Verification and validation of plane Poiseuille flow for low Mach, compressible, internal flow in SU2

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This paper is intended to provide verification and validation of the SU2-suite for simulation of low Mach, compressible, internal flow. The Poiseuille flow was chosen as a test case as it can be analytically derived from the Navier-Stokes equations for the domain of interest. Velocity profiles are compared, for various Reynolds numbers and grid spacings, to analytically reconstructed profiles based on local pressure drop in a grid refinement study. The results from SU2 turned out to be in good accordance with the analytical reconstructions.

Nomenclature

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Setup

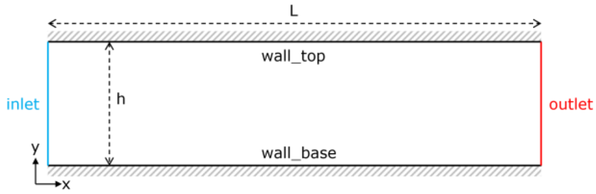

Figure 1 : Geometry

The problem was set up as a 2D channel with an inlet and an outlet, the fluid was allowed to move between the top and base wall. Both walls were of the no-slip kind. Riemann boundary conditions[x] were used for both inlet and outlet. At the inlet a uniform mass flow was prescribed. At the outlet a static back pressure was prescribed.

For the spatial integration, a second order MUSCL Roe with Venkatakrishnan-Wang slope limiter was used with implicit Euler for time stepping. Constant viscosity and ideal gas were assumed to describe the fluid. The parameters of the fluid model can be found in Table 1.

Table 1 : Parameters of fluid mode

Parameter	Value [SI]
Stagnation temperature	288.15
Stagnation pressure	101325.0
Ratio of specific heats	1.4
Dynamic viscosity	1.81E-5
Specific gas constant	287.058

Simulations of five meshes were run for three different Reynolds numbers. Each simulation is denoted by a letter signifying it's Reynolds number and a letter signifying the mesh. All meshes discussed in this paper are unidistant cartesian meshes. An overview of Reynolds number and the meshes can be found in Table 2 and Table 3 respectively.

Table 2 : Reynolds identifier

Identifier	Re
A	20
B	80
C	200

Table 3 : Mesh identifier

Identifier	No. of cells x-direction	No. of cells y-direction	No. of cells total
1	32	8	256
2	64	16	1024
3	128	32	4096
4	256	64	16384
5	512	128	65536

Method

By assuming a fully developed laminar incompressible Newtonian flow between two infinitely long plates, one can easily derive the following relation between the pressure drop and the velocity.

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}, y \in [0, h]$$

If one assumes the pressure drop and dynamic viscosity to be constant, the relation simplifies to a second order ordinary differential equation. By applying the no-slip condition to the upper and lower wall,

$$u(y = 0) = 0$$
$$u(y = h) = 0$$

and integrating with respect to y, one is left with

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - yh)$$

The spatial dependence on the variable y can be nondimensionalized by introducing $\eta = \frac{y}{h}$.

$$u(\eta) = \frac{h^2}{2\mu}\frac{dp}{dx}(\eta^2 - \eta), \eta \in [0,1]$$

A nondimensionalization of the velocity is chosen such that the mean of the non-dimensional velocity profile equals unity.

$$\bar{u} = -\frac{1}{6}\left(\frac{h^2}{2\mu}\frac{dp}{dx}\right)$$

Leading to the nondimensional profile

$$u^*(\eta) = \frac{u(\eta)}{\bar{u}} = -6(\eta^2 - \eta), \eta \in [0,1]$$

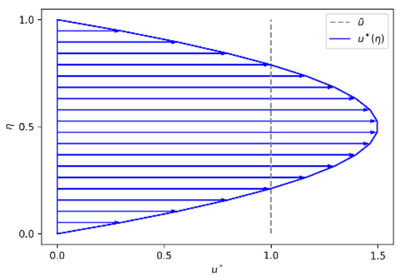

Figure 2 : Nondimensional velocity profile

The initial velocity distribution at the inlet was calculated based on the Reynolds number as

$$Re = \frac{\rho \bar{u} h}{\mu} \;\Rightarrow\; u_0(\eta) = \bar{u} = \frac{\mu}{\rho h} Re$$

