Swing-Up Methods For Inverted Pendulum

Conference Paper · September 2003

DOI: 10.6084/m9.figshare.697505

1 author:

Stefan Brock
Poznan University of Technology

Preprint of the paper from conference:
INTERNATIONAL CONFERENCE ON ELECTRICAL DRIVES AND POWER ELECTRONICS
SLOVAKIA 24 – 26 SEPTEMBER 2003

SWING-UP METHODS FOR INVERTED PENDULUM

Stefan BROCK
Poznań University of Technology
Institute of Control and Information Engineering
ul. Piotrowo 3A 60-965 Poznań/POLAND
e-mail: Stefan.Brock@put.poznan.pl

Abstract. This paper deals with the design of a swing up controller for the inverted pendulum, which brings the pendulum from any initial position to the unstable up position.
Classical method of swing-up control demands large range of movement of cart. Proposed method is assigned for systems with limited movement of cart. This method is based on position control of cart. Reference position is changing according to the state of pendulum. Simulation and experiments on a real pendulum illustrate the results.

Keywords: Motion control, Non-linear control, Simulation

1. INTRODUCTION

For several decades, inverted pendulum systems have served as excellent test beds for control theory. Because they exhibit nonlinear, unstable, non-minimum phase dynamics, and because the full-state is not often fully measured, control objectives are always challenging. In this paper the swing-up controller, which brings the pendulum from any initial position to the unstable up-position is studied.

A swing-up controller of an inverted pendulum system must diverge the pendulum from the stable position, that is, the hung down position, while a stabilization controller must stand the pendulum in the unstable position, that is, the standing position. Therefore, the swing-up controller and the stabilization controller have to be designed individually by many authors in the case of using a control technique based on a linear model, for example, an optimal control theory.

Astrom and Furuta [1] proposed swing up strategies based on energy control and time minimum. The position and the velocity of the pivot are not considered in this paper. Muškinja and Tovornik [2] proposed fuzzy logic controller for swinging up the pendulum. In Acosta et al. paper [3] new strategy based on a 3-dimension control law for the rotating pendulum is presented. In this case, the total movement of the cart can be ignored. Botroff [5] proposed a robust swing-up controller for double rotational pendulum. Zhong and Rock in [4] extended energy based control for the case of double inverted pendulum. Kawashima [6] has used only one controller for both swing-up and stabilization. The software limit switches are also proposed for the swing-up of inverted pendulum under the restriction of cart track length.

In this study a simple controller executing swing-up of linear inverted pendulum for the system with limited movement of the cart is developed. The swing-up strategy is based on position control of the cart. Reference position is changing according to the state of pendulum

2. CLASSICAL SWING - UP STRATEGY BASED ON ENERGY CONTROL.

Consider a single pendulum presented on fig. 1. Let its mass be m_p and let the moment of inertia with respect to the pivot point on the cart be J. Furthermore let $l_p/2$ be the distance from the pivot to the center of mass. The angle between the vertical and the pendulum is θ_p, where θ_p is positive in the clockwise direction. A cart with a mass m_c can implement its linear motion on a leading screw, which is driven by the motor. The position of a cart is x_c, and its speed is equal $v_c=dx_c/dt$.

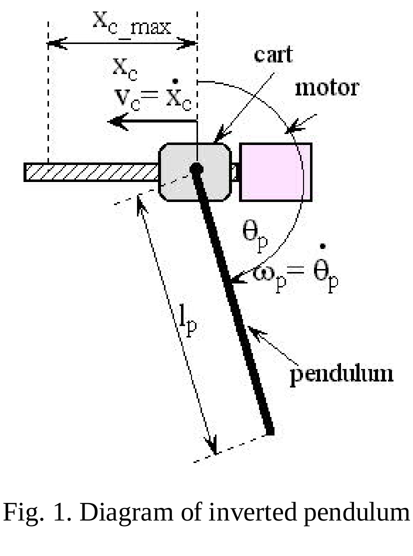

Fig. 1. Diagram of inverted pendulum

The acceleration of gravity is g and the acceleration of the cart is $a_c=dv_c/dt$. The acceleration a_c is positive if it is in the direction of the positive x axis.

