One Loop Matching for Quasi PDF

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1 Background

The definition of parton distribution function (PDF) is

$$q\left(x, \mu_{f}\right)=\frac{1}{2} \int \frac{d \eta^{-}}{2 \pi} e^{-i x P^{+} \eta^{-}}\left\langle P, S\left|\bar{\psi}\left(0, \eta^{-}, \mathbf{0}_{T}\right) \Gamma \mathcal{W}\left[\eta^{-} ; 0\right] \psi(0)\right| P, S\right\rangle \tag{1}$$

where with this unpolarized PDF case, $\Gamma=\gamma^{+}$. $\mathcal{W}$ is the gauge link defined as [Collins(2009)]

$$\mathcal{W}\left[w^{-}, 0\right]=P\left\{e^{-i g_{0} \int_{0}^{w^{-}} \mathrm{d} y^{-} A_{(0) \sigma}^{+}\left(0, y^{-}, \mathbf{0}_{\mathrm{T}}\right) t_{\sigma}}\right\} \tag{2}$$

The definition of quasi PDF is

$$\tilde{q}(x)=\frac{1}{2} \int \frac{\mathrm{d} z}{2 \pi} e^{i x P^{z} z}\langle P, S|\bar{\psi}(z) \tilde{\Gamma} \tilde{\mathcal{W}}[z, 0] \psi(0)| P, S\rangle \tag{3}$$

where

$$\tilde{\mathcal{W}}[z, 0]=\exp \left[i g \mathcal{P} \int_{0}^{z} \mathrm{d} z^{\prime} n \cdot A^{a}\left(z^{\prime}\right) \mathrm{t}^{a}\right], n^{\mu}=(0,0,0,-1) \tag{4}$$

and $\tilde{\Gamma}=\gamma^{z}$ in our case.

To make the gauge links equal to unity, we choose light cone gauge for PDF and axial gauge for quasi PDF.

2 Tree Level Matching

In axial gauge, the quasi PDF is

$$\tilde{q}(x)=\frac{1}{4 \pi} \int \mathrm{d} z e^{i x P^{z} z}\langle P|\bar{\psi}(z) \gamma^{z} \psi(0)| P\rangle \tag{5}$$

The frame is chosen such that $P^{\mu}=(P^{0}, \mathbf{0}, P^{z})$.

$$P^{0}=\sqrt{m^{2}+P^{z 2}} \tag{6}$$

Up to one loop, we can use quark state as the external state to complete the matching process. The quark field ψ reads

$$\psi(x)=\int \frac{\mathrm{d}^{3} \vec{k}}{(2 \pi)^{3}} \frac{1}{2 E_{k}}\left[u(k) e^{-i k \cdot x} b_{k}+v(k) e^{i k \cdot x} d_{k}^{\dagger}\right] \tag{7}$$

Insert it to (5)

$$\tilde{q}^{(0)}(x) = \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z} \langle 0|b_P \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{2E_p} \left[\bar{u}(p)e^{ip\cdot x} b_p^\dagger + \bar{v}(p) e^{-ip\cdot x} d_p\right] \gamma^z \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{1}{2E_k} \left[u(k)e^{-ik\cdot x} b_k + v(k) e^{ik\cdot x} d_k^\dagger\right] b_P^\dagger |0\rangle \tag{8}$$

Look at the creation-annihilation operators, we have the following combinations:

$$b_P b_p^\dagger b_k b_P^\dagger,\ b_P d_p b_k b_P^\dagger,\ b_P b_p^\dagger d_k^\dagger b_P^\dagger,\ b_P d_p d_k^\dagger b_P^\dagger \tag{9}$$

Apparently the latter three all go to zero by moving the anti-quark operators to the side:

$$\begin{aligned}
\tilde{q}^{(0)}(x) &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z} \langle 0| \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{1}{2E_p} \bar{u}(p) e^{ip\cdot z} b_P b_p^\dagger \gamma^z \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{1}{2E_k} u(k) e^{-ik\cdot 0} b_k b_P^\dagger |0\rangle \\
&= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z} \langle 0| \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{e^{ip\cdot z}}{2E_p} \bar{u}(p)(2\pi)^3 2E_{\mathbf{P}} \delta^{(3)}(\mathbf{p}-\mathbf{P}) \gamma^z \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{e^{-ik\cdot 0}}{2E_k} u(k)(2\pi)^3 2E_{\mathbf{P}} \delta^{(3)}(\mathbf{k}-\mathbf{P})|0\rangle \\
&= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z + iP\cdot z} \bar{u}(P)\gamma^z u(P)
\end{aligned} \tag{10}$$

