

# Two Loop Matching for Quasi PDF

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## 1 Renormalization

### 1.1 One loop diagrams

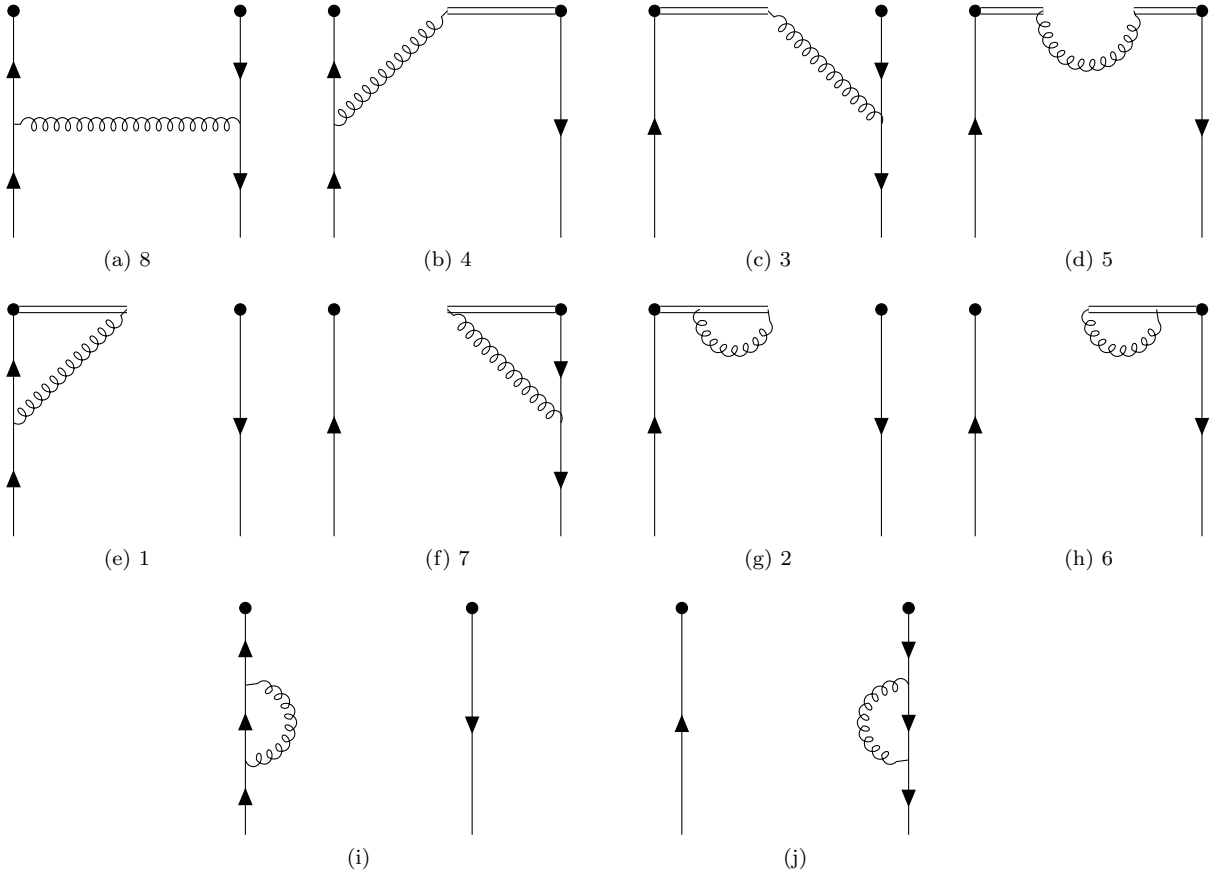
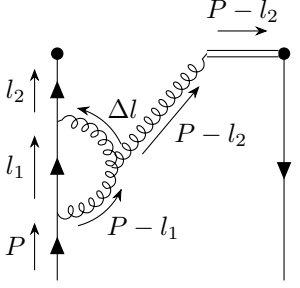


Figure 1: Diagrams of quasi PDF in Feynman gauge.

### 1.2 Vertex corrections

According to [Ji and Zhang(2015)], the vertex correction diagrams in axial gauge (which corresponds to varieties of diagrams in general covariant gauge) don't have total UV divergence. Rather, they only have subdivergence for sub-diagrams. For example the first column (which involves Figure 3), second row of Table 1 in [Ji and Zhang(2015)] is composed of  $\tilde{q}_{11}$  and  $\tilde{q}_{12}$ , thus we can find some representative diagrams and extract those components ( $l \equiv l_1 + l_2, \Delta l \equiv$

$l_1 - l_2$ )



$$P \propto \int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{1}{[l_1 - m][l_2 - m][(P - l_1)^2][(l_1 - l_2)^2][(P - l_2)^2][n \cdot (P - l_2)]} \quad (1)$$

Take the  $l_1 \gg l_2$  limit, the integrand becomes

$$\frac{1}{[l_1 - m][(P - l_1)^2][l_1^2][l_2 - m][(P - l_2)^2][n \cdot (P - l_2)]} \quad (2)$$

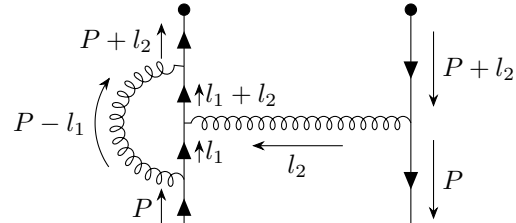
The integral involving  $l_2$  is exactly the integral of  $\tilde{q}_{12}$ . By adding the gluon self-interacting vertex we can see that the sub-diagram is logarithmic divergent.

Take the  $l_2 \gg l_1$  limit, the integrand becomes

$$\frac{1}{[l_1 - m][(P - l_1)^2][l_2^2][l_2 - m][(P - l_2)^2][n \cdot (P - l_2)]} \quad (3)$$

There's another limit where hard loop momentum flows through all paths except the one that's  $\Delta l$  in our current diagram. This configuration gives a finite integral and a power-divergent integral which happens to be a scaleless integral as well. Thus this configuration won't contribute.

What we extracted above is only the  $\tilde{q}_{12}$  part, now we will try on the  $\tilde{q}_{11}$  part



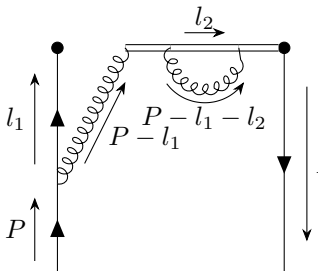
$$\propto \int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{1}{[l_1 - m][l_1 + l_2 - m][\not{P} + l_2 - m][\not{P} + l_2 - m][(P - l_1)^2][l_2^2]} \quad (4)$$

In the  $l_1 \gg l_2$  limit we have

$$\frac{1}{[l_1 - m][l_1 - m][(P - l_1)^2][\not{P} + l_2 - m][\not{P} + l_2 - m][l_2^2]} \quad (5)$$

and  $\tilde{q}_{11}$  is factorized out.

