

## Appendix

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In[ ]:= R1 = {{Cos[θ1], Sin[θ1], 0, 2 * L1 * (Sin[θ1/2])^2/θ1},
             {-Sin[θ1], Cos[θ1], 0, L1 * Sin[θ1]/θ1}, {0, 0, 1, 0}, {0, 0, 0, 1}};

In[ ]:= R2 = {{Cos[θ2], Sin[θ2], 0, 2 * L2 * (Sin[θ2/2])^2/θ2},
             {-Sin[θ2], Cos[θ2], 0, L2 * Sin[θ2]/θ2}, {0, 0, 1, 0}, {0, 0, 0, 1}};

In[ ]:= Rintm1 = Simplify[R1];
Rintm2 = Simplify[R1.R2];

In[ ]:= Rzdown = Simplify[{{Cos[-Pi/2], -Sin[-Pi/2], 0, pfx},
                          {Sin[-Pi/2], Cos[-Pi/2], 0, pfy}, {0, 0, 1, 0}, {0, 0, 0, 1}}];
Rpos1 = Simplify[Rzdown.Rintm1];
Rpos2 = Simplify[Rzdown.Rintm2];

In[ ]:= px1 = Rpos1[[1, 4]]; py1 = Rpos1[[2, 4]];
px2 = Rpos2[[1, 4]]; py2 = Rpos2[[2, 4]];
deltapx1 = py1 - pfy; deltapy1 = px1 - pfx;
deltapx2 = py2 - py1; deltapy2 = px2 - px1;
potentialgy1 = deltapy1/2; potentialgy2 = py1 + deltapy2/2;
Rcoord1 = Rpos1[[1 ;; 3, 4]];
Rcoord2 = Rpos2[[1 ;; 3, 4]];

In[ ]:= Jv1 = Simplify[D[{Rcoord1}, {{θ1, θ2}}]];
Jv2 = Simplify[D[{Rcoord2}, {{θ1, θ2}}]];
Jv1 = Join[{Jv1[[1, 1]], Jv1[[1, 2]], Jv1[[1, 3]]}];
Jv2 = Join[{Jv2[[1, 1]], Jv2[[1, 2]], Jv2[[1, 3]]}];

In[ ]:= I1 = 1/12 * m1 * (L1)^2; I2 = 1/12 * m2 * (L2)^2;
Rtwist1 = Rpos1[[1 ;; 3, 3]]; Rtwist2 = Rpos2[[1 ;; 3, 3]];
Jw1 = Simplify[D[{Rtwist1 * θ1/2}, {{θ1, θ2}}]];
Jw2 = Simplify[D[{Rtwist1 * θ1/2 + Rtwist2 * θ2/2}, {{θ1, θ2}}]];
Jw1 = Join[{Jw1[[1, 1]], Jw1[[1, 2]], Jw1[[1, 3]]}];
Jw2 = Join[{Jw2[[1, 1]], Jw2[[1, 2]], Jw2[[1, 3]]}];

(*Using the Jacobian matrices derived above,
we can work out the inertia matrix and Coriolis matrix respectively.*)

In[ ]:= M = Simplify[m1 * Transpose[Jv1].Jv1 +
                    I1 * Transpose[Jw1].Jw1 + m2 * Transpose[Jv2].Jv2 + I2 * Transpose[Jw2].Jw2];

In[ ]:= (*Simplify[Eigenvalues[M]];*)
(*If the Coriolis-Centripetal matrix was directly defined as C in Mathematica,
after computing the cell, it will display C is protected in workspace,
it's probably because capital C is a function,
so shall change the name of C matrix into "Cmatrix" instead.*)
Cmatrix = {{0, 0}, {0, 0}};
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ln[ ]:= Cmatrix[[1, 1]] = (D[M[[1, 1]], {θ1}] + D[M[[1, 1]], {θ1}] - D[M[[1, 1]], {θ1}]) * θ1dot / 2 +
        (D[M[[1, 1]], {θ2}] + D[M[[1, 1]], {θ1}] - D[M[[2, 1]], {θ1}]) * θ2dot / 2;
Cmatrix[[1, 2]] = (D[M[[1, 2]], {θ1}] + D[M[[1, 1]], {θ2}] - D[M[[1, 2]], {θ1}]) * θ1dot / 2 +
        (D[M[[1, 2]], {θ2}] + D[M[[1, 2]], {θ2}] - D[M[[2, 2]], {θ1}]) * θ2dot / 2;
Cmatrix[[2, 1]] = (D[M[[2, 1]], {θ1}] + D[M[[2, 1]], {θ1}] - D[M[[1, 1]], {θ2}]) * θ1dot / 2 +
        (D[M[[2, 1]], {θ2}] + D[M[[2, 2]], {θ1}] - D[M[[2, 1]], {θ2}]) * θ2dot / 2;
Cmatrix[[2, 2]] = (D[M[[2, 2]], {θ1}] + D[M[[2, 1]], {θ2}] - D[M[[1, 2]], {θ2}]) * θ1dot / 2 +
        (D[M[[2, 2]], {θ2}] + D[M[[2, 2]], {θ2}] - D[M[[2, 2]], {θ2}]) * θ2dot / 2;

