Chapter 4: Change of axes

November 5, 2021

1. Determine the equation of the curve $2x^2 + 3y^2 - 8x + 6y - 7 = 0$ when then origin is transferred to the point $(2, -1)$
2. Determine the equation of the parabola $x^2 - 2xy + y^2 + 2x - 4y + 3 = 0$ after rotating of axes through 45 degree.
3. The direction cosine of axes remaining the same, choose a new origin such that the new co-ordinates of the pair of points whose old co-ordinate are $(5, -13)$ and $(-3, 11)$ whose new co-ordinates are (h, k) and $(-h, -k)$.
4. Transform to parallel axes through the new origin of the equations.
   (a) Origin $(1, -2), 2x^2 + y^2 - 4x + 4y = 0$
   (b) Origin $(3, 1), x^2 - 6x + 2y^2 + 7 = 0$
5. Transform to axes inclined at 45 degree to the original axes the equations.
   (a) $x^2 - y^2 = a^2$
   (b) $x^2 - y^2 - 2\sqrt{2}x - 10\sqrt{2}y + 2 = 0$
6. Remove the first degree terms in each of the following equations.
   (a) $3x^2 - 4y^2 - 6x - 8y - 10 = 0$
   (b) $2x^2 + 5y^2 - 12x + 10y - 7 = 0$
   (c) $3x^2 - 4y^2 + 6x + 24y - 135 = 0$
7. Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point (2, -1) and inclined at an angle $\tan^{-1}\frac{4}{3}$.

8. Determine the angle through which the axes must be rotated to remove the xy term in the equation, $7x^2 - 6\sqrt{3}xy + 13y^2 = 16$.

9. The direction of axes remaining the same, choose a new origin such that the new coordinate of the pair of points whose old coordinate are (5, -13) and (-3, 11) may be the forms (h, k) and (-h, -k).

10. Transform the axes inclined at 30 degree to the original axes the equation, $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.

11. The equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ is transformed to $4x^2 + 2y^2 = 1$ when referred to rectangular axes through the point (2, 3). Find the inclination of the latter axes to the former.