Another example is the sixth row



$$P \propto \int \frac{d^d l_1}{(2\pi)^d} \frac{d^d l_2}{(2\pi)^d} \frac{1}{[l_1 - m][(P - l_1)^2][(P - l_1 - l_2)^2][n \cdot (P - l_1)][n \cdot l_2][n \cdot (P - l_1)]} \quad (6)$$

Take the  $l_2 \gg l_1$  limit, the integrand becomes

$$\frac{1}{[l_1 - m][(P - l_1)^2][n \cdot (P - l_1)][n \cdot (P - l_1)][n \cdot l_2][(P - l_2)^2]} \quad (7)$$

and the integral involving  $l_2$  should give something proportional to  $n \cdot (P - l_1)$ , thus cancels one eikonal propagator, the remainder is the integral of  $\tilde{q}_{12}$ .

## 2 Real Diagrams

### 2.1 All diagrams

Figure 2 lists all self-conjugated real diagrams, and Figure 3 lists all non-self-conjugated diagrams, excluding their conjugates.

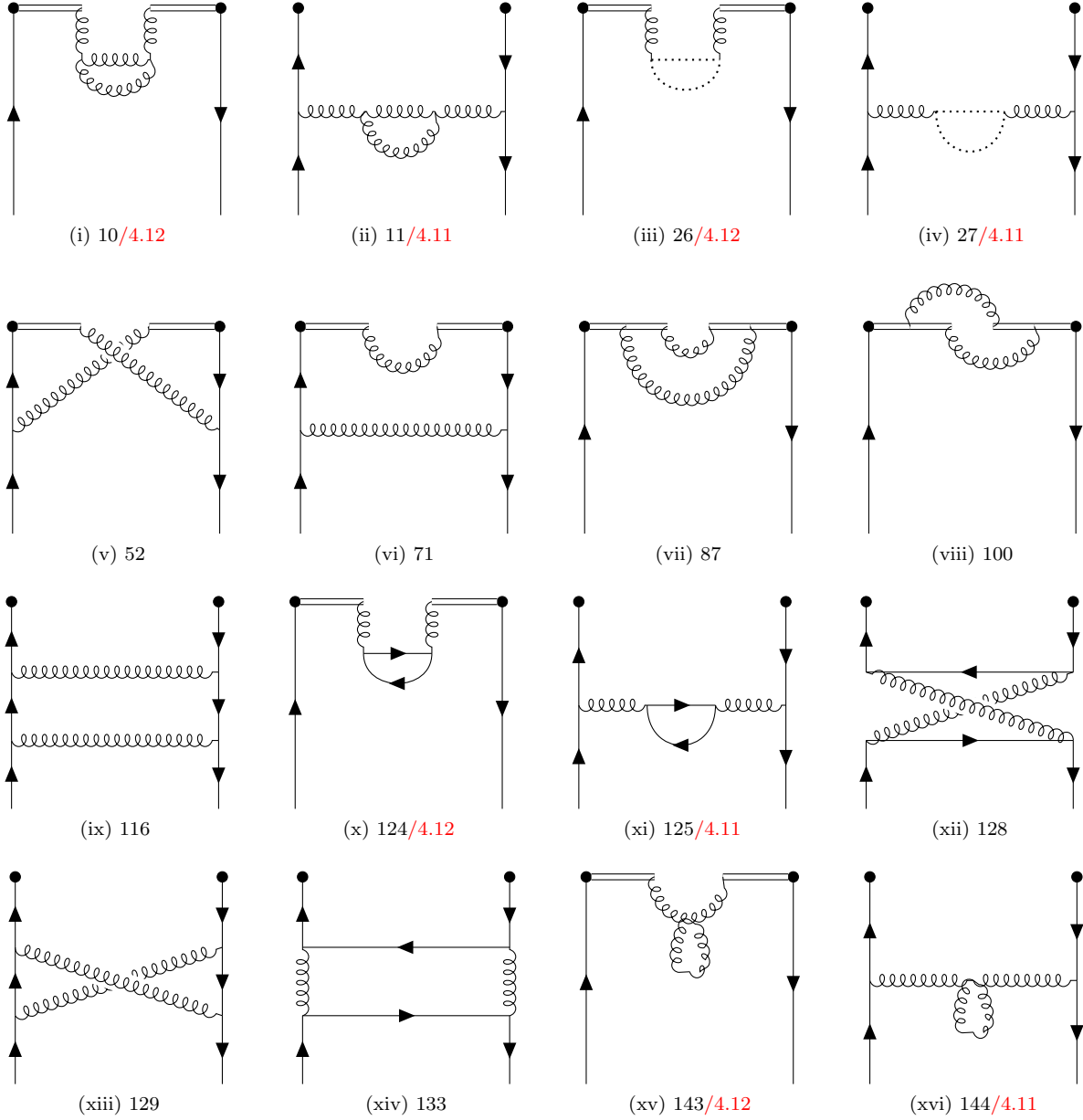


Figure 2: All self-conjugated diagrams, red n.i marks the diagram number in Ji&Zhang's paper.