If the friction can be neglected and it can be assumed that the pendulum is a rigid body, the equation of motion for the pendulum is:

$$2*J*\frac{d\ \omega_p}{dt}-m*g*l*\sin(\theta_p)+m*a_c*l*\cos(\theta_p)=0 \quad (1)$$

The total energy of the pendulum E is:

$$E=\frac{1}{2}*J*\omega_p{}^2+\frac{1}{2}*m*g*l_p*(\cos(\theta_p)-1) \quad (2)$$

The total energy is defined to be zero when the pendulum is in the upright position. The derivative of E can be computed as:

$$\frac{dE}{dt}=J*\omega_p*\frac{d\omega_p}{dt}-\frac{1}{2}*m*g*l_p*\sin(\theta_p) \quad (3)$$

From (1) and (3):

$$\frac{dE}{dt}=-\frac{1}{2}*m*a_c*l_p*\omega_p*\cos(\theta_p)=-k*a_c*\omega_p*\cos(\theta_p) \quad (4)$$

where k is positive constants. The swing-up method is based on energy control. The control strategy can be considered as a simple way of pumping energy into the pendulum. E is growing if:

$$a_c=-a_{c\_\max}*\operatorname{sgn}(\omega_p)*\operatorname{sgn}(\cos(\theta_p)) \quad (5)$$

where $a_{c\_\max}$ is the largest acceleration of the cart.

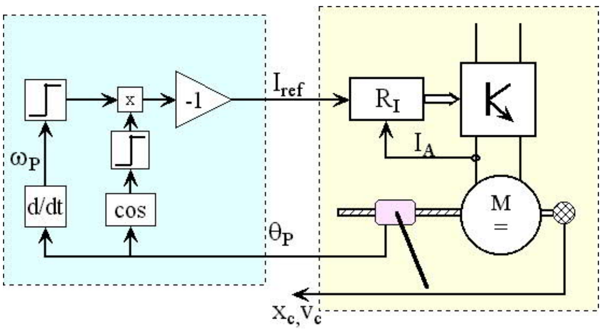

Fig. 2. Block diagram of classical swing-up control

Controllability is lost when the coefficient of a_c in the right hand side of (4) vanishes. This occurs for $\omega_p=0$ or $\theta_p=1/2*\pi$ or $\theta_p=1/2*\pi$, i.e., when the pendulum is horizontal or when it reverses its velocity. Control action is most effective when the angle θ_p is 0 or π and the velocity is large.

The simple block diagram of this classical swing-up control is presented on fig. 2. The information about position of cart is not used. The maximum cart position and speed depends only on acceleration and pendulum parameters.

3. PROPOSED SWING - UP STRATEGY UNDER THE RESTRICTION OF CART TRACK LENGTH

When way of movement of cart is to short, the classical swing-up method can not be used. The software limit switches are also proposed for the swing-up of inverted pendulum under the restriction of cart track length. The proposed control system is presented on fig. 3. First, the complete positions control system for the cart is designed. The input signal is reference position, switched by logic subsystem. After the change of reference position, the cart accelerates left or right. The pendulum energy grows up, if the reference signal is synchronized with pendulum position, according to equation 4. The control action is most effective, when the angle of pendulum during cart acceleration is close to π. In best case:

$$\pi-\alpha_{best}<\theta_p<\pi+\alpha_{best} \quad (6)$$

The best start angle α_{best} can be calculated as:

$$\alpha_{best}=\theta_{\max}*\sin\left(\frac{2*\pi}{T}*t_{acc}\right) \quad (7)$$

where θ_{max} is current amplitude of pendulum, T is the pendulum period and t_{acc} is the time of cart acceleration. On the end of way cart brakes. Deceleration can cause reduction of energy, but for whole cycle the energy grows. It the increase of energy is bigger than friction losses, the pendulum can swing -up.

4. SIMULATION STUDY OF SWING-UP

Simulation model of inverted pendulum was created using Matlab/Simulink system. In model the static and kinetic friction in cart and in pendulum are considered. The parameters are identified on the real laboratory stand. The simulation model was presented in [7]. The parameters of pendulum are listed in Appendix. Four pendulums were tested - from short to very long. In the paper results for short and very long only are presented.

