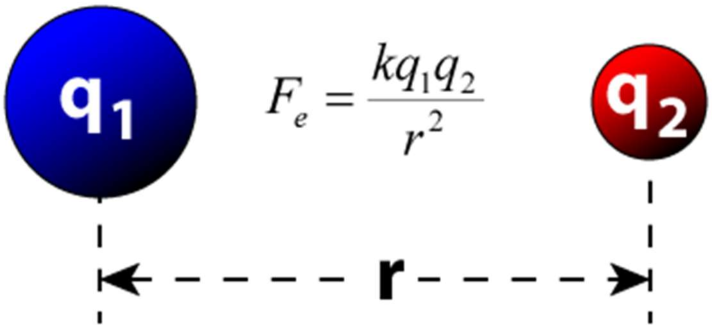
<u>Coulomb's Law</u>



Coulomb's law states that Force exerted between two point charges:

- Is inversely proportional to square of the distance between these charges and
- Is directly proportional to product of magnitude of the two charges
- Acts along the line joining the two point charges.

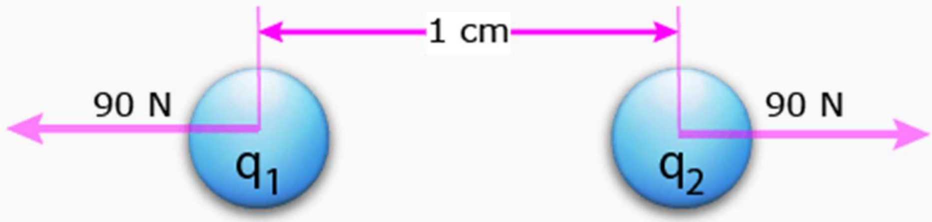
$$F = k\frac{|q_1 q_2|}{r^2} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\text{So, } F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

Here $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$ is called **permittivity of free space**.

Example Problem:
The force between two identical charges separated by 1 cm is equal to 90 N. What is the magnitude of the two charges?

Solution:
First, draw a force diagram of the problem.



Two charges separated by one centimeter experiencing a force of repulsion of 90 N.

Define the variables:
F = 90 N
q_1 = charge of first body
q_2 = charge of second body
r = 1 cm

Use the Coulomb's Law equation

$$F = k\frac{q_1 q_2}{r^2}$$

The problem says the two charges are identical, so

$q_1 = q_2 = q$

Substitute this into the equation

$$F = k\frac{q^2}{r^2}$$

Since we want the charges, solve the equation for q

$$q^2 = \frac{F \cdot r^2}{k}$$

$$q = \sqrt{\frac{F \cdot r^2}{k}}$$

Enter the values of the problem for each variable into this equation. Remember to convert 1 cm to 0.01 meters to keep the units consistent.

$$q = \sqrt{\frac{(90\ \mathrm{N})(0.01\ \mathrm{m})^2}{8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}}}$$

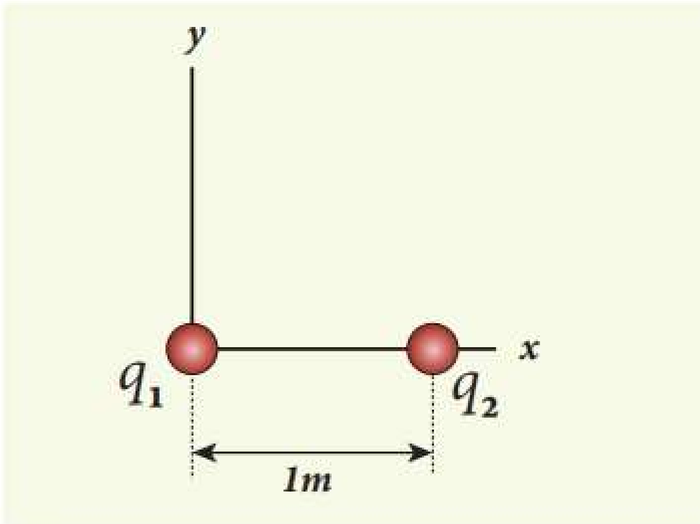
$q = \pm 1.00 \times 10^{-6}$ Coulombs

This equation has two possible answers. The charges can both be positive or both negative and the answer will be the same for the repulsive Columb force over a distance of 1 cm.