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ln[ ]:=

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(*Export the expressions of M and C matrices to MATLAB file*)
(*Sometimes it doesn't work for the following line of code,
we have to upload the external code manually by clicking "file → install"*)
(*Import[
  "D:\\R - Clemson\\SRA Program\\Weekly Report\\W10 - Dynamcis Derivation\\ToMatlab.m",
  "Package"];*)

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PrintMatlab[M]
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PrintMatlab[Cmatrix]
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[(1/48).*L1.^2.*m1+(1/48).*L2.^2.*m2+L1.^2.*m1.*θ1.^(-4).*(...
  2+θ1.^2+(-2).*cos(θ1)+(-2).*θ1.*sin(θ1))+m2.*θ1.^(-4).*θ2.^(...
  -2).*((θ1.*(L2.*θ1+(-1).*L1.*θ2).*cos(θ1)+(-1).*L2.*θ1.^2.*...
  cos(θ1+θ2)+L1.*θ2.*sin(θ1)).^2+(L1.*θ2+(-1).*L1.*θ2).*cos(θ1)...
  +θ1.*(L2.*θ1+(-1).*L1.*θ2).*sin(θ1)+(-1).*L2.*θ1.^2.*sin(θ1+...
  θ2)).^2),(1/48).*L2.*m2.*θ1.^(-2).*θ2.^(-2).*((-48).*L1+48.*...
  L2.*θ1.^2+L2.*θ1.^2.*θ2.^2+48.*L1.*cos(θ1)+48.*(L1+(-1).*...
  L2.*θ1.^2+L1.*θ1.*θ2).*cos(θ2)+(-48).*L1.*cos(θ1+θ2)+(-48).*...
  L1.*θ1.*sin(θ2)+48.*L1.*θ2.*sin(θ2)+(-48).*L1.*θ2.*sin(θ1+...
  θ2));(1/48).*L2.*m2.*θ1.^(-2).*θ2.^(-2).*((-48).*L1+48.*L2.*...
  θ1.^2+L2.*θ1.^2.*θ2.^2+48.*L1.*cos(θ1)+48.*(L1+(-1).*L2.*...
  θ1.^2+L1.*θ1.*θ2).*cos(θ2)+(-48).*L1.*cos(θ1+θ2)+(-48).*L1.*...
  θ1.*sin(θ2)+48.*L1.*θ2.*sin(θ2)+(-48).*L1.*θ2.*sin(θ1+θ2)),( ...
  1/48).*L2.^2.*m2.*θ2.^(-4).*(96+48.*θ2.^2+θ2.^4+(-96).*cos(...
  θ2)+(-96).*θ2.*sin(θ2))];