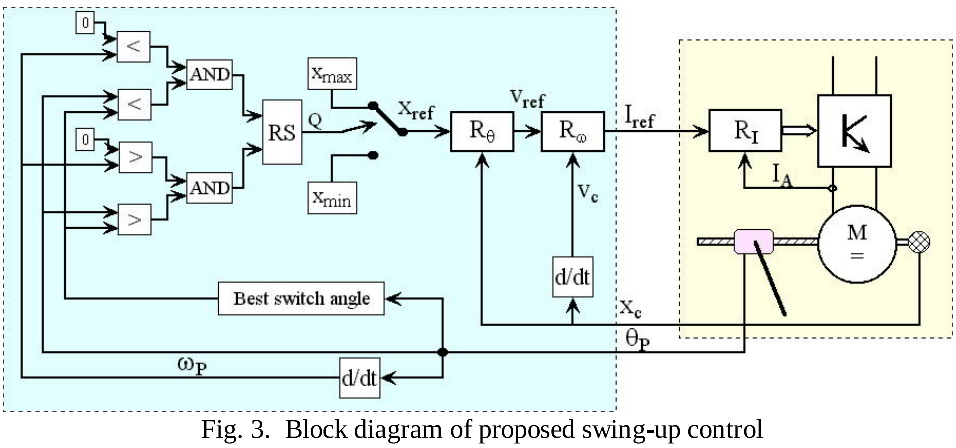

Fig. 3. Block diagram of proposed swing-up control

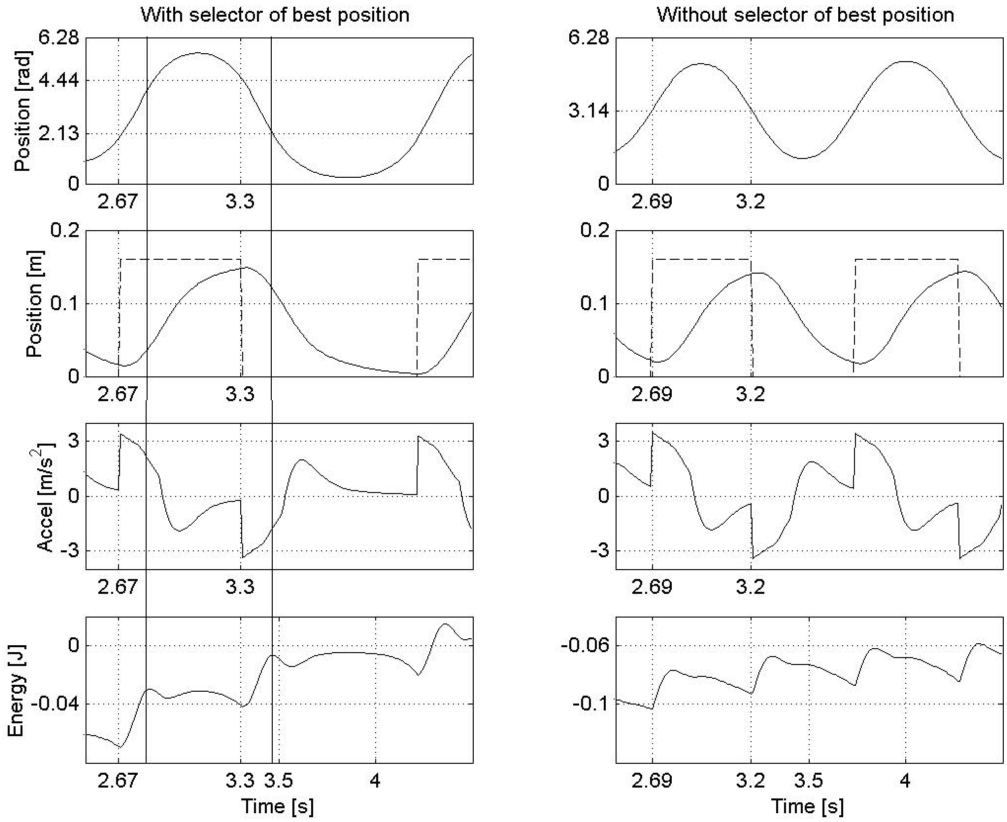

Fig. 4. Influence of selector of best position onto performance of system - short pendulum.

- position of pendulum,
- reference (dashed line) and real (solid line) position of cart
- acceleration of cart
- total energy of pendulum

First the position control system was designed and tuned. The anti wind-up PI speed and nonlinear P position controllers are used. The acceleration time is equal t_{acc}=0.15 s. Next, the best angle selector was tested. The results for short pendulum are presented on fig. 4. In this case the angle α_{best} =1 rad. Without best angle selector ($\alpha_{best} \equiv 0$ rad) the total energy of pendulum increases slowly and there is no possibility to reach upper position. If the best angle selector is used energy increases faster.

