Two Loop Matching for Quasi PDF

Yingsheng Huang

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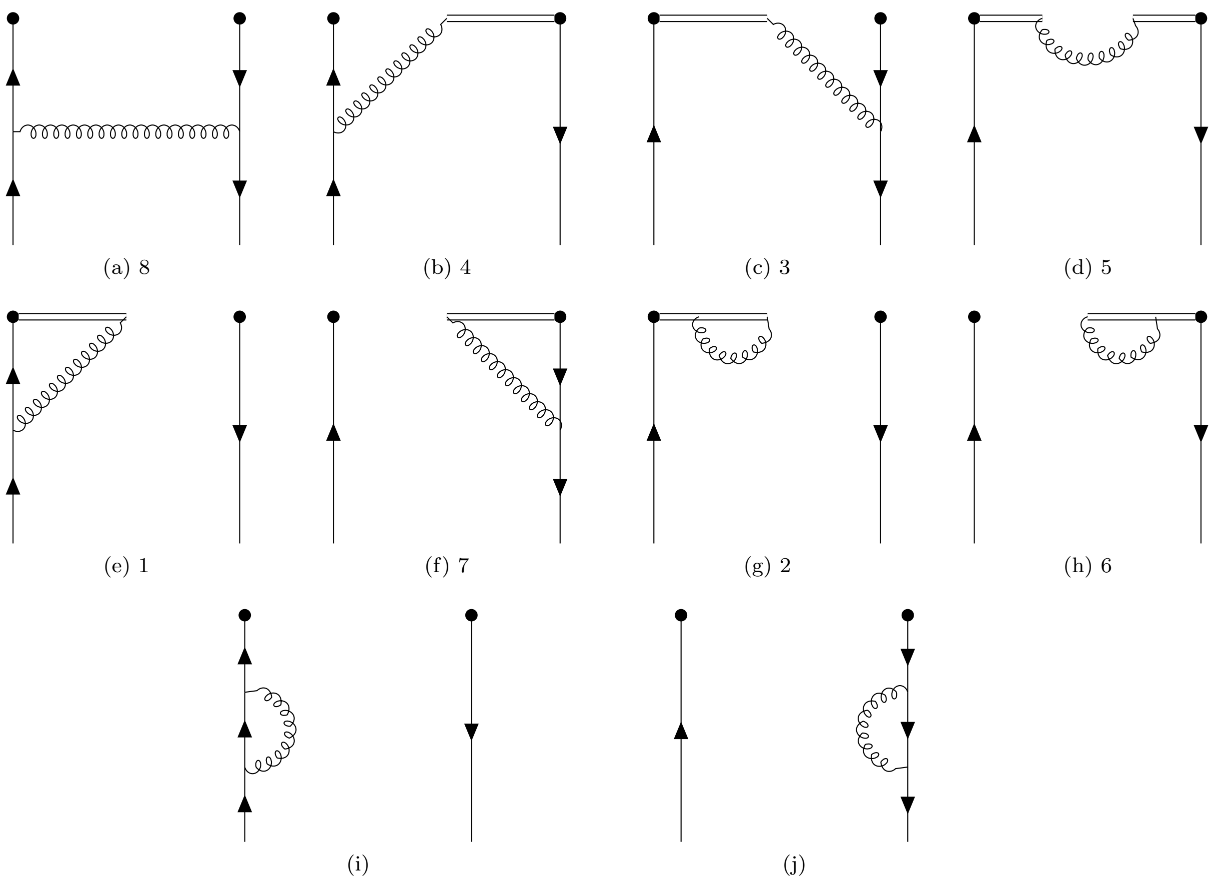
1 Renormalization

1.1 One loop diagrams



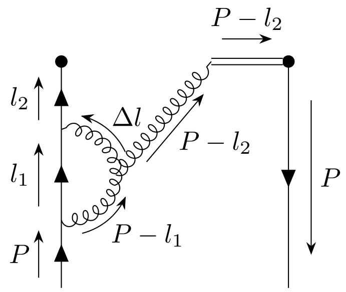
(a) 8 (b) 4 (c) 3 (d) 5
(e) 1 (f) 7 (g) 2 (h) 6
(i) (j)

Figure 1: Diagrams of quasi PDF in Feynman gauge.

