Dynamic Closest Color Warping to Sort and Compare Palettes

Supplemental Material #1

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1 ACRONYM TABLE

Table 1 lists the acronyms used in the paper.

Table 1. Acronyms

Acronym	Definition
ACO	Ant Colony Optimization
AP	Average Precision
BPS	Binary Palette Sorting
CEMD	Color-based Earth Mover's Distance
DCCW	Dynamic Closest Color Warping
DTW	Dynamic Time Warping
EMD	Earth Mover's Distance
FIA	Farthest Insertion Algorithm
FM100HT	Farnsworth-Munsell 100 Hue Test
FM100P	FM100HT Palette Dataset
GA	Genetic Algorithm
I-BPS	Improved BPS
KHT130	Kobayashi Hue & Tone System 130
KHTP	Kobayashi Hue & Tone Palette Dataset
KHTP-I	KHTP including interpolation only
KHTP-J	KHTP including jittering only
KHTP-J-I	KHTP including jittering and interpolation
KHTP-O	KHTP without jittering and interpolation
LD	Levenshtein Distance
LHSP	Lin and Hanrahan Palette Similarity Dataset
LKH	Lin-Kernighan Heuristic Algorithm
LLCS	Length of the Longest Common Subsequence
LLIS	Length of the Longest Increasing Subsequence
LTS-HD	Least Trimmed Square Hausdorff Distance
mAP	Mean Average Precision
MBME	Minimum Bipartite Matching Error
MCD	Minimum Color Difference
MHD	Modified Hausdorff Distance
MPD	Mean Pairwise Distance
MPHSM	Merged Palette Histogram Similarity Measure
SA	Simulated Annealing
SPS	Single Palette Sorting
SR	Success Rate
TSP	Traveling Salesman Problem

2 PALETTE SIMILARITY MEASUREMENTS

There are several ways to define the distance between two palettes $A = \{a_1, \cdots, a_{|A|}\}$ and $B = \{b_1, \cdots, b_{|B|}\}$, where a_i and b_i denote i-th color swatch for palette A and B, respectively. We discuss the major distance measures between sets in the subsequent sections.

2.1 Mean Pairwise Distance

The mean pairwise distance [Pan and Westland 2018] (MPD) is the average distance over all pairs of A and B. It is the most naive distance measure between two sets, and can be expressed as

$$D_{MPD}(A, B) = \frac{1}{|A| \cdot |B|} \sum_{a \in A} \sum_{b \in B} d(a, b). \tag{1}$$

2.2 Hausdorff Distance

The Hausdorff distance $D_{HD}(A, B)$ measures the discrepancy between two sets by finding a set of distances between the nearest point pairs from either set, and selecting the longest,

$$D_{HD}(A, B) = max(h(A, B), h(B, A)), \tag{2}$$

$$h(A, B) = \max_{a \in A} d(a, B), \tag{3}$$

$$d(a, B) = \min_{b \in B} d(a, b). \tag{4}$$

The greatest advantage of Hausdorff distance is computational simplicity and insensitivity to small perturbations [Huttenlocher et al. 1993]. Therefore, it has been widely used to measure similarity for characters [Wu and Shi 1999], images [Huttenlocher et al. 1993], and meshes [Borodin et al. 2003; Klein et al. 1996]. However, Hausdorff distance tends to be sensitive to noise or outliers [Felzenszwalb 2001; Sim et al. 1999]. Many alternative Hausdorff distances have been proposed to deal with this problem, and we discuss three alternatives that can be applied to palette comparison: modified Hausdorff distance, least trimmed square Hausdorff distance, and minimum color difference.

2.3 Modified Hausdorff Distance

The modified Hausdorff distance (MHD) [Dubuisson and Jain 1994], also known as chamfer distance [Barrow et al. 1977], calculates the average over all distances rather than maximum in Equation 3,

$$D_{MHD}(A, B) = max(h_{MHD}(A, B), h_{MHD}(B, A)), \tag{5}$$

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$$h_{MHD}(A,B) = \frac{1}{|A|}\sum_{a\in A} d(a,B). \tag{6}$$

Thus, MHD increases monotonically as the difference between the sets increases [Dubuisson and Jain 1994], and hence it is more robust to outliers than classical Hausdorff distance.