Using Gordon identity

$$\begin{aligned}
\tilde{q}^{(0)}(x) &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z - iP^z z} \bar{u}(P) \frac{P^z}{m} u(P) \\
&= \int \frac{\mathrm{d}z}{2\pi} e^{ixP^z z - iP^z z} P^z \\
&= \delta(1-x)
\end{aligned} \tag{11}$$

3 One Loop Quasi PDF (Axial Gauge)

First we consider the matrix element in the definition of quasi PDF

$$\langle P|\bar{\psi}(z)\gamma^z \psi(0)|P\rangle \tag{12}$$

and in leading order this one gives

$$e^{-iP^z z} \bar{u}(P)\gamma^z u(P) \tag{13}$$

as mentioned above. This, in higher orders, is embedded via a Fourier transform. The full form of quasi PDF can be considered as a momentum space matrix element with an $1/4\pi$ factor.

Two diagrams are required with one loop corrections to quasi PDF. Detailed derivation with rigorous Wick contraction is to be found in Section B.

The Feynman rule for the composite operator is

$$\overset{p_1,0}{\bullet} \cdots\cdots\cdots \overset{p_2,\mathrm{z}}{\bullet} = e^{-ip_2^z z} \gamma^z \tag{14}$$

and two external lines give $\bar{u}(P)$ and $u(P)$ respectively.

The first one is a quark self-energy correction

$$\bar{u}(P) e^{-iP^z z} \gamma^z \frac{i(\not{P}+m)}{P^2 - m^2} (-i\Sigma_2(P)) u(P) \tag{15}$$

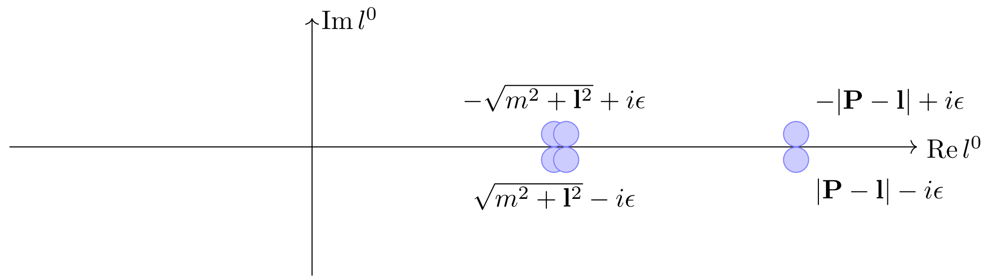
(a) (b)

Figure 1

The second one is

$$
\begin{aligned}
&\bar{u}(P)\int\frac{\mathrm{d}l^0}{2\pi}\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\,(-ig_st^a\gamma^\mu)\frac{i(\not{l}+m)}{l^2-m^2}\gamma^z\frac{i(\not{l}+m)}{l^2-m^2}(-ig_st^a\gamma^\nu)\tilde{D}^A_{G\mu\nu}(P-l)u(P)\Big|_{l^z=xP^z}\\
&=-g_s^2C_F\bar{u}(P)\int\frac{\mathrm{d}l^0}{2\pi}\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\,\gamma^\mu\frac{i(\not{l}+m)}{l^2-m^2}\gamma^z\frac{i(\not{l}+m)}{l^2-m^2}\gamma^\nu\tilde{D}^A_{G\mu\nu}(P-l)u(P)\Big|_{l^z=xP^z}
\end{aligned}
\tag{16}
$$

For the definition of $\tilde{D}^A_{G\mu\nu}$, see Section A. There're in total 6 poles:



For the result of numerator simplification, see Section C

4 One Loop Quasi PDF (Feynman Gauge)

In Feynman gauge, we must have the full definition of quasi PDF. For unpolarized quasi PDF

$$
\tilde{q}(x)=\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\langle P,S|\bar{\psi}(z)\gamma^z\exp\left[ig\mathcal{P}\int_0^z\mathrm{d}z'\,A^{a,z}\left(z'\right)\mathrm{t}^a\right]\psi(0)|P,S\rangle \tag{17}
$$

There're following 7 diagrams,

A Conventions

The quark field ψ reads

$$
\psi(x)=\int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\frac{1}{2E_k}\left[u(k)e^{-ik\cdot x}b_k+v(k)e^{ik\cdot x}d_k^\dagger\right] \tag{18}
$$

