DIRICHLET PROCESS MIXTURE MODEL FOR PROBABILISTIC REVERSAL LEARNING

Wiona Glänzer*, Katarzyna Krasnopolska*, Timofey Shpakov*
Supervisors: Tore Erdmann, Christoph Mathys, Alexander Hess

ETH Zurich
Institute for Biomedical Engineering
Translational Neuromodeling Unit
[wglaenzer, krasnopk, tshpakov]@ethz.ch

ABSTRACT

To study erroneous information processing in patients with delusions, a generative model based on a Dirichlet process mixture model has previously been constructed (Erdmann and Mathys, 2021). It is posited that a single parameter, the expected precision, can explain the difference between affected and non-affected subjects, by leading to the creation of superfluous explanatory hypotheses in Bayesian inference. We extended the model to study reversal learning, a process that has been shown to be impaired in patients with Schizophrenia (Reddy et al., 2016). We compared our model with the results of a probabilistic learning experiment conducted with individuals with Schizophrenia and found that we could mostly reproduce their behaviour with our model.

1. INTRODUCTION

We begin this report by giving an introduction to research about delusions and presenting a generative model that has previously been constructed. In section 3, we explain our modifications to the existing model. Afterward, we show the experiments we performed with our model and present the results. Finally, we discuss the outcomes of our project and how future work could build on it.

Motivation. Worldwide, Schizophrenia affects about 24 million people and accounts for roughly half of patients hospitalized for mental health disorders, creating an extreme burden for individuals and communities. While some treatment options exist, the causes and mechanisms of the disease are still relatively unknown, and a better understanding would aid the development of improved treatments (World Health Organization, 2022). As delusions are a very common symptom of Schizophrenia and likely share causative mechanisms with other symptoms, we employ computational methods to investigate their causes.

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*Equal contributions.

Related Work. Apart from the framework to study the formation of delusions proposed in Erdmann and Mathys (2021), there have been multiple other approaches applying Bayesian non-parametric models to tasks in human cognition ((Austerweil, 2013), (Collins, 2013)) and psychometric modeling ((Karabatsos and Walker, 2009), (Navarro et al., 2006)).

Our Contribution. We propose an alteration to the model implemented by Erdmann and Mathys (2021) to allow for its application to binary input data, which can be used to simulate probabilistic inference tasks as well as learning reversals.

2. BACKGROUND

In this section, we cover the relevant background knowledge from Psychology research as well as mathematical models.

Delusions. Delusions are a core symptom of Schizophrenia but can also be related to brain injuries or dementia. Capgras delusion, in which a patient believes that a person close to them has been replaced by an imposter, is one common delusion that has been studied in detail. When showing patients with a brain injury that typically causes Capgras delusion pictures of the same person photographed from different angles, it was found that their autonomic response (measured as the skin conductance) was different from that of healthy individuals, pointing to altered processing of the input. This conductance abnormality was found also patients with the brain injury who were not suffering from the delusion. This supports the so-called two-factor model of delusions, which postulates that both perceptions that differ from the norm and the affected processing of these perceptions are involved in the formation and maintenance of delusions (McKay, 2012). The model described below considers a possible explanation for the second factor.

Probabilistic Inference in Psychology. Probabilistic inference tasks are a means in psychology research to investigate an individuals ability to derive hypotheses about

their environment from uncertain data. One example would be the "beads" task, proposed by (Phillips and Edwards, 1966), which is structured in the following way: At the beginning of the experiment, participants know there are two urns containing beads of two different colors. One urn contains mostly red beads and the other mostly blue beads, the percentages being 15 and 85 % respectively. The study coordinator draws individual beads from the urns, and after each draw, the participant has to decide whether they are certain to know which urn the beads are coming from. The experimenters evaluate the number of draws it takes for a participant to reach a conclusion. Probabilistic reversal learning (PRL) tasks form a subgroup of probabilistic inference tasks. They usually involve a rule that participants have to learn, and that is reversed once they have done so. The participants learn this rule usually by making a choice between two options and receiving feedback on whether the chosen option was correct. The feedback is probabilistic, however, and as such, the participants might receive incorrect information. It is then evaluated whether, and how fast the participants learn the underlying rule and its reversal. The PRL task we use for the evaluation is described in detail in section 4. It has been suggested that probabilistic inference could be impaired in individuals with Schizophrenia (Ross et al., 2015), (Reddy et al., 2016), (Murray et al., 2008).

