HW2

1.

1. $L_0(x)=\frac{(x-100)(x-121)}{19\times 40}=\frac{(121-x)(100-x)}{760}$ $L_1(x)=\frac{(x-81)(x-121)}{19\times(-21)}=-\frac{(x-81)(x-121)}{399}$

$L_2(x)=\frac{(x-81)(x-100)}{40\times 21}=\frac{(x-81)(x-100)}{840}$

$\therefore L_2(x)=9\times\frac{(x-121)(x-100)}{760}-10\times\frac{(x-81)(x-121)}{399}+11\times\frac{(x-81)(x-100)}{840}$

$L_2(108)\approx 10.39398$ $R_2(x)=\frac{f^{(3)}(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)$

$\because f(x)=\sqrt{x}$ $\therefore f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$ $f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}$ $f^{(3)}(x)=\frac{3}{8}x^{-\frac{5}{2}}$

$\therefore R_2(x)=\frac{1}{16}\xi_x^{-\frac{5}{2}}(x-81)(x-100)(x-121)$ $\because \frac{1}{11^5}<\xi_x^{-\frac{5}{2}}<\frac{1}{10^5}$

$\therefore -0.00175<R_2(108)<-0.00109$

2、3.

i	x_i	$f(x_i)$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
0	1.0	4.0			
1	2.0	4.0	2.		
2	3.0	8.0	4	1	
3	4.0	5.0	-3.	$-\frac{7}{2}$	$-\frac{3}{2}$.

~~Newton插值式：~~

Newton多项式：$2+2(x-1)+x(x-1)-\frac{3}{2}x(x-1)(x-2)$.

$N(x)=-\frac{3}{2}x^3+\frac{11}{2}x^2-2x+2$.

$N(1.5)=6.3125$.

作差表：

0, $f(0)=2$

1, $f(1)=4.5$ $f[0,1]=-1.5$

3, $f(3)=0.25$ $f[1,3]=-0.125$ $f[0,1,3]=\frac{11}{24}$

3, $f'(3)=0.6$ $f[3,3]=0.6$ $f[1,3,3]=0.3625$ $f[0,1,3,3]=-0.03194$

~~$N(x)=2+(x-1.5)x+\frac{11}{24}$~~

$\therefore N(x)=2-1.5x+\frac{11}{24}x(x-1)-0.03194\times(x-1)(x-3)$

$f(2)=-0.01945$

$R(x)=\frac{f^{(4)}(\xi)}{4!}(x-0)(x-1)(x-3)(x-3)$.