Probability and risk management for sports betting

Applying principles of portfolio management to real world betting

First Edition

by The Portfolio Punter

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1. Introduction

This book has one aim: it is about how to take your skill in betting and turn it into a consistent profit by applying the portfolio management techniques described in this book. Regardless of the sport, or how you decide on your bets, this book outlines a system for managing your portfolio of bets by controlling risk and allocating your capital efficiently across the large range of available betting options.

The most important requirement to be successful in betting is to have an edge. The edge can be gained by having information that others don't have, or through superior analysis of publicly available information. To be clear about what an edge is: it is not about picking winners – most events have a clear favourite with the bookies and the favouritism is usually justified. An edge comes from you being a better judge of the probability of future sporting results than a bookmaker or other punters on a betting exchange. In other words, determining where the odds are wrong. Your edge is a necessary step for you to become a successful punter but it takes more to turn that edge into a consistent profit.

Betting is fundamentally risky and there is no way around the fact that even where you have an edge and get great odds on an event, any bet can lose as the events you are betting on are inherently random.

This book describes some of the tools for taking your edge and turning into a betting strategy – it does not detail a specific system for a specific sport. It is about understanding the types and level of risk you are taking on, and running a portfolio of bets with an eye to controlling risk while maximising your returns. It uses ideas both from betting/probability theory and from investment management.

The approach to risk management described here works regardless of whether you are using a quantitative or qualitative process to forecast odds. My experience in investment management has taught me that quantitative processes are of mixed benefit in picking investments but considerable benefit when constructing a portfolio. This comes with a strong caveat: the portfolio management process still has to be designed and used wisely as no quant process will work consistently when used without common sense.

Some readers may think this approach takes all of the fun out of betting, that this is betting for accountants. That's a fair comment.The system is about how to make a steadier profit from a successful betting idea – and that is likely to mean less adrenaline. One way in which this system works is to keep the profitable side of your edge while containing the painful losses. Even if you choose to keep some adrenaline in your betting, hopefully this book will offer you some ideas for how you might size your bets to avoid painful long runs of large losses.

To make money from betting you first need to identify a sport and a type of bet where you have an edge. Once you have found a sport/bet where you have an edge, this system will help you make a steadier return by control position sizes and downside risk through portfolio construction techniques. Most examples in this book are English Premier League because it is well known internationally and a very large betting market. Different markets have different characteristics so some other markets are considered but this book does not cover every sport or type of market.

Before you try any system with real money you should test your edge by running it with paper money. Type the market odds and the amount you would bet into a spreadsheet before the event starts and see how your system would go over a season. As a second step you

may want to try it live but with a small betting pool. This is a big money saver as many seemingly good ideas don't work in practice. As any experienced punter knows, some good ideas don't make money, some even lose money.

Rational betting

This book is written with the rational punter in mind. Unless you're a robot you will always have some behavioural biases but the book has a clear focus on head over heart punters. There are a number of obvious behavioural biases that some punters fall for such as "supporting your team" by betting on it, or trying to win back what you've just lost. There are many less obvious biases that undisciplined punters can fall for.

After winning a bet have you ever said "I should have put more on that bet?" Then why didn't you? This is an example of hindsight bias. With the benefit of hindsight it is obvious that the outsider was going to win the game and you should have bet your house. The reality is that before the game it was far from obvious that the outsider was going to win. That's why the odds were so long and it's probably also why you didn't put more on the bet. It's also why you probably shouldn't have put more on the bet.

You will find that over time you will have good runs and bad runs, and this will impact your confidence but this is often just chance messing with your head. You should never forget the importance of luck in betting. In the long run a skilful punter will win, but the fact that luck always plays a part in your results is another reason why you should not put all of your eggs in one basket. This is no different to investing – you can tilt the odds in your favour but there will always be times when you underperform.

Overconfidence: have you ever met anyone who claims to lose money betting? The reality is that most people do lose money betting – that's how bookmakers make their money. Betting exchanges take a commission from your winnings so on average, punters lose after costs and commissions there too. Most punters don't keep good enough records to know whether they are winning or losing and tend to remember the winning bets rather than the losers. Two messages here: be realistic – it's tough to come out in front after the bookies have taken their cut; and keep good records so you know how you are really doing.

Another bias relating to past bets is a tendency to believe bets that won were the good bets – this is a skill vs. luck bias. A good bet is not necessarily one that you won (it might have just been a dumb bet that got lucky). A good bet is one where the odds were mispriced and you positioned yourself to take advantage of the mispricing, regardless of whether you ended up winning or losing money. If you can reliably pick bets like this you will end up in front in the long run. Accepting odds of 1.5 on a coin toss it is a bad bet. If you win this bet then you make money but it still doesn't make it a good bet. It's very important to understand the difference between a good bet and a lucky bet. If you pursue bets like the coin toss above that you won, you will be pursuing bad bets where you have just been lucky in the past. Luck averages out over the long run and this will be a losing strategy.

A cornerstone of rational betting is a system for sizing positions. Most of these systems are based on the Kelly formula. The Kelly formula is really designed for card games where you put down a series of single bets with well defined and well known probabilities. Applying it to the real world is much more complex and difficult. While it is possible to solve a set of equations about how to optimally

size positions, a more practical approach is to use the Kelly formula as a guide. This idea is the core of this book and is expanded upon in Chapter 4.

Similarities with financial markets

There are many comparisons between betting markets and financial markets, especially if you are using a betting exchange. There are also some important differences. Take the stock market as one example of a financial market. With stocks, the whole market may be expensive but usually you can still find some stocks that are attractively priced. It's possible that all stocks are expensive and none of them represent value. In this instance, you may outperform the average investor but still lose money when the market returns to fair value.

With betting, the whole market can't be expensive as probabilities must add to 1 (ignoring the overround for now). Within this, individual bets can still be mispriced. If you outperform the average punter then, unlike for the stock market, you will make money subject to the overround and commissions not being too high. The idea is the same in both situations: buy cheap, sell expensive and you should make money as luck averages out.

Other strategies

To borrow the language of the financial markets, this book is about "investing" rather than "trading". Investing is about forming a view on the "correct" odds and placing your bets accordingly. Trading strategies typically don't have a view on the correct odds but aim to make money through other means such as finding risk-free opportunities (arbitrage), having a view on how odds will move from their current level in the short run or making a market.

Arbitrage is a valid strategy way to make money but significant opportunities don't turn up that often and you're not the only one looking for them. Arbitrage is about finding risk free opportunities such as one bookie offering $2.10 on Team A and another bookie offering $2.10 on Team B. Put a dollar on each and you make a $0.10 profit regardless of the outcome. There are many variations on how to implement arbitrage. You need to be wary of what financial markets call *basis risk*, that is, the risk that the two bets are not exactly the same and that neither wins. You also need to be careful that one side of the bet doesn't disappear as you're placing the bets and leave you exposed.

Making a market is only possible if you are using a betting exchange and relates to the situation where you offer odds for and against every bet. This is essentially what bookmakers do. Making a market is usually done with a view of what the correct odds should be, but is also impacted by the views of other punters through the "weight of money". In its purest form this is a sophisticated strategy but some elements of the approach can be incorporated into your strategy relatively easily.

A final point to remember is that if something looks like a guaranteed way to make money then chances are you've missed something. Occasionally there is free money within a market, between markets, or between different bookies/exchanges. When you see it, take it.

What to avoid

When someone tells you that buying their betting system will make you a fortune they are fibbing. If someone has a system that makes a fortune they will not sell it to anybody – and certainly not a complete stranger.

There are two main types of systems offered: arbitrage software and tipping services. In theory, each of these could work but in practice is unlikely to. Arbitrage systems can work but opportunities are small and hard to find because lots of people are looking for them. After paying the price of the software you will never make money. If you want to use this approach, you can build basic arbitrage software yourself in a spreadsheet in a few minutes.

2. Exchanges vs. bookies

Since the late 1990s it has been possible to bet via betting exchanges rather than betting with a bookie. When you use a betting exchange, you are betting against another punter and the exchange is merely facilitating the bet in return for a percentage of winnings. This differs from a traditional fixed odds bookie where you are betting against the bookie.

There are some other important differences between exchanges and bookies in the way your money is handled, and in the strategies that can be effectively used. When you bet against a bookie, the money you deposit goes into their bank account and if they go broke you are an unsecured creditor – meaning you won't be getting all of your money back. When you bet on an exchange, the money you deposit goes into a trust fund. The exchange takes commissions, fees or losses you owe from the trust but apart from this, generally can't touch your money. One thing that both bookies and exchanges have in common is that they keep any interest earned on your deposited funds so you should always keep unused funds to a minimum.

The most obvious difference between bookies and exchanges is that an exchange gives you the ability to lay as well as back a bet. This opens up many more possibilities in how you can bet and expands the range of strategies available to you. Of course, with a traditional bookie you can easily lay a bet in any two horse race, and with some effort and cost you can lay a bet in more complex situations through a technique known as "dutching".

Why is the ability to lay bets important? It doubles the number of potential bets at your disposal, it allows you to efficiently close out previous bets, and it means you can also make money when the available odds are shorter than your forecast odds. With a traditional

bookie, if you think the odds are too long then you bet. If you think the odds are too short then the rational thing to do is not bet. With a bookie, if you want to close out a previous bet then that's going to be expensive because bookies' margins are charged on all bets and not just your net winnings.

There are some other important differences. The overround (the sum of the implied probabilities) can be very close to 100% on an exchange but the exchange then takes a percentage of your winnings so you need to calculate the equivalent odds. If the exchange odds are, say, 3 and you are paying 5% of your winnings as commission then these odds are equivalent to 2.9 from a bookie. The formula to calculate the effective (bookie equivalent) odds where the commission is k, and exchange odds are d, is

$$\textit{Effective odds} = 1 + (1 - k) * (d - 1) \qquad (2.1)$$

In practice your effective odds are often better than this as the exchange only takes a percentage of *net* winnings. So where you have multiple bets on a market, the actual percentage can be less than k. A bookie effectively takes a percentage of each win separately. The exchange's commission arrangements are also more favourable if you bet and later want to lay the bet as you only pay a commission on your net winnings. Closing out a bet is typically very expensive to implement when using a bookie.

On an exchange there is an amount available to back or lay at each level of odds. This is the amount that other punters are currently offering to bet/lay. With a bookie there is also a fixed amount available before the odds change or they shut you off. This is usually more than on an exchange but it is never disclosed. Exchanges are more transparent in this sense as you have the same information as

everyone else. Bookies have an unfair information advantage as they can see what all the punters are doing but you can only see what you are doing.

There are other characteristics of betting exchanges that make them more like financial markets. The size of the spread (bet vs. lay), and the liquidity (amount available to bet) are two important characteristics. These are discussed in more detail in Chapter 5.

Even if you use a bookie rather than an exchange and you can't lay bets, the ideas in this book can still be applied in a two horse race, that is, any event where the only options are that a runner can either win or lose. Backing horse B is the same as laying horse A. And even when there are more than two runners it is possible to "lay" a runner by backing all of the others with the appropriate sized bet. This is known as dutching and the arithmetic can be performed simply using a spreadsheet. The calculations are not covered here – if you want to lay bets then it is far simpler and more efficient to use an exchange.

One thing that bookies provide as part of the service they offer is liquidity. Unless you are betting in a very popular sport such as the biggest English Premier League games, there are practical limits on how much you can bet on an exchange before you "move the market". That is to say, you exhaust the amount available to bet at your target price and either have to wait, or bet at a less favourable price. This is one advantage of using a bookie: they will usually match your entire bet (within reason) at the quoted price. If you are betting considerable amounts of money then this is important. Even if you are only betting modest amounts then liquidity can still be important depending on your chosen sport and markets.

2.1 Who offers the best odds?

As is so often the case, it depends. There is no golden rule as to where you will find the best odds, especially once you adjust exchange odds for commission on winnings and throw into the mix the impact of the strategies that you are playing. You probably should monitor odds at a few bookies and an exchange. There is no reason why you can't bet with more than one bookie/exchange – although for certain strategies it is more efficient to place all of your bets on a single exchange.

The availability of lays on exchanges can make them more attractive places to bet for the serious punter. It also opens up a range of other strategies that are not available with a traditional bookie. Some simple examples of strategies that are either impossible or difficult with a bookie include:

- cold trading – known as technical trading in the financial markets. This is short term trading purely on market information rather than a view of the correct odds;
- making a book – where you effectively act as a book maker and offer odds on each possible outcome; and
- laying off bets – where you take the opposite position to a previous bet to either lock in profits or limit losses.

Different markets have different characteristics, and those characteristics can determine what strategies will work. Some examples of different types of markets are:

- Wide spreads between bet and lay odds and limited liquidity. This type of market presents opportunities to

make the market. They offer better margins but there is often a small amount bet.