2.4 Least Trimmed Square Hausdorff Distance

The least trimmed square Hausdorff distance (LTS-HD) [Sim et al. 1999] ignores several large distance values using ranking,

$$D_{LTS}(A,B) = max(h_{LTS}(A,B), h_{LTS}(B,A)), \tag{7}$$

$$h_{LTS}(A,B) = \frac{1}{H}\sum_{i=1}^{H} d(\widehat{a_i}, B), \tag{8}$$

where $H = \lfloor h \cdot |A| \rfloor$, $h \in [0,1]$, and $\widehat{a_1}, \widehat{a_2}, \cdots, \widehat{a_H}$ are element of A that satisfy

$$d(\widehat{a_1}, B) \le d(\widehat{a_2}, B) \le \cdots \le d(\widehat{a_k}, B) \le \cdots \le d(\widehat{a_{|A|}}, B). \tag{9}$$

Since LTS-HD is only calculated for the top H candidates, it is robust to outliers that usually have large distance [Sim et al. 1999]. However, since the palette size is smaller than that of the image or mesh, it is dangerous to regard all outliers as noise. In some cases, outliers should not be treated as noise but should be adopted as valid factors for similarity. Experiments reported in this paper use $h = 0.6$ following Sim et al. [1999].

2.5 Minimum Color Difference

Minimum color difference (MCD) [Pan and Westland 2018] averages over two directed MHD rather than the maximum in Equation 5,

$$D_{MCD}(A,B) = \frac{h_{MHD}(A,B) + h_{MHD}(B,A)}{2}. \tag{10}$$

MCD was designed to generate a palette similarity model from psychophysical experiments. Pan and Westland [Pan and Westland 2018] confirmed MCD performance for 25-color palettes, and Yang et al. [2020] for 5-color palettes. However, they only compared MCD with a single average color, i.e., the average of all colors in the palette, and the mean pairwise distance. We evaluate MCD performance to test whether it is effective not only for psychophysical experiments, but also for quantitative evaluation.

2.6 Merged Palette Histogram Similarity Measure

The merged palette histogram similarity measure (MPHSM) [Po and Wong 2004] is a histogram-based image similarity measurement. MPHSM creates a common palette from the two descriptors, and converts the descriptors to the common palette's specification for comparison. Algorithm 1 shows the pseudo-code to compute the common palette from the two descriptors $F^A = \left\{\left(c_i^A, w_i^A\right), i = 1, \cdots, |A|\right\}$ and $F^B = \left\{\left(c_i^B, w_i^B\right), i = 1, \cdots, |B|\right\}$, which comprises a set of pairs of color c and weight w. T_d is the maximum distance for two colors to be considered similar. Po and Wong [2004] set $T_d = 15$ in CIELUV color space, and for our experiments we used $T_d = 15$ in CIELAB color space. For histogram intersection similarity measure, F^A is redefined to P^A,

$$P^A = \left\{\left(c_i^P, w_i\right), i = 1, \cdots, |P|\right\}, \tag{11}$$

ALGORITHM 1: Common palette generation

```
Input: Two given descriptors F^A, F^B
Output: Common palette P
P ← []
(x, y) ← a pair with the smallest color distance between F^A and F^B
while d(c_x, c_y) ≤ T_d do
    c_m ← (w_x c_x + w_y c_y)/(w_x + w_y)
    Add c_m to P.
    Delete a and b from F^A and F^B, respectively.
    (x, y) ← a pair with the smallest color distance, where x and y
      belong to different sets among F^A, F^B, and P.
end
Add remaining colors in F^A and F^B to P.
```

where c_i^P is same color as the common palette P, and $w_i = max(0, \sum_j w_j^A)$ for all j with $d(c_j^A, c_i^P) \le T_d$. F^B is converted to P^B in the same way. We assigned equal weights $w_i^A = w_j^B = 1$ for our experiments, and the distance between two converted descriptors is

$$D_{MPHSM}(P^A, P^B) = \sum_{i=1}^{|P|} min(w_i^A, w_i^B). \tag{12}$$

2.7 Earth Mover's Distance

The earth mover's distance (EMD) [Rubner and Tomasi 2001] measures the cost to transform one color palette into the other. It is defined as a minimum cost flow over all possible flows f_{ab},

$$D_{EMD} = \frac{min_{f_{ij}}(\sum_{i=1}^{|A|}\sum_{j=1}^{|B|} f_{ij} \cdot d(c_i^A, c_j^B))}{min(\sum_{i=1}^{|A|} w_i^A, \sum_{j=1}^{|B|} w_j^B)}, \tag{13}$$

subject to constraints:

