

# MAT205

## Fourier Analysis

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## Now we on. . .

- 1 Preamble
- 2 Basic Beyond
- 3 Illustration 1
- 4 Illustration 2
- 5 Ending Scene



# Start-UP

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# Basic Beyond

## What is Fourier series

### Fourier Series

FS is a special trigonometric periodic series derived by Joseph Fourier(1768 → 1830).



# Basic Beyond

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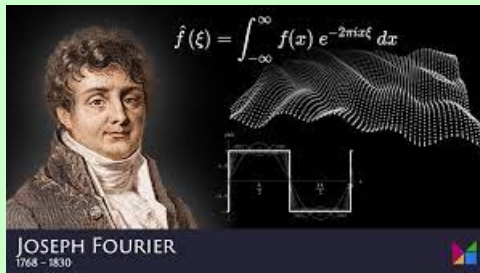


Figure: Joseph Fourier



# Basic Beyond

## Fourier series: Mathematical Representation

Our basic trigonometric series is:

$$\frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx)$$

It will called **Fourier Series** if the terms  $A_0, A_n, B_n$  is:

$$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \cdot dx$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx \cdot dx$$

$$B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx \cdot dx$$

Where  $f(x)$  is any single-valued function defined in interval  $(-\pi, \pi)$ .



# Basic Beyond

## What is Fourier transform

### Fourier Transform

FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.





# Basic Beyond

## What is Fourier transform

### Fourier Transform

FT Is a mathematical transform that decomposes functions depending on time or space into functions depending on spatial or temporal frequency such as the expression of a musical chord.

The Fourier transform of a function due to respect of time is a complex valued function of frequency whose magnitude represents the amount of that frequency present in the original function.



# Basic Beyond

## Some helpful equations

$$\textcircled{1} \sin 0^\circ = \sin \pi = 0$$



# Basic Beyond

## Some helpful equations

①  $\sin 0^\circ = \sin \pi = 0$

②  $\cos 0^\circ = \cos 2n\pi = (-1)^{2n} = 1$



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## Some helpful equations

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# Basic Beyond

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- ⑤  $\frac{d}{dx} \cos \theta = -\sin \theta$



# Basic Beyond

## Some helpful equations

$$\textcircled{1} \sin 0^\circ = \sin \pi = 0$$

$$\textcircled{2} \cos 0^\circ = \cos 2n\pi = (-1)^{2n} = 1$$

$$\textcircled{3} \cos n\pi = (-1)^n$$

$$\textcircled{4} \frac{d}{dx} \sin \theta = \cos \theta$$

$$\textcircled{5} \frac{d}{dx} \cos \theta = -\sin \theta$$

$$\textcircled{6} \int \sin \theta = -\cos \theta$$



# Basic Beyond

## Some helpful equations

- ❶  $\sin 0^\circ = \sin \pi = 0$
- ❷  $\cos 0^\circ = \cos 2n\pi = (-1)^{2n} = 1$
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- ❺  $\frac{d}{dx} \cos \theta = -\sin \theta$
- ❻  $\int \sin \theta = -\cos \theta$
- ❼  $\int \cos \theta = \sin \theta$





# Basic Beyond

## Some helpful equations

$$① \sin 0^\circ = \sin \pi = 0$$

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$$⑤ \frac{d}{dx} \cos \theta = -\sin \theta$$

$$⑥ \int \sin \theta = -\cos \theta$$

$$⑦ \int \cos \theta = \sin \theta$$

$$⑧ \int UV \cdot dx = U \int V \cdot dx + \int \frac{dU}{dx} (\int V \cdot dx) \cdot dx$$



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# Question 1

1

Use the method of Fourier transform to determine the displacement  $y(x, t)$  of an infinite string, given that the string is initially at rest and that the initial displacement is  $f(x)$ ,  $-\infty < x < \infty$ . Also show that the solution can be put in the form:

$$y(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)]$$



## Question 1

Trying to solve

[Trying to solve:]

