2011. а) $I_n=\int \sin^n x\,dx \quad (n \geqslant 2)$

$$\int \sin^n x\,dx=\int \underbrace{\sin^{n-1}x}_{f}\cdot\underbrace{\sin x\,dx}_{dg}=-\sin^{n-1}x\cdot\cos x-\int(-\cos x)\cdot(n-1)\sin^{n-2}x\cdot\cos x\,dx=$$

$$=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}x\,\underbrace{\cos^2 x}_{=1-\sin^2 x}\,dx=$$

$$=-\sin^{n-1}x\cdot\cos x+(n-1)\int \sin^{n-2}x\,dx-(n-1)\int \sin^n x\,dx$$, т.е.

$$\int \sin^n x\,dx=-\sin^{n-1}x\cdot\cos x+(n-1)\int \sin^{n-2}x\,dx-(n-1)\int \sin^n x\,dx$$

$$n\int \sin^n dx=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}x\,dx$$

$$\underline{\int \sin^n dx=-\frac{\sin^{n-1}x\cos x}{n}+\frac{n-1}{n}\int \sin^{n-2}x\,dx}$$

$$\int \sin^6 x\,dx=-\frac{\sin^5 x\cos x}{6}+\frac{5}{6}\int \sin^4 x\,dx=$$

$$=-\frac{\sin^5 x\cos x}{6}+\frac{5}{6}\left(-\frac{\sin^3 x\cos x}{4}+\frac{3}{4}\int \sin^2 x\,dx\right)=$$

$$=-\frac{\sin^5 x\cos x}{6}+\frac{5}{6}\left(-\frac{\sin^3 x\cos x}{4}+\frac{3}{4}\left(-\frac{\sin x\cos x}{2}+\frac{1}{2}x\right)\right)=$$

$$=-\frac{\sin^5 x\cos x}{6}-\frac{5\sin^3 x\cos x}{24}-\frac{15\sin x\cos x}{48}+\frac{15}{48}x+C.$$

б) $K_n=\int \cos^n x\,dx \quad (n \geqslant 2)$ (аналогично)

$$\int \cos^n x\,dx=\int \cos^{n-1}x\cdot\cos x\,dx=\cos^{n-1}x\sin x-\int \sin x\cdot(n-1)\cos^{n-2}x(-\sin x)\,dx=$$

$$=\cos^{n-1}x\sin x+(n-1)\int \cos^{n-2}x\sin^2 x\,dx=$$

$$=\cos^{n-1}x\sin x+(n-1)\int \cos^{n-2}x\,dx-(n-1)\int \cos^n x\,dx$$, т.е.

$$\int \cos^n x\,dx=\cos^{n-1}x\sin x+(n-1)\int \cos^{n-2}x\,dx-(n-1)\int \cos^n x\,dx$$

$$\int \cos^n x\,dx=\frac{\cos^{n-1}x\sin x}{n}+\frac{n-1}{n}\int \cos^{n-2}x\,dx.$$

$$\int \cos^8 x\,dx=\frac{\cos^7 x\sin x}{8}+\frac{7}{8}\left(\frac{\cos^5 x\sin x}{6}+\frac{5}{6}\left(\frac{\cos^3 x\sin x}{4}+\frac{3}{4}\left(\frac{\cos x\sin x}{2}+\frac{x}{2}\right)\right)\right)$$
$+C.$