Rao IIT Academy JEE - Main Level
OFFICIAL ONLINE
EXAM - 2018
Solutions

1.

Difficulty : Easy

Topics :

Current electricity,

$$R = \frac{\rho l}{A}, R' = \frac{\rho(l/2)}{4A}$$

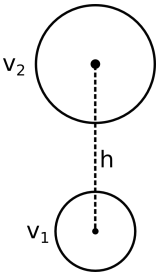
$$P = \frac{v^2}{R}; P' = \frac{v^2}{R'} = \frac{V^2}{R'} = \frac{8v^2}{R} = 8P$$

2.

Difficulty : Easy

Topics :

Surface tension,



$P_1 v_1 = P_2 v_2$

$\therefore (P_0 + h\rho_w g)\frac{4\pi}{3}r^3$

$= P_0 \times \frac{4}{3}\pi\left(\frac{5r}{4}\right)^3$

$\therefore (10 + h)\rho_w g = 10\rho_w g \times \frac{125}{64}$

$\therefore 640 + 64h = 1250 \therefore 64h = 610$

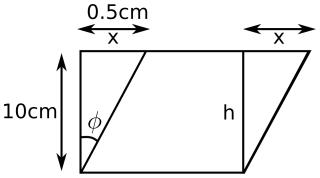
$\therefore h = 9.5\ m$

3.

Difficulty : Easy

Topics :

Elasticity,



$$\eta = \frac{F/A}{\phi} = \frac{Fh}{Ax}$$

$$\therefore x = \frac{Fh}{A\eta}$$

$$\therefore \frac{x_1}{x_2} = \frac{h_1}{h_2} \times \frac{A_2}{A_1}$$

$$= \frac{10}{20} \times \frac{400}{100}$$

$$= 2$$

$$\therefore \frac{x_2}{x_1} = \frac{1}{2}$$

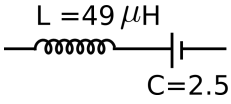
$$\therefore x_2 = \frac{x_1}{2} = \frac{0.5}{2} = 0.25\ cm$$

4.

Difficulty : Easy

Topics :

Physics,



$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{49 \times 10^{-6} 2.5 \times 10^{-9}}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{7 \times 5 \times 10^{-8} \times 2 \times \frac{22}{7}} = \frac{10^7}{22}$$

$$= 454.5\ kHz$$

$$(454.5 \pm 12)\ kHz$$

5.

Difficulty : Medium

Topics :

SOUND,

$$\frac{v}{2l_1} - f = 5$$

$$f - \frac{v}{2l_2} = 5$$

$$\therefore \frac{l_2}{l_1} = \frac{f+5}{f-5}; l_1 = 0.95\ m$$

$$l_2 = 1\ m$$

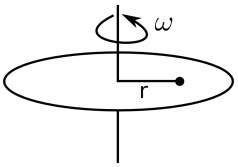
$$\therefore f = 195\ Hz$$

6.

Difficulty : Easy

Topics :

Circular motion,



$$\omega = 3.5 \times 2\pi\ rad/s$$

$$= 7\pi\ rad/s$$

In the frame of disc : $m\omega^2 r = \mu mg$

$$\therefore \mu = \frac{\omega^2 r}{g} = \frac{49\pi^2 \times 1.25 \times 10^{-2}}{10}$$

$$\simeq 49 \times 1.25 \times 10^{-2}$$

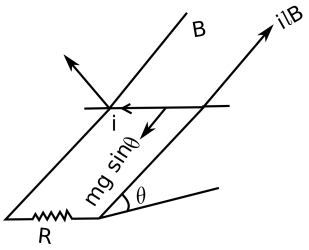
$$= 0.60$$

7.

Difficulty : Easy

Topics :

Electromagnetic Induction(EMI),



When terminal velocity is acquired net force on the rod becomes zero.

$$ilB = mg\sin\theta$$

$$\therefore \frac{vB^2l^2}{R} = mg\sin\theta$$

$$\therefore v = \frac{mgR\sin\theta}{B^2l^2}$$

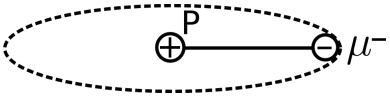
$$= \frac{mgR\sin\theta}{B^2l^2}$$

8.

Difficulty : Difficult

Topics :

Modern Physics,



$$r \propto \frac{1}{m}$$

v is independent of m

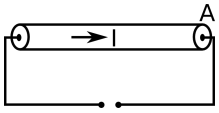
Energy α m

9.

Difficulty : Medium

Topics :

Current electricity,



$$I = neAv_d = \rho A v_d$$

$$\therefore t = \frac{d}{v_d} = \frac{d\rho A}{I}$$

10.

