

Chapter-06: Relevant Costs for Decision Making

4.1: Introduction:

Every decision involves choosing from among at least two alternatives. In making a decision, the costs and benefits of one alternative must be compared to the costs and benefits of other alternatives. Costs that differ between alternatives are called relevant costs. Distinguishing between relevant and irrelevant costs and benefits is critical for two reasons. First, irrelevant data can be ignored—saving decision makers tremendous amounts of time and effort. Second, bad decisions can easily result from erroneously including irrelevant costs and benefits when analyzing alternatives. To be successful in decision making, managers must be able to tell the difference between relevant and irrelevant data and must be able to correctly use the relevant data in analyzing alternatives. The purpose of this chapter is to develop these skills by illustrating their use in a wide range of decision-making situations. These decision-making skills are as important in your personal life as they are to managers.

4.2: Cost Concepts for Decision Making:

Identifying Relevant Costs and Benefits:

Only those costs and benefits that differ in total between alternatives are relevant in a decision. If the total amount of a cost will be the same regardless of the alternative selected, then the decision has no effect on the cost and it can be ignored. For example, if you are trying to decide whether to go to a movie or to rent a videotape for the evening, the rent on your apartment is irrelevant. Whether you go to a movie or rent a videotape, the rent on your apartment will be exactly the same and is therefore irrelevant to the decision. On the other hand, the cost of the movie ticket and the cost of renting the videotape would be relevant in the decision because they are avoidable costs.

An avoidable cost is a cost that can be eliminated in whole or in part by choosing one alternative over another. By choosing the alternative of going to the movie, the cost of renting the videotape can be avoided. By choosing the alternative of renting the videotape, the cost of the movie ticket can be avoided. Therefore, the cost of the movie ticket and the cost of renting the videotape are both avoidable costs. On the other hand, the rent on the apartment is not an avoidable cost of either alternative. You would continue to rent your apartment under either alternative. **Avoidable costs are relevant costs. Unavoidable costs are irrelevant costs.**

Two broad categories of costs are never relevant in decisions. These irrelevant costs are:

1. Sunk costs.
2. Future costs that do not differ between the alternatives.

Sunk cost is a cost that has already been incurred and cannot be avoided regardless of what a manager decides to do. For example, suppose a used car dealer purchased a five-year-old Toyota Camry for \$12,000. The amount paid for the Camry is a sunk cost because it has already been incurred and the transaction cannot be undone. Sunk costs are always the same no matter what alternatives are being considered; therefore, they are irrelevant and should be ignored when

making decisions. Future costs that do not differ between alternatives should also be ignored when making decisions. Continuing with the example discussed earlier, suppose you intend to order a pizza after you go to the movie theater or you rent a video. In that case, if you are going to buy the same pizza regardless of your choice of entertainment, its cost is irrelevant to the choice of whether you go to the movie theater or rent a video. Notice, the cost of the pizza is not a sunk cost because it has not yet been incurred. Nonetheless, the cost of the pizza is irrelevant to the entertainment decision because it is a future cost that does not differ between the alternatives. The term **differential cost** was also introduced in Chapter 2. In managerial accounting, the terms avoidable cost, differential cost, incremental cost, and relevant cost are often used interchangeably. To identify the costs that are avoidable in a particular decision situation and are therefore relevant, these steps should be followed:

1. Eliminate costs and benefits that do not differ between alternatives. These irrelevant costs consist of (a) sunk costs and (b) future costs that do not differ between alternatives.
2. Use the remaining costs and benefits that do differ between alternatives in making the decision. The costs that remain are the differential, or avoidable, costs.

4.3: Different Costs for Different Purposes:

We need to recognize a fundamental concept of managerial accounting from the outset of our discussion—costs that are relevant in one decision situation are not necessarily relevant in another. This means that managers need different costs for different purposes. For one purpose, a particular group of costs may be relevant; for another purpose, an entirely different group of costs may be relevant. Thus, each decision situation must be carefully analyzed to isolate the relevant costs.

a. An Example of Identifying Relevant Costs and Benefits:

Cynthia is currently a student in an MBA program in Boston and would like to visit a friend in New York City over the weekend. She is trying to decide whether to drive or take the train. Because she is on a tight budget, she wants to carefully consider the costs of the two alternatives. If one alternative is far less expensive than the other, that may be decisive in her choice. By car, the distance between her apartment in Boston and her friend's apartment in New York City is 230 miles. Cynthia has compiled the following list of items to consider:

Automobile Data

Item	Annual cost of fixed items	Cost per mile (based on 10,000 miles per year)
(a) Annual straight-line depreciation on car [(\$24,000 original cost - \$10,000 estimated resale value in 5 years)/5 years]	\$2,800	\$0.280
b) Cost of gasoline (\$1.60 per gallon / 32 miles per gallon)		0.050

(c) Annual cost of auto insurance and license	\$1,380	0.138
(d) Maintenance and repairs		0.065
(e) Parking fees at school (\$45 per month * 8 months)	\$360	0.036
(f) Total average cost per mile		<u>\$0.569</u>

Additional Data:

Item	Cost
(g) Reduction in the resale value of car due solely to wear and tear	\$0.026 per mile
(h) Cost of round-trip Amtrak ticket from Boston to New York City	\$104
(i) Benefit of relaxing and being able to study during the train ride rather than having to drive	?
(j) Cost of putting the dog in a kennel while gone	\$40
(k) Benefit of having a car available in New York City	?
(l) Hassle of parking the car in New York City	?
(m) Cost of parking the car in New York City	\$25 per day

Required: Which costs and benefits are relevant in this decision?

Solution:

Particulars	Costs
Relevant financial cost of driving to New York City:	
Gasoline (460 miles at \$0.050 per mile)	\$ 23.00
Maintenance and repairs (460 miles @ \$0.065 per mile)	29.90
Reduction in the resale value of car due solely to wear and tear (460 miles @ \$0.026 per mile)	11.96
Cost of parking the car in New York City (2 days @ \$25 per day)	50.00
Total	<u>\$114.86</u>
Relevant financial cost of taking the train to New York City:	
Cost of round-trip Amtrak ticket from Boston to New York City	<u>\$104.00</u>

Comments: Cynthia would consider the round trip of train from Boston to New York City rather than driving the car because of low costs.

b. Reconciling the Total and Differential Approaches:

Oak Harbor Woodworks is considering a new labor-saving machine that rents for \$3,000 per year. The machine will be used on the company's butcher block production line. Data concerning the company's annual sales and costs of butcher blocks with and without the new machine are shown below:

Particulars	Current situation	Situation with the new machine

Units produced and sold	5,000	5,000
Selling price per unit	\$40	\$40
Direct materials cost per unit	\$14	\$14
Direct labor cost per unit	\$8	\$5
Variable overhead cost per unit	\$2	\$2
Fixed costs, other	\$62,000	\$62,000
Fixed costs, rental of new machine		\$3,000

Required: Which situation should be recommended by the company and why?

Solution:

Particulars	Current Situation	Situation with New Machine	Differential Costs and Benefits
Sales (5,000 units @ \$40 per unit)	<u>\$2,00,000</u>	<u>\$2,00,000</u>	\$0
Variable expenses:			
Direct materials (5,000 units @ \$14 per unit)	70,000	70,000	0
Direct labor (5,000 units @ \$8 and \$5 per unit)	40,000	25,000	15,000
Variable overhead (5,000 units @ \$2 per unit)	<u>10,000</u>	<u>10,000</u>	0
Total variable expenses	<u>1,20,000</u>	<u>1,05,000</u>	
Contribution margin (Sales – Total variable expenses)	<u>80,000</u>	<u>95,000</u>	
Fixed expenses:			
Other	62,000	62,000	0
Rent of new machine	<u>0</u>	<u>3,000</u>	(3,000)
Total fixed expenses	<u>62,000</u>	<u>65,000</u>	
Net operating income (Contribution margin – Total fixed expenses)	<u>\$18,000</u>	<u>\$30,000</u>	<u>\$12,000</u>

Indeed, the only costs that do differ between the alternatives are direct labor costs and the fixed rental cost of the new machine. Hence, the two alternatives can be compared based only on these relevant costs:

Particulars	Amount
Net Advantage of Renting the New Machine:	
Decrease in direct labor costs (5,000 units at a cost savings of \$3 per unit)	\$15,000
Increase in fixed expenses	(3,000)
Net annual cost savings from renting the new machine	<u>\$12,000</u>

Those two relevant costs are both listed in the above analysis showing the net advantage of renting the new machine.