Here we have to solve the one-dimensional wave equation

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \quad [-\infty < x < \infty, t > 0]$$

subject to the following initial conditions

$$y(x, 0) = \text{Initial displacement} = f(x)$$

$$\text{and } y_t(x, 0) = \text{initial velocity} = 0$$

The given partial differential equation is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \quad (1)$$



## Question 1

Trying to solve

Taking the complex Fourier transform of both sides of (1), we have

$$\int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta t^2} e^{-iux} dx = c^2 \int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta x^2} e^{-iux} dx \quad (2)$$

By the Fourier transform of the derivative of a function we have if  $F^n(x)$  is the  $n$ th-derivative of  $F(x)$  and the first  $(n-1)$  derivatives of  $F(x)$  vanish as  $x \rightarrow \pm\infty$  then  $F\{F^n(x)\} = (-iu)^n F\{F(x)\}$ .

Thus from (2) we have

$$\frac{d^2}{dt^2} \int_{-\infty}^{+\infty} y e^{-iux} dx = c^2 (-iu)^2 F\{y(x, t)\}$$

$$\text{or, } \frac{d^2}{dt^2} \int_{-\infty}^{+\infty} y(x, t) e^{-iux} dx = c^2 (-u^2) F\{y(x, t)\}$$

$$\text{or, } \frac{d^2}{dt^2} F\{y(x, t)\} = -c^2 u^2 F\{y(x, t)\}$$

$$\text{or, } \frac{d^2 \bar{y}}{dt^2} = -c^2 u^2 \bar{y} \quad \left[ \text{where } \bar{y} = \bar{y}(u, t) = F\{y(x, t)\} = \int_{-\infty}^{+\infty} y(x, t) e^{-iux} dx \right]$$



## Question 1

Trying to solve

$$\therefore \frac{d^2 \bar{y}}{dt^2} + c^2 u^2 \bar{y} = 0 \quad (3)$$

Which is ordinary second order differential equation whose solution is

$$\bar{y} = \bar{y}(u, t) = A \cos(cut) + B \sin(cut) \quad (4)$$

Differentiating with both sides with respect to t

$$\text{we get, } \bar{y}_t(u, t) = -Acu \sin(cut) + Bcu \cos(cut) \quad (5)$$

Also from the initial given conditions, we have

$$y(x, 0) = f(x) \quad (6)$$

$$y_t(x, 0) = 0 \quad (7)$$



## Question 1

Trying to solve

Taking the Fourier transform of (6) and (7) we get,

$$\bar{y}(u, 0) = \int_{-\infty}^{+\infty} y(x, 0)e^{-iux} \cdot dx = \int_{-\infty}^{+\infty} f(x)e^{-iux} \cdot dx = \bar{f}(u) \quad (\text{say})$$

$$\therefore \quad \bar{y}(u, 0) = \bar{f}(u) \quad (8)$$

$$\text{and } \bar{y}_t(u, 0) = \int_{-\infty}^{+\infty} y_t(x, 0)e^{-iux} \cdot dx$$

$$= \int_{-\infty}^{+\infty} 0(e^{-iux}) \cdot du = 0$$

$$\therefore \quad \bar{y}_t(u, 0) = 0 \quad (9)$$

Putting  $t = 0$  in (5), we have  $\bar{y}_t(u, 0) = Bcu$   
or,  $Bcu = 0$  using (9)

$$\therefore \quad B = 0 \quad \text{Since, } cu \neq 0$$



## Question 1

Trying to solve

Again putting  $t = 0$  in (4), we get

$$\bar{y}(u, 0) = A$$

$$\therefore \bar{f}(u) = A \quad \text{using (8)}$$

$$\text{or, } A = \bar{f}(u)$$

Putting the values of A and B in (4), we get

$$\bar{y} = \bar{y}(u, t) = \bar{f}(u) \cos(cut) \quad (10)$$

Taking the inverse Fourier transform of (10) we have

$$y(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{f}(u) \cos(cut) e^{iux} \cdot du$$

$$\text{or, } y(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{f}(u) \left( \frac{e^{icut} + e^{-icut}}{2} \right) e^{iux} \cdot du$$

$$= \frac{1}{2} \left[ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{f}(u) e^{iu(x+ct)} \cdot du + \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{f}(u) e^{iu(x-ct)} \cdot du \right]$$

$$= \frac{1}{2} [f(x + ct) + f(x - ct)] \quad (\text{Using the definition of inverse Fourier transform})$$





