2012. а) $I_n = \int \frac{dx}{\sin^n x}$, $(n>2)$ пусть $k=n-2$ $(k>0)$ рассмотрим

$$\int \frac{dx}{\sin^k x} = \int \sin^{-k} x\,dx = \int \sin^{-k-1} x \cdot \sin x\,dx =$$

$$= -\sin^{-k-1} x \cos x + \int (-k-1)\sin^{-k-2} x \underbrace{\cos^2 x}_{=1-\sin^2 x}\, dx =$$

$$= -\sin^{-k-1}x\cos x + (-k-1)\int \sin^{-k-2}x\,dx - (-k-1)\int \sin^{-k}x\,dx, \text{ т.}$$

$$\int \sin^{-k}x\,dx = -\sin^{-k-1}x\cos x + (-k-1)\underline{\int\sin^{-k-2}x\,dx} - (-k-1)\int \sin^{-k}x\,dx$$

$$\int \sin^{-k-2}x\,dx = \frac{-k}{-k-1}\int\sin^{-k}x\,dx + \frac{\sin^{-k-1}x\cos x}{-k-1}. \leftarrow \underline{k=n-2}$$

$$\int \sin^{-n}x\,dx = \frac{n-2}{n-1}\int\sin^{-n+2}x\,dx - \frac{\sin^{-n+1}x\cos x}{n-1}$$

$$\int\frac{dx}{\sin^n x} = \frac{n-2}{n-1}\int\frac{dx}{\sin^{n-2}x} - \frac{\cos x}{(n-1)\sin^{n-1}x}.$$

$$\int\frac{dx}{\sin^5 x} = -\frac{\cos x}{4\sin^4 x} + \frac34\int\frac{dx}{\sin^3 x} = -\frac{\cos x}{4\sin^4 x} + \frac34\left(-\frac{\cos x}{2\sin^2 x} + \frac12\int\frac{dx}{\sin x}\right).$$

$$\int\frac{dx}{\sin x} = \int\frac{\sin x\,dx}{\sin^2 x} = \int\frac{\sin x\,dx}{1-\cos^2 x} = \left|\begin{matrix} t=\cos x\\ dt=-\sin x\,dx\end{matrix}\right| = -\int\frac{dt}{(1-t)(1+t)} = \int\frac{dt}{(t-1)(t+1)} =$$

$$= -\int\frac{\frac12 dt}{t+1} + \int\frac{\frac12 dt}{t-1} = -\frac12\ln(t+1) + \frac12\ln(t-1) = -\frac12\ln(\cos x+1) + \frac12\ln(\cos x-$$

$$\int\frac{dx}{\sin^5 x} = -\frac{\cos x}{4\sin^4 x} - \frac{3\cos x}{8\sin^3 x} - \frac{3}{16}\ln(\cos x+1) + \frac{3}{16}\ln(\cos x-1) + C.$$

б) $K_n = \int\frac{dx}{\cos^n x}$ $(n>2)$ (аналогично) пусть $k=n-2$ $(k>0)$

$$\int\frac{dx}{\cos^k x} = \int\cos^{-k-1}x\cdot\cos x\,dx = \cos^{-k-1}x\sin x + \int(-k-1)\cos^{-k-2}x\cdot\sin^2 x\,d$$

$$= \cos^{-k-1}x\sin x + (-k-1)\underline{\int\cos^{-k-2}x\,dx} - (-k-1)\int\cos^{-k}x\,dx = \int\frac{dx}{\cos^k}$$

$$\int\cos^{-k-2}x\,dx = -\frac{\cos^{-k-1}x\sin x}{-k-1} + \frac{-k}{-k-1}\int\cos^{-k}x\,dx$$

$$\int\cos^{-n}x\,dx = \frac{\sin x}{(n-1)\cos^{n-1}x} + \frac{n-2}{n-1}\int\cos^{-n+2}x\,dx.$$