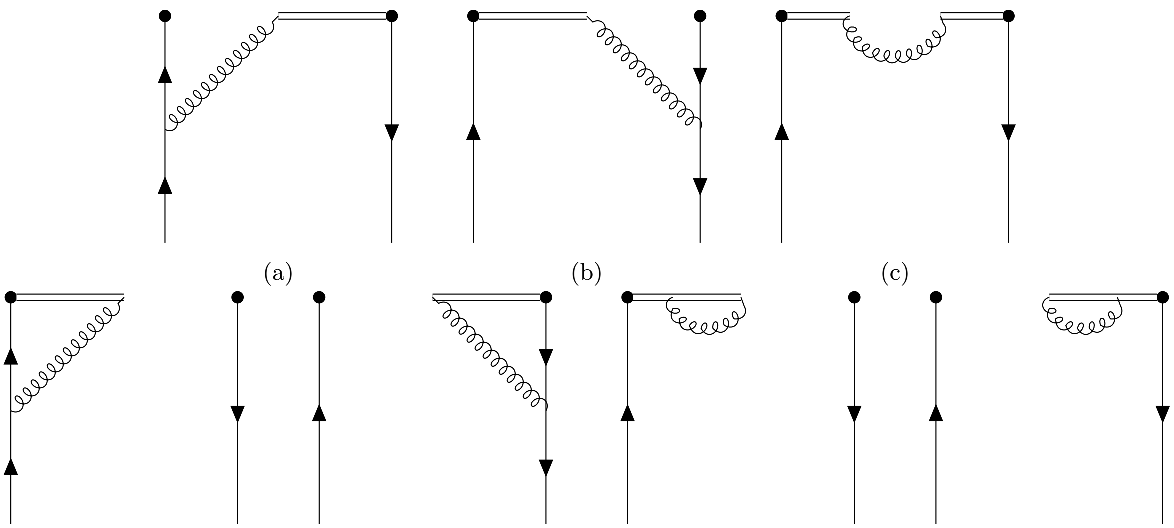


Figure 2: Diagrams of quasi PDF in Feynman gauge.

and the projection of single particle state is

$$|p\rangle = b_p^\dagger |0\rangle \tag{19}$$

$$\left\{b_{\mathbf{p}}^r, b_{\mathbf{q}}^{s\dagger}\right\} = (2\pi)^3 2E\delta^{(3)}(\mathbf{p}-\mathbf{q})\delta^{rs} \tag{20}$$

The Dirac spinor is normalized to

$$\bar{u}^s(p)u(p) = 2m\delta^{rs} \tag{21}$$

With Gordon identity, one can derive [Srednicki(2007)]

$$\bar{u}(P)\gamma^\mu u(P) = 2P^\mu \tag{22}$$

The axial gauge propagator is

$$\tilde{D}_G^{A\mu\nu}(p) = -i\delta_{ab}\left(g^{\mu\nu} - \frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p} + n\cdot n\frac{p^\mu p^\nu}{(n\cdot p)^2}\right)\frac{1}{p^2} \tag{23}$$

State contract with field:

$$\begin{aligned}\overbrace{\psi(x)|P}\rangle &= \int \frac{\mathrm{d}^3\mathbf{l}}{(2\pi)^3}\frac{1}{2E_\mathbf{l}}\Big[b_\mathbf{l}u(l)e^{-il\cdot x} + d_\mathbf{l}^\dagger v(l)e^{il\cdot x}\Big]b_\mathbf{P}^\dagger |0\rangle \\ &= \int \frac{\mathrm{d}^3\mathbf{l}}{(2\pi)^3}\frac{1}{2E_\mathbf{l}}u(l)e^{-il\cdot x}(2\pi)^3 2E\delta^{(3)}(\mathbf{l}-\mathbf{P})|0\rangle \\ &= u(P)e^{-iP\cdot x}\end{aligned} \tag{24}$$