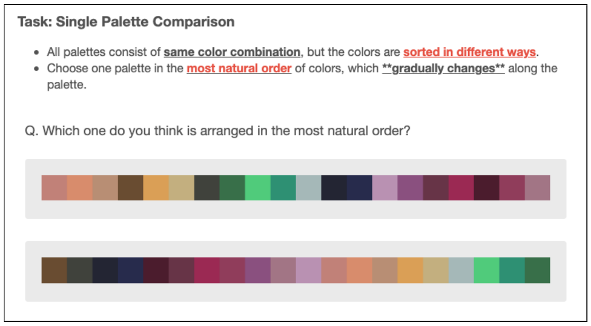
$$\begin{aligned} &\forall i, j : f_{ij} \ge 0, \\ &\forall i : \sum_{j=1}^{|B|} f_{ij} \le w_i^A, \\ &\forall j : \sum_{i=1}^{|A|} f_{ij} \le w_j^B, \\ &\sum_{i=1}^{|A|}\sum_{j=1}^{|B|} f_{ij} = min(\sum_{i=1}^{|A|} w_i^A, \sum_{j=1}^{|B|} w_j^B), \end{aligned} \tag{14}$$

where w_i^A and w_j^B are weights for colors in palette A and B respectively, and we assigned equal weights $w_i^A = w_j^B = 1$ for our experiments.

Beecks et al. [2010] adopted EMD to compare feature signatures and Skaff et al. [2011] employed EMD for color palettes comparison, using CIEDE2000 [Luo et al. 2001; Sharma et al. 2005] for the distance function between colors, calling this measure color-based EMD (CEMD).

2.8 Minimum Bipartite Matching Error

Lin and Hanrahan [2013] used the minimum bipartite matching error (MBME) to compare human extracted and machine generated palettes. MBME aims to match each color in one palette to a color



(a) SPS

(b) PPS

Fig. 1. Interfaces for perceptual study.

in the other palette with minimum total error. It is similar to finding a perfect match with minimum total cost in the complete bipartite graph comprising all color pairs (a, b) for palettes A and B. It is sometimes called the Hungarian algorithm [Kuhn 1955],

$$D_{MBME}(A, B) = min \sum_{a \in A} \sum_{b \in B} d(a, b) X_{ab}, \tag{15}$$

where $X_{ab} = 1$ if and only if color a matches b, and zero otherwise. The disadvantage of MBME is that it only works when the palettes have the same length.

3 PERCEPTUAL STUDY

We show the interface for our perceptual study on Amazon Mechanical Turk in Fig. 1. In the title of each assignment, we asked participants to select the palette that looks sorted in the most natural order. We have added the whole palettes used in the perceptual study to the Supplemental Material #2. Table 2 and Table 4 show the results of the within-subjects effects for SPS and PPS. For all cases,

	Type III SS	DF	MS	F	p
HSV vs. Ours					
Length	0.000	2	0.0000	0.00	1.000
Sorting method	69.352	1	69.3517	677.90	< .001
Length * Sorting method	0.465	2	0.2324	7.24	< .001
CIELAB vs. Ours					
Length	4.03e-31	2	2.01e-31	-2.64e-13	1.000
Sorting method	15.392	1	15.3917	32.60	< .001
Length * Sorting method	0.246	2	0.1228	1.92	0.149

Table 2. Within subject effects of the repeated measures ANOVA for SPS. Notes: SS = sum of squares, DF = degree of freedom, and MS = mean square

	MD	SE	DF	t	p_{tukey}
HSV vs. Ours					
Sorting method					
HSV - Ours	-0.687	0.0264	97.0	-26.0	< .001
Length					
10 - 15	6.13e-16	5.20e-16	194	1.18	0.467
10 - 20	-7.68e-16	5.20e-16	194	-1.48	0.304
15 - 20	-1.38e-15	5.20e-16	194	-2.66	0.023
CIELAB vs. Ours					
Sorting method					
CIELAB - Ours	-0.324	0.0567	97.0	-5.71	< .001
Length					
10 - 15	2.59e-16	5.22e-16	194	0.496	0.873
10 - 20	7.03e-17	5.22e-16	194	0.135	0.990
15 - 20	-1.89e-16	5.22e-16	194	-0.361	0.931

Table 3. Tukey post hoc tests of the repeated measures ANOVA for SPS. Notes: MD = mean difference, SE = standard errors, and DF = degree of freedom.

we found that the palette length does not affect the preference result, but that sorting methods do. The interaction between palette length and sorting method is significant in four cases: ours vs. CIELAB (SPS), ours vs. BPS, ours vs. I-BPS, ours vs. HSV (PPS).

