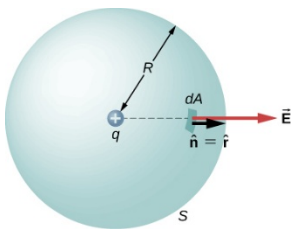
Electric Flux:

The net number of electric field lines leaving out of any closed surface is called electric flux of that surface.



$\phi = \oint \mathbf{E.dA}$

Where, ϕ = Electric flux on Gaussian Surface., E = Electric field

dA = Small surface area

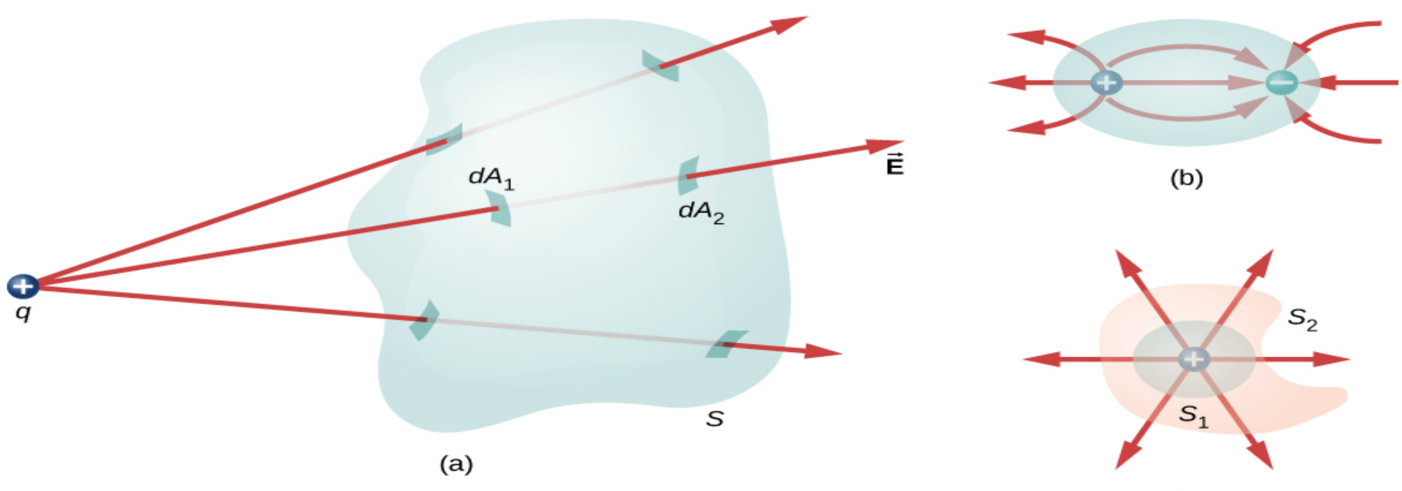
Gauss Law:

Gauss's law states the electric flux of any closed surface is proportional to the net electric charge inside that volume.

$\phi \propto q$ or, $\phi = q/\epsilon_0$.

Where, ϵ_0 = Electric constant (Permittivity of free space)

q = Total charge within the given surface,



Understanding the flux in terms of field lines. (a) The electric flux through a closed surface due to a charge outside that surface is zero. (b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero. (c) The shape and size of the surfaces that enclose a charge does not matter because all surfaces enclosing the same charge have the same flux

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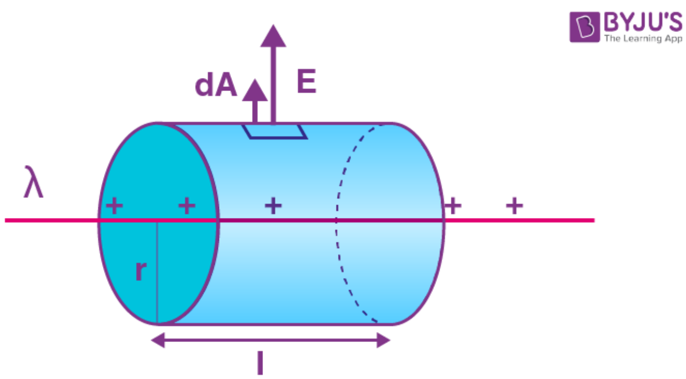
Application of Gauss Law

(1)_Electric Field due to Infinite Wire:

Consider an infinitely long line of charge with the charge per unit length being λ. We can take advantage of the cylindrical symmetry of this situation. The electric fields all point radially away from the line of charge, there is no component parallel to the line of charge.

We can use a cylinder (with an arbitrary radius (r) and length (l)) centered on the line of charge as our Gaussian surface.



As you can see in the above diagram, the electric field is perpendicular to the curved surface of the cylinder. Thus, the angle between the electric field and area vector is zero and $\cos\theta = 1$

The top and bottom surfaces of the cylinder lie parallel to the electric field. Thus the angle between area vector and the electric field is 90 degrees and $\cos\theta = 0$.

Thus, the electric flux is only due to the curved surface

According to Gauss Law,

$\phi = \phi_{\text{curved}} + \phi_{\text{left}} + \phi_{\text{right}}$

$\phi = \int \mathbf{E} \,.\, \mathbf{dA} \cos 0° + \int \mathbf{E} \,.\, \mathbf{dA} \cos 90° + \int \mathbf{E} \,.\, \mathbf{dA} \cos 90°$

$\phi = \int E\, dA = E\,(2\pi rl)$

According to Gauss's law

$\phi = q/\epsilon_0$

or, $\phi = \int E\, dA = E\,(2\pi rl) = q/\epsilon_0$

$E\,(2\pi rl) = q/\epsilon_0 = \lambda l/\epsilon_0$

$E = \lambda/2\pi r\varepsilon_0$

<u>Problems on Gauss Law</u>

Calculate the electric flux through each Gaussian surface shown in the following figures.