1.2 Vertex corrections

According to [Ji and Zhang(2015)], the vertex correction diagrams in axial gauge (which corresponds to varieties of diagrams in general covariant gauge) don't have total UV divergence. Rather, they only have subdivergence for subdiagrams. For example the first column (which involves Figure 3), second row of Table 1 in [Ji and Zhang(2015)] is composed of $\tilde{q}_{11}$ and $\tilde{q}_{12}$, thus we can find some representative diagrams and extract those components ($l \equiv l_1 + l_2, \Delta l \equiv$

$l_1 - l_2$)



$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{1}{\left[\not{l}_1 - m\right]\left[\not{l}_2 - m\right]\left[(P-l_1)^2\right]\left[(l_1-l_2)^2\right]\left[(P-l_2)^2\right][n\cdot(P-l_2)]} \tag{1}$$

Take the $l_1 \gg l_2$ limit, the integrand becomes

$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[l_1^2\right]\left[\not{l}_2 - m\right]\left[(P-l_2)^2\right][n\cdot(P-l_2)]} \tag{2}$$

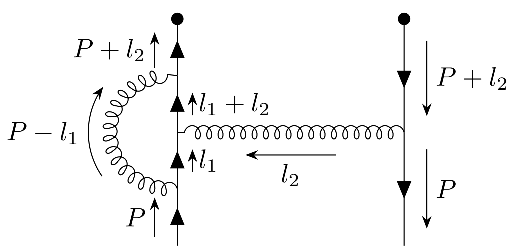
The integral involving l_2 is exactly the integral of $\tilde{q}_{12}$. By adding the gluon self-interacting vertex we can see that the sub-diagram is logarithmic divergent.

Take the $l_2 \gg l_1$ limit, the integrand becomes

$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[l_2^2\right]\left[\not{l}_2 - m\right]\left[(P-l_2)^2\right][n\cdot(P-l_2)]} \tag{3}$$

There's another limit where hard loop momentum flows through all paths except the one that's Δl in our current diagram. This configuration gives a finite integral and a power-divergent integral which happens to be a scaleless integral as well. Thus this configuration won't contribute.

What we extracted above is only the $\tilde{q}_{12}$ part, now we will try on the $\tilde{q}_{11}$ part



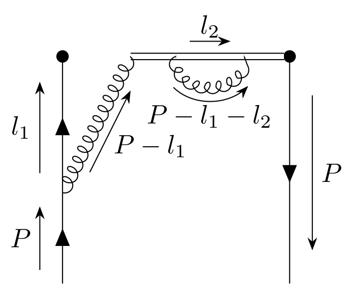
$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{1}{\left[\not{l}_1 - m\right]\left[\not{l}_1 + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[(P-l_1)^2\right]\left[l_2^2\right]} \tag{4}$$

In the $l_1 \gg l_2$ limit we have

$$\frac{1}{\left[\not{l}_1 - m\right]\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[\not{P} + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[l_2^2\right]} \tag{5}$$

and $\tilde{q}_{11}$ is factorized out.

Another example is the sixth row



$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[(P-l_1-l_2)^2\right][n\cdot(P-l_1)][n\cdot l_2][n\cdot(P-l_1)]} \tag{6}$$

Take the $l_2 \gg l_1$ limit, the integrand becomes

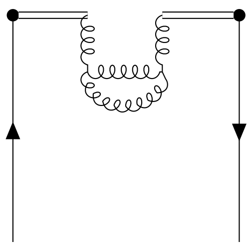
$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right][n\cdot(P-l_1)][n\cdot(P-l_1)][n\cdot l_2]\left[(P-l_2)^2\right]} \tag{7}$$

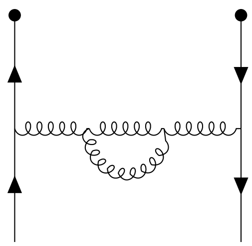
and the integral involving l_2 should give something proportional to $n\cdot(P-l_1)$, thus cancels one eikonal propagator, the remainder is the integral of $\tilde{q}_{12}$.