4.4: Why Isolate Relevant Costs?

In the preceding example, we used two different approaches to analyze the alternatives. First, we considered all costs, both those that were relevant and those that were not; and second, we considered only the relevant costs. We obtained the same answer under both approaches. It would be natural to ask, “Why bother to isolate relevant costs when total costs will do the job just as well?” Isolating relevant costs is desirable for at least two reasons. First, only rarely will enough information be available to prepare a detailed income statement for both alternatives. Assume, for example, that you are called on to make a decision relating to a portion of a single business process in a multi departmental, multiproduct company. Under these circumstances, it would be virtually impossible to prepare an income statement of any type. You would have to rely on your ability to recognize which costs are relevant and which are not in order to assemble the data necessary to make a decision. Second, mingling irrelevant costs with relevant costs may cause confusion and distract attention from the information that is really critical. Furthermore, the danger always exists that an irrelevant piece of data may be used improperly, resulting in an incorrect decision. The best approach is to ignore irrelevant data and base the decision entirely on relevant data. Relevant cost analysis, combined with the contribution approach to the income statement, provides a powerful tool for making decisions.

4.5: Adding and Dropping Product Lines and Other Segments:

Decisions relating to whether product lines or other segments of a company should be dropped and new ones added are among the most difficult that a manager has to make. In such decisions, many qualitative and quantitative factors must be considered. Ultimately, however, any final decision to drop a business segment or to add a new one is going to hinge primarily on the impact the decision will have on net operating income. To assess this impact, costs must be carefully analyzed.

An Illustration of Cost Analysis

Exhibit below provides sales and cost information for the preceding month for the Discount Drug Company and its three major product lines—drugs, cosmetics, and housewares. A quick review of this exhibit suggests that dropping the housewares segment would increase the company’s overall net operating income by \$8,000. However, this would be a flawed conclusion because the data in Exhibit 13–2 do not distinguish between fixed expenses that can be avoided if a product line is dropped and common fixed expenses that cannot be avoided by dropping any particular product line.

In this scenario, the two alternatives under consideration are keeping the housewares product line and dropping the housewares product line. Therefore, only those costs that differ between these two alternatives (i.e., that can be avoided by dropping the housewares product line) are relevant. In deciding whether to drop housewares, it is crucial to identify which costs can be avoided, and hence are relevant to the decision, and which costs cannot be avoided, and hence are irrelevant.

The decision should be analyzed as follows:

If the housewares line is dropped, then the company will lose \$20,000 per month in contribution margin, but by dropping the line it may be possible to avoid some fixed costs such as salaries or advertising costs. If dropping the housewares line enables the company to avoid more in fixed costs than it loses in contribution margin, then its overall net operating income will improve by eliminating the product line. On the other hand, if the company is not able to avoid as much in fixed costs as it loses in contribution margin, then the housewares line should be kept. In short, the manager should ask, “What costs can I avoid if I drop this product line?” As we have seen

from our earlier discussion, not all costs are avoidable. For example, some of the costs associated with a product line may be sunk costs. Other costs may be allocated fixed costs that will not differ in total regardless of whether the product line is dropped or retained.

Prepare an analysis showing whether a product line or other business segment should be dropped or retained.

E X H I B I T: Discount Drug Company Product Lines

Particulars	Product Lines			
	Total	Drugs	Cosmetics	Housewares
Sales	\$2,50,000	\$1,25,000	\$75,000	\$50,000
Less: Variable expenses	<u>1,05,000</u>	<u>50,000</u>	<u>25,000</u>	<u>30,000</u>
Contribution margin	<u>1,45,000</u>	<u>75,000</u>	<u>50,000</u>	<u>20,000</u>
Less: Fixed expenses:				
Salaries	50,000	29,500	12,500	8,000
Advertising	15,000	1,000	7,500	6,500
Utilities	2,000	500	500	1,000
Depreciation—fixtures	5,000	1,000	2,000	2,000
Rent	20,000	10,000	6,000	4,000
Insurance	3,000	2,000	500	500
General administrative	30,000	15,000	9,000	6,000
Total fixed expenses	<u>1,25,000</u>	<u>59,000</u>	<u>38,000</u>	<u>28,000</u>
Net operating income (loss)	<u>\$ 20,000</u>	<u>\$ 16,000</u>	<u>\$12,000</u>	<u>\$ (8,000)</u>