# Question 1

Trying to solve: Over

Finally

$$\therefore y(x, t) = \frac{1}{2} [f(x + ct) + f(x - ct)].$$



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## Question 2

2

A thin membrane of great extent is released from rest in the position  $z = f(x, y)$ , show that the displacement at any subsequent time is given by

$$z(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) \cos ct \sqrt{(u^2 + v^2)} \cdot e^{-i(ux+vy)} \cdot du \cdot dv$$

where  $F(u, v)$  is the double Fourier transform of  $f(x, y)$ .



## Question 2

Trying to proof

[Trying to proof:]

Here the displacement of the membrane is governed by two dimensional wave equation

$$\frac{\delta^2 z}{\delta t^2} = c^2 \left( \frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) \quad \text{where } c^2 = \frac{T}{\rho} \quad (1)$$

Taking the double Fourier transform of both sides of (1) we get

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\delta^2 z}{\delta t^2} e^{i(ux+vy)} \cdot dx \cdot dy = \frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( \frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot dy$$

$$\text{or, } \frac{d^2}{dt^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy =$$

$$\frac{c^2}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( \frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2} \right) e^{i(ux+vy)} \cdot dx \cdot dy$$



## Question 2

Trying to proof

$$\begin{aligned}\text{or, } \frac{d^2 \bar{z}}{dt^2} &= c^2 \{(-iu)^2 + (-iv)^2\} F\{z(x, y, t)\} \text{ where } \bar{z} = \bar{z}(u, v, t) = F\{z(x, y, t)\} \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy\end{aligned}$$

$$\text{or, } \frac{d^2 \bar{z}}{dt^2} = -c^2(u^2 + v^2)\bar{z}$$

$$\therefore \frac{d^2 \bar{z}}{dt^2} + c^2(u^2 + v^2)\bar{z} = 0 \quad (2)$$



## Question 2

Trying to proof

which is an ordinary differential equation whose solution is

$$\bar{z} = A \cos\{c\sqrt{(u^2 + v^2)}t\} + B \sin\{c\sqrt{(u^2 + v^2)}t\} \quad (3)$$

The given initial conditions are  $\bar{z} = f(x, y)$  and  $\frac{\delta z}{\delta t} = 0$  at  $t = 0$

Taking the Fourier transform of these initial conditions, we get

$$\bar{z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) e^{i(ux+vy)} \cdot dx \cdot dy = F(u, v) \quad (4)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\delta z}{\delta t} e^{i(ux+vy)} \cdot dx \cdot dy$$

$$= \frac{d}{dt} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy$$

$$= \frac{d\bar{z}}{dt} \text{ since } \bar{z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z e^{i(ux+vy)} \cdot dx \cdot dy$$



## Question 2

Trying to proof

$$\therefore \frac{d\bar{z}}{dt} = 0 \text{ at } t = 0$$

When  $t=0$ , combining (3) and (4), we get  $A = F(u, v)$

$$\text{Also } \frac{d\bar{z}}{dt} = -Ac\sqrt{(u^2 + v^2)} \sin c\sqrt{(u^2 + v^2)}t + Bc\sqrt{(u^2 + v^2)} \cos c\sqrt{(u^2 + v^2)}t$$

$$\therefore 0 = \left( \frac{d\bar{z}}{dt} \right)_{t=0} = Bc\sqrt{(u^2 + v^2)}$$

$$\text{or, } B = 0$$

Putting the values of A and B in (3), we get

$$\bar{z} = F(u, v) \cos\{c\sqrt{(u^2 + v^2)}t\} \quad (5)$$



## Question 2

Trying to proof: Over

### Ultimately

Now applying the inversion formula for double Fourier transform, we have

$$z(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) \cos\{c\sqrt{(u^2 + v^2)}t\} e^{-i(ux+vy)} \cdot du \cdot dv$$

Which is the required displacement at any subsequent time t.





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