2019 National Collegiate Programming Contest Preliminary Round

September 28, 2019

- Problems: There are 7 tasks (12 pages in all) in this packet.
- Program Input: Input to the program are through standard input. Program input may contain one or more test cases. Test cases may be separated by any delimiter as specified in the problem statements.
- Program Output: All output should be directed to the standard output (screen output).
- Time Limit: Judges will run each submitted program with certain time limit (given in the table below).

Table 1: **Task Information**

	Task Name	Time Limit
Problem A	Factoring Integers	1 sec.
Problem B	Watching a Movie	1 sec.
Problem C	Wireless Service in Fifth Avenue	1 sec.
Problem D	Enumerating Game	2 sec.
Problem E	Modulo Completeness	5 sec.
Problem F	Vector Addition in XOR-land	1 sec.
Problem G	The Word Puzzle	3 sec.

Problem A
Factoring Integers

Let n be an odd integer greater than 2. Write a program to repeatedly simulate the randomized algorithm (Monte Carlo algorithm) for factoring n:

1. randomly select an integer p.
2. **if** $(1 < \gcd(n, p) < n)$ print p **else** fail.

In the i-th simulation, the "random number" p_i should be computed by the following formula.

$$p_i = \begin{cases} 3 & \text{if } i = 1, \\ n - p_{i-1} & \text{if } i > 1 \text{ and } i \text{ is even,} \\ (3 + 5p_{i-1}) \bmod n & \text{if } i > 1 \text{ and } i \text{ is odd.} \end{cases} \tag{1}$$

The simulation stops when one of the following conditions occurs.

1. The integer n has been successfully factored, or
2. All the distinct "random numbers" described by Equation 1 have been tested.

Input Format

The first line of the input contains a positive integer m ($m < 40$) indicating the number of test cases that follows. Each test case contains an integer n in a line, where $2 < n < 2^{31}$.

Output Format

For each test case, print out the following in a line.

```
n = P * Q
```

In the above output, `P` is the $\gcd(n, p_i)$, and `Q`$= n/$`P`. If the given integer n is prime or it cannot be factored by the method described above, then print the following line.

```
n = 1 * n
```

Print exactly 1 space before and after the symbols "=" and "*".

Sample Input

```
3
221
187
13
```

Output for the Sample Input

```
221 = 13 * 17
187 = 11 * 17
13 = 1 * 13
```

Problem B
Watching a Movie

The ACM kingdom has $n \geq 2$ citizens, numbered from 0 to $n-1$. For each citizen $v \in \{0, 1, \ldots, n-1\}$, denote by $d(v)$ the number of mutual friends (in the ACM kingdom) of v. For each $v \in \{0, 1, \ldots, n-1\}$, assume $d(v) \geq 1$ and let $t(v) \in \{1, 2, \ldots, d(v)\}$. The ACM film studio invites a set of citizens, called the seeds, to watch a new movie. Whenever at least $t(v)$ mutual friends of a citizen $v \in \{0, 1, \ldots, n-1\}$ have watched the movie, v will also watch the movie. The process continues asynchronously until no more citizens will watch the movie. The question is: What is the minimum number of seeds needed so that all citizens will have watched the movie at the end?

Consider the undirected graph G with the vertex set $\{0, 1, \ldots, n-1\}$ and with the edge set containing precisely all unordered pairs (i, j) of mutual friends. Assume G to be connected and acyclic. So G is an undirected tree.

Technical Specification

1. $2 \leq n \leq 35$.
2. For all $v \in \{0, 1, \ldots, n-1\}$, v is not a mutual friend of v.

Input Format

The first line of input contains a positive integer m ($m \leq 10$) indicating the number of test cases that follows. For each test case, the first line contains serveral integers. The first integer is n, followed by n integers denoting $t(0)$, $t(1)$, ..., $t(n-1)$, respectively. The next line contains an integer p indicating the number of unordered pairs of mutual friends. The next p lines each contains two integers denoting a pair of mutual frineds. All integers are separated by white space(s).

Output Format

For each test case, output the minimum number of seeds needed so that all citizens will have watched the movie at the end.

Sample Input

```
5
6 2 2 1 2 1 1
5
0 1
5 3
0 3
4 1
2 3
```

```
3 1 1 1
2
0 1
0 2
5 1 2 1 1 1
4
0 2
2 4
2 1
1 3
5 1 2 2 1 1
4
0 2
2 4
2 1
1 3
8 3 2 2 1 1 1 1 1
7
0 1
0 2
0 7
1 3
1 4
2 5
2 6
```

Sample Output for the Sample Input

```
2
1
1
2
3
```

Problem C
Wireless Service in Fifth Avenue

Nowadays, there are base stations everywhere, so that people can use high-speed networks by connecting their devices to the base stations.

