ANSWER KEY

1.	(d)	11.	(c)	21.	(d)	31.	(b)	41.	(d)	51.	(b)	61.	(d)	71.
2.	(a)	12.	(d)	22.	(a)	32.	(c)	42.	(c)	52.	(c)	62.	(a)	72.
3.	(a)	13.	(d)	23.	(a)	33.	(b)	43.	(a)	53.	(b)	63.	(d)	73.
4.	(c)	14.	(d)	24.	(c)	34.	(c)	44.	(b)	54.	(c)	64.	(d)	74.
5.	(a)	15.	(c)	25.	(a)	35.	(a)	45.	(b)	55.	(b)	65.	(d)	75.
6.	(b)	16.	(a)	26.	(a)	36.	(d)	46.	(d)	56.	(a)	66.	(d)	76.
7.	(b)	17.	(a)	27.	(c)	37.	(b)	47.	(a)	57.	(d)	67.	(b)	77.
8.	(c)	18.	(c)	28.	(d)	38.	(b)	48.	(b)	58.	(b)	68.	(a)	78.
9.	(b)	19.	(a)	29.	(d)	39.	(a)	49.	(d)	59.	(a)	69.	(a)	79.
10.	(a)	20.	(c)	30.	(d)	40.	(d)	50.	(b)	60.	(a)	70.	(a)	80.

Hints & Solutions

PHYSICS

1. (d) Equation of motion when the mass slides down

$Mg\sin\theta - f = Ma$

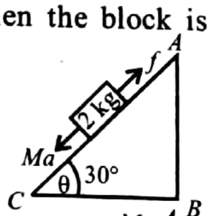
$\Rightarrow 10 - f = 6$

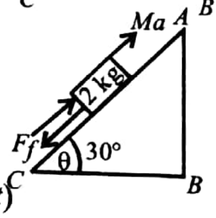
($M = 2$ kg, $a = 3$ m/s^2, $\theta = 30°$ given)

$\therefore f = 4$N

Equation of motion when the block is pushed up

Let the external force required to take the block up the plane with same acceleration be F





$F - Mg\sin\theta - f = Ma$

$\Rightarrow F - 10 - 4 = 6$

$F = 20$ N

2. (a) Using $E = E_0 - e^i(kz - \omega t)$

Given, at $t = t_1, z = z_1, E = 0$

the next zero that occurs in it's neighborhood is at z_2, the frequency of the electromagnetic wave at t_2

$$e^{i(kz_1 - \omega t_1)} = e^{i(kz_2 - \omega t_2)}$$

$$kz_1 - \omega t_1 = kz_2 - \omega t_2$$

$$(t_2 - t_1)\omega = k(z - z_1)$$

where $k = \dfrac{2\pi}{\lambda} = 2\pi v$

$$(t_2 - t_1) = \frac{2\pi}{\lambda \times 2\pi v}(z_2 - z_1)$$

$$= \frac{1}{\lambda \times v}(z_2 - z_1)$$

$$\Rightarrow \lambda \times v = \frac{(z_2 - z_1)}{(t_2 - t_1)} = C$$

$$(t_2 - t_1) = \frac{(z_2 - z_1)}{C}$$

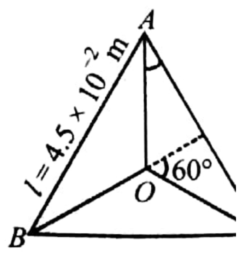
Frequency is $f \propto \dfrac{1}{t}$ then $\dfrac{}{(t}$

$\therefore$ Frequency, $f = \dfrac{3 \times 1}{(z_2 -}$

3. (a) Here, side of the triangl
current, I = 1A
magnetic field at the ce
'O' $B = ?$

From figure, $\tan 60° =$

$\Rightarrow d = \dfrac{l}{2\sqrt{3}} = \left(\dfrac{4.5 \times 1}{2\sqrt{}}\right.$



Magnetic field, $B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

Putting value of $\mu = 4\pi \times 10^{-7}$ and θ_1 and θ_2 we will get net magenetic field
$= 3 \times B = 4 \times 10^{-5}$ Wb/m^2

4. (c) Truth table of the circuit is as follows

x	y	$\bar{x}$	$a = x.y$	$b = \bar{x}.y$	$z = \overline{a.b}$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	0	1	0	1

5. (a) Rate of heat i.e., Power developed in the wire $= P = \frac{V^2}{R}$

Resistance of the wire of length, L

$$R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$$\therefore \text{ Power, } P_1 = \frac{V^2}{R_1}$$

Resistance of the wire when length is halved i.e., $L/2$

$$R_2 = \frac{\rho \frac{L}{2}}{\pi (2r)^2} = \frac{\rho L}{\pi 8 r^2} = \frac{R_1}{8}$$

$$\therefore \text{ Power, } P_2 = \frac{V}{\frac{R_1}{8}} = \frac{8V}{R_1}$$

or, $P_2 = 8P_1$ i.e., power increased 8 times of previous or original wire.

6. (b) Plank length is a unit of length, $l_p = 1.616229 \times 10^{-35}$ m

$$l_p = \sqrt{\frac{hG}{c^3}}$$

7. (b) For same material the ratio of stress to strain is same

For first cube

$$\text{Stress}_1 = \frac{\text{force}_1}{\text{area}_1} = \frac{10^5}{(0.1^2)}$$

$$\text{Strain}_1 = \frac{\text{change in length}_1}{\text{original length}_1} = \frac{0.5 \times 10^{-2}}{0.1}$$

For second block,

$$\text{stress}_2 = \frac{\text{force}_2}{\text{area}_2} = \frac{10^5}{(0.2^2)}$$

$$\text{strain}_2 = \frac{\text{change in length}_2}{\text{original length}_2} = \frac{x}{0.2}$$

x is the displacement for second block.

For same material, $\frac{\text{stress}_1}{\text{strain}_1} = \frac{\text{stress}_2}{\text{strain}_2}$

$$\text{or, } \frac{\frac{10.5}{(0.1)^2}}{\frac{0.5 \times 10^{-2}}{0.1}} = \frac{\frac{10^5}{(0.2)^2}}{\frac{x}{0.2}}$$

Solving we get, $x = 0.25$ cm

8. (c) Given : Inductance, $L = 49\ \mu\text{H} = 49 \times 10^{-6}$ H, capacitance $C = 2.5$ nF $= 2.5 \times 10^{-9}$ F

Using $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{49 \times 10^{-6} \times \frac{2.5}{10} \times 10^{-9}}}$$

$$= \frac{1}{7 \times 5 \times 10^{-8}} = \frac{10^8}{7 \times 5}$$

$$\text{or, } \frac{10^8}{7 \times 5} = 2\pi \times f = 2 \times \frac{22}{7} \times f$$

$(\because \omega = 2\pi f)$

$$\text{or, } f = \frac{10^7}{22} = \frac{10^4}{22}\text{kHz} = 454.54\text{kHz}$$

