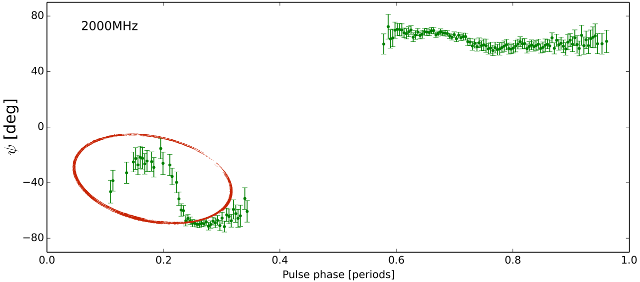
Emission beam geometry of PSR J2007+2722

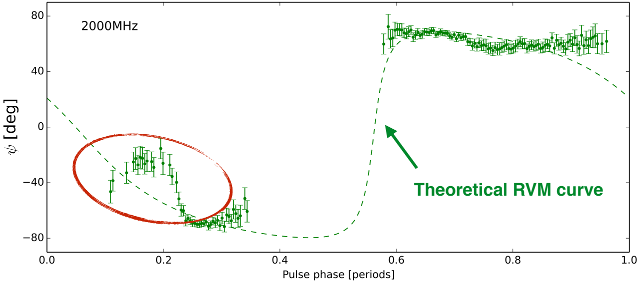
Tjark Miener

February 27, 2018

Motivation


2000MHz
80
40
0
−40
−80
ψ [deg]
0.0
0.2
0.4
0.6
0.8
1.0
Pulse phase [periods]

Motivation


2000MHz
Theoretical RVM curve
ψ [deg]
Pulse phase [periods]
80
40
0
−40
−80
0.0
0.2
0.4
0.6
0.8
1.0

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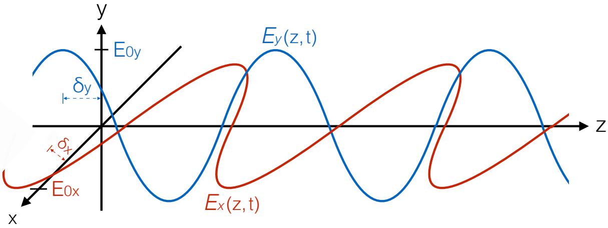
Electric field vector

- The polarization of a transverse wave refers to the geometrical orientation of the oscillating electric field:

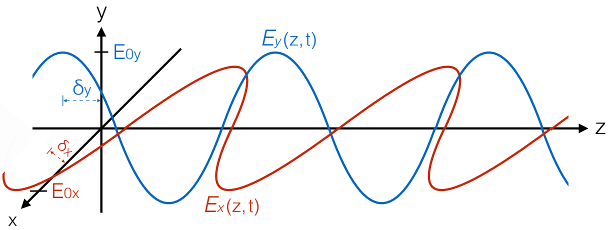
$$\vec{E}(z,t) = \vec{e}_x \cdot E_{0x} \cos(\tau + \delta_x) + \vec{e}_y \cdot E_{0y} \cos(\tau + \delta_y)$$

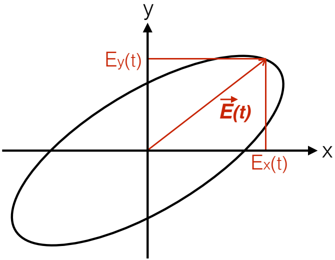
  - $\tau = \omega t - kz$ is the propagator
  - E_{0x} and E_{0y} are the maximum amplitudes
  - δ_x and δ_y are the corresponding phases

Electric field vector



Electric field vector





Polarization ellipse

- Equation of the polarization ellipse:

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x E_y}{E_{0x} E_{0y}} \cos\delta = \sin^2\delta$$

  - $\delta = \delta_y - \delta_x$ is the phase difference

Polarization ellipse

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x E_y}{E_{0x} E_{0y}} \cos\delta = \sin^2\delta$$

$$\Leftrightarrow$$

$$(\underbrace{E_{0x}^2 + E_{0y}^2}_{\equiv S_0})^2 - (\underbrace{E_{0x}^2 - E_{0y}^2}_{\equiv S_1})^2 - (\underbrace{2E_{0x}E_{0y}\cos\delta}_{\equiv S_2})^2 = (\underbrace{2E_{0x}E_{0y}\sin\delta}_{\equiv S_3})^2$$

Stokes vector

- Stokes vector:

$$\vec{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos\delta \\ 2E_{0x}E_{0y}\sin\delta \end{pmatrix}$$

Stokes vector

- Stokes vector:

$$\vec{S} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I_x + I_y \\ I_x - I_y \\ I_{+45^\circ} - I_{-45^\circ} \\ I_{\mathrm{RC}} - I_{\mathrm{LC}} \end{pmatrix}$$

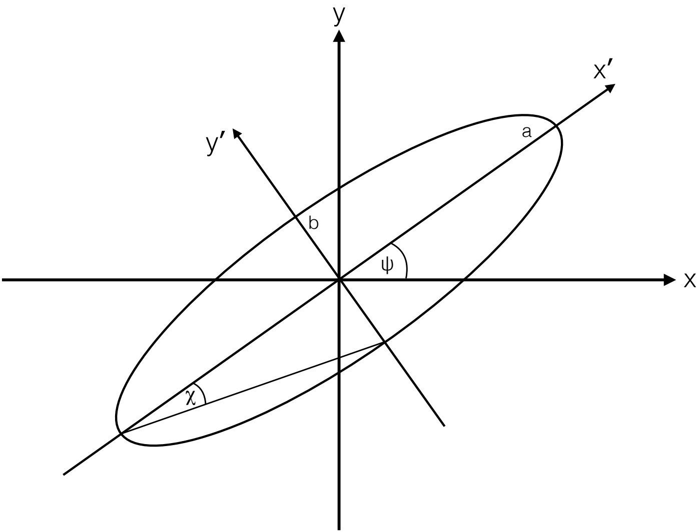
Stokes vector

- Stokes vector:

