点 (x_0, y_0) : 点到直线 $d=\dfrac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

线 $Ax+By+C=0$

线 $Ax+By+C_1=0$

线 $Ax+By+C_2=0$ $\Rightarrow$ 距离 $d=\dfrac{|C_1-C_2|}{\sqrt{A^2+B^2}}$

$\boxed{\text{几何知识}}$

$\boxed{\text{拉格朗日乘数法}}$: 原问题: $\mathrm{Min}\ f(x)$

$s.t.\ g_i(x) \geq b_i$ $\Rightarrow$ ~~Min 拉格朗日~~

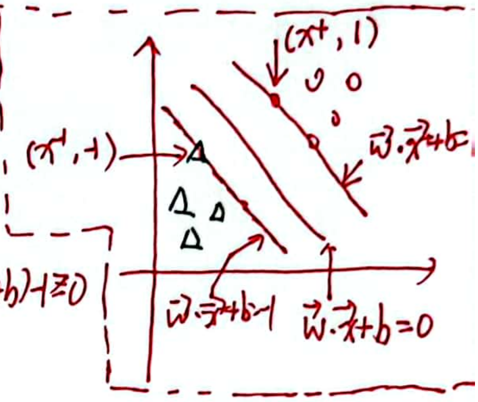
$$\mathrm{Min}\ f(x) - \sum_{i=1} \underbrace{\lambda_i}_{} [g_i(x)-b_i]$$

$\lambda_i \geq 0$: 拉格朗日乘子.

本来 $\begin{cases} \vec{w}\cdot\vec{x_i}+b \geq 0 \Rightarrow y_i=1 \\ \vec{w}\cdot\vec{x_i}+b<0 \Rightarrow y_i=-1 \end{cases} \Rightarrow \begin{cases} \vec{w}\cdot\vec{x_i}+b \geq 1 \Rightarrow y_i=1 \\ \vec{w}\cdot\vec{x_i}+b \leq -1,\ y_i=-1 \end{cases}$

目标: $\begin{cases} \mathrm{Max}\ margin = \dfrac{|1-(-1)|}{\|w\|} = \dfrac{2}{\|w\|} \\ s.t.\ y_i(\vec{w}\cdot\vec{x_i}+b)-1 \geq 0,\ i=1\dots n \end{cases} \Leftrightarrow \begin{cases} \cancel{\mathrm{Max}}\ \mathrm{Min}\ \dfrac{\|w\|^2}{2} \\ s.t.\ y_i(\vec{w}\cdot\vec{x_i}+b)-1 \geq 0 \end{cases}$

$\boxed{\text{SVM目标函数}}$



$\boxed{\text{拉格朗日变换}}$

$$\mathrm{Min}\ L_p = \frac{1}{2}\|w\|^2 - \sum_{i=1}^{n}\alpha_i[y_i(\vec{w}\cdot\vec{x_i}+b)-1]$$
$$= \frac{1}{2}\|w\|^2 - \sum_{i=1}^{n}\alpha_i y_i(\vec{w}\cdot\vec{x_i}+b) + \sum_{i=1}^{n}\alpha_i.$$

$\dfrac{\partial L_p}{\partial w} = w - \sum_{i=1}^{n} y_i\alpha_i\vec{x_i} = 0 \Rightarrow w = \sum_{i=1}^{n} y_i\alpha_i\vec{x_i}$

$\dfrac{\partial L_p}{\partial b} = -\sum_{i=1}^{n}\alpha_i y_i = 0 \Rightarrow \sum_{i=1}^{n}\alpha_i y_i = 0$

求偏导.

$y_i(\vec{w}\cdot\vec{x_i}+b)-1 \geq 0$ $\leftarrow$ 原来约束

$\alpha_i \geq 0$

$\alpha_i[y_i(\vec{w}\cdot\vec{x_i}+b)-1]=0$ $\leftarrow$ 互补条件 $\Rightarrow \begin{cases} \text{当 } \alpha_i>0 \text{ 时, } [y_i(\vec{w}\cdot\vec{x_i}+b)-1]=0 \\ \text{当 } [y_i(\vec{w}\cdot\vec{x_i}+b)-1] \neq 0 \text{ 时, } \alpha_i=0 \end{cases}$

$\boxed{\text{对偶问题}}$

$$\mathrm{Max}\ L_D = \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j \alpha_i \alpha_j \vec{x_i}\cdot\vec{x_j}$$