Therefore frequency range 454.54 ± 12kHz i.e., 442 kHz – 466 kHz

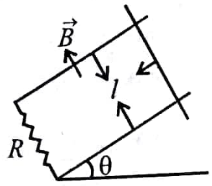
9. (b) From Faraday's law of electomagnetic induction,

$$e = \frac{d\phi}{dt} = \frac{d(BA)}{lt} = \frac{d(Bll)}{dt}$$

$$= \frac{Bdl \times l}{dt} = BVl$$

Also, $F = ilB = \left(\frac{BV}{R}\right)(l^2B) = \frac{B^2l^2V}{R}$

At equilibrium



$$mg\sin\theta = \frac{B^2lV}{R} \Rightarrow V = \frac{mgR\sin\theta}{B^2l^2}$$

10. (a) When the rod makes an angle α

Displacement of centre of mass $= \frac{l}{2}\cancel{\cos\alpha}$ sin α

$$mg\frac{l}{2}\cos\alpha = \frac{1}{2}I\omega^2$$

$mg\frac{l}{2}\cos\alpha = \frac{ml^2}{6}\omega^2$ ($\because$ M.I. of thin uniform rod about an axis passing through its centre of mass and perpendicular to the rod $I = \frac{ml^2}{12}$)

$$\Rightarrow \omega = \sqrt{\frac{3g\cos\alpha}{l}}$$

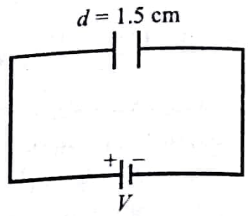
Speed of end $= \omega \times l = \sqrt{3g\cos\alpha l}$

i.e., Speed of end, $\omega \propto \sqrt{\cos\alpha}$ sin θ

11. (c) Given area of Parallel plate capacitor, $A = 200\ \text{cm}^2$
Separation between the plates, $d = 1.5$ cm
Force of attraction between the plates, $F = 25 \times 10^{-6}$N
$F = QE$

$F = \frac{Q^2}{2A\epsilon_0}$ (E due to parallel plate $= \frac{\sigma}{2\epsilon_0} = \frac{Q}{A2\epsilon_0}$)

But $Q = CV = \frac{\epsilon_0 A(V)}{d}$



$$\therefore F = \frac{(\epsilon_0 AV)^2}{d^2 \times 2A\epsilon_0}$$

$$= \frac{(\epsilon_0 A)^2 \times V^2}{d^2 \times 2 \times (A\epsilon_0)} = \frac{(\epsilon_0 A) \times V^2}{d^2 \times 2}$$

$$\text{or,}\ 25 \times 10^{-6} = \frac{(8.85 \times 10^{-12}) \times (200 \times 10^{-4}) \times V^2}{2.25 \times 10^{-4} \times 2}$$

$$\Rightarrow V = \frac{25 \times 10^{-6} \times 2.25 \times 10^{-4} \times 2}{8.85 \times 10^{-12} \times 200 \times 10^{-4}} \approx 250\ \text{V}$$

12. (d) Charge density, $\rho = \rho_0\left(1 - \frac{r}{R}\right)$

$$dq = \rho dv$$

$$q_{in} = \int dq = \rho dv$$

$$= \rho_0\left(1 - \frac{r}{R}\right)4\pi r^2 dr \quad (\because dv = 4\pi r^2 dr)$$

$$= 4\pi\rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2 dr$$

$$= 4\pi\rho_0 \int_0^R r^2 dr - \frac{r^2}{R} dr$$

$$= 4\pi\rho_0 \left[\left[\frac{r^3}{3}\right]_0^R - \left[\frac{r^4}{4R}\right]_0^R\right]$$

$$= 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$= 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^3}{4}\right] = 4\pi\rho_0 \left[\frac{R^3}{12}\right]$$

$$q = \frac{\pi\rho_0 R^3}{3}$$

$$E.4\pi r^2 = \left(\frac{\pi\rho_0 R^3}{3\epsilon_0}\right)$$

$\therefore$ Electric field outside the ball, $E = \dfrac{\rho_0 R^3}{12\epsilon_0 r^2}$

13. (d) Apply principle of conservation of momentum along x-direction,

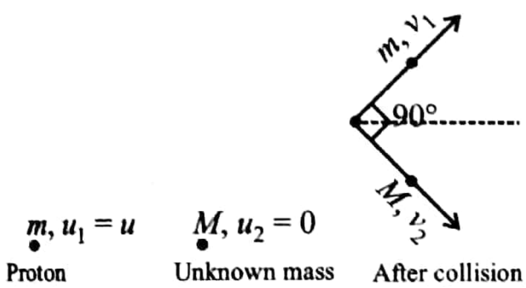
$mu = mv_1 \cos 45° + Mv_2 \cos 45°$

$$mu = \frac{1}{\sqrt{2}}(mv_1 + Mv_2) \quad .....(i)$$

Along y-direction,

$o = mv_1 \sin 45° - Mv_2 \sin 45°$

$$o = (mv_1 - Mv_2)\frac{1}{\sqrt{2}} \quad .....(ii)$$



Coefficient of restution $e = 1$

$$= \frac{v_2 - v_1\cos 90}{u\cos 45}$$

($\because$ Collision is elastic)

$$\Rightarrow \frac{v_2}{\frac{u}{\sqrt{2}}} = 1$$

$$\Rightarrow u = \sqrt{2}v_2 \quad .....(iii)$$

Solving eqs (i), (ii), & (iii), we get mass of unknown particle, $M = m$

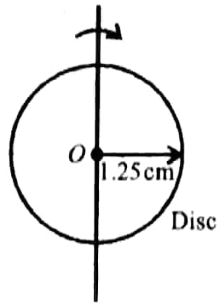
14. (d) Using, $\mu mg = \dfrac{mv^2}{r} = mr\omega^2$

$\omega = 2\pi n = 2\pi \times 3.5 = 7\pi$ rad/sec

Radius, $r = 1.25$ cm $= 1.25 \times 10^{-2}$ m

Coefficient of friction, $\mu = ?$

$$\mu mg = \frac{m(r\omega)^2}{r} \quad (\because v = r\omega)$$



$\mu mg = mr\omega^2$

$$\Rightarrow \mu = \frac{r\omega^2}{g} = \frac{1.25\times 10^{-2} \times \left(7\times\frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25\times 10^{-2}\times 22^2}{10} = 0.6$$

15. (c) According to Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt} \text{ and } \phi = BA\cos\omega t = B\pi r^2\cos\omega t$$

$$\Rightarrow e = -\frac{d}{dt}(\pi r^2 B\cos\omega t) = \pi r^2 B\sin\omega t(\omega)$$

$$\therefore e = \frac{\mu_0 I}{2R}\pi\omega r^2\sin\omega t \quad \left(\because B = \frac{\mu_0 I}{2R}\right)$$

16. (a) The sum of final charges on C_2 and C_3 is 36 μC.

17. (a) Probable frequencies of tuning fork be $n \pm 5$

Frequency of sonometer wire, $n \propto \dfrac{1}{l}$

$$\therefore \frac{n+5}{n-5} = \frac{100}{95} \Rightarrow 95(n+5) = 100(n-5)$$

or, $95n + 475 = 100n - 500$

or, $5n = 975$

or, $n = \dfrac{975}{5} = 195$ Hz

18. (c) From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A\sin(at+\delta)$
$y = B\sin(bt+r)$
Clearly $A \neq B$ hence ellipse.