Fifth Avenue is a good place for shopping. It is a long, straight street, along which there are n stores (coffee shops, restaurants, and so on) that provide base stations B_1, B_2, ..., B_n. Each base station B_i has a radius r_i. A tourist's device can connect to a base station B_i (for free) if and only if their distance is at most r_i. The collection of portions of Fifth Avenue at which tourists can get wireless service is the coverage of B_1, B_2, ..., B_n.

Consider the example in Figure 1. The length of Fifth Avenue is $L = 12$. For convenience, Fifth Avenue is regarded as an interval $[0, L]$ on the real line. There are $n = 4$ base stations B_1, B_2, B_3, B_4 located, respectively, at positions 1, 3, 6, and 9; and their radii are $r_1 = 2$, $r_2 = 1$, $r_3 = 1$, and $r_4 = 2$. In this example, tourists can get wireless service if and only if their positions are contained in $[0, 4] \cup [5, 11]$. Therefore, $[0, 4] \cup [5, 11]$ is the coverage of B_1, B_2, B_3, B_4, whose size is 10.

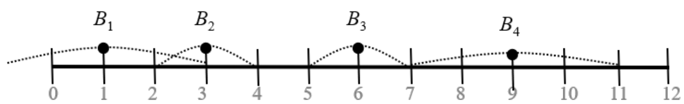

Figure 1: Four base stations at 1, 3, 6, 9

Given the length of Fifth Avenue and the locations and radii of the base stations, please write a program to compute the size of the coverage of the base stations.

Technical Specification

1. The length of Fifth Avenue is a positive integer $L \leq 10^7$.
2. The number of base stations is a positive integer $n \leq 10^5$.
3. The location of each base station is a non-negative integer $\leq L$.
4. The radius of each base station is a positive integer $\leq 10^7$.

Input Format

The first line is an integer t, $1 \leq t \leq 10$, indicating the number of test cases. The first line of each test case contains two integers L and n, $1 \leq L \leq 10^7$ and $1 \leq n \leq 10^5$, indicating the length of Fifth Avenue and the number of base stations. There are n pairs of integers in the following n lines, where the pair of integers b_i and r_i in ith line represents the location and radius of the ith base station. The given b_i and r_i satisfy the following: $0 \leq b_1 < b_2 < ... < b_n \leq L$ and $1 \leq r_i \leq 10^7$.

Output Format

For each test case, output in a line the size of the coverage of $B_1, B_2, \ldots, B_n$.

Sample Input

```
2
12 4
1 2
3 1
6 1
9 2
12 1
9 2
```

Sample Output for the Sample Input

```
10
4
```

Problem D
Enumerating Game

Cathy and Willy are playing a two-person game, where Cathy constructs an infinite binary tree and Willy tries to find out the order corresponding to a given pair of integers. Each node of the binary tree consists of a pair of ordered positive integers, i.e., the order of the integers matters. The node associated with a pair of positive integers (i, j) has two children: the ordered pair $(i, i + j)$ is for the left child, and $(i + j, j)$ is for the right child. The root of the binary tree is the first node and has a pair of positive integers (a, b), where a and b are relatively prime. The nodes of the binary tree are enumerated from the root and level-by-level from left to right, just as the ordering in a binary heap. Given a pair of positive integers (c, d), find its order in the binary tree.

Technical Specification

1. (a, b) is the pair of relatively prime positive integers for the root, $1 \leq a, b < 10$.
2. $1 \leq c, d < 64$.

Input Format

The first line of the input contains an integer m ($m \leq 20$), indicating the number of test cases. Each test consists of one line with four space-separated positive integers a, b, c and d.

Output Format

For each test case, the order of (c, d) in the binary tree. Output -1 if (c, d) does not appear in the tree.

Sample Input

```
5
1 1 1 1
1 1 1 5
1 1 3 6
3 2 3 2
5 2 1 2
```

Sample Output for the Sample Input

```
1
16
-1
1
-1
```

Problem E
Modulo Completeness

Given an integer sequence $a_1, a_2, a_3, \cdots, a_n$, we divide these integers into two groups A and B. Let S_A and S_B be the sum of integers in A and B respectively. Given a positive integer k, consider the remainder r of $S_A - S_B$ divided by k with $0 \leq r < k$, where we denote that $r = (S_A - S_B) \mod k$. For all possible A and B, we can gather all the different resulted remainders r as R. We are interested in the size of R. If the size of R is k, then the sequence is called k-modulo complete.

For example, given an integer sequence $2, 1, 1$, we can enumerate all the possible A and B as follows.

