1.19 $\int \frac{dx}{1+\sqrt{x^2+2x+2}} = \int \frac{1-\sqrt{x^2+2x+2}}{1-x^2-2x-2}dx = -\int \frac{1-\sqrt{x^2+2x+2}}{x^2+2x+1}dx =$

$= \underbrace{-\int \frac{dx}{(x+1)^2}}_{=\frac{1}{x+1}} + \int \frac{\sqrt{(x+1)^2+1}}{(x+1)^2}dx = \left| t=x+1;\ dt=dx \right| = \frac{1}{x+1} + \int \frac{\sqrt{t^2+1}}{t^2}dt =$

$= \frac{1}{x+1} + \int \frac{t^2+1}{t^2\sqrt{t^2+1}}dt = \frac{1}{x+1} + \underbrace{\int \frac{dt}{\sqrt{t^2+1}}}_{\substack{arsh(t)= \\ =arsh(x+1)}} + \int \frac{dt}{t^2\sqrt{t^2+1}} = \frac{1}{x+1} + arsh(x+1) + \int \frac{dt}{t^2\sqrt{t^2+1}}$

$= \left| k=\frac{1}{t^2};\ dk=-\frac{2dt}{t^3} \Rightarrow dt=-\frac{1}{2}t^3dk \quad t=\sqrt{\frac{1}{k}} \right| = \frac{1}{x+1} + arsh(x+1) - \frac{1}{2}\int \frac{t\,dk}{\sqrt{\frac{1}{k}+1}} =$

$= \frac{1}{x+1} + arsh(x+1) - \frac{1}{2}\int \frac{dk}{\sqrt{k+1}} = \frac{1}{x+1} + arsh(x+1) - \sqrt{k+1} + C =$

$= \frac{1}{x+1} + arsh(x+1) - \sqrt{\frac{1}{(x+1)^2}+1} = \frac{1+(x+1)arsh(x+1)-\sqrt{(x+1)^2+1}}{x+1}$

2.1.1

$f(x,y) = x^4+y^4-4x^2y^2$

$\frac{\partial f}{\partial x} = 4x^3-8y^2x;\ \frac{\partial^2 f}{\partial x^2} = 12x^2-8y^2;\ \frac{\partial^2 f}{\partial x\partial y} = -16xy$

$\frac{\partial f}{\partial y} = 4y^3-8x^2y;\ \frac{\partial^2 f}{\partial y^2} = 12y^2-8x^2;\ \frac{\partial^2 f}{\partial y\partial x} = -16xy$

2.1.5

$f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial f}{\partial x} = -\frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ $\quad \frac{\partial^2 f}{\partial x^2} = -\frac{(x^2+y^2+z^2)^{\frac{3}{2}} - \frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot 2x\cdot x}{(x^2+y^2+z^2)^3} = -\frac{(x^2+y^2+z^2)^{\frac{1}{2}}(-2x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$

$= \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$