MAT 220—Homework 7

1. Find the determinants of the following matrices. Show your work.

(a) $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

(b) Use trigonometric identities to simplify completely. $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(c) $C = \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$

(d) $D = \begin{bmatrix} b-2 & b \\ b & b+2 \end{bmatrix}$

(e) $E = \begin{bmatrix} -2 & 4 & -3 \\ -1 & 5 & -3 \\ 2 & -5 & -3 \end{bmatrix}$

(f) $F = \begin{bmatrix} 33 & 51 & 55 & 8 \\ 0 & 4 & 30 & 0 \\ 0 & 0 & 2 & 0 \\ 6 & 98 & 81 & 0 \end{bmatrix}$

2. Find a 3×3 matrix A such that $\det(A) = 1$ and A is not an identity matrix.

3. Find matrices A and B such that $\det(A + B)$ is not equal to $\det(A) + \det(B)$.

4. For the invertible matrices in Question 1, find the inverse.

5. Suppose P is an invertible 3×3 matrix, D is a diagonal matrix with diagonal entries $(3, 2, 4)$, and $A = PDP^{-1}$. Find $\det A$.

6. In each case, find the characteristic polynomial, eigenvalues, eigenvectors, and (if possible) an invertible matrix P such that $P^{-1}AP$ is diagonal.

(a) $A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

7. Define a sequence by $a_0 = 1$, $a_1 = 2$, and $a_k = 2a_{k-1} + 3a_{k-2}$ for all integers $k \geq 2$.

(a) Rewrite the recurrence relation to give a formula for a_{k+2} in terms of a_k and a_{k+1} for all integers $k \geq 0$.

(b) Rewrite the initial conditions and recurrence relation as an initial vector and a matrix recurrence.

(c) Use the initial vector and matrix recurrence to find an explicit formula for a_k.

(d) Use induction to show that your explicit formula is correct.

8. Define a sequence by $a_0 = 1$, $a_1 = 0$, $a_2 = 1$, and $a_{k+3} = -3a_k + a_{k+1} + 3a_{k+2}$ for all integers $k \geq 0$.

(a) Rewrite the initial conditions and recurrence relation as an initial vector and a matrix recurrence.

(b) Use the initial vector and matrix recurrence to find an explicit formula for a_k.

(c) Use induction to show that your explicit formula is correct.

9. Find the least squares approximating line $y = z_0 + z_1 x$ for each of the following sets of data points. Show your work.

(a) $(1, 1), (2, 3), (3, 7)$

(b) $(1, 21), (2, 17), (3, 12), (4, 7)$

(c) $(1, 1), (2, 3), (3, 4), (4, 8), (5, 11)$

10. Find the least squares approximating quadratic $y = z_0 + z_1 x + z_2 x^2$ for the data points $(1, 4), (2, 0), (3, 3), (4, 5)$. Show your work.