To show how to proceed in a product-line analysis, suppose that Discount Drug Company has analyzed the fixed costs being charged to the three product lines and has determined the following:

1. The salaries expense represents salaries paid to employees working directly on the product. All of the employees working in housewares would be discharged if the product line is dropped.
2. The advertising expense represents advertisements that are specific to each product line and are avoidable if the line is dropped.
3. The utilities expense represents utilities costs for the entire company. The amount charged to each product line is an allocation based on space occupied and is not avoidable if the product line is dropped.
4. The depreciation expense represents depreciation on fixtures used to display the various product lines. Although the fixtures are nearly new, they are custom-built and will have no resale value if the housewares line is dropped.
5. The rent expense represents rent on the entire building housing the company; it is allocated to the product lines on the basis of sales dollars. The monthly rent of \$20,000 is fixed under a long term lease agreement.
6. The insurance expense is for insurance carried on inventories within each of the three product lines. If housewares is dropped, the related inventories will be liquidated and the insurance premiums will decrease accordingly.
7. The general administrative expense represents the costs of accounting, purchasing, and general management, which are allocated to the product lines on the basis of sales dollars. These costs will not change if the housewares line is dropped.

With this information, management can determine that \$15,000 of the fixed expenses associated with the housewares product line are avoidable and \$13,000 are not:

Fixed Expenses	Total Cost Assigned to Housewares	Not Avoidable*	Avoidable
Salaries	\$ 8,000		\$ 8,000
Advertising	6,500		6,500
Utilities	1,000	1,000	
Depreciation—fixtures	2,000	2,000	
Rent	4,000		4,000
Insurance	500		500
General administrative	6,000	6,000	
Total	\$28,000	\$13,000	\$15,000

[*These fixed costs represent either sunk costs or future costs that will not change whether the housewares line is retained or discontinued.]

As stated earlier, if the housewares product line were dropped, the company would lose the product's contribution margin of \$20,000, but would save its associated avoidable fixed expenses. We now know that those avoidable fixed expenses total \$15,000. Therefore, dropping the housewares product line would result in a **\$5,000 reduction** in net operating income as shown below:

Contribution margin lost if the housewares line is discontinued.....	\$(20,000)
Less fixed costs that can be avoided if the housewares line is discontinued.....	<u>15,000</u>
Decrease in overall company net operating income.....	<u>\$(5,000)</u>

In this case, the fixed costs that can be avoided by dropping the housewares product line (\$15,000) are less than the contribution margin that will be lost (\$20,000). Therefore, based on the data given, the housewares line should not be discontinued unless a more profitable use can be found for the floor and counter space that it is occupying.

4.6: A Comparative Format:

This decision can also be approached by preparing comparative income statements showing the effects of either keeping or dropping the product line. Exhibit below contains such an analysis for the Discount Drug Company. As shown in the last column of the exhibit, if the housewares line is dropped, then overall company net operating income will decrease by \$5,000 each period. This is the same answer, of course, as we obtained when we focused just on the lost contribution margin and avoidable fixed costs.

Particulars	Keep Housewares	Drop Housewares	Difference: Net Operating Income Increase (or Decrease)
Sales	\$50,000	\$ 0	\$(50,000)
Less: Variable expenses	30,000	0	30,000

Contribution margin	<u>20,000</u>	<u>0</u>	<u>(20,000)</u>
Fixed expenses:			
Salaries	8,000	0	8,000
Advertising	6,500	0	6,500
Utilities	1,000	1,000	0
Depreciation—fixtures	2,000	2,000	0
Rent	4,000	4,000	0
Insurance	500	0	500
General administrative.	<u>6,000</u>	<u>6,000</u>	<u>0</u>
Total fixed expenses	<u>28,000</u>	<u>13,000</u>	<u>15,000</u>
Net operating income (loss) [Contribution Margin – Total Fixed expenses]	<u>\$(8,000)</u>	<u>\$(13,000)</u>	<u>\$(5,000)</u>