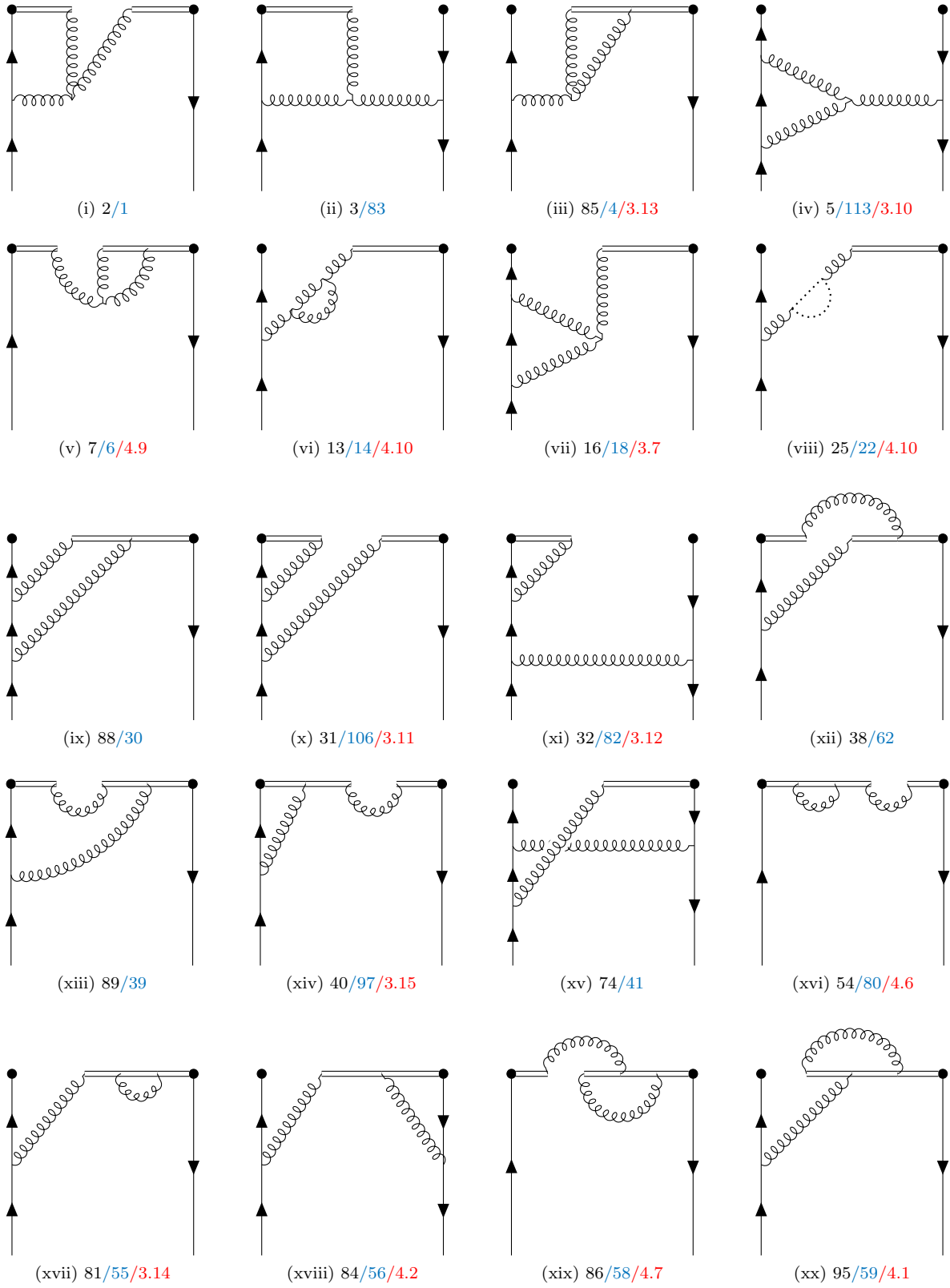


Figure 3: All real diagrams (excluding conjugated diagrams), xxx marks conjugated diagram number, red n.i marks the diagram number in Ji&Zhang's paper.

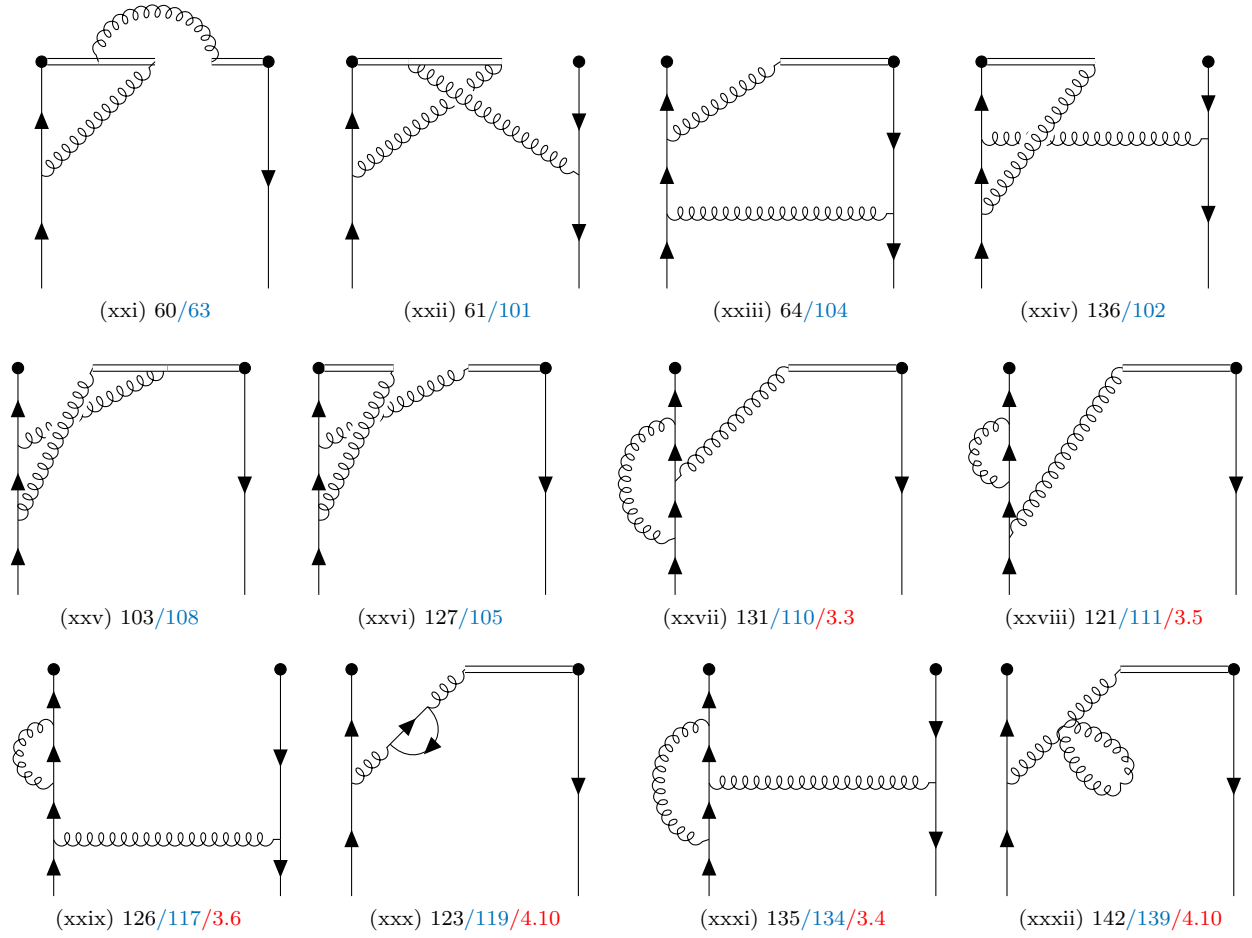


Figure 3: All real diagrams (excluding conjugated and self-conjugated diagrams), **xxx** marks conjugated diagram number, red n.i marks the diagram number in Ji&Zhang's paper.

## 2.2 Amplitude test

First we take diagram 2xv to test if the type of diagrams that is a sub-diagram involving only QCD Feynman rules on top of one loop diagram consist with our manual input.

