HW3

1.

1. 用M关系式相连：

$h_0=1 \quad h_1=2 \quad h_2=1$

$\therefore \lambda_1=\frac{2}{3} \quad M_1=\frac{1}{3} \quad \lambda_2=\frac{1}{3} \quad M_2=\frac{2}{3}$

插值表：

i	x_i	$f(x_i)$	$f[x_{i+1},x_i]$	$f[x_{i-2},x_{i-1},x_i]$
0	-2	-4		
1	-1	3	7	
2	1	5	1	-2
3	2	10	5	$\frac{4}{3}$

$\therefore d_1=-12 \quad d_2=8 \quad M_0=M_3=0$

$\therefore \begin{pmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{pmatrix}\begin{pmatrix} M_1 \\ M_2 \end{pmatrix}=\begin{pmatrix} -12 \\ 8 \end{pmatrix} \quad \therefore \begin{cases} M_1=-8.25 \\ M_2=6.75 \end{cases}$

$$\therefore S(x)=\begin{cases} 1.125(x+2)^3+1.875(x+2)+4(x+1) & x\in[-2,-1] \\ 0.5625(x+1)^3-0.6875(x-1)^3+0.25(x+1)-4.25(x-1) & x\in[-1,1] \\ -1.125(x-2)^3+10(x-1)-3.875(x-2) & x\in[1,2] \end{cases}$$

$S(0)=0.5625-0.6875+0.25+4.25=4.375.$

2.

2、 $S_0(x)=\frac{x-x_1}{x_0-x_1}f(x_0)+\frac{x-x_0}{x_1-x_0}f(x_1)=2\frac{x-1.1}{-0.05}+2.2\frac{x-1.05}{0.05}$

$=44(x-1.05)-40(x-1.1)$ $\quad 1.05\le x\le 1.10$

$S_1(x)=\frac{x-x_2}{x_1-x_2}f(x_1)+\frac{x-x_1}{x_2-x_1}f(x_2)=2.2\cdot\frac{x-1.15}{-0.05}+2.17\frac{x-1.1}{0.05}$

$=43.4(x-1.1)-44(x-1.15)$ $\quad x\in[1.10,1.15]$

$S_2(x)=\frac{x-x_3}{x_1-x_3}f(x_2)+\frac{x-x_2}{x_3-x_2}f(x_3)=2.17\frac{x-1.2}{-0.05}+2.35\frac{x-1.15}{0.05}$

$=47(x-1.15)-43.4(x-1.20)$ $\quad x\in[1.15,1.20]$

∴ 插值函数：$S=\begin{cases}44(x-1.05)-40(x-1.1) & x\in[1.05,1.10]\\ 43.4(x-1.1)-44(x-1.15) & x\in[1.10,1.15]\\ 47(x-1.15)-43.4(x-1.2) & x\in[1.15,1.20]\end{cases}$

$\therefore f(1.075)\approx S(1.075)=2.100$

$f(1.175)\approx S(1.175)=2.260$

3.4.

3. $f(x)=10x^3+3x+2020$

i	x_i	$f(x_i)$	$f[x_{i-1},x_i]$	$f[x_{i-2},x_{i-1},x_i]$	$f[x_{i-3},x_{i-2},x_{i-1},x_i]$
0	1	2033			
1	2	2106	73		
2	3	2299	193	60	
3	4	2672	373	90	10

$\therefore f[1,2]=73 \quad f[1,2,3,4]=10.$

4. $L_i(x)=\prod_{\substack{j=0 \\ j\neq i}}^{6}\frac{x-x_j}{x_i-x_j}$

$$\therefore \sum_{i=0}^{6}(x_i^3+x_i^2+1)L_i(x)=\sum_{i=0}^{6}((2i)^3+(2i)^2+1)\prod_{\substack{j=0 \\ j\neq i}}^{6}\frac{x-x_j}{x_i-x_j}$$

$$=x^3+x^2+1.$$

$$\sum_{i=0}^{6}(x_i^3+x_i^2+1)L_i'(x)=3x^2+2x.$$