

# Basic Counting Principle

## Combinatorics

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14 April, 2020



# Fulfilment

1 5.1

2 5.2



Now we on. . .

1 5.1

2 5.2



# Two Basic Counting Principle

## 1 Product Rule



# Two Basic Counting Principle

- 1 Product Rule
- 2 Sum Rule



# Two Basic Counting Principle

## ① Product Rule

## ② Sum Rule

**Product Rule:** Suppose that a procedure can be broken down into a sequence of two tasks. If there are  $n_1$  ways to do the first task and for each of these ways of doing the first task, there are  $n_2$  ways to do the second task, then there are  $n_1 n_2$  ways to do the procedure.



# Two Basic Counting Principle

## ① Product Rule

## ② Sum Rule

**Product Rule:** Suppose that a procedure can be broken down into a sequence of two tasks. If there are  $n_1$  ways to do the first task and for each of these ways of doing the first task, there are  $n_2$  ways to do the second task, then there are  $n_1 n_2$  ways to do the procedure.

**Sum Rule:** If a task can be done either in one of  $n_1$  ways or in one of  $n_2$  ways, where none of the set of  $n_1$  ways is same as the any of set  $n_2$  ways. Then there are  $n_1 + n_2$  ways to do the task.



# Inclusion-Exclusion Principle

Suppose that a task can be done in  $n_1$  ways or in  $n_2$  ways, but that some of the set of  $n_1$  ways is same as some of the set of  $n_2$  ways.

To **correctly count** the ways to do the two task:

We add

- The number of ways to do it in one way

then subtract.

So...

The number of ways to do the task in a way that is both among the set of  $n_1$  ways and the set of  $n_2$  ways.

This technique is called **Inclusion-Exclusion**.





# Inclusion-Exclusion Principle

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To **correctly count** the ways to do the two task:

We add

- The number of ways to do it in one way
- The number of ways to do it in another way

then subtract.

So...

The number of ways to do the task in a way that is both among the set of  $n_1$  ways and the set of  $n_2$  ways.

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# Now we on...

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2 5.2



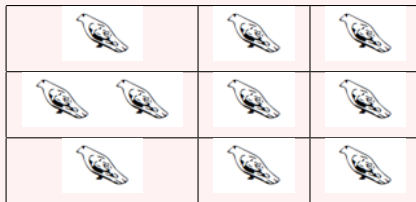
# The pigeonhole principle

Let's consider that, there are 10 pigeon and 9 pigeonhole at your home.  
So, there must a hole that contains two pigeon.



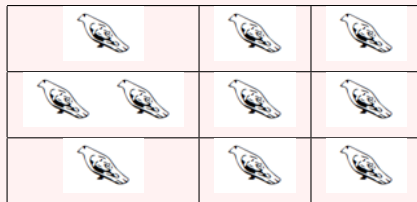
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## Principle:

$k$  is a positive integer. There are  $n$  objects ( $n > k$ ) placed in  $k$  boxes, then there are at least one box containing two or more objects.



### Corollary 1

A function ( $f$ ) from set with  $k + 1$  of more elements to a set of  $k$  elements, not *one – to – one*.

**Proof:** We can proof this by pigeonhole principle. Suppose elements of  $x$  is pigeon(Domain),  $y$  elements are pigeonhole(Co-domain). Then there are at least one pigeonhole(Co-domain) that contains more than one element. That's mean, the function is not *one – to – one*.



# Generalized Pigeonhole Principle

If  $N$  objects are placed into  $K$  boxes, then there is at least one box that containing at least  $\text{ceil}(N/K)$  elements.



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