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MAT205
Fourier Analysis

Bangobandhu Sheikh Mujibur Rahman Science and Technology University

24 August, 2021

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Now we on. . .

Start-UP

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Basic Beyond
What is Fourier series

Fourier Series

FS is a special trigonometric periodic series derived by Joseph Fourier$(1768 \rightarrow 1830)$.

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Basic Beyond
What is Fourier series

Fourier Series

FS is a special trigonometric periodic series derived by Joseph
Fourier($1768 \rightarrow 1830$).

Figure: Joseph Fourier

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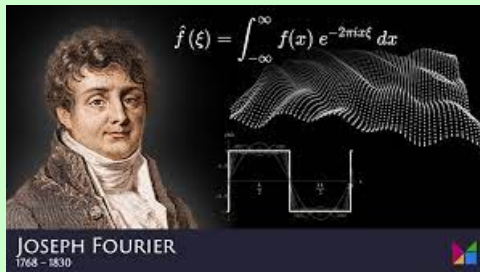○○

Basic Beyond
Fourier series: Mathematical Representation

Our basic trigonometric series is:

$$\frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n \cos nx + B_n \sin nx)$$

It will called **Fourier Series** if the terms A_0, A_n, B_n is:

$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) \cdot dx$$

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot \cos nx \cdot dx$$

$$B_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot \sin nx \cdot dx$$

Where $f(x)$ is any single-valued function defined in interval $(-\pi, \pi)$.

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Basic Beyond
What is Fourier transform

Fourier Transform

FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.

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Basic Beyond
What is Fourier transform

Fourier Transform

FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.

The Fourier transform of a function due to respect of time is a complex valued function of frequency whose magnitude represents the amount of that frequency present in the original function.

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Basic Beyond
Some helpful equations

1. $\sin 0° = \sin \pi = 0$

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Some helpful equations

1. $\sin 0° = \sin \pi = 0$
2. $\cos 0° = \cos 2n\pi = (-1)^{2n} = 1$

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Some helpful equations

1. $\sin 0° = \sin \pi = 0$
2. $\cos 0° = \cos 2n\pi = (-1)^{2n} = 1$
3. $\cos n\pi = (-1)^n$

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Some helpful equations

1. $\sin 0° = \sin \pi = 0$
2. $\cos 0° = \cos 2n\pi = (-1)^{2n} = 1$
3. $\cos n\pi = (-1)^n$
4. $\frac{d}{dx} \sin \theta = \cos \theta$

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Some helpful equations

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3. $\cos n\pi = (-1)^n$
4. $\frac{d}{dx} \sin \theta = \cos \theta$
5. $\frac{d}{dx} \cos \theta = -\sin \theta$

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Some helpful equations

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3. $\cos n\pi = (-1)^n$
4. $\frac{d}{dx} \sin \theta = \cos \theta$
5. $\frac{d}{dx} \cos \theta = -\sin \theta$
6. $\int \sin \theta = -\cos \theta$

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Some helpful equations

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4. $\frac{d}{dx} \sin \theta = \cos \theta$
5. $\frac{d}{dx} \cos \theta = -\sin \theta$
6. $\int \sin \theta = -\cos \theta$
7. $\int \cos \theta = \sin \theta$

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Basic Beyond
Some helpful equations

① $\sin 0° = \sin \pi = 0$

② $\cos 0° = \cos 2n\pi = (-1)^{2n} = 1$

③ $\cos n\pi = (-1)^n$

④ $\frac{d}{dx} \sin \theta = \cos \theta$

⑤ $\frac{d}{dx} \cos \theta = -\sin \theta$

⑥ $\int \sin \theta = -\cos \theta$

⑦ $\int \cos \theta = \sin \theta$

⑧ $\int UV \cdot dx = U \int V \cdot dx + \int \frac{dU}{dx} (\int V \cdot dx) \cdot dx$

Now we on. . .