Dirichlet Process Mixture Models. A Dirichlet Process (DP) describes a distribution over distributions. It is described by a base distribution H and a parameter α, which characterizes how distributions are drawn from the base distribution. DPs are used in Bayesian non-parametric models of data and as non-parametric methods allow for a potentially infinite number of parameters. As such they are a viable candidate to be used in modeling open-ended human learning (Erdmann and Mathys, 2021).

To explain how the processing of perceptions could differ in patients with delusions, we employed computational methods based on Bayesian inference as a model for decision-making in the brain. Erdmann and Mathys (2021) present a generative model for delusions that explains the affected processing via a difference in the expected precision in the context of Bayesian inference. For this purpose, they use a Dirichlet process mixture model, which for each observation that a subject makes, either integrates their observation into an existing explanatory hypothesis or creates a new one for the observation. The model allows for infinitely many different explanatory hypotheses to exist.

Each hypothesis is represented by a Gaussian $F(y, \phi_k) = \mathcal{N}(y|\mu_k, \tau_k^{-1})$ with the prior-distributions for μ_k being normal and for τ_k being half-normal distributions. The mean μ_τ of the prior distribution of τ_k, is the expected precision. This is the parameter that Erdmann and Mathys (2021) use to explain the difference in delusional patients. If these patients have a smaller μ_τ, they expect their explanations to fit the observations more closely, which leads to the creation of more additional explanatory hypotheses. To study whether this effect could (partially) account for affected processing in Schizophrenia, we adapted their model to simulate decision-making processes during experiments in which a difference between healthy and schizophrenic subjects has been found. The experiment we chose investigates probabilistic reversal learning and is described in detail in section 4.

3. MODEL

In the following section, we present the proposed extension of the Dirichlet mixture model (DMM) for simulating PRL tasks.

Model Inputs. The DMM requires continuous inputs, however, in the used experiments the agent only makes discrete observations. We justify the use of Gaussians as hypotheses during PRL tasks by mapping the discrete observations to a continuous scale. By not just looking at a single observation but at a summary over a history of observations, we can relax the requirement for the input data of the base DMM. Using the beads experiment as an example, we can compute the log odds over the observed beads to create continuous input data for the model. In this case, we go from the binary hypothesis space to continuous parameter space where the agents try to estimate the parameter of the Bernoulli urn.

A negative side effect of the proposed mapping is that an agent with perfect memory will always find the best estimate in the case of probabilistic learning. In the case of PRL the agent will fail to adapt to a changing rule as the beads seen during initial phases will outweigh the contribution of the most recent observations to the hypothesis. As a remedy, we consider only the most recent observations when performing inference on a new observation. It also makes intuitive sense to do so, as a human subject would in time learn to discard old information in favor of more recent observations.

Model Parameters. The model makes use of two parameters to simulate agents solving PRL tasks. The expected precision μ_τ is used to control the degree of "delusion formation" and n_{history} for controlling the forgetfulness of the agent.

Extension to Sequential Data. We further extend the model by allowing sequential input. We achieve this by initializing the mixture model as one cluster with initial parameters θ_0 and initial observations. We then fit a DMM over the most recent observations and draw a new observation. Then, the mixture is updated by considering the new observation and we read out the cluster parameters that the new observation is assigned to. We consider the mean of

this cluster to be the explanation that the model offers for the new data point. The resulting updated mixture is then used as the initial mixture for the iteration. Subsequently, the oldest observation used to fit the cluster is discarded and the newest observation is added to the previously seen observations. An illustration of this procedure can be seen in figure 1 and the pseudo-code for the simulation of our PRL task is given in Algorithm 1.

