One Loop Gauge Link Self Energy

Yingsheng Huang

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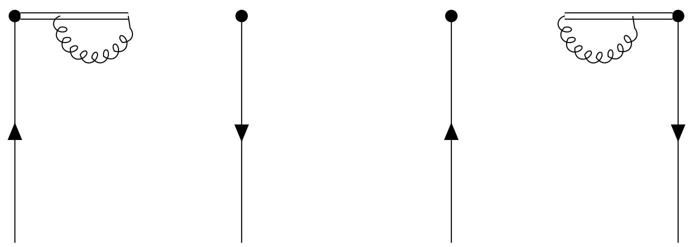
1 One Loop



(i) g (ii) h

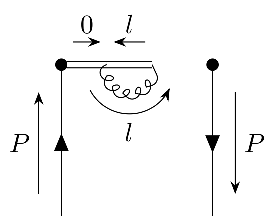
Figure 1: Diagrams of quasi PDF in Feynman gauge.

The definition of the gauge link self energy diagram (diagram g) is

$$\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle P,S|\bar{\psi}(z)\gamma^z\frac{\mathcal{P}\left[-ig_s\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right]\left[-ig_s\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\mathrm{t}^a\right]}{2}\psi(0)|P,S\rangle \tag{1}$$

Applying Feynman rule straightaway gives (the overall 1/2 factor has been counted in)

$$\Gamma_g(l) =$$



$$= -g_s^2 C_F\delta(1-x)\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{i\epsilon}\frac{i}{-l^z+i\epsilon} \tag{2}$$

1.1 Direct Contraction

1.1.1 Left

$$\begin{aligned}
&\frac{1}{2!}\mathcal{P}\left[\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\right]\\
&=\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\theta(z'-z'')\\
&=\int \mathrm{d}z' A^{a,z}\left(z'\right)\int \mathrm{d}z'' A^{a,z}\left(z''\right)\theta(z'-z'')\theta(z'')\\
&=\int \mathrm{d}z' \contraction{}{A}{^{a,z}\left(z'\right)\int \mathrm{d}z''}{A}A^{a,z}\left(z'\right)\int \mathrm{d}z'' A^{a,z}\left(z''\right)\theta(z'-z'')\theta(z'')\\
&=\int \mathrm{d}z'\int \mathrm{d}z''\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}e^{-il\cdot(z''-z')}\theta(z'-z'')\theta(z'')
\end{aligned}$$

1.1.2 Right

$$
\begin{aligned}
&\frac{1}{2!}\mathcal{P}\left[\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''+z\right)\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'+z\right)\right]\\
&=\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''+z\right)\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'+z\right)\theta(z''-z')\\
&=\int \mathrm{d}z'' A^{a,z}\left(z''+z\right)\int \mathrm{d}z' A^{a,z}\left(z'+z\right)\theta(z''-z')\theta(z')\\
&=\int \mathrm{d}z'' A^{a,z}\left(z''+z\right)\int \mathrm{d}z' A^{a,z}\left(z'+z\right)\theta(z''-z')\theta(z')\\
&=\int \mathrm{d}z'\int \mathrm{d}z''\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}e^{-il\cdot(z''-z')}\theta(z''-z')\theta(z')
\end{aligned}
$$

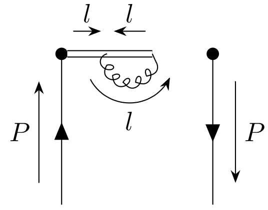
1.1.3 Summing together

$$
\int \mathrm{d}z'\int \mathrm{d}z''\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}e^{-il\cdot(z''-z')}[\theta(z'-z'')\theta(z'')+\theta(z''-z')\theta(z')]
$$

1.2 Adding two path order together

$$
\begin{aligned}
&\mathcal{P}\left[\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\right]\\
&=\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)[\theta(z'-z'')+\theta(z''-z')]\\
&=\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\\
&=\int \mathrm{d}z' A^{a,z}\left(z'\right)\int \mathrm{d}z'' A^{a,z}\left(z''\right)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\
&=\int \mathrm{d}z' A^{a,z}\left(z'\right)\int \mathrm{d}z'' A^{a,z}\left(z''\right)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\
&=\int \mathrm{d}z'\int \mathrm{d}z''\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}e^{-il\cdot(z''-z')}\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\
&=\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\int \mathrm{d}z'\int \mathrm{d}z''\int\frac{\mathrm{d}w}{2\pi}\frac{i}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{i}{h+i\epsilon}e^{-i(w-l)\cdot z'}e^{-i(h+l)\cdot z''}\\
&=\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{l^z+i\epsilon}\frac{i}{-l^z+i\epsilon}
\end{aligned}
$$