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[(1/2).*θ1dot.*(L1.^2.*m1.*θ1.^(-4).*(2.*θ1+(-2).*θ1.*cos(...
  θ1))+(-4).*L1.^2.*m1.*θ1.^(-5).*(2+θ1.^2+(-2).*cos(θ1)+(-2)...
  .*θ1.*sin(θ1))+(-4).*m2.*θ1.^(-5).*θ2.^(-2).*((θ1.*(L2.*θ1+...
  -1).*L1.*θ2).*cos(θ1)+(-1).*L2.*θ1.^2.*cos(θ1+θ2)+L1.*θ2.*...
  sin(θ1)).^2+(L1.*θ2+(-1).*L1.*θ2).*cos(θ1)+θ1.*(L2.*θ1+(-1).*...
  L1.*θ2).*sin(θ1)+(-1).*L2.*θ1.^2.*sin(θ1+θ2)).^2+m2.*θ1.^(...
  -4).*θ2.^(-2).*(2.*(θ1.*(L2.*θ1+(-1).*L1.*θ2).*cos(θ1)+(-1)...
  .*L2.*θ1.^2.*cos(θ1+θ2)+L2.*θ1.*sin(θ1)+L1.*θ2.*sin(θ1)+(...
  L2.*θ1+(-1).*L1.*θ2).*sin(θ1)+(-2).*L2.*θ1.*sin(θ1+θ2)).*(...
  L1.*θ2+(-1).*L1.*θ2.*cos(θ1)+θ1.*(L2.*θ1+(-1).*L1.*θ2).*sin(...
  θ1)+(-1).*L2.*θ1.^2.*sin(θ1+θ2))+2.*(θ1.*(L2.*θ1+(-1).*L1.*...
  θ2).*cos(θ1)+(-1).*L2.*θ1.^2.*cos(θ1+θ2)+L1.*θ2.*sin(θ1)).*(...
  L2.*θ1.*cos(θ1)+L1.*θ2.*cos(θ1)+L2.*θ1+(-1).*L1.*θ2).*cos(...
  θ1)+(-2).*L2.*θ1.*cos(θ1+θ2)+(-1).*θ1.*(L2.*θ1+(-1).*L1.*θ2)...
  .*sin(θ1)+L2.*θ1.^2.*sin(θ1+θ2)))+(1/2).*θ2dot.*(L1.^2.*...
  m1.*θ1.^(-4).*(2.*θ1+(-2).*θ1.*cos(θ1))+(-4).*L1.^2.*m1.*...