The program gives

$$\begin{aligned} & (\delta_{\text{CI}(9)} \text{CI}(10) \delta_{\text{CI}(11)} \text{CI}(12) \delta_{\text{CI}(13)} \text{CI}(14) g^{\text{LI}(9) \text{LI}(10)} g^{\text{LI}(11) \text{LI}(12)} g^{\text{LI}(13) \text{LI}(14)} g_s^4 \text{MomC}(-\mathbf{k}_1) n_1^{\text{LI}(9)} n_2^{\text{LI}(11)} \text{ColorLine}(T_{\text{CI}(11)}, T_{\text{CI}(9)}, \{p, p\}) \\ & ((g^{\text{LI}(10) \text{LI}(13)} g^{\text{LI}(12) \text{LI}(14)} - g^{\text{LI}(10) \text{LI}(14)} g^{\text{LI}(12) \text{LI}(13)}) f_{\text{eS19 CI}(13) \text{CI}(14)} f_{\text{CI}(10) \text{CI}(12) \text{eS19}} + (g^{\text{LI}(10) \text{LI}(12)} g^{\text{LI}(13) \text{LI}(14)} - g^{\text{LI}(10) \text{LI}(14)} g^{\text{LI}(13) \text{LI}(12)}) \\ & f_{\text{eS20 CI}(12) \text{CI}(14)} f_{\text{CI}(10) \text{CI}(13) \text{eS20}} + (g^{\text{LI}(10) \text{LI}(12)} g^{\text{LI}(14) \text{LI}(13)} - g^{\text{LI}(10) \text{LI}(13)} g^{\text{LI}(14) \text{LI}(12)}) f_{\text{eS21 CI}(12) \text{CI}(13)} f_{\text{CI}(10) \text{CI}(14) \text{eS21}}) \\ & \text{SpinLine}(\gamma \cdot n, \{p, p\}) / (2 k_2^2 (-p - p_e)^2 (k_1 + p + p_e)^2 n_1 \cdot (p + p_e) n_2 \cdot (p + p_e)) \end{aligned}$$

which translates to

$$\begin{aligned} & g_s^4 \delta(-k_1) \delta_{c13c14} g^{l13l14} n_1^{l10} n_1^{l12} t^{c10} t^{c12} \frac{\bar{u}(P) \not{p} u(P)}{2k_2^2 (-p - p_e)^2 (k_1 + p + p_e)^2 n_1 \cdot (p + p_e) n_2 \cdot (p + p_e)} \\ & [(g^{l10l13} g^{l12l14} - g^{l10l14} g^{l12l13}) f^{e19c13c14} f^{c10c12e19} + (g^{l10l12} g^{l13l14} - g^{l10l14} g^{l13l12}) f^{e20c12c14} f^{c10c13e20} \\ & + (g^{l10l12} g^{l14l13} - g^{l10l13} g^{l14l12}) f^{e21c12c13} f^{c10c14e21}] \Big|_{p_e = -xP^z \rightarrow -p, p=P, n \rightarrow z, k_2=l_2, n_1=n_2=n} \end{aligned}$$

Taking  $k_1 = p + p_e - l_1$ , the first line (that's excluding the four-gluon vertex) becomes

$$g_s^4 \delta(-k_1) \delta_{c13c14} g^{l13l14} n_1^{l10} n_1^{l12} t^{c10} t^{c12} \frac{\bar{u}(P) \not{p} u(P)}{2k_2^2 (-p - p_e)^2 (k_1 + p + p_e)^2 n_1 \cdot (p + p_e) n_2 \cdot (p + p_e)} \quad (8)$$

$$= g_s^4 \delta(l_1 - p - p_e) \delta_{c13c14} g^{l13l14} n_1^{l10} n_1^{l12} t^{c10} t^{c12} \frac{\bar{u}(P) \not{p} u(P)}{2l_2^2 (-p - p_e)^2 (2p + 2p_e - l_1)^2 n_1 \cdot (p + p_e) n_2 \cdot (p + p_e)} \quad (9)$$

$$= g_s^4 \delta(l_1 - p - p_e) \delta_{c13c14} g^{l13l14} n_1^{l10} n_1^{l12} t^{c10} t^{c12} \frac{\bar{u}(P) \not{p} u(P)}{2l_2^2 l_1^2 l_1^2 n_1 \cdot l_1 n_2 \cdot l_1} \quad (10)$$

Diagram 2xv gives

$$\begin{aligned} & \frac{-ig_s^4}{2} \bar{u}(P) \not{p} u(P) \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} n_\tau t^i \tilde{D}_G^{\tau\mu, ia}(l_1) \tilde{D}_G^{\sigma\lambda, dj}(l_1) \tilde{D}_G^{\nu\rho, bc}(l_2) n_\lambda t^j \frac{i}{n \cdot l_1 + i\epsilon} \frac{i}{-n \cdot l_1 + i\epsilon} \delta(l_1^z - (1-x)P^z) \\ & [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \end{aligned} \quad (11)$$

$$\begin{aligned} & = \frac{(-1)^3 i^6 g_s^4}{2} \bar{u}(P) \not{p} u(P) \int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \frac{n^\mu n^\sigma g^{\nu\rho} t^i \delta^{ia} t^j \delta^{dj}}{[l_1^2]^2 [l_2^2] [n \cdot l_1]^2} \delta(l_1^z - (1-x)P^z) \\ & [f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \end{aligned} \quad (12)$$

Let's compare the color indices:

## 2.3 Numerical test (ordered as Figure 2 and Figure 3, $z = 1/4$ )

### 2.3.1 Self-conjugated

10

$$\begin{aligned} & (0.135095 -1.68869/\text{s})/\text{ep}-(13.0007 -5.30516 \text{ I})/\text{s}+(0.991423 -0.424413 \text{ I})+(1.68869 \\ & \rightarrow \log(\text{s}))/\text{s}-0.135095 \log(\text{s}) \end{aligned}$$

11

$$\begin{aligned} & -((2.71097 -1.0865 \text{ I})/\text{s}^3)+(13.6036 -5.53435 \text{ I})/\text{s}^2+(0.351595 -0.203718 \\ & \rightarrow \text{I})/\text{s}+(-(0.345843/\text{s}^3)+1.76164/\text{s}^2+0.0648456/\text{s})/\text{ep}+(0.345843 \log(\text{s}))/\text{s}^3-(1.76164 \\ & \rightarrow \log(\text{s}))/\text{s}^2-(0.0648456 \log(\text{s}))/\text{s}-7.57481*10^{-7} \end{aligned}$$

26

$$\rightarrow \frac{(0.991423 - 0.424413 I)}{s - (0.563259 - 0.212207 I) + (0.135095/s - 0.0675475)/\epsilon_p - (0.135095 \log(s))/s + 0.0675475 \log(s)}$$

27

$$\rightarrow \frac{-((1.24375 - 0.509296 I)/s^2) - (0.24875 - 0.101859 I)/s + (- (0.162114/s^2) - 0.0324228/s)/\epsilon_p + (0.162114 \log(s))/s^2 + (0.0324228 \log(s))/s - 3.8192 \cdot 10^{-7}}{s}$$

52 Not handled eqn.

71 Divergent integrand.

87 Divergent integrand.

100

$$-0.180127$$

116 Zero FIRE.py input file.

124

$$\rightarrow \frac{((0.180127 \text{ CV}(1,3))/s + 0.180127 \text{ CV}(1,3))/\epsilon_p + (0.939647 \text{ CV}(1,3))/s - (0.180127 \text{ CV}(1,3) \log(s))/s - 0.180127 \text{ CV}(1,3) \log(s) + 0.939647 \text{ CV}(1,3)}{s}$$

125

$$\rightarrow \frac{(0.194537 \text{ CV}(1,3))/(\epsilon_p s) + (0.885127 \text{ CV}(1,3))/s - (0.194537 \text{ CV}(1,3) \log(s))/s - 4.09664 \cdot 10^{-15} \text{ CV}(1,3)}{s}$$