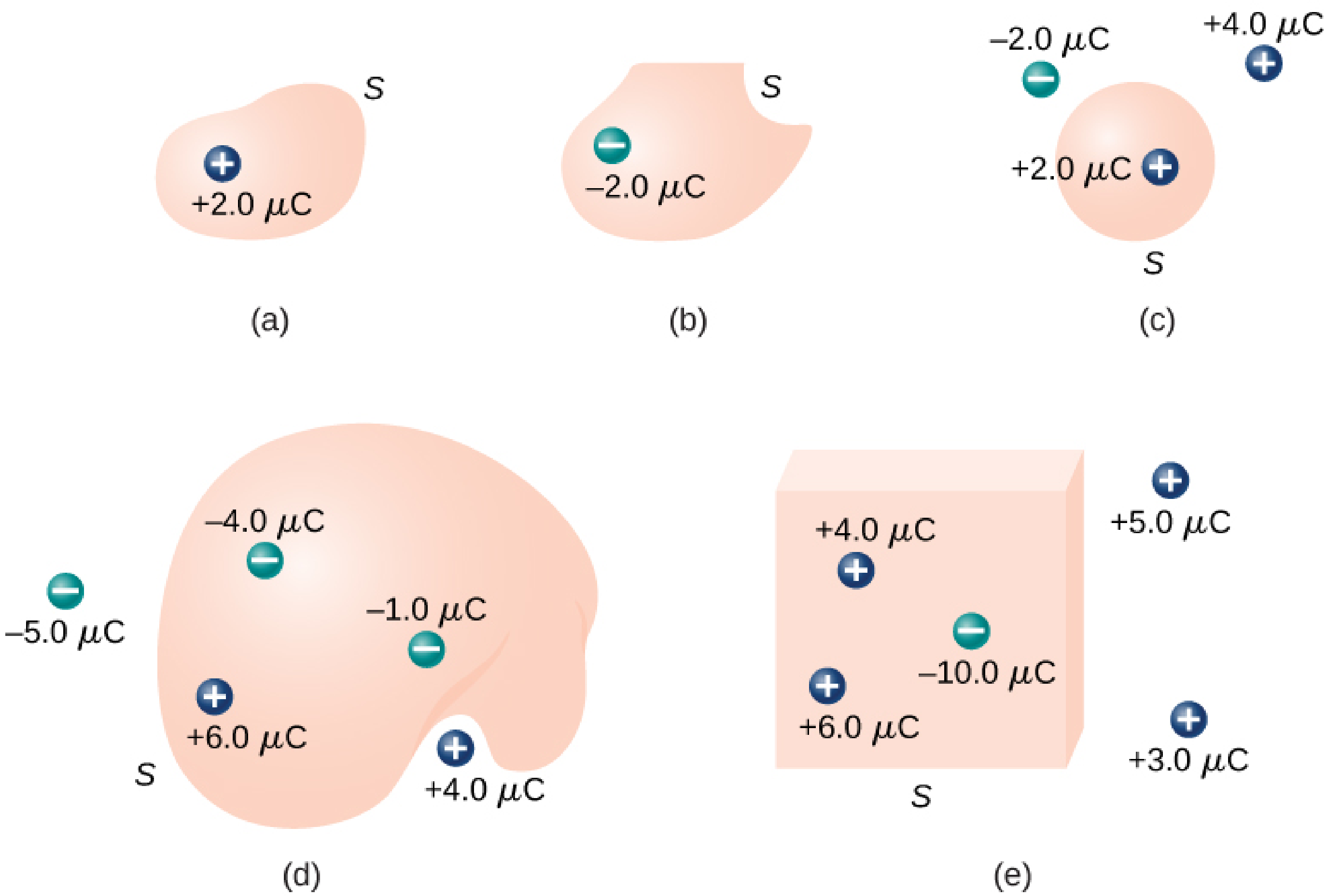


Figure 6.3.76.3.7: Various Gaussian surfaces and charges.
Solution: Using the formula $\phi = q/\epsilon_0$ we find

a. $\phi = 2.0\mu C/\epsilon 0 = 2.3\times10^5$ N·m²/C.

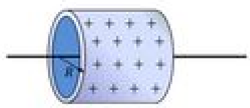
b. $\phi = -2.0\mu C/\epsilon 0 =-2.3\times10^5$ N·m²/C.

c. $\phi = 2.0\mu C/\epsilon 0 = 2.3\times10^5$ N·m²/C.

d. $\phi = (-4.0\mu C+6.0\mu C-1.0\mu C)/\epsilon 0=1.1\times10^5$ N·m²/C.

e. $\phi = (4.0\mu C+6.0\mu C-10.0\mu C)/\epsilon 0 = 0$.

Problem solving - Flux and Gauss' law



Consider an infinitely long, very thin metal tube with radius $R=2.90\ \mathrm{cm}$. The above figure shows a section of it. If the linear charge density of the cylinder is $\lambda=1.50\times10^{-8}\ \mathrm{C/m}$, what is the approximate magnitude of the electric field at radial distance $r=2R$?