19. (a) Let the car turn of the highway at a distance 'x' from the point M. So, RM $= x$
And if speed of car in field is v, then time taken by the car to cover the distance $QR = QM - x$ on the highway,

OR use Snell's law $v_0 \cdot \sin(90°) = v_0 \cdot \sin\theta$ / v_{field} ... $\theta = 30°$

$$t_1 = \frac{QM - x}{2v} \quad .....(i)$$

Time taken to travel the distance 'RP' in the field

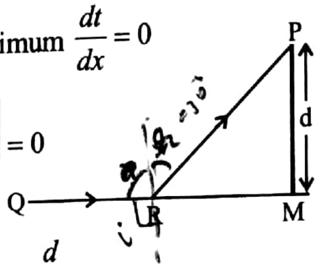
$$t_2 = \frac{\sqrt{d^2 + x^2}}{v} \quad .....(ii)$$

Total time elapsed to move the car from Q to P

$$t = t_1 + t_2 = \frac{QM - x}{2v} + \frac{\sqrt{d^2+x^2}}{v}$$

For 't' to be minimum $\frac{dt}{dx} = 0$

$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2+x^2}}\right] = 0$$



or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

20. (c) Charge density

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{q}{Ad} \Rightarrow q = \rho Ad$$

Also, $q = IT \Rightarrow T = \frac{q}{I} = \frac{\rho Ad}{I}$

21. (d) Efficiency of engine A, $n_A = \frac{T_1 - T_2}{T_1}$

and $n_B = \frac{T_2 - T_3}{T_2}$; $T_2 = \frac{T_1 + T_3}{2} = 350\,K$

or $\frac{n_A}{n_B} = \frac{\frac{600-350}{600}}{\frac{350-100}{350}} = \frac{7}{12}$

22. (a) For minimum spherical aberration separation,
$d = f_1 - f_2 = 2$ cm

Resultant focal length $=$ F $= 10$ cm

Using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ and solving

we get f_1, f_2 18 cm and 20 cm respectively

23. (a)

24. (c) Let heavy nucleus breaks into two nuclei of mass m_1 and m_2 and move away with velocities V_1 and V_2 respectively.

According to question, $\frac{V_1}{V_2} = \frac{8}{27}$

$m_1 V_1 = m_2 V_2$ (Law of momentum conservation)

$$\Rightarrow \frac{m_1}{m_2} = \frac{V_2}{V_1} = \frac{27}{8}$$

$$\frac{\rho \times \frac{4}{3}\pi R_1^3}{\rho \times \frac{4}{3}\pi R_2^3} \quad \left(\because \text{ density } \rho = \frac{\text{mass}}{\text{volume}}\right)$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right) = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \left(\frac{3}{2}\right)^{3\times\frac{1}{3}}$$

$$\therefore \frac{R_1}{R_2} = \frac{3}{2}$$

25. (a) According to Newton's law of cooling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1+\theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60-50}{10}\right) = K\left(\frac{60+50}{2} - 25\right) \quad .....(i)$$

and, $\left(\frac{50-\theta}{10}\right) = K\left(\frac{50+\theta}{2} - 25\right) \quad .....(ii)$

Dividing eq. (i) by (ii),

$$\frac{10}{(50-\theta)} = \frac{60}{\theta} \Rightarrow \theta = 42.85°C \cong 43°C$$

26. (a)

27. (c) According to question, $\lambda_p = \lambda_\alpha$

Using, $\lambda = \frac{h}{p} = \frac{h}{mv}$

So, $\frac{h}{m_p \times v_p} = \frac{h}{m_\alpha \times v_\alpha}$

$$\Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p}$$

($\because$ mass of α-particle is 4 times of mass of proton)

So, $\dfrac{v_p}{v_\alpha} = \dfrac{4}{1}$; i.e., 4 : 1

28. (d) Using $P_1V_1 = P_2V_2$

$$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\pi\frac{125r^3}{64}$$

$$\frac{\rho g(10)+\rho gh}{\rho g(10)} = \frac{125}{64}$$

$640 + 64\,h = 1250$

On solving we get $h = 9.5$ m

29. (d)

(A) Radius of muon $= \dfrac{\text{Radius of hydrogen}}{200}$

Radius of H atom $= r = \dfrac{\epsilon_0 n^2h^2}{\pi me^2}$

Radius of muon $= r_\mu = \dfrac{\epsilon_0 n^2h^2}{\pi \times 200\, me^2}$

$$r_\mu = \frac{r}{200}$$

(B) Velocity relation given is wrong

(C) Ionization energy in e^-H atom

$$E = \frac{+me^4}{8\epsilon_0^2 n^2h^2}$$

$$E_\mu = \frac{200me^4}{8\epsilon_0^2 n^2h^2} = 200E$$

(D) Momentum of H-atom

$$mvr = \frac{nh}{2\pi}$$

Hence (A), (C), (D) are correct.

30. (d)

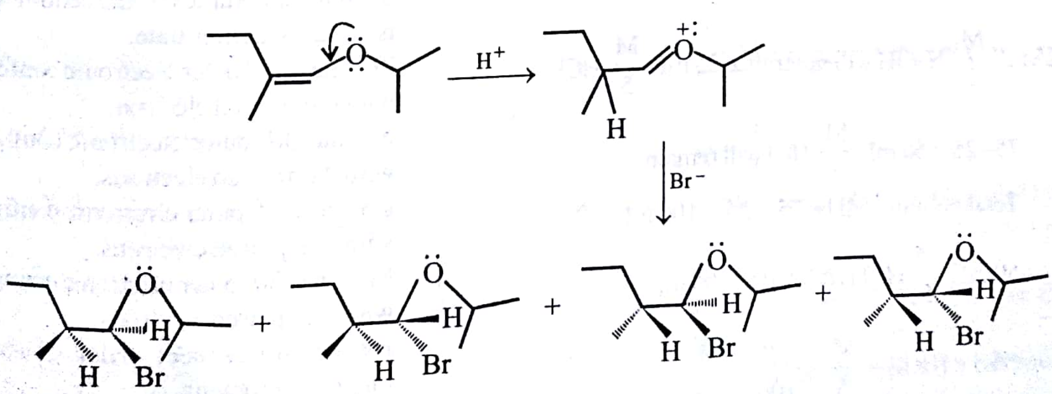
$$v = \sqrt{3k_BT/m_{He}} = \sqrt{\frac{3\times1.4\times10^{-23}\times300}{7\times10^{-27}}}$$

$= 1.34 \times 10^3$ m/s

CHEMISTRY

31. (b) Aromatic diazonium salts are more stable than aliphatic diazonium salts. The higher stability of aryl diazonium salts is due to resonance. Electron donating substituents increase electron density on benzene ring. Hereby they increase the stability of diazonium salts. Electron withdrawing substituents decrease electron density on benzene ring. Hereby they decrease the stability of diazonium salts. $-COCH_3$ group is electron withdrawing and hence, diazonium salts from (D) is less stable than that from (B). Although $-O-COCH_3$ is electron donating substituent, but it is present in meta position. Hence, it will not have significant effect on stability.
The increasing order of diazotisation is (A) < (D) < (B) < (C).

32. (c) The total number of optically active compounds formed is four. The product has two chiral C atoms. Thus, it has $2^n = 2^2 = 4$ stereoisomers.



33. (b) In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are superoxide (superoxide ion is O_2^-) and $-1/2$ respectively.
Let x be oxidation state of oxygen. The oxidation state of K is +1. Hence

$$+1+2(x)=0$$
$$2x=-1$$
$$x=-\frac{1}{2}$$

34. (c) $\Delta G^o_{rxn} = \Delta_f G°(\text{vapour}) - \Delta_f G°(\text{liquid})$

$\Delta G^o_{rxn} = 103 - 100.7 = 2.3$ kcal/mol
$= 2300$ cal/mol

$\Delta G^o_{rxn} = -RT \ln K$

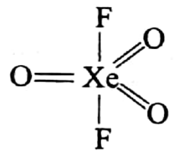
$2300 \text{ cal/mol} = -2 \text{ cal/mol K} \times 500 \text{ K} \times \ln K$

$\ln K = 2.3$

$K = 10$ atm = Vapour pressure of liquid 'S'

∴ Vapour pressure of liquid 'S' at 500 K is approximately equal to 10 atm.

35. (a) Structure of XeO_3F_2



36. (d) An antibonding π orbital best describes the diagram of a molecular orbital. Two orbitals laterally overlap to form pi bond. Out of phase combination, of these two *p*-orbitals, give π^* MO.