Answer:
Two identical charges of $\pm 1.00 \times 10^{-6}$ Coulombs separated by 1 cm produce a repulsive force of 90 N.

EXAMPLE 1.2

Consider two point charges q1 and q2 at rest as shown in the figure.



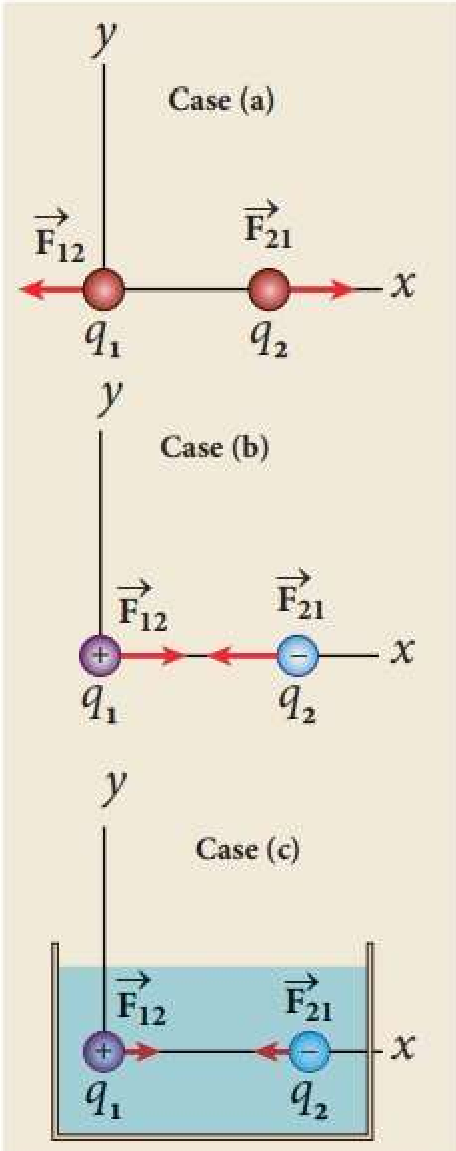
They are separated by a distance of 1m. Calculate the force experienced by the two charges for the following cases:

(a) $q_1 = +2\mu C$ and $q_2 = +3\mu C$

(b) $q_1 = +2\mu C$ and $q_2 = -3\mu C$

(c) $q_1 = +2\mu C$ and $q_2 = -3\mu C$ kept in water ($\varepsilon_r = 80$)

Solution



Case (a)
$\vec{F}_{12}$
$\vec{F}_{21}$
q_1
q_2
x
y
Case (b)
$\vec{F}_{12}$
$\vec{F}_{21}$
q_1
q_2
x
y
Case (c)
$\vec{F}_{12}$
$\vec{F}_{21}$
q_1
q_2
x
y

(a) $q_1 = +2\ \mu C$, $q_2 = +3\ \mu C$, and $r = 1m$. Both are positive charges. so the force will be repulsive

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_\circ}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Here $\hat{r}_{12}$ is the unit vector from q_1 to q_2. Since q_2 is located on the right of q_1, we have

$$\hat{r}_{12} = \hat{i}\text{ , so that}$$

$$\vec{F}_{21} = \frac{9\times10^{9}\times2\times10^{-6}\times3\times10^{-6}}{1^2}\hat{i}\quad\left[\frac{1}{4\pi\varepsilon_\circ} = 9\times10^{9}\right]$$

$$= 54\times10^{-3}N\hat{i}$$

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F}_{12} = -\vec{F}_{21}$

So that $\vec{F}_{12} = -54\times10^{-3}N\hat{i}$.

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the above figure in case (a)

(b) q_1 = +2 μC, q_2 = -3 μC, and r = 1m. They are unlike charges. So the force will be attractive.

Force experienced by the charge q_2 due to q_1 is given by

$$\vec{F}_{21} = \frac{9\times10^9\times\left(2\times10^{-6}\right)\times\left(-3\times10^{-6}\right)}{1^2}\hat{r}_{12}$$
$$= -54\times10^{-3} N\hat{i} \quad (\text{Using } \hat{r}_{12} = \hat{i})$$

The charge q_2 will experience an attractive force towards q_1 which is in the negative x direction.

According to Newton's third law, the force experienced by the charge q_1 due to q_2 is

$$\vec{F}_{12} = -\vec{F}_{21}$$

so that $\vec{F}_{12} = 54\times10^{-3} N\hat{i}$

The directions of $\vec{F}_{21}$ and $\vec{F}_{12}$ are shown in the figure (case (b)).