The amplitude is



$$
=-\frac{g_s^2 C_F}{2}\delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{l^z+i\epsilon}\frac{i}{-l^z+i\epsilon} \tag{3}
$$

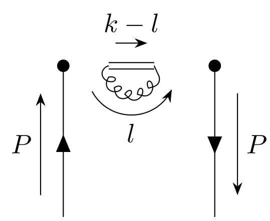
1.3 Adding $\Gamma_g(l)$ and $\Gamma_g(-l)$

$$\begin{aligned}
\Gamma_g(l) = \frac{\Gamma_g(l)+\Gamma_g(-l)}{2} &= -\frac{1}{2}\left[g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{i\epsilon}\frac{i}{-l^z+i\epsilon} + g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{i\epsilon}\frac{i}{l^z+i\epsilon}\right] \\
&= -\frac{1}{2} g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{i\epsilon}\left[\frac{i}{-l^z+i\epsilon} + \frac{i}{l^z+i\epsilon}\right] \\
&= -\frac{1}{2} g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{i\epsilon}\frac{2\epsilon}{{l^z}^2+\epsilon^2} \\
&= g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{l^z+i\epsilon}\frac{i}{l^z-i\epsilon}
\end{aligned}$$

There's an overall factor of 1/2 missing.

1.4 Taking derivatives

Add a small momentum to the gauge link line and consider an actual self energy diagram



$$= -g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{k^z-l^z+i\epsilon} \tag{4}$$

take the derivative

$$-g_s^2 C_F \delta(1-x)\lim_{k^z\to 0}\frac{\partial}{\partial k^z}\left[(i)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{k^z-l^z}\right] \tag{5}$$

$$= i g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{(l^z)^2} \tag{6}$$

Adding $i\epsilon$ by hand and we got

$$g_s^2 C_F \delta(1-x)\int \frac{\mathrm{d}^4 l}{(2\pi)^4}\frac{i}{l^2+i\epsilon}\frac{i}{l^z+i\epsilon}\frac{i}{l^z-i\epsilon} \tag{7}$$

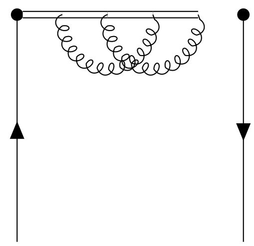
2 Two Loop

In coordinate space

$$\langle P,S|\bar\psi(z)\gamma^z\mathcal{P}\frac{\left[-ig_s n_\mu\int_0^\infty \mathrm{d}z_1 A^{a,\mu}(z_1)t^a\right]\left[-ig_s n_\nu\int_0^\infty \mathrm{d}z_2 A^{b,\nu}(z_2)t^b\right]\left[-ig_s n_\rho\int_0^\infty \mathrm{d}z_3 A^{c,\rho}(z_3)t^c\right]\left[-ig_s n_\sigma\int_0^\infty \mathrm{d}z_4 A^{d,\sigma}(z_4)t^d\right]}{4!} \psi(0)|P,S\rangle \tag{8}$$

$$\begin{aligned}
&\frac{1}{4!}\mathcal{P}\left[\int_0^\infty \mathrm{d}z_1 A^{a,\mu}(z_1)\right]\left[\int_0^\infty \mathrm{d}z_2 A^{b,\nu}(z_2)\right]\left[\int_0^\infty \mathrm{d}z_3 A^{c,\rho}(z_3)\right]\left[\int_0^\infty \mathrm{d}z_4 A^{d,\sigma}(z_4)\right] \\
&= \left[\int_0^\infty \mathrm{d}z_1 A^{a,\mu}(z_1)\right]\left[\int_0^\infty \mathrm{d}z_2 A^{b,\nu}(z_2)\right]\left[\int_0^\infty \mathrm{d}z_3 A^{c,\rho}(z_3)\right]\left[\int_0^\infty \mathrm{d}z_4 A^{d,\sigma}(z_4)\right]\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)
\end{aligned}$$

The amplitude for



is related to the color ordering $t^a t^b t^a t^b$.

