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MAT 220—Homework 6

1. **Verify that**

$$x = 7t - 8$$
$$y = 2 - 5t$$
$$z = t$$

are solutions of

$$2x + 3y + z = -10$$
$$x + y - 2z = -6$$

for all values of t.

$2x + 3y + z = -10$
$2x = -10 - 3y - z \rightarrow x = -5 - 3/2y - ½z$
$= -5 - (3/2)(2 - 5t) - (1/2)(t) = -5 - 3 + 15/2t - 1/2t = 7t - 8$
$3y = -10 - 2x - z \rightarrow y = -10/3 - 2/3x - 1/3z$
$= -10/3 - (2/3)(7t - 8) - (1/3)(t) = -10/3 - 14/3t + 16/3 - 1/3t = -5t + 2 = 2 - 5t$
$z = -10 - 2x - 3y$
$= -10 - 2(7t - 8) - 3(2 - 5t) = -10 - 14t + 16 - 6 + 15t = t$
$2(7t - 8) + 3(2 - 5t) + t = -10$
$14t - 16 + 6 - 15t + t = -10$
$0t = 0$
Works for all values of t.

$x + y - 2z = -6$
$x = -6 - y + 2z = -6 - (2 - 5t) + 2t = -6 - 2 + 5t + 2t = 7t - 8$
$y = -6 - x + 2z = -6 - (7t - 8) + 2t = -6 - 7t + 8 + 2t = 2 - 5t$
$2z = x + y + 6 \rightarrow z = (7t - 8) + (2 - 5t) + 6 = 2t$
$(7t - 8) + (2 - 5t) - 2(t) = -6$
$0t - 6 = -6$
$0t = 0$
$0t = 0$
Works for all values of t.

2. **Regarding 7z = 9 as the equation 0x + 0y + 7z = 9 in three variables, find all solutions in parametric form.**

$0x_1 + 0y + 7z = 9$
$x = s$
$y = t$
$7z = 9 - 0s - 0t \rightarrow z = 9/7 - 0s - 0t$

3. **Write the augmented matrix for the following system of linear equations. (Hint: Pay close attention to the variables.)**

6x – z = -5
-5x – 6y = -2
-7y – 3z = 3

6x + 0y – z = -5
-5x – 6y + 0z = -2
0x – 7y – 3z = 3

$$\begin{bmatrix} 6 & 0 & -1 & | -5 \\ -5 & -6 & 0 & | -2 \\ 0 & -7 & -3 & | \quad 3 \end{bmatrix}$$

4. Write a system of linear equations that has the following augmented matrix.

$$\begin{bmatrix} \mathbf{5} & \mathbf{2} & \mathbf{0} & | \ \mathbf{3} \\ \mathbf{3} & \mathbf{2} & \mathbf{-4} & | \ \mathbf{9} \\ \mathbf{-2} & \mathbf{-1} & \mathbf{4} & | \ \mathbf{6} \end{bmatrix}$$

5x + 2y + 0 = 3
3x + 2y – 4z = 9
-2x – y + 4z = 6

5. Find the quadratic ax^2 + bx + c such that the graph of y = ax^2 + bx + c contains the points (-1, 11), (1, 5), and (3, 7). Show all work, including a system of equations, augmented matrix, and Gauss-Jordan elimination.

a(-1)^2 + b(-1) + c = 11
a(1)^2 + b(1) + c = 5
a(3)^2 + b(3) + c = 7

$$\begin{bmatrix} 1 & -1 & 1 & | \ 11 \\ 1 & 1 & 1 & | \quad 5 \\ 9 & 3 & 1 & | \quad 7 \end{bmatrix} \qquad \begin{matrix} -\mathrm{R1} + \mathrm{R2} \rightarrow \ \mathrm{R2} \\ -9\mathrm{R1} + \mathrm{R3} \rightarrow \ \mathrm{R3} \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & | \quad 11 \\ 0 & 2 & 0 & | -6 \\ 0 & 12 & -8 & | -92 \end{bmatrix} \qquad \frac{1}{2}\mathrm{R2} \rightarrow \ \mathrm{R2}$$

$$\begin{bmatrix} 1 & -1 & 1 & | \quad 11 \\ 0 & 1 & 0 & | -3 \\ 0 & 12 & -8 & | -92 \end{bmatrix} \qquad \begin{matrix} R2 + R1 \rightarrow R1 \\ \\ -12R2 + R3 \rightarrow R3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & | \quad 8 \\ 0 & 1 & 0 & | -3 \\ 0 & 0 & -8 & | -128 \end{bmatrix} \qquad -\frac{1}{8}R3 \rightarrow R3$$