The complete swing up process is presented on fig. 5. In the case of classical, optimal controller the pendulum swing up in 2.74 s, and the cart move to 0.26 m. Proposed controller swing up the pendulum in 4.65 s, but the cart move only to 0.16 m.

On the next figure 6 the same signals are presented for very long pendulum. The optimal controller swing up the pendulum in 11.58 s, and the cart move to 1.25 m. Proposed controller swing up the pendulum in 53.47 s, but the cart move again only to 0.16 m. In this case there is a phase of waiting for the optimal pendulum position before the reference cart position is changed.

The results for medium and long pendulums are listed in Appendix.

5. EXPERIMENTAL VALIDATION OF CONTROL METHOD

The functional scheme of pendulum control system is illustrated in fig. 7. DC motor M is supplied from 4-quadrant transistor converter with current regulation loop. The measurement signals θ_p and $\Delta\theta_m$ (motor angle position change) are introduced to PC based control system. The system has two measurement converters: increment rotary-impulse converter (1024 impulses per rotation), connected to driven motor shaft and absolute code rotary converter (10-bit) connected to pendulum rotary axe. The experimental results are presented on fig. 8. After 10 s the pendulum swing-up, and the cart is moving only to 8 cm. Every deceleration cause reduction of pendulum energy, but for the whole cycle energy increases.

6. CONCLUSION

Presented simple swing-up method for inverted pendulum can be used, if the movement of cart should be limited. In such case there is no possibility to use the classical methods. Presented method can by use as first stage controller in hybrid control of inverted pendulum.

Simulation and real experiment confirmed proposed method.

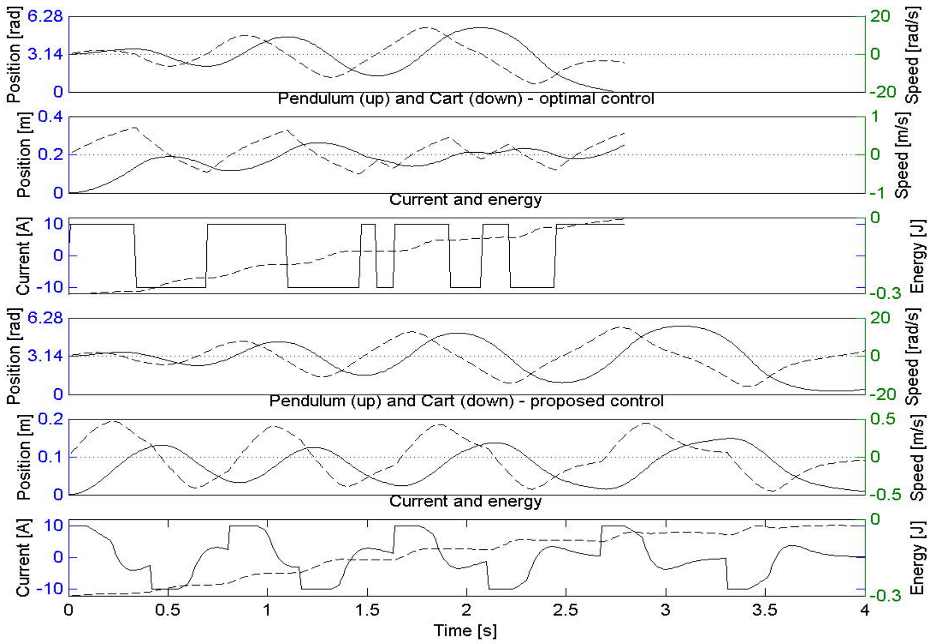

Fig. 5. Comparisons of swing-up for classical (up) and proposed (down) controls - short pendulum (numerical simulation)

- position (solid line) and speed (dashed line) of pendulum
- position (solid line) and speed (dashed line) of cart
- reference current (solid line) and total energy of pendulum (dashed line)