- Tight spreads between bet and lay odds. In these markets you usually need to take a view, although cold trading can also work. There is typically less mispricing but you are more likely to be able to bet the amount you want to bet.
- Liquid markets are better as you can get your bets set, but liquidity and spread are inversely correlated so this may compromise your preferred strategy.

In practice, you will need to monitor markets closely and experiment to find what works for you and where it works.

3. Odds and probability

3.1 What are odds?

Odds are simply another way of representing the probability of an event occurring. Odds tell you how much you get back if you pick the outcome of the event correctly. Long odds imply a low probability, short odds imply a high probability. The relationship is plotted below for decimal odds.

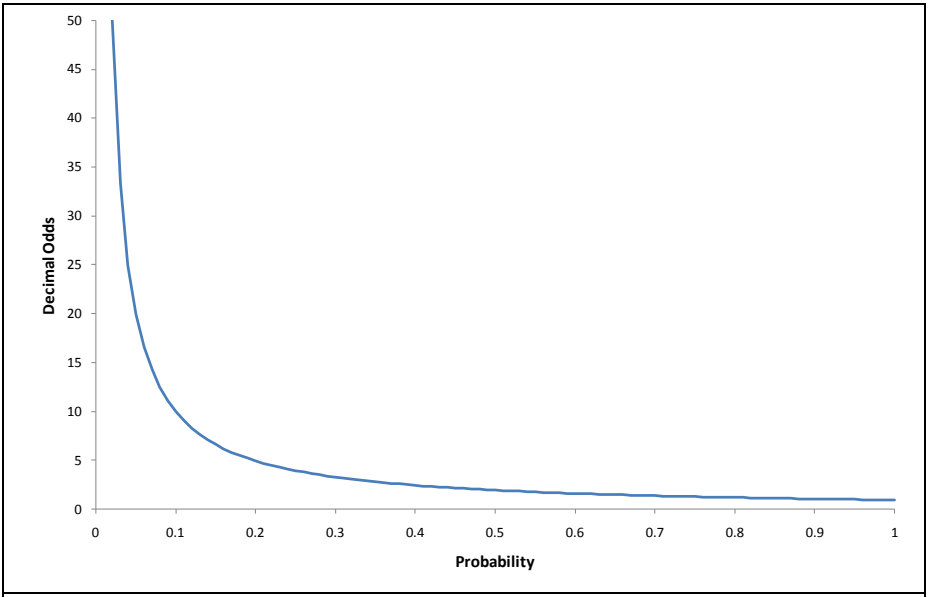


Chart 3-1 showing the relationship between probability and the corresponding decimal odds.

Based on a bookmaker's odds, it is possible to calculate the *implied probability* that an event will occur, at least in the view of the bookie. There are a number of different ways to represent odds

and they can all be transformed to give an implied probability. Most online bookmakers use decimal odds as their default. Decimal odds refer to how much you win for each dollar that you bet. So if the odds are 3, you get 3 dollars if you win – that is the dollar that you bet plus another two dollars that you have won. A traditional (fractional) odds system would call these odds 2-to-1. American bookies have their own way of representing odds called moneyline odds which would show these odds as 200. Odds represented in any one of these systems can be transformed to any other system using simple formulas. If you are used to using a particular type of odds then you can put these transformation formulas into a spreadsheet so that you can convert them quickly. Some bookies' websites allow you to choose how you want the odds to be represented so you can stick with whatever system you feel most comfortable with.

The formulas to convert from probabilities to odds are:

$$\left.\begin{array}{ll} \text{Decimal odds} = & 1/p \\ \text{Traditional odds} = & (1-p)/p \\ & \frac{1}{d_1}+\frac{1}{d_2}+\ldots+\frac{1}{d_n}>1 \end{array}\right\} \quad \textbf{(3.1)}$$

$$\begin{array}{ll} \text{Moneyline odds} = & (1-p)/p*100 \text{ if } p<1/2 \\ & -p/(1-p)*100 \text{ if } p>1/2 \end{array}$$

Probability	Decimal odds	Traditional odds	Moneyline odds
0.1	10	9/1	900
0.25	4	3/1	300
0.5	2	Evens	100

			(or -100)
0.75	1.33	1/3	-300
0.9	1.11	1/9	-900

This book uses decimal odds. Decimal odds can be thought of as inverse probabilities. More technically, they are the reciprocal of the probability, $1/p$. If you are used to thinking in a different odds system then you can set up a spreadsheet that quickly converts between odds types. All of the formulas shown in this book for decimal odds can be transformed to use the other odds formats. You may want to set up a spreadsheet to automatically do the conversion for you.

Do you really need to know about probabilities?

Many punters aim to pick the winner without giving any thought to calculating probabilities – it seems way too theoretical. Perhaps you don't estimate probabilities when you bet, or at least you don't consciously estimate them. But every punter has at least a rough understanding of probability and an implicit view on the probabilities of the events they are betting on.

For example, what price would you bet on a fair coin landing heads? If you are a rational punter, you wouldn't bet at 1.5 but you would definitely bet at 3. For a rational punter, the cutoff point where you would be indifferent is 2, because the probability of heads is 1/2. If you are offered odds above 2 you should bet and for odds below 2 you should pass.

Even when you don't think you are estimating probabilities, and you are relying more on gut feel, you have an idea of what the

probabilities are– your cutoff odds for the event. This is the point where slightly higher you would bet on the event and slightly lower you would pass or bet against it. If your (decimal) cutoff odds are 4 then your implied probability of the event happening is 1/4.

This is a very important point. Betting markets can be thought of as markets in probabilities. As in any market, a fundamental "investing" strategy to make a profit is to buy cheap (below fair value), and sell expensive (above fair value). In betting terms, you bet when odds are too high and lay when odds are too low. This is sometimes referred to as value betting. There are alternative strategies, which are built more around trading odds as markets move. The techniques in this book are built around investing strategies where all bets have an expected return greater than 1, that is, value betting.

Another critical part of deciding when to bet and how much to bet is risk management and this will be covered in more detail in Chapter 4. It is even more important when using a betting exchange where market liquidity and the ability to get set complicate the risk of your total portfolio of bets.

3.2 What is probability?

Probability is a difficult concept to explain in words. Most people have an understanding of chance and randomness from their experiences and this is the best way to understand probability. The probability of a future uncertain (random) event is the chance that the event occurs. It is always between 0 (impossible) and 1 (certain). An easy example is that the probability of a coin turning up heads is ½. In the sporting context what does probability mean? What is the probability of Manchester City beating Fulham (the lowest ranked

team to be promoted to the Premier League at time of writing). Is it even chance that determines the outcome? Clearly Manchester City is the stronger team and would be expected to win. But they may rest their stronger players, be over confident or be focussed on a bigger Champions' League game in a few days' time. This could give the weaker team a better shot at winning and change the "probability". But is any of this random? Based on all of the information you could think of to collect, you can get a more accurate estimate of the factors determining the outcome. But there is still a random element involved in any sporting event. Who wins the toss, every pass and every shot at goal has an element of chance to it. There are no certainties. Your aim as a punter is to get the best possible estimate of the underlying probability and then bet based on that – you're never going to get them all right but if you're good at it you can tilt things in your favour.

Probability is a difficult concept to fully understand – outcomes often appear more certain than they are. In particular, the probability of past events is often confused by punters looking back and seeing events as certain, or at least much more certain than they were before the event. The outcome of yesterday's games is known now but it was far from certain before yesterday. Why did anyone take the long odds on Fulham when it was always obvious that Manchester City were going to win? There is always a chance that the weakest team beats the strongest team and it happens often enough to give faithful fans and punters hope that it will happen again this game.

The real problem with understanding probability of sporting events is that we only have one repetition of each game. With a coin we can toss it 100 times and we'll get about 50 heads and 50 tails. With a football game, we can never play the same game again so,

looking back, the result that we observed was always certain. One way to get a feel for probabilities of sporting events is to write down each week in advance what you think is the probability of the result in each game. Over time you'll see that if you take all of the results that you thought were unlikely, say less than 20%, a reasonable proportion of these end up with the unlikely result. By writing down your guess of the probability before the game you remove the look back bias where the outcome looks more certain than it was beforehand.

3.3 Different types of probabilities

Probability relates to the future and the uncertainty surrounding future events, so there can be different views of the probability of each event. Sometimes we can be reasonably confident about estimating probabilities, for example with fair dice or coins, as we have good (but not perfect) knowledge of the factors affecting the outcome. At other times there are a lot of unknown factors that impact the outcome of an event, for example the winner of an election, so confidence in any probability forecasts should be low. In the latter situation, forecasts by different individuals will differ more than forecasts for the toss a coin.

True probability is generally unknown. One example where the actual probability of an event is known is tossing an unbiased coin and getting Heads. In this case we know the actual probability is ½. We can demonstrate what probability means by tossing the coin a large number of times and showing that the proportion of Heads is close to ½. There are two main differences between this event and a sporting event: the event depends just on physics and not on teams of people interacting in complex environments; and the event is

repeatable so we could estimate the probability even if we couldn't calculate it. In most controlled circumstances such as cards or dice games it is possible to calculate the theoretical true probability. In more complex situations where there are many influences on the probability of the event, such as sporting matches, we don't know what the actual probabilities are. If Manchester City is playing at home to Chelsea we will never know the true probability – we have to forecast it.

Forecast probability. This is an estimate of the actual probabilities. For the example mentioned above with Manchester City at home to Chelsea my forecast odds on a recent game were roughly 46% for Manchester City, 28% for a draw and 26% for Chelsea. As the game gets closer ad more detailed information becomes available, the forecast can be refined based on the most up-to-date information. Has either team lost players to injury or suspension? Does weather affect the teams differently? Has one team had a Champions' league fixture during the week? Has either team just fired its manager? There are numerous other factors that could influence the outcome. Even if all of these match specific factors are taken into account, any forecast is still only a best estimate of the true probability. And no forecast will tell you the result – just the probability of each possible outcome.

Implied probability. This is the probability of an event occurring implied by the market/bookie odds. For decimal odds it is given by $p_d = 1/d$. Depending what you are using the implied probability for, you may need to adjust the calculation for the bookmaker's margin but it is close enough for now.

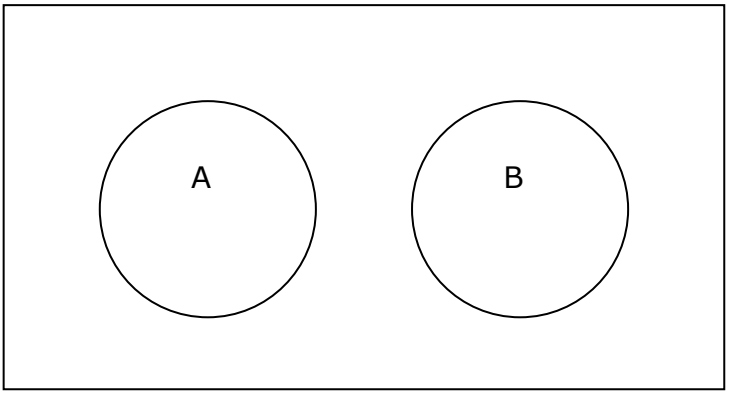
The opportunity in betting is created when you can tilt the playing field in your favour because your forecast probability for an event is more accurate than the bookmaker's forecast, or the market's forecast if you're using an exchange.

3.4 Some background on probability

This section is a little theoretical but it is important to understand this background material.

I. Mutually exclusive

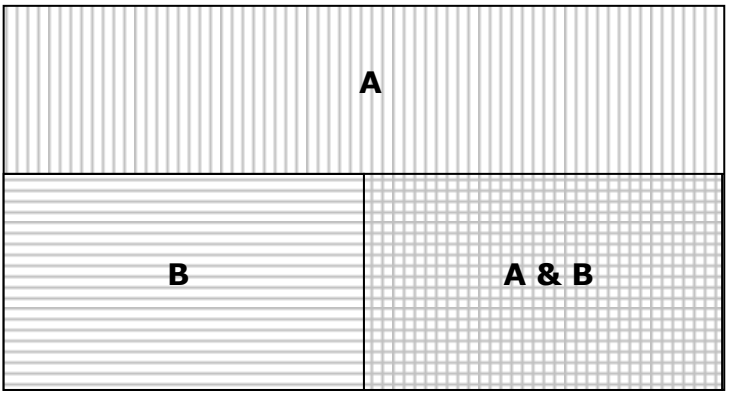
Two or more events are mutually exclusive if at most one of the events can occur. It may be that none of the events occur. This is shown in the diagram below.



If you are looking at a Manchester City vs Chelsea game then you could think of A as Manchester City winning and B as Chelsea winning. These events are mutually exclusive as at most one of them can occur but if there is a draw then neither of A nor B occur.