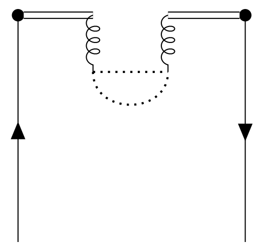
2 Real Diagrams

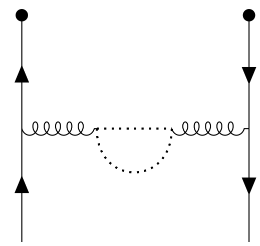
2.1 All diagrams

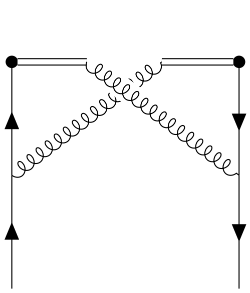
Figure 2 lists all self-conjugated real diagrams, and Figure 3 lists all non-self-conjugated diagrams, excluding their conjugates.

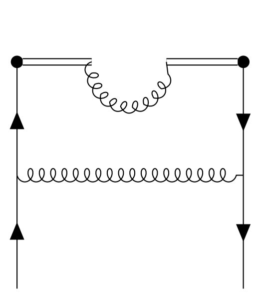

(i) 10

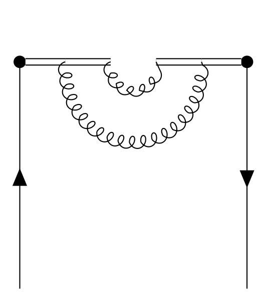

(ii) 11

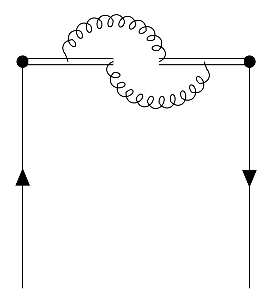

(iii) 26

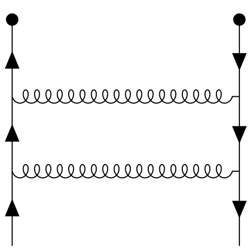

(iv) 27

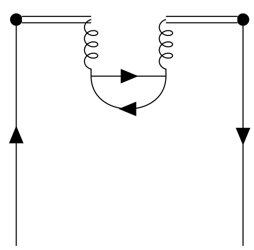

(v) 52

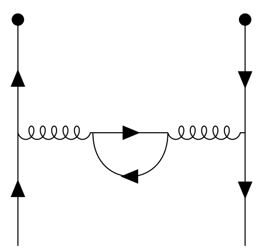

(vi) 71

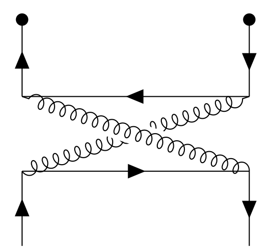