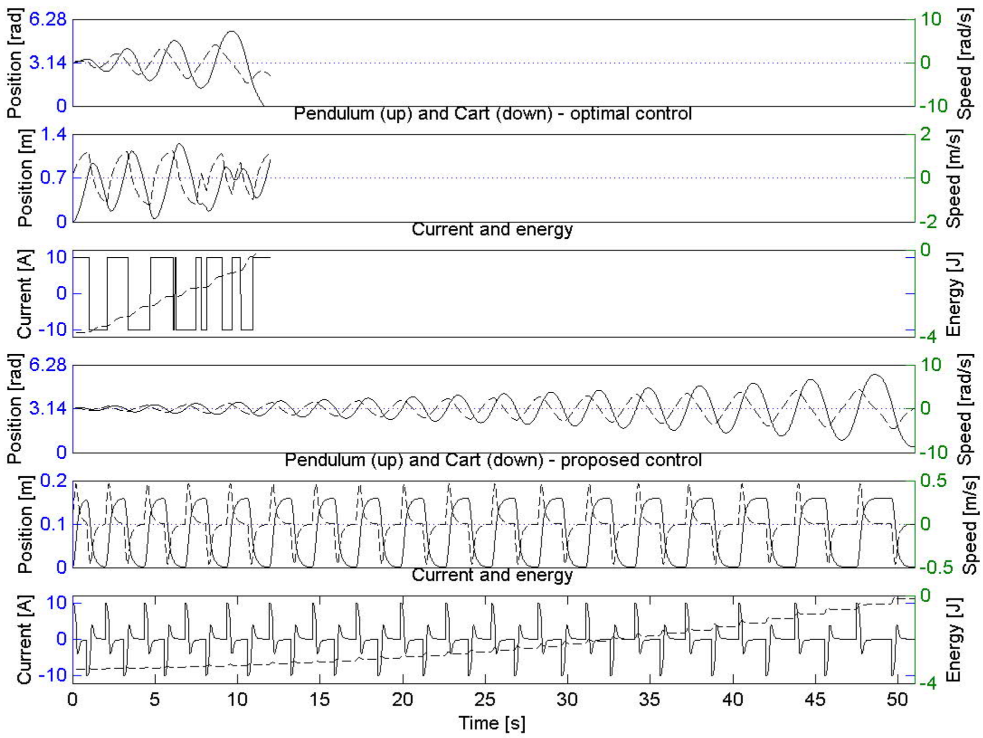

Fig. 6. Comparison of swing-up for classical (up) and proposed (down) controls - very long pendulum (numerical simulation) - - the same signals as on fig. 5.

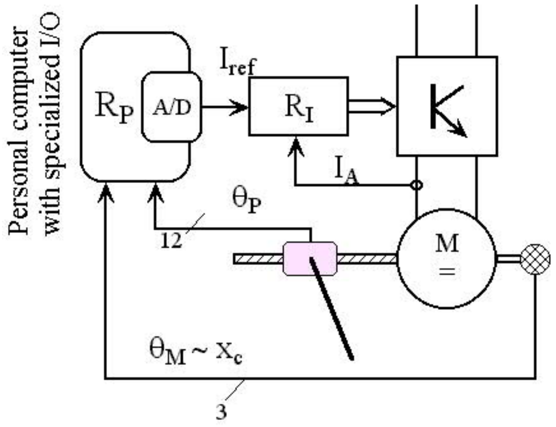

Fig. 7. Block diagram of experimental system

7. REFERENCES

[1] Åström , K.J. - Furuta, K. : Swinging Up a Pendulum by Energy Control. Proceedings of the 1996 IFAC World Congress, (San Francisco, CA). July, 1996

[2] Muškinj, N. - Tovornik, B.: Controlling of real inverted pendulum by fuzzy logic. http://www.au.feri.uni-mb.si/~nenad/.

[3] Acosta, J. A. - Gordillo, F. - Aracil, J.: Swinging Up the Furuta Pendulum by the Speed Gradient Method. European Control Conference (ECC). 2001

[4] Zhong, W. - Rock, H.: Energy and Passivity Based Control of the Double Inverted Pendulum on a Cart. IEEE Conference On Control Applications, 2001

[5] Botroff, S.A.: Robust Swing-up control for a rotational double pendulum. , Proceedings of the 1996 IFAC World Congress, Volume F, (San Francisco, CA). July, 1996

[6] Kawashima, T.: Swing-Up and Stabilization of Inverted Pendulum Using Only One Sliding Mode Controller with Nonlinear Model Observer.

[7] Brock, S.: Identification of the parameters in inverted pendulum model. 7th International Workshop on Advanced Motion Control, Maribor, Slovenia. 3-5 July 2002, pp.: 316 - 321

.

