

$$\text{In[]:= CL11 =} \\ -\text{Lthetadot1} * \left(\frac{14427 * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{25600} + \frac{12027 * \text{Cos}[\text{Ltheta1}]}{25600} \right) - \\ \text{Lthetadot2} * \left(\frac{11493 * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{20480} - \right. \\ \left. \frac{81 * \text{Cos}[\text{Ltheta1}]}{51200} + \frac{27 * \text{Sin}[\text{Ltheta2}]}{160} \right)$$

$$\text{Out[]:=} -\text{Lthetadot1} \left(\frac{12027 * \text{Cos}[\text{Ltheta1}]}{25600} + \frac{14427 * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{25600} \right) - \\ \text{Lthetadot2} \left(-\frac{81 * \text{Cos}[\text{Ltheta1}]}{51200} + \frac{11493 * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{20480} + \frac{27 * \text{Sin}[\text{Ltheta2}]}{160} \right)$$

$$\text{In[]:= CL12 = FullSimplify} \left[\frac{81 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1}]}{51200} - \right. \\ \frac{114957 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{102400} - \\ \frac{12027 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1}]}{25600} - \\ \frac{14427 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{12800} - \\ \frac{27 * \text{Lthetadot1} * \text{Sin}[\text{Ltheta2}]}{160} - \frac{27 * \text{Lthetadot2} * \text{Sin}[\text{Ltheta2}]}{80} - \\ \left. \frac{81 * \text{Lthetadot2} * \text{Sin}[2 * \text{Ltheta1} + 2 * \text{Ltheta2}]}{2560} \right]$$

$$\text{Out[]:=} -\frac{1}{102400} * \\ 3 \left(2 \left(8018 * \text{Lthetadot1} - 27 * \text{Lthetadot2} \right) \text{Cos}[\text{Ltheta1}] + 3 \left(12824 * \text{Lthetadot1} + 12773 * \text{Lthetadot2} \right) \right. \\ \left. \text{Cos}[\text{Ltheta1} + \text{Ltheta2}] + 5760 \left(\text{Lthetadot1} + 2 * \text{Lthetadot2} \right) \text{Sin}[\text{Ltheta2}] + \right. \\ \left. 1080 * \text{Lthetadot2} \text{Sin}[2 * (\text{Ltheta1} + \text{Ltheta2})] \right)$$

$$\text{In[]:= CL21 = FullSimplify} \left[\frac{81 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1}]}{51200} - \right. \\ \frac{23013 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{20480} - \\ \frac{3027 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1}]}{6400} - \\ \left. \frac{57897 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{51200} - \frac{27 * \text{Lthetadot2} * \text{Sin}[\text{Ltheta2}]}{80} \right]$$

$$\text{Out[]:=} -\frac{1}{102400} * \\ 3 \left(2 \left(8072 * \text{Lthetadot1} - 27 * \text{Lthetadot2} \right) \text{Cos}[\text{Ltheta1}] + \left(38598 * \text{Lthetadot1} + 38355 * \text{Lthetadot2} \right) \right. \\ \left. \text{Cos}[\text{Ltheta1} + \text{Ltheta2}] + 11520 * \text{Lthetadot2} \text{Sin}[\text{Ltheta2}] \right)$$

$$\text{In[]:= CL22 = FullSimplify} \left[\frac{81 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1}]}{51200} - \right. \\ \frac{115173 * \text{Lthetadot2} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{102400} - \\ \frac{3027 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1}]}{6400} - \\ \frac{23229 * \text{Lthetadot1} * \text{Cos}[\text{Ltheta1} + \text{Ltheta2}]}{20480} - \\ \frac{27 * \text{Lthetadot2} * \text{Sin}[\text{Ltheta2}]}{80} + \frac{81 * \text{Lthetadot1} * \text{Sin}[2 * \text{Ltheta1} + 2 * \text{Ltheta2}]}{2560} + \\ \left. \frac{81 * \text{Lthetadot2} * \text{Sin}[2 * \text{Ltheta1} + 2 * \text{Ltheta2}]}{2560} \right]$$

$$\text{Out[]:=} \frac{1}{102400} * 3 \left(\left(-16144 * \text{Lthetadot1} + 54 * \text{Lthetadot2} \right) \text{Cos}[\text{Ltheta1}] - \right. \\ 3 \left(12905 * \text{Lthetadot1} + 12797 * \text{Lthetadot2} \right) \text{Cos}[\text{Ltheta1} + \text{Ltheta2}] + 360 \\ \left. \left(-32 * \text{Lthetadot2} \text{Sin}[\text{Ltheta2}] + 3 \left(\text{Lthetadot1} + \text{Lthetadot2} \right) \text{Sin}[2 * (\text{Ltheta1} + \text{Ltheta2})] \right) \right)$$