$s.t.\ \sum_{i=1}^{n} y_i\alpha_i = 0$

$\alpha_i \geq 0.$

$\boxed{\text{最终的决策边界(极大边距超平面)}}$

$$\vec{w}\cdot\vec{x}+b=0 \Leftrightarrow \sum y_i\alpha_i \underbrace{\langle \vec{x_i}, \vec{x} \rangle}_{\text{核函数 } K(\vec{x_i}, \vec{x})} + b = 0$$

草稿纸

$L_P \Rightarrow L_D$ 推导过程. 学号

$$L_P = \frac{1}{2}\langle w\cdot w\rangle - \sum_{i=1}^{n}\alpha_i[y_i(\langle w\cdot x_i\rangle + b) - 1]$$

$$= \frac{1}{2}\langle w\cdot w\rangle - \sum_{i=1}^{n}\alpha_i y_i(\langle w\cdot x_i\rangle + b) + \sum_{i=1}^{n}\alpha_i$$

$$= \frac{1}{2}\langle w\cdot w\rangle - w\sum_{i=1}^{n}\alpha_i y_i x_i - b\sum_{i=1}^{n}\alpha_i y_i + \sum_{i=1}^{n}\alpha_i$$

$$= \frac{1}{2}\langle w\cdot w\rangle - \langle w\cdot w\rangle - b\cdot 0 + \sum_{i=1}^{n}\alpha_i$$

$$= \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\langle w\cdot w\rangle$$

$$L_D = \sum_{i=1}^{n}\alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i\alpha_j y_i y_j\langle x_i\cdot x_j\rangle$$

$$w = \sum_{i=1}^{n}\alpha_i y_i x_i$$

$$\sum_{i=1}^{n}\alpha_i y_i = 0$$

例子计算过程.

$$L_D = \alpha_1+\alpha_2 - \frac{1}{2}\alpha_1\alpha_1 y_1 y_1\langle x_1\cdot x_1\rangle - \frac{1}{2}\alpha_1\alpha_2 y_1 y_2\langle x_1\cdot x_2\rangle - \frac{1}{2}\alpha_2\alpha_1 y_2 y_1\langle x_2\cdot x_1\rangle - \frac{1}{2}\alpha_2\alpha_2 y_2 y_2\langle x_2\cdot x_2\rangle$$

$$= \alpha_1 + \alpha_2 - \alpha_1^2$$

$$= 2\alpha_1 - \alpha_1^2$$

$\alpha_1 y_1 + \alpha_2 y_2 = 0 = \alpha_1 - \alpha_2 \Rightarrow \alpha_1 = \alpha_2.$

$\sum_{i=1}^{2} y_i\alpha_i = 0$

当 $\alpha_1 = 1$ 时, L_D 最大.

$\therefore \alpha_1 = \alpha_2 = 1. \Rightarrow \vec{w} = \sum_{i=1}^{2}\alpha_i y_i x_i = 1\times 1\times[1,1] + 1\times(-1)\times[0,0] = [1,1]$

将 $\langle x_1, y_1\rangle$ 代入满足 H^+ 方程. 即 $\langle \vec{w}\cdot x_1\rangle + b = 1.$

$\Rightarrow [1,1]\cdot[1,1] + b = 1$

$\Rightarrow 2 + b = 1$

$\Rightarrow b = -1.$

$\therefore g(x) = wx + b = [1,1]\cdot x + b = x_1 + x_2 + b.$

补充:

求出w后求b的值.

由互补条件知 $\alpha_i[y_i(\vec{w}\cdot\vec{x_i}+b)-1]=0$

$$\Rightarrow y_i(\vec{w}\cdot\vec{x_i}+b)-1=0$$

$$\Rightarrow y_i(\vec{w}\cdot\vec{x_i}+b)=1$$

$$\Rightarrow y_i^2(\vec{w}\cdot\vec{x_i}+b)=y_i$$

$y_i^2=1 \Rightarrow \vec{w}\cdot\vec{x_i}+b=y_i$

$\Rightarrow b=y_i-\vec{w}\cdot\vec{x_i}$. $i\in SV=$[所有的支持向量, 即$\alpha_i>0$的点]

最终取平均值:即可得 $b=\frac{1}{N_s}\sum_{i\in SV}(y_i-\vec{w}\cdot\vec{x_i})$