algorithm

Joseph Wang

2020/1/30

Algorithm

1. parameters:

- σ^2
- $\boldsymbol{M} \succcurlyeq \mathbf{0}$
- $K > 0$
- $\delta = \frac{\tau^2}{\sigma^2}$
- $\boldsymbol{\theta} = (\boldsymbol{\beta}, \nu, \rho)'$
  - $\boldsymbol{\beta} \in \mathcal{R}^L$
  - $\nu > 0$
  - $\rho > 0$

2. objective function

$$n \log 2\pi + \frac{\boldsymbol{z}'\boldsymbol{R}^{-1}(\boldsymbol{\theta}, \delta)\boldsymbol{z}}{\sigma^2} + \sum_{k=1}^{K} \left\{ \log(\eta_{K,k} + \sigma^2) - \frac{\eta_{K,k}}{\sigma^2(\eta_{K,k} + \sigma^2)} \right\} + (n - K)\log(\sigma^2) + \log|\boldsymbol{R}(\boldsymbol{\theta}, \delta)|$$

3. Estimation

-
$$\hat{\sigma}^2 = \max\left\{ \frac{1}{n - K^*} \left(\boldsymbol{z}^\top \boldsymbol{R}^{-1}(\boldsymbol{\theta}, \delta)\mathbf{z} - \sum_{k=0}^{K^*} \eta_{K,k} \right), 0 \right\}$$

-
$$\hat{M}_K = \left[\boldsymbol{F}_K^\top \boldsymbol{R}^{-1}(\boldsymbol{\theta}, \delta)\boldsymbol{F}_K\right]^{-1/2} \boldsymbol{U} \mathrm{diag}\big(\hat{\eta}_{K,1}, \ldots, \hat{\eta}_{K,K}\big) \boldsymbol{U}^\top \left[\boldsymbol{F}_K^\top \boldsymbol{R}^{-1}(\boldsymbol{\theta}, \delta)\boldsymbol{F}_K\right]^{-1/2}$$

  - $\mathbf{R}(\boldsymbol{\theta}, \delta) := \delta \mathrm{Cov}(\boldsymbol{\xi}) + \boldsymbol{I}_n$
  - $\mathbf{R}(a, \lambda) := \frac{1}{\lambda} \Phi \mathbf{B}^{-1}(a) \mathbf{B}^{-\top}(a) \Phi^\top + \mathbf{I}_n$
  - $\mathbf{R}^{-1}(a, \lambda) = \mathbf{I}_n - \Phi \left(\Phi^\top \Phi + \lambda \mathbf{B}^\top(a) \mathbf{B}(a) \right)^{-1} \Phi^\top$
  - Wendland basis: $\Phi = (\phi(\mathbf{s}_1), \ldots, \phi(\mathbf{s}_n))^\top$
    * $\phi(\mathbf{s}) = (\phi_1(\mathbf{s}), \ldots, \phi_J(\mathbf{s}))^\top$
    * $\phi_j(\mathbf{s}) = \frac{1}{3} \left(1 - \frac{\|\mathbf{s} - \mathbf{c}_j\|}{\Delta_j} \right)^6 \left(35 \frac{\|\mathbf{s} - \mathbf{c}_j\|^2}{\Delta_j} + 18 \frac{\|\mathbf{s} - \mathbf{c}_j\|}{\Delta_j} + 3 \right) I(\|\mathbf{s} - \mathbf{c}_j\| \le \Delta_j)$

3.

$$a^{(k+1)} = \arg\min_a \frac{\mathbf{z}^\top \mathbf{R}^{-1}(a, \lambda^{(k)})\mathbf{z}}{(\sigma^2)^{(k)}} + \log|\Phi^\top \Phi + \lambda^{(k)} \mathbf{B}^\top(a) \mathbf{B}(a)| - \log|\mathbf{B}^\top(a) \mathbf{B}(a)|,$$

4.
$$\lambda^{(k+1)} = \arg\min_{\lambda} \frac{\mathbf{z}^\top \mathbf{R}^{-1}(a^{(k)}, \lambda^{(k)})\mathbf{z}}{(\sigma^2)^{(k)}} - J \log \lambda + \log |\Phi^\top \Phi + \lambda \mathbf{B}^\top (a^{(k)}) \mathbf{B}(a^{(k)})|$$

- Ref: https://asreml.kb.vsni.co.uk/wp-content/uploads/sites/3/2018/02/ASReml-R-3-Reference-Manual.pdf