II. Exhaustive

Two or more events are exhaustive if it is always the case that at least one of the events occurs. It may be that more than one of the events occurs. This is shown in this diagram.



Think of the outcome when you toss two coins. A is the event "at least one tail" and B is the event "the second coin is a head". The diagram here shows the possible outcomes. You can think of the four possible outcomes to see that the areas are A: HT, TT, B: HH, A & B: TH. So the events are exhaustive – at least one of the events always occurs. Note that in this case, Pr(A) + Pr(B) = ¾ + ½ =1¼ > 1.

III. Probabilities add to 1

One common misunderstanding about probabilities is that they always add to 1. This is the same as the misunderstanding that percentages always add to 100%. Both of these statements are true under certain conditions. Where events are exhaustive and mutually exclusive, the sum of the true probabilities of the events adds to 1 (and the sum of the percentages adds to 100%).

Where there are three possible outcomes to a game – for example Team A wins, Team B wins, Draw – and there are no other possible outcomes – then the true probabilities of these three events must add to 1. Your forecast probabilities should also add to 1. The probabilities implied by a bookmaker's odds are likely to add to more than 1 as they include the bookie's margin, also known as the overround.

When you are betting, it is very important to understand the detailed rules for each bet with the bookmaker/exchange you are using. Are there any quirks in the rules? What happens to your bet if the game is abandoned or finished early due to crowd trouble/weather? Are the events exhaustive, that is, should the probabilities add to 1 or something less than 1? Put another way, what if there is some other possible outcome not listed by the bookmaker/exchange? In Australian Rules Football draws are uncommon so odds are typically quoted for a win/lose with no draw odds. But draws do occur and happen roughly 1 in every 70 games. There will probably be two or three draws during most AFL seasons so you need to know what happens to your bet in this situation. In

compound/exotic bets this also matters – an example is the bet "Geelong has a perfect season", that is that it wins all 22 games. Does "perfect season" include the scenario where they have 21 wins and one draw? If you are looking at arbitrage bets it becomes even more important to ensure there are no subtle rule differences between the two bets that mean the odds should be different and it is not strictly arbitrage. In financial markets, this difference between the two bets is called *basis risk*. These details are very important and will cost you money if you don't understand them.

There are some markets where probabilities will add to a number other than 1. For example, if you are looking at the market where teams finish in the Top 3 of the Premier League, there are 3 "winners" so the probabilities will add to 3. For simplicity, we will refer to probabilities summing to 1 for the rest of this section but you need to remember that the sum may be different for some bets.

IV. Independence

Two events are independent if the occurrence of one event has no impact on the probability that the other event will occur. For example, each week in the premier league there are ten games and the outcomes of each of these games should be independent of each other or very close to it. You could contrive an example where independence is not strictly true – such as where Manchester City plays Chelsea in an FA Cup game during the week so they each play a weaker lineup on the prior Saturday against lower ranking teams. If a team is tired or carrying injuries this is also important to estimating match probabilities. In practice though, this example would have little impact on the independence of games.

In sports betting, independence is important where you are managing a portfolio of bets. Independence is an assumption that is often made subconsciously. Independence between games is usually a reasonable assumption but it is still important to acknowledge that the assumption has been made, and to understand under what circumstances it may not be met and what the impact would be. There are examples where the assumption of independence is not met and this has implications for probabilities in a total portfolio of bets. This is expanded on in Section 6.7.

Independence can also have an impact in the calculation of probabilities in compound/exotic bets. You need to take this into consideration when you are calculating the odds for your exotic bet.

V. Expected value

Expected value is a theoretical concept but with an important practical application. Expected value is very important in deciding when and how much to bet. One way to think of it is the average return we would expect if we could run the same event infinitely many times. The coin example is again useful here. If the quoted odds are 2 that a coin lands on heads then the expected value is 2 x Pr(Heads) = 2 x ½ = 1. These odds are called fair odds as they don't advantage either side. If the quoted odds are less than 2 then the expected value is less than 1 so this bet is almost certain to lose money in the long run. Odds greater than 2 give an expected value greater than 1 which means there is an edge.

In sports betting, the concept of expected value is harder to define as each game is played only once so there can only be one observation of the event. You could think of the expected value as

the frequency of wins if we had an infinite number of parallel universes and could observe the event in each universe.

At fair odds, the expected value is exactly 1.

Forecast probabilities

Regardless of what method you use to forecast probabilities, the sum of your forecast probabilities should be the same as the sum of the actual probabilities, typically 1. Your forecast of the expected value for each bet is simply your forecast probability multiplied by the odds offered. If your forecast of the expected value of a bet is more than 1, then you should place a bet as your forecast probability of an event is greater than the probability that the bookie is pricing into the odds. Conversely, if you forecast the expected value of a bet to be less than 1 then don't bet as the odds are too short.

Implied probabilities

The implied probability is what the bookie/market has priced in as the probability of the event (adjusted for commissions). In the context of the financial markets, this would be thought of as what information the market has priced in and incorporates the collective view of all market participants on the future prospects of returns.

Implied probabilities calculated using odds from a single bookmaker will always add to more than one (unless they've made a mistake). The sum of the implied probabilities is

$$\frac{1}{d_1}+\frac{1}{d_2}+\ldots+\frac{1}{d_n}>1 \qquad (3.1)$$

The amount that this is above 1 is called the overround. This may be as little as 5% for some markets, but is often closer to 20%. In general, the more choices there are in a market, the higher the overround. Overrounds for betting exchanges are typically, but not always, lower than for bookmakers, especially for popular markets. The overround is the bookmaker's margin and it is a hurdle to

making a profit, so generally you want this number to be as small as possible.

When using a betting exchange there are two implied probabilities for each bet – at the bet price and at the lay price. The sum of the implied probabilities that you can bet at if you cross the spread is the equivalent of odds offered by a bookie and is typically more than 1. The sum of implied probabilities that you can lay at is typically less than 1.

You can use the implied probability to decide whether to bet. If the implied probability is less than your forecast probability then you should bet as you believe the event is more likely than the market/bookie does. Conversely, if the implied probability is more than your forecast probability then you should not bet. In this situation, it may even be desirable to lay the event. If the implied probability is close to your forecast probability then you may choose to sit this one out as there is little benefit in betting.

Combining the different types of probabilities

The different types of probabilities can be shown on a graph.

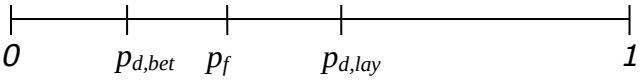


Figure 3-2 Diagramatic representation of the types of odds.

where

$p_{d,bet}$ is the implied probability of the market's current lay odds,
$p_{d,lay}$ is the implied probability of the market's current bet odds,
p_f is your forecast odds.

3.5 Forecasting probabilities

The first question you should ask yourself before you bet is "What do I know that others don't?"

This is what is sometimes known in financial markets as your "alpha source". It is your strategy, system, ideas, or knowledge that enable you to make a profit. The system presented here does not provide an alpha source. It is designed to manage a portfolio of bets based on your alpha source, whatever it is, as long as it genuinely is able to forecast probabilities more accurately than other punters in the market.

You can use any basis at all for forecasting – from a team's recent form to astrology. Generating a forecast is easy. Generating useful (profitable) forecasts is not so easy and there is no method to guarantee profitable forecasts. Not all of your forecasts will be right but you need to be able to consistently generate profitable forecasts if you are going to make money from betting. Picking that the top team is going to beat the bottom team is probably correct and will help you in a tipping competition, but it's also pretty obvious and the bookie's odds will already reflect the strong chance that the top team is going to win. Your forecast needs to help you determine where the market odds are wrong. Has the bookie over- or under-estimated the difference between the teams and hence the chance of the strong team winning?

A forecasting system can be qualitative, quantitative or some combination of the two. Regardless of how it works, it will use currently available information to form a view about the future. As new information comes to light, your forecast should change to reflect this and the market odds are also likely to change. There are

many factors that could be considered in a forecasting model – different things work for different markets. Two completely different systems could both work, although probably at different times. There is no right answer and some seemingly sensible systems don't work – possibly because the information being used is already factored in by the market as everyone else knows about it. Systems don't have to be complex. You may want to experiment with a paper betting system for a while to test if you can make money in a market before you start betting with real money.

Some factors to consider in constructing your forecast odds:

- Current position of each team on the league table
- Most recent results for each team (and strength of opponents)
- Goal difference (or percentage for AFL)
- Attacking/Defensive record
- Home ground advantage (or distance travelled)
- How well a team travels
- Each team's line-up for this game (consider player ratings)
- Any injuries or suspensions
- Time since last game and recent workload (fatigue)
- The budget for each team
- The depth of each team's player roster
- The weather forecast
- Several of these measures could be combined into a "team strength" indicator.

You can use a quantitative system to measure some or all of your factors, or you can use your judgment. You should understand why each factor you consider contributes to providing a more useful probability forecast. You may also want to test your factors on a

previous season's results but there is no substitute for doing a live run, either with a shadow portfolio or with a small amount of money. There are many more factors that you could consider and you are really only limited by your imagination.

As you add more obscure factors it is likely that they will each have less impact on making an accurate forecast of the probability of an event and may not be worth the effort. A few key factors will usually get you pretty close in terms of estimating the probability. This idea is known as the Principle of Parsimony – keep the model as simple as is necessary to do the job.

You should aim to make your system robust. A small change in the inputs should produce a small change in the forecast probability and not a dramatic change in your suggested bet. If this is not the case then forecasts are likely to be over-influenced by unimportant factors or noise, and your forecasts may not be reliable.

In designing a forecasting methodology you are only limited by your imagination. Qualitative and quantitative forecasters are typically suspicious of each other but both can work well in the right hands and can even be combined to produce a hybrid system which may be more powerful than either system separately. When it comes to risk management, some basic quant tools are very helpful. To use quant risk management, your forecasting method doesn't have to be quantitative. Even if your forecasts are quantitative, mathematically sophisticated models are not necessarily better than simple models. Your insights into the sport are far more important than mathematical or statistical theory. One quantitative element you do need is a number as the outcome of your system (at least an approximate estimate) and that is your forecast probability.

Financial market quant analysts have a saying – "garbage in, garbage out". In other words, a bad model with poor data is useless and exactly the same applies in betting as in financial markets. You need to ensure that your data source is free from errors.

When forecasting probabilities you may want to combine estimates from more than one system to improve reliability. You will probably need to combine the team strength scores with other factors to arrive at your forecast probability. The challenge is that you are converting an open ended scale to a 0 to 1 scale.

A common way to convert a variable to a 0 to 1 scale is with a logistic function. It takes an open ended strength scale and transforms it to the probability scale with a minimum of 0 and a maximum of 1. Again, this could be a quantitatively or qualitatively derived strength score but the conversion is quantitative. If you decide to use a logistic function you'll need to calibrate it.

The chart below shows a logistic function. There are no values on the *x*-axis as this is intended to show the general shape of the relationship and, as mentioned above, the curve must still be fitted to your data.

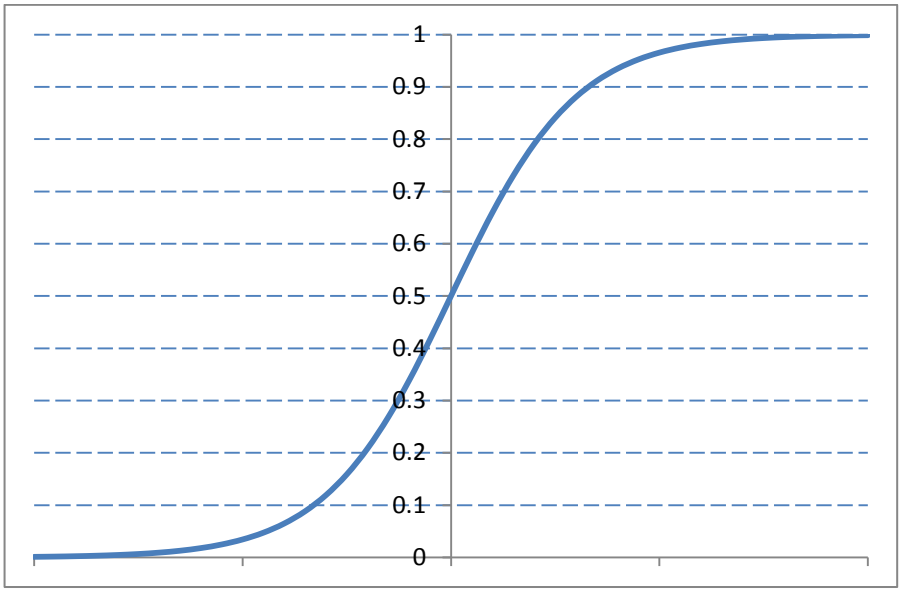


Chart 3-1 Logistic function

The next big question is: How do you calculate the parameters and fit the curve to your data? This is beyond the scope of this book and there are many online resources that can help with this. It is usually done using what is, in effect, a trial and error method. You start with a rough estimate and then try moving each parameter up then down and find which of these changes gives you the best fit to the data. This can be automated in a spreadsheet using a simple macro.

