1.19 $\int \frac{dx}{1+\sqrt{x^2+2x+2}} = \int \frac{1-\sqrt{x^2+2x+2}}{1-x^2-2x-2}\,dx = -\int \frac{1-\sqrt{x^2+2x+2}}{x^2+2x+1}\,dx =$

$= \underbrace{-\int \frac{dx}{(x+1)^2}}_{=\frac{1}{x+1}} + \int \frac{\sqrt{(x+1)^2+1}}{(x+1)^2}\,dx = \left| t = x+1;\ dt = dx \right| = \frac{1}{x+1} + \int \frac{\sqrt{t^2+1}}{t^2}\,dt =$

$= \frac{1}{x+1} + \int \frac{t^2+1}{t^2\sqrt{t^2+1}}\,dt = \frac{1}{x+1} + \underbrace{\int \frac{dt}{\sqrt{t^2+1}}}_{\substack{arsh(t)= \\ =arsh(x+1)}} + \int \frac{dt}{t^2\sqrt{t^2+1}} = \frac{1}{x+1} + arsh(x+1) + \int \frac{dt}{t^2\sqrt{t^2+1}}$

$= \left| k = \frac{1}{t^2};\ dk = -\frac{2dt}{t^3} \Rightarrow dt = -\frac{1}{2}t^3 dk;\ t = \sqrt{\frac{1}{k}} \right| = \frac{1}{x+1} + arsh(x+1) - \frac{1}{2}\int \frac{t\,dk}{\sqrt{\frac{1}{k}+1}} =$ $t = \sqrt{\frac{1}{k}}$

$= \frac{1}{x+1} + arsh(x+1) - \frac{1}{2}\int \frac{dk}{\sqrt{k+1}} = \frac{1}{x+1} + arsh(x+1) - \sqrt{k+1} + C =$

$= \frac{1}{x+1} + arsh(x+1) - \sqrt{\frac{1}{(x+1)^2}+1} = \frac{1+(x+1)arsh(x+1) - \sqrt{(x+1)^2+1}}{x+1}$

2.1.1

$f(x,y) = x^4 + y^4 - 4x^2y^2$

$\frac{\partial f}{\partial x} = 4x^3 - 8y^2x;\quad \frac{\partial^2 f}{\partial x^2} = 12x^2 - 8y^2;\quad \frac{\partial^2 f}{\partial x \partial y} = -16xy$

$\frac{\partial f}{\partial y} = 4y^3 - 8x^2y;\quad \frac{\partial^2 f}{\partial y^2} = 12y^2 - 8x^2;\quad \frac{\partial^2 f}{\partial y \partial x} = -16xy$

2.1.5

$f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$\frac{\partial f}{\partial x} = -\frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ $\quad \frac{\partial^2 f}{\partial x^2} = -\frac{(x^2+y^2+z^2)^{\frac{3}{2}} - \frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot 2x\cdot x}{(x^2+y^2+z^2)^3} = -\frac{(x^2+y^2+z^2)^{\frac{1}{2}}(-2x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$

$= \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$

$$\frac{\partial^2 f}{\partial x \partial y} = -x \cdot \left(-\frac{3}{2}\right) \frac{1}{(x^2+y^2+z^2)^{\frac{5}{2}}} \cdot 2y = \frac{3xy}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

ОСТАЛЬНОЕ ПО АНАЛОГИИ

2.3.1.

$$\frac{\partial^3 u}{\partial^2 x \partial y} \qquad u = x \ln x\, y \qquad \frac{\partial u}{\partial x} = y(\ln x + 1).$$

$$\frac{\partial^2 u}{\partial^2 x} = \frac{y}{x} \qquad \frac{\partial^3 u}{\partial^2 x \partial y} = \frac{1}{x}.$$

2.5.2

$$\partial^3 u \qquad u = \sin(x^2+y^2)$$

$$\frac{\partial u}{\partial x} = \cos(x^2+y^2) \cdot 2x \quad ; \quad \frac{\partial^2 u}{\partial x^2} = -\sin(x^2+y^2) \cdot 2x \cdot 2x + 2\cos(x^2+y^2) =$$

$$= 2\cos(x^2+y^2) - 4x^2 \sin(x^2+y^2)$$

$$\frac{\partial^3 u}{\partial x^3} = -2\sin(x^2+y^2)^{\cdot 2x} - 4\left(\cancel{\sin(x^2+y^2)} + \cos(x^2+y^2) \cdot 2x \cdot x\right) =$$

$$= \cancel{-8\sin(x^2+y^2) - 8}\ 2x \sin(x^2+y^2) + \cos(x^2+y^2) \cdot 2x \cdot x^2) =$$

$$= -2\sin(x^2+y^2)_{\cdot 2x} - 8x\sin(x^2+y^2) - 8x^3\cos(x^2+y^2)$$

$$-12x\sin(x^2+y^2) - 8x^3\cos(x^2+y^2) =$$

$$= -4x(3\sin(x^2+y^2) + 2x^2\cos(x^2+y^2))$$

$$\frac{\partial^3 u}{\partial x^2 \partial y} = -2\sin(x^2+y^2) \cdot 2y - 4x^2\cos(x^2+y^2) \cdot 2y =$$

$$= -4y(\sin(x^2+y^2) + 2x^2\cos(x^2+y^2)).$$

$$\partial^3 u = -4x(3\sin(x^2+y^2) + 2x^2\cos(x^2+y^2))(\partial x)^3 - 12y(\sin(x^2+y^2) + 2x^2\cos(x^2+y^2))(\partial x)^2 \partial y$$

$$-12x(\sin(x^2+y^2) + 2y^2\cos(x^2+y^2))\, \partial x (\partial y)^2 - 4y(3\sin(x^2+y^2) + 2y^2\cos(x^2+y^2))(\partial y)^3.$$