Algorithm 1: PRL Simulation

Input : log odds vector log_odds, history n_{h}
Output: vector of probabilities p

```
p = []
x_prev = log_odds[1]
clusters_prev = clusters_init
for x_new in log_odds[2:]
    clusters = init_mixture(x_new, x_prev,
      clusters_prev)
    x = [x_prev; x_new]
    clusters = update_mixture(x)
    pred_log_odds = last_cluster_mean(clusters)
    prob = prob_from_log_odds(pred_log_odds)
    p.append(prob)
    x_prev = x[2:n_h]
    clusters = forget_oldest_point(clusters)
    clusters_prev = clusters
end for
return p
```

4. EXPERIMENTAL RESULTS

To evaluate our model, we implemented the PRL task proposed by Cools et al. (2002) in combination with the probabilistic inference task by Phillips and Edwards (1966). We compare the outcomes of our simulation to experimental results obtained in a clinical study by Reddy et al. (2016). The model and experiments were implemented and run in Julia. [1]

Experimental Setup. The probabilistic inference task was already described in 2. The other experiment (Cools et al., 2002) is a task designed to measure the participants' ability to perform a learning reversal. It is comprised of three phases. In each phase, participants are shown two distinct patterns. The participants choose one of the patterns and receive negative or positive feedback, based on which pattern they chose. This trial is repeated ten times, with the same two patterns. During the whole phase there is one correct, and one incorrect pattern, this does not change for the duration of the phase. However, the feedback on whether the chosen pattern is correct is probabilistic, and with a probability of 0.2, the participant will receive incorrect feedback. After ten trials the decisions of the participant are evaluated and if nine out of ten trials were decided correctly, the next phase of the experiment begins. In the next phase, the "correct" pattern is switched. The experiment proceeds as before; the participant again needs to answer correctly nine out of ten times. If they succeed, the correct solution is then again swapped and again the participants need to learn the reversal. If the participant is not able to learn the correct pattern after five repeats of ten trials in one phase, the experiment does not proceed to the next phase. The setup of the experiment is illustrated in Figure 2.

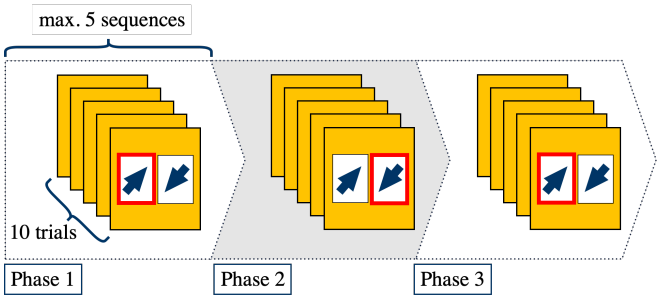

Fig. 2: Probabilistic Reversal Learning task

For our purpose, we reduce the task by making the following assumptions:

- Instead of pictures, our input is a series of 1s and 0s, where in phase 1 we have 1s with a probability of 0.8d.
- Our model is not choosing a correct pattern, instead, intuitively, it is building the hypothesis from which urn the current sequence is drawn.
- The model gains information not by receiving feedback on a choice, but by including the next data point in the computation.
- We interpret the output probability of our model in the following way: If the probability is over 0.5, the model considers the sequence to come from urn 1, and if it is below 0.5, it is considered to come from urn 0. From now on we call this the "choice" of urn 0 or 1.

Qualitative Results. In the following we evaluate and compare the performance of two models with a different μ_τ parameter. With the exception of Figures 3 and 4 each comparison was obtained by executing 200 runs of the PRL experiment described above on each of the two models. Model 1 was initialized with $\mu_\tau = 100$ and model 2 with $\mu_\tau =$

