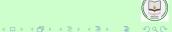
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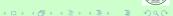
Team Members

Preamble

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Dr. Saleh Ahmed Supervisor Associate Professor Department of CSE.





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 Basic Beyond
 Illustration 1
 Illustration 2
 Ending Scene

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Start-UP

Name: Md. Kazi Iqbal Hossen ID: 18ICTCSE065 Department of CSE SHIICT, BSMRSTU



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To obtain beamer source code, click to:

Link 1: **Ubuntu pastebin**Link 2: **Github**(recommended)

Or, scan(clickable in pdf) the following quick response code:



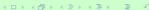
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What is Fourier series

Fourier Series

FS is a special trigonometric periodic series derived by Joseph Fourier(1768 \rightarrow 1830).





Fourier series: Mathematical Representation

Our basic trigonometric series is:

$$\frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx)$$

It will called **Fourier Series** if the terms A_0, A_n, B_n is:

$$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cdot dx$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx \cdot dx$$

$$B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx \cdot dx$$

Where f(x) is any single-valued function defined in interval $(-\pi, \pi)$.



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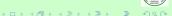


What is Fourier transform

Fourier Transform

FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.





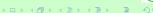
What is Fourier transform

Fourier Transform

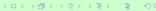
FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.

The Fourier transform of a function due to respect of time is a complex valued function of frequency whose magnitude represents the amount of that frequency present in the original function.



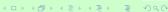






$$cos 0^{\circ} = cos 2n\pi = (-1)^{2n} = 1$$



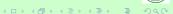


$$\sin 0^{\circ} = \sin \pi = 0$$

$$\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$$

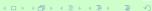
$$\cos n\pi = (-1)^n$$





- $\sin 0^{\circ} = \sin \pi = 0$
- $\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$
- $\cos n\pi = (-1)^n$





Some helpful equations

$$0 \sin 0^{\circ} = \sin \pi = 0$$

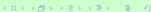
$$\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$$

$$os n\pi = (-1)^n$$



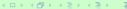
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Ending Scene



- $\sin 0^{\circ} = \sin \pi = 0$
- $\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$
- $\cos n\pi = (-1)^n$
- $\frac{d}{dx}\sin\theta = \cos\theta$





- $\sin 0^{\circ} = \sin \pi = 0$
- $\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$
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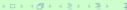




$$\cos 0^{\circ} = \cos 2n\pi = (-1)^{2n} = 1$$

$$\cos n\pi = (-1)^n$$





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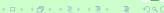


1

Use the method of Fourier transform to determine the displacement y(x,t) of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x), -\infty < x < \infty$. Also show that the solution can be put in the form:

$$y(x,t) = \frac{1}{2} [f(x+ct) + f(x-ct)]$$





Trying to solve

[Trying to solve:]

Here we have to solve the one-dimensional wave equation

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \qquad [-\infty < x < \infty, t > 0]$$

subject to the following initial conditions

$$y(x, 0) = \text{Initial displacement} = f(x)$$

and $y_t(x, 0) = \text{initial velocity} = 0$

The given partial differential equation is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \tag{1}$$





Trying to solve

Taking the complex Fourier transform of both sides of (1), we have

$$\int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta t^2} e^{-iux} dx = c^2 \int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta x^2} e^{-iux} dx \tag{2}$$

By the Fourier transform of the derivative of a function we have if $F^n(x)$ is the nth-derivative of F(x) and the first (n-1) derivatives of F(x) vanish as $x \to \pm \infty$ then $F\{F^n(x)\} = (-iu)^n F\{F(x)\}.$ Thus from (2) we have

$$\frac{d^2}{dt^2} \int_{-\infty}^{+\infty} y e^{-iux} dx = c^2 (-iu)^2 F\{y(x,t)\}$$
or,
$$\frac{d^2}{dt^2} \int_{-\infty}^{+\infty} y(x,t) e^{-iux} dx = c^2 (-u^2) F\{y(x,t)\}$$

or,
$$\frac{d}{dt^2} \int_{-\infty}^{\infty} y(x,t)e^{-iux} dx = c^2(-u^2)F\{y(x,t)\}$$

or,
$$\frac{d^2}{dt^2}F\{y(x,t)\} = -c^2u^2F\{y(x,t)\}$$

or,
$$\frac{d^2\overline{y}}{dt^2} = -c^2u^2\overline{y}$$
 [where $\overline{y} = \overline{y}(u,t) = F\{y(x,t)\} = \int_{-\infty}^{+\infty} y(x,t)e^{-iux}dx$]

Trying to solve

$$\therefore \frac{d^2 \overline{y}}{dt^2} + c^2 u^2 \overline{y} = 0 \tag{3}$$

Which is ordinary second order differential equation whose solution is

$$\overline{y} = \overline{y}(u,t) = A\cos(cut) + B\sin(cut)$$
 (4)

Differentiating with both sides with respect to t

we get,
$$\overline{y}_t(u,t) = -Acu\sin(cut) + Bcu\cos(cut)$$
 (5)

Also from the initial given conditions, we have

$$y(x,0) = f(x) \tag{6}$$

$$y_t(x,0) = 0 (7)$$



Trying to solve

Taking the Fourier transform of (6) and (7) we get,

$$\overline{y}(u,0) = \int_{-\infty}^{+\infty} y(x,0)e^{-iux} \cdot dx = \int_{-\infty}^{+\infty} f(x)e^{-iux} \cdot dx = \overline{f}(u) \quad (\text{say})$$

$$\therefore \quad \overline{y}(u,0) = \overline{f}(u) \tag{8}$$

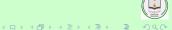
and
$$\overline{y}_t(u,0) = \int_{-\infty}^{+\infty} y_t(x,0)e^{-iux} \cdot dx$$