As shown in Table 3 and Table 5, we performed Tukey post hoc tests for pairwise differences across all treatment combinations. Fig. 2 and Fig. 3 indicate that there is a clear preference for the results of our algorithm against all methods. Also, there is no common tendency of the interactions between palette length and sorting method.

4 ADDITIONAL IMAGE RECOLORING RESULTS

Fig. 4 presents additional recoloring results comparing with simple approach of Tan et al.'s [2018; 2017] convex hull-based palette. The simple convex-hull based transformation recolors an image with a new palette by transforming a convex hull enclosing the colors of the source image into a new hull enclosing the colors of a new palette. When the volume of the new hull is smaller than that of the original image, the range of colors that can be expressed is significantly reduced, which leads to overfitting to the target palette. Our DCCW supports the choice of buffer colors to mitigate the drastic volume change while still representing the target palette.

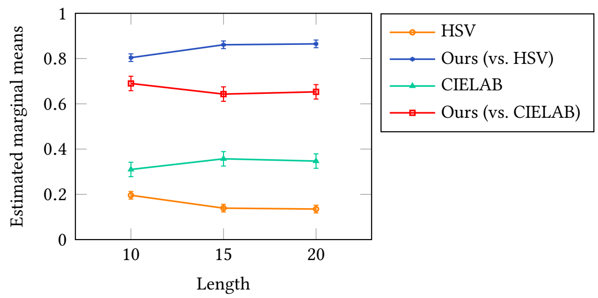


Fig. 2. Estimated marginal means of the repeated measures ANOVA for SPS with 95% confidence interval.

	Type III SS	DF	MS	F	p
HSV vs. Ours					
Length	1.61e-30	2	8.05e-31	5.28e-13	1.000
Sorting method	84.104	1	84.1040	737.70	< .001
Length * Sorting method	0.884	2	0.4420	7.28	< .001
CIELAB vs. Ours					
Length	5.13e-30	2	2.56e-30	8.23e-13	1.000
Sorting method	7.785	1	7.785	20.117	< .001
Length * Sorting method	0.203	2	0.101	0.766	0.466
BPS vs. Ours					
Length	3.90e-31	2	1.95e-31	1.42e-13	1.000
Sorting method	88.964	1	88.9641	714.69	< .001
Length * Sorting Method	0.301	2	0.1507	2.48	0.086
I-BPS vs. Ours					
Length	2.79e-30	2	1.39e-30	3.15e-13	1.000
Sorting method	19.204	1	19.204	96.58	< .001
Length * Sorting method	0.304	2	0.152	1.01	0.367

Table 4. Within subject effects of the repeated measures ANOVA for PPS. Notes: SS = sum of squares, DF = degree of freedom, and MS = mean square

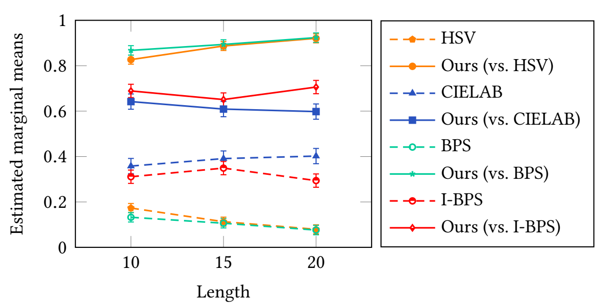


Fig. 3. Estimated marginal means of the repeated measures ANOVA for PPS with 95% confidence interval.

	MD	SE	DF	t	p_{tukey}
HSV vs. Ours					
Sorting method					
HSV - Ours	-0.756	0.0278	97.0	-27.2	< .001
Length					
10 - 15	8.00e-16	5.85e-16	194	1.37	0.359
10 - 20	-7.16e-16	5.85e-16	194	-1.23	0.440
15 - 20	-1.52e-15	5.85e-16	194	-2.59	0.027
CIELAB vs. Ours					
Sorting method					
CIELAB - Ours	-0.233	0.0518	95.0	-4.49	< .001
Length					
10 - 15	-1.41e-16	4.09e-16	190	-0.344	0.937
10 - 20	-1.92e-16	4.09e-16	190	-0.470	0.885
15 - 20	-5.14e-17	4.09e-16	190	-0.126	0.991
BPS vs. Ours					
Sorting method					
BPS - Ours	-0.790	0.0296	94.0	-26.7	< .001
Length					
10 - 15	-1.08e-15	6.77e-16	188	-1.590	0.253
10 - 20	-6.55e-16	6.77e-16	188	-0.968	0.598
15 - 20	4.21e-16	6.77e-16	188	0.622	0.808
I-BPS vs. Ours					
Sorting method					
I-BPS - Ours	-0.363	0.0370	96.0	-9.83	< .001
Length					
10 - 15	-4.79e-17	3.08e-16	192	-0.155	0.987
10 - 20	-2.89e-16	3.08e-16	192	-0.937	0.618
15 - 20	-2.41e-16	3.08e-16	192	-0.782	0.715

Table 5. Tukey post hoc tests of the repeated measures ANOVA for PPS. Notes: MD = mean difference, SE = standard errors, and DF = degree of freedom.

