



**UTM**  
UNIVERSITI TEKNOLOGI MALAYSIA

# **ASSIGNMENT 2**

**SECI1013**

**STRUKTUR DISKRIT (DISCRETE STRUCTURE)**

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**SECTION: 02**

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## Assignment 2

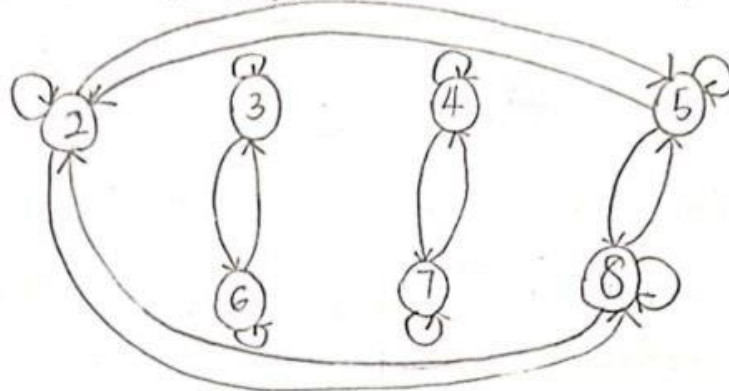
1.  $A = \{2, 3, 4, 5, 6, 7, 8\}$ ,  $x - y = 3n$

when  $x - y = 3n$  ( $n = \text{multiple of } 3$ ),

$$R = \{(8, 5), (5, 8), (7, 4), (4, 7), (6, 3), (3, 6), (5, 2), (2, 5), (8, 2), (2, 8)\},$$

when  $x - y = 0$  ( $3n = 0$ ),

$$R = \{(2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8)\},$$



Domain =  $\{2, 3, 4, 5, 6, 7, 8\}$

Range =  $\{2, 3, 4, 5, 6, 7, 8\}$

Reflexive, Symmetric,  
Transitive

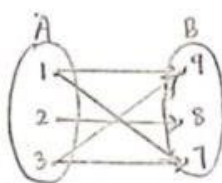
$\therefore$  Equivalence

2.  $R = \{(1, 9), (1, 8), (1, 7), (2, 9), (2, 8), (2, 7), (3, 9), (3, 8), (3, 7)\}$

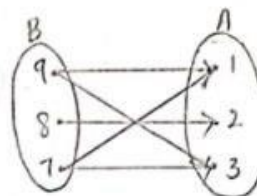
(a)  $R = \{(1, 9), (1, 7), (2, 8), (3, 9), (3, 7)\}$

$$R^{-1} = \{(9, 1), (7, 1), (8, 2), (9, 3), (7, 3)\}$$

(b)  $R$



$R^{-1}$

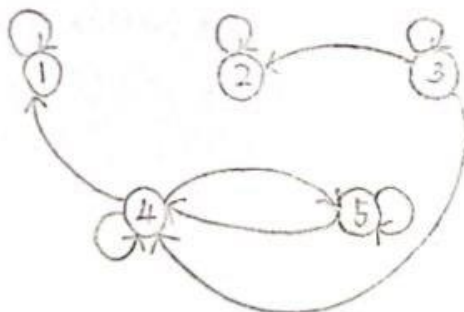


(c)  $R^{-1} : B \text{ to } A$

For all  $(b, a) \in B \times A$ ,  $b R a \iff b + a$  is an even number.

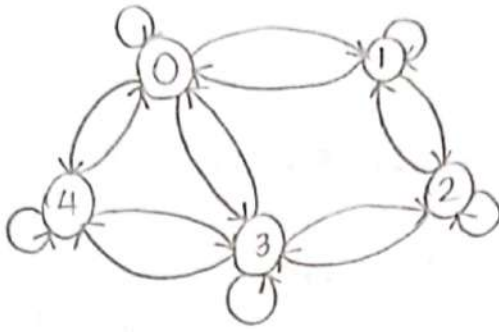
3.  $A = \{1, 2, 3, 4, 5\}$

$$R = \{(1, 1), (2, 2), (3, 2), (3, 3), (3, 4), (4, 1), (4, 4), (4, 5), (5, 4), (5, 5)\}$$



|   | In-degree | Out-degree |
|---|-----------|------------|
| 1 | 2         | 1          |
| 2 | 2         | 1          |
| 3 | 1         | 3          |
| 4 | 3         | 3          |
| 5 | 2         | 2          |

4.