$$\begin{bmatrix} 1 & 0 & 1 & | \quad 8 \\ 0 & 1 & 0 & | -3 \\ 0 & 0 & 1 & | \quad 16 \end{bmatrix} \qquad -R3 + R1 \rightarrow R1$$

$$\begin{bmatrix} 1 & 0 & 0 & | \quad -8 \\ 0 & 1 & 0 & | -3 \\ 0 & 0 & 1 & | \quad 16 \end{bmatrix} \rightarrow$$

y = -8x^2 – 3x + 16

6. **Determine whether or not the vector x can be written as a linear combination of the vectors u, v, w. If so, give such a linear combination. Show all work, including a system of equations, augmented matrix, and Gauss Jordan elimination.**
   a. **x = (-9, -4, 3), u = (1, -4, -1), v = (-2, 2, -5), w = (-4, 3, -1)**

$$\begin{bmatrix} 1 & -2 & -4 & | -9 \\ -4 & 2 & 3 & | -4 \\ -1 & -5 & -1 & | \quad 3 \end{bmatrix} \begin{matrix} 4R1 + R2 \rightarrow R2 \\ R1 + R3 \rightarrow R3 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & -4 & | -9 \\ 0 & -6 & -13 & | -40 \\ 0 & -7 & -5 & | \quad -6 \end{bmatrix} -\frac{1}{6}R2 \rightarrow R2$$

$$\begin{bmatrix} 1 & -2 & -4 & | -9 \\ 0 & 1 & \frac{13}{6} & | \quad \frac{20}{3} \\ 0 & -7 & -5 & | \quad -6 \end{bmatrix} \begin{matrix} 2R2 + R1 \rightarrow R1 \\ \\ 7R2 + R3 \rightarrow R3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{3} & | & \frac{13}{3} \\ 0 & 1 & \frac{13}{6} & | & \frac{20}{3} \\ 0 & 0 & \frac{61}{6} & | & \frac{122}{3} \end{bmatrix} \frac{6}{61}R3 \rightarrow R3$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{3} & | & \frac{13}{3} \\ 0 & 1 & \frac{13}{6} & | & \frac{20}{3} \\ 0 & 0 & 1 & | & 4 \end{bmatrix} \begin{matrix} -\frac{1}{3}R3 + R1 \rightarrow R1 \\ -\frac{13}{6}R3 + R2 \rightarrow R2 \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 4 \end{bmatrix}$$

3u -2v + 4w = x

b. x = (2, 14, 6), u = (2, -12, -6), v = (4, 10, 6), w = (-5, 13, 6)

$$\begin{bmatrix} 2 & 4 & -5 & | & 2 \\ -12 & 10 & 13 & | & 14 \\ -6 & 6 & 6 & | & 6 \end{bmatrix} \begin{matrix} 6\text{R1} + \text{R2} \to \text{R2} \\ 3R1 + R3 \to R3 \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & -5 & | & 2 \\ 0 & 34 & -17 & | & 26 \\ 0 & 18 & -9 & | & 12 \end{bmatrix} \frac{1}{34}\text{R2} \to \text{R2}$$

$$\begin{bmatrix} 2 & 4 & -5 & | & 2 \\ 0 & 1 & -\frac{1}{2} & | & \frac{13}{17} \\ 0 & 18 & -9 & | & 12 \end{bmatrix} \begin{matrix} -4R2 + R1 \to R1 \\ \\ -18R2 + R3 \to R3 \end{matrix}$$

$$\begin{bmatrix} 2 & 0 & -3 & | & -\frac{18}{17} \\ 0 & 1 & -\frac{1}{2} & | & \frac{13}{17} \\ 0 & 0 & 0 & | & -\frac{30}{17} \end{bmatrix} \quad \rightarrow \text{no solution}$$

c. x = (0, -9, 3), u = (-2, -15, 2), v = (4, 3, 5), w = (4, 12, 2)