(c) If these two charges are kept inside the water, then the force experienced by q_2 due to q_1

$$\vec{F}_{21}^{W} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

since $\varepsilon = \varepsilon_r \varepsilon_o$,

we have $$\vec{F}_{21}^{W} = \frac{1}{4\pi\varepsilon_r\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{\vec{F}_{21}}{\varepsilon_r}$$

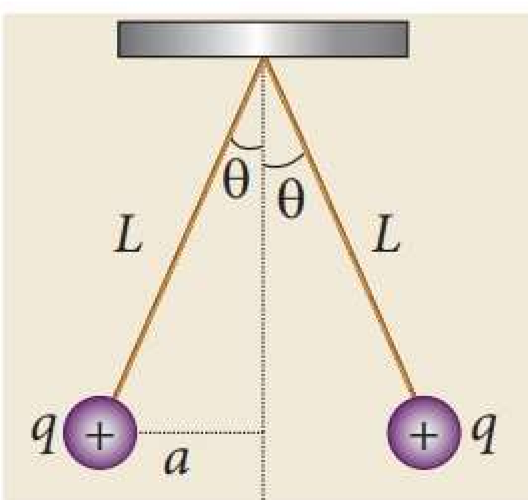
Therefore,

$$\vec{F}_{21}^{W} = -\frac{54\times10^{-3}N}{80}\hat{i} = -0.675\times10^{-3}N\hat{i}$$

EXAMPLE 1.3

Two small-sized identical equally charged spheres, each having mass 1 mg are hanging in equilibrium as shown in the figure. The length of each string is 10 cm and the angle θ is 7° with the vertical. Calculate the magnitude of the charge in each sphere.

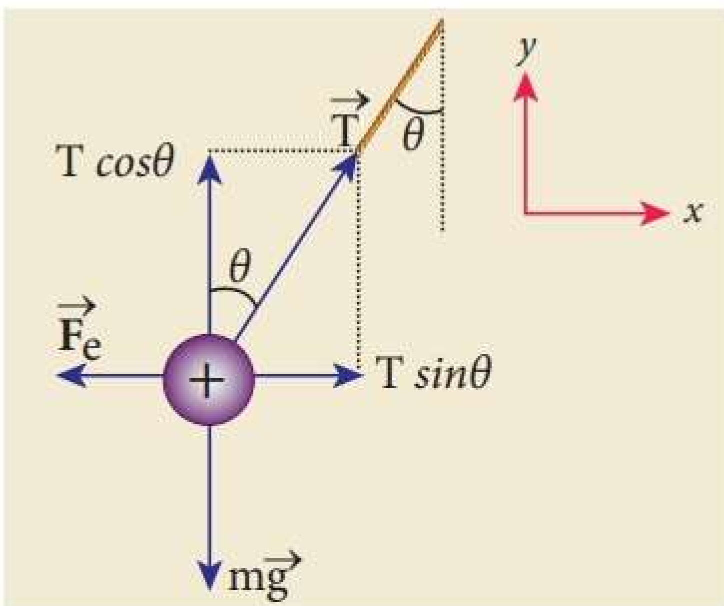
(Take g = 10 ms–2)



Solution

If the two spheres are neutral, the angle between them will be 0o when hanged vertically. Since they are positively charged spheres, there will be a repulsive force between them and they will be at equilibrium with each other at an angle of 7° with the vertical. At equilibrium, each charge experiences zero net force in each direction. We can draw a free body diagram for one of the charged spheres and apply Newton's second law for both vertical and horizontal directions.

The free body diagram is shown below.



In the x-direction, the acceleration of the charged sphere is zero.

Using Newton's second law $\left(\vec{F}_{tot} = m\vec{a}\right)$, we have

$$T\sin\theta\hat{i} - F_e\hat{i} = 0$$

$$T\sin\theta = F_e \qquad (1)$$

Here T is the tension acting on the charge due to the string and F_e is the electrostatic force between the two charges.