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$$\begin{aligned}
& \theta_1.^{(-5)}.* (2+\theta_1.^2+(-2).*\cos(\theta_1)+(-2).*\theta_1.*\sin(\theta_1))+(-1/48) \dots \\
& . * L_2.*m_2.*\theta_1.^{(-2)}.*\theta_2.^{(-2)}.* (96.*L_2.*\theta_1+2.*L_2.*\theta_1.*\theta_2.^2+ \dots \\
& 48.*((-2). * L_2.*\theta_1+L_1.*\theta_2).*\cos(\theta_2)+(-48). * L_1.*\theta_2.*\cos(\theta_1+\theta_2) \dots \\
& +(-48). * L_1.*\sin(\theta_1)+(-48). * L_1.*\sin(\theta_2)+48.*L_1.*\sin(\theta_1+\theta_2))+ ( \dots \\
& 1/24). * L_2.*m_2.*\theta_1.^{(-3)}.*\theta_2.^{(-2)}.* ((-48). * L_1+48.*L_2.*\theta_1.^2+ \dots \\
& L_2.*\theta_1.^2.*\theta_2.^2+48.*L_1.*\cos(\theta_1)+48.* (L_1+(-1). * L_2.*\theta_1.^2+ \dots \\
& L_1.*\theta_1.*\theta_2).*\cos(\theta_2)+(-48). * L_1.*\cos(\theta_1+\theta_2)+(-48). * L_1.*\theta_1.* \dots \\
& \sin(\theta_2)+48.*L_1.*\theta_2.*\sin(\theta_2)+(-48). * L_1.*\theta_2.*\sin(\theta_1+\theta_2))+(-2) \dots \\
& . *m_2.*\theta_1.^{(-4)}.*\theta_2.^{(-3)}.* ((\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos( \dots \\
& \theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L_1.*\theta_2.*\sin(\theta_1)).^2+(L_1.*\theta_2+ \dots \\
& (-1). * L_1.*\theta_2.*\cos(\theta_1)+\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin(\theta_1)+ ( \dots \\
& -1). * L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2)).^2+(-4). *m_2.*\theta_1.^{(-5)}.*\theta_2.^{(-2)} \dots \\
& . * ((\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos(\theta_1)+(-1). * L_2.*\theta_1.^2.*\cos( \dots \\
& \theta_1+\theta_2)+L_1.*\theta_2.*\sin(\theta_1)).^2+(L_1.*\theta_2+(-1). * L_1.*\theta_2.*\cos(\theta_1)+ \dots \\
& \theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin(\theta_1)+(-1). * L_2.*\theta_1.^2.*\sin(\theta_1+ \dots \\
& \theta_2)).^2+m_2.*\theta_1.^{(-4)}.*\theta_2.^{(-2)}.* (2.* (L_1+(-1). * L_1.*\cos(\theta_1)+ ( \dots \\
& -1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+(-1). * L_1.*\theta_1.*\sin(\theta_1)). * (L_1.*\theta_2+ ( \dots \\
& -1). * L_1.*\theta_2.*\cos(\theta_1)+\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin(\theta_1)+(-1) \dots \\
& . * L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))+2.* (\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos( \dots \\
& \theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L_1.*\theta_2.*\sin(\theta_1)). * ((-1). * \dots \\
& L_1.*\theta_1.*\cos(\theta_1)+L_1.*\sin(\theta_1)+L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))+m_2.* \dots \\
& \theta_1.^{(-4)}.*\theta_2.^{(-2)}.* (2.* (\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos(\theta_1)+ \dots \\
& (-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L_2.*\theta_1.*\sin(\theta_1)+L_1.*\theta_2.*\sin(\theta_1)+ \dots \\
& (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin(\theta_1)+(-2). * L_2.*\theta_1.*\sin(\theta_1+\theta_2)). * ( \dots \\
& L_1.*\theta_2+(-1). * L_1.*\theta_2.*\cos(\theta_1)+\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin( \dots \\
& \theta_1)+(-1). * L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))+2.* (\theta_1.* (L_2.*\theta_1+(-1). * L_1.* \dots \\
& \theta_2).*\cos(\theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L_1.*\theta_2.*\sin(\theta_1)). * ( \dots \\
& L_2.*\theta_1.*\cos(\theta_1)+L_1.*\theta_2.*\cos(\theta_1)+(L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos( \dots \\
& \theta_1)+(-2). * L_2.*\theta_1.*\cos(\theta_1+\theta_2)+(-1). * \theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2) \dots \\
& . * \sin(\theta_1)+L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))) , (1/2). * \theta_2 \text{dot} . * ((1/24). * \dots \\
& L_2.*m_2.*\theta_1.^{(-2)}.*\theta_2.^{(-2)}.* (2.*L_2.*\theta_1.^2.*\theta_2+48.*L_1.*\theta_2.* \dots \\
& \cos(\theta_2)+(-48). * L_1.*\theta_2.*\cos(\theta_1+\theta_2)+48.*L_1.*\sin(\theta_2)+(-48). * ( \dots \\
& L_1+(-1). * L_2.*\theta_1.^2+L_1.*\theta_1.*\theta_2).*\sin(\theta_2))+(-1/12). * L_2.*m_2.* \dots \\
& \theta_1.^{(-2)}.*\theta_2.^{(-3)}.* ((-48). * L_1+48.*L_2.*\theta_1.^2+L_2.*\theta_1.^2.* \dots \\
& \theta_2.^2+48.*L_1.*\cos(\theta_1)+48.* (L_1+(-1). * L_2.*\theta_1.^2+L_1.*\theta_1.*\theta_2). * \dots \\
& \cos(\theta_2)+(-48). * L_1.*\cos(\theta_1+\theta_2)+(-48). * L_1.*\theta_1.*\sin(\theta_2)+48.* \dots \\
& L_1.*\theta_2.*\sin(\theta_2)+(-48). * L_1.*\theta_2.*\sin(\theta_1+\theta_2)))+(1/2). * \theta_1 \text{dot} . * ( \dots \\
& -2). *m_2.*\theta_1.^{(-4)}.*\theta_2.^{(-3)}.* ((\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2). * \dots \\
& \cos(\theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L_1.*\theta_2.*\sin(\theta_1)).^2+( \dots \\
& L_1.*\theta_2+(-1). * L_1.*\theta_2.*\cos(\theta_1)+\theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\sin( \dots \\
& \theta_1)+(-1). * L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2)).^2+m_2.*\theta_1.^{(-4)}.*\theta_2.^{(-2)} \dots \\
& . * (2.* (L_1+(-1). * L_1.*\cos(\theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+(-1) \dots \\
& . * L_1.*\theta_1.*\sin(\theta_1)). * (L_1.*\theta_2+(-1). * L_1.*\theta_2.*\cos(\theta_1)+\theta_1.* (L_2.* \dots \\
& \theta_1+(-1). * L_1.*\theta_2).*\sin(\theta_1)+(-1). * L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))+2.* ( \dots \\
& \theta_1.* (L_2.*\theta_1+(-1). * L_1.*\theta_2).*\cos(\theta_1)+(-1). * L_2.*\theta_1.^2.*\cos(\theta_1+ \dots \\
& \theta_2)+L_1.*\theta_2.*\sin(\theta_1)). * ((-1). * L_1.*\theta_1.*\cos(\theta_1)+L_1.*\sin(\theta_1)+ \dots \\
& L_2.*\theta_1.^2.*\sin(\theta_1+\theta_2))) ; (1/2). * \theta_1 \text{dot} . * ((1/24). * L_2.*m_2.* \dots \\
& \theta_1.^{(-2)}.*\theta_2.^{(-2)}.* (96.*L_2.*\theta_1+2.*L_2.*\theta_1.*\theta_2.^2+48.*((-2). * \dots \\
& L_2.*\theta_1+L_1.*\theta_2).*\cos(\theta_2)+(-48). * L_1.*\theta_2.*\cos(\theta_1+\theta_2)+(-48). * \dots \\
& L_1.*\sin(\theta_1)+(-48). * L_1.*\sin(\theta_2)+48.*L_1.*\sin(\theta_1+\theta_2))+(-1/12). * \dots \\
& L_2.*m_2.*\theta_1.^{(-3)}.*\theta_2.^{(-2)}.* ((-48). * L_1+48.*L_2.*\theta_1.^2+L_2.* \dots
\end{aligned}$$