$$\left[\begin{array}{ccc|c} -2 & 4 & 4 & 0 \\ -15 & 3 & 12 & -9 \\ 2 & 5 & 2 & 3 \end{array}\right] -\frac{1}{2}R1 \to R1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ -15 & 3 & 12 & -9 \\ 2 & 5 & 2 & 3 \end{array}\right] \begin{array}{l} 15R1 + R2 \to R2 \\ -2R1 + R3 \to R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & -27 & -18 & -9 \\ 0 & 9 & 6 & 3 \end{array}\right] -\frac{1}{27}R2 \to R2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 9 & 6 & 3 \end{array}\right] \begin{array}{l} 2R2 + R1 \to R1 \\ -9R2 + R3 \to R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{10}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 12 & 6 \end{array}\right] \frac{1}{12}R3 \to R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{10}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 12 & 6 \end{array}\right] \frac{1}{12}R3 \to R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{10}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{2}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right] \begin{array}{l} \frac{10}{3}R3 + R1 \to R1 \\ \frac{2}{3}R3 + R2 \to R2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

u + ½w = x

7. **Determine whether the vectors u, v, w are linearly dependent or linearly independent. Show all work, including a system of homogeneous equations and the augmented matrix. You can reference your Gauss-Jordan elimination in the previous problem to shorten the process. If the vectors are linearly dependent, find all solutions to the system of homogeneous equations.**
   a. **u = (1, -4, -1), v = (-2, 2, -5), w = (-4, 3, -1)**

$$\left[\begin{array}{ccc|c} 1 & -2 & -4 & -9 \\ -4 & 2 & 3 & -4 \\ -1 & -5 & -1 & 3 \end{array}\right] \qquad \begin{aligned} x_1 - 2x_2 - 4x_3 &= -9 \\ -4x_1 + 2x_2 + 3x_3 &= -4 \\ -x_1 - 5x_2 - x_3 &= 3 \end{aligned}$$

$$\mathrm{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2s + 4t - 9 \\ s \\ t \end{bmatrix} = \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix} + \mathrm{s}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

$\mathrm{x}_1 = \begin{bmatrix} -9 \\ 0 \\ 0 \end{bmatrix}$ when s = 0 = t, $\mathrm{x}_0 = \mathrm{s}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$ gives all solution to associated homogeneous system

linearly independent

   b. **u = (2, -12, -6), v = (4, 10, 6), w = (-5, 13, 6)**

$$\left[\begin{array}{ccc|c} 2 & 4 & -5 & 2 \\ -12 & 10 & 13 & 14 \\ -6 & 6 & 6 & 6 \end{array}\right] \qquad \begin{aligned} 2x_1 + 4x_2 - 5x_3 &= 2 \\ -12x_1 + 10x_2 + 13x_3 &= 14 \\ -6x_1 + 6x_2 + 6x_3 &= 6 \end{aligned}$$

$$\mathrm{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 - 2s + \frac{5}{2}t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \mathrm{s}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} \frac{5}{2} \\ 0 \\ 1 \end{bmatrix}$$

$\mathrm{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ when s = 0 = t, $\mathrm{x}_0 = \mathrm{s}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} \frac{5}{2} \\ 0 \\ 1 \end{bmatrix}$ gives all solution to associated homogeneous system

linearly independent

   c. **u = (-2, -15, 2), v = (4, 3, 5), w = (4, 12, 2)**

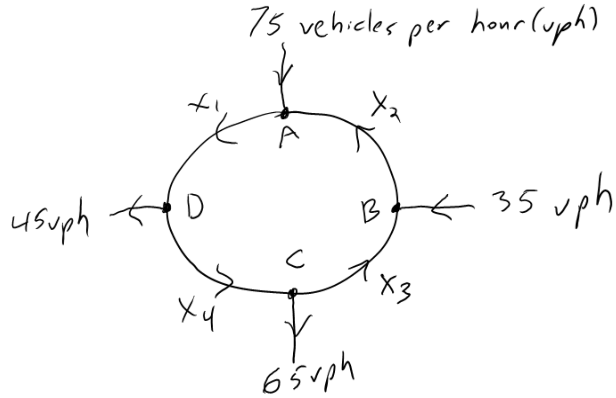
$$\left[\begin{array}{ccc|c} -2 & 4 & 4 & 0 \\ -15 & 3 & 12 & -9 \\ 2 & 5 & 2 & 3 \end{array}\right] \qquad \begin{aligned} -2x_1 + 4x_2 + 4x_3 &= 0 \\ -15x_1 + 3x_2 + 12x_3 &= -9 \\ 2x_1 + 5x_2 + 2x_3 &= 3 \end{aligned}$$

$$\mathrm{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2s + 2t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \mathrm{s}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$\mathrm{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ when s = 0 = t, $\mathrm{x}_0 = \mathrm{s}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ gives all solution to associated homogeneous system

linearly dependent

8. Find the possible traffic flows in the roundabout below. Show all work, including a system of equations, augmented matrix, and Gauss-Jordan elimination.



