非相对论性有效场论在原子物理和 QCD 中的应用

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Outlines

1. Operator Product Expansion for Atomic Wave-functions

2. NRQCD Factorization for Fully-heavy Tetraquark Production

Operator Product Expansion for Atomic Wave-functions

Motivation

- Universal behaviors in Coulombic wave-functions, near-origin divergence in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} + \cdots \; (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} + \cdots \; (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} + \cdots \; (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} + \cdots \; (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{KG}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

- ☐ Universal behaviors in Coulombic wave-functions, near-origin divergence in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} + \cdots \; (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} + \cdots \; (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} + \cdots \; (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} + \cdots \; (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{Dirac}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} - \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} - \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} - \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} - \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

Attack the problem with OPE & EFT: Construct EFT

- Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus.

- Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \mathcal{L}_{\mathrm{Max}} + \mathcal{L}_{\mathrm{NRQED}} + \mathcal{L}_{\mathrm{HNET}} + \delta\mathcal{L}_{\mathrm{contact}} \tag{1}$$

where

$$\begin{aligned}
\mathcal{L}_{\mathrm{Max}} &= -\frac{1}{4} d_\gamma F_{\mu\nu} F^{\mu\nu} + \cdots \, , , \\
\mathcal{L}_{\mathrm{NRQED}} &= \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\boldsymbol{\nabla} \cdot \mathbf{E}]}{8m^2} + \cdots \right\}, \\
\mathcal{L}_{\mathrm{HNET}} &= N^\dagger i D_0 N + \cdots \, , \\
\delta\mathcal{L}_{\mathrm{contact}} &= \frac{c_4}{m^2} \psi^\dagger \psi N^\dagger N + \cdots \, ,
\end{aligned}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: Construct EFT

- ☐ Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus, keep only Coulomb potential.
- ☐ Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \cancel{\mathcal{L}_{\mathrm{Max}}}\ \mathcal{L}_{\mathrm{Coul}} + \mathcal{L}_{\mathrm{NRQED}} + \mathcal{L}_{\mathrm{HNET}} + \delta\mathcal{L}_{\mathrm{contact}} \tag{1}$$

where

$$\mathcal{L}_{\mathrm{Coul}} = \frac{1}{2}\left(\nabla A^0\right)^2,$$

$$\mathcal{L}_{\mathrm{NRQED}} = \psi^\dagger\left\{iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e\frac{[\mathbf{\nabla}\cdot\mathbf{E}]}{8m^2} + \cdots\right\},$$

$$\mathcal{L}_{\mathrm{HNET}} = N^\dagger iD_0 N + \cdots\,,$$

$$\xcancel{\delta\mathcal{L}_{\mathrm{contact}} = \frac{c_4}{m^2}\psi^\dagger\psi N^\dagger N + \cdots\,,}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

☐ Operator Product Expansion (OPE): The limit when product of local operators at different points approach each other.

$$T\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\mathcal{O}}(x^\mu)[\mathcal{O}(0)]_R \tag{2}$$

Correct OPE relation in coordinate space

$$\psi(\mathbf{r})N(\mathbf{0}) = (1 - mZ\alpha|\mathbf{r}|)\,[\psi N](\mathbf{0}) + (1 - mZ\alpha|\mathbf{r}|/2)\mathbf{r}\cdot[\nabla\psi N](\mathbf{0}) + \cdots$$

Correct OPE relation in momentum space

$$\begin{aligned}\widetilde{\psi}(\mathbf{q})N(\mathbf{0}) &\equiv \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\psi(\mathbf{r})N(\mathbf{0}) \\ &= \frac{8\pi Z\alpha m}{\mathbf{q}^4}[\psi N](\mathbf{0}) - \frac{16i\pi Z\alpha m}{\mathbf{q}^6}\boldsymbol{q}\cdot[\nabla\psi N](\mathbf{0}) + \cdots .\end{aligned}$$

Reproduce Wave-function origin

- ☐ With operator definition of the wave-functions

$$\Psi_{nlm}(\mathbf{r}) = \langle 0 \,|\psi(\mathbf{r}) N(\mathbf{0})|\, nlm\rangle \tag{3}$$

Wave-function origin (Schrödinger equation)

$$R_{n0}(r) = R_{n0}(0)\left[1 - \frac{r}{a_0} + \mathcal{O}\left(r/a_0\right)^2\right] \tag{4}$$

- ☐ Add relativistic corrections in OPE with higher order Lagrangian to account for the logarithms in relativistic wave-functions (Klein-Gordon, Dirac).