$\therefore R$  is reflexive because  $(0,0), (1,1), (2,2), (3,3), (4,4) \in R$

$R$  is symmetric because  $(0,1)$  and  $(1,0) \in R$ ,  
 $(1,2)$  and  $(2,1) \in R$ ,  
 $(2,3)$  and  $(3,2) \in R$ ,  
 $(3,4)$  and  $(4,3) \in R$ .

$R$  is not transitive because  $(0,1)$  &  $(1,2) \in R$ ,  
 but  $(0,2) \notin R$

5.  $R = \{(1,3), (2,6), (3,9), (4,12)\}$

(a)  $R$  is irreflexive because  $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10), (11,11), (12,12), (13,13), (14,14) \notin R$ .  $(x,x) \notin R$

(b)  $R$  is antisymmetric because  $(1,3) \in R$  but  $(3,1) \notin R$ .

(c)  $R$  is not transitive because  $(1,3)$  and  $(3,9) \in R$  but  $(1,9) \notin R$ .

$$6. (a) RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

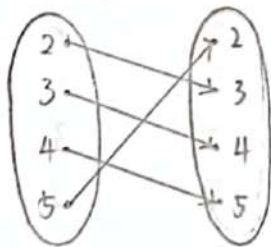
$$(b) SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

7. Relation is a set of ordered pairs, function is a specific type of relation where each input value (element of the domain) is associated with exactly one output value (element of the codomain).

8.  $A = \{2, 3, 4, 5\}$

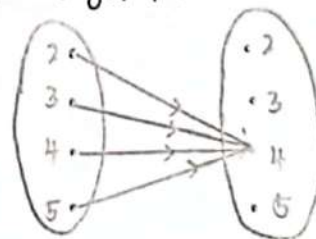
(i)  $\{(2,3), (3,4), (4,5), (5,2)\}$

is a function because  $(x, y_1) \in f$   
 and  $(x, y_2) \notin f$ .

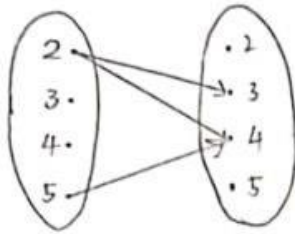


(ii)  $\{(2,4), (3,4), (5,4), (4,4)\}$

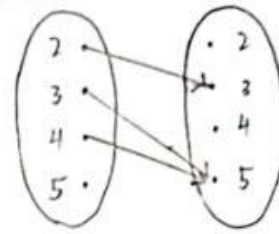
is a function because  $(x, y_1) \in f$   
 and  $(x, y_2) \notin f$



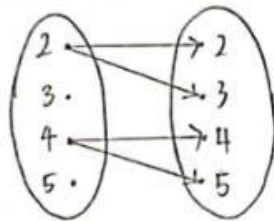
(iii)  $\{(2,3), (2,4), (5,4)\}$   
 is not a function because  $(2,3)$   
 and  $(2,4) \in f$ ,  $3 \neq 4$



(iv)  $\{(2,3), (3,5), (4,5)\}$   
 is not a function because  $(5,y) \notin f$ ,  
 there is no arrow from 5



(v)  $\{(2,2), (2,3), (4,4), (4,5)\}$   
 is not a function because  $(2,2)$   
 and  $(2,3) \in f$ ,  $2 \neq 3$



9.  $R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$   
 Domain =  $\{1, 2, 3, 4, 5\}$   
 Range =  $\{6, 7, 8, 9, 10\}$

10. (v)  $f: R \rightarrow R$ ,  $f(x) = 1-2x$   
 $f(x_1) = f(x_2)$   
 $1-2x_1 = 1-2x_2$   
 $-2x_1 = -2x_2$

$x_1 = x_2 \therefore$  one to one

$f(x) = 1-2x$   
 let  $f(x) = y$   
 $2x = 1-y$   
 $x = \frac{1-y}{2}$

$\therefore$  there exist an  $x$  for every  $y$ , hence it is onto

$\therefore$  Since the function is both one to one and onto, it is bijective

(vi)  $f: R \rightarrow R$ ,  $f(x) = 5x^2 - 1$   
 $f(x_1) = f(x_2)$   
 $5x_1^2 - 1 = 5x_2^2 - 1$   
 $5x_1^2 = 5x_2^2$   
 $x_1^2 = x_2^2$

$x_1 \neq x_2 \therefore$  not one to one

$f(x) = 5x^2 - 1$   
 $5x^2 = y + 1$   
 $x^2 = \frac{y+1}{5}$

$\therefore$  there do not exist  $x$  for every  $y$ ,  
 hence the function is not onto