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Question 1

1

Use the method of Fourier transform to determine the displacement $y(x,t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x), -\infty < x < \infty$. Also show that the solution can be put in the form:

$$y(x,t) = \frac{1}{2} \left[f(x+ct) + f(x-ct) \right]$$

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Question 1
Trying to solve

[Trying to solve:]
Here we have to solve the one-dimensional wave equation

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \qquad [-\infty < x < \infty, t > 0]$$

subject to the following initial conditions

$$y(x, 0) = \text{Initial displacement} = f(x)$$
$$\text{and } y_t(x, 0) = \text{initial velocity} = 0$$

The given partial differential equation is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \qquad (1)$$

Question 1
Trying to solve

Taking the complex Fourier transform of both sides of (1), we have

$$\int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta t^2} e^{-iux} dx = c^2 \int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta x^2} e^{-iux} dx \qquad (2)$$

By the Fourier transform of the derivative of a function we have if $F^n(x)$ is the nth-derivative of $F(x)$ and the first $(n-1)$ derivatives of F(x) vanish as $x \to \pm\infty$ then $F\{F^n(x)\} = (-iu)^n F\{F(x)\}$.

Thus from (2) we have

$$\frac{d^2}{dt^2} \int_{-\infty}^{+\infty} y e^{-iux} dx = c^2(-iu)^2 F\{y(x,t)\}$$

or, $\dfrac{d^2}{dt^2} \displaystyle\int_{-\infty}^{+\infty} y(x,t)e^{-iux} dx = c^2(-u^2)F\{y(x,t)\}$

or, $\dfrac{d^2}{dt^2} F\{y(x,t)\} = -c^2 u^2 F\{y(x,t)\}$

or, $\dfrac{d^2\overline{y}}{dt^2} = -c^2 u^2 \overline{y}$ $\quad \left[\text{where } \overline{y} = \overline{y}(u,t) = F\{y(x,t)\} = \displaystyle\int_{-\infty}^{+\infty} y(x,t)e^{-iux} dx\right]$

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Question 1
Trying to solve

$$\therefore \quad \frac{d^2\overline{y}}{dt^2} + c^2 u^2 \overline{y} = 0 \tag{3}$$

Which is ordinary second order differential equation whose solution is

$$\overline{y} = \overline{y}(u, t) = A\cos(cut) + B\sin(cut) \tag{4}$$

Differentiating with both sides with respect to t

$$\text{we get, } \overline{y}_t(u, t) = -Acu\sin(cut) + Bcu\cos(cut) \tag{5}$$

Also from the initial given conditions, we have

$$y(x, 0) = f(x) \tag{6}$$

$$y_t(x, 0) = 0 \tag{7}$$

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Question 1
Trying to solve

Taking the Fourier transform of (6) and (7) we get,

$$\overline{y}(u,0) = \int_{-\infty}^{+\infty} y(x,0)e^{-iux} \cdot dx = \int_{-\infty}^{+\infty} f(x)e^{-iux} \cdot dx = \overline{f}(u) \quad \text{(say)}$$

$$\therefore \quad \overline{y}(u,0) = \overline{f}(u) \tag{8}$$

$$\text{and } \overline{y}_t(u,0) = \int_{-\infty}^{+\infty} y_t(x,0)e^{-iux} \cdot dx$$

$$= \int_{-\infty}^{+\infty} 0(e^{-iux}) \cdot du = 0$$

$$\therefore \quad \overline{y}_t(u,0) = 0 \tag{9}$$

Putting t = 0 in (5), we have $\overline{y}_t(u,0) = Bcu$

or, $Bcu = 0$ using (9)

$$\therefore \quad B = 0 \quad \text{Since, } cu \neq 0$$

Question 1
Trying to solve

Again putting t = 0 in (4), we get

$$\overline{y}(u,0) = A$$

$$\therefore \ \overline{f}(u) = A \quad \text{using (8)}$$

$$\text{or, } A = \overline{f}(u)$$

Putting the values of A and B in (4), we get

$$\overline{y} = \overline{y}(u,t) = \overline{f}(u)\cos(cut) \tag{10}$$

Taking the inverse Fourier transform of (10) we have

$$y(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{f}(u)\cos(cut)e^{iux}\cdot du$$

$$\text{or, } y(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{f}(u)\left(\frac{e^{icut}+e^{-icut}}{2}\right)e^{iux}\cdot du$$

$$= \frac{1}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{f}(u)e^{iu(x+ct)}\cdot du + \frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{f}(u)e^{iu(x-ct)}\cdot du\right]$$

$$= \frac{1}{2}\left[f(x+ct)+f(x-ct)\right] \text{ (Using the definition of inverse Fourier transform)}$$

