HW8

PB17111614

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1.

1. $a=1.0 \quad b=1.8 \quad n=4$

复化梯形：$T(h)=h\left[\frac{1}{2}f(a)+\sum_{i=1}^{n-1}f(a+ih)+\frac{1}{2}f(b)\right]$

$h=\frac{b-a}{n}=0.2$

$\therefore T(h)=0.2\times\left(\frac{1}{2}\times3.2+3.5+5.0+5.2+\frac{1}{2}\times4.8\right)$

$=3.54$

复化Simpson：$S_n(f)=\frac{h}{3}\left[f(a)+4\sum_{i=0}^{m-1}f(x_{2i+1})+2\sum_{i=1}^{m-1}f(x_{2i})+f(b)\right]$

$m=2$.

$\therefore S_4(f)=\frac{0.2}{3}\times\left[3.2+4\times(3.5+5.2)+2\times5.0+4.8\right]$

$=3.52$

2.

2、0阶：$I_3(x^0)=I(x^0)$

$\therefore 3h=a_{-1}+a_0+a_1$

1阶：$I_3(x^1)=I(x^1)$

$\therefore \frac{3}{2}h^2=-ha_{-1}+2ha_1$

2阶：$I_3(x^2)=I(x^2)$

$3h^3=h^2a_{-1}+4h^2a_1$

$$\Rightarrow \begin{cases} 3h=a_{-1}+a_0+a_1 \\ \frac{3}{2}h^2=-ha_{-1}+2ha_1 \\ 3h^3=h^2a_{-1}+4h^2a_1 \end{cases}$$

$$\therefore \begin{cases} a_{-1}=0 \\ a_0=\frac{9}{4}h \\ a_1=\frac{3}{4}h \end{cases}$$

$\therefore$ 至少有2次代数精度

$\because \int_{-h}^{2h}x^3dx \neq \frac{3}{4}h(h)^3$

$\therefore$ 具有2次代数精确度 $\int_{-h}^{2h}f(x)dx=\frac{9}{4}hf(0)+\frac{3}{4}hf(2h)$.

3.

(a).

3、

(1) 分别取 $f(x)=1, x, x^2, x^3, x^4$

$$\therefore \begin{cases} 4=A+B+C \\ 0=-Aa+Ca \\ \frac{16}{3}=Aa^2+Ca^2 \\ 0=-Aa^3+Ca^3 \\ \frac{64}{5}=Aa^4+Ca^4 \end{cases} \quad \therefore \begin{cases} A=\frac{10}{9} \\ B=\frac{16}{9} \\ C=\frac{10}{9} \\ a=\frac{2\sqrt{15}}{5} \end{cases}$$

$\because \int_{-2}^{2}x^5dx=\frac{10}{9}(-a)^5+\frac{10}{9}a^5$

$\int_{-2}^{2}x^6dx \neq \frac{10}{9}(-a)^6+\frac{10}{9}a^6$

$\therefore \int_{-2}^{2}f(x)dx=\frac{10}{9}f(-\frac{2\sqrt{15}}{5})+\frac{16}{9}f(0)+\frac{10}{9}f(\frac{2\sqrt{15}}{5})$ 具有5次代数精确度

(b).

(5). ∵ $I(f(x))$ 是 Gauss 型积分

∴ $E_n(f) = I(f) - G_n(f)$

$= \frac{f^{(2n)}(\xi)}{(2n)!}\int_a^b W(x)\omega_n^2(x)dx, \xi \in [a,b]$

$n=3$, $E_3(f) = \frac{f^{(6)}(\xi)}{6!}\int_{-2}^{2}(x+\frac{2\sqrt{15}}{5})x(x-\frac{2\sqrt{15}}{5})dx, \xi \in [a,b]$