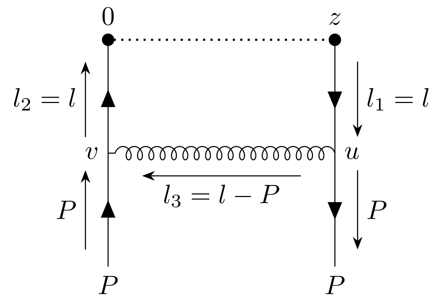
and correspondingly

$$\langle\overbrace{P|\bar{\psi}}(x) = \bar{u}(P)e^{iP\cdot x} \tag{25}$$

B Wick Contraction

B.1 Axial Gauge

Take diagram 1b as an example



This corresponds to

$$
\begin{aligned}
&\frac{1}{4\pi}\int \mathrm{d}z e^{ixP^z z}\langle P|\int \mathrm{d}^4u\bar{\psi}_u\psi_u A_u\bar{\psi}(z)\gamma^z\psi(0)\int \mathrm{d}^4v\bar{\psi}_v\psi_v A_v|P\rangle && (26)\\
=&\frac{1}{4\pi}\int \mathrm{d}z e^{ixP^z z}\int \mathrm{d}^4u\mathrm{d}^4v\bar{u}(P)e^{iP\cdot u}\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\tilde{D}_F(l_1)e^{-il_1\cdot(u-z)}\gamma^z\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\tilde{D}_F(l_2)e^{-il_2\cdot(-v)}\\
&\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}\tilde{D}_G(l_3)e^{-il_3\cdot(v-u)}u(P)e^{-iP\cdot v} && (27)\\
=&\frac{1}{4\pi}\int \mathrm{d}z\int \mathrm{d}^4u\mathrm{d}^4v\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\int\frac{\mathrm{d}^4l_3}{(2\pi)^4}e^{ixP^z z+il_1\cdot z}e^{i(P-l_1+l_3)\cdot u}e^{i(l_2-l_3-P)\cdot v}\bar{u}(P)\tilde{D}_F(l_1)\gamma^z\tilde{D}_F(l_2)\\
&\tilde{D}_G(l_3)u(P) && (28)\\
=&\frac{1}{4\pi}\int \mathrm{d}z\int\frac{\mathrm{d}^4l}{(2\pi)^4}e^{ixP^z z+il\cdot z}\bar{u}(P)\tilde{D}_F(l)\gamma^z\tilde{D}_F(l)\tilde{D}_G(l-P)u(P) && (29)\\
=&\frac{1}{4\pi}\int \mathrm{d}z\int\frac{\mathrm{d}^4l}{(2\pi)^4}e^{i(xP^z-l^z)z}\bar{u}(P)\tilde{D}_F(l)\gamma^z\tilde{D}_F(l)\tilde{D}_G(l-P)u(P) && (30)\\
=&\frac{1}{4\pi}\int\frac{\mathrm{d}l^0}{2\pi}\int\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\ \bar{u}(P)\tilde{D}_F(l)\gamma^z\tilde{D}_F(l)\tilde{D}_G(l-P)u(P)\Big|_{l^z=xP^z} && (31)
\end{aligned}
$$

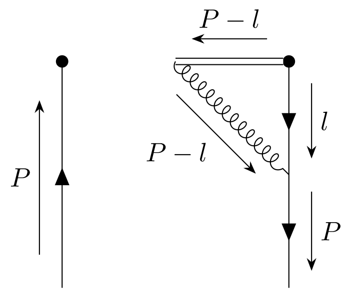
where

$$\int \mathrm{d}z e^{i(xP^z-l^z)z} = 2\pi\delta(l^z - xP^z) \tag{32}$$

This indicates what the Feynman diagram actually means: a normal Feynman diagram with only 3-momentum integration, where the axial momentum is fixed by $l^z = xP^z$, and with an extra $1/4\pi$ factor.

B.2 Feynman Gauge

Let's take diagram 2b as an example:



The one loop quasi PDF is

$$\tilde{q}_1^{(1)}(x) = \frac{1}{2}\int \frac{\mathrm{d}z}{2\pi} e^{ixP^z z}\langle P,S|\int \mathrm{d}^4u(-ig_s t^a)\bar{\psi}_u\psi_u A_u\bar{\psi}(z)\gamma^z\tilde{\mathcal{W}}[z,0]\psi(0)|P,S\rangle \tag{33}$$

where

$$\tilde{\mathcal{W}}[z,0] = \exp\left[ig_s\mathcal{P}\int_0^z \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right] \tag{34}$$

We should rewrite the gauge link to the product of two gauge links connect to infinity

$$\tilde{\mathcal{W}}[z,0] = \tilde{\mathcal{W}}[z,+\infty]\tilde{\mathcal{W}}[\infty,0] \tag{35}$$

and in one loop level it equals to

$$\exp\left[ig_s\mathcal{P}\int_\infty^z \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right]\exp\left[ig_s\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right] = \left[ig_s\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right] - \left[ig_s\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'+z\right)\mathrm{t}^a\right]$$

The path ordering gives

$$\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right) = \int \mathrm{d}z' A^{a,z}\left(z'\right)\theta(z') = \int \mathrm{d}z' A^{a,z}\left(z'\right)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon} \tag{36}$$

and

$$\left[\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\right]^2 = \int \mathrm{d}z' A^{a,z}\left(z'\right)\theta(z')\int \mathrm{d}z'' A^{a,z}\left(z''\right)\theta(z''-z') \tag{37}$$

with all momenta involved with z' must be in the z-direction (the exponent is actually $z'n\cdot w$ if a four-vector w actually exists). Consider the second gauge link first, the matrix element is then (discarding all couplings)