3.6 Elo rating systems

You may have heard other punters talk about Elo systems for rating teams. Arpad Elo was a Hungarian mathematician who designed a rating system for chess players in the 1950s. His system has been adapted and refined over time but the basic methodology that he outlined is still in use today. Each time a game is played the

ratings of the player will be adjusted to reflect the outcome of the game and strength of the opponent. If a player wins a game they are expected to win against a much weaker opponent their rating will gain slightly. If a player wins a game they are expected to lose against a much stronger opponent then their rating may increase substantially. The system needs to be calibrated so that a lucky win won't turn the ratings upside down but consistent wins against strong players will move a player's rating significantly. Over time the system adjusts each player's rating to the appropriate level.

Some measure of the relative strength of the teams is likely to be a factor in your probability forecast. A system similar to the Elo chess rating system can be used to maintain team strength ratings. It is more complicated to implement than simpler systems based on league table position but can give better results as it incorporates factors such as strength of opponents and the winning margin. Calibrating the system can be difficult sports, particularly for sports where teams are fairly evenly balanced. For some sports there are Elo-like ratings freely available on the net. You still need to decide whether this information is useful to you and if so, how to use it.

4. Risk management

Anybody can win a bet. It could be through skill or it could just be through dumb luck on a bet with terrible odds. To consistently win more than you lose is much harder and requires skill rather than luck. Even if you have skill in estimating what the odds for an event should be you are likely to lose many of your bets, as betting by definition exposes you to uncertain future outcomes. You will have periods where everything goes right and periods where nothing goes right – all due to luck.

To survive all of this, you need a consistent approach to decide when and how much to bet to ensure you get the most from your skill and minimise the negative impact of bad luck.

4.1 When should you bet?

At the risk of stating the obvious, you bet when you believe the odds being offered are too high, or put another way, you think an event is more likely than the bookies are implying in their odds. In the mathematical terminology introduced earlier, you should bet when $p_f > p_d$ where p_f is your forecast probability and p_d is the implied probability.

Another way of saying this is you should bet when the expected value of the bet based on your probability forecast is greater than 1. If the odds are d then you should bet where $d.p_f > 1$.

These two rules are mathematically equivalent but it can sometimes be helpful to think about things in a different way. The idea in both formulas is the same: bet when you think the bookie/exchange has got it wrong in your favour.

If you already have a bet on a particular team then you may want to consider whether to add to your existing bet. To decide this you need a way to determine how much you should bet. If you've already bet your target amount, then you don't need to place any further bets for now.

4.2 How much should you bet?

There are many different systems for determining the size of your bets and each system has a different risk/return profile. Some of the key characteristics to look at are:

- the risk of going bust;
- the risk of losing money;
- the volatility of returns and the size of your betting pool; and
- the magnitude of winnings when you get things right.

Each of these characteristics is in turn impacted by the level of edge that you have, the size of the bookie's overround and various other factors. There is no approach to position sizing that will cause a bad forecasting system to make money – as a starting point, you have to have an edge over the bookies/exchange if you want to win consistently. Unfortunately, a bad approach to position sizing can cause a good forecasting system to lose money. That is, even where you have an edge over the bookie, you can lose money if you choose your stake sizes poorly.

Some of the simplest approaches to position sizing are: fixed bet, fixed winnings, edge and Martingale. This section summarises each of the systems, outlines their main characteristics and lists some key pros and cons for each. These staking systems are covered in

considerable detail in Joseph Buchdahl's book *Fixed Odds Sports Betting.* Following this, some desirable characteristics for a position sizing system will be determined and a system that has these characteristics will be outlined.

It's also worth noting that most of the theory about staking systems implicitly assumes that you will be betting sequentially – such as when you are playing cards and you bet on successive hands, one at a time. In practice when you bet on sports you are likely to have multiple bets on at the same time. If you are playing longer term bets, say on the outcome of a season, it is possible to have hundreds or even thousands of bets on by the end of the season. These bets are also likely to be placed at numerous different times throughout the season but will mostly settle around the same time at the end of the season. There is also the issue of correlation between the bets to complicate things further. A naive approach to position sizing will not work effectively in managing a large, complex portfolio of bets in this situation.

Fixed bet is where you bet a fixed dollar amount on each bet regardless of the size of your pool, the odds being offered or the amount by which you think the bet is mispriced (provided you believe it is mispriced). This is the simplest system but it doesn't take into account valuable information about risk and return. The system has low volatility in the pool size.

Fixed percentage involves betting a fixed percentage of your pool on each bet. This is similar to the fixed bet system but you wind back your bets when you have lost money and increase your bets as your pool gets bigger. Again, this system doesn't take into account

the odds being offered or the amount by which you think the bet is mispriced. This system has a low probability of bankruptcy and increases your returns by fully utilising all of your available funds at each point in time. The downside is that volatility in the size of the pool will increase as the pool itself increases.

Fixed winnings is where you set your stake so that your winnings will be equal to a fixed amount if you win the bet. This system tends to bet more on bets that are more certain. An example of how fixed winnings would work in practice: if you bet $1 at odds of 2 then your winnings is $1. If you bet $2 at 1.50, or $10 at 1.10 then if you win the bet your winnings will also be $1.

If C is the amount you want to win when you are successful then you can calculate the size of your stake, s, as:

$$s = \frac{C}{d-1}. \qquad (4.1)$$

In effect this is a risk adjustment to your stake size and is a natural safety mechanism because you end up betting less on long shots (which usually don't pay off) and more on favourites. As a result, this system has less volatile outcomes than the fixed bets or fixed percentage systems. Note that the bet size doesn't depend on your forecast probability or edge. The system has low volatility in the pool size.

Edge is where you bet an amount proportional to your estimated winnings, or edge. If C is your nominal bet size then your stake, s, would be:

$$s = C(d.p_f - 1). \qquad (4.2)$$

Obviously, if $d.p_f$ is less than 1 then this gives a negative number. In this case, your expected return is negative because you don't have an edge and this formula confirms that you shouldn't bet in this situation.

Also note that in theory there is no upper limit to your bet size in cases where your forecast probability is very different from the implied probability. If d =10 and p_f = 0.5 then s = 4.C.

This formula can be restated as

$$s = C(\frac{p_f}{p_d} - 1). \qquad (4.3)$$

So where the ratio of forecast to implied probability is high then you should bet a large amount. If p_f = 0.01 and p_d = 0.001 then this system says you should bet nine times your nominal bet – if you can find a bookie willing to take such a large bet at 1000 to 1. The bet size appears rash when you believe the bet has only a 1% chance of working out, and these numbers are very sensitive to forecasting error. This system can put very large bets on low probability events as it does not adjust for the risk of a bet. In other words, this system does not take account of the odds being offered, other than through the expected return. The system can have a volatile pool size and a high risk of bankruptcy.

Martingale is sometimes described as a staking system that ensures you will end up in the black, even where the odds are against you. Of course there are no certainties when you're gambling and this system has not found a magic way to guarantee a profit. So how does the Martingale system work and when does it not work? The

real problem with the system is that you keep winning a small amount until you go broke. In *The Mathematics of Gambling*, Thorp makes the point that staking systems don't work because "a series of negative expectation bets must have a negative expectation".

In order to keep the calculations simple, let's look at a simple case where we toss a coin and the odds for each coin toss are 2. All of these calculations can be adapted to the case where the odds are different for each bet. On the first toss you bet $1. If you win then you are up $1. If you lose you are down $1 so on the second toss you bet $2. If you win you are up $1 ($2-$1). If you lose you are down $3 so on the third toss you bet $4. If you win you are up $1 ($4-$2-$1) and if you lose you are down $7. Continue this process until you win – which has to happen eventually, at which point you will be up $1. This sounds too good to be true, so what's the catch? The catch is that you could run out of money before your first win. If it takes ten coin tosses to win then you will be $1,023 down, and have to bet $1,024 before you end up $1. If your standard bet is $10 then you have had to risk $10,230 for your profit of $10. This is the real problem with this system – you win small and lose big. In fact when you lose, you lose your entire betting pool. Note that the bet size doesn't take account of risk or depend on your forecast probability but instead depends on an arbitrary factor: how long since you last won a bet. This system has a high likelihood of bankruptcy.

4.3 What system should you use?

So which is the best system? Or is there some other system that does a better job than these? Is it possible to extract the best parts of each system and combine them? Some characteristics you may want to have in a position sizing system are:

- Place more money on bets that are more certain/lower risk. This reduces the volatility of returns.
- Place more money on bets where you have a bigger edge. This puts more money where you believe the bookies/market have got it more wrong, and where you have the most information.
- The system should be fairly tolerant of forecast errors.
- Bet size shouldn't depend on arbitrary factors such as what happened with your last bet – play each bet on its merits.
- Maximise returns, subject to managing risk.
- You don't want volatility of your betting book to be too great.

A system that satisfies these criteria is the Kelly formula. This formula was derived by Kelly in the 1950s. The original mathematical derivation is based on Information Theory and uses some assumptions that not everyone agrees with, particularly the assumptions relating to the punters return preferences – technically, their "utility function". So we'll get to the formula in a far less mathematical way, with fewer assumptions about what a punter does or doesn't want, and just use the guidelines listed above.

First, you want to put more money on bets that are more certain. The edge system does this by making the stake size, *s*, proportional to your edge, $d.p_f - 1$.

Second, you want to put more money on less risky bets. The fixed winnings system achieves this by dividing by the potential winnings when you stake $1. This might seem like an odd thing to do but $\frac{1}{d-1}:1$ are the odds of the event not occurring and so is a measure of the risk you are taking on board.

Third, in order to maximise returns you need to utilise your full pool of funds, so any bet size should be proportional to the amount of money, *C*, currently in your betting pool.

Combining these three factors together gives a stake size, *s*, of:

$$s = C\frac{(d.p_f - 1)}{(d-1)} \qquad (4.4)$$

This is the Kelly formula, although possibly in a different form to the way it is presented elsewhere. It favours bets where you have a greater edge, favours more certain bets, and utilises the full amount of capital at your disposal.

The issue that hasn't been addressed by this system is the volatility of the pool size and this is one of the main objections to using the Kelly system. One way to reduce this is to use a fractional Kelly system – such as betting half the amount suggested by the system. If your probability forecasts are correct, it can be shown mathematically that this will halve your volatility while maintaining 75% of your winnings. If you are running a portfolio of bets, rather than sequential bets then you automatically run a fractional Kelly system. Rather than trying to tweak the formula, Chapter 6 will look at how to work with these limitations in a practical setting.

No system is perfect but the Kelly system ticks most of the boxes with less drawbacks than other systems.

Pros

- The Kelly system takes bigger bets where your edge is greater which helps to maximise returns.

- The Kelly system takes bigger bets where odds are shorter (probability is higher) which reduces the volatility of your pool size.
- Kelly leads to the highest expected winnings (not the highest possible winnings) and it can be shown mathematically that no other system has a higher expected outcome. This is "expected" winnings in the mathematical sense of the word expected – the weighted average of all possible outcomes multiplied by their probabilities.

Cons

- The Kelly system can lead to a volatile bank balance. Anybody who has implemented this formula exactly as shown above would understand that it can be a pretty exciting ride. As was mentioned, some punters recommend using a fractional Kelly system where you bet a fraction of the amount recommended by the Kelly formula. This will lead to lower profits but less volatile outcomes. Betting more than the amount suggested by Kelly is never a good idea – it will produce a worse outcome with more volatility. In extreme cases it can wipe you out very quickly.

4.4 Practical issues with Kelly

What is the most important thing when choosing how much to bet? Is it the greatest expected outcome? What about stability in the size of your betting pool? Or minimising the risk of going broke? Or some balance between these? The answer depends on the combination of your personal return goals and risk tolerance.

The Kelly system gives the highest expected outcome but is volatile and needs to be adapted for use in practice. Equation 4.4 essentially assumes that you have a series of bets, one after the other but this is unlikely to be the case unless you're playing cards. Some punters playing longer term bets may have hundreds or even thousands of bets on at the same time. There is the complication of correlation between some of these bets which the simple formula doesn't deal with. The practical modifications to the Kelly system suggested here have the added benefit that they reduce some of the excess volatility that the system can cause.

The proposed adaptation reduces the problem of excessive volatility by diversifying bets. Diversification is a well known concept in investment management but is often ignored by punters, particularly those driven more by adrenaline or intuition. Some in the financial markets also don't like it and call it Di-worse-ification – they argue that you should just put all of your money on your best idea or your best few ideas to maximise your returns. If you have perfect foresight then this is excellent advice but in financial markets, just as in betting, there are many unforeseen events that could cause your best ideas to fail. The argument also misses the point: it is not suggesting you start putting money on bad bets just to diversify – you need to find more good bets and spread your money across these good bets. There are so many bets available that you should always be able to find enough good bets, otherwise just sit on your cash and be patient.