[1] GitLab repository:
https://gitlab.ethz.ch/tn_projects_fs2022/project_group_1

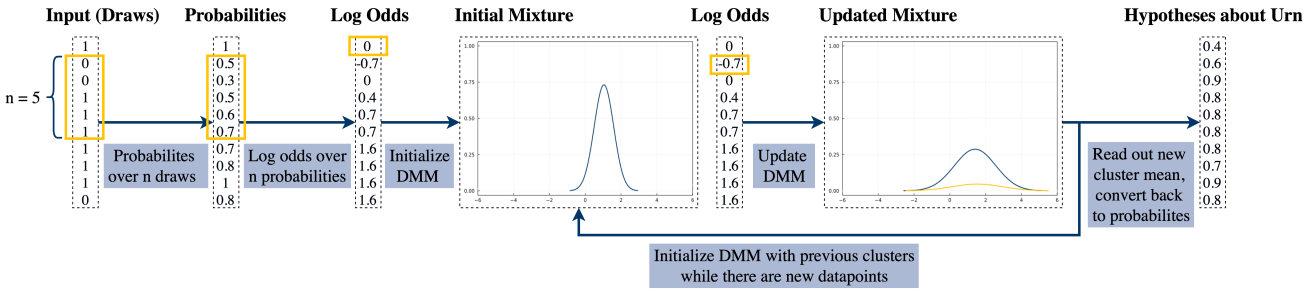

Fig. 1: Diagram of our model

$1/100$. In Figures 3 and 4 two runs of the PRL experiment are shown.

Figure 3 shows the learning behaviour of an agent with a high μ_τ, which we call part of the control group, as we expect its behaviour to resemble the human control group, i.e. participants without delusional symptoms. In 4 we show the learning behaviour of an agent with a low parameter μ_τ, which we analogously call part of the patient group, as we expect its behaviour to coincide with the human patient group. The topmost plot in Figures 3 and 4 shows the output of our model, that is the estimated probability of the bead sequence coming from urn 1 at each time step. The corresponding bead sequence is shown in the lowermost plot. The two middle plots show the inner workings of the model during the inference task. The second plot from the top shows the instances, where the newly drawn bead was assigned to a new cluster ("hypothesis") in the model, and the third plot from the top shows the total number of clusters at any given step. When comparing the two plots it can be seen that overall the output of the control model shows a much smoother progression, even though the underlying bead sequences are reasonably similar. The number of generated clusters is about the same in both cases, the rate at which the cluster number changes is higher in the patient model however. Moreover, new beads get assigned to new clusters more often in the patient model.

A more comprehensive analysis of this behaviour can be seen in Figure 5. For the purpose of this analysis, the number of trials per phase was fixed to ten trials. The two agents were evaluated on 200 different bead sequences of fixed length and the number of cluster switches at each step of the experiment was summed up, which is shown in the top plot of Figure 5. The bottom plot shows an example of a bead sequence, from which the number of cluster switches for a particular run of the experiment was obtained. Please note that since the bead sequences are generated randomly, each bead sequence for each run of the experiment was different; we show this particular one simply for illustration purposes. In this graph, it can be observed that both agents tend to assign data points to new clusters at the beginning of the experiment. Nevertheless, the patient model assigns new data points to new clusters to a higher degree even at this stage. It is also of interest to note, that as the experiment progresses, agents in the control group tend to assign new data points to new clusters most notably at the reversal points, whereas agents in the patient group tend to assign to new clusters uniformly throughout the rest of the experiment.