In Out	
A: $x_2 + 75 = x_1$ →	$x_1 - x_2 + 0x_3 + 0x_4 = 75$
B: $x_3 + 35 = x_2$ →	$0x_1 + x_2 - x_3 + 0x_4 = 35$
C: $x_4 = x_3 + 65$ →	$0x_1 + 0x_2 - x_3 + x_4 = 65$
D: $x_1 = x_4 + 45$ →	$x_1 + 0x_2 + 0x_3 - x_4 = 45$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 75 \\ 0 & 1 & -1 & 0 & 35 \\ 0 & 0 & -1 & 1 & 65 \\ 1 & 0 & 0 & -1 & 45 \end{bmatrix} -R1 + R4 \to R4$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 75 \\ 0 & 1 & -1 & 0 & 35 \\ 0 & 0 & -1 & 1 & 65 \\ 0 & 0 & 0 & -1 & -30 \end{bmatrix} R2 + R1 \to R1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 110 \\ 0 & 1 & -1 & 0 & 35 \\ 0 & 0 & -1 & 1 & 65 \\ 0 & 0 & 0 & -1 & -30 \end{bmatrix} -R3 \to R3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 110 \\ 0 & 1 & -1 & 0 & 35 \\ 0 & 0 & 1 & -1 & -65 \\ 0 & 0 & 0 & -1 & -30 \end{bmatrix} R3 + R2 \to R2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 110 \\ 0 & 1 & 0 & 0 & -30 \\ 0 & 0 & 1 & -1 & -65 \\ 0 & 0 & 0 & -1 & -30 \end{bmatrix} -R4 \to R4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 110 \\ 0 & 1 & 0 & 0 & -30 \\ 0 & 0 & 1 & -1 & -65 \\ 0 & 0 & 0 & 1 & 30 \end{bmatrix} R4 + R3 \to R3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 110 \\ 0 & 1 & 0 & 0 & -30 \\ 0 & 0 & 1 & 0 & -35 \\ 0 & 0 & 0 & 1 & 30 \end{bmatrix}$$

$x_1 = 110$, $x_2 = -30$, $x_3 = -35$, $x_4 = 30$

9. Use Gauss-Jordan elimination to determine whether the matrices are invertible. If the matrix is invertible, give the inverse. Then use matrix multiplication to check that your answer is correct.

a. $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-3} \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 2 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{array}{l} -2\mathrm{R1} + \mathrm{R2} \to \mathrm{R2} \\ -\mathrm{R1} + \mathrm{R3} \to \mathrm{R3} \end{array}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -4 \\ 0 & -3 & -6 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad -\frac{1}{2}\mathrm{R2} \to \mathrm{R2}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & -3 & -6 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & -\frac{1}{2} & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{array}{l} -R2 + R1 \to R1 \\ \\ 3R2 + R3 \to R3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -\frac{1}{2} & 0 \\ 2 & -\frac{3}{2} & 1 \end{bmatrix}$$

I think that if a matrix has a row of 0's or a column of 0's its not invertible. Otherwise I messed up somewhere.