To reduce the volatility of your returns, ideally you want a good spread of bets each week or season. The more markets you bet on, the more you reduce the volatility of your returns, provided those markets are uncorrelated and you have an edge in each of the games you bet on. If you stray away from sports that you know, where you don't have an edge, then diversification won't help. If you follow a sound bet sizing system in a disciplined manner then some weeks you may end up not betting much, if at all. If you feel the need to bet anyway then maybe you are betting for the adrenaline rather than the profit. If you really must bet where you don't have much of an edge then keep it small and save your powder for bets where you have an edge.

The idea behind diversification is simple. If you diversify across a number of bets where you have an edge on each and the bets are

uncorrelated, you can maintain a positive expected outcome while reducing the spread of possible outcomes. That means you reduce the maximum amount that you could make compared with a single bet and perfect foresight, but more importantly you also reduce the amount you could lose and reduce your chance of going broke. The real benefit of this approach is that it reduces the chance that you lose money through randomness – it won't help if you don't actually have the edge that you thought you had. An example of where diversification can help is the completely unpredictable events that can occur, such as where sports administrators take arbitrary action against teams for breaches of various rules.

If you are use a betting exchange rather than a bookmaker there are other practical issues to take into account when implementing a position sizing system based on the Kelly system. When you bet with a bookie you immediately know what your exposure is and what odds you have bet at (provided the bookie accepts your bet at the odds and quantity you want). If you bet on an exchange and cross the spread then you also know exactly how much you are betting. However, if you offer a bet on the market and wait for someone to take it, then you only know your maximum potential exposure. It may be that nobody accepts your odds before the event closes, or someone else offers better odds so you then have to change your offer. This is especially the case in illiquid markets with wide back-lay spreads. Either way, if your offer lapses then you don't have a bet on the event. This may mean that while you offered a diversified range of bets, you may end up with a more concentrated group of bets that have actually been matched. All of these issues can be managed in practice but there is no simple rule or equation that gives you all of the answers.

4.5 Making Kelly work in practice

Once you move away from the theoretical case of successive single bets there is no longer a simple, correct theoretical answer for how to size your positions using the Kelly system. This is even more true if you are using a betting exchange. The ideas here are one suggestion of how to adapt the Kelly system and this is taken further in the next section. There are many variations and alternatives that would work equally as well and you may want to build on these ideas to suit your particular strategy and circumstances. The rationale for the approach presented here is that bet sizes are broadly based on the Kelly system, risk is effectively managed (diversified) and it works in practice. In other words, it should turn an edge into a consistent profit if followed in a disciplined manner. The system will be refined further in the next section to incorporate other practical issues such as whether you can place bets of the size and odds you desire. This addresses the issue known in the finance industry as the liquidity of the market.

Standard Bet Size

When you have multiple bets on a market simultaneously you can't use the Kelly formula with the multiplier *C* equal to your entire pool as this may suggest betting more than you have in your pool. Rather than staking the Kelly percentage of your total bank, the concept of a *Standard Bet Size, S*, is introduced to ensure your total bets are less than your total pool. Note that as *S* is less than your total pool, this is a fractional Kelly system which means it has lower volatility compared with the full Kelly system, and also lower returns.

Where you want to bet across a number of different markets, you can divide your pool into a subpool for each market. This reduces the risk of blowing up your whole pool, regardless of which position sizing system you are using. Betting on uncorrelated pools in this way is a powerful way to reduce risk in your portfolio. The greater the number of markets and bets that risk is spread across, the smaller fraction is used on each bet, and the less volatility there will be in your betting pool. Of course, this assumes that you have an edge on each bet. There will still be volatility as many bets don't work out but the potential for a large negative impact from a single bad bet is dramatically reduced.

Selecting the appropriate standard bet size is a mixture of science and art. You can do some initial calculations to determine the optimal size but you may end up refining this based on your experience. As an example of where you might choose to start, if you are likely to have a maximum of ten bets simultaneously and your bank is $1,000 then you could set your standard bet at $100. In practice you could set it higher than this as you will never end up betting 100% of your standard bet on each of your bets. It may take a little time to find the right level so that you are fully utilising your capital. If you are unsure, it is better to start low and build up until you find the right level. If you are betting on games each week then you may find some weeks you don't bet much if the odds are close to your forecasts. This is a good thing as it means that you are putting less on the table where the opportunities are less attractive, and saving it for better odds in the future. There is a natural calibration of bet size that occurs: if the market odds are about right (or about the same as your forecasts) then you place only small bets, conserving your capital for later when your view differs from the market and

you have a greater edge. If your forecasts are very different to the market then you bet a much larger percentage of your pool. This means you are automatically sizing your bets to reflect your edge. Ideally you would like to arrive at a value for *S* so that you bet most of your pool in weeks where there are substantial mispricings. If you never end up betting all of your pool, or at least most of it, then you may as well take the part of your pool that you never use and stick it in the bank. If you run out of money mid-way through a round then you have set *S* too high and you are gaining the full benefits of diversification.

Once you have chosen *S*, equation $s = C\dfrac{(d.p_f - 1)}{(d-1)}$ **(4.4)** can be adapted to get the formulas for the amount you should put at risk when you back or lay:

Back amount = $S(d.p_f - 1)/(d-1)$ (4.5)

Lay amount (at risk) = $S(1 - d.p_f)$ (4.6)

As before, if the suggested back amount from this formula is negative it means you don't have an edge so you shouldn't bet. Similarly for the lay amount, if the amount is negative then don't lay a bet.

The lay amount given here is the amount that you are putting at risk, that is, the amount you have to pay the punter if you lose. This is not the same as the amount that you need to lay. To get the number

of dollars that you need to lay to put this much at risk, you divide the above amount by (d - 1) giving:

$$\text{Lay amount} = S(1 - d.p_f)/(d - 1) \qquad (4.7)$$

If the back and lay amounts are close to zero this means the odds are about fairly priced and you may be best leaving it and looking for a bet where you have a clearer edge.

If you are betting on an exchange then you have the choice of crossing and backing at d_{lay} or waiting and offering to back at d_{bet}. You can calculate the back amounts for both odds values depending on whether you decide to cross or wait. We will discuss whether you should cross or wait more in Chapter 5.

Adding to a bet

There are times that you may want to add to an existing bet or lay. This may happen, for example, where you have placed a range of bets in a market but wish to increase your exposures further. In determining the size of your additional bet, it doesn't matter whether the forecast or market odds have changed, the only factors that influence the size of the additional bet are the current forecast and market odds, and your current risk exposure, r. The *Risk Contribution* is defined as $r = w - l$ where w is the amount that we win if an event occurs and l is the amount we lose if the event doesn't occur.

The individual previous bets or expected return don't influence the additional bet size other than through the impact they have on

your risk exposure. The formulas for back and lay amounts from above are then adjusted for the amount that you already have at risk.

Back amount = $S(d.p_f - 1)/(d-1) - r/d$ (4.8)

Lay amount = $S(1 - d.p_f)/(d-1) + r/d$ (4.9)

Kelly Formula in terms of probabilities

The Kelly formula can also be represented in terms of forecast and implied probabilities. While this doesn't change the amount you would bet, it is another way of thinking about the bet sizes that come out of the Kelly formula which and this give you more insight in some cases.

Back amount = $S\dfrac{p_f - p_d}{1 - p_d}$ (4.10)

Lay amount = $S\dfrac{p_d - p_f}{p_d}$ (4.11)

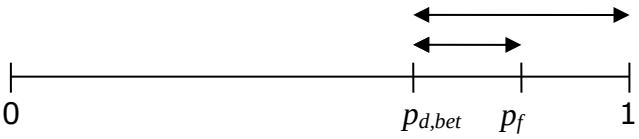


Figure **4-1** Kelly formula in terms of probabilities.

The chart above gives a visual interpretation of the position sizing formula for backing a bet. In this example the forecast probability is greater than the implied probability from the available odds. This means we should bet. The amount to bet is given by the ratio of the length of the short line to the length of the long line in the chart.

What does the chart say about the factors that impact position size? Bets will be larger when p_f is close to 1. The biggest back bets will occur when $p_{d,bet}$ is close to 0 and p_f is close to 1. That is, when the odds are long but we believe the event is very likely.

In practice bet sizing is much more complex. Your forecast probability, p_f, is an estimate and in some cases small differences in this estimate can make a big difference to the suggested bet size. Where p_d is small and p_f is close to p_d then bet size is very sensitive to your estimate. As an example, if p_d is 0.01 and p_f is 0.02 has an expected return of twice your outlay but even a small change in your forecast means that you might not want to bet at all. It's important to consider the impact of estimation error on your system as sometimes bets that look great at first glance are best left alone. This is discussed further in Section 6.6.

The formula for position size for lays can be viewed in the same way as for bets. Lays will be larger when p_f is close to 0. The biggest lays will occur when $p_{d,lay}$ is close to 1 and p_f is close to 0. That is, when the odds are short but we believe the event is very unlikely.

5. Capital management

The Kelly formula will get you a long way but there are also many practical issues that must be managed when you are betting. One issue is that you have finite capital to bet with but an almost endless array of bets to choose from.

This brings us to the second part of managing your portfolio of bets: capital management. At any point in time there are thousands of different bets available. Even if you target only a few sports, it's likely that there are still too many bets to bet on all of them at the full amount suggested by the Kelly formula. This was addressed to some extent by setting a standard bet size but the question of how you can most efficiently allocate your capital while also managing your portfolio risk needs further consideration.

If you are betting through a bookie then you can spread your betting pool across those bets with the most attractive odds and guarantee that you can place the bets, remembering that if the odds aren't attractive then the best use for your capital is to find another market or leave it in the bank.

If you are betting on an exchange then the capital allocation decision is more complex in a number of ways. First, you can back or lay bets. A second more important distinction with exchanges is that you can choose to cross the spread and accept odds that are already available on the exchange or you can place an order at odds more favourable to you and wait to see if someone bets against you by accepting your odds. This means that some bets may not get matched before game time and you end up with fewer bets than you wanted. On the positive side, exchange betting provides opportunities to employ a greater variety of betting strategies.

A third and important point is that with exchanges, the amount of capital set aside for unsettled bets is netted off in the case where not all of your bets can lose.

5.1 Cross or wait?

One of the biggest differences when betting on an exchange is that you can either cross the spread and bet at the odds already available on the exchange, or you can offer more favourable odds and wait to see if someone accepts your odds and bets against you. The benefit of the first approach is that you can be certain about how much you will end up betting and what your risk exposure is. The benefit of the second approach is that where someone matches your bet, you get better odds than if you had crossed – but it's possible that nobody matches your odds and then you don't have a bet at any odds. This can lead to the situation where you have put on a selection of bets that would have diversified your risk but if only your biggest bet gets matched then your actual risk is far more concentrated.

The decision about whether to cross or wait is typically influenced by the spread between the current back and lay odds, and by the amount of liquidity in the market. Markets with narrow spreads also tend to have high liquidity and vice versa. If there are a lot of punters in the market then there is a good chance that someone will cross the spread, but if it is an obscure market then it is more likely that a bet will be left unmatched. This is an important consideration as while the bet remains unmatched, a betting exchange sets aside the amount that your bet places at risk. If you only put bets on at favourable odds in obscure markets then you can end up with all of your capital set aside awaiting matches but very few, if any, bets actually matched. This question requires a pragmatic solution and is explored further in

the next four sections – liquidity, spread, total amount bet on a market and maximum at risk.

5.2 Liquidity

The liquidity of a market can be defined as the ease with which you can match a bet of your desired size. In the most popular markets you can typically bet a large amount with ease. Liquidity can vary over time and between runners in the same market. It's not possible to precisely quantify liquidity but there are some measures that help to measure it in practice.

Two quick measures of liquidity are: the amount that has been matched on a market to date; and the amount which is currently waiting to be matched on the market. While these measures have value in most circumstances, they have limitations. The amount matched to date can be less relevant where a market dries up – for example a "season winner" market may lose interest for punters once the outcome is nearly certain. In many markets, as events unfold and an outcome becomes unlikely, odds will move out there will usually be few backers at odds of greater than 50 or 100. The amount waiting to be bet may also not be a good indicator in some circumstances as the liquidity may not be where you are looking to bet, or may be at very unfavourable odds.

