444. $\lim\limits_{x\to 0} \dfrac{\sqrt[n]{x+1}-1}{x}$ n-целое число. т.е. $\frac{1}{n}$ – дробное

$$\sqrt[n]{1+x}=(1+x)^{\frac{1}{n}}=1+\frac{1}{n}x+\frac{\frac{1}{n}(\frac{1}{n}-1)}{2!}x^2+\frac{\frac{1}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}x^3+\ldots$$

(биномиальная формула для дробной степени)

т.е.

$$\lim_{x\to 0}\frac{\sqrt[n]{1+x}-1}{x}=\lim_{x\to 0}\frac{1+\frac{1}{n}x+\frac{\frac{1}{n}(\frac{1}{n}-1)}{2!}x^2+\frac{\frac{1}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}x^3+\ldots-1}{x}=$$

$$=\lim_{x\to 0}\frac{x\cdot\frac{1}{n}+\frac{\frac{1}{n}(\frac{1}{n}-1)}{2!}x^2+\frac{\frac{1}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}x^3+\ldots}{x}=\lim_{x\to 0}\left(\frac{1}{n}+\frac{\frac{1}{n}(\frac{1}{n}-1)}{2!}x+\frac{\frac{1}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}x^2+\ldots\right)=\underline{\frac{1}{n}}.$$

452. m, n-целые.

$$\lim_{x\to 0}\frac{\sqrt[m]{1+\alpha x}-\sqrt[n]{1+\beta x}}{x}=\lim_{x\to 0}\frac{1+\frac{1}{m}\alpha x+\frac{\frac{1}{m}(\frac{1}{m}-1)}{2!}(\alpha x)^2+\ldots-\left(1+\frac{1}{n}\beta x+\frac{\frac{1}{n}(\frac{1}{n}-1)}{2!}(\beta x)^2+\ldots\right)}{x}=$$

$$=\lim_{x\to 0}\frac{x\left(\frac{\alpha}{m}-\frac{\beta}{n}\right)+x^2\left(\frac{\frac{\alpha}{m}(\frac{1}{m}-1)}{2!}-\frac{\frac{\beta}{n}(\frac{1}{n}-1)}{2!}\right)+x^3\left(\frac{\frac{\alpha}{m}(\frac{1}{m}-1)(\frac{1}{m}-2)}{3!}-\frac{\frac{\beta}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}\right)+\ldots}{x}=$$

$$=\lim_{x\to 0}\left(\left(\frac{\alpha}{m}-\frac{\beta}{n}\right)+x\left(\frac{\frac{\alpha}{m}(\frac{1}{m}-1)}{2!}-\frac{\frac{\beta}{n}(\frac{1}{n}-1)}{2!}\right)+x^2\left(\frac{\frac{\alpha}{m}(\frac{1}{m}-1)(\frac{1}{m}-2)}{3!}-\frac{\frac{\beta}{n}(\frac{1}{n}-1)(\frac{1}{n}-2)}{3!}\right)\right)=\underline{\frac{\alpha}{m}-\frac{\beta}{n}}.$$

525. Долго пытался решить, не справился без правила Лопиталя...

$$\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x=\lim_{x\to\infty}e^{x\ln(\sin\frac{1}{x}+\cos\frac{1}{x})}=\lim_{x\to\infty}e^{\frac{\ln(\sin\frac{1}{x}+\cos\frac{1}{x})}{\frac{1}{x}}}=$$

$$=\lim_{x\to\infty}e^{\left(\frac{\frac{1}{\sin\frac{1}{x}+\cos\frac{1}{x}}\left(\cos\frac{1}{x}-\sin\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}\right)}=\lim_{x\to\infty}e^{\frac{\cos\frac{1}{x}-\sin\frac{1}{x}}{\cos\frac{1}{x}+\sin\frac{1}{x}}}=e^{\frac{1-0}{1+0}}=e^1=e.$$