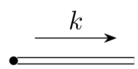
$$\begin{aligned}
&\langle P,S|\bar{\psi}_u\psi_u A_u\bar{\psi}(z)\gamma^z\int \mathrm{d}z' A^{a,z}\left(z'+z\right)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\psi(0)|P,S\rangle\\
=&\int \mathrm{d}^4u\langle P,S|\bar{\psi}_u\psi_u A_u\bar{\psi}(z)\gamma^z\int \mathrm{d}z' A^{a,z}\left(z'+z\right)\psi(0)|P,S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\\
=&\int \mathrm{d}^4u\langle P,S|\bar{\psi}_u\psi_u A_u\bar{\psi}(z)\gamma^z\int \mathrm{d}z' A^{a,z}\left(z'+z\right)\psi(0)|P,S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\\
=&\int \mathrm{d}z'\int \mathrm{d}^4u\bar{u}(P)e^{iP\cdot u}\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\tilde{D}_F(l_1)e^{-il_1\cdot(u-z)}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\tilde{D}_G(l_2)e^{-il_2\cdot(u-z'-z)}u(P)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\\
=&\int \mathrm{d}z'\int \mathrm{d}^4u\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\int\frac{\mathrm{d}^4l_2}{(2\pi)^4}\int\frac{\mathrm{d}w}{2\pi}\bar{u}(P)e^{i(P-l_1-l_2)\cdot u}e^{i(l_1+l_2)\cdot z}e^{-iwz'-il_2\cdot z'}\tilde{D}_F(l_1)\tilde{D}_G(l_2)u(P)\frac{i}{w+i\epsilon}\\
=&\int \mathrm{d}z'\int\frac{\mathrm{d}^4l}{(2\pi)^4}\int\frac{\mathrm{d}w}{2\pi}\bar{u}(P)e^{iP\cdot z}e^{-iwz'+i(P-l)^z z'}\tilde{D}_F(l)\tilde{D}_G(P-l)u(P)\frac{i}{w+i\epsilon}
\end{aligned}$$

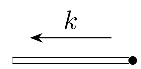
$$=\bar{u}(P)e^{iP\cdot z}\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_F(l)\tilde{D}_G(P-l)\frac{i}{P^z-l^z+i\epsilon}u(P)$$

The complete quasi PDF at one loop is

$$\begin{aligned}&\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\bar{u}(P)e^{iP\cdot z}\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_F(l)\tilde{D}_G(P-l)\frac{i}{P^z-l^z+i\epsilon}u(P)\\ =&\frac{1}{2P^z}\bar{u}(P)\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_F(l)\tilde{D}_G(P-l)\frac{i}{P^z-l^z+i\epsilon}u(P)\delta(1-x)\end{aligned}$$

multiplied by those couplings. This basically established that the momentum of a gluon equals to the momentum of the eikonal line it attaches to. We then have the Feynman rule:

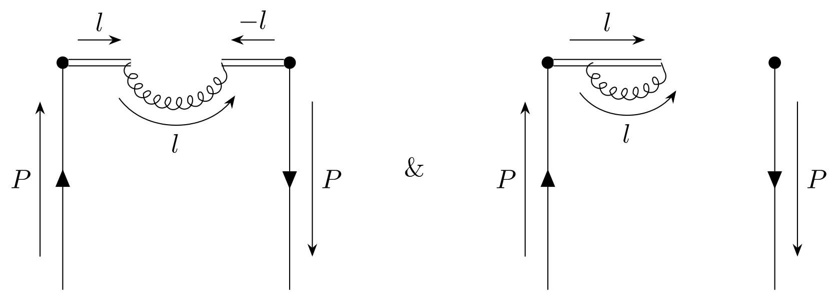


$$=\frac{i}{n\cdot k+i\epsilon};$$


$$=\frac{i}{n\cdot k+i\epsilon}\tag{38}$$

and for the gluon-eikonal vertex on the r.h.s., an extra minus sign is added for the normal (ig_st^a).

The next job is to determine the Feynman rule for



The first one is

$$\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\langle P,S|\bar{\psi}(z)\gamma^z\left[-ig_s\mathcal{P}\int_0^\infty\mathrm{d}z'A^{a,z}(z'+z)\,\mathrm{t}^a\right]\left[ig_s\mathcal{P}\int_0^\infty\mathrm{d}z'A^{a,z}(z')\,\mathrm{t}^a\right]\psi(0)|P,S\rangle\tag{39}$$

