当 $x=1$ 时, $x^3=1$

$\therefore \frac{1}{2}(x-1)^3+a(x-1)^2+b(x-1)+c=1$

$\therefore c=1$

$$S''(x)=\begin{cases}6x & 0\le x\le 1\\ 3x+2a-3 & 1\le x\le 3\end{cases}$$

$\because$ 二阶导连续 $\therefore S''(1)=S''(1)$

$\therefore 6=3+2a-3 \quad \therefore a=3$

且一阶导连续 $S'(x)=\begin{cases}3x^2 & \\ \frac{3}{2}x^2+(2a-3)x+\frac{3}{2}-2a+b & \end{cases}$

$S'(1)=S'(1)$

$\therefore 3=\frac{3}{2}+2\times 3-3+\frac{3}{2}-2\times 3+b$

$\therefore b=3$

$\therefore a=3, b=3, c=1$