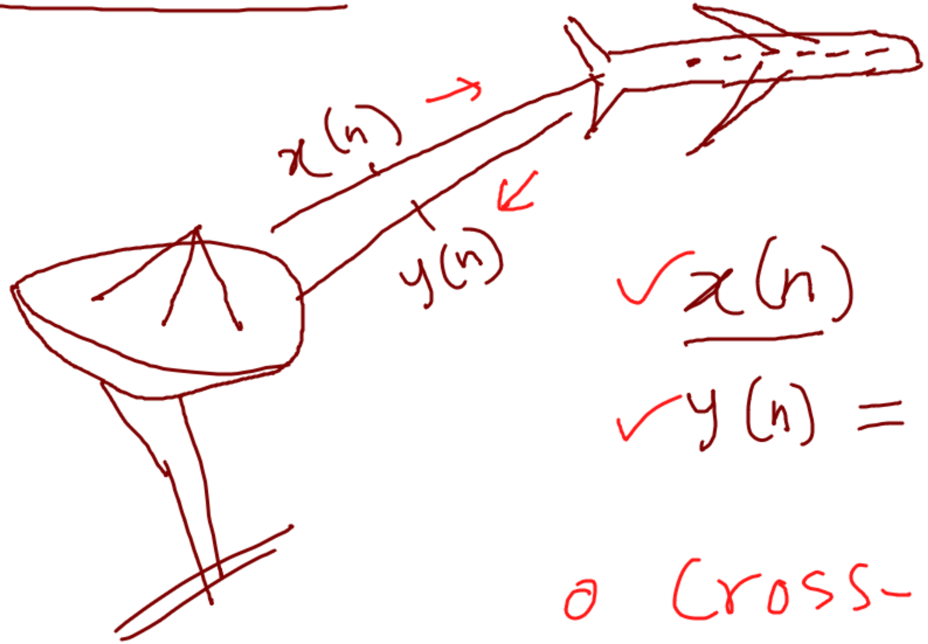
2.6 Corellation of Discrete-Time Signals



$x(n)$

$y(n) = \alpha x(n-D) + w(n)$

- Cross-corellation
- Auto-correlation

Cross-Correlation

The cross-correlation of $x(n)$ and $y(n)$ is a sequence $r_{xy}(l)$, which is defined as

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l),$$

folded

$$y(n) = \sum x(k)\, h(n-k)$$

convolution

cross-corretion

$$\boxed{y(n) = x(n) * h(-n)} \qquad y(n) = x(n) * h(n)$$

Ex. 2.6.1 $\qquad r_{xy}(l) = \sum_{n} x(n)\, y(n-l)$

$x(n) = \{\cdots 0, 0, 2, -1, 3, 7, 1, 2, -3, 0 \cdots\}$

$y(n) = \{\cdots 0, 0, 1, -1, 2, -2, 4, 1, -2, 5, \cdots\}$

$r_{xy}(0) = 2 + 1 + 6 - 14 + 4 + 2 + 6 = 7$ ✓

$r_{xy}(1) = -1 - 3 + 14 - 2 + 8 - 3 = 13, \quad r_{xy}(2) = -18 \cdots$

$r_{xy}(-1) = -2 - 2 - 6 + 28 + 1 - 4 - 15 = 0, \quad r_{xy}(-2) = 33 \cdots$

Similarities btw^n cross-corr and convolution

In convolution: folding operation is applied on the signal before ~~mu~~ the multiplication step.

In cross-correlation: folding operation is not applied.

$$y(n) = x(n) * y(n) \longrightarrow \text{convolution}$$

$$y(n) = x(n) * y(-n) \longrightarrow \text{cross-correation}$$

Auto-correation

The auto-correlation of $x(n)$ is defined as the sequence

$$r_{xx}(l) = \sum_{l=-\infty}^{\infty} x(n)\, x(n-l)$$

Cross-Correation : Relationns bet^n two different signals.

Auto-Correation : Relations with the signal itself.

$$\frac{1}{1-r}$$

Ex. 2.6.2

$$x(n) = a^n u(n), \quad 0 < a < 1$$

$$r_{xx}(l) = \sum_{n=0}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=} a^n a^{n-l} = a^{-l} \sum_{n=0}^{\infty} (a^2)^n = a^{-l} \cdot \frac{1}{1-a^2}$$