128 Zero FIRE.py input file.

129

$$\rightarrow \frac{((0.500352 \text{ CV}(1,3))/s + 0.500352 \text{ CV}(1,3) + (0.0180127 - 2.54985 \cdot 10^{-8} I)/s - (9.42776 \cdot 10^{-8} + 7.15663 \cdot 10^{-8} I))/\epsilon_p + (2.78522 \text{ CV}(1,3))/s - (0.500352 \text{ CV}(1,3) \log(s))/s - 0.500352 \text{ CV}(1,3) \log(s) + 2.78522 \text{ CV}(1,3) + (0.526293 - 0.0565893 I)/s + (0.0725309 - 4.81307 \cdot 10^{-8} I) \log(s) + (0.253132 - 0.452708 I) + (0.148304 \log(s))/s}{s}$$

133 Zero FIRE.py input file.

143

$$0$$

144

$$0$$



### 2.3.2 Remaining diagrams

#### 2 Zero FIRE.py input file.

1

$$\rightarrow (0.210926 - 1.87371 \cdot 10^{-8} I) / s + (0.052147 - 2.27485 \cdot 10^{-8} I) + (0.165116 \log(s)) / s - 0.082558 \log(s)$$

3

$$\begin{aligned} & -((2.67042 + 2.12028 I) / s^2) - ((0.260912 - 1.01859 I) \log(s)) / s^2 + (14.6734 + 1.31932 \\ & \rightarrow I) / s - ((0.324228 + 3.72868 \cdot 10^{-9} I) \log^2(s)) / s + (0.222907 - 5.53015 \cdot 10^{-9} I) \log^2(s) + ((0.881442 \\ & \rightarrow -3.43775 I) \log(s)) / s + (0.673891 - 1.68704 I) \log(s) + (13.4354 - 26.5232 \\ & \rightarrow I) + (-((4.70467 \cdot 10^{-9} + 1.04672 \cdot 10^{-9} I) / s^2) - (1.49279 \cdot 10^{-8} + 4.88207 \cdot 10^{-9} \\ & \rightarrow I) / s + (-3.02109 \cdot 10^{-8} - 6.73934 \cdot 10^{-9} I)) / \text{ep}^2 + (1 / \text{ep}) (-((2.61261 \cdot 10^{-8} + 1.68009 \cdot 10^{-8} \\ & \rightarrow I) / s^2) - ((4.70467 \cdot 10^{-9} + 1.04672 \cdot 10^{-9} I) \log(s)) / s^2 - (1.30934 \cdot 10^{-7} + 9.62535 \cdot 10^{-8} \\ & \rightarrow I) / s - ((1.49279 \cdot 10^{-8} + 4.88207 \cdot 10^{-9} I) \log(s)) / s - (3.02109 \cdot 10^{-8} + 6.73934 \cdot 10^{-9} I) \\ & \rightarrow \log(s) - (3.63837 \cdot 10^{-7} + 2.4728 \cdot 10^{-7} I)) - ((3.64128 \cdot 10^{-9} + 8.50597 \cdot 10^{-10} I) \log^2(s)) / s^2 \end{aligned}$$

83 Not handled eqn.

85 Not handled eqn.

4

$$\begin{aligned} & -((2.42703 - 0.625114 I) / s) + ((0.0810569 - 3.18897 \cdot 10^{-9} I) \log^2(s)) / s - (0.0405285 \\ & \rightarrow + 2.8617 \cdot 10^{-9} I) \log^2(s) + ((0.0994902 + 1.01859 I) \log(s)) / s + (0.0313118 - 0.509296 I) \\ & \rightarrow \log(s) + (1.1822 + 0.196737 I) + ((-5.16401 \cdot 10^{-10} - 3.70072 \cdot 10^{-9} I) - (5.50793 \cdot 10^{-10} + 3.94743 \cdot 10^{-9} \\ & \rightarrow I) / s) / \text{ep}^2 + (-((2.78596 \cdot 10^{-8} + 7.69087 \cdot 10^{-8} I) / s) - ((5.50793 \cdot 10^{-10} + 3.94743 \cdot 10^{-9} I) \\ & \rightarrow \log(s)) / s - (5.16401 \cdot 10^{-10} + 3.70072 \cdot 10^{-9} I) \log(s) - (3.69889 \cdot 10^{-8} + 6.98559 \cdot 10^{-8} I)) / \text{ep} \end{aligned}$$

#### 5 Zero FIRE.py input file.

113 Zero FIRE.py input file.

#### 7 Zero FIRE.py input file.

Part::partw: Part 1 of {} does not exist.

#### 6 Zero FIRE.py input file.

Part::partw: Part 1 of {} does not exist.

#### 13 Zero FIRE.py input file.

14

$$(0.108076 / s^2 - 0.770041 / s) / \text{ep} + 1.09363 / s^2 - 6.67758 / s - 1.08404 \cdot 10^{-6}$$

16

$$\begin{aligned} & -((0.211474 + 1.27324 I) / s^2) + (2.28897 - 1.08225 I) / s - (4.7286 - 0.668451 \\ & \rightarrow I) + (-((6.7167 \cdot 10^{-16} / s^2) + 0.486342 / s - 0.729513) / \text{ep} - (3.00378 \cdot 10^{-16} \log(s)) / s^2 - (0.486342 \\ & \rightarrow \log(s)) / s + 0.729513 \log(s) \end{aligned}$$

18 Zero FIRE.py input file.

25 Zero FIRE.py input file.

22

$$(0.0810569/s-0.054038/s^2)/\epsilon p-0.411721/s^2+0.617579/s$$

88 Zero FIRE.py input file.

30

$$\rightarrow -((0.394011 +1.25922*10^{-8} I)/s)+(0.782248 -2.46415*10^{-8} I)+(0.168118 \log(s))/s-0.354249 \log(s)$$

31 Numbers overflow.

$$\rightarrow -((0.162114 +1.91875*10^{-8} I)/\epsilon p^3)+(-0.324228 \log(s)-(92.0766 +2.65448 I))/\epsilon p^{2+1}/\epsilon p$$
$$\rightarrow (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) \log(s)-0.0405285 \log^2(s)-(1.58061$$
$$\rightarrow \log(s))/s-(1.91238*10^{10}+1.98727*10^{10} I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46$$
$$\rightarrow I)/s-(16.9633 +0.31831 I) \log^2(s)-(4.74183 \log(s))/s^2+0.148604 \log^3(s)+(0.790305$$
$$\rightarrow \log^2(s))/s+(81.2751 \log(s))/s+(4.05197*10^{10}-6.80923*10^{10} I)$$
$$\rightarrow \log(s)+(6.59315*10^{13}-1.55809*10^{14} I)$$