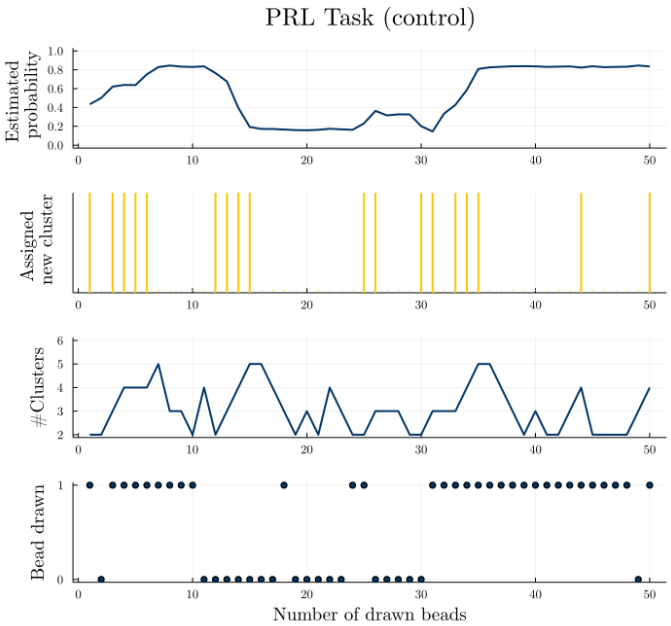

Fig. 3: Experiment with $\mu_\tau = 100$

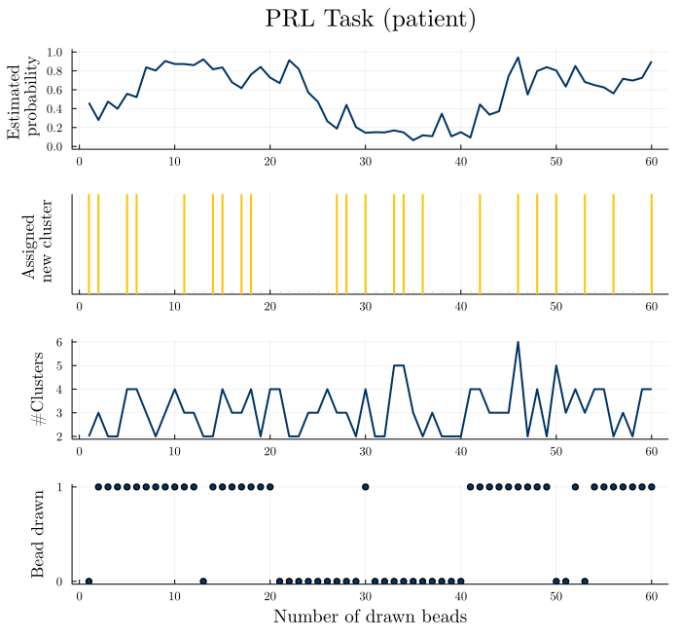

Fig. 4: Experiment with $\mu_\tau = 1/100$

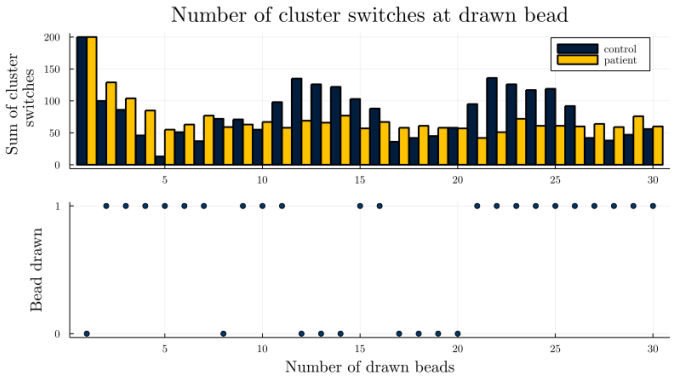

Fig. 5: Sum of cluster switches at each step in experiment

In Figures 6, 7 and 8 the phase learning behaviour is evaluated in more detail. It can be seen that both agents learn almost all phases correctly, with the patient model learning slightly fewer phases. Notably, the patient model needs a higher or equal number of trials to correctly learn a phase in the mean and median.

