Mid 2.1

Sofiullah Iqbal Kiron

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CSE201: Data Structure. `(20 August, 2021)`

Algorithm to convert infix notation to prefix notation

Step 1: Reverse the infix expression. While reversing, each '(' will become ')' and each ')' become '('. e.g.: $A + B * C \rightarrow C * B + A$

Step 2: Make it postfix. e.g.: $CB * A+$

Step 3: Reverse the final expression. e.g.: $+A * BC$

MAT205: Fourier Analysis. (24 August, 2021)

1. Use the method of Fourier transform to determine the displacement $y(x,t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x)$, $-\infty < x < \infty$. Also show that the solution can be put in the form:

$$y(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right]$$

[Trying to solve:]
Here we have to solve the one-dimensional wave equation

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \qquad [-\infty < x < \infty, t > 0]$$

subject to the following initial conditions

$$\begin{aligned} y(x,0) &= \text{Initial displacement} = f(x) \\ \text{and } y_t(x,0) &= \text{initial velocity} = 0 \end{aligned}$$

The given partial differential equation is

$$\frac{\delta^2 y}{\delta t^2} = c^2 \frac{\delta^2 y}{\delta x^2} \tag{1}$$

Taking the complex Fourier transform of both sides of (1), we have

$$\int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta t^2} e^{-iux} dx = c^2 \int_{-\infty}^{+\infty} \frac{\delta^2 y}{\delta x^2} e^{-iux} dx \tag{2}$$

By the Fourier transform of the derivative of a function we have if $F^n(x)$ is the nth-derivative of $F(x)$ and the first $(n-1)$ derivatives of F(x) vanish as $x \to \pm\infty$ then $F\{F^n(x)\} = (-iu)^n F\{F(x)\}$.

Thus from (2) we have

$$\begin{aligned} &\frac{d^2}{dt^2}\int_{-\infty}^{+\infty} y e^{-iux} dx = c^2(-iu)^2 F\{y(x,t)\} \\ \text{or, } &\frac{d^2}{dt^2}\int_{-\infty}^{+\infty} y(x,t) e^{-iux} dx = c^2(-u^2) F\{y(x,t)\} \\ \text{or, } &\frac{d^2}{dt^2} F\{y(x,t)\} = -c^2u^2 F\{y(x,t)\} \\ \text{or, } &\frac{d^2\overline{y}}{dt^2} = -c^2u^2\overline{y} \qquad \left[\text{where } \overline{y} = \overline{y}(u,t) = F\{y(x,t)\} = \int_{-\infty}^{+\infty} y(x,t)e^{-iux}dx\right] \end{aligned}$$

$$\therefore \quad \frac{d^2\overline{y}}{dt^2} + c^2u^2\overline{y} = 0 \tag{3}$$

Which is ordinary second order differential equation whose solution is

$$\overline{y} = \overline{y}(u,t) = A\cos(cut) + B\sin(cut) \tag{4}$$

Differentiating with both sides with respect to t

$$\text{we get, } \overline{y}_t(u,t) = -Acu\sin(cut) + Bcu\cos(cut) \tag{5}$$

Also from the initial given conditions, we have

$$y(x,0) = f(x) \tag{6}$$
$$y_t(x,0) = 0 \tag{7}$$

Taking the Fourier transform of (6) and (7) we get,

$$\overline{y}(u,0) = \int_{-\infty}^{+\infty} y(x,0)e^{-iux} \cdot dx = \int_{-\infty}^{+\infty} f(x)e^{-iux} \cdot dx = \overline{f}(u) \quad \text{(say)}$$

$$\therefore \quad \overline{y}(u,0) = \overline{f}(u) \tag{8}$$

$$\begin{aligned} \text{and } \overline{y}_t(u,0) &= \int_{-\infty}^{+\infty} y_t(x,0)e^{-iux} \cdot dx \\ &= \int_{-\infty}^{+\infty} 0(e^{-iux}) \cdot du = 0 \end{aligned}$$

$$\therefore \quad \overline{y}_t(u,0) = 0 \tag{9}$$

Putting t = 0 in (5), we have $\overline{y}_t(u,0) = Bcu$
or, $Bcu = 0$ using (9)
$\therefore \quad B = 0 \quad$ Since, $cu \neq 0$
Again putting t = 0 in (4), we get

$$\begin{aligned} &\overline{y}(u,0) = A \\ \therefore\ &\overline{f}(u) = A \quad \text{using (8)} \\ \text{or, } &A = \overline{f}(u) \end{aligned}$$

Putting the values of A and B in (4), we get

$$\overline{y} = \overline{y}(u,t) = \overline{f}(u)\cos(cut) \tag{10}$$

Taking the inverse Fourier transform of (10) we have

$$\begin{aligned} y(x,t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{f}(u)\cos(cut)e^{iux} \cdot du \\ \text{or, } y(x,t) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{f}(u)\left(\frac{e^{icut}+e^{-icut}}{2}\right)e^{iux} \cdot du \\ &= \frac{1}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{f}(u)e^{iu(x+ct)} \cdot du + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{f}(u)e^{iu(x-ct)} \cdot du\right] \\ &= \frac{1}{2}\left[f(x+ct) + f(x-ct)\right] \quad \text{(Using the definition of inverse Fourier transform)} \end{aligned}$$

$\therefore \quad y(x,t) = \frac{1}{2}\left[f(x+ct) + f(x-ct)\right].$

2. A thin membrane of great extent is released from rest in the position $z = f(x,y)$, show that the displacement at any subsequent time is given by

$$z(x,y,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)\cos ct\sqrt{(u^2+v^2)}\cdot e^{-i(ux+vy)}\cdot du\cdot dv$$

where $F(u,v)$ is the double Fourier transform of $f(x,y)$.

