HW6

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1,2.

1.(a) $\|A\|_1 = \max\limits_{1\le j\le 3}\sum_{i=1}^{3}|a_{ij}| = \max\{7,7,8\} = 8$

$\|A\|_\infty = \max\limits_{1\le i\le 3}\sum_{j=1}^{3}|a_{ij}| = \max\{8,5,9\} = 9.$

(b). $\|A\|_1 = \max\limits_{1\le j\le 3}\sum_{i=1}^{3}|a_{ij}| = \max\{7,5,8\} = 8$

$\|A\|_\infty = \max\limits_{1\le i\le 3}\sum_{j=1}^{3}|a_{ij}| = \max\{6,6,8\} = 8$

2、对于矩阵 $A = \begin{pmatrix} 4 & -2 \\ -1 & 1 \end{pmatrix}$

特征方程: $\det(\lambda E - A) = \begin{vmatrix} \lambda-4 & 2 \\ 1 & \lambda-1 \end{vmatrix} = (\lambda-1)(\lambda-4)-2 = 0.$

$\therefore$ 两个单特征值 $\lambda_1 = \frac{5-\sqrt{17}}{2}$ $\lambda_2 = \frac{5+\sqrt{17}}{2}$

$\therefore$ A的谱半径 $\rho(A) = \max\limits_{1\le i\le 2}|\lambda_i| = \frac{5+\sqrt{17}}{2}$

$\|A\|_2 = \sqrt{\rho(A^TA)}$

设 $B = A^TA = \begin{pmatrix} 20 & -6 \\ -6 & 2 \end{pmatrix}$ $\therefore \det(\lambda E - B) = \begin{vmatrix} \lambda-20 & 6 \\ 6 & \lambda-2 \end{vmatrix} = \lambda^2 - 22\lambda + 4 = 0$

单特征值 $\lambda_1 = 11+3\sqrt{13}$ $\lambda_2 = 11-3\sqrt{13}$

$\therefore \|A\|_2 = \sqrt{\rho(A^TA)} = \sqrt{\max\limits_{1\le i\le 2}|\lambda_i|} = \sqrt{11+3\sqrt{13}} = \frac{3\sqrt{2}+\sqrt{26}}{2}$

3.

3、整理为 $Ax=b$

$$\therefore \quad A=\begin{pmatrix} 5 & 1 & 2 \\ 1 & 3 & -1 \\ 2 & 1 & 5 \end{pmatrix} \qquad b=\begin{pmatrix} 10 \\ 5 \\ 20 \end{pmatrix}$$

用Doolittle分解法:

$$A = LU$$

$$= \begin{pmatrix} 1 & & \\ l_{21} & 1 & \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} 5 & 1 & 2 \\ & u_{22} & u_{23} \\ & & u_{33} \end{pmatrix}$$

$$\begin{cases} 5l_{21}=1 \\ 5l_{31}=2 \\ l_{21}+u_{22}=3 \\ 2l_{21}+u_{23}=-1 \\ l_{31}+l_{32}u_{22}=1 \\ 2l_{31}+l_{32}u_{23}+u_{33}=5 \end{cases}$$

$$\therefore \cancel{u}L=\begin{pmatrix} 1 & & \\ \frac{1}{5} & 1 & \\ \frac{2}{5} & \frac{3}{14} & 1 \end{pmatrix}$$

$$U=\begin{pmatrix} 5 & 1 & 2 \\ & \frac{14}{5} & -\frac{7}{5} \\ & & \frac{9}{2} \end{pmatrix}$$

$$\therefore \begin{cases} Ly=b \\ Ux=y \end{cases} \qquad \begin{cases} y_1 = 10 \\ \frac{1}{5}y_1+y_2=5 \\ \frac{2}{5}y_1+\frac{3}{14}y_2+y_3=20 \end{cases} \qquad \begin{cases} y_1=10 \\ y_2=3 \\ y_3=\frac{215}{14} \end{cases}$$

$$\therefore \begin{cases} 5x_1+x_2+2x_3=10 \\ \frac{14}{5}x_2-\frac{7}{5}x_3=3 \\ \frac{9}{2}x_3=\frac{215}{14} \end{cases} \qquad x=\left(\frac{5}{63},\ \frac{25}{9},\ \frac{215}{63}\right)^T$$