North East University Bangladesh
Department of Computer Science and Engineering
Program: BSc(Engg) in CSE
Semester Final Examination, Fall - 2021

Course Code: MAT 201 Course Title: Numerical Methods
Time: 2 hours Maximum Marks: 40

Square bracketed numbers in the margin indicate marks for each part of a question.
Boxed numbers in the margin indicate total marks of the question.

(Answer all of the following questions)

Question 1 ..... Total Marks on Question 1 is: 10

Answer **any 5 questions** from the following:

(a) True or False: "Polynomial Interpolation using Newton's basis results in a lower triangular matrix". If false, write the correct answer. [2]

(b) What is the computational complexity of the **basis evaluation step** of the **Newton basis** for polynomial interpolation in terms of $\Theta(.)$ notation? Assume k represents the number of given data samples. [2]

(c) True or False: "A condition for applying Newton-Cotes quadrature rules is to have input samples spaced evenly in the specified interval of integration". If false, write the correct answer. [2]

(d) Write the equation for the **midpoint quadrature rule** *directly acting over an* interval $[a, b]$. [2]

(e) Mention whether the following ordinary differential equations are **explicit** or **implicit**: [2]

i.

$$y''' + 2y - 3 = 0$$

ii.

$$y''' = 3 - 2y$$

(f) Write the general formula for the initial value problem of ordinary differential equation. [2]

(g) Write the equation of Lipschitz regularity condition for root-finding problems. [2]

Question 2 ..... Total Marks on Question 2 is: 15

Answer **any 5 questions** from the following:

(a) Write the first 4 Monomial basis used for polynomial interpolation. [3]

(b) i. Write the interpolation function for **polynomial interpolation with rational basis.** [2]

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ii. What condition does the highest order of both the numerator and the denominator must maintain in the **rational basis** of polynomial interpolation? [1]

(c) Derive the equation of interpolatory quadrature from the general equation of polynomial interpolation. [Hint: the general equation of polynomial interpolation is a linear combination of basis functions). [3]

(d) Using the **midpoint quadrature rule**, integrate the following function numerically in the interval $[1, 7]$: [3]

$$f(x) = -x^3 + 8x^2 - 2x + 3$$

(e) Write the following equation as a system of the 1^{st}-order explicit ordinary differential equation in the matrix form: [3]

$$y''' = 2y'' - y$$

(f) Express the same ordinary differential equation given in question **(2.e)** in the equation form. [3]

(g) Using a single figure, explain the difference between a root-finding problem and an optimization problem over a single-variable non-linear equation. Make sure your textual explanation do not exceed 2-3 lines. [3]

Question 3 ..... Total Marks on Question 3 is: 15

Answer **any 3 questions** from the following:

(a) Using **Polynomial Interpolation with Lagrange Basis**, *find the unique* degree-3 polynomial that goes through the following four data points: *[5]*

$$(-1, 3), (0, -4), (1, 5), (2, -6)$$

Note that, each data point is given in the format (x_i, y_i) where x_i denotes the sample points and y_i denotes the corresponding exact values of the polynomial.

(b) Using the **trapezoidal quadrature rule**, integrate the following function numerically in the interval $[1, 7]$: [5]

$$f(x) = -x^3 + 8x^2 - 2x + 3$$

(c) Consider you are given the following Ordinary Differential Equation (ODE): [5]

$$y' = 2 - e^{-4t} - 2y$$

Also given, the initial value for this equation at time $t = 0$ is $y = 1$. Now use **Forward Euler's Method** with step-size $h = 0.1$ to approximate values of the solution of this ODE at time-step $t = 0.1, 0.2, 0.3, 0.4, 0.5$.

(d) Using two figures, explain when a fixed point iteration method for root-finding will converge, and when it will diverge. [5]

End of Exam Questions

09.01.22