Name: ______________________________ Student ID: __________________

MAT 220—Homework 6

1. Verify that

$$\begin{aligned} x &= 7t - 8 \\ y &= 2 - 5t \\ z &= t \end{aligned}$$

are solutions of

$$\begin{aligned} 2x + 3y + z &= -10 \\ x + y + 2z &= -6 \end{aligned}$$

for all values of t.

2. Regarding $7z = 9$ as the equation $0x + 0y + 7z = 9$ in three variables, find all solutions in parametric form.

3. Write the augmented matrix the following system of linear equations. (Hint: Pay close attention to the variables.)

$$\begin{aligned} 6x - z &= -5 \\ -5x - 6y &= -2 \\ -7y - 3z &= 3 \end{aligned}$$

4. Write a system of linear equations that has the following augmented matrix.

$$\left[\begin{array}{rrr|r} 5 & 2 & 0 & 3 \\ 3 & 2 & -4 & 9 \\ -2 & -1 & 4 & 6 \end{array}\right]$$

5. Find the quadratic $ax^2 + bx + c$ such that the graph of $y = ax^2 + bx + c$ contains the points $(-1, 11)$, $(1, 5)$, and $(3, 7)$. Show all work, including a system of equations, augmented matrix, and Gauss-Jordan elimination.

6. Determine whether or not the vector x can be written as a linear combination of the vectors u, v, w. If so, give such a linear combination. Show all work, including a system of equations, augmented matrix, and Gauss-Jordan elimination.

(a) $x = (-9, -4, 3),\ u = (1, -4, -1),\ v = (-2, 2, -5),\ w = (-4, 3, -1)$

(b) $x = (2, 14, 6),\ u = (2, -12, -6),\ v = (4, 10, 6),\ w = (-5, 13, 6)$

(c) $x = (0, -9, 3),\ u = (-2, -15, 2),\ v = (4, 3, 5),\ w = (4, 12, 2)$

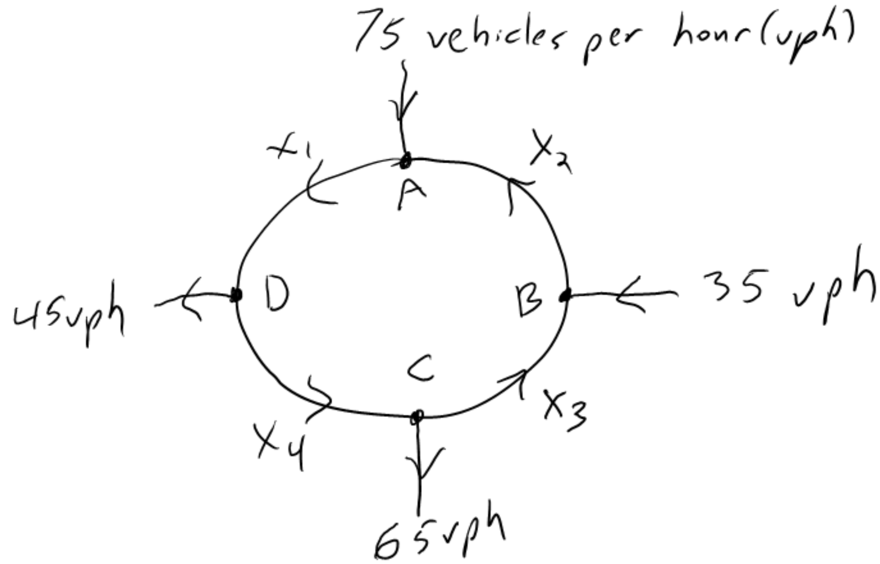
7. Determine whether the vectors u, v, w are linearly dependent or linearly independent. Show all work, including a system of homogeneous equations and the augmented matrix. You can reference your Gauss-Jordan elimination in the previous problem to shorten the process. If the vectors are linearly dependent, find all solutions to the system of homogeneous equations.

(a) $u = (1, -4, -1),\ v = (-2, 2, -5),\ w = (-4, 3, -1)$

(b) $u = (2, -12, -6)$, $v = (4, 10, 6)$, $w = (-5, 13, 6)$

(c) $u = (-2, -15, 2)$, $v = (4, 3, 5)$, $w = (4, 12, 2)$

8. Find the possible traffic flows in the roundabout below. Show all work, including a system of equations, augmented matrix, and Gauss-Jordan elimination.



9. Use Gauss-Jordan elimination to determine whether the matrices are invertible. If the matrix is invertible, give the inverse. Then use matrix multiplication to check that your answer is correct.

(a)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}$$

10. Find the LU-factorization for each A in the previous question. (Find L, find U, and verify by multiplying them together.)

Can you use the LU-decomposition to solve the equation $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$? If so, use the LU-factorization to find the solution x and then plug in x to verify it is correct. If you cannot use the LU-factorization to solve the equation, explain why not.

(a)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 0 & 2 \\ 1 & -2 & -3 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 1 & -1 & -3 \end{bmatrix}$$