E X H I B I T: Discount Drug Company Product Lines—Recast in Contribution Format

Particulars	Product Lines			
	Total	Drugs	Cosmetics	Housewares
Sales	\$2,50,000	\$1,25,000	\$75,000	\$50,000
Less: Variable expenses	<u>1,05,000</u>	<u>50,000</u>	<u>25,000</u>	<u>30,000</u>
Contribution margin	<u>1,45,000</u>	<u>75,000</u>	<u>50,000</u>	<u>20,000</u>
Less: Traceable fixed expenses:				
Salaries	50,000	29,500	12,500	8,000
Advertising	15,000	1,000	7,500	6,500
Depreciation—fixtures	5,000	1,000	2,000	2,000
Insurance	<u>3,000</u>	<u>2,000</u>	<u>500</u>	<u>500</u>
Total traceable fixed expenses	<u>73,000</u>	<u>33,500</u>	<u>22,500</u>	<u>17,000</u>
Product-line segment margin	<u>\$ 72,000</u>	<u>\$ 41,500</u>	<u>\$ 27,500</u>	<u>\$ 3,000*</u>
Common fixed expenses:				
Utilities	2,000			
Rent	20,000			
General administrative	30,000			
Total common fixed expenses	<u>52,000</u>			
Net operating income	<u>\$ 20,000</u>			

[*If the housewares line is dropped, this \$3,000 in segment margin will be lost to the company. In addition, we have seen that the \$2,000 depreciation on the fixtures is a sunk cost that cannot be avoided. The sum of these two figures (\$3,000 _ \$2,000 _ \$5,000) would be the decrease in the company's overall profits if the housewares line were discontinued. Of course, the company may later choose to drop the product if circumstances change—such as a pending decision to replace the fixtures.]

By keeping the line, the company's overall net operating income will be higher than if the product line were dropped. Additionally, managers may choose to retain an unprofitable product line if the line helps sell other products or if it serves as a “magnet” to attract customers. Bread, for example, may not be an especially profitable line in some food stores, but customers expect it

to be available, and many of them would undoubtedly shift their buying elsewhere if a particular store decided to stop carrying it.

EXERCISE**Exc-01: Review Problem:**

Charter Sports Equipment manufactures round, rectangular, and octagonal trampolines. Sales and expense data for the past month follow:

Particulars	Trampoline			
	Total	Round	Rectangular	Octagonal
Sales	\$1,000,000	\$140,000	\$500,000	\$360,000
Less: Variable expenses	<u>410,000</u>	<u>60,000</u>	<u>200,000</u>	<u>150,000</u>
Contribution margin	<u>590,000</u>	<u>80,000</u>	<u>300,000</u>	<u>210,000</u>
Less: Fixed expenses:				
Advertising-traceable	216,000	41,000	110,000	65,000
Depreciation of special equipment	95,000	20,000	40,000	35,000
Line supervisors' salaries	19,000	6,000	7,000	6,000
General factory overhead*	200,000	28,000	100,000	72,000
Total fixed expenses	<u>530,000</u>	<u>95,000</u>	<u>257,000</u>	<u>178,000</u>
Net operating income (loss)	<u>\$60,000</u>	<u>\$(15,000)</u>	<u>\$43,000</u>	<u>\$32,000</u>

Management is concerned about the continued losses shown by the round trampolines and wants a recommendation as to whether or not the line should be discontinued. The special equipment used to produce the trampolines has no resale value. If the round trampoline model is dropped, the two line supervisors assigned to the model would be discharged.

Required:

- Should production and sale of the round trampolines be discontinued? The company has no other use for the capacity now being used to produce the round trampolines. Show computations to support your answer.
- Recast the above data in a format that would be more useful to management in assessing the profitability of the various product lines.

Solution:

- No, production and sale of the round trampolines should not be discontinued. Computations to support this answer follow:

Contribution margin lost if the round trampolines are discontinued.....\$(80,000)

Less: fixed costs that can be avoided:

Advertising-traceable.....\$41,000

Line supervisors' salaries..... 6,000 47,000

Decrease in net operating income for the company as a whole.... \$(33,000)

The depreciation of the special equipment represents a sunk cost, and therefore it is not relevant to the decision. The general factory overhead is allocated and will presumably continue regardless of whether or not the round trampolines are discontinued; thus, it is not relevant.