In the y-direction also, the net acceleration experienced by the charge is zero.

$$T\cos\theta\,\hat{j} - mg\,\hat{j} = 0$$

Therefore, $T\cos\theta = mg$. (2)

By dividing equation (1) by equation (2),

$$\tan\theta = \frac{F_e}{mg} \qquad (3)$$

Since they are equally charged, the magnitude of the electrostatic force is

$$F_e = k\frac{q^2}{r^2} \text{ where } k = \frac{1}{4\pi\varepsilon_0}$$

Here r = 2a = 2Lsinθ. By substituting these values in equation (3),

$$\tan\theta = k\frac{q^2}{mg\left(2L\sin\theta\right)^2} \qquad (4)$$

Rearranging the equation (4) to get q

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k}}$$

$$= 2\times 0.1\times\sin 7^\circ\times\sqrt{\frac{10^{-3}\times 10\times\tan 7^\circ}{9\times 10^9}}$$

$q = 8.9 \times 10^{-9}$ C = 8.9 nC

EXAMPLE 1.4

Calculate the electrostatic force and gravitational force between the proton and the electron in a hydrogen atom. They are separated by a distance of 5.3×10^{-11} m. The magnitude of charges on the electron and proton are 1.6×10^{-19} C. Mass of the electron is $m_e = 9.1 \times 10^{-31}$ kg and mass of proton is $m_p = 1.6 \times 10^{-27}$ kg.

Solution

The proton and the electron attract each other. The magnitude of the electrostatic force between these two particles is given by

$$F_e = \frac{ke^2}{r^2} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{\left(5.3\times10^{-11}\right)^2}$$

$$= \frac{9\times2.56}{28.09}\times10^{-7} = 8.2 \times 10^{-8}\ \text{N}$$

The gravitational force between the proton and the electron is attractive. The magnitude of the gravitational force between these particles is

$$F_G = \frac{Gm_e m_p}{r^2}$$

$$= \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{\left(5.3\times10^{-11}\right)^2}$$

$$= \frac{97.11}{28.09}\times10^{-47} = 3.4 \text{ x } 10^{-47}\ \text{N}$$

The ratio of the two forces $\frac{F_e}{F_G} = \frac{8.2\times10^{-8}}{3.4\times10^{-47}}$

$$= 2.41 \times 10^{39}$$

Note that $F_e \approx 10^{39} F_G$

The electrostatic force between a proton and an electron is enormously greater than the gravitational force between them. Thus the gravitational force is negligible when compared with the electrostatic force in many situations such as for small size objects and in the atomic domain. This is the reason why a charged comb attracts an uncharged piece of paper with greater force even though the piece of paper is attracted downward by the Earth. This is shown in Figure 1.3

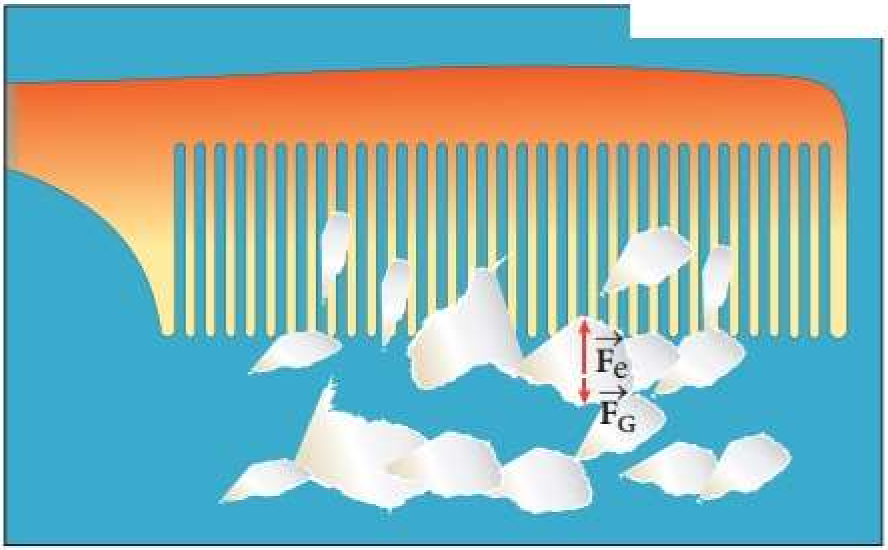
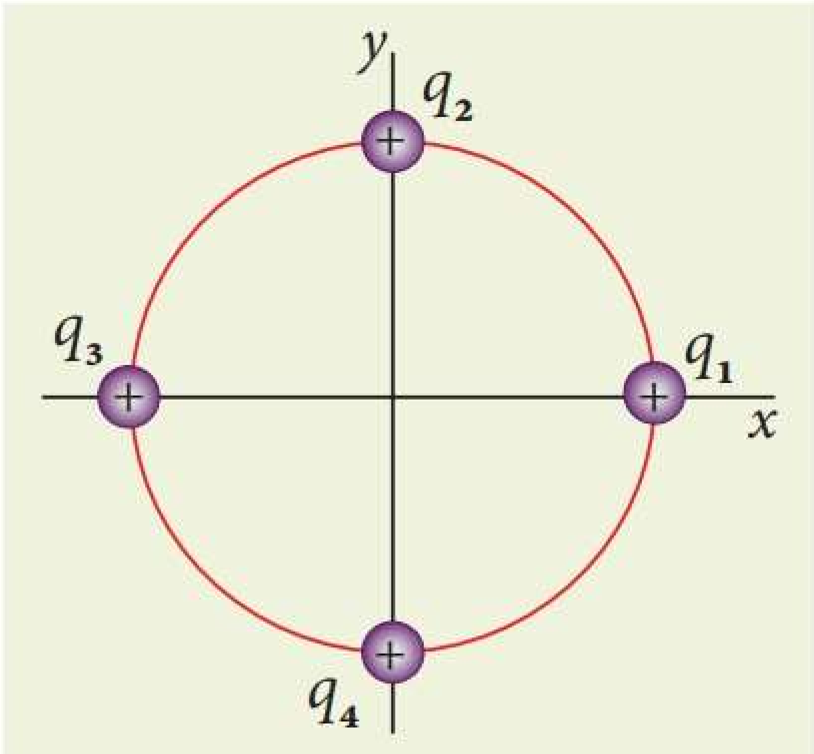