In practice liquidity is variable and unpredictable and must be monitored closely. The impact of liquidity on your strategy will depend on the type of strategy that you are trying to implement. For strategies that involve crossing the spread, more liquid markets are generally more attractive as the cost of crossing the spread is lower because the spread is narrow. This is analogous to financial markets where the greatest cost in buying or selling an investment can be the

cost of crossing the spread. It is a cost that can make a strategy that appears profitable lose money. The downside of more liquid markets with narrower spreads is that they are typically more efficient and are harder to make money out of. In the most liquid markets there are usually a number of professional players and you should be wary of betting against them – they are professional for a reason. Depending on your strategy, the markets with good but not great liquidity can provide the sweet spot for profitability. These "goldilocks" markets provide a nice balance between liquidity and efficiency. Obscure, illiquid markets can be difficult to run a systematic betting approach as you will struggle to match enough bets but there are ways of managing risks within these markets.

There is a saying in financial markets that liquidity creates liquidity. The same applies in betting markets - punters will favour markets that are liquid because that is where they can bet a reasonable amount without distorting the market odds. This then attracts more punters to those markets. That is, the liquidity attracts more liquidity.

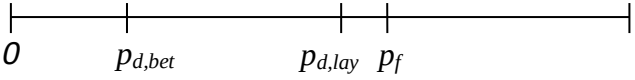
5.3 Spread

If liquidity determines how much can be bet, the spread indicates how much it will cost to bet now by crossing the spread rather than waiting. A simple measure of the spread for an individual runner in a market is the ratio of the odds at which you can bet to the odds at which you can lay. You can measure it for an individual bet as

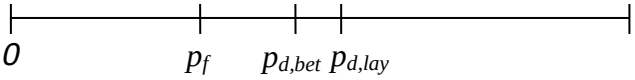
$$\text{Spread} = d_{b,i} / d_{l,i} = p_{d,lay} / p_{d,bet} \qquad (5.1)$$

The Spread must always be greater than 1. If you are using more than one bookie/exchange then you may find examples where it is

less than 1. This means you have found an arbitrage opportunity where in theory you can make a riskless profit – if the odds don't move before you get set on both sides. In practice the spread can be very close to 1 for popular, liquid markets. The spread measure can make things appear worse (or better) than they are. A wide spread doesn't necessarily mean that crossing the spread will give you bad odds as it depends where your probability forecast sits relative to the implied probabilities. The diagram below shows an example where the spread is wide (about 3) but it is still possible to bet at favourable odds when crossing the spread and betting at $p_{d,lay}$ as the market odds are well above the estimate of where they should be. If you are prepared to wait then the odds could be even more favourable, if you are matched.



Similarly, a narrow spread doesn't mean that you will be able to bet at good odds. This diagram shows an example where the spread is tight but you can't bet at favourable odds, although you may be able to lay at favourable odds.



This is helpful in looking at a single runner within a market but typically we are looking at whole markets with multiple runners.

There is an analogous formula for measuring the spread of the market as a whole. The market spread is given by

$$\text{Market Spread} = \frac{\sum_{i=1}^{n} 1/d_{l,i}}{\sum_{i=1}^{n} 1/d_{b,i}} = \frac{\sum_{i=1}^{n} p_{d,lay,i}}{\sum_{i=1}^{n} p_{d,bet,i}} \qquad (5.2)$$

This formula may look a bit intimidating but it is simply the total of the implied probabilities using current market lay odds (the overround for a traditional bookie), divided by the total of the implied probabilities using current market back odds. Note that as with the spread for a single runner, a large spread doesn't mean that you can't bet at good odds. If you are betting only then the important factor is the top line of the equation. If this is close to one (or the number of winners) then this is the same as a bookie having a small overround. The fact that the bottom line of the ratio may be much smaller than one is of no concern to you if you are only betting as it describes the opportunity to lay. Similarly, if you are only laying bets then it is the bottom line of the equation that is of interest. You require that the bottom line is close to one (or the number of winners) and have no interest in the value of the top line.

Market spread is a rough measure of the liquidity of a market: it shows how tight the market is on average across all of the runners and roughly how much you will have to sacrifice if you want to cross the spread to ensure your bet is placed, rather than wait. The individual spread gives you an idea of how tight the market is for an individual bet. Neither of these spread formulas measure the amount available at either side of the spread so they are imperfect measures of liquidity but useful inputs nonetheless. In practice, markets with a

tight spread will typically also have higher amounts available to bet and vice versa.

Betting on illiquid markets can be frustrating and is often not very fruitful as many bets go unmatched. If your favourite sport has little liquidity then you may be better off either using a bookie who offers odds on the sport, or finding another sport to bet on.

5.4 Total amount bet on a market

The total amount already bet on a market by all participants is displayed by betting exchanges and is another measure of a market's liquidity. This is a purely historical measure but is still usually a good guide to how much will be bet in the future. Typically markets that have had a lot bet on them to date will have tighter spreads and these will be the easiest markets to get a bet matched. There may be a point in the season/event where betting dries up and you should consider this before betting. All exchanges provide the amount that has been bet on a market. Bookies don't usually provide this information but that doesn't matter as much in their case as they are, in effect, a liquidity provider.

5.5 Maximum exposure to a market

No matter how good your knowledge of a sport, you can still be wrong. This makes it important to spread your capital across a number of bets and markets, rather than have everything riding on a single event working out the way you want it to. The natural inclination is to put the majority of your capital in the most liquid markets but this can lead to concentrated risk in the most efficiently

priced markets as market liquidity can fall off rapidly outside of the biggest markets.

In order to spread your bets you need to have an accurate measure of your total exposure to each market, that is, the maximum you could lose on the market if *everything* goes against you. Your total exposure is not simply the sum of all of your bets. The algorithm for calculating this amount is given below has a number of steps.

The algorithm is best demonstrated with an example. This example is for the case when there is one winner – such as the winner of a game or the winner of a league or tournament.

The first step in calculating how much capital is set aside is to put the outcome of your bets into a table like the one below. The "Bet wins" column is your outcome if that bet wins (in this case the home team wins) and the "Bet loses" column is your outcome if the bet loses. In this example, the punter has bet on a draw and bet against both a home win and an away win.

	Bet Wins	Bet Loses	Risk Contribution	
Home	-18.00	11.25	-29.25	*
Draw	50.00	-12.50	62.50	
Away	-9.60	6.96	-16.56	
Sum		*5.71*	*-29.25*	
Total at risk			**-23.54**	

The second step is to calculate the Risk Contribution for each bet. This is simply the Bet Wins amount minus the Bet Loses amount for each bet. In this example there is only one winner, as only one of

these results can occur. The biggest risk contribution comes from the largest negative number (or the smallest positive number if all numbers are positive). In this example it is -$29.25.

To get the total risk exposure for the market, sum the Bet Loses amounts which gives $5.71 here. Then add this amount to the smallest (generally the largest negative) risk contribution. This gives a total of -$23.54. In other words, if everything goes against you, the most you can lose on this market is $23.54 which occurs if the home team wins and then you make -$18.00 + -$12.50 + $6.96.

How can this calculation be adapted to the case where there are multiple "winners", such as picking who will finish in the Top 4 positions at the end of the AFL season? The algorithm is the same except that you need to identify the four largest risk contributions and sum them. In the case of the Top 4 in the AFL, you first put all 18 teams into a table with the Bet Wins and Bet Loses data, then calculate Risk Contributions. The second step is to star the four biggest Risk Contributions. The Total at Risk is the sum of the four starred Risk Contributions minus the sum of all numbers from the "Bet loses" column. This is described in mathematical terminology below.

In mathematical terms, consider a market that has n teams and m winners/place getters. We define w_i as the amount that paid by a win if event i occurs and l_i as the amount lost if event i doesn't occur.

Individual Risk Contributions are defined as:

$$r_i = w_i - l_i \qquad (5.3)$$

Set $r_{(i)}$ to be the r_i values sorted from smallest to largest. Then the total Risk Contribution is:

$$R = \sum_{i=1}^{m} r_{(i)} \tag{5.4}$$

The total from losing bets is:

$$L = \sum_{i=1}^{n} l_i \tag{5.5}$$

Then the total maximum possible loss is:

$$\boldsymbol{C = R + L} \tag{5.6}$$

This is the amount of capital you have at risk on the market for all of your bets that have already been matched. Typically *C* will be a negative number as *R* is typically negative and *L* is rarely positive enough to offset *R*. If *C* is positive it means that you have locked in a guaranteed profit regardless of the outcome of the event and the betting exchange should not require you to set aside any capital for this market.

5.6 Capital required

There may also be bets that you have placed that have not been matched. When you are deciding which bets to place, you need to know how much capital is required by the exchange for all of the bets that haven't yet been matched. The approach used by the exchanges is to look at the worst possible outcome and set aside capital to cover that. This is complex as you may have both bets and lays waiting on the same outcome at different odds: how do you allow for this? There is a simple algorithm to calculate the worst possible outcome which can be fairly easily set up in a spreadsheet.

The algorithm is an adaptation of that which is used to calculate the amount of capital at risk for matched bets. As before, there are n teams and m winners/place getters. We define w_i^* as the amount that you win if event i occurs assuming any lays that you have waiting are matched. This means that in each case $w_i^* \leq w_i$. Similarly, l_i^* is the amount that you lose if event i doesn't occur assuming any ebts you have waiting are matched, so $l_i^* \leq l_i$.

We then define individual Risk Contributions for each event as:

$$r_i^* = w_i^* - l_i^* \qquad (5.7)$$

Set $r_{(i)}^*$ to be the r_i^* values sorted from smallest to largest. Then total risk contribution is:

$$R^* = \sum_{i=1}^{m} r_{(i)}^* \tag{5.8}$$

The total from losing bets is:

$$L^* = \sum_{i=1}^{n} l_i^* \tag{5.9}$$

The total capital allocated for the market (maximum loss) is:

$$\boldsymbol{C^* = R^* + L^*} \tag{5.10}$$

The amount of capital required to be set aside for unmatched bets is $C - C^*$ which is a positive amount if further capital is required to be set aside.

Extending the example from Section 5.55.5, we offer to lay an additional further $15 at odds of 1.40. The Bet Wins value for the Away team, $w_{(3)}^*$, is now -$24.60. This makes $r_{(3)}^*$, the Risk Contribution from the Away outcome, -$31.56.

	Bet wins	Bet loses	Risk contribution	
Home	-18.00	11.25	-29.25	
Draw	50.00	-12.50	62.50	
Away	-24.60	6.96	-31.56	*
Sum		*5.71*	*-31.56*	
Total at risk			**-25.85**	

The maximum total at risk with the additional bet is $25.85, so the further capital required to offer lay this bet is -23.54 – (-25.85) = $2.31.

Placing additional bets does not necessarily increase the amount of capital at risk, and the capital required may be zero. This is a "free bet" in the capital allocation sense and this concept is discussed further in Section 5.8.

5.7 Closing out bets

One of the main benefits of using an exchange rather than a bookie is that you can efficiently close out bets that you have previously made. There are a number of reasons that you might want to close out a bet. You may want to lock in a profit or limit your losses. Or there may be new information that means your view on the fair odds has changed and you no longer want to take this bet.

Closing out a bet means removing all of your exposure to a bet so that your financial outcome no longer depends on the result of the event you are betting on. You will usually end up not completely square and will either lock in a profit or a loss. The main reasons to close out a bet are: to remove risk; to lock in a particular result; or to get your capital back so you can use it more productively elsewhere. You should consider closing a bet when the odds for doing so are favourable regardless of whether you are taking a profit or a loss on the closed bet.

In some instances, it may also make sense to close a bet when the odds are slightly against you in order to reduce your risk exposure and release your capital. This is discussed further in Chapter 6.

To calculate what you need to lay to close out a bet, you use the individual risk contribution r_i from equation $r_i = w_i - l_i$ **(5.3)**. If r_i is positive this means your current net position is that you have a

bet on runner *i*. To close this bet where the available odds are *d* you need to lay an amount of

$$c = r_i / d \tag{5.11}$$

and your profit/loss will be $w_i + c$.

Similarly, if r_i is negative your current net position is that you have a lay on runner *i* and you need to bet an amount of

$$c = -r_i / d \tag{5.12}$$

at odds *d* to give you a profit/loss of $w_i - c$.

5.8 "Free" bets

Most bets that you place will either add to or subtract from your net exposure to a market, and hence to the amount of capital that needs to be allocated. Most bets also increase the total amount that you have bet on a market. There are some bets that increase the total amount that you have bet but do not increase the amount of your capital required to be allocated. These bets still have a cost in the risk sense as they can lose you money but there is no need to allocate additional capital so they are "free" in the sense of requiring a capital allocation. In effect they are making your money work harder – you are placing multiple bets using the same amount of allocated capital. You can also use a variation on this idea to free up capital to invest elsewhere if you can reduce the capital allocated to a market.

To understand how this concept works in practice, refer back to the formula for the maximum exposure to a market described in Section 5.5. If you reduce the risk contribution, *R,* or increase the sum of bet losses, *L,* or some combination of the two then you can reduce your total capital allocated to a market. The idea is to equalise,

as far as possible, the risk contribution from each runner. In general the more you equalise the risk contributions from different runners, the more efficiently you can use your capital.