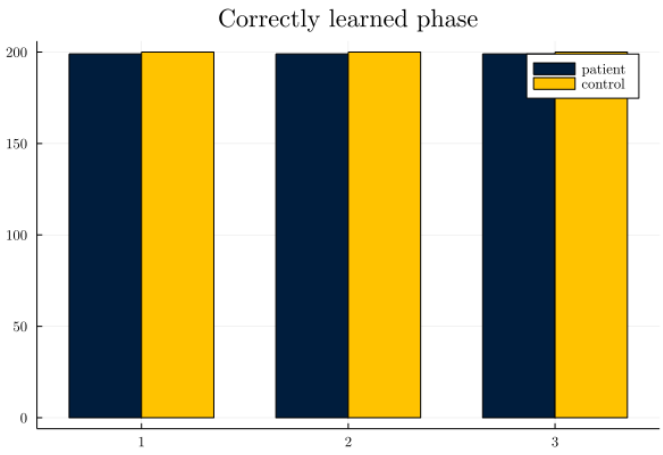

Fig. 6: Number of phases learned correctly

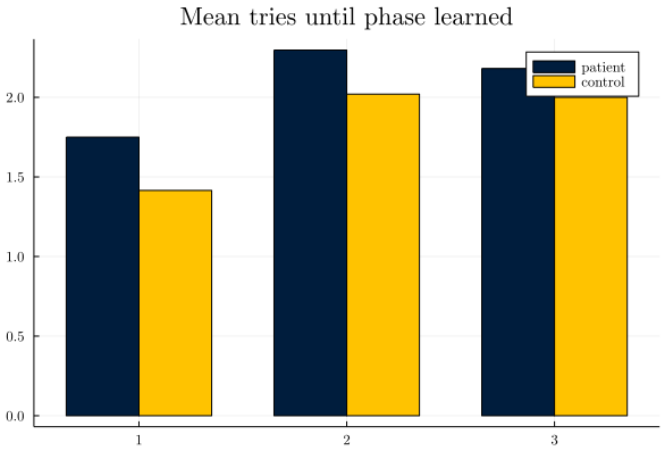

Fig. 7: Mean tries until a phase was learned

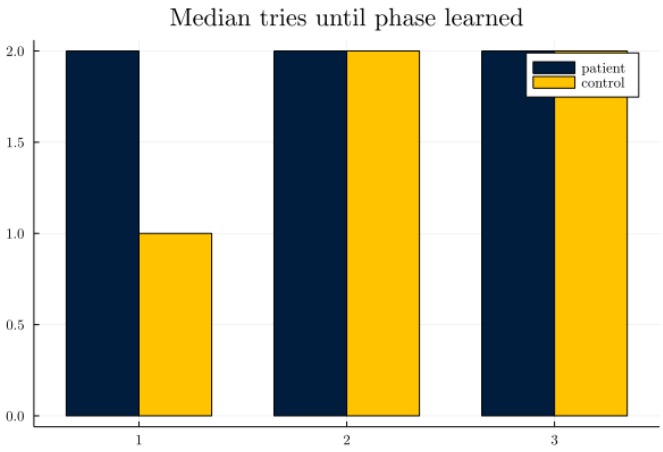

Fig. 8: Median tries until a phase was learned

For a more in-depth analysis of the agents' behaviour,

we use some of the same metrics as Reddy et al. (2016), which are shown in Figures 9 and 10. In the leftmost column, we count the number of correct guesses per attempt to learn a phase. In the other columns, we evaluate the influence of a previous state on the next state of the model, or, how large an influence a draw from the sequence has on the next hypothesis. If the previous hypothesis had a probability of below 0.5, and the next has a probability above 0.5 (and vice versa), it is counted as a shift, otherwise as a stay. A shift following a correct hypothesis (for example, the beads are currently drawn from urn 1, the current model hypothesis is above 0.5, the model sees a 1, and despite this, the next calculated hypothesis is below 0.5), is called a Win-shift. A shift following an incorrect hypothesis is called a Lose-shift. Analogously a stay following a correct hypothesis is a Win-stay, and a stay following an incorrect hypothesis is a Lose-stay. If the bead seen by the model is a 1 (0), which is drawn from urn 1 (0) (probability 0.8 of drawing a 1 (0)), the following stay/shift behaviour is called Valid, if the bead seen by the model is a 0 (1) drawn from urn 1 (0) (probability 0.2 of drawing a 0 (1)), the stay/shift behaviour is called Invalid. The detailed analysis from our experiment can be seen in Figures 9 and 10.

