3.3. Rational z-Transform

$$X(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

roots

$$= c\,\frac{(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_N)}$$

roots

2nd order

$ax^2 + bx + c = 0$

$(x - x_1)(x - x_2) = 0$

$x = x_1$

$x = x_2$

$$X(z) = C\,\frac{(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_N)}$$

$$\frac{?}{0} = \infty$$

Poles and zeros

The zeros of z-transform $X(z)$ are the values of z for which $X(z)=0$. [zeros: $z_1, z_2, z_3, \ldots z_M$]

↳ roots of the numerator

The poles of a z-transform are the values of z for which $X(z)=\infty$. [poles: $p_1, p_2, p_3, \ldots p_N$]

↳ roots of the denominator

Ex. 3.3.1

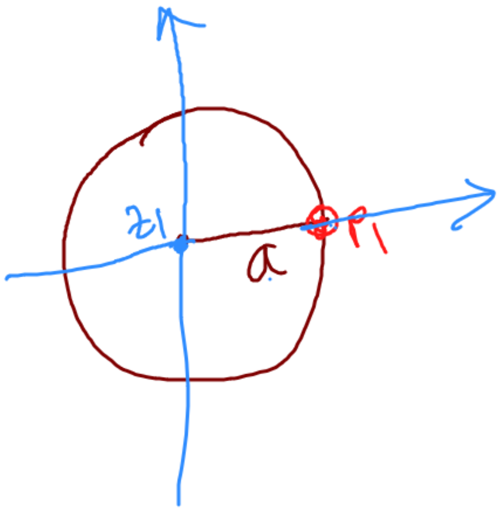
Determine the pole-zero plot for the signal:

$$x(n) = a^n u(n)$$

$$X(z) = \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{1}{\frac{z-a}{z}} = \frac{(z-0)}{(z-a)}$$

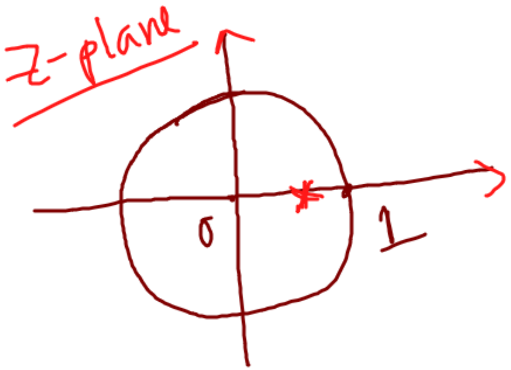
zero: $z_1 = 0$

Pole: $P_1 = a$



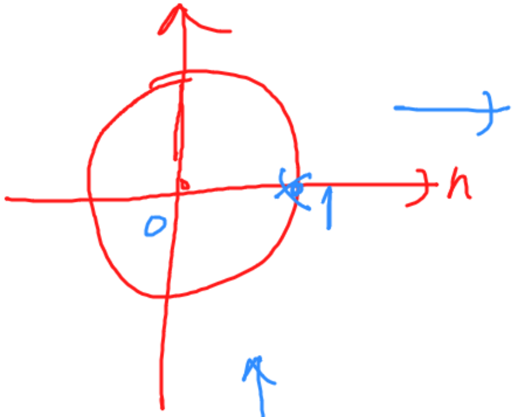
3.3.2 Pole-zero location and Time-domain Behaviour for causal Signals

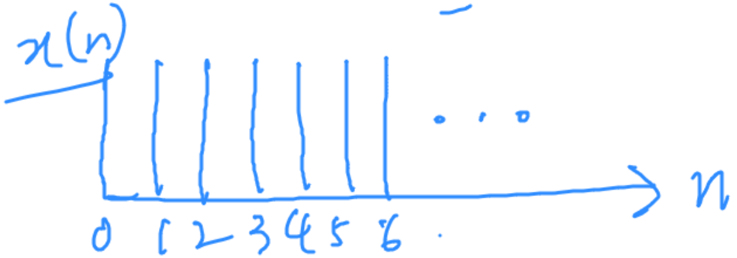
$$x(n) = a^n u(n) \longleftrightarrow X(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$