Let's look at the coupling-free form:

$$\begin{aligned}&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\langle P,S|\bar{\psi}(z)\gamma^z\left[\mathcal{P}\int_0^\infty\mathrm{d}z'A^{a,z}(z'+z)\right]\left[\mathcal{P}\int_0^\infty\mathrm{d}z''A^{a,z}(z'')\right]\psi(0)|P,S\rangle\\ =&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\langle P,S|\bar{\psi}(z)\gamma^z\int\mathrm{d}z'A^{a,z}(z'+z)\int\mathrm{d}z''A^{a,z}(z'')\,\psi(0)|P,S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\ =&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\langle P,S|\bar{\psi}(z)\gamma^z\int\mathrm{d}z'A^{a,z}(z'+z)\int\mathrm{d}z''A^{a,z}(z'')\,\psi(0)|P,S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\ =&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^zz}\bar{u}(P)e^{iP\cdot z}\gamma^z\int\mathrm{d}z'\mathrm{d}z''\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_G(l)e^{-il\cdot(z''-z'-z)}u(P)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ihz''}}{h+i\epsilon}\\ =&\bar{u}(P)\int\frac{\mathrm{d}z}{2\pi}e^{i(x-1)P^zz+il^zz}\gamma^z\int\mathrm{d}z'\mathrm{d}z''\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_G(l)\int\frac{\mathrm{d}w}{2\pi}\frac{i}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{i}{h+i\epsilon}e^{-i(w-l)\cdot z'}e^{-i(l+h)\cdot z''}u(P)\\ =&\bar{u}(P)\int\frac{\mathrm{d}z}{2\pi}e^{-i(1-x)P^zz+il^zz}\gamma^z\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_G(l)\frac{i}{l^z+i\epsilon}\frac{i}{-l^z+i\epsilon}u(P)\\ =&\bar{u}(P)\gamma^z\int\frac{\mathrm{d}^4l}{(2\pi)^4}\tilde{D}_G(l)\frac{i}{l^z+i\epsilon}\frac{i}{-l^z+i\epsilon}\delta(l^z-(1-x)P^z)u(P)\end{aligned}$$

The second one is

$$\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle P,S|\bar{\psi}(z)\gamma^z\frac{\left[ig_s\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\mathrm{t}^a\right]\left[ig_s\mathcal{P}\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\mathrm{t}^a\right]}{2}\psi(0)|P,S\rangle \tag{40}$$

The coupling-free form is

$$\begin{aligned}
&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle P,S|\bar{\psi}(z)\gamma^z\left[\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'\right)\right]\left[\mathcal{P}\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''\right)\right]\psi(0)|P,S\rangle\\
=&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle P,S|\bar{\psi}(z)\gamma^z\int \mathrm{d}z' A^{a,z}\left(z'\right)\int \mathrm{d}z'' A^{a,z}\left(z''\right)\psi(0)|P,S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ih(z''-z')}}{h+i\epsilon}\\
=&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle \overline{P,S|\bar{\psi}}(z)\gamma^z\int \mathrm{d}z' \overline{A^{a,z}\left(z'\right)\int \mathrm{d}z'' A}^{a,z}\left(z''\right)\overline{\psi(0)|P},S\rangle\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ih(z''-z')}}{h+i\epsilon}\\
=&\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\bar{u}(P)e^{iP\cdot z}\gamma^z\int \mathrm{d}z'\int \mathrm{d}z''\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\tilde{D}_G(l)e^{-il\cdot(z''-z')}u(P)\int\frac{\mathrm{d}w}{2\pi}\frac{ie^{-iwz'}}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{ie^{-ih(z''-z')}}{h+i\epsilon}\\
=&\bar{u}(P)\int\frac{\mathrm{d}z}{2\pi}e^{-i(1-x)P^z z+il^z z}\gamma^z\int \mathrm{d}z'\int \mathrm{d}z''\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\tilde{D}_G(l)\int\frac{\mathrm{d}w}{2\pi}\frac{i}{w+i\epsilon}\int\frac{\mathrm{d}h}{2\pi}\frac{i}{h+i\epsilon}e^{-i(w-l-h)\cdot z'}e^{-i(h+l)\cdot z''}u(P)\\
=&\bar{u}(P)\gamma^z\int\frac{\mathrm{d}^4 l}{(2\pi)^4}\tilde{D}_G(l)\frac{i}{0+i\epsilon}\frac{i}{-l^z+i\epsilon}\delta(l^z-(1-x)P^z)u(P)
\end{aligned}$$

Diagram 2g is

$$\frac{1}{2}\int\frac{\mathrm{d}z}{2\pi}e^{ixP^z z}\langle P,S|\bar{\psi}(z)\gamma^z\frac{\left[-ig_s\mathcal{P}\int_0^\infty \mathrm{d}z' A^{a,z}\left(z'+z\right)\mathrm{t}^a\right]\left[-ig_s\mathcal{P}\int_0^\infty \mathrm{d}z'' A^{a,z}\left(z''+z\right)\mathrm{t}^a\right]}{2}\psi(0)|P,S\rangle \tag{41}$$

and it behaves exactly like 2f, since those extra $(+z)$s will be cancelled in the Wick contraction.

