Ch. 3: z-Transform

$$\underline{\underline{X}}(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}, \quad \underline{z = re^{jw}}$$

Ex. $x(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

$n = -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

$$X(z) = x(-2)\cdot z^{2} + x(-1)\cdot z^{1} + x(0)\cdot z^{-0} + x(1)\cdot z^{-1} + x(2)\cdot z^{-2} + x(3)\, z^{-3}$$

$$X(z) = z^{2} + 2z + 5 + 7z^{-1} + z^{-3}$$

ROC: all values of z except $z = 0, \infty$.

Ex. 3.1.2

Determine the z-transform of the signal

$$x(n) = \left(\frac{1}{2}\right)^n u(n) \qquad u(n) = \underset{n=0\ 1\ 2}{\{1, 1, 1, \cdots\}}$$

$$x(n) = \left\{1, \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \cdots\right\}$$

$$n = -1, 0, 1, 2, 3, \cdots$$

$$X(z) = 1 + \frac{1}{2}z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \cdots$$

$$= 1 + A + A^2 + A^3 + \cdots$$

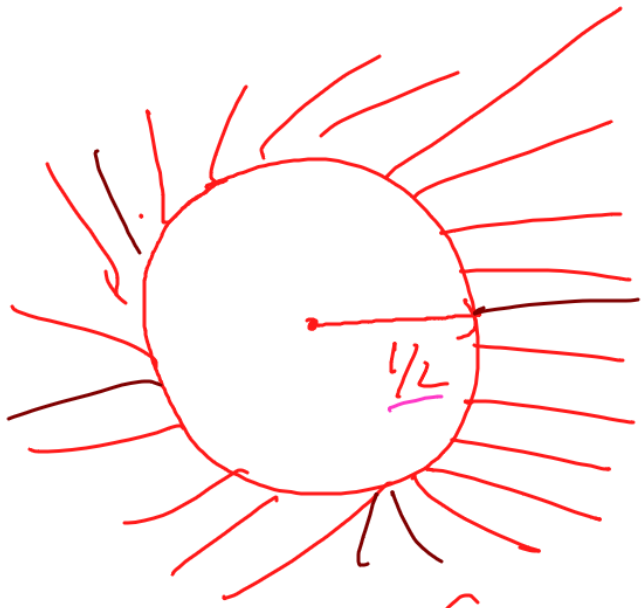
$$= \frac{1}{1-A}, \quad \text{for } A < 1,$$

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$|A| < 1$

$= \left|\frac{1}{2}z^{-1}\right| < 1$

$= z > \frac{1}{2} \rightarrow$ ROC



ROC

[For a right-hand signal the ROC is outside the circle]

3.1.3

Ex. Determine the z-Transform of the signal

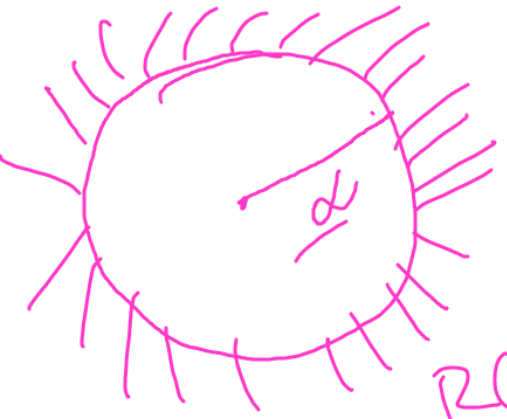
$$x(n) = \alpha^n u(n) \quad \begin{cases} \alpha^n, & n \geq 0 \\ 0, & n < 0 \end{cases} \rightarrow \text{right-hand side signal}$$

$$X(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$

$$1 + A + A^2 + A^3 + \cdots$$

$$\frac{1}{1-A}$$

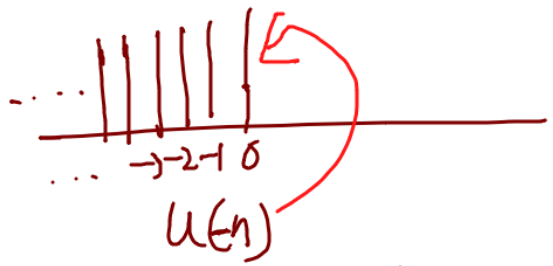
$$X(z) = \frac{1}{1-\alpha z^{-1}}$$



$|\alpha z^{-1}| < 1 \rightarrow |z| > \alpha$. [ROC]