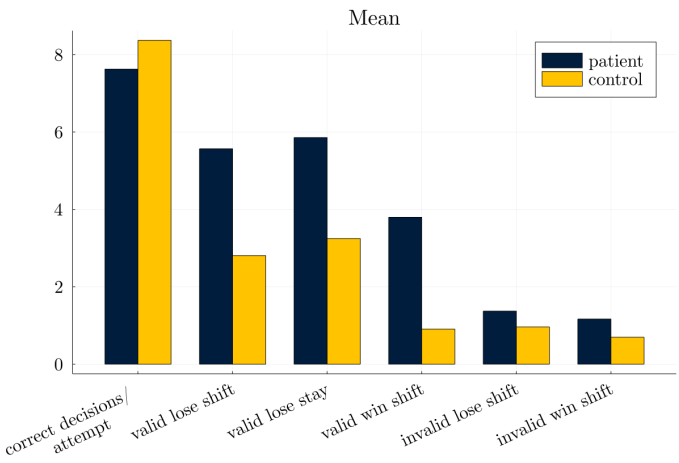

Fig. 9: Evaluation of decisions taken by model in mean

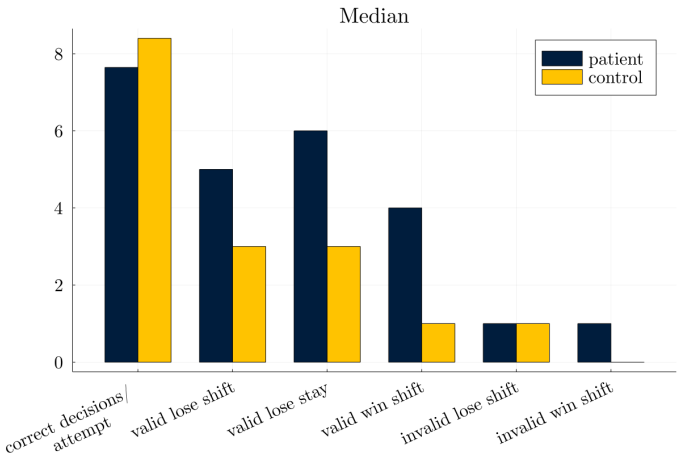

Fig. 10: Evaluation of decisions taken by model in median

5. DISCUSSION

Qualitative comparison with clinical data. In the following, we conduct a qualitative comparison of our results with the findings of Reddy et al. (2016).

1. Primary PRL Task variables: In our experiment, patients and controls achieved about the same number of phases learned, which is not consistent with the actual findings of Reddy et al. (2016), where patients managed to learn significantly fewer reversals. It is consistent, however, that the number of trials to learn a phase was higher in the patient group than in the control group. Moreover, Figures 9 and 10 show that the number of correct "decisions" made by agents in the control group is higher than the one by the patient group, which was also found by Reddy et al. (2016).

2. Comparison of response selection: In Reddy et al. (2016) patients tended to make more mistakes overall (stayed more when they lost and shifted more when they won), which can also be seen for our modelled patients in Figures 9 and 10. Moreover, both groups had a higher rate of Valid Lose-stays than Valid Win-shifts in the real-life experiment, which can also be seen for our model. In the experiment, patients shifted more than controls for Valid Lose and Valid Win events, but there was no group difference for Invalid Lose-shifts. We could replicate the first point with our model. For Invalid Lose-shifts however, only the median value is the same in our simulation.

3. We also investigated, whether the finding, that individuals with delusional symptoms show a "jumping to conclusions" behaviour summarized by Ross et al. (2015), could be replicated with our model. A qualitative analysis suggests that our model did not reproduce this phenomenon.

Limitations. The model's explanatory power with regard to the experiment is limited due to several simplifying assumptions that were made. Firstly, we interpret the estimated probability of the urns as the decision of the study participant. In reality, a more complex decision-making process could be performed by the participant, especially if the probability is close to 0.5. However, using the probability should at least be a reasonable approximation. Secondly, we represent the input that subjects are given as 0 and 1, while in the actual reversal learning task complex images were used. Therefore the processing of the stimulus and possible uncertainty is not taken into account. The fact that study participants might not remember every data point they received is modeled inherently through the DMM that always processes 5 inputs directly and does not sum up at the end as one might expect a person with perfect memory to do. More complex memory processes are however not taken into account. Another difference between our model and the experiment is that we do not give the subject a positive or negative feedback for their choice, instead, we just show the next data points which confirm or contradict the subject's previous hypotheses. While these assumptions differentiate our simulation from the real-life experiment, they appear to be reasonable and necessary for translational modeling and we do not expect them to bias the model in any specific direction.

6. CONCLUSIONS

Overall, the results of the implemented experiment suggest that a smaller expected precision μ_τ in schizophrenic patients could partially explain observed differences in information processing and decision making. This is in agreement with the results from Erdmann and Mathys (2021). The next step in verifying the plausibility of the suggested mechanism would be to implement additional experiments and compare results with available measured data. It would further be interesting to investigate which biological factors might influence the expected precision of Bayesian interference in the brain to bridge the gap to medical research.

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