37. (b) $75 \text{ mL} \frac{M}{5} HCl + 25 \text{ mL} \frac{M}{5} NaOH$

$25 \text{ mL} \frac{M}{5}$ NaOH will neutralise $25 \text{ mL} \frac{M}{5}$ HCl

$75-25 = 50 \text{ mL} \frac{M}{5}$ HCl will remain.

Total volume will be $75 + 25 = 100$ mL

$50 \text{ mL} \frac{M}{5}$ HCl is diluted to 100 mL

$$[H^+] = [HCl] = \frac{M}{5} \times \frac{50}{100} = \frac{M}{10}$$

$$pH = -\log_{10}[H^+] = -\log_{10}\frac{M}{10} = 1$$

38. (b) (i) $2Fe_2O_3(s) \rightarrow 4Fe(s) + 3O_2(g)$;
$\Delta_r G° = +1487.0 \text{ kJ mol}^{-1}$

(ii) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$;
$\Delta_r G° = -514.4 \text{ kJ mol}^{-1}$

Multiple reaction (ii) with 3, we get

(iii) $6CO(g) + 3O_2(g) \rightarrow 6CO_2(g)$;
$\Delta_r G° = 3 \times -514.4 = -1543.2 \text{ kJmol}^{-1}$

When we add reaction (i) and reaction (iii), we get reaction (iv)

(iv) $2Fe_2O_3(s) + 6CO(g) \longrightarrow 4Fe(s) + 6CO_2(g)$

Free energy change, $\Delta_r G°$ for the reaction will be
$1487.0 - 1543.2 = -56.2 \text{ kJ mol}^{-1}$

39. (a) Initially 2 moles of CO are present.
At equilibrium, 1 mole of CO is present
Hence, $2-1=1$ moles of CO has reacted.
1 mole of CO will react with 1 mole of Cl_2 to form 1 mole of $COCl_2$.
$3-1=2$ moles of Cl_2 remains at equilibrium
The equilibrium constant

$$K_c = \frac{[COCl_2]}{[CO][Cl_2]} = \frac{\frac{1 \text{ mol}}{5L}}{\frac{1 \text{ mol}}{5L} \times \frac{2 \text{ mol}}{5L}} = 2.5$$

40. (d) The complex having higher number of unpaired electrons will have higher value of spin only magnetic moment.
In all these complexes, the central metal ion is in +2 oxidation state.
Zn^{2+} has $3d^{10}$ outer electronic configuration with 0 unpaired electron.
Ni^{2+} has $3d^8$ outer electronic configuration with 2 unpaired electrons.
Co^{2+} has $3d^7$ outer electronic configuration with 3 unpaired electrons.
Mn^{2+} has $3d^5$ outer electronic configuration with 5 unpaired electrons
Hence the correct order of spin-only magnetic moments is
$[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$.

41. (d) When 2-butyne is treated with H_2/Lindlar's catalyst, compound X (*cis*-2-butene) is produced as the major product; and when treated with Na/liq NH_3 it produces Y (*trans* -2- butene) as the major product. *Cis*-isomer(X) will have higher dipole moment and higher boiling point than *trans* (Y).

42. (c) The half life $t_{1/2} = 10$ days

The decay constant

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{10\text{days}} = 0.0693\ \text{days}^{-1}$$

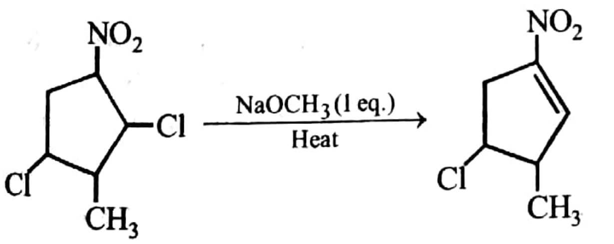
The time required for one fourth conversion

$$t = \frac{2.303}{k}\log_{10}\frac{a}{a-x}$$

$$= \frac{2.303}{0.0693\ \text{day}^{-1}}\log_{10}\frac{1}{1-(1/4)} = 4.1\ \text{days}$$

43. (a) Note : In the given reaction a molecule of HCl is lost and C = C double bond is formed. Thus it is dehydrohalogenation reaction.

Nitro group is electron withdrawing group. Hence increases the acidity of H atom (attached to C atom bearing nitro group) which is removed easily. Further the newly formed double bond is in conjugation with nitro group.



44. (b) First Bohr orbit of H atom has radius $r = 0.529$ Å

Also, the angular momentum is quantised.

$$mvr = \frac{h}{2\pi}$$

$$2\pi r = \frac{h}{mv} = \lambda$$

$$\lambda = 2\pi \times 0.529\ \text{Å}$$

45. (b) $SiCl_4 + LiAlH_4 \rightarrow LiCl + AlCl_3 + SiH_4$

46. (d) On moving from left to right across a period, the electron affinity becomes more negative. On moving from top to bottom in a group, the electron affinity becomes less negative.

Chlorine has more negative electron affinity than fluorine, Because adding an electron to fluorine (2*p* orbital) causes greater repulsion than adding an electron to chlorine (3*p* orbital) which is larger in size.

47. (a) The relationship between molar masses of the two solvents is

$$M_X = \frac{3}{4}M_Y \qquad \ldots(i)$$

The relative lowering of vapour pressure of the two solutions is

$$\left(\frac{\Delta P}{P}\right)_X = m\left(\frac{\Delta P}{P}\right)_Y$$

But, the relative lowering of vapour pressure of solutions is directly proportional to the mole fraction of solute.

Given 5 molal solution, means 5 moles of solute are dissolved in 1 kg (or 1000 g) of solvent.

The number of moles of solvent $= \dfrac{1000\ g}{M}$

The mole fraction of solute $= \dfrac{5}{1000/M}$

$$= M \times \frac{5}{1000}$$

hence $M_X \times \dfrac{5}{1000} = m \times M_Y \times \dfrac{5}{1000} \ldots(ii)$

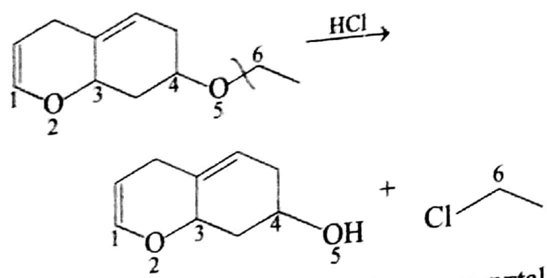
Substitute equation (i) in equation (ii)

$$\frac{3}{4} \times M_Y \times \frac{5}{1000} = m \times M_Y \times \frac{5}{1000}$$

$$m = \frac{3}{4}$$

48. (b) The lone pair of electrons on O2 is involved in resonance with C = C. Hence O2 will not be protonated.

The lone pair of electrons on O5 is not involved in resonance with C = C. Hence, O5 will be protonated. Chloride ion will then attack least substituted C atom (C6)

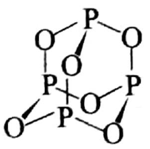


49. (d) RbCl, NaCl and CsCl share the same crystal structure except LiCl.
LiCl is deliquescent. It crystalises as a hydrated LiCl. $2H_2O$. Other alkali metal chlorides do not form hydrates.