- ☐ Use renormalization group equation to reproduce all leading logarithms.

Relativistic OPE relations and Resummation

OPE relation

$$\lim_{r \to \frac{1}{m}} \psi(\mathbf{r}) N(0) = \mathcal{C}(r)[\psi N]_R(0) + \cdots$$

Wilson Coefficient

Klein-Gordon: $\mathcal{C}(r) = 1 - mZ\alpha r - Z^2\alpha^2 (\ln \mu r + \text{const}) + \mathcal{O}(Z^3\alpha^3)$

Dirac: $\mathcal{C}(r) = 1 - mZ\alpha r - \frac{Z^2\alpha^2}{2} (\ln \mu r + \text{const}) + \mathcal{O}(Z^3\alpha^3)$

RGE for Dirac

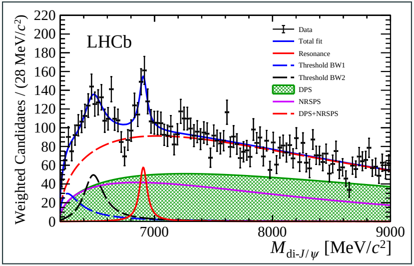
$$\mu \frac{\partial \mathcal{C}(r,\mu)}{\partial \mu} + \gamma_{\mathcal{S}} \mathcal{C}(r,\mu) = 0, \quad \gamma_{\mathcal{S}} \equiv \frac{d \ln Z_{\mathcal{S}}}{d \ln \mu} = \frac{Z^2\alpha^2}{2}.$$

$$\mathcal{C}(r,\mu) = \mathcal{C}(r_0,\mu) \left(\frac{r}{r_0}\right)^{-\frac{Z^2\alpha^2}{2}}.$$

NRQCD Factorization for Fully-heavy Tetraquark Production

Factorization theorem for $T_{4c/b}$ production at LHC

- ☐ LHCb discovered a narrow structure near 6.9 GeV in the di-J/ψ invariant mass spectrum ($> 5\sigma$): $X(6900)$.
- ☐ Strong candidate for fully-charmed tetraquark.



- ☐ QCD factorization theorem for fully-heavy tetraquark ($T_{4c/b}$) exclusive production at high-p_T

$$\mathrm{d}\sigma\left(pp \to T_{4c/b}\,(p_{\mathrm{T}}) + X\right) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z\; f_{a/p}(x_a,\mu) f_{b/p}(x_b,\mu)$$
$$\times \mathrm{d}\hat{\sigma}(ab \to i(p_T/z) + X, \mu) D_{i\to T_{4c/b}}(z,\mu) + \mathcal{O}(1/p_T). \quad (5)$$

- ☐ Dominate partonic channel is $gg \to gg$, rather than $gg \to q\bar{q}$.

Fragmentation Function

Collins-Soper definition of fragmentation function:

$$D_{g\to T_{4c}}(z,\mu) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ \left(N_c^2-1\right)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-}$$

$$\times \sum_X \left\langle 0 \left| G_c^{+\mu}(0)\mathcal{E}^\dagger\left(0,0,\mathbf{0}_\perp\right)_{cb} \left|T_{4c}(P)+X\right\rangle\left\langle T_{4c}(P)+X\right| \mathcal{E}\left(0,x^-,\mathbf{0}_\perp\right)_{ba} G_a^{+\nu}\left(0,x^-,\mathbf{0}_\perp\right) \right| 0 \right\rangle$$

- ☐ NRQCD factorization:

$$D_{g\to H}(z) = \sum_n d_n(z) \left\langle 0 \left| \mathcal{O}_n^H \right| 0 \right\rangle$$

- ☐ Perturbative matching to determine short distance coefficients.
- ☐ Use wave-function origin (S-wave) from potential models to determine long range matrix elements in order to yield a phenomenological result.
- ☐ More details in Jia-Yue Zhang's talk this afternoon.