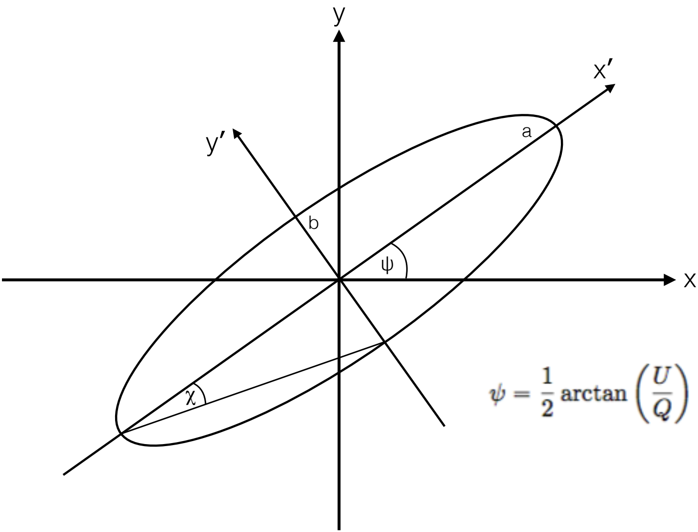
$$\vec{S} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I_x + I_y \\ I_x - I_y \\ I_{+45^\circ} - I_{-45^\circ} \\ I_{\mathrm{RC}} - I_{\mathrm{LC}} \end{pmatrix}$$

  - I is the total intensity
  - $L = \sqrt{Q^2 + U^2}$ is the linearly polarized intensity
  - V is the circularly polarized intensity

Orientation angle ψ

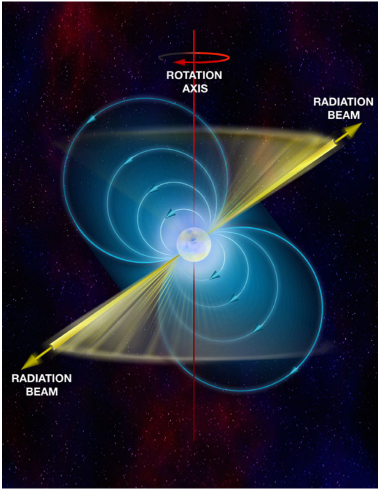


Orientation angle ψ



$$\psi = \frac{1}{2} \arctan\left(\frac{U}{Q}\right)$$

Basic radio pulsar model



Credit: Bill Saxton, NRAO/AUI/NSF

Goldreich-Julian model

- Inside the rotating superconducting sphere a force-free state is obtained:

$$\vec{E} + \frac{1}{c}\left(\vec{\Omega} \times \vec{r}\right) \times \vec{B} = 0$$

- External quadrupole field:

$$\Phi_{ext}(r, \theta) = -\frac{\Omega B_0 R^5}{6cr^3}(3\cos^2\theta - 1)$$

Goldreich-Julian model

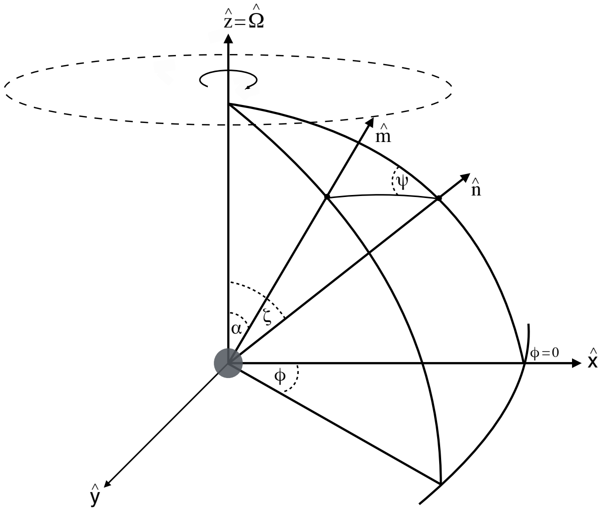
- Corresponding electric field at the stellar surface:

$$E_{\parallel} \equiv \left. \frac{\vec{E}_{ext} \cdot \vec{B}}{|\vec{B}|} \right|_{r=R} = \frac{-2\Omega B_0 R}{c} \frac{\cos^3\theta}{\sqrt{3\cos^2\theta + 1}}$$

- *Goldreich-Julian charge density*:

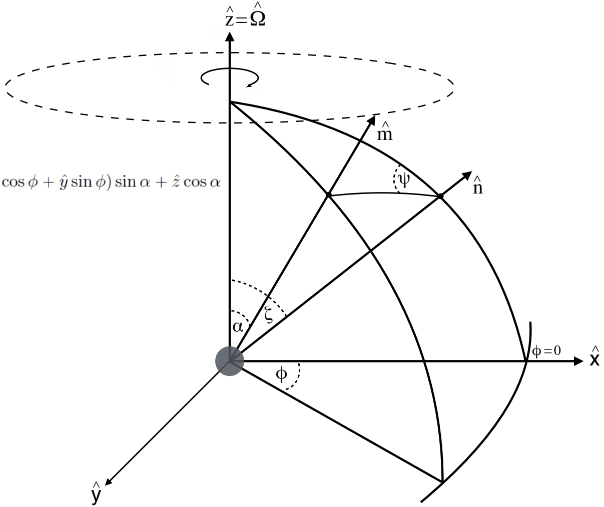
$$\rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c} = -\frac{\Omega B_0}{4\pi c} \frac{R^3}{r^3} \left(3\cos^2\theta - 1\right)$$