106 Not handled eqn. Failed in GiNaC\_Parallel!

32 Numbers overflow.

$$\rightarrow -((0.162114 +1.91875*10^{-8} I)/\epsilon p^3)+(-0.324228 \log(s)-(92.0766 +2.65448 I))/\epsilon p^{2+1}/\epsilon p$$
$$\rightarrow (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) \log(s)-0.0405285 \log^2(s)-(1.58061$$
$$\rightarrow \log(s))/s-(1.91238*10^{10}+1.98727*10^{10} I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46$$
$$\rightarrow I)/s-(16.9633 +0.31831 I) \log^2(s)-(4.74183 \log(s))/s^2+0.148604 \log^3(s)+(0.790305$$
$$\rightarrow \log^2(s))/s+(81.2751 \log(s))/s+(4.05197*10^{10}-6.80923*10^{10} I)$$
$$\rightarrow \log(s)+(6.59315*10^{13}-1.55809*10^{14} I)$$

82 Not handled eqn.

38 Zero FIRE.py input file.

62 Zero FIRE.py input file.

89 Divergent integrand.

39 Divergent integrand.

40 Zero FIRE.py input file.

97 Zero FIRE.py input file.

74 Zero FIRE.py input file.

41 Failed in GiNaC\_Parallel! power::eval(): division by zero.

54 Zero FIRE.py input file.

80 Zero FIRE.py input file.

81

$$\begin{aligned} & ((0.180127 - 9.13154 \cdot 10^{-9} I) \log(s) - (0.231946 + 7.6754 \cdot 10^{-8} I)) / \text{ep} + (1.02169 - 0.565884 I) \\ \rightarrow & \log(s) - (0.815813 - 0.728676 I) - (5.2095 \cdot 10^{-10} + 1.21468 \cdot 10^{-8} I) / \text{ep}^2 + (-0.0900633 - 4.02323 \cdot 10^{-9} I) \\ \rightarrow & \log^2(s) \end{aligned}$$

55

$$\begin{aligned} & ((0.288202 - 2.40402 \cdot 10^{-8} I) / s - (0.432304 + 3.26984 \cdot 10^{-8} I)) / \text{ep} + (1.81157 + 0.905414 \\ \rightarrow & I) / s - (2.71736 + 1.35812 I) \end{aligned}$$

3.14

$$(0.180127 \log(s) - 0.231946) / \text{ep}$$

84

$$\begin{aligned} & ((0.226837 - 4.91292 \cdot 10^{-7} I) - (0.0900633 + 3.62168 \cdot 10^{-8} I) \log(s)) / \text{ep} + (0.27463 + 0.141471 I) \\ \rightarrow & \log^2(s) - (1.20417 + 0.36434 I) \log(s) + (1.63449 + 0.234572 I) - (4.43277 \cdot 10^{-8} + 3.77952 \cdot 10^{-8} \\ \rightarrow & I) / \text{ep}^2 - (7.40521 \cdot 10^{-8} + 3.24692 \cdot 10^{-8} I) / s \end{aligned}$$

56

$$\begin{aligned} & ((0.216152 - 1.26134 \cdot 10^{-7} I) - (0.144101 + 4.52496 \cdot 10^{-8} I) / s) / \text{ep} - (1.36468 - 0.29147 \\ \rightarrow & I) / s + ((0.367358 - 0.226354 I) \log(s)) / s - (0.696665 - 0.962003 I) \log(s) + (2.44396 + 0.289131 I) \end{aligned}$$

86 Zero FIRE.py input file.

58 Zero FIRE.py input file.

95

$$\begin{aligned} & ((0.0225158 - 9.13144 \cdot 10^{-9} I) \log(s) - (0.0289933 + 1.11581 \cdot 10^{-7} I)) / \text{ep} + (0.127711 - 0.0707356 \\ \rightarrow & I) \log(s) - (0.101978 - 0.0910826 I) - (3.18099 \cdot 10^{-10} + 1.21466 \cdot 10^{-8} \\ \rightarrow & I) / \text{ep}^2 + (-0.0112579 - 4.02319 \cdot 10^{-9} I) \log^2(s) \end{aligned}$$

59

$$\begin{aligned} & ((0.0360253 - 2.54985 \cdot 10^{-8} I) / s - (0.054038 + 3.46821 \cdot 10^{-8} I)) / \text{ep} + (0.226446 + 0.113176 \\ \rightarrow & I) / s - (0.33967 + 0.169767 I) \end{aligned}$$

60 Zero FIRE.py input file.

63 Zero FIRE.py input file.

61

$$\begin{aligned} & ((0.0249708 + 0.0565884 I) / s - (0.0409356 + 0.113177 I) - (0.0180127 \log(s)) / s + 0.0360253 \\ \rightarrow & \log(s)) / \text{ep} + (0.121805 + 0.302986 I) / s - ((0.0819104 + 0.0282942 I) \log(s)) / s + (0.123545 + 0.12025 I) \\ \rightarrow & \log(s) - (0.132015 + 0.412085 I) + ((-1.70997 \cdot 10^{-8} - 7.51261 \cdot 10^{-9} I) - (3.80204 \cdot 10^{-10} + 2.72486 \cdot 10^{-9} \\ \rightarrow & I) / s) / \text{ep}^2 - 1.51762 \cdot 10^{-7} / s^2 + (0.00900633 \log^2(s)) / s - 0.0180127 \log^2(s) \end{aligned}$$

101

$$\begin{aligned} & ((0. - 5.44316 \cdot 10^{-9} I) \log(s) - (5.06726 \cdot 10^{-8} + 1.09474 \cdot 10^{-7} I)) / \text{ep}^2 + ((0.0301033 - 0.0353678 \\ \rightarrow & I) \log(s) - (0.0200969 - 0.045542 I) - (1.32299 \cdot 10^{-8} + 4.49679 \cdot 10^{-9} I) / s + (-0.0112579 - 3.05202 \cdot 10^{-9} I) \\ \rightarrow & \log^2(s)) / \text{ep} + (0.0112579 - 9.07187 \cdot 10^{-10} I) \log^3(s) - (0.0593367 - 0.0353677 I) \log^2(s) + (0.0923378 \\ \rightarrow & - 0.190433 I) \log(s) - (0.0423955 - 0.117755 I) - (1.47432 \cdot 10^{-10} + 8.25276 \cdot 10^{-9} \\ \rightarrow & I) / \text{ep}^3 - (4.6093 \cdot 10^{-7} + 4.84678 \cdot 10^{-8} I) / s \end{aligned}$$

64 Zero FIRE.py input file.

104 Zero FIRE.py input file.

136

$$\begin{aligned} & (-(0.00499433 + 0.0113178 I)/s) + (0.000695404 + 0.00565863 I) + (0.00360253 \log(s))/s - 0.00180128 \\ \rightarrow & \log(s)/\text{ep} + (0.232843 - 0.0984924 I)/s + ((0.0363448 + 0.0226354 I) \log(s))/s + (0.0153548 \\ \rightarrow & -1.05329 \cdot 10^{-7} I) \log(s) - (0.653916 - 3.42529 I) + ((-1.16122 \cdot 10^{-8} - 2.31214 \cdot 10^{-9} \\ \rightarrow & I) - (4.75779 \cdot 10^{-9} + 2.54986 \cdot 10^{-9} I)/s)/\text{ep}^2 - 2.41702 \cdot 10^{-7}/s^2 - (0.0054038 \log^2(s))/s + 0.00270189 \\ \rightarrow & \log^2(s) \end{aligned}$$