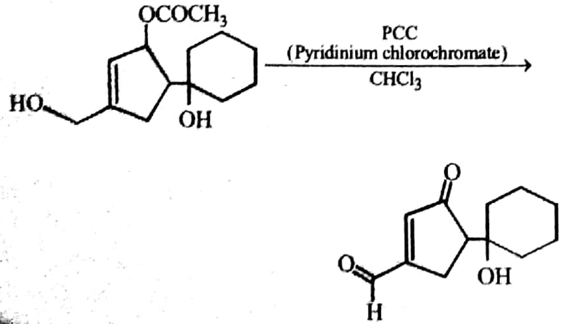
50. (b) The square planar complex of the type $[Mabcd]^{n\pm}$, where all four ligands are different, has 3 geometrical isomers. But if one of the ligands is ambidentate, then $2 \times 3 = 6$ geometrical isomers are possible. But if two ligands are ambidentate, then $4 \times 3 = 12$ geometrical isomers are possible.
In the given example, NO_2^- and SCN^- are ambidentate ligands.

51. (b) Since, adsorption of I > II, I is firmly attached to column (stationary phase). Hence, it will move slowly and will move little distance. Also II is loosely attached to column (stationary phase). Hence, it will move faster and will move large distance.

52. (c) The number of P–O bonds in $P_4O_6 = 12$



53. (b) PCC oxidizes primary alcohols to aldehydes and secondary alcohols to ketones.
In the above reaction, $-OCOCH_3$ group is hydrolyzed to secondary alcohol which is then oxidised (with PCC) to ketone.



54. (c)
(a) Molar mass of $Ba(N_3)_2(s) = 221$ g/mol
1 mole of $Ba(N_3)_2(s)$ will give 3 moles of N_2

hence $\frac{1\text{ g}}{221\text{ g/mol}}$ moles of $Ba(N_3)_2(s)$ will give $3 \times \frac{1}{221}$

$= 0.014$ moles of N_2

(b) Molar mass of $(NH_4)_2Cr_2O_7 = 252$ g/mol.
1 mole of $(NH_4)_2Cr_2O_7$ will give 1 mole of N_2

hence $\frac{1\text{ g}}{252\text{ g/mol}}$ moles of $(NH_4)_2Cr_2O_7$ will give

$1 \times \frac{1}{252} = 0.0039$ moles of N_2

(c) Molar mass of NH_3=17 g/mol.
2 mole of NH_3 will give 1 mole of N_2

hence $\frac{1\text{ g}}{17\text{ g/mol}}$ moles of NH_3 will give

$\frac{1}{2 \times 17} = 0.0297$ moles of N_2.

(d) Molar mass of $NH_4NO_3 = 80$ g/mol.
1 mole of NH_4NO_3 will give 1 mole of N_2

hence $\frac{1\text{ g}}{80\text{ g / mol}}$ moles NH_4NO_3 will give

$1 \times \frac{1}{80} = 0.0125$ moles of N_2

Hence Thermal decomposition of NH_3 will produce maximum amount of N_2.

55. (b) According to Freundlich adsorption isotherm.

$$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$\log_{10}\frac{x}{m} = \frac{1}{n}\log_{10}P + \log_{10}k$$

This is the equation of straight line of type $y = mx + c$
Hence slope is 1/n (m) and intercept is $\log_e k$.

56. (a) Clean water has BOD value less than 5 ppm. Polluted water has BOD value higher than 10 ppm. .

57. (d) $Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l)$
Alumina

$$\xrightarrow[35-36\text{ bar}]{473-523\text{ K}} \underset{\substack{\text{Sodium aluminate}\\ (X)}}{2Na[Al(OH)_4](aq)}$$

$$2Na[Al(OH)_4](aq) + CO_2(g) \longrightarrow$$

$$\underset{\substack{\text{Hydrated alumina}\\ (Y)}}{Al_2O_3xH_2O\,(s)} + 2NaHCO_3(aq)$$

58. (b) The statement (b) is not true. Chain growth polymerisation (or addition polymerisation) involves homopolymerisation only Examples of such polymers include polythene, orlon and teflon.

59. (a) Amino group of glutamine is acetylated. Amide group of glutamine is not acetylated.
Note :
Acetylation of amide requires activation of amides and /or acyl donors, since the nitrogen atom of amides is less basic than that of the corresponding amines due to amide resonance.

60. (a) The increasing order of the acidity of the carboxylic acids is III < II < IV < I. In aromatic acids, electron withdrawing groups like – Cl, –CN, $-NO_2$ increases the acidity, whereas electron releasing groups like – CH_3, –OH, $-OCH_3$, $-NH_2$ decreases the acidity.

MATHEMATICS

61. (d) Suppose $y = f(x)$

$$\Rightarrow y = \frac{x-1}{x-2}$$

$$\Rightarrow yx - 2y = x - 1$$

$$\Rightarrow (y-1)x = 2y - 1$$

$$\Rightarrow x = f^{-1}(y) = \frac{2y-1}{y-1}$$

As the function is invertible on the given domain and its inverse can be obtained as above.

62. (a) $\because (1+x)^2 = 1 + 2x + x^2$,

$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$,

and $(1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

So, the possible combinations for x^{10} are:

$x.x^9, x.x^6.x^3, x^2.x^2.x^6, x^4.x^6$

Corresponding coefficients are 2×4, $2\times1\times4$, $1\times3\times6$, 3×6 or 8, 8, 18, 18.

$\therefore$ Sum of the coefficient is

$8+8+18+18=52$

Therefore, the coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is 52.

63. (d) As the system of equations has no solution then Δ should be zero and at least one of Δ_1, Δ_2 and Δ_3 should not be zero.

$$\therefore \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$$

$$\Rightarrow -a - 1 = 0 \Rightarrow a = -1$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow b \neq 9$$

64. (d) If a and -1 are the roots of the polynomial, then we get

$$f(x) = x^2 + (1-a)x - a.$$

$$\therefore \quad f(1) = 2 - 2a$$

and $f(2) = 6 - 3a$

As, $f(1) + f(2) = 0$

$$\Rightarrow 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$$

Therefore, the other root is $\frac{8}{5}$

65. (d) If all four letters are different then the number of words ${}^5C_4 \times 4! = 120$

If two letters are R and other two different letters are chosen from B, A, C, K then the

number of words = ${}^4C_2 \times \frac{4!}{2!} = 72$

If two letters are A and other two different letters are chosen from B, R, C, K then the

number of words = ${}^4C_2 \times \frac{4!}{2!} = 72$

If word is formed using two R's and two A's

then the number of words $= \dfrac{4!}{2!2!} = 6$

Therefore, the number of four-letter words that can be formed $= 120 + 72 + 72 + 6 = 270$

66. (d) $\sin 3x = \cos 2x$

$\Rightarrow 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$

$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow \sin x = 1, \dfrac{-2 \pm 2\sqrt{5}}{8}$

In the interval $\left(\dfrac{\pi}{2}, \pi\right)$, $\sin x = \dfrac{-2 + 2\sqrt{5}}{8}$

So, there is only one solution.

67. (b) $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy}$

Let $y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\Rightarrow v + x\dfrac{dv}{dx} = \dfrac{v^2x^2 - x^2}{2vx^2}$

$\Rightarrow v + x\dfrac{dv}{dx} = \dfrac{v^2 - 1}{2v}$

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{-v^2 - 1}{2v}$

$\Rightarrow \dfrac{2v\,dv}{v^2 + 1} = -\dfrac{dx}{x}$

After integrating, we get

$\ln|v^2 + 1| = -\ln|x| + \ln c$

$\dfrac{y^2}{x^2} + 1 = \dfrac{c}{x}$

As curve passes through the point (1, 1), so
$1 + 1 = c \Rightarrow c = 2$
$x^2 + y^2 - 2x = 0$, which is a circle of radius one.

68. (a) If the outcome is one of the following: $H, TTH, TTTTH, \ldots$, then X wins

As subsequent tosses are independent, so the probability that X wins is

$p + \dfrac{p}{4} + \dfrac{p}{16} + \ldots = \dfrac{4p}{3}$.