Rotating vector model



Rotating vector model

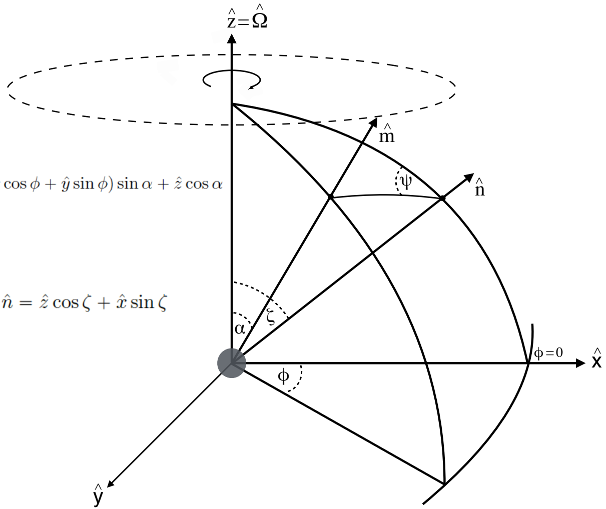
$$\hat{m} = (\hat{x}\cos\phi + \hat{y}\sin\phi)\sin\alpha + \hat{z}\cos\alpha$$



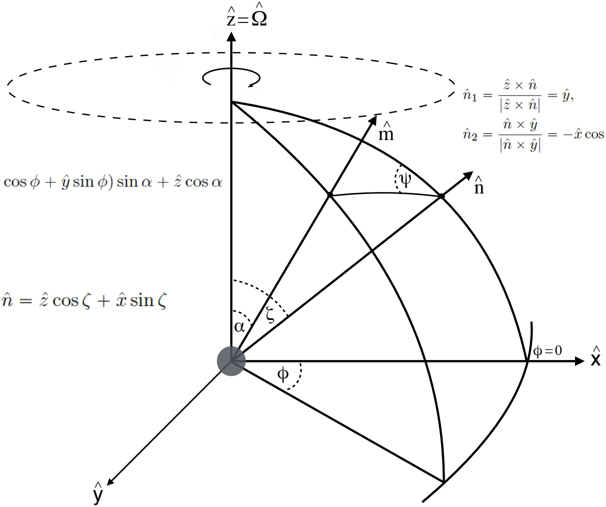
Rotating vector model

$$\hat{m} = (\hat{x}\cos\phi + \hat{y}\sin\phi)\sin\alpha + \hat{z}\cos\alpha$$

$$\hat{n} = \hat{z}\cos\zeta + \hat{x}\sin\zeta$$



Rotating vector model



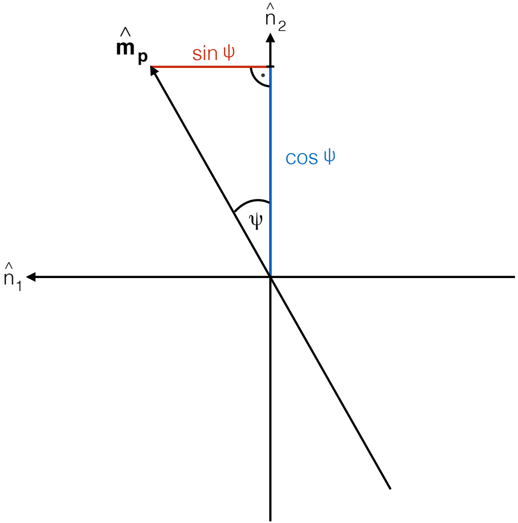
$$\hat{n}_1 = \frac{\hat{z}\times\hat{n}}{|\hat{z}\times\hat{n}|} = \hat{y},$$

$$\hat{n}_2 = \frac{\hat{n}\times\hat{y}}{|\hat{n}\times\hat{y}|} = -\hat{x}\cos\zeta + \hat{z}\sin\zeta$$

$$\hat{m} = (\hat{x}\cos\phi + \hat{y}\sin\phi)\sin\alpha + \hat{z}\cos\alpha$$

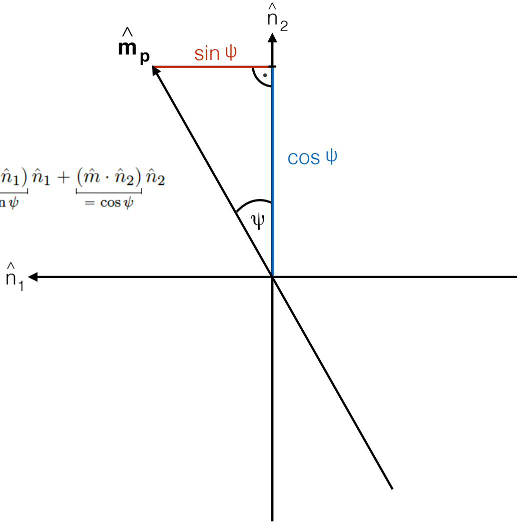
$$\hat{n} = \hat{z}\cos\zeta + \hat{x}\sin\zeta$$

Rotating vector model



Rotating vector model

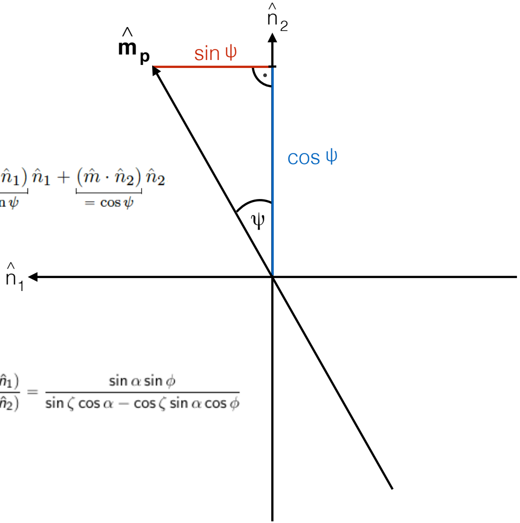
$$\hat{m}_p = \underbrace{(\hat{m} \cdot \hat{n}_1)}_{= \sin\psi} \hat{n}_1 + \underbrace{(\hat{m} \cdot \hat{n}_2)}_{= \cos\psi} \hat{n}_2$$



