MAT 220—Homework 4

Part A:

Chapter 7 #3, 6, 7, 9, 12, 15 (Use https://www.random.org/integers/ to get the random $+1$'s and -1's for your answer in part b. You might need to get random 0's and 1's and then transform them into -1's and $+1$'s.)
Chapter 8 #1, 4
Chapter 10 #2, 4, 6

Part B:

1. *Rainfall and river height.* The T-vector r gives the daily rainfall in some region over a period of T days. The vector h gives the daily height of a river in the region (above its normal height). By careful modeling of water flow, or by
fitting a model to past data, it is found that these vectors are (approximately) related by convolution: $h = g * r$, where

$$g = (0.2, 0.7, 0.4, 0.1).$$

Give a short story in English (with no mathematical terms) to approximately describe this relation. For example, you might mention how many days after a day of heavy rainfall the river height is most affected or how many days it takes for the river height to return to the normal height once the rain stops.

2. Verify that $f(x_1, x_2, x_3) = (x_1 + 2x_2, 3x_3 - 2x_2)$ satisfies $f(x + y) = f(x) + f(y)$ for all vectors $x, y \in \mathbb{R}^3$ and satisfies $f(\alpha x) = \alpha f(x)$ for all $\alpha \in \mathbb{R}$ and all $x \in \mathbb{R}^3$. Then find a matrix A such that $f(x) = Ax$, where x is a column vector.

3. $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & -1 \\ 0.5 & -0.5 & 0 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 1 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$

Calculate the following or state that it is not defined: **(a)** AB **(b)** BA **(c)** BC **(d)** CB **(e)** ABC **(f)** AD **(g)** DA **(h)** BD **(i)** DB **(j)** CD **(k)** DC **(l)** $\mathrm{tr}(AB)$ **(m)** C^T **(n)** $(ABA)^T$.