$$\begin{aligned}
&\int_0^\infty \mathrm{d}z_1 \int_0^\infty \mathrm{d}z_2 \int_0^\infty \mathrm{d}z_3 \int_0^\infty \mathrm{d}z_4 A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)A^{d,\sigma}(z_4)\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\\
&=\int_0^\infty \mathrm{d}z_1 \int_0^\infty \mathrm{d}z_2 \int_0^\infty \mathrm{d}z_3 \int_0^\infty \mathrm{d}z_4 \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4}\frac{-ig^{\mu\rho}\delta^{ac}}{l_1^2+i\epsilon}e^{-il_1\cdot(z_3-z_1)}\int \frac{\mathrm{d}^4 l_2}{(2\pi)^4}\frac{-ig^{\nu\sigma}\delta^{bd}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_4-z_2)}\\
&\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\\
&=\int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4}\frac{-ig^{\mu\rho}\delta^{ac}}{l_1^2+i\epsilon}e^{-il_1\cdot(z_3-z_1)}\int \frac{\mathrm{d}^4 l_2}{(2\pi)^4}\frac{-ig^{\nu\sigma}\delta^{bd}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_4-z_2)}\\
&\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)
\end{aligned}$$

The exponent is (for simplicity we assume vectors $z_i = (0,0,0,z_i)$ and $k_i = (0,0,0,-k_i)$)

$$\begin{aligned}
&-il_1\cdot(z_3-z_1)-il_2\cdot(z_4-z_2)-ik_1\cdot(z_1-z_2)-ik_2\cdot(z_2-z_3)-ik_3\cdot(z_3-z_4)-ik_4\cdot z_4\\
&=-iz_3\cdot(l_1+k_3-k_2)-iz_1\cdot(k_1-l_1)-iz_4\cdot(l_2+k_4-k_3)-iz_2\cdot(k_2-k_1-l_2)
\end{aligned}$$

which gives 4 delta functions. The propagators involved are then

$$\int \frac{\mathrm{d}^4 l_1}{(2\pi)^4}\int \frac{\mathrm{d}^4 l_2}{(2\pi)^4}\frac{-ig^{\mu\rho}\delta^{ac}}{l_1^2+i\epsilon}\frac{-ig^{\nu\sigma}\delta^{bd}}{l_2^2+i\epsilon}\frac{i}{-l_1^z+i\epsilon}\frac{i}{-l_1^z-l_2^z+i\epsilon}\frac{i}{-l_2^z+i\epsilon}\frac{i}{i\epsilon}$$

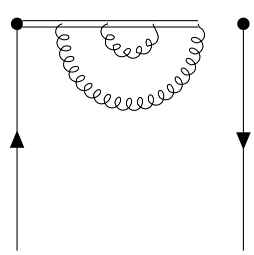
$$\begin{aligned}
&\int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)A^{d,\sigma}(z_4)\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)\\
&=\frac{1}{2}\int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)A^{d,\sigma}(z_4)[\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)\\
&+\theta(z_3-z_4)\theta(z_4-z_1)\theta(z_1-z_2)\theta(z_2)]\\
&=\int \frac{\mathrm{d}^4 l_1}{(2\pi)^4}\int \frac{\mathrm{d}^4 l_2}{(2\pi)^4}\frac{-ig^{\mu\rho}\delta^{ac}}{l_1^2+i\epsilon}\frac{-ig^{\nu\sigma}\delta^{bd}}{l_2^2+i\epsilon}\frac{-\left(l_1^{z\,2}+3l_1^z l_2^z+l_2^{z\,2}-\epsilon^2\right)}{(l_1^z-i\epsilon)(l_1^z+i\epsilon)(l_2^z+i\epsilon)(l_2^z-i\epsilon)(l_1^z+l_2^z-i\epsilon)(l_1^z+l_2^z+i\epsilon)}
\end{aligned}$$

Taking the derivative of the former expression, we can also arrive at a divergence-free form

$$\frac{i}{2}\lim_{p\to 0}\frac{\partial}{\partial p}\left[\frac{i}{p+l_1^z}\frac{i}{p+l_1^z+l_2^z}\frac{i}{p+l_2^z}\right]=\frac{-\left(l_1^{z\,2}+3l_1^z l_2^z+l_2^{z\,2}\right)}{l_1^{z\,2}l_2^{z\,2}(l_1^z+l_2^z)^2}$$

which is equivalent to above expression.