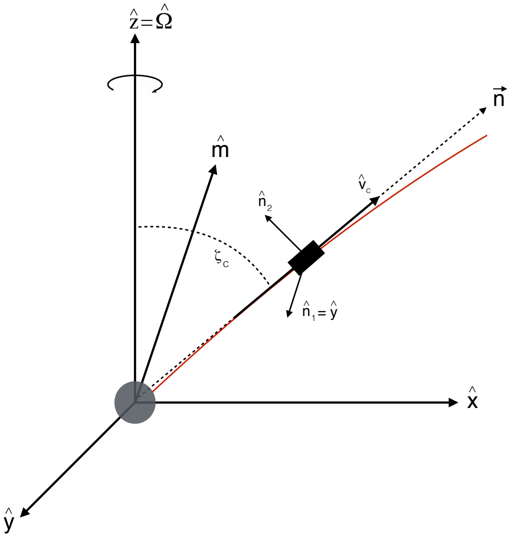
Rotating vector model

$$\hat{m}_p = \underbrace{(\hat{m}\cdot\hat{n}_1)}_{=\sin\psi}\hat{n}_1 + \underbrace{(\hat{m}\cdot\hat{n}_2)}_{=\cos\psi}\hat{n}_2$$

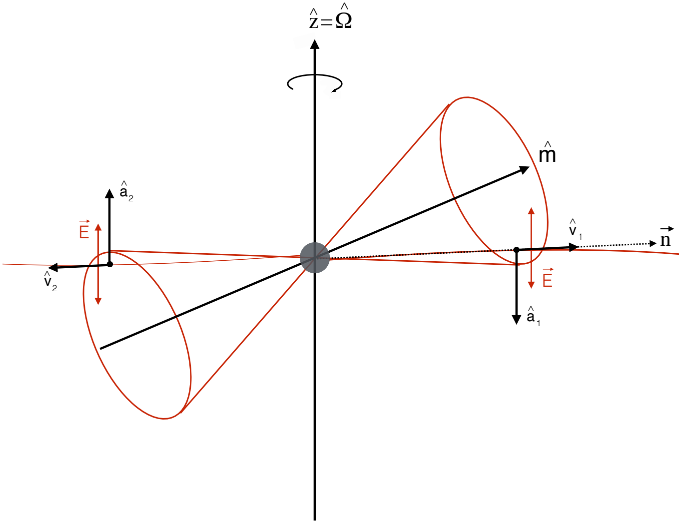
$$\tan\psi = \frac{(\hat{m}\cdot\hat{n}_1)}{(\hat{m}\cdot\hat{n}_2)} = \frac{\sin\alpha\sin\phi}{\sin\zeta\cos\alpha - \cos\zeta\sin\alpha\cos\phi}$$



Curvature radiation



Interpulse studies



Interpulse studies

- The geometrical values of the IP:

$$\alpha^{\mathrm{IP}} = 180^\circ - \alpha^{\mathrm{MP}}$$
$$\zeta^{\mathrm{IP}} = \zeta^{\mathrm{MP}}$$
$$\phi_0^{\mathrm{IP}} = \phi_0^{\mathrm{MP}} + 180^\circ$$
$$\psi_0^{\mathrm{IP}} = \psi_0^{\mathrm{MP}}$$

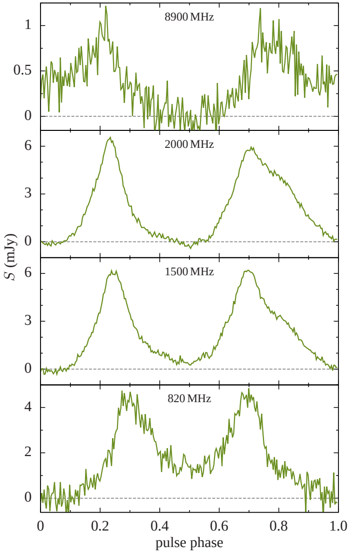
- The RVM predicts the same theoretical curve for the MP and IP:

$$\psi_{\mathrm{RVM}}\left(\alpha^{\mathrm{IP}}, \zeta^{\mathrm{IP}}, \phi_0^{\mathrm{IP}}, \psi_0^{\mathrm{IP}}\right) = \psi_{\mathrm{RVM}}\left(\alpha^{\mathrm{MP}}, \zeta^{\mathrm{MP}}, \phi_0^{\mathrm{MP}}, \psi_0^{\mathrm{MP}}\right)$$

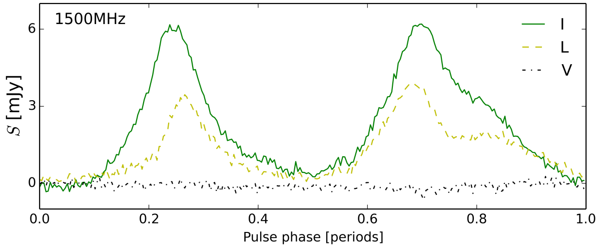
PSR J2007+2722

- PSR J2007+2722 was the first discovery of the *Einstein@Home* project
- Discovery date: July 11, 2010
- Location: Vulpecula constellation
  - *right ascension* $\alpha = 20^{\mathrm{h}}07^{\mathrm{m}}15^{\mathrm{s}}.83$
  - *declination* $\delta = +27^\circ 22' 47''.91$ (J2000)
  - angular frequency $\nu = 40.821\,\mathrm{Hz}$
- Distance to earth: $5.4\,\mathrm{kpc}$ ($\approx 17600\,\mathrm{ly}$)
- Reference: Allen, B., 2013. *ApJ*, **773**, 91.