= $\int_{-\infty}^{+\infty} 0(e^{-iux}) \cdot du = 0$

$$\therefore \quad \overline{y}_t(u,0) = 0 \tag{9}$$

Putting t = 0 in (5), we have $\overline{y}_t(u,0) = Bcu$ or, Bcu = 0 using (9)

$$\therefore$$
 $B=0$ Since, $cu \neq 0$



Trying to solve

Again putting t = 0 in (4), we get

$$\overline{y}(u,0) = A$$

 $\therefore \overline{f}(u) = A \quad \text{using (8)}$
or, $A = \overline{f}(u)$

Putting the values of A and B in (4), we get

$$\overline{y} = \overline{y}(u,t) = \overline{f}(u)\cos(cut)$$
 (10)

Taking the inverse Fourier transform of (10) we have

$$\begin{split} y(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{f}(u) \cos(cut) e^{iux} \cdot du \\ \text{or, } y(x,t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{f}(u) \left(\frac{e^{icut} + e^{-icut}}{2} \right) e^{iux} \cdot du \\ &= \frac{1}{2} \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{f}(u) e^{iu(x+ct)} \cdot du + \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{f}(u) e^{iu(x-ct)} \cdot du \right] \\ &= \frac{1}{2} \left[f(x+ct) + f(x-ct) \right] \text{ (Using the definition of inverse Fourier transform)} \end{split}$$



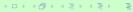


Question 1 Solved

Finally

$$y(x,t) = \frac{1}{2} [f(x+ct) + f(x-ct)].$$

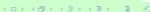




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2

A thin membrane of great extent is released from rest in the position z=f(x,y), show that the displacement at any subsequent time is given by

$$z(x,y,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u,v) \cos ct \sqrt{(u^2 + v^2)} \cdot e^{-i(ux + vy)} \cdot du \cdot dv$$

where F(u, v) is the double Fourier transform of f(x, y).





Trying to proof

[Trying to proof:]

Here the displacement of the membrane is governed by two dimensional wave equation

$$\frac{\delta^2 z}{\delta t^2} = c^2 \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) \quad \text{where } c^2 = \frac{T}{\rho}$$
 (1)

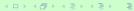
Taking the double Fourier transform of both sides of (1) we get

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\delta^2 z}{\delta t^2} e^{i(ux=vy)} \cdot dx \cdot dy = \frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot dy$$

or,
$$\frac{d^2}{dt^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy =$$

$$\frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot dy$$





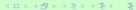
Trying to proof

or,
$$\frac{d^2 \overline{z}}{dt^2} = c^2 \{ (-iu)^2 + (-iv)^2 \} F\{ z(x, y, t) \} \text{ where } \overline{z} = \overline{z}(u, v, t) = F\{ z(x, y, t) \}$$
$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux + vy)} \cdot dx \cdot dy$$

or,
$$\frac{d^2 \overline{z}}{dt^2} = -c^2 (u^2 + v^2) \overline{z}$$

$$\therefore \frac{d^2\overline{z}}{dt^2} + c^2(u^2 + v^2)\overline{z} = 0$$
 (2)





Ending Scene

Trying to proof

which is an ordinary differential equation whose solution is

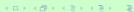
$$\overline{z} = A\cos\{c\sqrt{(u^2 + v^2)}t\} + B\sin\{c\sqrt{(u^2 + v^2)}t\}$$
 (3)

The given initial conditions are $\overline{z}=f(x,y)$ and $\frac{\delta z}{\delta t}=0$ at t=0 Taking the Fourier transform of these initial conditions, we get

$$\overline{z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) e^{i(ux + vy)} \cdot dx \cdot dy = F(u, v) \tag{4}$$

$$\begin{split} &=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\delta z}{\delta t}e^{i(ux+vy)}\cdot dx\cdot dy\\ &=\frac{d}{dt}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}ze^{i(ux+vy)}\cdot dx\cdot dy\\ &=\frac{d\overline{z}}{dt}\ \mathrm{since}\ \overline{z}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}ze^{i(ux+vy)}\cdot dx\cdot dy \end{split}$$





Trying to proof

$$\therefore \frac{d\overline{z}}{dt} = 0 \text{ at } t = 0$$

When t=0, combining (3) and (4), we get A = F(u, v)

Also
$$\frac{d\overline{z}}{dt} = -Ac\sqrt{(u^2 + v^2)}\sin c\sqrt{(u^2 + v^2)t} + Bc\sqrt{(u^2 + v^2)}\cos c\sqrt{(u^2 + v^2)t}$$

$$\therefore 0 = \left(\frac{d\overline{z}}{dt}\right)_{t=0} = Bc\sqrt{(u^2 + v^2)}$$
or, $B = 0$

 $\overline{z} = F(u, v)cos\{c\sqrt{(u^2 + v^2)t}\}$

Putting the values of A and B in (3), we get



(5)



Ultimately

Now applying the inversion formula for double Foruier transform, we have

$$z(x,y,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u,v) \cos\{c\sqrt{(u^2+v^2)}t\} e^{-i(ux+vy)} \cdot du \cdot dv$$

Which is the required displacement at any subsequent time t.

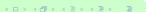




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