Similarly Y wins if the outcome is one of the following: $TH, TTTH, TTTTTH, \ldots$

Therefore, the probability that Y wins is

$\dfrac{1-p}{2} + \dfrac{1-p}{8} + \dfrac{1-p}{32} = \dfrac{2(1-p)}{3}$

Since, the probability of winning the game by both the players is equal then, we have

$\dfrac{4p}{3} = \dfrac{2(1-p)}{3} \Rightarrow p = \dfrac{1}{3}$

69. (a) **Statement p:**

$\sin 120° = \cos 30° = \dfrac{\sqrt{3}}{2} \Rightarrow 2\sin 120° = \sqrt{3}$

So, $\sqrt{1 + \sin 240°} - \sqrt{1 - \sin 240°} =$

$\sqrt{\dfrac{1 - \sqrt{3}}{2}} - \sqrt{\dfrac{1 + \sqrt{3}}{2}} \neq \sqrt{3}$

Statement q:

So, $A + B + C + D = 2\pi$

$\Rightarrow \dfrac{A + C}{2} + \dfrac{B + D}{2} = \pi$

$\Rightarrow \cos\left(\dfrac{A + C}{2}\right) + \cos\left(\dfrac{B + D}{2}\right)$

$= \cos\left(\dfrac{A + C}{2}\right) - \cos\left(\dfrac{A + C}{2}\right) = 0$

Therefore, statement p is false and statement q is true.

70. (a) $\because 7 - 6x - x^2 = 16 - (x + 3)^2$

and $\dfrac{d}{dx}(7 - 6x - x^2) = -2x - 6$

So, $\displaystyle\int \frac{2x + 5}{\sqrt{7 - 6x - x^2}}\,dx = \int \frac{2x + 6}{\sqrt{7 - 6x - x^2}}\,dx$

$\displaystyle - \int \frac{1}{\sqrt{16 - (x + 3)^2}}\,dx$

71. (a) Since the plane bisects the line joining the points (1, 2, 3) and (−3, 4, 5) then the plane passes through the midpoint of the line which is :

$$\left(\frac{1-3}{2}, \frac{2+4}{2}, \frac{5+3}{2}\right) \equiv \left(\frac{-2}{2}, \frac{6}{2}, \frac{8}{2}\right)$$

$$\equiv (-1, 3, 4).$$

As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are $(-3-1, 4-2, 5-3) = (-4, 2, 2)$

So the equation of the plane is :

$-4x + 2y + 2z = \lambda$

As plane passes through (− 1, 3, 4) so

$-4(-1) + 2(3) + 2(4) = \lambda \Rightarrow \lambda = 18$

Therefore, equation of plane is :

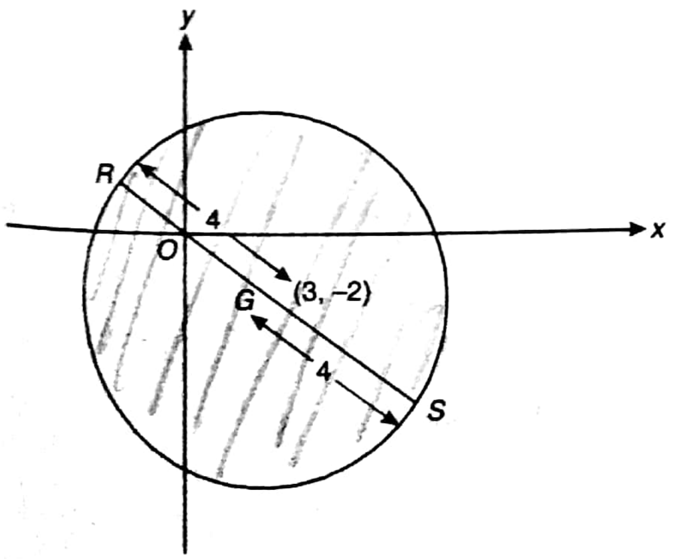
$-4x + 2y + 2z = 18$

Now, only (− 3, 2, 1) satisfies the given plane as $-4(-3) + 2(2) + 2(1) = 18$

a circular region

72. (b) $|z - (3 - 2i)| \leq 4$ represents a circle whose centre is (3, − 2) and radius = 4.

$|z| = |z - 0|$ represents the distance of point 'z' from origin (0, 0)



Suppose RS is the normal of the circle passing through origin 'O' and G is its center (3, − 2).

As, $RG = GS = 4$

$$OG = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

From (1), $OR = 4 - \sqrt{13}$ and $OS = 4 + \sqrt{13}$

So, required difference

$$= \left(4 + \sqrt{13}\right) - \left(4 - \sqrt{13}\right) \quad = 4+\sqrt{13}$$

$$= \sqrt{13} + \sqrt{13} = 2\sqrt{13}$$

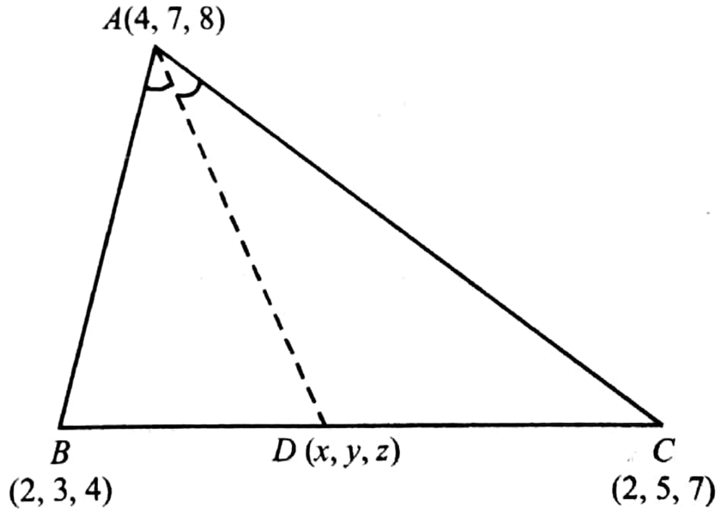
73. (b) Suppose angular bisector of A meets BC at D (x, y, z)

Using angular bisector theorem,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{BD}{DC} = \frac{\sqrt{(4-2)^2 + (7-3)^2 + (8-4)^2}}{\sqrt{(4-2)^2 + (7-5)^2 + (8-7)^2}}$$

$$= \frac{\sqrt{2^2 + 4^2 + 4^2}}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{6}{3} = 2$$



$$\text{So, } D(x, y, z) = \left(\frac{(2)(2) + (1)(2)}{2+1}, \frac{(2)(5) + (1)(3)}{2+1}, \frac{(2)(7) + (1)(4)}{2+1}\right)$$

$$D(x, y, z) = \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$$

Therefore, position vector of point

$$P = \frac{1}{3}(6i + 13j + 18k)$$

74. (d) Equation of the line, which is perpendicular to the line, $3x + y = \lambda(\lambda \neq 0)$ and passing through origin, is given by

$$\frac{x-0}{3} = \frac{y-0}{1} = r$$

For foot of perpendicular

$$r = \frac{-((3\times 0)+(1\times 0)-\lambda)}{3^2+1^2} = \frac{\lambda}{10}$$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Given the line meets X-axis at $A = \left(\frac{\lambda}{3}, 0\right)$

and meets Y-axis at $B = (0, \lambda)$

So,

$$BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10} - \lambda\right)^2} \Rightarrow BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$$

$$\Rightarrow BP = \sqrt{\frac{90\lambda^2}{100}}$$

Now, $PA = \sqrt{\left(\frac{\lambda}{3} - \frac{3\lambda}{10}\right)^2 + \left(0 - \frac{\lambda}{10}\right)^2}$

$$\Rightarrow PA \sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}} \Rightarrow PA = \sqrt{\frac{10\lambda^2}{900}}$$