Difficulty : Easy

Topics :

Heat and thermodynamics,

$$v_{rms} = \sqrt{\frac{3 \times 1.4 \times 10^{-23} \times 300}{7 \times 10^{-27}}}$$

$$= 3 \times 2\sqrt{5} \times 10^2$$

$$= 13.42 \times 10^2$$

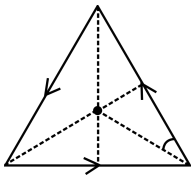
$$= 1.3 \times 10^3$$

11.

Difficulty : Medium

Topics :

MAGNETISM,



$$\frac{\mu_0 i}{4\pi d}(\cos\alpha + \cos\beta) \times 3$$

$$= \left(\frac{4\pi \times 10^{-7} \times 1 \times 2\sqrt{3} \times \sqrt{3}}{4\pi \times 4.5 \times 10^{-2}}\right) \times 3$$

$$= \frac{2 \times 10^{-5} \times 2 \times 3 \times 3}{4.5 \times 2}$$

$$= 4 \times 10^{-5} Wb/m^2$$

12.

Difficulty : Easy

Topics :

Units,Dimensions & Error analysis,

$$l = f(Ghc)$$

$$[l] = [M^{-1}L^3T^{-2}]^x[ML^2T^{-1}]^y[LT^{-1}]^z$$

$$= [M^{-x+y}][L^{3x+2y+z}][T^{-2x-y} - z]$$

$$-x + y = 0 \left| \begin{matrix} -2x - y - z = 0 \\ \therefore -3x - z = 0 \\ \therefore z = -3x \end{matrix} \right| \begin{matrix} 3x + 2y + z = 1 \\ 3x + 2x - 3x = 0 \\ \therefore x = \frac{1}{2} = y \end{matrix}$$

$$z = -3/2$$

$$\therefore l = \left(G^{1/2}h^{1/2}c^{-3/2}\right) = \sqrt{\frac{Gh}{c^3}}$$

13.

Difficulty : Medium

Topics :

Ray Optics,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{10} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_1 f_2}$$

First data satisfies above relation.

14.

Difficulty : Easy

Topics :

Rotational Mechanics,

$$\frac{1}{2} \times \frac{1}{3}ml^2 \times \omega^2 = mg\frac{l}{2}\sin\alpha$$

$$\therefore \omega = \sqrt{\frac{3g\sin\alpha}{l}}$$

$$\therefore v = \omega l = \sqrt{3gl\sin\alpha}$$

15.

Difficulty : Medium

Topics :

Simple harmonic motion,

(1) $x = 4\left(1 - \frac{2y}{b^2}\right)$ (parabola)

(2) $x = A\sin at$

$y = B\sin at$

(3) $\frac{x}{A} = \cos at$

$\frac{y}{B} = \sin at$

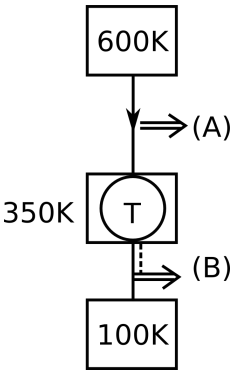
$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$

16.

Difficulty : Difficult

Topics :

Thermodynamics,



$\eta_A = 1 - \frac{T}{600}$

$\eta_B = 1 - \frac{100}{T}$

Assuming $T = 350\ K$ (in series)

$\frac{\eta_B}{\eta_A} = \frac{T - 100}{T} \times \frac{600}{600 - T}$

$= \frac{250}{350} \times \frac{600}{250}$

$= 12/7$

17.

Difficulty : Medium

Topics :

Capacitors,

$$F = \frac{Q^2}{2A\epsilon_0} = \frac{c^2v^2}{2A\epsilon_0}$$

$$= \frac{\epsilon_0^2 A^2}{d^2} \times \frac{v^2}{2A\epsilon_0}$$

$$= \frac{\epsilon_0 A v^2}{2d^2}$$

$$25 \times 10^{-6} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-2} \times v^2}{2 \times 2.25 \times 10^{-4}}$$

$$v = 250v$$

18.

Difficulty : Easy

Topics :

Collisions,

Elastic collision between two masses leads to such a situation.

19.

Difficulty : Medium

Topics :

Electrostatics,

$$E \times 4\pi r^2 = \int_0^R \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr \times \frac{1}{\epsilon_0}$$

$$= \frac{\rho_0 \times 4\pi}{\epsilon_0} \times \left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$\therefore Er^2 = \rho_0 \times \frac{R^3}{12\epsilon_0}$$

$$\therefore E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$$

20.

Difficulty : Easy

Topics :

HEAT,

$$\frac{60-50}{10}=k[55-25]$$

$$\frac{60-\theta}{20}=k\left[\frac{60+\theta}{2}-25\right]$$

$$\frac{20}{60-\theta}=\frac{30}{30+\frac{\theta}{2}-25}$$

$$\therefore \frac{2}{60-\theta}=\frac{3}{5+\theta/2}$$

$$\therefore 10+\theta=180-3\theta$$

$$\therefore 4\theta=170$$

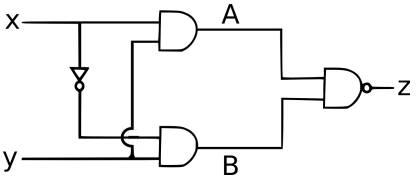
$$\therefore=\frac{85}{2}=42.5^{o}C$$

21.