102 Not handled eqn.

103 Zero FIRE.py input file.

108 Zero FIRE.py input file.

127 Zero FIRE.py input file.

105 Zero FIRE.py input file.

131

$$(0.027019 - 0.0180127/s)/\text{ep} + 0.0890396/s + (0.0180127 \log(s))/s - 0.027019 \log(s) + 0.197988$$

110 Zero FIRE.py input file.

121 Zero FIRE.py input file.

111 Zero FIRE.py input file.

126

$$-(0.478416/(\text{ep } s)) - 3.57259/s + (0.478416 \log(s))/s$$

117

$$\begin{aligned} & (0.337737 \log(s) + (12.5141 + 0.065117 I))/\text{ep} + (4.9247 + 0.043133 I)/s - (7.6631 + 0.248286 I) \\ \rightarrow & \log(s) + (960.52 - 2.012 I) - 0.371511 \log^2(s) + (0.0506606 \log(s))/s \end{aligned}$$

123 Zero FIRE.py input file.

119

$$\begin{aligned} & ((0.108076 \text{ CV}(1,3))/s - (0.0720506 \text{ CV}(1,3))/s^2)/\text{ep} - (0.621011 \text{ CV}(1,3))/s^2 - (0.221294 \\ \rightarrow & \text{ CV}(1,3))/s + 0.51236 \text{ CV}(1,3) \log(s) + 0.289007 \text{ CV}(1,3) \end{aligned}$$

135 Zero FIRE.py input file.

134 Zero FIRE.py input file.

142 Zero FIRE.py input file.

$$\begin{aligned}
& ((0.226837 - 4.76795 \cdot 10^{-7} I) + (2.53148 \cdot 10^{-36} + 9.52697 \cdot 10^{-70} I) / s + (-0.0900633 - 5.77113 \cdot 10^{-8} \\
\hookrightarrow & I) \log(s)) / \epsilon p + (0.27463 + 0.141471 I) \log^2(s) - (1.20417 + 0.36434 I) \log(s) + (1.63449 + 0.234572 \\
\hookrightarrow & I) - (6.78712 \cdot 10^{-8} + 5.87408 \cdot 10^{-8} I) / \epsilon p^2 - (7.40521 \cdot 10^{-8} + 3.24692 \cdot 10^{-8} I) / s
\end{aligned}$$

### 3 Virtual Diagrams (Excluding Gauge Link Self-Energy Diagrams)

#### 3.1 All diagrams

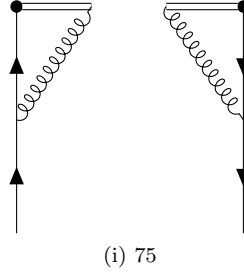


Figure 4: All self-conjugated virtual diagrams (actually there's only one).

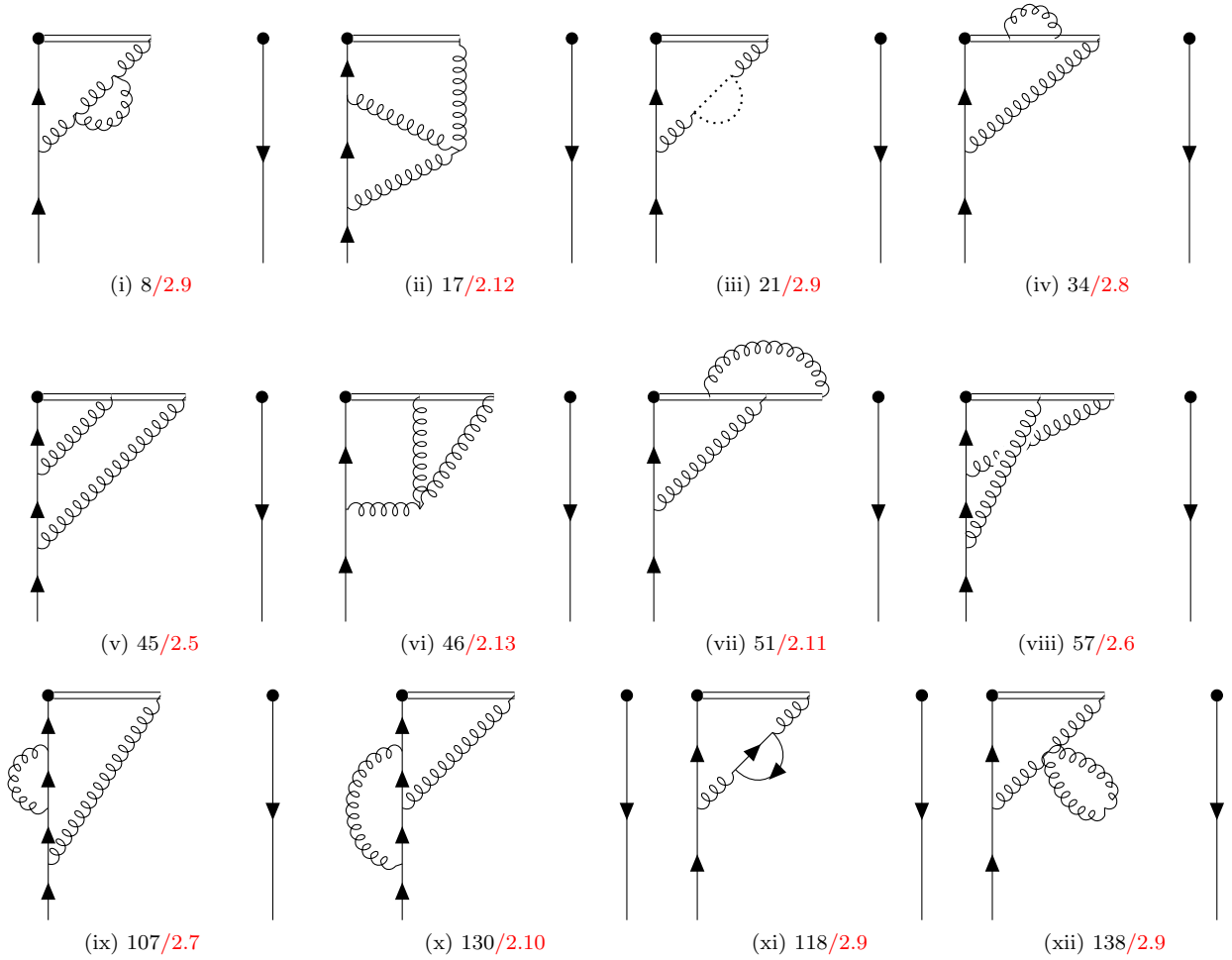


Figure 5: All virtual diagrams (excluding conjugated and self-conjugated diagrams), red n.i marks the diagram number in Ji&Zhang's paper.

## 3.2 Numerical test (ordered as Figure 4 and Figure 3, $z = 1/4$ )

### 3.2.1 Self-conjugated

75

### 3.2.2 Remaining diagrams (exclude conjugated ones)

## 4 Gauge Link Self-Energy Diagrams

### 4.1 All diagrams

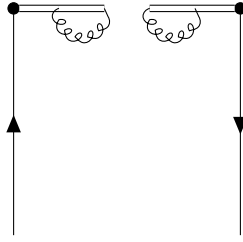


Figure 6: All self-conjugated gauge link self-energy diagrams (actually there's only one).