Figure 1.3 Electrostatic attraction between a comb and pieces of papers

EXAMPLE 1.5

Consider four equal charges q_1,q_2, q_3 and q_4 = q = +1µC located at four different points on a circle of radius 1m, as shown in the figure. Calculate the total force acting on the charge q_1 due to all the other charges.

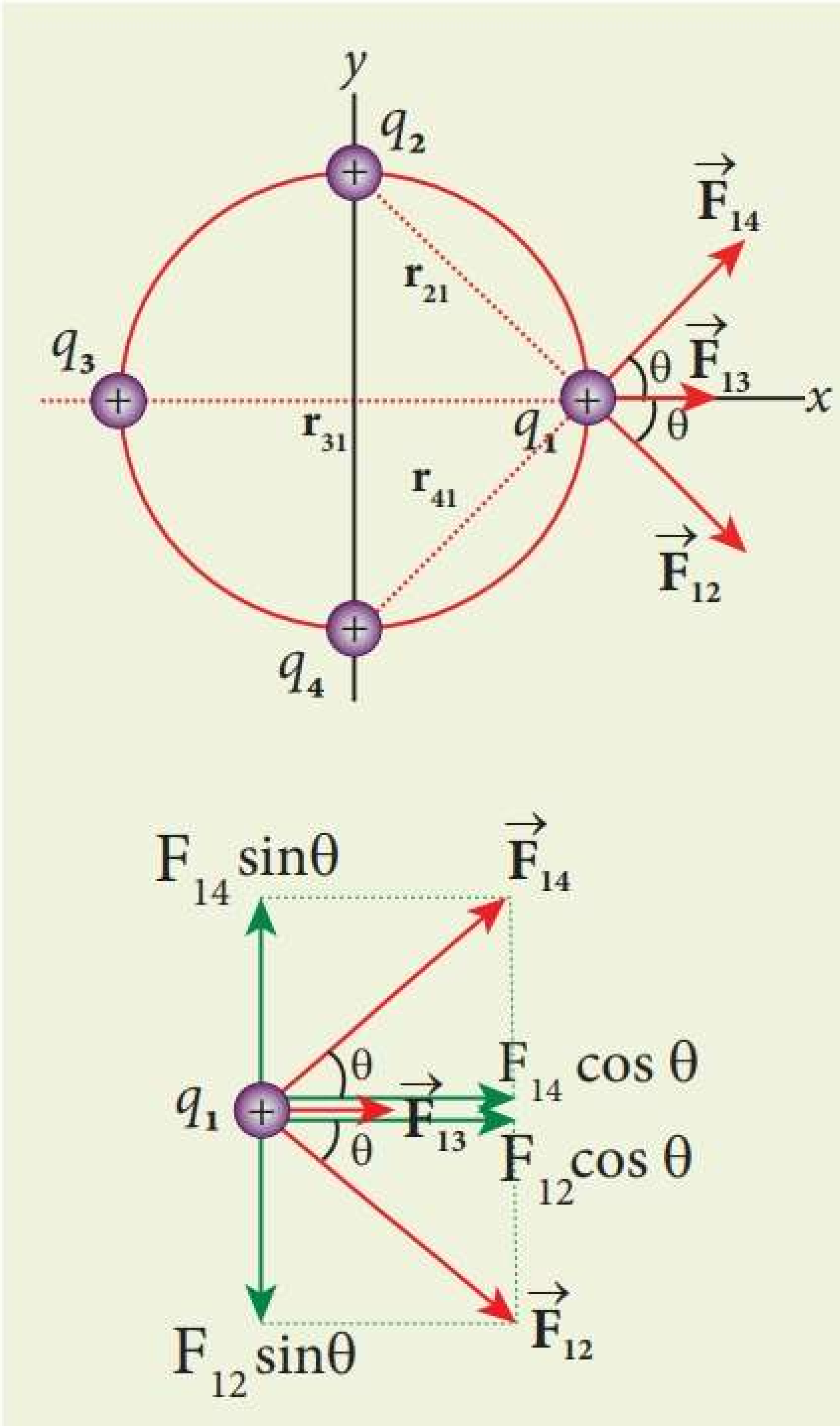


Solution

According to the superposition principle, the total electrostatic force on charge q1 is the vector sum of the forces due to the other charges,

$$\vec{F}_1^{tot} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

The following diagram shows the direction of each force on the charge q1.



The charges q_2 and q_4 are equi-distant from q_1. As a result the strengths (magnitude) of the forces $\vec{F}_{12}$ and $\vec{F}_{14}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F}_{13}$ is lesser than that of forces $\vec{F}_{12}$ and $\vec{F}_{14}$. Hence the vector representing the force $\vec{F}_{13}$ is drawn with smaller length compared to that for forces $\vec{F}_{12}$ and $\vec{F}_{14}$.

From the figure, $r_{21} = \sqrt{2}\,m = r_{41}$ and $r_{31} = 2m$

The magnitudes of the forces are given by

$$F_{13} = \frac{kq^2}{r_{31}^2} = \frac{9 \times 10^9 \times 10^{-12}}{4}$$

$$F_{13} = 2.25 \times 10^{-3}\ N$$

$$F_{12} = \frac{kq^2}{r_{21}^2} = F_{14} = \frac{9 \times 10^9 \times 10^{-12}}{2}$$

