1890. $\int \frac{ax^2+bx+c}{x^3(x-1)^2}dx$.

$\frac{ax^2+bx+c}{x^3(x-1)^2} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x-1}+\frac{E}{(x-1)^2}$. При взятии интеграла получится: $\int \frac{ax^2+bx+c}{x^3()(-1)^2}dx = A\ln x - \frac{B}{x} - \frac{C}{2x^2} + D\ln(x-1) - \frac{E}{x-1}$.

Ф-ция рациональна если членов $A\ln x$ и $D\ln(x-1)$ не будет, т.е. если $\underline{A=D=0}$.

$C = \lim_{x\to 0} \frac{ax^2+bx+c}{(x-1)^2} = \underline{c}$; $B = \lim_{x\to 0}\left(\frac{ax^2+bx+c}{x(x-1)^2} - \frac{c}{x}\right) = \lim_{x\to 0}\frac{ax^2+bx+c-cx^2+2cx-c}{x(x-1)^2}$

$= \lim_{x\to 0}\frac{(a-c)x+b+2c}{(x-1)^2} = \underline{b+2c}$

$A = \lim_{x\to 0}\left(\frac{ax^2+bx+c}{x^2(x-1)^2} - \frac{c}{x^2} - \frac{b+2c}{x}\right) = \lim_{x\to 0}\frac{ax^2+bx+c-(x^2+2cx-c)-(b+2c)(x^3-2x^2+x)}{x^2(x-1)^2}$

$= \lim_{x\to 0}\frac{(a-c)x^2+(b+2c)x-(b+2c)x^3+2(b+2c)x^2-(b+2c)x}{x^2(x-1)^2} = \lim_{x\to 0}\frac{-(b+2c)x+a-c+2b}{(x-1)^2}$

$= \underline{a+2b+3c}$.

$E = \lim_{x\to 1}\frac{ax^2+bx+c}{x^3} = a+b+c$,

$D = \lim_{x\to 1}\left(\frac{ax^2+bx+c}{x^3(x-1)} - \frac{a+b+c}{x-1}\right) = \lim_{x\to 1}\frac{ax^2+bx+c-ax^3-bx^3-cx^3}{x^3(x-1)} =$

$= \lim_{x\to 1}\frac{(x-1)(-(a+b+c)x^2-(b+c)x-c)}{x^3(x-1)} = -a-b-c-b-c-c = -a-2b-3c$.

$\begin{cases} A=0 \\ D=0 \end{cases}$ $\begin{cases} a+2b+3c=0 \\ -a-2b-3c=0 \end{cases}$ $\boxed{a+2b+3c=0}$