`In[]:= CR11 = FullSimplify[`

$$\begin{aligned} & \left(9 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1 + \theta_2] \right) / 32 - \left(9 * R_{\theta 2} \dot{\theta}_2 * \cos[\theta_1 + \theta_2] \right) / 16 - \\ & \left(9 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1 + \theta_2] \right) / 16 + \\ & \left(9 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_1 + \theta_2] \right) / 32 - \left(15 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1] \right) / 32 + \\ & \left(15 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1] \right) / 64 - \left(27 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_2] \right) / 160 \end{aligned}$$

`Out[]:=`
$$\frac{1}{320} \left(-150 R_{\theta 1} \dot{\theta}_1 \cos[\theta_1] - \right.$$

$$180 (R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \cos[\theta_1 + \theta_2] + 75 R_{\theta 1} \dot{\theta}_1 \sin[\theta_1] -$$

$$54 R_{\theta 2} \dot{\theta}_2 \sin[\theta_2] + 90 (R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \sin[\theta_1 + \theta_2] \Big)$$

`In[]:= CR12 = FullSimplify[`

$$\begin{aligned} & \left(9 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1 + \theta_2] \right) / 16 - \left(9 * R_{\theta 2} \dot{\theta}_2 * \cos[\theta_1 + \theta_2] \right) / 16 - \\ & \left(9 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1 + \theta_2] \right) / 8 + \left(9 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_1 + \theta_2] \right) / 32 - \\ & \left(15 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1] \right) / 32 + \left(15 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1] \right) / 64 - \\ & \left(27 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_2] \right) / 160 - \left(27 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_2] \right) / 160 - \\ & \left(81 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_1 + \theta_2 + \theta_1 + \theta_2] \right) / 320 \end{aligned}$$

`Out[]:=`
$$-\frac{3}{320} \left(50 R_{\theta 1} \dot{\theta}_1 \cos[\theta_1] + 60 (2 R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \cos[\theta_1 + \theta_2] - \right.$$

$$25 R_{\theta 1} \dot{\theta}_1 \sin[\theta_1] + 18 R_{\theta 1} \dot{\theta}_1 \sin[\theta_2] + 18 R_{\theta 2} \dot{\theta}_2 \sin[\theta_2] -$$

$$60 R_{\theta 1} \dot{\theta}_1 \sin[\theta_1 + \theta_2] - 30 R_{\theta 2} \dot{\theta}_2 \sin[\theta_1 + \theta_2] +$$

$$27 R_{\theta 2} \dot{\theta}_2 \sin[\theta_1 + \theta_2 + \theta_1 + \theta_2] \Big)$$

`In[]:= CR21 = FullSimplify[`

$$\begin{aligned} & \left(9 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1 + \theta_2] \right) / 32 - \left(9 * R_{\theta 2} \dot{\theta}_2 * \cos[\theta_1 + \theta_2] \right) / 16 - \\ & \left(9 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1 + \theta_2] \right) / 16 + \\ & \left(9 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_1 + \theta_2] \right) / 32 - \left(15 * R_{\theta 1} \dot{\theta}_1 * \cos[\theta_1] \right) / 32 - \\ & \left(27 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1 + \theta_2 + \theta_1] \right) / 160 + \\ & \left(15 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1] \right) / 64 - \left(27 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_2] \right) / 160 - \\ & \left(81 * R_{\theta 1} \dot{\theta}_1 * \sin[\theta_1 + \theta_2 + \theta_1 + \theta_2] \right) / 320 - \\ & \left(81 * R_{\theta 2} \dot{\theta}_2 * \sin[\theta_1 + \theta_2 + \theta_1 + \theta_2] \right) / 320 \end{aligned}$$

`Out[]:=`
$$\frac{1}{320} \left(-3 (50 R_{\theta 1} \dot{\theta}_1 \cos[\theta_1] + \right.$$

$$60 (R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \cos[\theta_1 + \theta_2] - 25 R_{\theta 1} \dot{\theta}_1 \sin[\theta_1] +$$

$$18 R_{\theta 1} \dot{\theta}_1 \sin[\theta_1 + \theta_2 + \theta_1] + 18 R_{\theta 2} \dot{\theta}_2 \sin[\theta_2] \Big) +$$

$$90 (R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \sin[\theta_1 + \theta_2] -$$

$$81 (R_{\theta 1} \dot{\theta}_1 + R_{\theta 2} \dot{\theta}_2) \sin[\theta_1 + \theta_2 + \theta_1 + \theta_2] \Big)$$