$$= 4.5 \times 10^{-3}\ N$$

From the figure, the angle $\theta = 45°$. In terms of the components, we have

$$\vec{F}_{12} = F_{12}\cos\theta\,\hat{i} - F_{12}\sin\theta\,\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} - 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

$$\vec{F}_{13} = F_{13}\,\hat{i} = 2.25\times10^{-3}\,N\,\hat{i}$$

$$\vec{F}_{14} = F_{14}\cos\theta\,\hat{i} + F_{14}\sin\theta\,\hat{j}$$

$$= 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{i} + 4.5\times10^{-3}\times\frac{1}{\sqrt{2}}\hat{j}$$

Then the total force on q_1 is,

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta\,\hat{i} - F_{12}\sin\theta\,\hat{j}\right) + F_{13}\hat{i} + \left(F_{14}\cos\theta\,\hat{i} + F_{14}\sin\theta\,\hat{j}\right)$$

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i} + \left(-F_{12}\sin\theta + F_{14}\sin\theta\right)\hat{j}$$

Since $F_{12} = F_{14}$, the j_{th} component is zero.

Hence we have

$$\vec{F}_1^{tot} = \left(F_{12}\cos\theta + F_{13} + F_{14}\cos\theta\right)\hat{i}$$

substituting the values in the above equation,

$$= \left(\frac{4.5}{\sqrt{2}} + 2.25 + \frac{4.5}{\sqrt{2}} \right) \hat{i} = \left(4.5\sqrt{2} + 2.25 \right) \hat{i}$$

$$\vec{F}_1^{tot} = 8.61 \times 10^{-3} N \hat{i}$$

The resultant force is along the positive x axis.