[Trying to proof:]
Here the displacement of the membrane is governed by two dimensional wave equation

$$\frac{\delta^2 z}{\delta t^2} = c^2\left(\frac{\delta^2 z}{\delta x^2} + \frac{\delta^2 z}{\delta y^2}\right) \qquad \text{where } c^2 = \frac{T}{\rho} \tag{1}$$

Taking the double Fourier transform of both sides of (1) we get

$$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\delta^2 z}{\delta t^2}e^{i(ux=vy)}\cdot dx\cdot dy = \frac{c^2}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\delta^2 z}{\delta x^2}+\frac{\delta^2 z}{\delta y^2}\right)e^{i(ux+vy)}\cdot dx\cdot dy$$

$$\text{or, } \frac{d^2}{dt^2}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ze^{i(ux+vy)}\cdot dx\cdot dy = \frac{c^2}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{\delta^2 z}{\delta x^2}+\frac{\delta^2 z}{\delta y^2}\right)e^{i(ux+vy)}\cdot dx\cdot dy$$

$$\text{or,} \frac{d^2\overline{z}}{dt^2} = c^2\{(-iu)^2 + (-iv)^2\}F\{z(x,y,t)\} \text{ where } \overline{z} = \overline{z}(u,v,t) = F\{z(x,y,t)\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ze^{i(ux+vy)}\cdot dx\cdot dy$$

$$\text{or, } \frac{d^2\overline{z}}{dt^2} = -c^2(u^2+v^2)\overline{z}$$

$$\therefore \quad \frac{d^2\overline{z}}{dt^2} + c^2(u^2+v^2)\overline{z} = 0 \tag{2}$$

which is an ordinary differential equation whose solution is

$$\overline{z} = A\cos\{c\sqrt{(u^2+v^2)}t\} + B\sin\{c\sqrt{(u^2+v^2)}t\} \tag{3}$$

The given initial conditions are $\overline{z} = f(x,y)$ and $\frac{\delta z}{\delta t} = 0$ at $t = 0$
Taking the Fourier transform of these initial conditions, we get

$$\overline{z} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{i(ux+vy)}\cdot dx\cdot dy = F(u,v) \tag{4}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\delta z}{\delta t}e^{i(ux+vy)}\cdot dx\cdot dy$$

$$= \frac{d}{dt}\frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ze^{i(ux+vy)}\cdot dx\cdot dy$$

$$= \frac{d\overline{z}}{dt} \text{ since } \overline{z} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ze^{i(ux+vy)}\cdot dx\cdot dy$$

$\therefore \quad \dfrac{d\overline{z}}{dt} = 0$ at $t = 0$

When t=0, combining (3) and (4), we get $A = F(u, v)$

Also $\dfrac{d\overline{z}}{dt} = -Ac\sqrt{(u^2+v^2)}\sin c\sqrt{(u^2+v^2)t} + Bc\sqrt{(u^2+v^2)}\cos c\sqrt{(u^2+v^2)t}$

$\therefore\ 0 = \left(\dfrac{d\overline{z}}{dt}\right)_{t=0} = Bc\sqrt{(u^2+v^2)}$

or, $B = 0$

Putting the values of A and B in (3), we get

$$\overline{z} = F(u, v)cos\{c\sqrt{(u^2+v^2)t}\} \tag{5}$$

Now applying the inversion formula for double Foruier transform, we have

$$z(x, y, t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u, v)\cos\{c\sqrt{(u^2+v^2)}t\}e^{-i(ux+vy)} \cdot du \cdot dv$$

Which is the required displacement at any subsequent time t.

Present State		Input	Next State		Flip-Flop Inputs			
Q_A	Q_B	x	Q_A+	Q_B+	J_A	K_A	J_B	K_B
0	0	0	0	0	0	$\times$	0	$\times$
0	1	0	0	1	0	$\times$	$\times$	0
1	0	0	1	0	$\times$	0	0	$\times$
1	1	0	1	1	$\times$	0	$\times$	0
0	0	1	0	1	0	$\times$	1	$\times$
0	1	1	1	1	1	$\times$	$\times$	0
1	0	1	0	0	$\times$	1	0	$\times$
1	1	1	1	0	$\times$	0	$\times$	1

a	f	b	0	0
b	d	c	0	0
c	f	e	0	0
d	g	a	1	0
e	d	c	0	0
f	f	b	1	1
g	g	h	0	1
h	g	a	1	0

CSE203: Digital Logic Design. `(21 August, 2021)`

CSE205: Java Technology. (22 August, 2021)

STA205: Statistics. (23 August, 2021)