Therefore $BP : PA = 3 : 1$

75. (a) Since $f(x) = \sin\left(\frac{2\times 3^x}{1+9^x}\right)$

Suppose $3^x = \tan t$

$$\Rightarrow f(x) = \sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)$$

$$= \sin^{-1}(\sin 2t) = 2t = 2\tan^{-1}(3x)$$

So, $f'(x) = \frac{2}{1+(3^x)^2} \times 3^x . \log_e 3$

$$\therefore \quad f'\left(-\frac{1}{2}\right) = \frac{2}{1+\left(3^{-\frac{1}{2}}\right)^2} \times 3^{-\frac{1}{2}} . \log_e 3$$

$$= \frac{1}{2} \times \sqrt{3} \times \log_e 3 = \sqrt{3} \times \log_e \sqrt{3}$$

76. (b)

$$A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$$

Which is a G.P. with $a = \frac{3}{4}, r = \frac{-3}{4}$ and number of terms $= n$

$$\therefore A_n = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{1 - \left(\frac{-3}{4}\right)} = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$$

$$\Rightarrow A_n = \frac{3}{7}\left[1 - \left(\frac{-3}{4}\right)^n\right] \quad (1)$$

As, $B_n = 1 - A_n$

For least odd natural number p, such that $B_n > A_n$

$$\Rightarrow 1 - A_n > A_n \quad \Rightarrow 1 > 2 \times A_n \quad \Rightarrow A_n < \frac{1}{2}$$

From eqn. (1), we get

$$\frac{3}{7} \times \left[1 - \left(\frac{-3}{4}\right)^n\right] < \frac{1}{2} \quad \Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6}$$

$$\Rightarrow 1 - \frac{7}{6} < \left(\frac{-3}{4}\right)^n \Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$$

As n is odd, then $\left(\frac{-3}{4}\right)^n = -\frac{3^n}{4}$

So $\frac{-1}{6} < -\left(\frac{3}{4}\right)^n \Rightarrow \frac{1}{6} > \left(\frac{3}{4}\right)^n$

$\log\left(\frac{1}{6}\right) = n\log\left(\frac{3}{4}\right) \Rightarrow 6.228 < n$

Hence, n should be 7.

77. (c) Given, $4x^2 - 9y^2 = 36$

After differentiating w.r.t. x, we get

$$4.2.x - 9.2.y.\frac{dy}{dx} = 0$$

$$\Rightarrow \text{Slope of tangent} = \frac{dy}{dx} = \frac{4x}{9y}$$

$$\text{So, slope of normal} = \frac{-9y}{4x}$$

Now, equation of normal at point (x_0, y_0) is given by

$$y - y_0 = \frac{-9y_0}{4x_0}(x - x_0)$$

As normal intersects X axis at A, Then

$$A \equiv \left(\frac{13x_0}{9}, 0\right)$$

$$\text{and } B \equiv \left(0, \frac{13y_0}{4}\right)$$

As $OABP$ is a parallelogram

$\therefore$ midpoint of $OB \equiv \left(0, \frac{13y_0}{8}\right) \equiv$ Midpoint of AP

$$\text{So, } P(x, y) \equiv \left(\frac{-13x_0}{9}, \frac{13y_0}{4}\right) \quad \text{...(i)}$$

$\because (x_0, y_0)$ lies on hyperbola, therefore

$$4(x_0)^2 - 9(y_0)^2 = 36 \quad \text{...(ii)}$$

From equation (i): $x_0 = \frac{-9x}{13}$ and $y_0 = \frac{4y}{13}$

From equation (ii), we get

$9x^2 - 4y^2 = 169$

Hence, locus of point P is : $9x^2 - 4y^2 = 169$

78. (d) $\because f(x)$ has extremum values at $x = 1$ and $x = 2$

$\because f'(1) = 0$ and $f'(2) = 0$

As, $f(x)$ is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \lim_{x\to 0}\left(\frac{f(x)}{x^2} + 1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{Ax^4 + Bx^3 + Cx^2 + Dx + E}{x^2} + 1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(Ax^2 + Bx + C + \frac{D}{x} + \frac{E}{x^2} + 1\right) = 3$$

As limit has finite value, so $D = 0$ and $E = 0$

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow 4A + 3B = -4$ ...(1)

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow 8A + 3B = -2$ ...(2)

From equations (1) and (2), we get

$$A = \frac{1}{2} \text{ and } B = -2$$

$$\text{So, } f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

$$\text{Therefore, } f(-1) = \frac{(-1)^4}{2} - 2(-1)^3 + 2(-1)^2$$

$$= \frac{1}{2} + 2 + 2 = \frac{9}{2}$$

$$\text{Hence } f(-1) = \frac{9}{2}$$

79. (d) $\bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \frac{54+\lambda}{8} = 8 \Rightarrow \lambda = 10$

Now variance $= \sigma^2$

$$= \frac{(7-8)^2+(8-8)^2+(9-8)^2+(7-8)^2+(8-8)^2 + (7-8)^2+(10-8)^2+(8-8)^2}{8}$$

$$\Rightarrow \sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$$

Hence, the variance is 1.

80. (b) Given

$l + 3m + 5n = 0$ (1)

and $5lm - 2mn + 6nl = 0$ (2)

From eq. (1) we have

$l = -3m - 5n$

Put the value of l in eq. (2), we get;

$5(-3m-5n)m - 2mn + 6n(-3m-5n) = 0$

$\Rightarrow 15m^2 + 45mn + 30n^2 = 0$

$\Rightarrow m^2 + 3mn + 2n^2 = 0$

$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$

$\Rightarrow (m+n)(m+2n) = 0$

$\therefore m = -n$ or $m = -2n$

For $m = -n, l = -2n$

And for $m = -2n, l = n$

$\therefore (l, m, n) = (-2n, -n, n)$ Or (l, m, n)
$= (n, -2n, n)$

$\Rightarrow (l, m, n) = (-2, -1, 1)$ Or (l, m, n)
$= (1, -2, 1)$

Therefore, angle between the lines is given as:

$$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}.\sqrt{6}}$$

$$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

81. (a) Here, equation of tangent on C_1 at (2, 1) is:

$2x + y - (x+2) - 1 = 0$

Or $x + y = 3$

If it cuts off the chord of the circle C_2 then the equation of the chord is:

$x + y = 3$

$\therefore$ distance of the chord from $(3, -2)$ is :

$$d = \left|\frac{3-2-3}{\sqrt{2}}\right| = \sqrt{2}$$

Also, length of the chord is $l = 4$

$$\therefore \text{ radius of } C_2 = r = \sqrt{\left(\frac{l}{2}\right)^2 + d^2}$$

$$= \sqrt{(2)^2 + (\sqrt{2})^2} = \sqrt{6}$$

82. (a) We have

$(A-3I)(A-5I) = O$

$\Rightarrow A^2 - 8A + 15I = O$

Multiplying both sides by A^{-1}, we get;