The amplitude for



is related to the color ordering $t^a t^b t^b t^a$.

$$\int_0^\infty \mathrm{d}z_1 \int_0^\infty \mathrm{d}z_2 \int_0^\infty \mathrm{d}z_3 \int_0^\infty \mathrm{d}z_4 A^{a,\mu}(z_1) A^{b,\nu}(z_2) A^{c,\rho}(z_3) A^{d,\sigma}(z_4)\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)$$
$$= \int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} e^{-il_1\cdot(z_3-z_1)} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} e^{-il_2\cdot(z_4-z_2)}$$
$$\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)$$

The propagators involved are

$$\int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} \frac{i}{-l_1^z+i\epsilon} \frac{i}{-l_1^z-l_2^z+i\epsilon} \frac{i}{-l_1^z+i\epsilon} \frac{i}{i\epsilon}$$

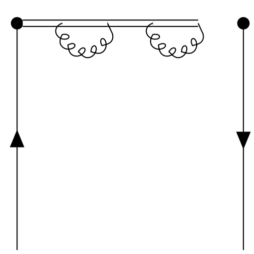
$$\frac{1}{2} \int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 A^{a,\mu}(z_1) A^{b,\nu}(z_2) A^{c,\rho}(z_3) A^{d,\sigma}(z_4)[\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)$$
$$+\theta(z_4-z_3)\theta(z_3-z_2)\theta(z_2-z_1)\theta(z_1)]$$
$$= \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} \frac{-3{l_1^z}^2 - 2l_1^z l_2^z + \epsilon^2}{\left({l_1^z}^2+\epsilon^2\right)^2 \left({l_1^z}^2 + 2l_1^z l_2^z + {l_2^z}^2 + \epsilon^2\right)}$$

Taking the derivative of the former expression, we can also arrive at a divergence-free form

$$\frac{i}{2} \lim_{p\to 0} \frac{\partial}{\partial p} \left[\frac{i}{p+l_1^z} \frac{i}{p+l_1^z+l_2^z} \frac{i}{p+l_1^z} \right] = \frac{-(3l_1^z + 2l_2^z)}{{l_1^z}^3 (l_1^z+l_2^z)^2}$$

which is equivalent to above expression.

The amplitude for



is related to the color ordering $t^a t^a t^b t^b$.

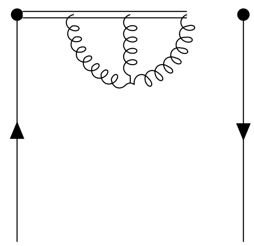
$$\int_0^\infty \mathrm{d}z_1 \int_0^\infty \mathrm{d}z_2 \int_0^\infty \mathrm{d}z_3 \int_0^\infty \mathrm{d}z_4 A^{a,\mu}(z_1) A^{b,\nu}(z_2) A^{c,\rho}(z_3) A^{d,\sigma}(z_4)\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)$$
$$= \int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} e^{-il_1\cdot(z_3-z_1)} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} e^{-il_2\cdot(z_4-z_2)}$$
$$\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)$$

The propagators involved are

$$\int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} \frac{i}{-l_1^z+i\epsilon} \frac{i}{i\epsilon} \frac{i}{-l_2^z+i\epsilon} \frac{i}{i\epsilon}$$

$$\frac{1}{4} \int \mathrm{d}z_1 \int \mathrm{d}z_2 \int \mathrm{d}z_3 \int \mathrm{d}z_4 A^{a,\mu}(z_1) A^{b,\nu}(z_2) A^{c,\rho}(z_3) A^{d,\sigma}(z_4)[\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3-z_4)\theta(z_4)$$
$$+\theta(z_2-z_1)\theta(z_1-z_3)\theta(z_3-z_4)\theta(z_4) + \theta(z_1-z_2)\theta(z_2-z_4)\theta(z_4-z_3)\theta(z_3) + \theta(z_2-z_1)\theta(z_1-z_4)\theta(z_4-z_3)\theta(z_3)]$$
$$= \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 l_2}{(2\pi)^4} \frac{-ig^{\mu\sigma}\delta^{ad}}{l_1^2+i\epsilon} \frac{-ig^{\nu\rho}\delta^{bc}}{l_2^2+i\epsilon} \frac{1}{\left({l_1^z}^2+\epsilon^2\right)\left({l_2^z}^2+\epsilon^2\right)}$$

The amplitude for



is related to

$$\langle P,S|\bar{\psi}(z)\gamma^z V_3\mathcal{P}\frac{\left[-ig_s n_\mu\int_0^\infty \mathrm{d}z_1 A^{a,\mu}(z_1)t^a\right]\left[-ig_s n_\nu\int_0^\infty \mathrm{d}z_2 A^{b,\nu}(z_2)t^b\right]\left[-ig_s n_\rho\int_0^\infty \mathrm{d}z_3 A^{c,\rho}(z_3)t^c\right]}{3!}\psi(0)|P,S\rangle \tag{9}$$

where

$$V_3=-\frac{g_s}{2}f^{def}\int \mathrm{d}^4t\left(\partial^\alpha A_d^\beta-\partial^\beta A_d^\alpha\right)A_\alpha^e A_\beta^f$$