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Question 1
Solved

Finally

$\therefore \qquad y(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right].$

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Question 2

2

A thin membrane of great extent is released from rest in the position $z = f(x, y)$, show that the displacement at any subsequent time is given by

$$z(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) \cos ct \sqrt{(u^2 + v^2)} \cdot e^{-i(ux + vy)} \cdot du \cdot dv$$

where $F(u, v)$ is the double Fourier transform of $f(x, y)$.

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Question 2
Trying to proof

[Trying to proof:]
Here the displacement of the membrane is governed by two dimensional wave equation

$$\frac{\delta^2 z}{\delta t^2} = c^2 \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) \quad \text{where } c^2 = \frac{T}{\rho} \tag{1}$$

Taking the double Fourier transform of both sides of (1) we get

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\delta^2 z}{\delta t^2} e^{i(ux=vy)} \cdot dx \cdot dy = \frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot$$

$$\text{or, } \frac{d^2}{dt^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy =$$

$$\frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot dy$$

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Question 2
Trying to proof

$$\text{or,} \frac{d^2\overline{z}}{dt^2} = c^2\{(-iu)^2 + (-iv)^2\}F\{z(x,y,t)\} \text{ where } \overline{z} = \overline{z}(u,v,t) = F\{z(x,y,t)\}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy$$

$$\text{or,} \ \frac{d^2\overline{z}}{dt^2} = -c^2(u^2 + v^2)\overline{z}$$

$$\therefore \qquad \frac{d^2\overline{z}}{dt^2} + c^2(u^2 + v^2)\overline{z} = 0 \qquad\qquad (2)$$

Question 2
Trying to proof

which is an ordinary differential equation whose solution is

$$\overline{z} = A\cos\{c\sqrt{(u^2 + v^2)}t\} + B\sin\{c\sqrt{(u^2 + v^2)}t\} \qquad (3)$$

The given initial conditions are $\overline{z} = f(x, y)$ and $\frac{\delta z}{\delta t} = 0$ at $t = 0$
Taking the Fourier transform of these initial conditions, we get

$$\overline{z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y)e^{i(ux+vy)} \cdot dx \cdot dy = F(u, v) \qquad (4)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\delta z}{\delta t} e^{i(ux+vy)} \cdot dx \cdot dy$$

$$= \frac{d}{dt} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ze^{i(ux+vy)} \cdot dx \cdot dy$$

$$= \frac{d\overline{z}}{dt} \text{ since } \overline{z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} ze^{i(ux+vy)} \cdot dx \cdot dy$$

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Question 2
Trying to proof

$$\therefore \quad \frac{d\overline{z}}{dt} = 0 \text{ at } t = 0$$

When t=0, combining (3) and (4), we get $A = F(u,v)$

Also $\frac{d\overline{z}}{dt} = -Ac\sqrt{(u^2+v^2)}\sin c\sqrt{(u^2+v^2)}t + Bc\sqrt{(u^2+v^2)}\cos c\sqrt{(u^2+v^2)}t$

$$\therefore \ 0 = \left(\frac{d\overline{z}}{dt}\right)_{t=0} = Bc\sqrt{(u^2+v^2)}$$

or, $B = 0$

Putting the values of A and B in (3), we get

$$\overline{z} = F(u,v)cos\{c\sqrt{(u^2+v^2)}t\} \qquad (5)$$

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Question 2
Proved

Ultimately

Now applying the inversion formula for double Fouier transform, we have

$$z(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) \cos\{c\sqrt{(u^2 + v^2)}t\}e^{-i(ux+vy)} \cdot du \cdot dv$$

Which is the required displacement at any subsequent time t.

Now we on. . .

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