Ex. 3.1.4

$$x(n) = -\alpha^n u(-n-1) \begin{cases} 0, & n \geq 0 \\ -\alpha^n, & n \leq -1 \end{cases}$$



$$X(z) = \sum_{n=-\infty}^{-1} (-\alpha^n) z^{-n}$$

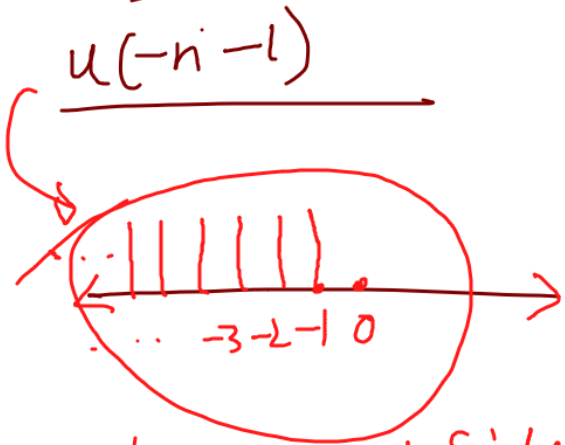
$$= -\sum_{n=-\infty}^{-1} (\alpha^{-1} z)^{-n} = \sum_{l=1}^{\infty} (\alpha^{-1} z)^{l} \qquad l = -n$$

$$A = \alpha^{-1} z$$

$$= A + A^2 + A^3 + \cdots$$

$$= A \cdot (1 + A + A^2 + A^3 + \cdots)$$

$$= \frac{A}{1 - A}.$$



$$X(z) = -\frac{A}{1-A}$$

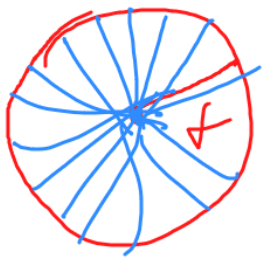
$$= \frac{-\alpha^{-1}z}{1-\alpha^{-1}z} = -\frac{1}{1-\alpha^{-1}z}$$

$$= \frac{1}{1-\alpha z^{-1}}, \quad |\alpha^{-1}z| < 1$$

$$\text{ROC}: |z| < \alpha$$

$x(n)$

ROC



[ROC of a left-hand side signal is the interior of a circle]

Ex. 3.1.5

$$x(n) = \alpha^n u(n) + b^n u(-n-1)$$

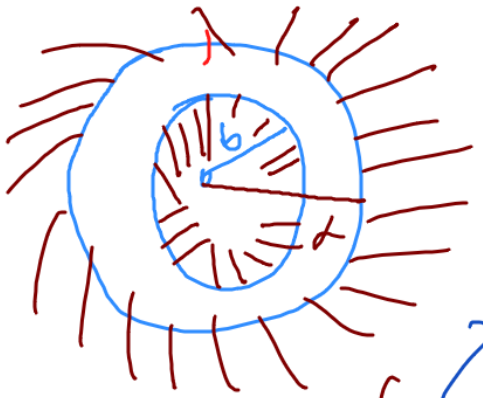
$$X(z) = \frac{1}{1-\alpha z^{-1}} - \frac{1}{1-b z^{-1}}$$

ROC: $|z| > \alpha$

ROC: $|z| < b$

Case 1: $|b| < |\alpha|$

ROC does not exist!



ROC → does not exist.

Case 2: $|b| > |\alpha|$

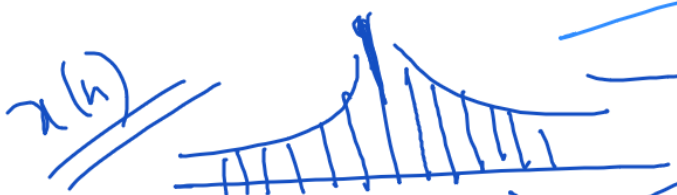
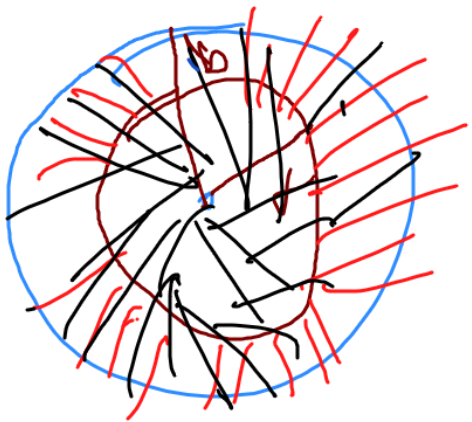


Table 3?



ROC: $|\alpha| < |z| < |b|$