Using the example in Section 5.5, if you back an additional bet on a home win then you can reduce your maximum loss on the market by as much as $12.69 (= $29.25 –$16.56).

To understand how this works, first look at your lays. You can reduce your maximum potential loss by laying any runner that is not included in the *m* lowest ranked risk contributors. For an event with a single winner, look first at the bet with the largest risk contribution, $r_{(1)}$. You can increase the risk contribution from all of your other bets to equal $r_{(1)}$ without increasing your Total at Risk. In fact, because you are laying bets, *L* will decrease by the total amount of your new lays and this will decrease your maximum potential loss, *C*. If you lay

$$c_i = \frac{(r_i - r_{(1)})}{d} \tag{5.13}$$

for each *i*, you get the greatest possible reduction in your capital allocation from an additional lay on this runner. Your total at risk on the market reduces by $r_i - r_{(1)}$ and your expected return changes by $c_i(d.p_f - 1)$.This reduces your risk but does not necessarily increase your expected return.

As a variation on this idea, if you lay

$$c_i^* = \frac{(r_i - r_{(1)})}{(d - 1)} \tag{5.14}$$

then your total at risk on the market remains unchanged. This is in effect a “free bet” in a capital allocation sense as it does not change

the amount of capital that is allocated to the market. In this case, your risk on the market remains unchanged and your expected return changes by $c_i^*(d.p_f-1)$. Any amount you lay beyond c_i^* will increase your maximum potential loss on the market relative to your current position, and hence increase the amount of capital allocated to this market. If this lay has a positive expected outcome you are getting an increased expected return for no increase in risk – possibly even a decrease in risk.

The same theory applies for backs and can be applied to all runners that are included in the m lowest ranked risk contributors. Look at the largest negative (or smallest positive) Risk Contribution, $r_{(i)}$, that is not currently included in your maximum exposure calculation, in the case of a single winner it is $r_{(2)}$. Any bet that lowers the included Risk Contributions towards this amount reduces your total exposure. The maximum reduction from betting on runner i is achieved by betting

$$c_i = \frac{(r_i - r_{(2)})}{d}. \qquad (5.15)$$

This reduces your maximum loss by $r_i - r_{(2)}$ and changes your expected return by $c_i(1-d.p_f)$, which will be an increase if you have bet at favourable odds.

Any amount you bet beyond c_i will increase your potential maximum loss on the market.

Calculation of these amounts is fiddly and is best automated using a spreadsheet or other software. The use of a spreadsheet can be helpful as it can also highlight risk reducing bets that you otherwise may not have identified.

General Formulas for total at risk

The general formulas where there are m winners are presented here. For lays, to get the maximum reduction in allocated capital, lay

$$c_i = \frac{(r_i - r_{(m)})}{d} \text{ for } i > m \tag{5.16}$$

and to get a "free bet" in the capital allocation sense lay

$$c_i* = \frac{(r_i - r_{(m)})}{(d-1)} \text{ for } i > m \tag{5.17}$$

For backs, to get the maximum reduction in allocated capital, bet

$$c_i = \frac{(r_i - r_{(m+1)})}{d} \text{ for } i \leq m \tag{5.18}$$

6. Bringing it all together

6.1 Portfolio management & practical issues

This chapter pulls together all of the material presented so far and provides a guide to implementing the staking system in the real world. It looks at common issues that arise with the system and what you might do to address them. Most markets suffer from mixed liquidity and the chapter also covers this and other practical issues confronted by punters when betting on exchanges.

6.2 Risk tolerance

It is important to understand the theory of risk management but it is also critical that you understand your own tolerance for risk. Psychology is very important in betting (and investing) and if you feel uncomfortable with the volatility in your betting returns it will impact on your ability to maintain the discipline necessary for a successful betting system. Maintaining a disciplined approach to position sizing when you have lost your nerve, or alternatively when you are feeling invincible, is very challenging. All punters have great runs where they feel unbeatable and believe they may just have discovered the secret to free money. And all punters also have periods where nothing goes right and they feel like giving up. Never underestimate the impact of luck on your short term returns. Ensuring you operate within your risk tolerance will help you stay focussed.

Everyone goes into betting hoping to win or at least break even but many people don't. So how much are you prepared to lose? How do you feel when you lose half of your book? These considerations and others feed into the determination of your standard bet size

which in turn determines how fast you could lose your money. You need to adapt your system so that you are comfortable with the volatility.

6.3 Choosing your standard bet size

There are a number of factors to take into account when you are determining your standard bet size. If you are betting on multiple markets it doesn't need to be the same between sports, leagues or even individual markets within a league. It can also increase over time. There is no theoretically correct answer for this question and your final choice will be a pragmatic decision. Here are some factors to consider in determining the size of your standard bet:

Your total pool

An obvious upper limit to your standard bet is determined by the amount in your account. If you plan to bet on each of the ten games in an English Premier League round then on average you can bet a maximum of 10% of your pool on each game. In practice many games are priced close to your forecast odds so if you the sizing system presented in this book it doesn't make sense to bet the same percentage of your pool on those games. Even in games where you bet, you are will rarely go close to your limit on an individual game when using this system. You may choose to set a standard bet at 25% or even 50% of your pool. If you set the standard bet at a higher level then you may need to scale it back some weeks where there are a lot of attractive opportunities.

If you are betting on multiple leagues then you may want to start by dividing the pool up between the different leagues, then dividing

it up within each league. Refining your approach to this comes with experience of what works best for you.

Confidence

Another consideration is your degree of confidence in your forecast. Is this a market where you have a clear edge? How confident are you in your forecasts? Do you have past experience betting on this market? Have you fully researched or are you still learning about the market?

Where you are more confident of your forecasts, you should bet larger sums. For markets where you are less confident, or if this is your first foray into a new market, you may want to start with a small standard bet size and build up from there.

Liquidity

Exchanges have practical limits on how much you can bet that depend on how many other punters are also playing in the market. Some markets have very little liquidity and this limits the amount you are able to bet, or at least the amount that you are able to bet at favourable prices. In these markets, it is best to start with small bets and build up for both risk management and capital management reasons. The risk management aspect is discussed in the next point on diversification. The capital management issue is that you will be placing a large amount of bets on the market which will be waiting for someone to take the other side of the bet, especially if your standard bet is large. As this market is illiquid, you are likely to end up with very few bets matched despite having allocated a lot of capital to the market while it was live.

A lack of liquidity can also be a danger sign, depending on your strategy, as it will make it harder to place more bets later if you want to diversify your exposure or close a bet.

Diversification

Diversification is closely linked to liquidity, as your ability to diversify in a market is impacted by the market's liquidity. To maintain diversification in your portfolio, you may choose to build up your bets in stages. For example, if you are aiming to have a standard bet of, say, $500 on the winner of the Premier League and you place bets based on this standard bet size then it's possible that you end up with only one $500 bet matched giving you an undiversified total exposure. Alternatively, you can build up your standard bet size gradually. For example, you could initially set your standard bet at $100, then when you have a good spread of bets at this level and you have a reasonably diversified portfolio, you increase the standard bet to $200. As your bets are matched and your portfolio has reasonable diversification, you can repeat this until you get to your target standard bet size – or until your capital is all allocated.

Combining these factors is a matter of experimentation and depends on the types of markets you play, the strategies you use and how these interact. If you are looking only at individual games you should get a feel for setting your standard bet after a few weeks. If you are betting on season bets such as the winner of the Premier League then it may take longer to get the level right so you may wish to start with a cautious approach. It is also worth noting that this approach to sizing season bets soaks up more money as the season goes on so building up your standard bet size gradually helps to

ensure that you have capital available throughout the season. If in doubt, start below where you think your standard bet should be and build up to it.

Cut your losses, move on

It's important that your capital is employed where it can earn the highest return. If one of your current bets is going bad and unlikely to turn around then the best thing to do is cut your losses, escape with some of your money intact, and use that capital more productively elsewhere. When a bet is going against you it is usually better to take a small but painful loss now rather than the even more painful 90% (or 100%) loss later on. Learning to cut losers is one of the best lessons that you can learn in betting and something that good traders in the financial markets know how to do. Financial markets traders will often have stop losses – where they automatically sell anything that drops, say, 10% in price. This limits the loss and stops them from falling into the trap of justifying to themselves why they should continue to hold it only to watch it continue to fall. Stop losses can also make sense in betting markets, depending on the strategies you are using, and you should consider adapting the idea to work with your strategies.

6.4 Time value of money

This section is really only relevant if you bet on longer term markets such as season outcomes. Some bets are nearly certain and have correspondingly low odds. For example, Manchester United may be clearly top of the league table half way through the season and looking strong. You could bet on them finishing in the Top 6 but you are only getting odds of 1.01 at best and it is four months until the bet settles. Even with such small odds, this could be a well priced bet if you believe the outcome is virtually certain. Some position sizing systems will recommend a large bet if you have a very high forecast probability. The catch is that a 1% return in four months is a

poor return on your money and you could probably do better by pulling the money out of your bookmaker's account and sticking it in the bank. Money has a time value – a rate of return that you can earn with no risk.

This is a purely practical issue, but you may decide to set a minimum level of odds which you won't bet below (and a maximum level of odds which you won't lay above). Of course, if the bet with odds of 1.01 is settling tomorrow then you don't need to worry about the time value of money – just assess it on its merits as you would for any other bet. So your minimum odds cutoff will vary depending on the time frame of the bet and if you only ever bet on next weekend's game then you probably don't even need to consider this issue. There are adjustments to the Kelly formula available to take the time value of money into account. In practice these tell you little other than you need better odds if you're betting far into the future. It is easier just to adjust the lower limit on your betting odds depending on the time frame and this achieves a very similar result.

The formula taking time value into account is

$$s = \frac{S(d.p_f - (1+r)^t)}{(d - (1+r)^t)} \qquad (6.1)$$

where r is your cost of capital (the risk free annual interest rate that you can earn) and t is the fraction of a year until the bet settles. Where $d.p_f < (1+r)^t$ you shouldn't bet as you would be better leaving your money in the bank.

Taking money off the table

The other side of the time cost of money is that once a bet is almost certain, you aren't going to make any more out of it and it is effectively costing you money. So if there is still a reasonable time until a backed bet settles then you can lay it off to release your capital allocation. For example, if a team is two wins clear at the top of the table with three games to go, it may be possible to close the bet out by laying at 1.01, releasing 99% of your capital up to three weeks earlier than you would otherwise have had access to it. The true probability is possibly even lower than implied by the odds of 1.01 so you are giving up a small amount of return but you are now able to use the capital more profitably than earning the last 1% of return. If this is a long term bet with time to run and you can earn more than 1% elsewhere in the same time that it takes the bet to settle then you should close the bet at 1.01. Even if it is relatively short term, the same logic applies – take a small clip off your profit and use the capital more effectively elsewhere. This rule can be applied more generally: once the odds for a bet get close to fair, then you don't expect to make any further profit so you can close it and use your capital elsewhere.

If you close out a bet at odds equal to your current forecast odds then you are locking in a return (positive or negative) equal to the current expected return of your bet but less than you could make if the bet pays off. If you close out a bet at odds better than your current forecast odds then you are increasing your profit and freeing up capital that you can put to use elsewhere.

The concept of taking a profit applies to both bookies and exchanges but is much harder and more expensive to implement through a bookie unless you are betting on a two horse race. The ability to simply and cheaply take money off the table – either

locking in a profit or cutting a loss – is a big argument for using an exchange. This is discussed further in Section 6.8.

6.5 Optimising risk/return

As mentioned earlier, the Kelly system in its pure form can lead to an uncomfortable level of volatility in the size of your pool. Placing a number of independent bets at the same time can substantially reduce the high volatility of the Kelly system without necessarily reducing your return significantly.

The Kelly system is really designed for sizing bets when you are placing a single bet at a time and in its simplest form is ideal for card games. The system could be adapted to give you a theoretical solution for use where you have many concurrent bets that are all put on at the same time. This has been done for financial markets where liquidity is high but cannot be readily applied in most sports betting markets which have far less liquidity. The more complex your portfolio of bets is, the more you have to use Kelly as a guide rather than a neat solution to position sizing. It is also hard to adapt where you have placed both short term and long term bets concurrently. If you are using an exchange and don't cross the spread then your actual bets are typically unknown at the time you place the bets and will depend on which of your bets are matched making it even more difficult to apply a simple Kelly system.

