Local Operator Divergence

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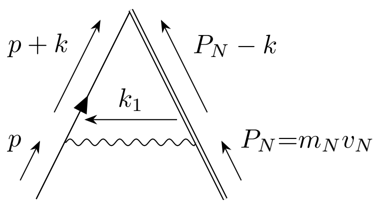
December 10, 2017

NRQED matrix element at NLO

$$\langle 0|\psi_e(0)N(0)(-ie)\int \mathrm{d}^4y\bar{\psi}_e\psi_e A^0(-ie)\int \mathrm{d}^4z\bar{N}NA^0|eN\rangle =$$



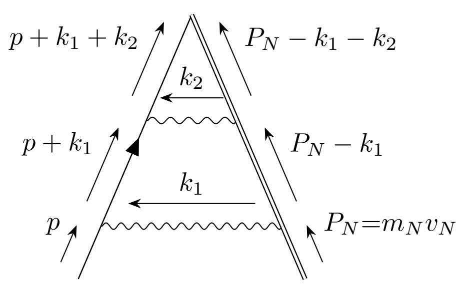
$$= ie^2u_N(v_N)\int[\mathrm{d}k]\frac{1}{\mathbf{k}^2(-k^0+i\epsilon)(^0+k^0-m-\frac{(\mathbf{p}+\mathbf{k})^2}{2m}+\frac{(\mathbf{p}+\mathbf{k})^4}{8m^3}+i\epsilon)}\psi(p)$$

$$= e^2u_N(v_N)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{(\mathbf{k}-\mathbf{p})^2(E-\frac{\mathbf{k}^2}{2m}+\frac{\mathbf{k}^4}{8m^3})}\psi(p)$$

$$= e^2u_N(v_N)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{(\mathbf{k}-\mathbf{p})^2(E-\frac{\mathbf{k}^2}{2m})}(1-\frac{\mathbf{k}^4}{8m^3(E-\frac{\mathbf{k}^2}{2m})})\psi(p)$$

$$= e^2u_N(v_N)\psi(p)[\frac{\pi}{2p}] = e^2u_N(v_N)\psi(p)[\frac{\pi}{2mv}]$$

At NNLO (where we're only interested in divergent parts)

$-$ 

$$=e^4\int[dk_1][dk_2]\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}|^2}\frac{1}{-k_1^0-k_2^0+i\epsilon}\frac{1}{-k_1^0+i\epsilon}\frac{1}{p^0+k_1^0-m-\frac{(\mathbf{p}+\mathbf{k_1})^2}{2m}+i\epsilon}\frac{1}{p^0+k_1^0+k_2^0-m-\frac{(\mathbf{p}+\mathbf{k_1}+\mathbf{k_2})^2}{2m}+i\epsilon}\psi_e(p)u_N(v_N)$$

do the shift as above

$$=-e^4\int\frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}-\mathbf{p}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{1}{E-\frac{|\mathbf{k_1}|^2}{2m}+2i\epsilon}\frac{1}{E-\frac{|\mathbf{k_2}|^2}{2m}+2i\epsilon}\psi_e(p)u_N(v_N)$$

drop $\mathbf{p}$

$$=-e^4\int\frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{1}{-\frac{|\mathbf{k_1}|^2}{2m}+2i\epsilon}\frac{1}{-\frac{|\mathbf{k_2}|^2}{2m}+2i\epsilon}\psi_e(p)u_N(v_N)$$

if we add higher reltivistic correction

$$=-e^4\int\frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{2m}{|\mathbf{k_1}|^2-\frac{|\mathbf{k_1}|^4}{4m^2}}\frac{2m}{|\mathbf{k_2}|^2-\frac{|\mathbf{k_2}|^4}{4m^2}}\psi_e(p)u_N(v_N)$$

$$=-4m^2e^4\int\frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_1}|^2}(1+\frac{|\mathbf{k_1}|^{\mathbf{2}}}{4m^2})\frac{1}{|\mathbf{k_2}|^2}(1+\frac{|\mathbf{k_2}|^{\mathbf{2}}}{4m^2})\psi_e(p)u_N(v_N)$$

The integral

$$
\begin{aligned}
&\int \frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_1}|^2}(1+\frac{|\mathbf{k_1}|^{\mathbf{2}}}{4m^2})\frac{1}{|\mathbf{k_2}|^2}(1+\frac{|\mathbf{k_2}|^{\mathbf{2}}}{4m^2})\\
=&\int \frac{\mathrm{d}^3\mathbf{k_1}}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{k_2}}{(2\pi)^3}\frac{1}{|\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_2}-\mathbf{k_1}|^2}\frac{1}{|\mathbf{k_1}|^2}(1+\frac{|\mathbf{k_1}|^{\mathbf{2}}}{4m^2})\frac{1}{|\mathbf{k_2}|^2}(1+\frac{|\mathbf{k_2}|^{\mathbf{2}}}{4m^2})
\end{aligned}
$$

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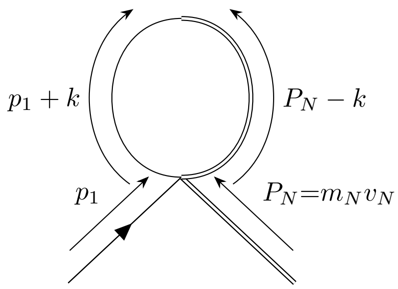

$$
= \frac{4\pi e^4}{3m}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{1}{p_1^0 + k^0 - m - \frac{(\mathbf{p_1}+\mathbf{k})^2}{2m} + i\epsilon}\frac{1}{-k^0 + i\epsilon}\psi_e(p_1)u_N(v_N)
$$

$$
= i\frac{4\pi e^4}{3m}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{1}{p_1^0 - m - \frac{(\mathbf{p_1}+\mathbf{k})^2}{2m} + 2i\epsilon}\psi_e(p_1)u_N(v_N)
$$

drop $\mathbf{p_1}$

$$
= i\frac{4\pi e^4}{3m}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{1}{-\frac{\mathbf{k}^2}{2m} + 2i\epsilon}\psi_e(p_1)u_N(v_N)
$$

if the dispersion relation is up to $\mathbf{k}^4$ then

$$
= \frac{4e^4 m}{3}
$$