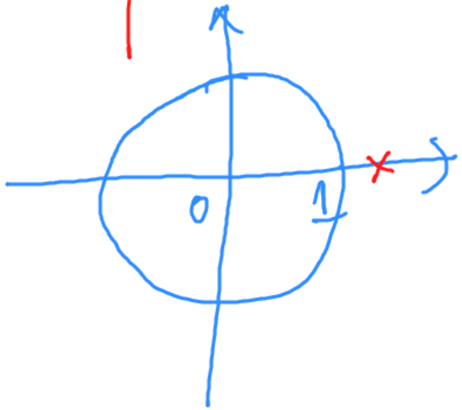


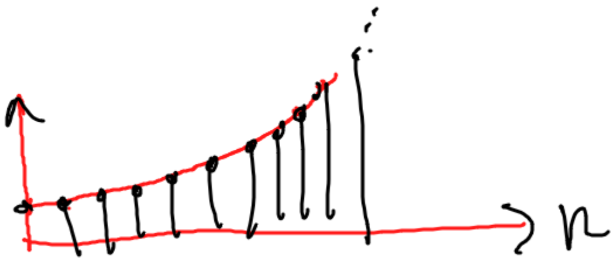
$\longleftrightarrow$

$$x(n) = a^n u(n) \longrightarrow \frac{z}{z-a}$$

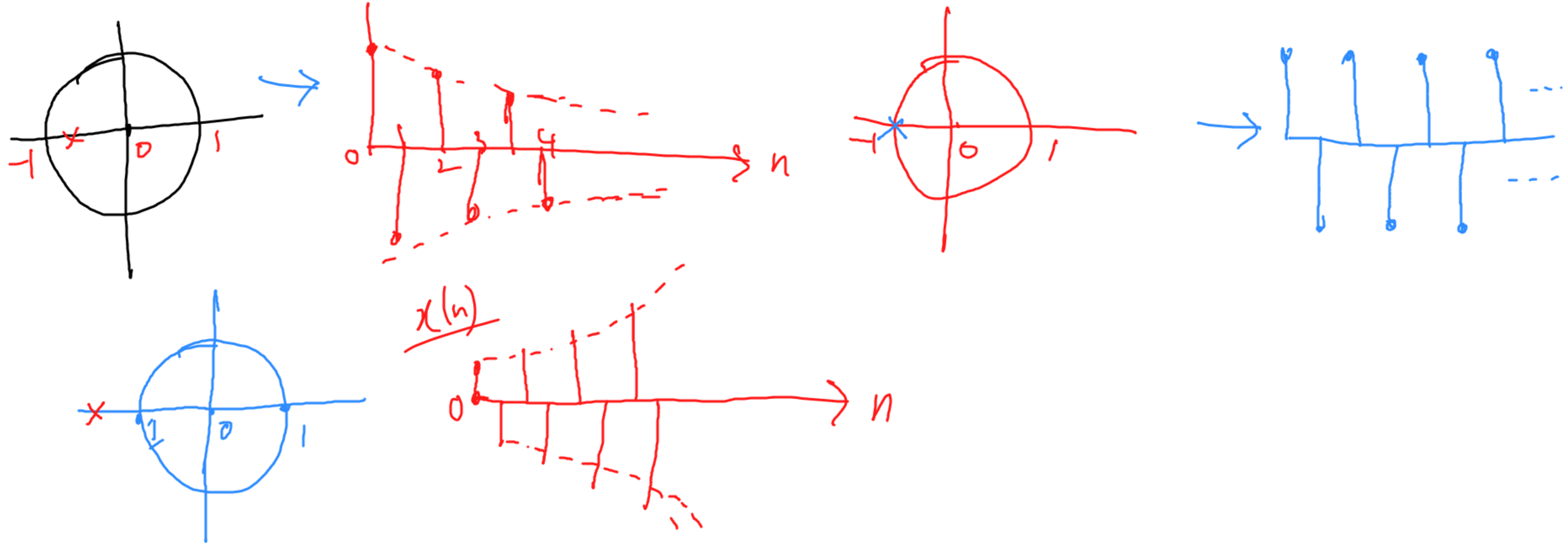








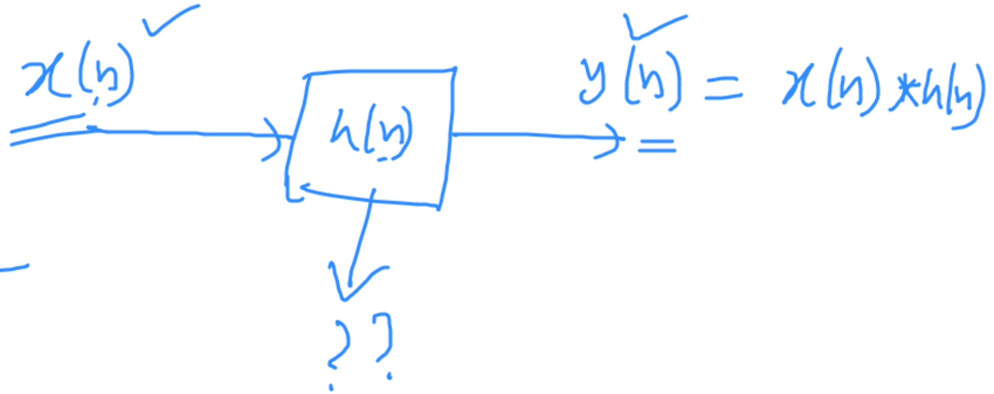
$$x(n) = a^n u(n) \longrightarrow \frac{z}{z-a}$$



3.3.3 The System function of an LTI system

$$y(n) = x(n) * h(n)$$

$$Y(z) = X(z)\cdot H(z)$$



$$H(z) = \frac{Y(z)}{X(z)}$$

$Y(z) \rightarrow$ output

$X(z) \rightarrow$ Input

$\hookrightarrow$ System function

Ex. Determine the system function and unit sample response:

$$y(n) = \frac{1}{2} y(n-1) + 2x(n)$$

$$Y(z) = \frac{1}{2} z^{-1} Y(z) + 2X(z)$$

$$2X(z) = Y(z)\left[1 - \frac{1}{2} z^{-1}\right]$$

$$\frac{Y(z)}{X(z)} = \frac{2}{1-\frac{1}{2}z^{-1}} \Rightarrow H(z) = \frac{2}{1-\frac{1}{2}z^{-1}}$$

$$h(n) = 2\left(\frac{1}{2}\right)^n u(n)$$

z

$H(z) = ?$

$h(n) = ?$

$$H(z) = \frac{Y(z)}{X(z)}$$

Inverse z-Transform

Next-time