Project 3

The textbook discussed matrix representations of directed graphs. A directed graph can also have labels on the edges that correspond to the probability of transitioning from one state to another. For example, suppose there are car rental branches in city A and city B. After one day, a car rented in city A has a 90% chance of being returned to the same branch and a 10% chance of being returned in city B. A car rented in city B has an 80% chance of staying there and a 20% chance of being returned in city A. Here is a directed graph representing this situation:

We can associate a matrix with this graph as in the book, except we put the edge labels in the entries instead of 1's.

$$M = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$$

Here I've filled in the matrix as if it were the table

	A	B
A		
B		

The column label corresponds to the starting city and the row label corresponds to the ending city.

Suppose we start off with 20 cars in city A and 80 cars in city B. We can represent that with the vector $V_0 = \begin{bmatrix} 20 \\ 80 \end{bmatrix}$. Notice what happens when we multiply M by v_0.

$$v_1 = Mv_0 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 80 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 \times 20 + 0.2 \times 80 \\ 0.1 \times 20 + 0.8 \times 80 \end{bmatrix}$$

Cars that start in A and end in A

Cars that start in B and end in A.

Cars that start in A and end in B.

Cars that start in B and end in B.

$$= \begin{bmatrix} 34 \\ 66 \end{bmatrix}$$

cars that are returned to A

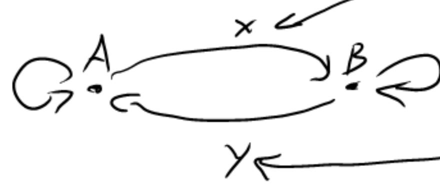
cars that are returned to B

This is why M is called a <u>transition matrix</u>.

So after one day, there are 34 cars in city A and 66 cars in city B. We can simulate how many cars are in each city after two days by multiplying by M again.

$$v_2 = Mv_1 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} 34 \\ 66 \end{bmatrix} \cdots$$

For your project: 1. Use the (pseudo)random number generator to get two numbers between 0 and 1. These numbers will give you the transition probabilities from A to B and from B to A.



2. Find the labels on the other two arrows.

3. Use the edge labels to find M.

4. Suppose you start with 50 cars in each city. Find v_0.

5. Use Python to find $v_1, \dots, v_{10}$ (at minimum), where $v_1 = Mv_0$, $v_2 = Mv_1$, $v_3 = Mv_2$, etc.. ($v_{n+1} = Mv_n \; \forall n \geq 0$)
   You may do this with a loop, but have it print out intermediate results, not just v_{10}.

6. Explain what you think v_n looks like for large n.

7. Repeat steps 4-6 with the following situations.
   Start out with 10 cars in city A and 90 cars in city B.
   Start out with 90 cars in city A and 10 cars in city B.

8. How do you think the long-term stock of cars in the two cities depends on how many cars start out in each city? Explain.