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In[ ]:= CR22 = FullSimplify[
  (9 * Rthetadot1 * Sin[Rtheta1 + Rtheta2]) / 32 - (9 * Rthetadot2 * Cos[Rtheta1 + Rtheta2]) / 16 -
  (9 * Rthetadot1 * Cos[Rtheta1 + Rtheta2]) / 16 +
  (9 * Rthetadot2 * Sin[Rtheta1 + Rtheta2]) / 32 - (15 * Rthetadot1 * Cos[Rtheta1]) / 32 -
  (27 * Rthetadot1 * Sin[Ltheta1 + Ltheta2 + Rtheta1]) / 160 +
  (15 * Rthetadot1 * Sin[Rtheta1]) / 64 - (27 * Rthetadot2 * Sin[Rtheta2]) / 160 -
  (81 * Rthetadot1 * Sin[Ltheta1 + Ltheta2 + Rtheta1 + Rtheta2]) / 320 -
  (81 * Rthetadot2 * Sin[Ltheta1 + Ltheta2 + Rtheta1 + Rtheta2]) / 320]
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Out[ ]:= 
$$\frac{1}{320} \left( -3 \left( 50 \text{Rthetadot1} \cos[R\theta_1] + \right. \right. \\ \left. \left. 60 \left( \text{Rthetadot1} + \text{Rthetadot2} \right) \cos[R\theta_1 + R\theta_2] - 25 \text{Rthetadot1} \sin[R\theta_1] + \right. \right. \\ \left. \left. 18 \text{Rthetadot1} \sin[L\theta_1 + L\theta_2 + R\theta_1] + 18 \text{Rthetadot2} \sin[R\theta_2] \right) + \right. \\ \left. 90 \left( \text{Rthetadot1} + \text{Rthetadot2} \right) \sin[R\theta_1 + R\theta_2] - \right. \\ \left. 81 \left( \text{Rthetadot1} + \text{Rthetadot2} \right) \sin[L\theta_1 + L\theta_2 + R\theta_1 + R\theta_2] \right)$$

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In[ ]:= PkL1 =
  FullSimplify[- (10 * 2^(1/2) * ((9 * Cos[Ltheta1 + Ltheta2]) / 20 + (3 * Cos[Ltheta1]) / 20) *
    ((2^(1/2) * (27 * Cos[Ltheta2] - 90 * Sin[Ltheta1 + Ltheta2] - 30 * Sin[Ltheta1] + 95)^(1/2)) / 20 - 3/5)) /
    (27 * Cos[Ltheta2] - 90 * Sin[Ltheta1 + Ltheta2] - 30 * Sin[Ltheta1] + 95)^(1/2)]
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Out[ ]:= 
$$\left( 3 \left( \cos[L\theta_1] + 3 \cos[L\theta_1 + L\theta_2] \right) \right. \\ \left. \left( 6\sqrt{2} - \sqrt{95 + 27 \cos[L\theta_2] - 30 \sin[L\theta_1] - 90 \sin[L\theta_1 + L\theta_2]} \right) \right) / \\ \left( 20 \sqrt{95 + 27 \cos[L\theta_2] - 30 \sin[L\theta_1] - 90 \sin[L\theta_1 + L\theta_2]} \right)$$

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In[ ]:= PkL2 =
  FullSimplify[- (10 * 2^(1/2) * ((9 * Cos[Ltheta1 + Ltheta2]) / 20 + (27 * Sin[Ltheta2]) / 200) *
    ((2^(1/2) * (27 * Cos[Ltheta2] - 90 * Sin[Ltheta1 + Ltheta2] - 30 * Sin[Ltheta1] + 95)^(1/2)) / 20 - 3/5)) /
    (27 * Cos[Ltheta2] - 90 * Sin[Ltheta1 + Ltheta2] - 30 * Sin[Ltheta1] + 95)^(1/2)]
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Out[ ]:= 
$$\left( 9 \left( 10 \cos[L\theta_1 + L\theta_2] + 3 \sin[L\theta_2] \right) \right. \\ \left. \left( 6\sqrt{2} - \sqrt{95 + 27 \cos[L\theta_2] - 30 \sin[L\theta_1] - 90 \sin[L\theta_1 + L\theta_2]} \right) \right) / \\ \left( 200 \sqrt{95 + 27 \cos[L\theta_2] - 30 \sin[L\theta_1] - 90 \sin[L\theta_1 + L\theta_2]} \right)$$