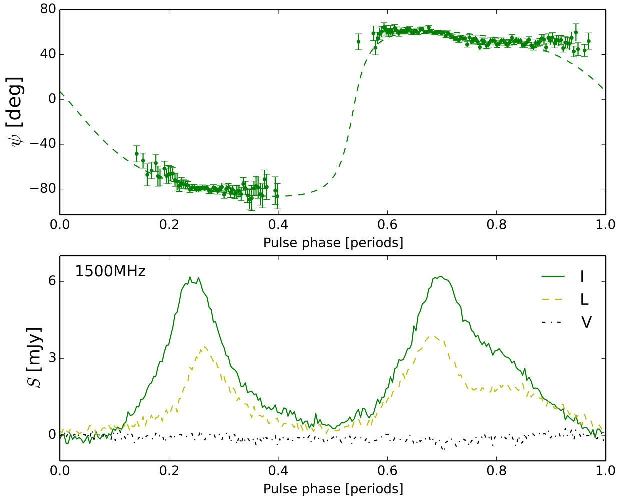
Pulse profile



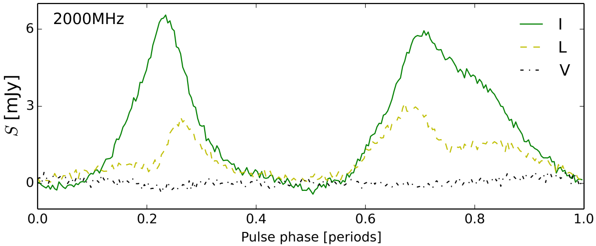
Stokes parameters



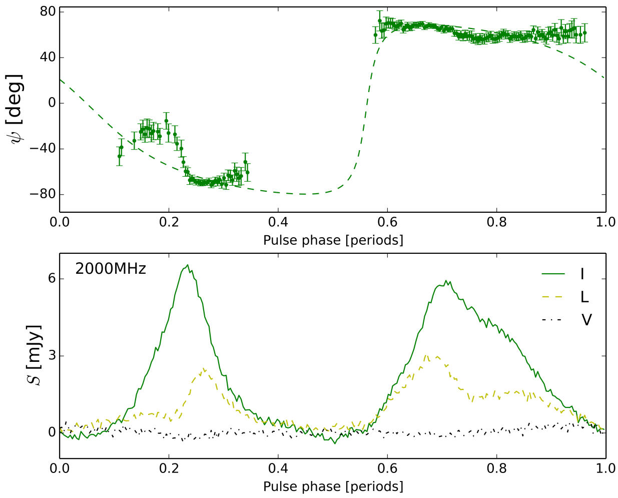
Polarization position angle



Stokes parameters



Polarization position angle



χ^2 method

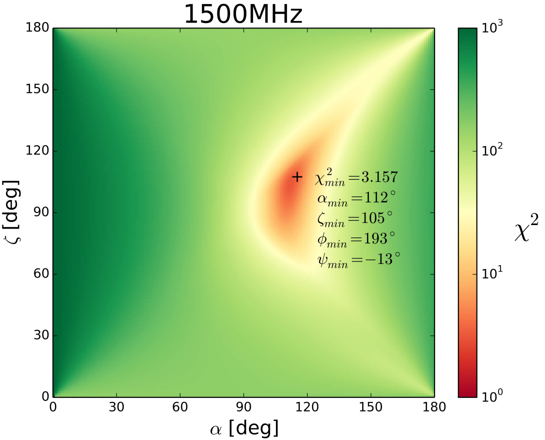
$$\chi^2 = \frac{1}{N-4} \sum_{i=1}^{N} \frac{(\psi(\phi_i) - \psi_{\mathrm{RVM}}(\phi_i))^2}{(\Delta\psi_i)^2}$$

χ^2 method

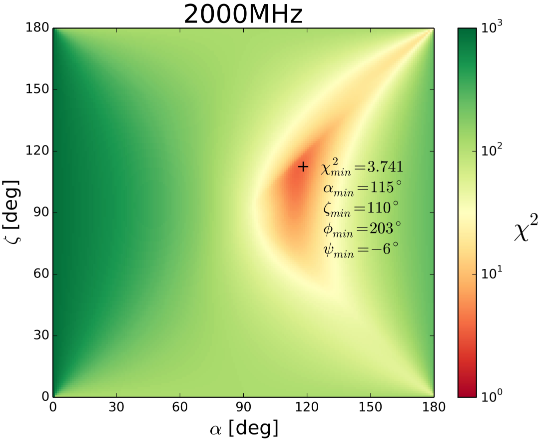
$$\chi^2 = \frac{1}{N-4} \sum_{i=1}^{N} \frac{\left(\psi(\phi_i) - \psi_{\mathrm{RVM}}(\phi_i)\right)^2}{(\Delta\psi_i)^2}$$

- $\psi = \frac{1}{2} \arctan\left(\frac{U}{Q}\right)$ is the measured value
- $\psi_{\mathrm{RVM}} = \arctan\left(-\frac{\sin\alpha \sin(\phi-\phi_0)}{\sin\zeta \cos\alpha - \cos\zeta \sin\alpha \cos(\phi-\phi_0)}\right) + \psi_0$ is the theoretical value
- $\Delta\psi_i$ is the measurement uncertainty
- N is the number of measurements

χ^2 method


1500MHz
$\chi^2_{min} = 3.157$
$\alpha_{min} = 112^\circ$
$\zeta_{min} = 105^\circ$
$\phi_{min} = 193^\circ$
$\psi_{min} = -13^\circ$
α [deg]
ζ [deg]
χ^2
0
30
60
90
120
150
180
10^0
10^1
10^2
10^3

χ^2 method


2000MHz
$\chi^2_{min} = 3.741$
$\alpha_{min} = 115^\circ$
$\zeta_{min} = 110^\circ$
$\phi_{min} = 203^\circ$
$\psi_{min} = -6^\circ$
ζ [deg]
α [deg]
0
30
60
90
120
150
180
10^0
10^1
10^2
10^3
χ^2

Best-fit RVM parameters

ν	α	ζ	ϕ_0	ψ_0	χ^2
1500 MHz	111.6°	104.1°	193°	-12.9°	3.137
2000 MHz	115.1°	109.7°	202°	-5.5°	3.736

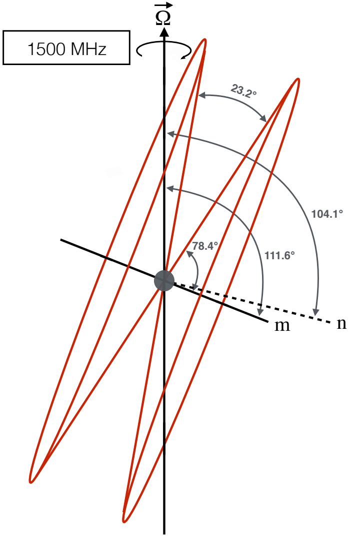
Emission beam geometry

- Emission cone half-opening angle:

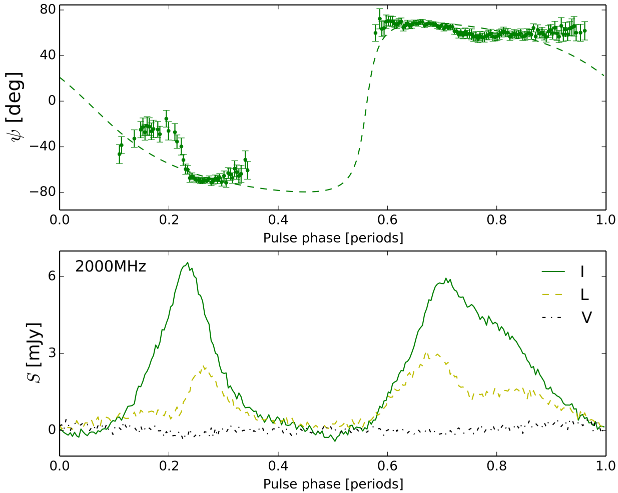
$$\rho = \arccos\left(\cos\alpha\cos\zeta + \sin\alpha\sin\zeta\cos\left(\frac{W}{2}\right)\right)$$

  - $W_{1500} = 165.9^\circ \Rightarrow \rho_{1500} = 78.4^\circ$
  - $W_{2000} = 170.2^\circ \Rightarrow \rho_{2000} = 77.5^\circ$