A more pragmatic solution is needed. An approach outlined in Section 4.5 is to set a standard bet size for each market then multiply this by your Kelly percentage. Allocating capital between markets can be based on the liquidity of each market, the spread of each market, your ability to diversify within a market, your desire to diversify between markets or any other factor that makes sense for your system. This approach has a number of advantages. You can put bigger bets on markets that are more liquid or where you have

greater confidence, and use a small standard bet size if you are testing out a new market. If you want to control your risk, you can build up your standard bet on each market, particularly where you are offering bets rather than crossing the spread. This approach ensures you have a reasonable spread of bets at the current standard bet size before increasing it and adding to your largest bets. Ideally you want to keep the amount that you bet reasonably even across runners in a market to avoid concentrating risk in a single runner.

6.6 Allowing for error in your forecasts

One thing which is certain is that all of your forecasts contain some level of forecasting error and some may be materially wrong. So what does this mean for your betting strategy and in particular for position sizing in practice? If your forecast probability is wrong then your bets will be either too big or too small and this is likely to reduce your returns and possibly also increase your risk in some circumstances.

If your system works *on average* and you have a diversified portfolio of bets then the risk created by forecasting errors will be lower at the portfolio level. If your errors are random then sometimes errors will work against you and sometimes they may work in your favour.

A second issue is that if you are using the Kelly system then you don’t want to bet an amount above the Kelly stake based on the (unknown) true probability of the event. This will lead to higher risk and reduced expected return. If you are using a full Kelly system then an error in your forecast could lead to you betting above the Kelly stake. Where your bets size is relative to the standard bet size rather than your total pool, you always bet less than the Kelly stake

as a percentage of your total pool, often much less. This means that if you have overestimated the probability, it is likely that you will still be betting less than the full Kelly stake and so you will not be in the zone with higher risk and lower return.

Sensitivity analysis

Sensitivity analysis is frequently used in the financial services to assess the impact if an assumption, estimate or forecast is wrong. In a sensitivity analysis, you redo your analysis using forecasts for each parameter that are a small amount higher and a small amount lower, for example, how does a small change in the level of forecast sales impact a company's value?

Sensitivity analysis can be applied in a similar way to sports betting. What if your probability forecast is out by 1%? Or 5%? If your team rating is wrong by a small amount then what impact would it have on your bet size? It may be that you still run with your original bet size but you will better understand the impact of any errors in your system. By conducting a sensitivity analysis on each of your key variables, you may find that some factors are more important drivers of risk and return, and you may choose to stick to more robust bets that are less dependent on small variations in your assumptions.

If you "stress" your probability forecast you will notice that the impact varies depending on the context. In particular, where the implied and forecast probabilities are both small, the recommended bet size (and even whether you should bet) is extremely sensitive to small changes in the level of these probabilities. Where the probabilities are large, there is much less sensitivity to changes in the

probabilities. The best way to understand the sensitivity to parameters for the types of bets that you favour is to put the formulas from Section 4.4 into a spreadsheet and vary each of the numbers, noting the impact. This analysis may lead you to tone down your bet sizes where a small forecast error could make a bet much riskier.

6.7 Correlation of bets

An issue that sometimes arises in financial markets is where an investment manager thinks they have a diversified range of positions but then finds that when it matters most, their positions are actually highly correlated. In financial markets this is even more likely to happen when you don't want it to through "contagion" where a problem in one market spreads to other markets via the psychology of market participants and capital flows. Most financial markets change their overall level based on a general risk premium which means that in down markets, even the best investments lose in an absolute sense. Where the risk premium changes rapidly, all financial markets tend to move together. Where risk tolerance is high (that is, the risk premium is low) or there is lots of money entering the system, markets all tend to rise together.

Betting markets have many similarities with financial markets but there is an important difference: the overall level of betting markets can't rise and fall like financial markets. In betting markets the probabilities always have to add to 1. For this reason, betting markets can't crash across the board like financial markets – although the odds on an individual team can certainly collapse when something unexpected happens. Sports betting markets can "dry up" if liquidity is withdrawn from the system: there may be nobody for you to bet against, market spreads will widen and bookmakers may demand a

higher risk premium (overround). In this situation it will be harder to place bets and harder to diversify your bets or close out bets as you had planned.

It is also possible that your bets within a sport or market are more correlated than you think. This can be the case if, for example, you are betting on both the Winner and the Top 2 of the Premier League – clearly the outcomes of these two markets are dependent on each other, so bets placed in the different markets will be correlated. Typically the main source of correlation between your bets will be through bets on different markets within the same sporting league.

In most sport betting situations it is less clear how correlation may arise across sports or leagues but you still need to consider it. If you bet on the ten premier league games each week then bets on individual games should be uncorrelated with each other. However, if you bet on Manchester United winning the premier league and also on them finishing in the Top 2 then these bets are clearly related. You need to think about whether there is any way in which your bets could be correlated. Could they be correlated in a particular scenario, even if it is highly unlikely? It's important to explore and understand this as it will enhance your understanding of your risk factors.

While it would be possible to analyse the combined impact of all of your bets on all of the markets where you have bet, this is a very sophisticated approach which would require considerable effort and is unlikely to be worthwhile. An example where it might pay to look across markets is if you were betting on each of the winner, Top 2 and Top 3 in the English Premier League. This means that essentially you can bet on each position in the league by, for example betting that a team will finish in the Top 3 but laying them finishing in the Top 2. This is analogous to a strip trade in interest rate markets. You

may also find inconsistencies between these markets if you calculate implied probabilities from each of the strips but this is getting complex and is for the serious punter. A simpler and more pragmatic approach is to keep an eye on where you are betting on the same team doing well in a number of similar markets and constrain your total exposure. Better still, if there are a number of similar markets then just choose the one whose characteristics suit you best and ignore the others.

6.8 Taking a profit (or a loss)

In betting, as in financial markets, the ideal situation is to lock in a profit regardless of how future events unfold. The challenge in both cases is that you can lock in a small profit early, or you can wait and potentially lock in an even bigger profit later but with greater risk that things go against you. How do you decide when to take your profit?

The same question applies to losing bets. Every participant in financial markets has made a loss but the smart ones are good at knowing when they have got things wrong and cut their losses before things get worse. Some traders in financial markets use a stop loss limit where they automatically sell an investment once it falls by a set percentage regardless of why it has happened. They then review the situation and, if it is justified, may put the same bet back on.

The reason for doing this is psychology. You can become too convinced that your view is right and too wedded to the bet. Closing out a bet limits your losses before they get worse and recoups some of your capital. Putting a bet back on is then a separate decision where you consider whether you would place the bet without the complication of any current positions. You should get used to losing

bets as it will happen very often. You need to identify any lessons that can be learned from the bet, take the loss, then find your next opportunity. Put simply, you close out your losses and move on to more profitable opportunities rather than wasting time trying to fix problems that have occurred in the past, or hoping that a miracle will save a bad bet. This can be difficult to do in practice but mastering the art of acknowledging a loss, closing the bet and moving on will actually reduce your losses and focus your remaining capital where it can have the most positive impact.

There will be occasions where, due to moves in the available odds, you are able to lock in a profit regardless of the outcome of an event. Let's start with a very simple example of how this can happen. Several days before a game the odds are Team A: $2.10, Team B: $1.80 and you bet $1 on Team A. The day before the game Team B loses their star player to injury and the odds shift to Team A $1.80, Team B: $2.10. You now bet $1 on Team B. Whichever team wins you are guaranteed a profit of $0.10 (you get the $2.10 winnings and you have bet $2). This is a certain profit of 5% of the total amount you have bet. Situations where you can lock in a profit are common but the odds could just as easily move against you.

So if you see an opportunity to lock in a profit, should you do it? When should you not lock in a profit? And are there situations where you should lock in a loss? (The phrase "limiting a loss" sounds less painful and is usually a more accurate description of the situation.)

There are two reasons why you should consider closing out a bet at either a profit or a loss:

1. there is new information about the event which has changed your forecast of the probability of the event occurring; or

2. there is no new information but the market odds have changed.

It doesn't matter why the opportunity to close a bet has arisen, the rule of thumb for locking in a profit or loss is the same. If you have bet at above fair odds and you can now lay at below fair odds then, all else being equal, you should do so. This approach applies regardless of whether you are locking in a profit or a loss. Locking in a profit can be done as in the example above – you make a profit regardless of the outcome of the event. Locking in a loss is exactly the same but you make a loss regardless of the outcome of the event.

Why would lock in a loss? It's better to think of it as limiting the downside if the bet goes against you but sacrificing any upside. Is it better to have a guaranteed loss of $100 or a probable loss of $400? The reason you would lock in a loss is because new information has come to light which has changed the fair odds. Usually you will be closing out a bet when it has already started to go against you and you are simply containing any further damage. Assuming you are laying the bet at below where fair odds now sit, your expected result actually improves (but remains negative). The point of this approach is to behave rationally and reject the irrational hope that a very unlikely outcome will occur and save you.

6.9 Chasing odds and getting whipsawed

The odds that you see on markets are based on what market participants *expect* to happen in an event. When unexpected events occur, such as very long winning/losing streaks then odds for a runner can move dramatically. While unlikely events can make your odds wrong, biases in your forecasts can also lead your results to be wrong.

Where there is a bias in your forecasts, a team may always look expensive – that is the odds may always be too short. For example, you may never end up betting on the favourite winning the premiership because the odds look too short at the start of the season and as they keep winning the odds continue to look to short. With the benefit of hindsight, the odds at the start of the season start to look very attractive but you have missed the boat.

The opposite problem can also occur. This is where a team goes through extremes of form during a season and you overreact to the changes in form. You can end up getting whipsawed by betting at long odds on a weak team, then laying at low odds after the team has had a strong run, then seeing the team have a weak run and the odds getting longer again. The problem is, you never know when it is just a lucky run and when it is a sustained change. You can end up locking in losses several times over a season. This is very frustrating when it happens, so what can you do about it? Experience will help you to identify situations where this is likely to happen. This is a place where you may want to use a subjective override. You can't avoid losing bets some times and this is often one of those situations.

6.10 Software

Good software will make your life much easier. Even if you have a qualitative approach and don't use software to forecast probabilities, the risk management system outlined here is far easier to implement if you use some simple software. The spreadsheet included with this book covers a basic risk management system for betting on season results in a 20 team competition and for betting on a soccer game with three outcomes. The 20 team template can be adapted to any number of teams, a horse race or any other event.

If you want to keep it simple, just adapt the spreadsheet and enter your forecast probabilities for your chosen market. Alternatively you can use this spreadsheet as a basis and expand it further to add in other information that is useful to you. You are really only limited by your spreadsheet skills. For spreadsheet experts and experienced programmers, Betfair provides a spreadsheet template that enables you to pull data directly from the exchange into a spreadsheet via the API. The API is a big labour and time saver and worth a bit of effort to integrate into your spreadsheet if you can. Some other bookies and exchanges may offer similar facilities.

To use the included spreadsheet you first need to set the parameters. What is your standard bet for each market? What is the minimum bet allowed by the bookie/exchange (this varies by currency)? Next you need to enter your probability forecasts for each runner. The forecasts must add to 1 (or to the number of runners). You can either enter the forecast odds for each market directly or you can enter the probabilities for each team finishing in each position and the spreadsheet will then calculate odds for the separate markets. The second approach is only practical for those using a quantitative process.

6.11 Constant evaluation & improvement

No matter how much you know about a sport and how successful you are at betting, it is critically important that you constantly evaluate your track record in detail. There are a number of reasons why this is important:

1. It can highlight what type of bets are making you money and what type of bets are losing you money;

2. It can alert you to changes in the pattern of returns. Markets are dynamic and if others are catching on to the same idea that you have then this tells you that it's time to review your strategy or cut this market;
3. It can highlight patterns over time, for example, you may be more successful at different stages of the season.

If you discover that there are types of bets where you make money and others where you lose money, you can give more weight to your winning strategies and try to fix your losing strategies.

The first step in understanding your strengths and weaknesses is that you need to collect the data on all of the bets that you make. At a minimum, the data you need is the type of bet, the odds, your forecast probability, back or lay, time of the bet. You may want to collect other information, such as why you are making the bet, as it's always easier to collect information as you go along rather than trying to reconstruct it after the event. It also keeps you honest and stops you from justifying bets after the event.

The type of analysis that you conduct will depend on the strategies you are using. The first things to look at are:

- success at different times of the season or times of the week for individual games;
- success in different odds ranges;
- success on backs vs lays;

In order to get meaningful output from your analysis, you need some spreadsheet skills to adapt the basic analysis to your particular strategies and circumstances. If you don't know how to use pivot

tables in spreadsheets, then it is worth learning about them as they will make analysis much simpler. Pivot tables allow you to easily calculate the total or average returns for a range of categories, for example, comparing lays vs bets or comparing long odds vs short odds. You could achieve the same result without pivot tables but it is more time consuming, harder and gives you less flexibility.

Once you have analysed your results, you will get a much better idea of your strengths and weaknesses. And once you know this, you should stick to what you're good at.

7. References

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