$A^{-1}A.A - 8A^{-1}A + 15A^{-1}I = A^{-1}O$

$\Rightarrow A - 8I + 15A^{-1} = O$

$A + 15A^{-1} = 8I$

$$\frac{A}{2} + \frac{15A^{-1}}{2} = 4I$$

$$\therefore \alpha + \beta = \frac{1}{2} + \frac{15}{2} = \frac{16}{2} = 8$$

83. (a) Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$

also let $K = \frac{x}{1+\sin x}$

Multiplying numerator and denominator by $(1-\sin x)$, we get;

$$K = \frac{x(1-\sin x)}{1-(\sin x)^2} = \frac{x(1-\sin x)}{(\cos x)^2}$$

$= x(1-\sin x)\sec^2 x$

$= x\sec^2 x - x\sin x\sec^2 x = x\sec^2 x - x\tan x\sec x$

$$\text{Now, } I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec^2 x dx - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec x\tan x dx$$

$$= \left[x\tan x - \int\frac{dx}{dx}\tan x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} - \left[x\sec x - \int\frac{dx}{dx}\sec x dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

84.

$$= \left[x\tan x - \ln|\sec x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$-\left[x\sec x - \ln|\sec x + \tan x|\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + c$$

$$\Rightarrow I = \left\{\left[\frac{3\pi}{4}\tan\frac{3\pi}{4} - \ln\left|\frac{3\pi}{4}\right|\right.\right.$$

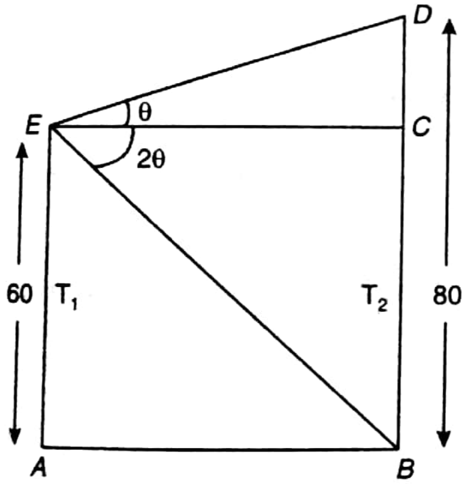
$$\left.\left.-\left[\frac{3\pi}{4}\sec\frac{3\pi}{4} - \ln\left|\sec\frac{3\pi}{4} + \tan\frac{3\pi}{4}\right|\right]\right]\right\}$$

$$-\left\{\left[\frac{\pi}{4}\tan\frac{\pi}{4} - \ln\left|\frac{\pi}{4}\right|\right.\right.$$

$$\left.\left.-\left[\frac{\pi}{4}\sec\frac{\pi}{4} - \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right|\right]\right]\right\}$$

$$= \frac{\pi}{2}(\sqrt{2}+1)$$

84. (d) Let the distance between T_1 and T_2 be x



From the figure

$EA = 60\text{ m}(T_1)$ and $DB = 80\text{ m}(T_2)$

$\angle DEC = \theta$ and $\angle BEC = 2\theta$

Now in ΔDEC,

$$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$$

and in ΔBEC,

$$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$$

We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

$$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$$

$$\Rightarrow x^2 = 1200 \Rightarrow x = 20\sqrt{3}$$

85. (d) Given:

$$I_1 = \int_0^1 e^{-x}\cos^2 x\,dx;$$

$$I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx \text{ and}$$

$$I_3 = \int_0^1 e^{-x^3}dx$$

For $x \in (0, 1)$

$\Rightarrow x > x^2$ or $-x < -x^2$

and $x^2 > x^3$ or $-x^2 < -x^3$

$\therefore e^{-x^2} < e^{-x^3}$ and $e^{-x} < e^{-x^2}$

$$\Rightarrow e^{-x} < e^{-x^2} < e^{-x^3}$$

$$\Rightarrow e^{-x^3} > e^{-x^2} > e^{-x}$$

$$\Rightarrow I_3 > I_2 > I_1$$

86. (d) Let the coordinate A be $(0, c)$

Equations of the given lines are

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines; $y = x + 2$ and $y = 7x + 3$

$\therefore$ equation of angle bisectors is given as:

$$\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}}$$

$$5x - 5y + 10 = \pm(7x - y + 3)$$

$\therefore$ Parallel equations of the diagonals are $2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\therefore$ slopes of diagonals are $\frac{-1}{2}$ and 2.

Now, slope of the diagonal from $A(0, c)$ and passing through $P(1, 2)$ is $(2 - c)$

$\therefore 2 - c = 2 \Rightarrow c = 0$ (not possible)

$$\therefore 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$$

$\therefore$ ordinate of A is $\frac{5}{2}$.

87. (d) Let,

$$L=\lim_{x\to 0}\frac{(x\tan 2x-2x\tan x)}{(1-\cos 2x)^2}=\lim_{x\to 0}K\text{ (say)}$$

$$\Rightarrow K=\frac{x\left[\frac{2\tan x}{1-(\tan x)^2}\right]-2x\tan x}{(1-(1-2\sin^2 x))^2}$$

$$=\frac{2x\tan x-[2x\tan x-2x\tan^3 x]}{4\sin^4 x\times(1-\tan^2 x)}$$

$$=\frac{2x\tan^3 x}{4\sin^4 x\times(1-\tan^2 x)}$$

$$=\frac{2x\tan^3 x}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$=\frac{2x\frac{\sin^3 x}{\cos^3 x}}{4\sin^4 x\times\left(\frac{\cos^2 x-\sin^2 x}{\cos^2 x}\right)}$$

$$\Rightarrow K=\frac{x}{2\sin x\times(\cos^2 x-\sin^2 x)\cos x}$$

$$\therefore L=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos x(\cos^2 x-\sin^2 x)}$$

$$=\lim_{x\to 0}\frac{x}{2\sin x}\times\lim_{x\to 0}\frac{1}{\cos 0(\cos^2 0-\sin^2 0)}$$

$$=\frac{1}{2}$$

88. (c) Since $f(x)$ is continuous at $x=2$.

$\therefore \lim_{x\to 2} f(x)=f(2)$

$\Rightarrow \lim_{x\to 2}(x-1)^{\frac{1}{2-x}}=k$ $(1^\infty$ form)

$\therefore e^l=k$

where

$$l=\lim_{x\to 2}(x-1-1)\times\frac{1}{2-x}=\lim_{x\to 2}\frac{x-2}{2-x}$$

$$=\lim_{x\to 2}\left(\frac{x-2}{x-2}\right)$$

$\Rightarrow k=e^{-1}$

89. (d) $\because a, b, c$ are in A.P. then

$a+c=2b$

also it is given that,

$a+b+c=\frac{3}{4}$ ...(1)

$\Rightarrow 2b+b=\frac{3}{4}\Rightarrow b=\frac{1}{4}$ ...(2)

Again it is given that, a^2, b^2, c^2 are in G.P. then

$(b^2)^2=a^2c^2\Rightarrow ac=\pm\frac{1}{16}$ ...(3)

From (1), (2) and (3), we get;

$a\pm\frac{1}{16a}=\frac{1}{2}\Rightarrow 16a^2-8a\pm 1=0$

Case I: $16a^2-8a+1=0$

$\Rightarrow a=\frac{1}{4}$ (not possible as $a<b$)

Case II: $16a^2-8a-1=0\Rightarrow a=\frac{8\pm\sqrt{128}}{32}$

$\Rightarrow a=\frac{1}{4}\pm\frac{1}{2\sqrt{2}}$

$\therefore a=\frac{1}{4}-\frac{1}{2\sqrt{2}}$ $(\because a<b)$

90. (a) Equation of the chord of contact PQ is given by:

$T=0$

or $T\equiv yy_1-4(x+x_1)$,

where $(x_1,y_1)\equiv(-8,0)$

$\therefore$ Equation becomes: $x=8$

& Chord of contact is $x=8$

$\therefore$ Coordinates of point P and Q are (8, 8) and (8, –8)

and focus of the parabola is F (2, 0)

$\therefore$ Area of triangle $PQF=\frac{1}{2}\times(8-2)\times(8+8)$

$=48$ sq. units