level	Klein-Gordon eq	1st-order perturbation	error	Numerical results	error
1S	-0.0506385	-0.050625	0.000266002	-0.0501571	0.0095065
2S	-0.0126034	-0.0126016	0.000143573	-0.0125255	0.00617853
3S	-0.00558847	-0.00558796	0.0000911318	-0.00556369	0.00443476
4S	-0.00313937	-0.00313916	0.0000654067	-0.00312855	0.00344427
5S	-0.0020075	-0.0020074	0.0000505745	-0.00200186	0.00281209
6S	-0.00139329	-0.00139323	0.0000410503	-0.00138998	0.00237488
7S	-0.0010232	-0.00102317	0.0000344628	-0.00102110	0.00205486
8S	-0.000783135	-0.000783112	0.0000296546	-0.000781717	0.00181064
9S	-0.000618615	-0.000618599	0.0000260001	-0.000617614	0.00161819
10S	-0.000500974	-0.000500962	0.0000231334	-0.000500241	0.00146265
11S	-0.000413957	-0.000413949	0.0000208272	-0.000413405	0.00133436
12S	-0.000347789	-0.000347783	0.0000189334	-0.000347363	0.00122673
13S	-0.000296305	-0.0002963	0.0000173514	-0.000295969	0.00113515
14S	-0.000255461	-0.000255457	0.0000160108	-0.000255191	0.00105629
15S	-0.000222514	-0.000222511	0.0000148605	-0.000222295	0.000987666
16S	-0.000195554	-0.000195551	0.0000138631	-0.000195372	0.000927384
17S	-0.000173212	-0.000173209	0.0000129901	-0.000173061	0.000871789
18S	-0.000154491	-0.000154489	0.0000122198	-0.000154370	0.000782066
19S	-0.000138649	-0.000138647	0.0000115351	-0.000138515	0.000963144
20S	-0.000125124	-0.000125123	0.0000109227	-0.000125031	0.000745458

$K-G$	per	$error$
-0.0506385	-0.050625	0.000266002
-0.0126034	-0.0126016	0.000143573
-0.00558847	-0.00558796	0.0000911318
-0.00313937	-0.00313916	0.0000654067
-0.0020075	-0.0020074	0.0000505745
-0.00139329	-0.00139323	0.0000410503
-0.0010232	-0.00102317	0.0000344628
-0.000783135	-0.000783112	0.0000296546
-0.000618615	-0.000618599	0.0000260001
-0.000500974	-0.000500962	0.0000231334
-0.000413957	-0.000413949	0.0000208272
-0.000347789	-0.000347783	0.0000189334
-0.000296305	-0.0002963	0.0000173514
-0.000255461	-0.000255457	0.0000160108
-0.000222514	-0.000222511	0.0000148605
-0.000195554	-0.000195551	0.0000138631
-0.000173212	-0.000173209	0.0000129901
-0.000154491	-0.000154489	0.0000122198
-0.000138649	-0.000138647	0.0000115351
-0.000125124	-0.000125123	0.0000109227

$$\nabla^2\psi_{true}(r) = \overline{\gamma}(r)\int d^3r\,\psi_{eff}\delta_a^3(\mathbf{r}) + \overline{\gamma'}(r)\int d^3r\,\nabla^2\psi_{eff}\delta_a^3(\mathbf{r}) + \overline{\eta}(r)a^2\int d^3r\,\psi_{eff}\nabla^2\delta_a^3(\mathbf{r})$$
$$+ \overline{\eta'}(r)a^2\int d^3r\,\nabla^2\psi_{eff}\nabla^2\delta_a^3(\mathbf{r}) + \mathcal{O}(a^3) \tag{1}$$