Advanced Management Accounting

- b) If management wants a clearer picture of the profitability of the segments, the general factory overhead should not be allocated. It is a common cost and therefore should be deducted from the total product-line segment margin. A more useful income statement format would be as follows:

Particulars	Trampoline			
	Total	Round	Rectangular	Octagonal
Sales	\$1,000,000	\$140,000	\$500,000	\$360,000
Less: Variable expenses	<u>410,000</u>	<u>60,000</u>	<u>200,000</u>	<u>150,000</u>
Contribution margin	<u>590,000</u>	<u>80,000</u>	<u>300,000</u>	<u>210,000</u>
Less: Fixed expenses:				
Advertising-traceable	216,000	41,000	110,000	65,000
Depreciation of special equipment	95,000	20,000	40,000	35,000
Line supervisors' salaries	<u>19,000</u>	<u>6,000</u>	<u>7,000</u>	<u>6,000</u>
Total traceable fixed expenses	<u>330,000</u>	<u>67,000</u>	<u>157,000</u>	<u>106,000</u>
Product-line segment margin	260,000	<u>\$13,000</u>	<u>\$143,000</u>	<u>\$104,000</u>
General factory overhead*	200,000			
Net operating income (loss)	<u>\$60,000</u>			

Exc-2: Dropping or Retaining a Segment:

Jackson County Senior Services is a nonprofit organization devoted to providing essential services to seniors who live in their own homes within the Jackson County area. Three services are provided for seniors—home nursing, meals on wheels, and housekeeping. In the home nursing program, nurses visit seniors on a regular basis to check on their general health and to perform tests ordered by their physicians. The meals on wheels program deliver a hot meal once a day to each senior enrolled in the program. The housekeeping service provides weekly housecleaning and maintenance services. Data on revenue and expenses for the past year follow:

Particulars	Total	Home Nursing	Meals on Wheels	House-keeping
Revenues	\$9,00,000	\$260,000	\$4,00,000	\$2,40,000
Less: Variable expenses	<u>4,90,000</u>	<u>1,20,000</u>	<u>2,10,000</u>	<u>1,60,000</u>
Contribution margin	<u>4,10,000</u>	<u>1,40,000</u>	<u>1,90,000</u>	<u>80,000</u>
Less: Fixed expenses:				
Depreciation	68,000	8,000	40,000	20,000
Liability insurance	42,000	20,000	7,000	15,000
Program administrators' salaries	1,15,000	40,000	38,000	37,000
General administrative overhead*	1,80,000	52,000	80,000	48,000
Total fixed expenses	<u>4,05,000</u>	<u>1,20,000</u>	<u>1,65,000</u>	<u>1,20,000</u>
Net operating income (loss)	<u>\$5,000</u>	<u>\$20,000</u>	<u>\$25,000</u>	<u>\$(40,000)</u>

*Allocated on the basis of program revenues.

The head administrator of Jackson County Senior Services, Judith Miyama, is concerned about the organization's finances and considers the net operating income of \$5,000 last year to be a

thin. (Last year's results were very similar to the results for previous years and are representative of what would be expected in the future.) She feels that the organization should be building its financial reserves at a more rapid rate in order to prepare for the next inevitable recession. After seeing the above report, Ms. Miyama asked for more information about the financial advisability of perhaps discontinuing the housekeeping program. The depreciation in housekeeping is for a small van that is used to carry the housekeepers and their equipment from job to job. If the program were discontinued, the van would be donated to a charitable organization. Depreciation charges assume zero salvage value. None of the general administrative overhead would be avoided if the housekeeping programs were dropped, but the liability insurance and the salary of the program administrator would be avoided.

Required:

- Should the housekeeping program be discontinued? Explain. Show computations to support your answer.
- Recast the above data in a format that would be more useful to management in assessing the long-run financial viability of the various services.

Exc-3: Sell or Process Further:

Solex Company manufactures three products from a common input in a joint processing operation. Joint processing costs up to the split-off point total \$100,000 per year. The company allocates these costs to the joint products on the basis of their total sales value at the split-off point. These sales values are as follows: **product X, \$50,000; product Y, \$90,000; and product Z, \$60,000.** Each product may be sold at the split-off point or processed further. Additional processing requires no special facilities. The additional processing costs and the sales value after further processing for each product (on an annual basis) are shown below:

Product	Additional Processing Costs	Sales Value
X	\$35,000	\$80,000
Y	\$40,000	\$150,000
Z	\$12,000	\$75,000

Required: Which product or products should be sold at the split-off point, and which product or products should be processed further? Show computations.