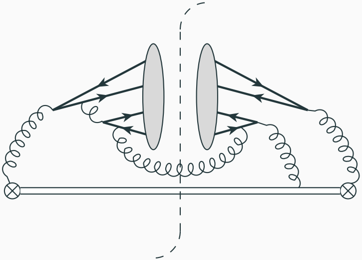
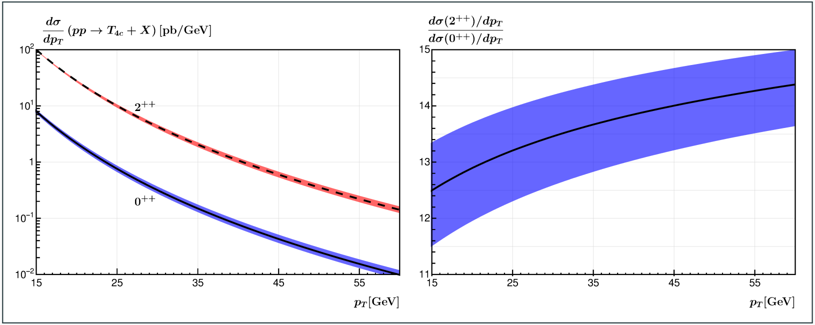
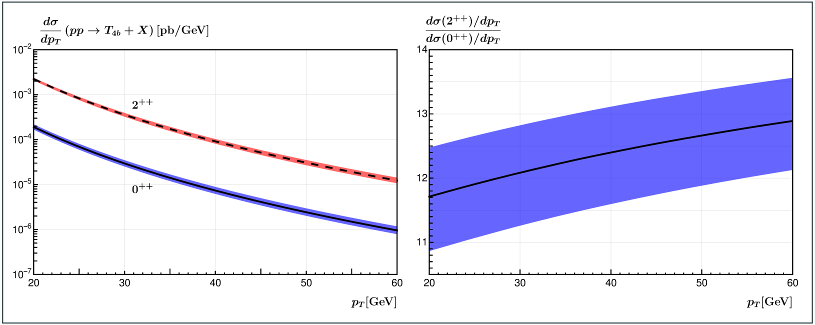


Figure 1: A representative Feynman diagram for the fragmentation function of gluon into T_{4c}. The grey blob indicates the C-even tetraquark. Horizontal double line denotes the eikonal line.

Phenomenology for $T_{4c/b}$ production at LHC





- 2^{++} cross section is about 10 times larger than 0^{++}.
- We obtain the yields of the accumulated event number for T_{4c} at HL-LHC are a hundred million for 0^{++} and 8 hundreds million for 2^{++} (with integrated luminosity 3000 fb^{-1}).
- The prediction for T_{4b} is highly suppressed, mainly due to the relative larger bottom mass suppression.
- The total cross section we obtained is unreliable mainly due to the fact that fragmentation only works at high-p_T, and our integration is done within approximately $15 \leq p_T \leq 60\mathrm{GeV}$.

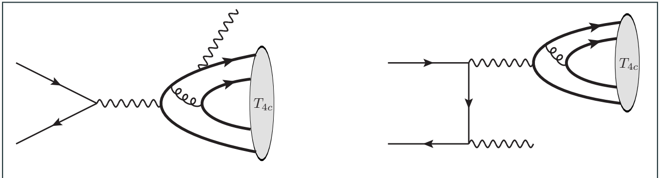
☐ Factorization Formula (amplitude)

$$\mathcal{M}^{J}_{\lambda_1,\lambda_2} = \frac{\mathcal{A}^{3[J]}_{\lambda_1,\lambda_2}}{m^4}\sqrt{2M_{T_{4c}}}\langle T^{J}_{4c}|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}|0\rangle + \frac{\mathcal{A}^{6[J]}_{\lambda_1,\lambda_2}}{m^4}\sqrt{2M_{T_{4c}}}\langle T^{J}_{4c}|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}|0\rangle + \mathcal{O}(v^2),$$

