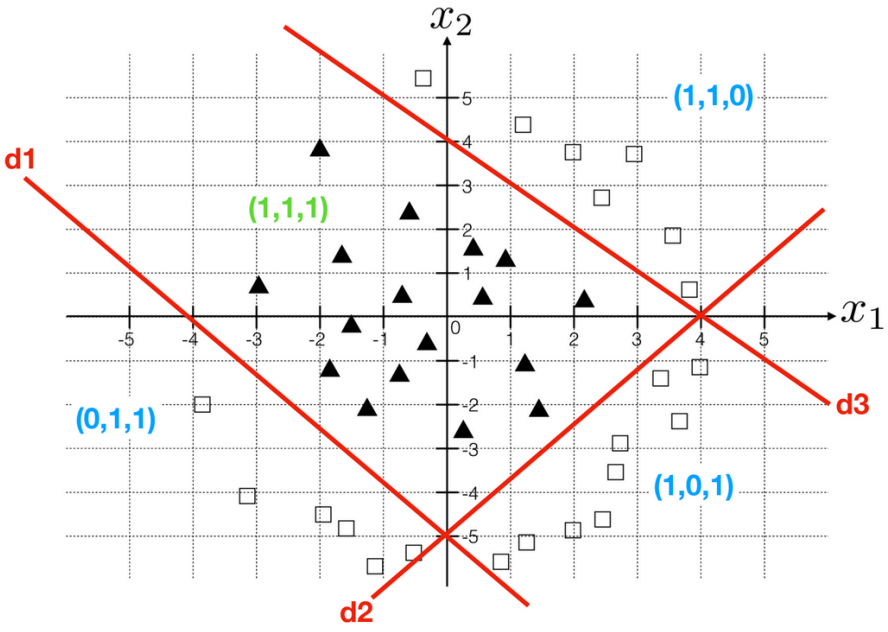

Figure 2: Hint

$d1, d2, d3$ (Figure 2) seem to define decent decision boundaries. Points that are above $d1, d2$ and below $d3$ are going to be classified as triangles (the network should output 1 for these points).

Test point: $(0, 0)$

$d1 : (-4, 0), (0, -5)$

$d1 : \frac{5}{4}x_1 + x_2 + 5 = 0$

(x_1, x_2) which satisfy $\frac{5}{4}x_1 + x_2 + 5 > 0$ are above $d1$.

Hence

$$\begin{cases} w_{11} = \frac{5}{4} \\ w_{21} = 1 \\ b_1 = 5 \end{cases}$$

$d2 : (4, 0), (0, -5)$

$d2 : \frac{5}{4}x_1 - x_2 - 5 = 0$

In this case, the points that satisfy $\frac{5}{4}x_1 - x_2 - 5 < 0$ are above d2.

$$\begin{cases} w_{12} = -\frac{5}{4} \\ w_{22} = 1 \\ b_2 = 5 \end{cases}$$

Similarly for $d3$:
$d3 : x_1 + x_2 - 4 = 0$

$$\begin{cases} w_{13} = -1 \\ w_{23} = -1 \\ b_3 = 4 \end{cases}$$

z_1, z_2, z_3 results from the hidden neurons
output triangles (1) : $u_1 z_1 + u_2 z_2 + u_3 z_3 - 3 \geq 0$
output squares (0) : $u_1 z_1 + u_2 z_2 + u_3 z_3 - 3 < 0$

$$\begin{cases} u_1 = 1 \\ u_2 = 1 \\ u_3 = 1 \\ c = -3 \end{cases}$$

□