C Diagram 1b Comparing

Let's start with

$$\begin{aligned}
&\bar{u}(P)\int\frac{\mathrm{d}l^0}{2\pi}\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\left(-ig_s t^a\gamma^\mu\right)\frac{i(\not{l}+m)}{l^2-m^2}\gamma^z\frac{i(\not{l}+m)}{l^2-m^2}\left(-ig_s t^a\gamma^\nu\right)\tilde{D}^A_{G\mu\nu}(P-l)u(P)\Big|_{l^z=xP^z}\\
=&-g_s^2 C_F\bar{u}(P)\int\frac{\mathrm{d}l^0}{2\pi}\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\gamma^\mu\frac{i(\not{l}+m)}{l^2-m^2}\gamma^z\frac{i(\not{l}+m)}{l^2-m^2}\gamma^\nu\tilde{D}^A_{G\mu\nu}(P-l)u(P)\Big|_{l^z=xP^z}\\
=&-ig_s^2 C_F\bar{u}(P)\int\frac{\mathrm{d}l^0}{2\pi}\frac{\mathrm{d}^2\mathbf{l}_T}{(2\pi)^2}\gamma^\mu\frac{\not{l}+m}{l^2-m^2}\gamma^z\frac{\not{l}+m}{l^2-m^2}\gamma^\nu\frac{1}{(P-l)^2}u(P)\\
&\left[\bar{g}^{\mu\nu}-\frac{n^\nu\left(P^\mu-l^\mu\right)+n^\mu\left(P^\nu-l^\nu\right)}{n\cdot(P-l)}+\frac{n^2\left(P^\mu-l^\mu\right)\left(P^\nu-l^\nu\right)}{\left(n\cdot P-n\cdot l\right)^2}\right]\Bigg|_{l^z=xP^z}
\end{aligned} \tag{42}$$

We consider the numerator as a first step

$$\bar{u}(P)\gamma^\mu(\not{l}+m)\gamma^z(\not{l}+m)\gamma^\nu\left[(P-l)^2\tilde{D}^A_{G\mu\nu}(P-l)\right]u(P) \tag{43}$$

We can separate the gluon propagator into there parts. The first one gives a metric tensor and the final result

$$4l^3\left(m\bar{u}(P)u(P)-\bar{u}(P)\not{l}u(P)\right)-2\left(m^2-l^2\right)\bar{u}(P)\gamma^3 u(P) \tag{44}$$

The combined result can be further separated with respect to the structure of gamma matrices. The first one is for $\bar{u}(P)\not{l}u(P)$:

$$\begin{aligned}
&\frac{2l^z\left(2l^z\left(P^z-l^z\right)-l^2+m^2\right)}{\left(l^2-m^2\right)^2(P-l)^2\left(l^z-P^z\right)^2}\\
=&-\frac{4(l^z)^2}{\left(l^2-m^2\right)^2(P-l)^2\left(l^z-P^z\right)}-\frac{2l^z}{\left(l^2-m^2\right)(P-l)^2\left(l^z-P^z\right)^2}
\end{aligned}$$

for $\bar{u}(P)u(p)$:

$$\begin{aligned}
&\frac{2ml^z\left(-6l^zP^z+4\left(l^z\right)^2+2\left(P^z\right)^2+l^2-m^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{2ml^z\left(-4l^zP^z+4\left(l^z\right)^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}+\frac{2ml^z\left(-2l^zP^z+2\left(P^z\right)^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}+\frac{2ml^z\left(l^2-m^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{8m\left(l^z\right)^2\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}-\frac{4ml^zP^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}+\frac{2ml^z\left(l^2-m^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{8m\left(l^z\right)^2}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}-\frac{4ml^zP^z}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{2ml^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{4m\left(l^z\right)^2}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{4m\left(l^z\right)^2-4ml^zP^z}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{2ml^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{4m\left(l^z\right)^2}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{4ml^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{2ml^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{4m\left(l^z\right)^2}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}+\frac{4ml^z}{\left(l^2-m^2\right)^2(l-P)^2}+\frac{2ml^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2}
\end{aligned}$$

for $\bar{u}(P)\gamma^z u(p)$:

$$\begin{aligned}
&\frac{(l-P)^2\left(2l^z\left(P^z-l^z\right)-l^2+m^2\right)+2\left(m^2-l^2\right)P^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{(l-P)^2\left(2l^z\left(P^z-l^z\right)-l^2+m^2\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}+\frac{2\left(m^2-l^2\right)P^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)^2}\\
=&\frac{2l^z\left(P^z-l^z\right)-l^2+m^2}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)^2}-\frac{2P^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)}\\
=&\frac{2l^z\left(P^z-l^z\right)}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)^2}-\frac{l^2-m^2}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)^2}-\frac{2P^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)}\\
=&-\frac{2l^z}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)}-\frac{1}{\left(l^2-m^2\right)\left(l^z-P^z\right)^2}-\frac{2P^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)}
\end{aligned}$$