☐ Factorization Formula (cross section)

$$\begin{aligned}\sigma(e^+e^- \to T^{J}_{4c} + \gamma) = {} & \frac{F^{[J]}_{3,3}}{m^8}(2M_{T_{4c}})\left|\left\langle T^{(J)}_{4c}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\right|^2 \\ & + \frac{F^{[J]}_{6,6}}{m^8}(2M_{T_{4c}})\left|\left\langle T^{(J)}_{4c}\left|\mathcal{O}^{(J)}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|0\right\rangle\right|^2 \\ & + \frac{F^{[J]}_{3,6}}{m^8}(2M_{T_{4c}})2\mathrm{Re}\left[\left\langle T^{(J)}_{4c}\left|\mathcal{O}^{(J)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\right|0\right\rangle\left\langle 0\left|\mathcal{O}^{(J)\dagger}_{\mathbf{6}\otimes\bar{\mathbf{6}}}\right|T^{(J)}_{4c}\right\rangle\right] + \cdots .\end{aligned}$$

☐ Typical Feynman diagrams:



$a)$ $b)$

Phenomenology for T_{4c} production at B factory

- ☐ Choosing $\sqrt{s} = 10.58$ GeV:

$$\sigma\left[T_{4c}^{0}+\gamma\right] \approx 0.0026 \text{ fb},$$
$$\sigma\left[T_{4c}^{2}+\gamma\right] \approx 0.020 \text{ fb}.$$

- ☐ Integrated luminosity of `Belle 2` experiment: $50\ \mathrm{ab}^{-1}$.
- ☐ Estimate: $130\ \gamma + T_{4c}^{0}$ events and $1020\ \gamma + T_{4c}^{2}$ events.
- ☐ Unreliable total cross section: lack of sextet contribution, model dependence of diquark potential model.

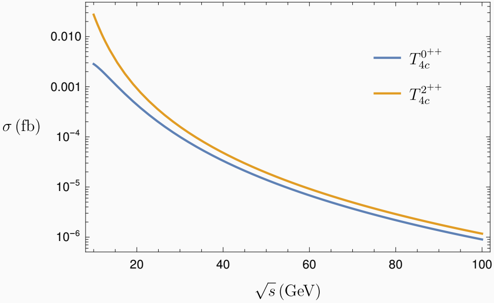


Figure 2: The $\sqrt{s}$-dependence of the cross sections (J=0,2)

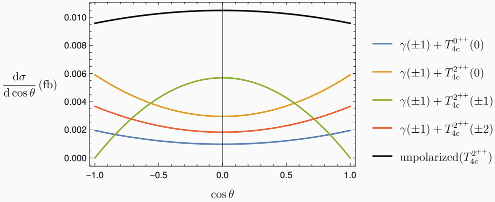


Figure 3: The polarized differential cross sections

Publications

Publications?

- Huang, Y., Jia, Y., & Yu, R. (2018a). Deciphering the coalescence behavior of Coulomb-Schrödinger atomic wave functions from operator product expansion.arXiv 1809.09023 (rejected by PRL, waiting for resubmission)
- Huang, Y., Jia, Y., & Yu, R. (2018b). Near-the-origin divergence of Klein-Gordon wave functions for hydrogen-like atoms and operator product expansion.arXiv 1812.11957 (Submitted to PRD, referee comments received)
- Huang, Y., Jia, Y., & Yu, R. (2019). Near-the-origin divergence of Dirac wave functions of hydrogen and operator product expansion.arXiv 1901.04971 (rejected by PRL, waiting for appeal)
- Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391 (Submitted to PRD and received positive response.)
- Feng, F., Huang, Y., Jia, Y., Sang, W.-L., Xiong, X., & Zhang, J.-Y. (2020). Fragmentation production of fully-charmed tetraquarks at lhc.arXiv 2009.08450 (To be submitted to PRL)
- Feng, F., Huang, Y., Jia, Y., Sang, W.-L., & Zhang, J.-Y. (2020). Exclusive radiative production of fully-charmed tetraquarks at b factory.arXiv 2011.03039 (To be submitted to PLB)

Questions?

Backup

References

Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391.

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