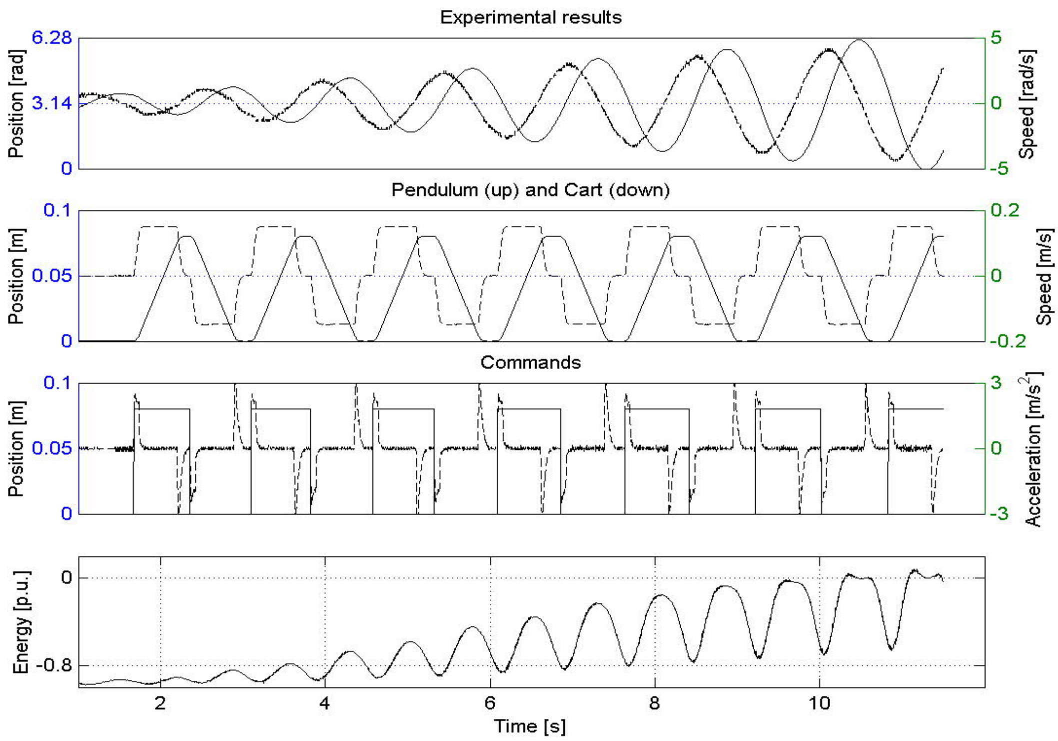

Fig. 8. Experimental results for proposed control

- position (solid line) and speed (dashed line) of pendulum
- position (solid line) and speed (dashed line) of cart
- reference position (solid line) and acceleration (dashed line) of cart
- total energy of pendulum (relative)

APPENDIX

Most important parameters:

Mass of the cart m_c=1 kg

Mass of the pendulum m_p=0.13 kg

Motor torque constant k_m=0.105 Nm/A

Maximal cart speed vmax=0.5 m/s

Inertia of screw J_s=2.6*10^{-4} kg*m^2

Translation gain of leading screw ks=5/(2* π) mm/rad

Maximal current I_{max} =10 A

Maximal cart position x_{max}=0.16 m

<table>
<tr><th colspan="2">Pendulum</th><th>Natural</th><th colspan="3">Classical swing-up control</th><th colspan="3">Proposed swing-up control</th></tr>
<tr><th></th><th>length [m]</th><th>period [s]</th><th>swing-up time [s]</th><th>max cart position [m]</th><th>max cart speed [m/s]</th><th>swing-up time [s]</th><th>max cart position [m]</th><th>max cart speed [m/s]</th></tr>
<tr><td>Short</td><td>0.22</td><td>0.80</td><td>2.74</td><td>0.26</td><td>0.71</td><td>4.65</td><td rowspan="4">0.16</td><td rowspan="4">0.50</td></tr>
<tr><td>Medium</td><td>0.44</td><td>1.21</td><td>3.12</td><td>0.41</td><td>0.81</td><td>5.78</td></tr>
<tr><td>Long</td><td>1.32</td><td>3.01</td><td>6.30</td><td>0.78</td><td>1.08</td><td>18.40</td></tr>
<tr><td>Very Long</td><td>2.64</td><td>5.48</td><td>11.58</td><td>1.25</td><td>1.24</td><td>53.47</td></tr>
</table>

THE AUTHOR

Stefan Brock received the M.S. and Ph.D. degrees from Poznań University of Technology in Poznań Poland in 1990 and 1997, respectively. Currently he is working in Poznań University of Technology as assistant professor. He is interested in motion control and application of soft computing in industrial electronic.

ACKONWLEDGEMENT

This work was supported by a grant **PB-45-085/03 /BW**