The gluon related contraction (one out of three, others can be obtained by exchanging $d,e,f.$)

$$\begin{aligned}
&\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\big(\partial^\alpha A^{d,\beta}-\partial^\beta A^{d,\alpha}\big)A_\alpha^e A_\beta^f A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)\\
&=\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\left[g^{\beta\rho}\partial^\alpha e^{-il_1\cdot(z_3-t)}-g^{\alpha\rho}\partial^\beta e^{-il_1\cdot(z_3-t)}\right]\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\frac{-ig_\alpha^\mu\delta^{ea}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_1-t)}\\
&\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-ig_\beta^\nu\delta^{fb}}{l_3^2+i\epsilon}e^{-il_3\cdot(z_2-t)}\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)\\
&=\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\left[g^{\beta\rho}il_1^\alpha-g^{\alpha\rho}il_1^\beta\right]e^{-il_1\cdot(z_3-t)}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\frac{-ig_\alpha^\mu\delta^{ea}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_1-t)}\\
&\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-ig_\beta^\nu\delta^{fb}}{l_3^2+i\epsilon}e^{-il_3\cdot(z_2-t)}\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)\\
&=\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}[g^{\nu\rho}il_1^\mu-g^{\mu\rho}il_1^\nu]e^{-il_1\cdot(z_3-t)}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\frac{-i\delta^{ea}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_1-t)}\\
&\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-i\delta^{fb}}{l_3^2+i\epsilon}e^{-il_3\cdot(z_2-t)}\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)
\end{aligned}$$

Multiplied by n

$$n_\mu n_\nu n_\rho[g^{\nu\rho}il_1^\mu-g^{\mu\rho}il_1^\nu]=in^2[l_1^z-l_1^z]=0$$

Contracting with different fields in the parenthesis

$$\partial^\alpha A^{d,\beta}A_\alpha^e A_\beta^f A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)-\partial^\beta A^{d,\alpha}A_\alpha^e A_\beta^f A^{a,\mu}(z_1)A^{b,\nu}(z_2)A^{c,\rho}(z_3)$$

$$\begin{aligned}
&\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\frac{-ig_\alpha^\mu\delta^{ea}}{l_2^2+i\epsilon}e^{-il_2\cdot(z_1-t)}\frac{-ig_\beta^\nu\delta^{fb}}{l_3^2+i\epsilon}\\
&\left[e^{-il_3\cdot(z_2-t)}g^{\beta\rho}\partial^\alpha e^{-il_1\cdot(z_3-t)}-e^{-il_1\cdot(z_3-t)}g^{\alpha\rho}\partial^\beta e^{-il_3\cdot(z_2-t)}\right]\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)\\
&=\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\frac{-i\delta^{ea}}{l_2^2+i\epsilon}\frac{-i\delta^{fb}}{l_3^2+i\epsilon}e^{-il_2\cdot(z_1-t)}e^{-il_3\cdot(z_2-t)}e^{-il_1\cdot(z_3-t)}\\
&[g^{\nu\rho}il_1^\mu-g^{\mu\rho}il_3^\nu]\theta(z_1-z_2)\theta(z_2-z_3)\theta(z_3)
\end{aligned}$$

Make $z_1\to -z_1, t\to -t, l_2\to -l_2$,

$$\begin{aligned}
&\int \mathrm{d}z_1\int \mathrm{d}z_2\int \mathrm{d}z_3\int \mathrm{d}^4t\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\frac{-i\delta^{ea}}{l_2^2+i\epsilon}\frac{-i\delta^{fb}}{l_3^2+i\epsilon}e^{-il_2\cdot(z_1-t)}e^{-il_3\cdot(z_2-t)}e^{-il_1\cdot(z_3-t)}\\
&[g^{\nu\rho}il_1^\mu-g^{\mu\rho}il_3^\nu]\theta(-z_1-z_2)\theta(z_2-z_3)\theta(z_3)\\
&=\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\frac{-i\delta^{dc}}{l_1^2+i\epsilon}\frac{-i\delta^{ea}}{(l_1+l_3)^2+i\epsilon}\frac{-i\delta^{fb}}{l_3^2+i\epsilon}\frac{i}{-l_1^z+i\epsilon}\frac{i}{-2l_1^z+i\epsilon}\frac{i}{-2l_1^z-l_3^z+i\epsilon}
\end{aligned}$$