HW4

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1.

1. 设 $\because y=ae^{bx}$

$\therefore \ln y=\ln a+bx$

$\therefore$ 设 $Y=\ln y,\ A=\ln a$

$\therefore Y=A+bx$

设 $P(x)=A+bx$，对此求线性拟合：

$$Q(A,b)=\sum_{i=1}^{n}(P(x_i)-Y_i)^2=\sum_{i=1}^{n}(A+bx_i-Y_i)^2$$

由 $\begin{cases}\frac{\partial Q}{\partial A}=0\\ \frac{\partial Q}{\partial b}=0\end{cases}$ $\therefore$ $\begin{cases}2\sum_{i=1}^{n}(A+bx_i-Y_i)=0\\ 2\sum_{i=1}^{n}(A+bx_i-Y_i)x_i=0\end{cases}$

解得 $\begin{cases}A=0.672295\\ b=1.028275\end{cases}$

$\therefore Y=0.672295+1.028275x$

$\because y=e^Y,\ a=e^A$

$\therefore y=1.958727e^{1.028275x}$

2.

2. 对于方程组 $\begin{cases} x_1-2x_2=4 \\ x_1+6x_2=14 \\ 3x_1+x_2=7.5 \\ x_1+x_2=4.5 \end{cases}$

写成矩阵：$\begin{pmatrix} 1 & -2 \\ 1 & 6 \\ 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 4 \\ 14 \\ 7.5 \\ 4.5 \end{pmatrix}$

法方程为：

$$\begin{pmatrix} 1 & 1 & 3 & 1 \\ -2 & 6 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 1 & 6 \\ 3 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 45. \\ 88 \end{pmatrix}$$

$$\therefore \begin{cases} x_1= \\ x_2= \end{cases} \sim \begin{pmatrix} 12 & 8 \\ 8 & 42 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 45 \\ 88 \end{pmatrix} \quad \therefore \begin{cases} x_1=2.695455 \\ x_2=1.581818 \end{cases}$$

3.

3、设 $x'=x-2010$ $y'=y-134091$

∴

x'	0	1	2	3	4
y'	0	644	1313	1981	2691

设 $P(x')=a+bx'+cx'^2$

∴
$$\begin{pmatrix}1&0&0\\1&1&1\\1&2&4\\1&3&9\\1&4&16\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}0\\644\\1313\\1981\\2691\end{pmatrix}$$

∴法线：
$$\begin{pmatrix}1&1&1&1&1\\0&1&2&3&4\\0&1&4&9&16\end{pmatrix}\begin{pmatrix}1&0&0\\1&1&1\\1&2&4\\1&3&9\\1&4&16\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}6629\\19977\\66781\end{pmatrix}$$

即
$$\begin{pmatrix}5&10&30\\10&30&100\\30&100&354\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}6629\\19977\\66781\end{pmatrix}$$

∴
$$\begin{cases}a=0.378788\\b=634.763636\\c=9.30303\end{cases}$$

∴ $y'=0.378788+634.7636x'+9.30303x'^2$

∴ $y=134091.378788+634.7636(x-2010)+9.30303(x-2010)^2$

$=134091.4+634.8(x-2010)+9.3(x-2010)^2$ $y(2015)=137497.8$