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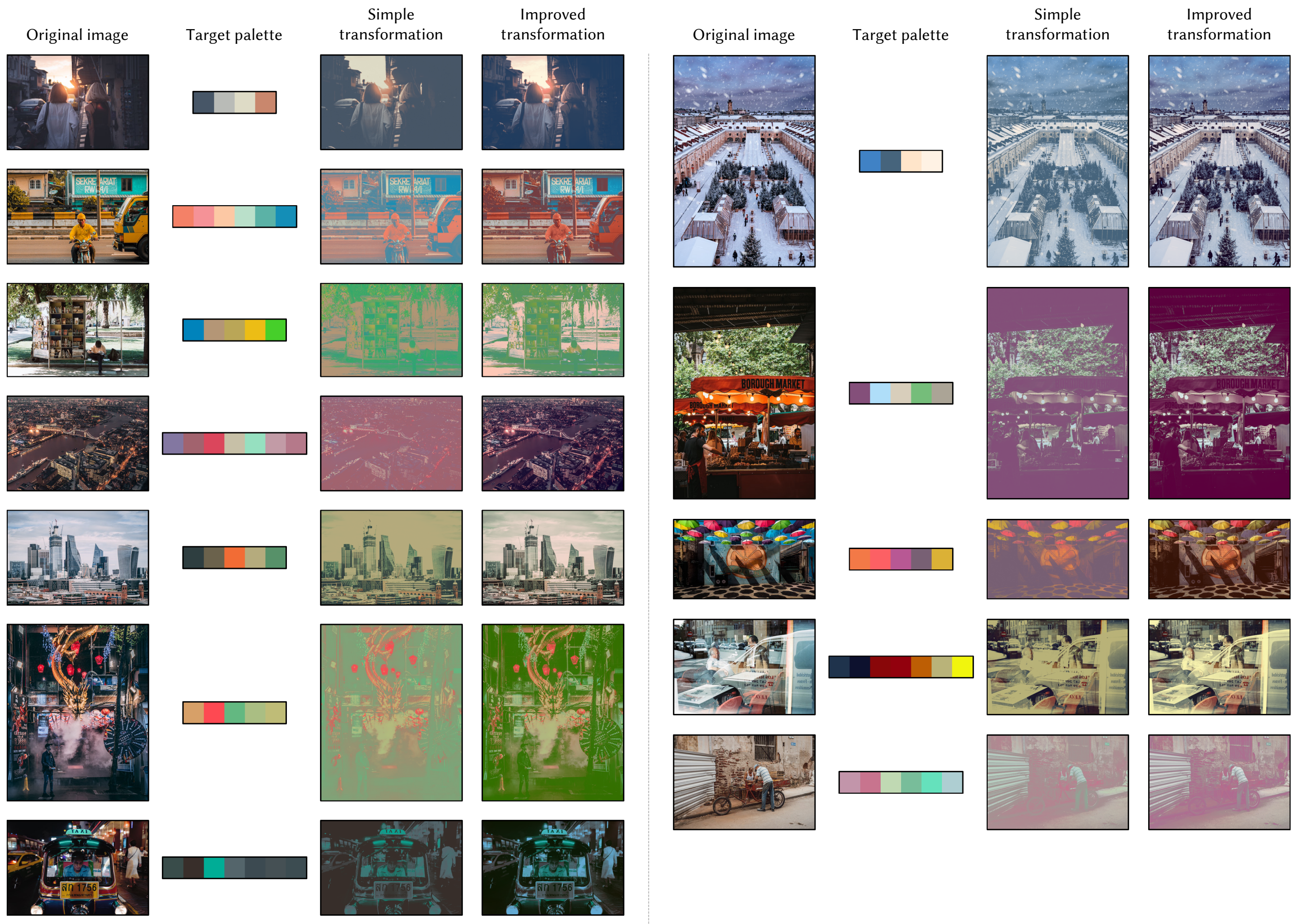

Fig. 4. Additional recoloring results.

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