HW10

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1.

1. 设 $x_{n+1}-x_n = x_n - x_{n-1} = x_{n-1} - x_{n-2} = h$

∴ 对 $y(x_{n+1})$, $y'(x_n)$, $y'(x_{n-2})$ 在 x_{n-1} 作 Taylor 展开

$$y(x_{n+1}) = y(x_{n-1}) + 2h y'(x_{n-1}) + 2h^2 y''(x_{n-1}) + \frac{4}{3}h^{3} y^{(3)}(x_{n-1}) + \frac{2}{3}h^4 y^{(4)}(x_{n-1}) + O(h^5)$$

$$y'(x_n) = y'(x_{n-1}) + h y''(x_{n-1}) + \frac{h^2}{2} y^{(3)}(x_{n-1}) + \frac{h^3}{6} y^{(4)}(x_{n-1}) + O(h^4)$$

$$y'(x_{n-2}) = y'(x_{n-1}) - h y''(x_{n-1}) + \frac{h^2}{2} y^{(3)}(x_{n-1}) - \frac{h^3}{6} y^{(4)}(x_{n-1}) + O(h^4)$$

$$\therefore y_{n+1} = y(x_{n-1}) + \frac{h}{3}\left[7y'(x_n) - 2y'(x_{n-1}) + y'(x_{n-2})\right]$$

$$= y(x_{n-1}) + 2h y'(x_{n-1}) + 2h^2 y''(x_{n-1}) + \frac{4}{3}h^3 y^{(3)}(x_{n-1}) + \frac{1}{3}h^4 y^{(4)}(x_{n-1}) + O(h^5)$$

$$\therefore y(x_{n+1}) - y_{n+1} = \frac{1}{3}h^4 y^{(4)}(x_{n-1}) + O(h^5)$$

$$\therefore T_{n+1} = \frac{1}{3}h^4 y^{(4)}(x_{n-1}) + O(h^5).$$

2.

2、积分区间 $[x_{n-1}, x_{n+1}]$ 积分节点 $\{x_{n+1}, x_n, x_{n-1}\}$.

设 $h = x_n - x_{n-1} = x_{n+1} - x_n$.

$\therefore y_{n+1} = y_{n-1} + [\beta_0 f(x_{n+1}, y_{n+1}) + \beta_1 f(x_n, y_n) + \beta_2 f(x_{n-1}, y_{n-1})]$.

可求得 $\beta_0 = \int_{x_{n-1}}^{x_{n+1}} \frac{(x-x_n)(x-x_{n-1})}{(x_{n+1}-x_n)(x_{n+1}-x_{n-1})} dx = \frac{1}{3}h$

$\beta_1 = \int_{x_{n-1}}^{x_{n+1}} \frac{(x-x_{n+1})(x-x_{n-1})}{(x_n-x_{n+1})(x_n-x_{n-1})} dx = \frac{4}{3}h$.

$\beta_2 = \int_{x_{n-1}}^{x_{n+1}} \frac{(x-x_{n+1})(x-x_n)}{(x_{n-1}-x_{n+1})(x_{n-1}-x_n)} dx = \frac{1}{3}h$.

$y_{n+1} = y_{n-1} + \frac{h}{3}[f(x_{n+1}, y_{n+1}) + 4f(x_n, y_n) + f(x_{n-1}, y_{n-1})]$ —— 差分格式.

对 $y(x_{n+1})$, $y'(x_{n+1})$; $y'(x_n)$ 在 x_{n-1} 做 Taylor 展开:

$y(x_{n+1}) = y(x_{n-1}) + 2hy'(x_{n-1}) + 2h^2y''(x_{n-1}) + \frac{4}{3}h^3y^{(3)}(x_{n-1}) + \frac{2}{3}h^4y^{(4)}(x_{n-1}) + \frac{4}{15}h^5y^{(5)}(x_{n-1}) + O(h^6)$

$y'(x_{n+1}) = y'(x_{n-1}) + 2hy''(x_{n-1}) + 2h^2y^{(3)}(x_{n-1}) + \frac{4}{3}h^3y^{(4)}(x_{n-1}) + \frac{2}{3}h^4y^{(5)}(x_{n-1}) + O(h^5)$

$y'(x_n) = y'(x_{n-1}) + hy''(x_{n-1}) + \frac{1}{2}h^2y^{(3)}(x_{n-1}) + \frac{1}{6}h^3y^{(4)}(x_{n-1}) + \frac{1}{24}h^4y^{(5)}(x_{n-1}) + O(h^5)$.

$\therefore y(x_{n+1}) - y_{n+1} = -\frac{h^5}{90}y^{(5)}(x_{n-1}) + O(h^6)$

$\therefore$ 误差主项 $-\frac{h^5}{90}y^{(5)}(x_{n-1})$ 4阶

先计算 $p=1$, $q=1$ 的显式

积分区间 $[x_{n-1}, x_{n+1}]$ 节点 $\{x_n, x_{n-1}\}$

$\therefore y_{n+1} = y_{n-1} + [\alpha_0 f(x_n, y_n) + \alpha_1 f(x_1, y_1)]$

$\alpha_0 = \int_{x_{n-1}}^{x_{n+1}} \frac{(x-x_{n-1})}{(x_n-x_{n-1})} dx = 2h$ $\alpha_1 = \int_{x_{n-1}}^{x_{n+1}} \frac{(x-x_n)}{(x_{n-1}-x_n)} dx = 0$

$\therefore y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$

$\therefore$ 预估一校正格式:

$$\begin{cases} \bar{y}_{n+1} = y_{n-1} + 2hf(x_n, y_n) \\ y_{n+1} = y_{n-1} + \frac{h}{3}[f(x_{n+1}, \bar{y}_{n+1}) + 4f(x_n, y_n) + f(x_{n-1}, y_{n-1})] \end{cases}$$