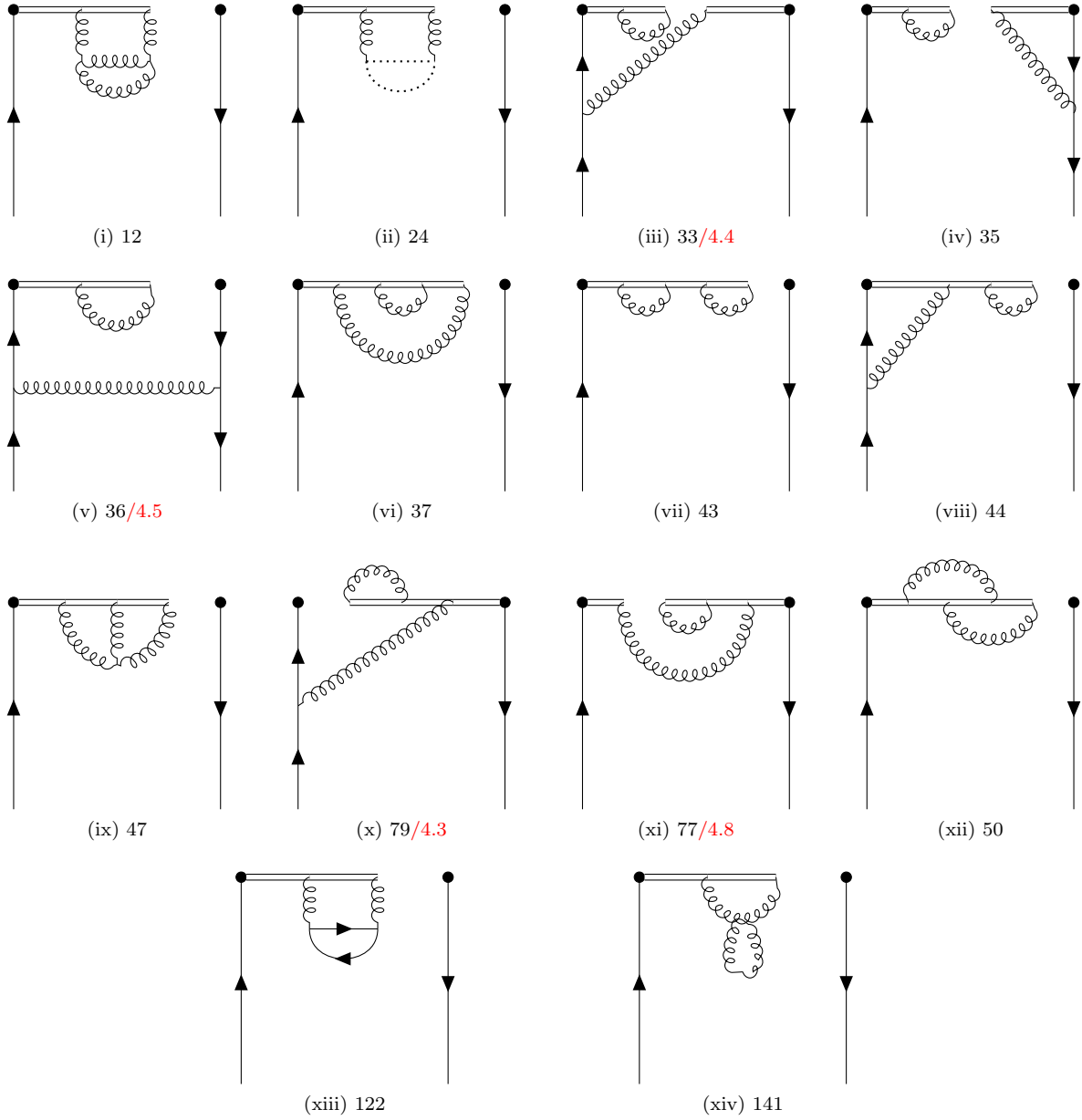


Figure 7: All gauge link self-energy diagrams (excluding conjugated and self-conjugated diagrams), red n.i marks the diagram number in Ji&Zhang's paper.

## 5 Diagrams with Direct Contracting $\bar{\psi}(z)\psi(0)$

### 5.1 All diagrams

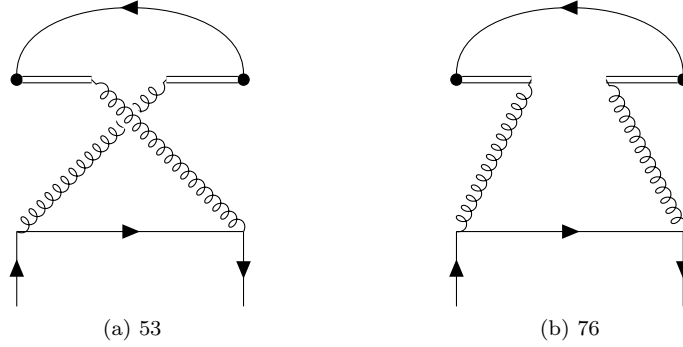


Figure 8: All self-conjugated quark contraction diagrams

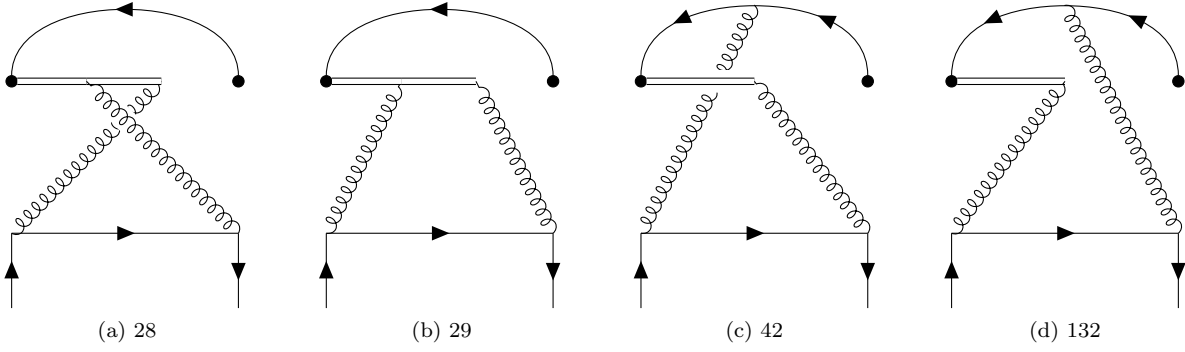


Figure 9: All quark contraction diagrams (excluding conjugated and self-conjugated diagrams).

## 6 HQET Correspondence

## References

[Ji and Zhang(2015)] X. Ji and J.-H. Zhang, [Phys. Rev. \*\*D92\*\*, 034006 \(2015\)](#), [arXiv:1505.07699 \[hep-ph\]](#) .