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PkR1 = FullSimplify[ - (20 * 5^(1/2) * ((3 * Cos[Rtheta1 + ArcTan[1/2]])) / 40 +
  (9 * Cos[Rtheta1 + Rtheta2 + ArcTan[1/2]]) / 40) *
  ((9 * Cos[Rtheta1 + Rtheta2] + 3 * Cos[Rtheta1] - 5)^2 +
    (9 * Sin[Rtheta1 + Rtheta2] + 3 * Sin[Rtheta1] - 10)^2)^(1/2) / (20 - 3/5)) /
  ((9 * Cos[Rtheta1 + Rtheta2] + 3 * Cos[Rtheta1] - 5)^2 +
    (9 * Sin[Rtheta1 + Rtheta2] + 3 * Sin[Rtheta1] - 10)^2)^(1/2) ]

Out[2]= - ((3 (Cos[Rtheta1 + ArcCot[2]] + 3 Cos[Rtheta1 + Rtheta2 + ArcCot[2]])
  (-12 + Sqrt((-5 + 3 Cos[Rtheta1] + 9 Cos[Rtheta1 + Rtheta2])^2 +
    (-10 + 3 Sin[Rtheta1] + 9 Sin[Rtheta1 + Rtheta2])^2))) /
  (8 Sqrt(5) Sqrt((-5 + 3 Cos[Rtheta1] + 9 Cos[Rtheta1 + Rtheta2])^2 +
    (-10 + 3 Sin[Rtheta1] + 9 Sin[Rtheta1 + Rtheta2])^2)))

In[3]:= PkR2 = FullSimplify[
  - (20 * ((27 * Sin[Rtheta2]) / 200 + (9 * 5^(1/2) * Cos[Rtheta1 + Rtheta2 + ArcTan[1/2]])) /
    40) * ((9 * Cos[Rtheta1 + Rtheta2] + 3 * Cos[Rtheta1] - 5)^2 +
      (9 * Sin[Rtheta1 + Rtheta2] + 3 * Sin[Rtheta1] - 10)^2)^(1/2) / (20 - 3/5)) /
  ((9 * Cos[Rtheta1 + Rtheta2] + 3 * Cos[Rtheta1] - 5)^2 +
    (9 * Sin[Rtheta1 + Rtheta2] + 3 * Sin[Rtheta1] - 10)^2)^(1/2) ]

Out[3]= - ((9 (5 Sqrt(5) Cos[Rtheta1 + Rtheta2 + ArcCot[2]] + 3 Sin[Rtheta2])
  (-12 + Sqrt((-5 + 3 Cos[Rtheta1] + 9 Cos[Rtheta1 + Rtheta2])^2 +
    (-10 + 3 Sin[Rtheta1] + 9 Sin[Rtheta1 + Rtheta2])^2))) /
  (200 Sqrt((-5 + 3 Cos[Rtheta1] + 9 Cos[Rtheta1 + Rtheta2])^2 +
    (-10 + 3 Sin[Rtheta1] + 9 Sin[Rtheta1 + Rtheta2])^2)))

In[5]:= Pcl1 = FullSimplify[(9 * Lthetadot1) / 40 + (81 * Lthetadot2) / 400 +
  (27 * Lthetadot1 * Cos[Ltheta2]) / 200 + (27 * Lthetadot2 * Cos[Ltheta2]) / 400]

Out[5]= 9/400 (10 Lthetadot1 + 9 Lthetadot2 + 3 (2 Lthetadot1 + Lthetadot2) Cos[Ltheta2])

In[6]:= Pcl2 = FullSimplify[
  (81 * Lthetadot1) / 400 + (81 * Lthetadot2) / 400 + (27 * Lthetadot1 * Cos[Ltheta2]) / 400]

Out[6]= 27/400 (3 (Lthetadot1 + Lthetadot2) + Lthetadot1 Cos[Ltheta2])

In[7]:= Pcr1 = FullSimplify[(9 * Lthetadot1) / 40 + (81 * Lthetadot2) / 400 +
  (27 * Lthetadot1 * Cos[Ltheta2]) / 200 + (27 * Lthetadot2 * Cos[Ltheta2]) / 400]

Out[7]= 9/400 (10 Lthetadot1 + 9 Lthetadot2 + 3 (2 Lthetadot1 + Lthetadot2) Cos[Ltheta2])

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In[8]:= PcR2 = FullSimplify[
  (81 * Lthetadot1) / 400 + (81 * Lthetadot2) / 400 + (27 * Lthetadot1 * Cos[Ltheta2]) / 400]
Out[8]:=  $\frac{27}{400} (3 (Lthetadot1 + Lthetadot2) + Lthetadot1 \cos[Ltheta2])$ 
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