Name:____________________ Class Test [8 : 30]

Applied Differential Equations
Class Test on Chapters 8, 9 and 10

Date: Monday, 15 Dec, 2015; Time: 8 : 30am

Instructions: This in-class exam in 50 minutes. No calculators, notes, tables or books. Details count 75%. The answer counts 25%. The sample exam has extra problems to show different problem types.

1. Find the fundamental matrix e^{At} and report the solution $u = e^{At}u(0)$ for the initial value problem
$$u'(t) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} u(t)$$
$$u(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

2. Solve the general solution $u = u_h + u_p$, finding the particular solution u_p by variation of parameters
$$u_p(t) = e^{At} \int_0^t e^{-Au} F(u) \cdot du$$

3. Identify the predator and the prey variables in the predator-prey system. Find the equilibrium points and identify the unique equilibrium which corresponds to coexistence with periodic populations oscillating about the two carrying capacities.
$$x' = 0.005x(40 - y) \tag{1}$$
$$y' = 0.01y(-50 + x) \tag{2}$$

4. It is assumed that you have memorized the basic 4-item Laplace integral table and know the 6 basic rules for Laplace integrals. No other tables or theory are required to solve problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.
Apply Laplace's method to solve the system. Find a 2×2 system for $\mathcal{L}(x), \mathcal{L}(y)$. Solve it for $\mathcal{L}(x), \mathcal{L}(y)$. Find formulae for $x(t), y(t)$.