Difficulty : Easy

Topics :

Semiconductors,



x	y	$\bar{x}$	A	B	z
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	0	1

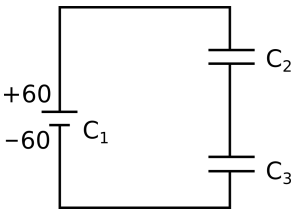
22.

Difficulty : Easy

Topics :

Capacitors,

$Q_1 = 60\mu C$



Equivalent capcitance of C_2 and C_3

$$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{3 \times 6}{3 \times 6}\mu F = 2\mu F$$

Common potential difference to C_1 and C' combined.

$$v' = \frac{C_1 v_1 + 0}{C_1 + C'} = \frac{60}{1+2} = 3\ volts$$

Charge of C_2, C_3 system $= C'v = 2 \times 20 = 40\mu C$

23.

Difficulty : Easy

Topics :

Electromagnetic waves,

$$v = \frac{v}{\lambda}$$

Here $\frac{\lambda}{2} = |z_1 - z_2|$

$$v = \frac{v}{2|(z_2 - z_1)|}$$

$$= \frac{3 \times 10^8}{2|(z_2 - z_1)|}$$

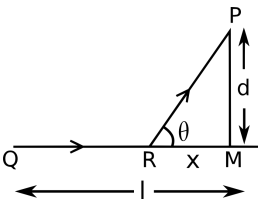
$$= \frac{1.5 \times 10^8}{|z_2 - z_1|}$$

24.

Difficulty : Medium

Topics :

Kinematics,



$$t = \frac{l-x}{v} + \frac{\sqrt{x^2+d^2}}{v/2}$$

$$\frac{dt}{dx} = \frac{1}{v}(0-1) + \frac{2}{v}\frac{2x}{2\sqrt{x^2+d^2}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^o$$

$$x = \frac{d}{\sqrt{3}}$$

25.

Difficulty : Easy

Topics :

Atomic nucleus,

$$m_1 v_1 = m_2 v_2$$

$$\therefore \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8}$$

$$\therefore \frac{r_1^3}{r_2^3} = \frac{27}{8}$$

$$\therefore \frac{r_1}{r_2} = \frac{3}{2}$$

26.

Difficulty : Easy

Topics :

Electromagnetic Induction(EMI),

Flux $\phi = BA$

$\pi = \frac{\mu_o I}{2R}\pi r^2 \cos\theta$

e.n.f $e = \frac{-d\phi}{dl} = \frac{\mu_0 I}{2R}\pi\omega \sin\theta$

So $\frac{\mu_o I}{2R}\omega\pi r^2 \sin\theta$

27.

Difficulty : Easy

Topics :

Modern Physics,

$\lambda = \frac{h}{mv}$

λ is same so

mv is also same

$m_\alpha v_\alpha = m_p v_p$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4}{1} = 4$

28.

Difficulty : Easy

Topics :

Physics,

By cross checking

29.

Difficulty : Medium

Topics :

Rotational Mechanics,

After collision centre of mass ramain at O

$m_1 r_1 = m_2 r_2$

$2m\frac{L}{6} = mL/3$

Using conservation of angular momentum

$m(2v)\frac{L}{3} + 2mv\frac{L}{6} = I\omega$ .....(1)

when $I = 8m\frac{L^2}{12} + 2m\left(\frac{L}{6}\right)^2 + m\left(\frac{L}{3}\right)^2$

$= \frac{5}{6}mL^2$

From (1) $\left(\frac{2}{3} + \frac{2}{6}\right) mvL = \frac{5}{6}mL^2\omega$

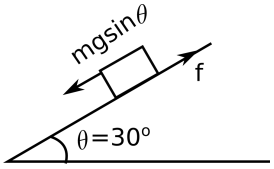
or $\omega = \frac{6}{5}\frac{v}{l}$

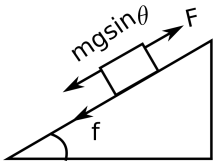
30.

Difficulty : Easy

Topics :

Friction,





For downward motion

$mg\sin\theta - f = ma$

or $f = mg\sin\theta - ma$

$= 2 \times 10 \times \frac{1}{2} - 2 \times 3$

$= 4N$

For upward motion

$F - mg\sin\theta - f = ma$

or $F = mg\sin\theta + f + ma$

$= 2 \times 10 \times 1/2 + 4 + 2 \times 3$

$= 10 + 4 + 6 = 20N$