$A = \{2, 1, 1\}, B = \{\}$, and $S_A - S_B = 4$.
$A = \{2, 1\}, B = \{1\}$, and $S_A - S_B = 2$.
$A = \{2\}, B = \{1, 1\}$, and $S_A - S_B = 0$.
$A = \{1, 1\}, B = \{2\}$, and $S_A - S_B = 0$.
$A = \{1\}, B = \{2, 1\}$, and $S_A - S_B = -2$.
$A = \{\}, B = \{2, 1, 1\}$, and $S_A - S_B = -4$.

If $k = 3$, we can get $R = \{(-4 \mod 3), (-2 \mod 3), (0 \mod 3), (2 \mod 3), (4 \mod 3)\} = \{0, 1, 2\}$ and its size is 3. If $k = 4$, we can get $R = \{(-4 \mod 4), (-2 \mod 4), (0 \mod 4), (2 \mod 4), (4 \mod 4)\} = \{0, 2\}$ and its size is 2. If $k = 5$, we can get $R = \{(-4 \mod 5), (-2 \mod 5), (0 \mod 5), (2 \mod 5), (4 \mod 5)\} = \{0, 1, 2, 3, 4\}$ and its size is 5.

Technical Specification

1. $0 < k < 10,000$
2. $0 < n < 600,000$
3. $\forall_{1 \leq i \leq n}, 0 \leq a_i \leq k$

Input Format

The first line contains a number m ($m \leq 10$) indicating the number of test cases. For each test case, the first line has 2 numbers k and n. The following n lines contains n integers a_i.

Output Format

For each test case, please output the size of the diferent remander set R.

Sample Input

```
2
5 3
2 1 1
```

```
4 3
2 1 1
```

Sample Output for the Sample Input

```
5
2
```

Problem F
Vector Addition in XOR-land

The ACM Company recently decided to extend their business to a scenery tribe named XOR-land in a nearby galaxy. As the name suggests, people in XOR-land use bitwise exclusive-OR $\oplus$ as a replacement of the arithmetic addition. For example, the result of $12 \oplus 4$ is 8.

Given a set of n-dimensional integer vectors $v_1, v_2, \ldots, v_m$, the ACM Company would like you to calculate the result of $v_1 \oplus v_2 \oplus \ldots \oplus v_m$.

Technical Specification

1. $1 \leq n, m \leq 100$
2. The coordinates of the input vectors are integers between 0 and 10^6.

Input Format

First line of the input contains an integer t ($t \leq 15$) denoting the number of test cases. For each test case, the first line contains two integer n and m, denoting there are m n-dimensional integer vectors. Each of the following m lines contains n integers, denoting the coordinates of that vector.

Output Format

For each test case, print the resulting vector in a line.

Sample Input

```
1
4 2
4 8 12 3
8 4 12 2
```

Sample Output for the Sample Input

```
12 12 0 1
```

Problem G
The Word Puzzle

In this word puzzle, you are given several English words. Your job is to 'link' these words together. The rule is very simple. Initially, you choose an arbitrary word as the starting point. Then, you can extend it by attaching another word in the end as long as the first letter of this word is identical to the last letter of the previous word. Two consecutive words overlap exactly one letter for the sake that you need to tight them together. In addition, each word can only be used exactly once. For example, suppose we have four words: `eagle, apple, eat, tree`. One way to connect them is in this order `apple, eat, tree, eagle`. In this case, the result is `appleatreeagle`. Another way to connect them is in this order `apple, eagle, eat, tree`, and the result is `appleagleatree`. Due to some unknown reason, the correct answer is the one whose lexicographic order is the smallest if it is possible to link them altogether. In our example, the first resultant string `appleatreeagle` is larger than the second string `appleagleatree`, because at the seventh position the first one has letter 't' and the second one has letter 'g', and 't' is larger than 'g'. Therefore, the first string should be dropped. In fact, the second string is the correct answer in this case.

Technical Specification

1. There are only lowercase English letters in the words.
2. Each word has at least 2 characters and at most 20 characters.
3. The total number of words is at least 1 and at most 10^5.

Input Format

The first line contains an integer m ($m \leq 20$) denoting the number of test cases. For each test case, the first line contains a single integer w ($1 \leq w \leq 10^5$) denoting the total number of words in this test case. In the next w lines, each line contains exactly one word of this test case.

Output Format

For each test case, output lexicographically the smallest linked word, or output `IMPOSSIBLE` is the given words cannot be linked.

Sample Input

```
3
3
eve
eagle
age
4
```

```
ago
oh
home
ema
2
ago
dig
```

Sample Output for the Sample Input

```
ageagleve
agohomema
IMPOSSIBLE
```