The total result is

$$\begin{aligned}
&\bar{u}(P)\Bigg\{-\frac{4(l^z)^2 \not{l}}{\left(l^2-m^2\right)^2 (l-P)^2 \left(l^z-P^z\right)}-\frac{2l^z \not{l}}{\left(l^2-m^2\right)(l-P)^2 \left(l^z-P^z\right)^2} \\
&\quad+\frac{4m\left(l^z\right)^2}{\left(l^2-m^2\right)^2 (l-P)^2 \left(l^z-P^z\right)}+\frac{4ml^z}{\left(l^2-m^2\right)^2 (l-P)^2}+\frac{2ml^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2} \\
&\quad-\frac{2l^z\gamma^z}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)}-\frac{\gamma^z}{\left(l^2-m^2\right)\left(l^z-P^z\right)^2}-\frac{2P^z\gamma^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)}\Bigg\}u(P)
\end{aligned} \tag{45}$$

$$\begin{aligned}
&=\bar{u}(P)\Bigg\{\frac{-4\left(l^z\right)^2\left(\not{l}-m\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}-\frac{2l^z\left(\not{l}-m\right)}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2}+\frac{4ml^z}{\left(l^2-m^2\right)^2(l-P)^2} \\
&\quad-\frac{2l^z\gamma^z}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)}-\frac{\gamma^z}{\left(l^2-m^2\right)\left(l^z-P^z\right)^2}-\frac{2P^z\gamma^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)}\Bigg\}u(P)
\end{aligned} \tag{46}$$

$$\begin{aligned}
&=\bar{u}(P)\Bigg\{\frac{-4\left(l^z\right)^2\left(\not{l}-m\right)+4ml^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}-\frac{2l^z\left(\not{l}-m\right)+2P^z\gamma^z\left(l^z-P^z\right)}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2} \\
&\quad-\frac{2l^z\gamma^z}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)}-\frac{\gamma^z}{\left(l^2-m^2\right)\left(l^z-P^z\right)^2}\Bigg\}u(P)
\end{aligned} \tag{47}$$

$$\begin{aligned}
&=\bar{u}(P)\Bigg\{-\frac{4\left(l^z\right)^2\left(\not{l}-2m\right)+4ml^zP^z}{\left(l^2-m^2\right)^2(l-P)^2\left(l^z-P^z\right)}-\frac{2l^z\left(\not{l}-m+P^z\gamma^z\right)-2(P^z)^2\gamma^z}{\left(l^2-m^2\right)(l-P)^2\left(l^z-P^z\right)^2} \\
&\quad-\frac{2l^z\gamma^z}{\left(l^2-m^2\right)^2\left(l^z-P^z\right)}-\frac{\gamma^z}{\left(l^2-m^2\right)\left(l^z-P^z\right)^2}\Bigg\}u(P)
\end{aligned} \tag{48}$$

Xiong's result is

$$\begin{aligned}
&-ig_s^2C_F\int\frac{d^4k}{(2\pi)^4}\bar{u}(P)\Bigg[\frac{2\gamma^z}{\left(k^2-m^2\right)(P-k)^2}+\frac{4\left(2m-k^z\right)\not{k}}{\left(k^2-m^2\right)^2(P-k)^2} \\
&+\frac{2\left(k^z\gamma^z+\not{k}-m\right)}{\left(k^2-m^2\right)(P-k)^2\left(P^z-k^z\right)}-\frac{\gamma^z}{(P-k)^2\left(P^z-k^z\right)^2}\Bigg]P^z\delta\left(k^z-xP^z\right)u(P)
\end{aligned} \tag{49}$$

As we discussed earlier, it can be dissected into

$$\frac{4\left(2m-k^z\right)\not{k}}{\left(k^2-m^2\right)^2(P-k)^2}+\frac{2\not{k}}{\left(k^2-m^2\right)(P-k)^2\left(P^z-k^z\right)} \qquad \bar{u}(P)\not{k}u(P) \tag{50}$$

$$\frac{-2m}{\left(k^2-m^2\right)(P-k)^2\left(P^z-k^z\right)} \qquad \bar{u}(P)u(P) \tag{51}$$

$$\frac{2\gamma^z}{\left(k^2-m^2\right)(P-k)^2}+\frac{2k^z\gamma^z}{\left(k^2-m^2\right)(P-k)^2\left(P^z-k^z\right)}-\frac{\gamma^z}{(P-k)^2\left(P^z-k^z\right)^2} \qquad \bar{u}(P)\gamma^zu(P) \tag{52}$$

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