From this, the assumed constant pressure gradient could be estimated.

$$\left(\frac{dp}{dx}\right)_0 = -\frac{12\mu}{h^2} u_0$$

Which was then used to prescribe the back pressure relative to the stagnation pressure.

$$p_b = p_0 + L\left(\frac{dp}{dx}\right)_0$$

The lower resolution meshes had deviations in the fully developed gradient. Therefore, a renormalization of the velocity profile was attempted using a new pressure gradient constructed from the fully developed region. The renormalization led to greater deviation between the expected velocity profile and the simulated profile. Nonetheless, the reconstructed gradient can be of use as its own monitor.

$$\left(\frac{dp}{dx}\right)' = \frac{p_b - p_{x=L/2}}{L/2}$$

$$\phi = \left(\frac{dp}{dx}\right)' \Big/ \left(\frac{dp}{dx}\right)_0$$

Results

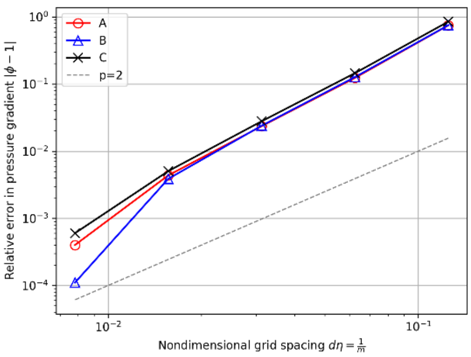

Figure 3 : Convergence plot of relative error in pressure gradient

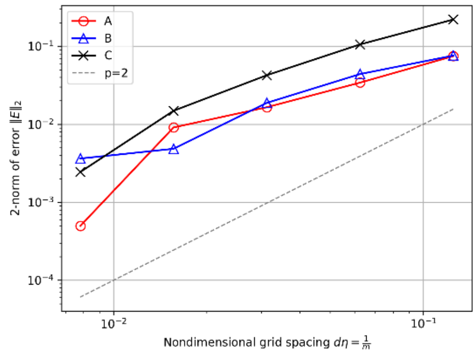

Figure 4 : Convergence plot of relative error in midpoint velocity profile

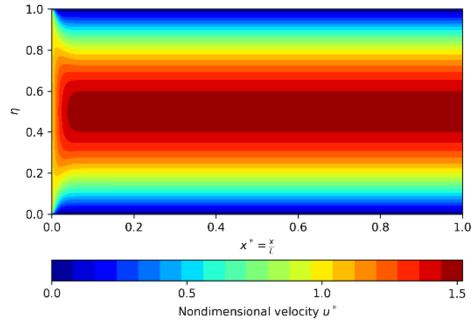

Figure 5 : Velocity, B5

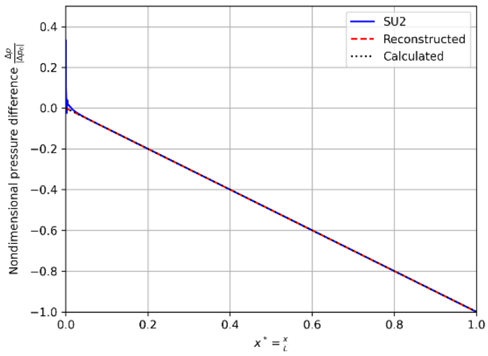

Figure 6 : Centerline pressure, B5

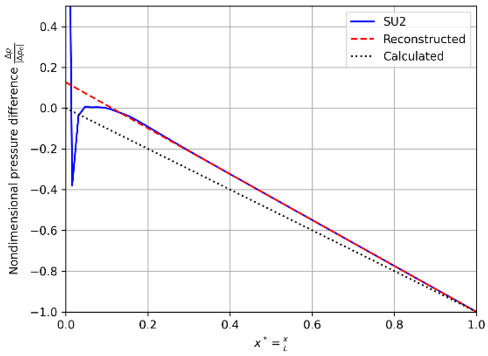

Figure 7 : Centerline pressure, B2