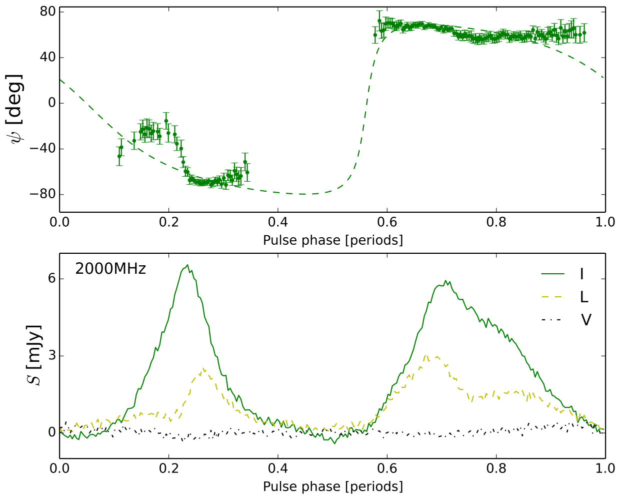
Emission beam geometry



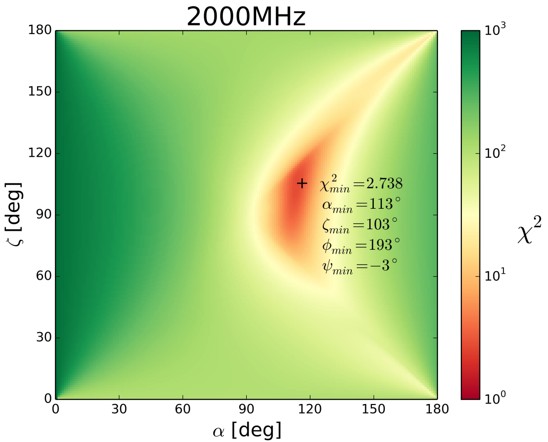
Extending the RVM with interpulse



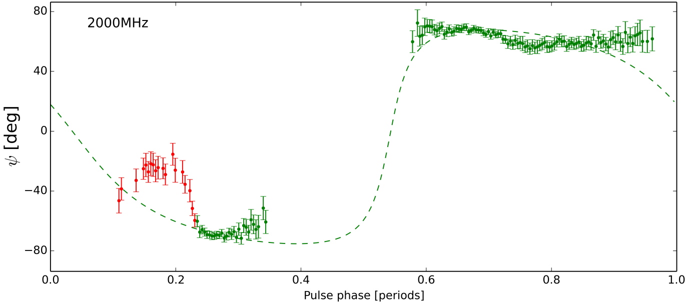
Disregard the bump at 2000 MHz



Disregard the bump at 2000 MHz



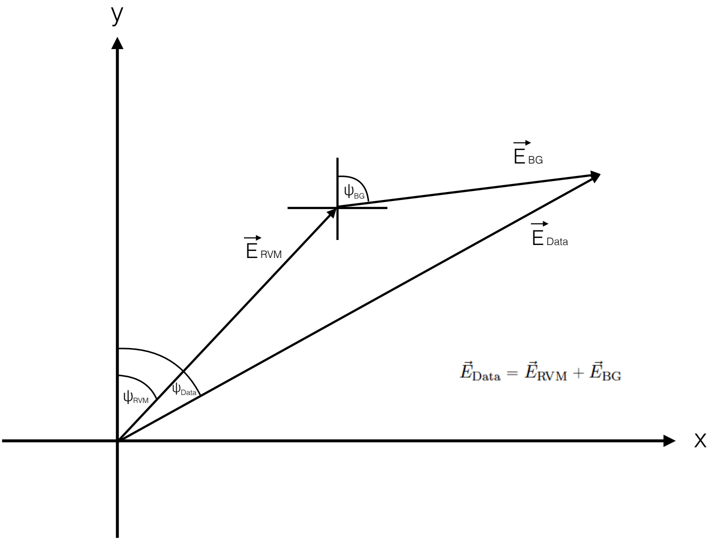
Disregard the bump at 2000 MHz



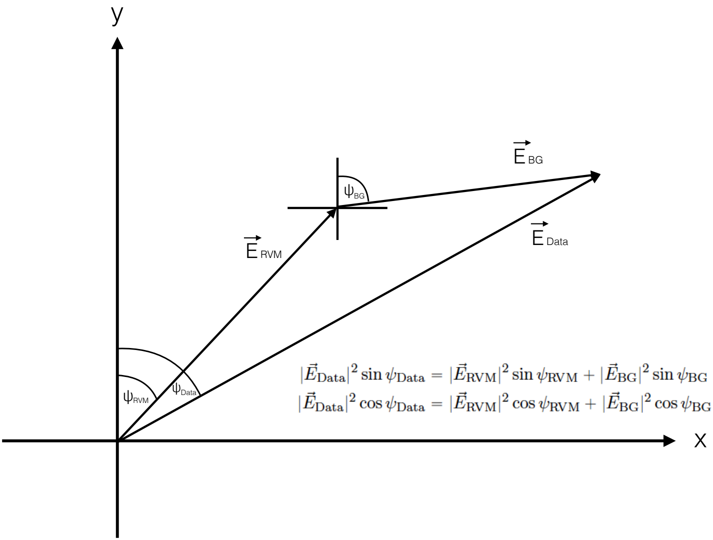
Disregard the bump at 2000 MHz

ν	α	ζ	ϕ_0	ψ_0	χ^2
1500 MHz	111.6°	104.1°	193°	-12.9°	3.137
2000 MHz	113.2°	104.3°	195°	-3.7°	2.728

Background radiation model



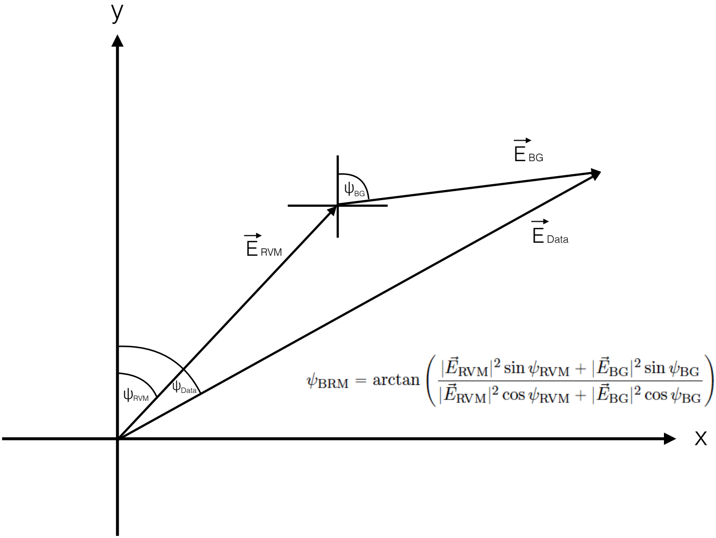
Background radiation model



$$|\vec{E}_{\mathrm{Data}}|^2 \sin\psi_{\mathrm{Data}} = |\vec{E}_{\mathrm{RVM}}|^2 \sin\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \sin\psi_{\mathrm{BG}}$$

$$|\vec{E}_{\mathrm{Data}}|^2 \cos\psi_{\mathrm{Data}} = |\vec{E}_{\mathrm{RVM}}|^2 \cos\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \cos\psi_{\mathrm{BG}}$$

Background radiation model



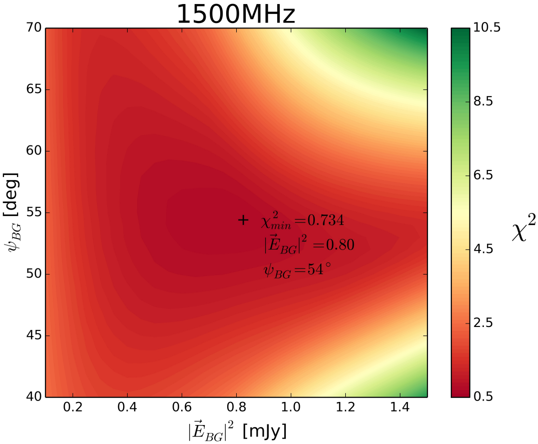
$$\psi_{\mathrm{BRM}} = \arctan\left(\frac{|\vec{E}_{\mathrm{RVM}}|^2 \sin\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \sin\psi_{\mathrm{BG}}}{|\vec{E}_{\mathrm{RVM}}|^2 \cos\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \cos\psi_{\mathrm{BG}}}\right)$$

χ^2 method

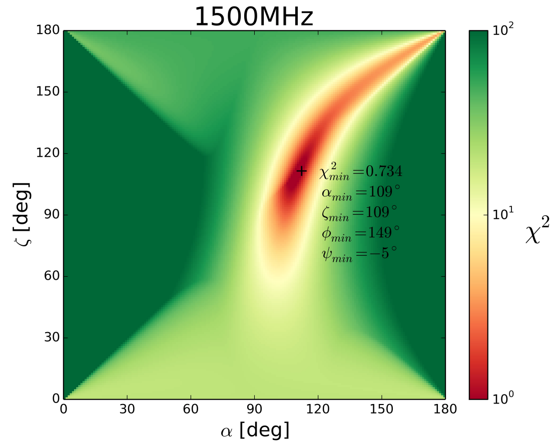
$$\chi^2 = \frac{1}{N-6} \sum_{i=1}^{N} \frac{\left(\psi(\phi_i) - \psi_{\mathrm{BRM}}(\phi_i)\right)^2}{(\Delta\psi_i)^2}$$

- $\psi = \frac{1}{2} \arctan\left(\frac{U}{Q}\right)$ is the measured value
- $\psi_{\mathrm{BRM}} = \arctan\left(\frac{|\vec{E}_{\mathrm{RVM}}|^2 \sin\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \sin\psi_{\mathrm{BG}}}{|\vec{E}_{\mathrm{RVM}}|^2 \cos\psi_{\mathrm{RVM}} + |\vec{E}_{\mathrm{BG}}|^2 \cos\psi_{\mathrm{BG}}}\right)$ is the theoretical value
- $\Delta\psi_i$ is the measurement uncertainty
- N is the number of measurements