$\therefore$  Since the function is not one to one and not onto, it is not bijective.

$$(vii) f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^4$$

$$f(x_1) = f(x_2)$$

$$x_1^4 = x_2^4$$

$$x_1 \neq x_2 \quad \therefore \text{not one to one}$$

$$y = x^4$$

$\therefore$  there do not exist  $x$  for every  $y$ ,  
hence the function is not onto

$\therefore$  Since the function is not one to one and not onto, it is not bijective.

$$(viii) f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \left(\frac{x-2}{x-3}\right)$$

$$f(x_1) = f(x_2)$$

$$\left(\frac{x_1-2}{x_1-3}\right) = \left(\frac{x_2-2}{x_2-3}\right)$$

$$(x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$$x_1 \neq x_2 \quad \therefore \text{not one to one}$$

$$y = \frac{x-2}{x-3}$$

$$y(x-3) = x-2$$

$$xy - 3y = x - 2$$

$$x(y-1) = 3y-2$$

$$x = \frac{3y-2}{y-1} \quad \therefore \text{onto}$$

$\therefore$  Since the function is not both one to one and onto, it is not bijective.

$$11. (ix) f(g(x)) = f(x^2-1)$$

$$= 3(x^2-1)-1$$

$$= 3x^2-4$$

$$x=0$$

$$fg(0) = 3(0)^2-4$$

$$= -4$$

$$x=1$$

$$fg(1) = 3(1)^2-4$$

$$= -1$$

$$x=2$$

$$fg(2) = 3(2)^2-4$$

$$= 8$$

$$x=3$$

$$fg(3) = 3(3)^2-4$$

$$= 23$$

$$(xi) f(g(x)) = f(x^3+1)$$

$$= x^3+1-1$$

$$= x^3$$

$$x=0$$

$$fg(0) = 0^3$$

$$= 0$$

$$x=1$$

$$fg(1) = 1^3$$

$$= 1$$

$$x=2$$

$$fg(2) = 2^3$$

$$= 8$$

$$x=3$$

$$fg(3) = 3^3$$

$$= 27$$

$$(x) f(g(x)) = f(5x-6)$$

$$= (5x-6)^2$$

$$= 25x^2 - 60x + 36$$

$$= 25x^2 - 60x + 36$$

$$x=0$$

$$fg(0) = 25(0)^2 - 60(0) + 36$$

$$= 36$$

$$x=1$$

$$fg(1) = 25(1)^2 - 60(1) + 36$$

$$= 1$$

$$x=2$$

$$fg(2) = 25(2)^2 - 60(2) + 36$$

$$= 16$$

$$x=3$$

$$fg(3) = 25(3)^2 - 60(3) + 36$$

$$= 81$$



12. (xii)  $a_n = 6a_{n-1} - 9a_{n-2}$ ;  $a_0 = 1$  and  $a_1 = 6$

$$a_2 = 6(6) - 9(1) = 27$$

$$a_3 = 6(27) - 9(6) = 108 \quad \therefore 1, 6, 27, 108, 405, \dots$$

$$a_4 = 6(108) - 9(27) = 405$$

(xiii)  $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ ;  $a_0 = 2, a_1 = 5, a_2 = 15$

$$a_3 = 6(15) - 11(5) + 6(2) = 47$$

$$a_4 = 6(47) - 11(15) + 6(5) = 147 \quad \therefore 2, 5, 15, 47, 147, 455, \dots$$

$$a_5 = 6(147) - 11(47) + 6(15) = 455$$

(xiv)  $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$ ;  $a_0 = 1, a_1 = -2, a_2 = -1$

$$a_3 = -3(-1) - 3(-2) + 1 = 10$$

$$a_4 = -3(10) - 3(-1) + (-2) = -29 \quad \therefore 1, -2, -1, 10, -29, 56, \dots$$

$$a_5 = -3(-29) - 3(10) + (-1) = 56$$

13. (i)  $a_{n+1} = 5a_n - 3$ ;  $a_1 = k$

$$a_2 = 5k - 3$$

$$a_3 = 5(5k - 3) - 3$$

$$= 25k - 18$$

$$a_4 = 5(25k - 18) - 3$$

$$= 125k - 90 - 3$$

$$= 125k - 93 //$$

(ii)  $a_4 = 7$

$$7 = 125k - 93$$

$$k = 0.8 //$$