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 $\theta_1.^2.*\theta_2.^2+48.*L1.*\cos(\theta_1)+48.*(L1+(-1).*L2.*\theta_1.^2+L1.*...$ 
 $\theta_1.*\theta_2).*\cos(\theta_2)+(-48).*L1.*\cos(\theta_1+\theta_2)+(-48).*L1.*\theta_1.*\sin(...$ 
 $\theta_2)+48.*L1.*\theta_2.*\sin(\theta_2)+(-48).*L1.*\theta_2.*\sin(\theta_1+\theta_2))+2.*m2.*...$ 
 $\theta_1.^{(-4)}.*\theta_2.^{(-3)}.*((\theta_1.*(L2.*\theta_1+(-1).*L1.*\theta_2)).*\cos(\theta_1)+(...$ 
 $-1).*L2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+L1.*\theta_2.*\sin(\theta_1)).^2+(L1.*\theta_2+(-1)...$ 
 $.*L1.*\theta_2.*\cos(\theta_1)+\theta_1.*(L2.*\theta_1+(-1).*L1.*\theta_2)).*\sin(\theta_1)+(-1).*...$ 
 $L2.*\theta_1.^2.*\sin(\theta_1+\theta_2)).^2+(-1).*m2.*\theta_1.^{(-4)}.*\theta_2.^{(-2)}.*((...$ 
 $2.*(L1+(-1).*L1.*\cos(\theta_1)+(-1).*L2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+(-1).*...$ 
 $L1.*\theta_1.*\sin(\theta_1)).*(L1.*\theta_2+(-1).*L1.*\theta_2.*\cos(\theta_1)+\theta_1.*(L2.*\theta_1+...$ 
 $(-1).*L1.*\theta_2)).*\sin(\theta_1)+(-1).*L2.*\theta_1.^2.*\sin(\theta_1+\theta_2))+2.*( \theta_1.*...$ 
 $(L2.*\theta_1+(-1).*L1.*\theta_2)).*\cos(\theta_1)+(-1).*L2.*\theta_1.^2.*\cos(\theta_1+\theta_2)+...$ 
 $L1.*\theta_2.*\sin(\theta_1)).*((-1).*L1.*\theta_1.*\cos(\theta_1)+L1.*\sin(\theta_1)+L2.*...$ 
 $\theta_1.^2.*\sin(\theta_1+\theta_2)))$ ), (1/2).* $\theta_2$ dot.*( (1/48).*L2.^2.*m2.* $\theta_2.^{(-4)}$ .*
(96.* $\theta_2+4.*\theta_2.^3+(-96).*\theta_2.*\cos(\theta_2))$ )+(-1/12).*L2.^2.*...
m2.* $\theta_2.^{(-5)}$ .*(96+48.* $\theta_2.^2+\theta_2.^4+(-96).*\cos(\theta_2)+(-96).*\theta_2.*...$ 
sin( $\theta_2$ )))]];

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