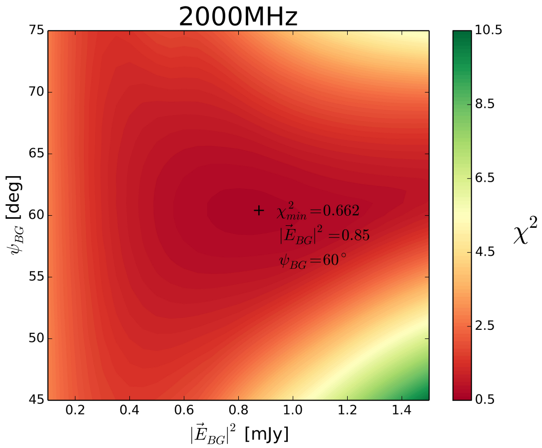
Background radiation model



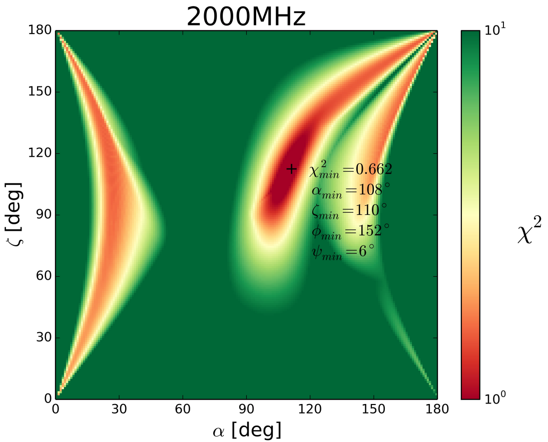
Background radiation model



Background radiation model



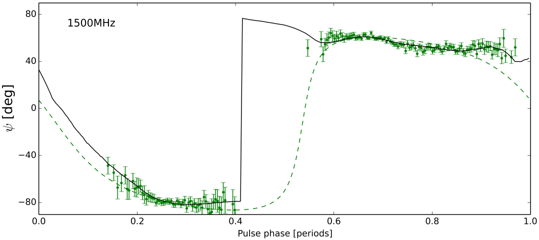
Background radiation model



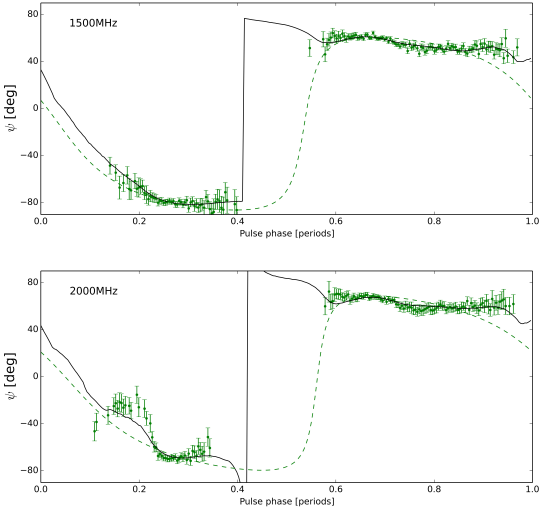
Background radiation model

ν	α	ζ	ϕ_0	ψ_0	$\lvert\vec{E}_{BG}\rvert^2$	ψ_{BG}	χ^2
1500	109°	109°	149°	-5°	$0.8\,\mathrm{mJy}$	54°	0.734
2000	108°	110°	152°	6°	$0.85\,\mathrm{mJy}$	60°	0.662

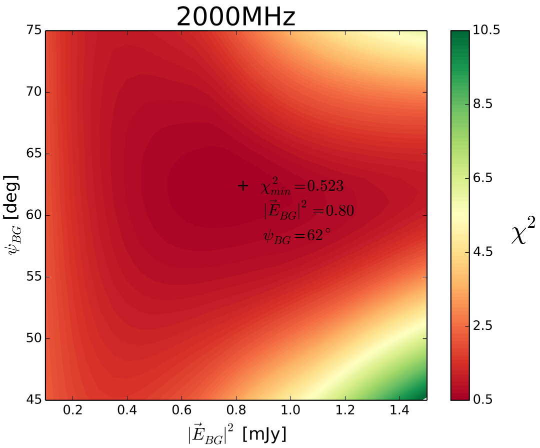
Background radiation model



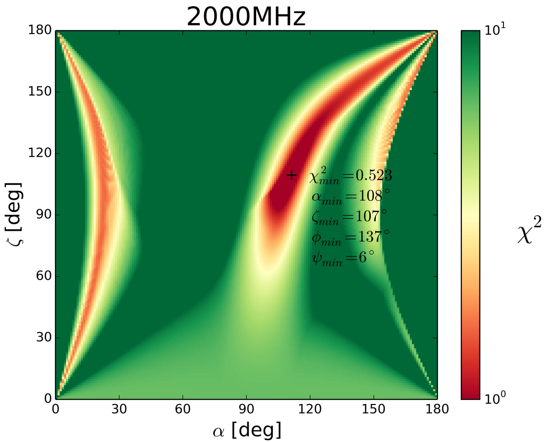
Background radiation model



Disregard the bump in the BRM



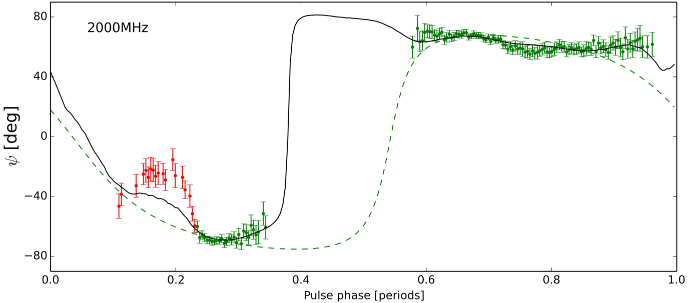
Disregard the bump in the BRM



Disregard the bump in the BRM

ν	α	ζ	ϕ_0	ψ_0	$\lvert\vec{E}_{\mathrm{BG}}\rvert^2$	ψ_{BG}	χ^2
1500	109°	109°	149°	-5°	$0.8\,\mathrm{mJy}$	54°	0.734
2000	108°	107°	137°	6°	$0.8\,\mathrm{mJy}$	62°	0.523

Disregard the bump in the BRM



What have we learned?

- Fundamentals of polarization
- Several pulsar models
- Confirmation of the results by B. Allen and B. Knispel
- Developing new model that describes the data well

Thank you for your attention.