b. $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{3} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{-3} \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} -3R1 + R2 \to R2 \\ -R1 + R3 \to R3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 7 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad \tfrac{1}{2}R2 \to R2$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & \frac{7}{2} \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad R2 + R3 \to R3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & \frac{7}{2} \\ 0 & 0 & \frac{5}{2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{5}{2} & \frac{1}{2} & 1 \end{bmatrix} \quad \tfrac{2}{5}R3 \to R3$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & \frac{7}{2} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & \frac{1}{2} & 0 \\ -1 & \frac{1}{5} & \frac{2}{5} \end{bmatrix} \quad \begin{matrix} 2R3 + R1 \to R1 \\ -\frac{7}{2}R3 + R2 \to R2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & \frac{2}{5} & \frac{4}{5} \\ 2 & -\frac{1}{5} & -\frac{7}{5} \\ -1 & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} -1 & \frac{2}{5} & \frac{4}{5} \\ 2 & -\frac{1}{5} & -\frac{7}{5} \\ -1 & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} (1*-1)+(0*2)+(-2*-1) & \left(1*\frac{2}{5}\right)+\left(0*-\frac{1}{5}\right)+\left(-2*\frac{1}{5}\right) & \left(1*\frac{4}{5}\right)+\left(0*-\frac{7}{5}\right)+\left(-2*\frac{2}{5}\right) \\ (3*-1)+(2*2)+(1*-1) & \left(3*\frac{2}{5}\right)+\left(2*-\frac{1}{5}\right)+\left(1*\frac{1}{5}\right) & \left(3*\frac{4}{5}\right)+\left(2*-\frac{7}{5}\right)+\left(1*\frac{2}{5}\right) \\ (1*-1)+(-1*2)+(-3*-1) & \left(1*\frac{2}{5}\right)+\left(-1*-\frac{1}{5}\right)+\left(-3*\frac{1}{5}\right) & \left(1*\frac{4}{5}\right)+\left(-1*-\frac{7}{5}\right)+\left(-3*\frac{2}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+2 & \frac{2}{5}+0+\left(-\frac{2}{5}\right) & \frac{4}{5}+0+\left(-\frac{4}{5}\right) \\ -3+4+(-1) & \frac{6}{5}+\left(-\frac{2}{5}\right)+\frac{1}{5} & \frac{12}{5}+\left(-\frac{14}{5}\right)+\frac{2}{5} \\ -1+(-2)+3 & \frac{2}{5}+\frac{1}{5}+\left(-\frac{3}{5}\right) & \frac{4}{5}+\frac{7}{5}+\left(-\frac{6}{5}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & \frac{2}{5} & \frac{4}{5} \\ 2 & -\frac{1}{5} & -\frac{7}{5} \\ -1 & \frac{1}{5} & \frac{2}{5} \end{bmatrix} = A^{-1}$$

10. **Find the LU-factorization for each A in the previous question. (Find L, find U, and verify by multiplying them together.)**

Can you use the LU-decomposition to solve the equation Ax = $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$? If so, use the LU-factorization to find the solution x and then plug in x to verify it is correct. If you cannot use the LU-factorization to solve the equation, explain why not.

I don't think you can use LU decomposition to solve Ax = $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ because you need square

a. **A =** $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-3} \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 2 \\ 1 & -2 & -3 \end{bmatrix}$ Row2: $-2R_1 + R_2$ $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ & & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -4 \\ 1 & -2 & -3 \end{bmatrix}$ Row3: $-R_1 + R_3$ $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -4 \\ 0 & -3 & -6 \end{bmatrix}$ Row3: $-3/2R_2 + R_3$ $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \frac{3}{2} & 1 \end{bmatrix} = L$

$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix} = U$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1*1)+(0*0)+(0*3) & (1*1)+(0*-2)+(0*0) & (1*3)+(0*-4)+(0*0) \\ (2*1)+(1*0)+(0*0) & (2*1)+(1*-2)+(0*0) & (2*3)+(1*-4)+(0*0) \\ (1*1)+*\left(\frac{3}{2}*0\right)+(1*0) & (1*1)+\left(\frac{3}{2}*-2\right)+(1*0) & (1*3)+\left(\frac{3}{2}*-4\right)+(1*0) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 1+0+0 & 3+0+0 \\ 2+0+0 & 2+(-2)+0 & 6+(-4)+0 \\ 1+0+0 & 1+(-3)+0 & 3+(-6)+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

b. $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{3} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{-3} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}$ Row2: $-3R_1 + R_2$ $\qquad \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ & & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 7 \\ 1 & -1 & -3 \end{bmatrix}$ Row3: $-R_1 + R_3$ $\qquad \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 7 \\ 0 & -1 & -1 \end{bmatrix}$ Row3: $1/2R_2 + R_3$ $\qquad \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -\frac{1}{2} & 1 \end{bmatrix} = L$

$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 7 \\ 0 & 0 & \frac{5}{2} \end{bmatrix} = U$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 7 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$= \begin{bmatrix} (1*1)+(0*0)+(0*0) & (1*0)+(0*2)+(0*0) & (1*-2)+(0*7)+(0*\frac{5}{2}) \\ (3*1)+(1*0)+(0*-2) & (3*0)+(1*2)+(0*0) & (3*-2)+(1*7)+(0*\frac{5}{2}) \\ (1*1)+\left(-\frac{1}{2}*0\right)+(1*0) & (1*0)+\left(-\frac{1}{2}*2\right)+(1*0) & (1*-2)+\left(-\frac{1}{2}*7\right)+(1*\frac{5}{2}) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & -2+0+0 \\ 3+0+0 & 0+2+0 & -6+7+0 \\ 1+0+0 & 0+(-1)+0 & -2+\left(-\frac{7}{2}\right)+\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}$$