(vii) 87

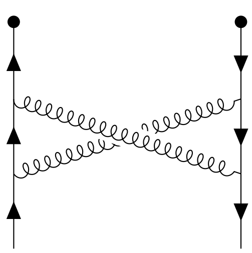

(viii) 100

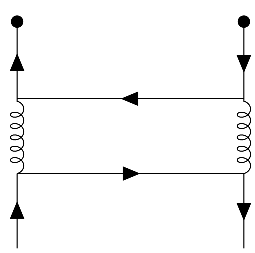

(ix) 116

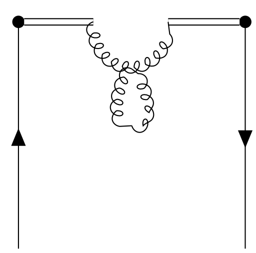

(x) 124


(xi) 125


(xii) 128


(xiii) 129


(xiv) 133


(xv) 143

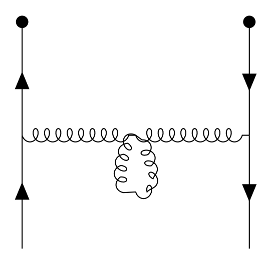
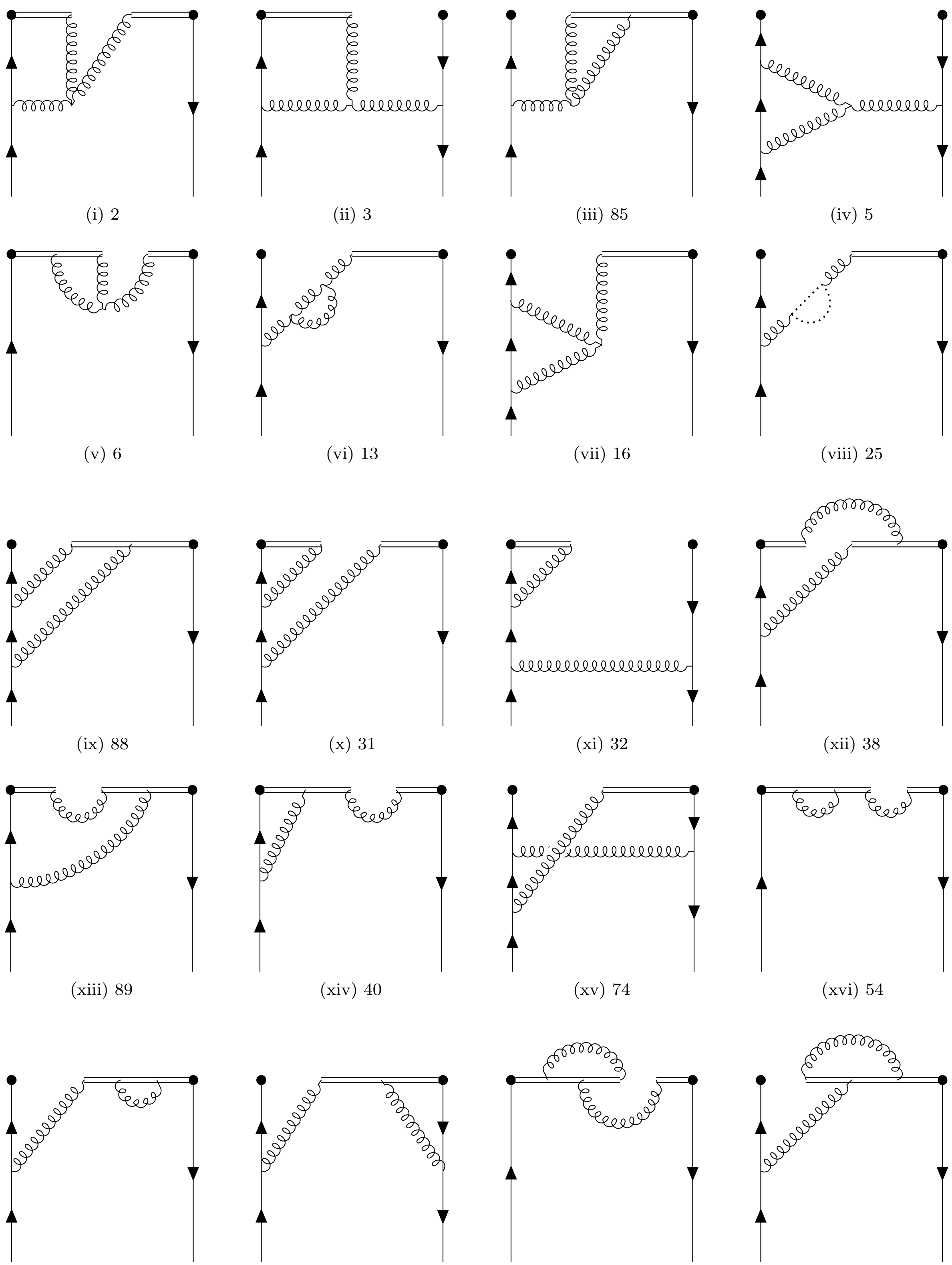

(xvi) 144

Figure 2: All self-conjugated diagrams.



(i) 2 (ii) 3 (iii) 85 (iv) 5

(v) 6 (vi) 13 (vii) 16 (viii) 25

(ix) 88 (x) 31 (xi) 32 (xii) 38

(xiii) 89 (xiv) 40 (xv) 74 (xvi) 54

(xvii) 81 (xviii) 84 (xix) 58 (xx) 95

Figure 3: All real diagrams (excluding conjugated diagrams).

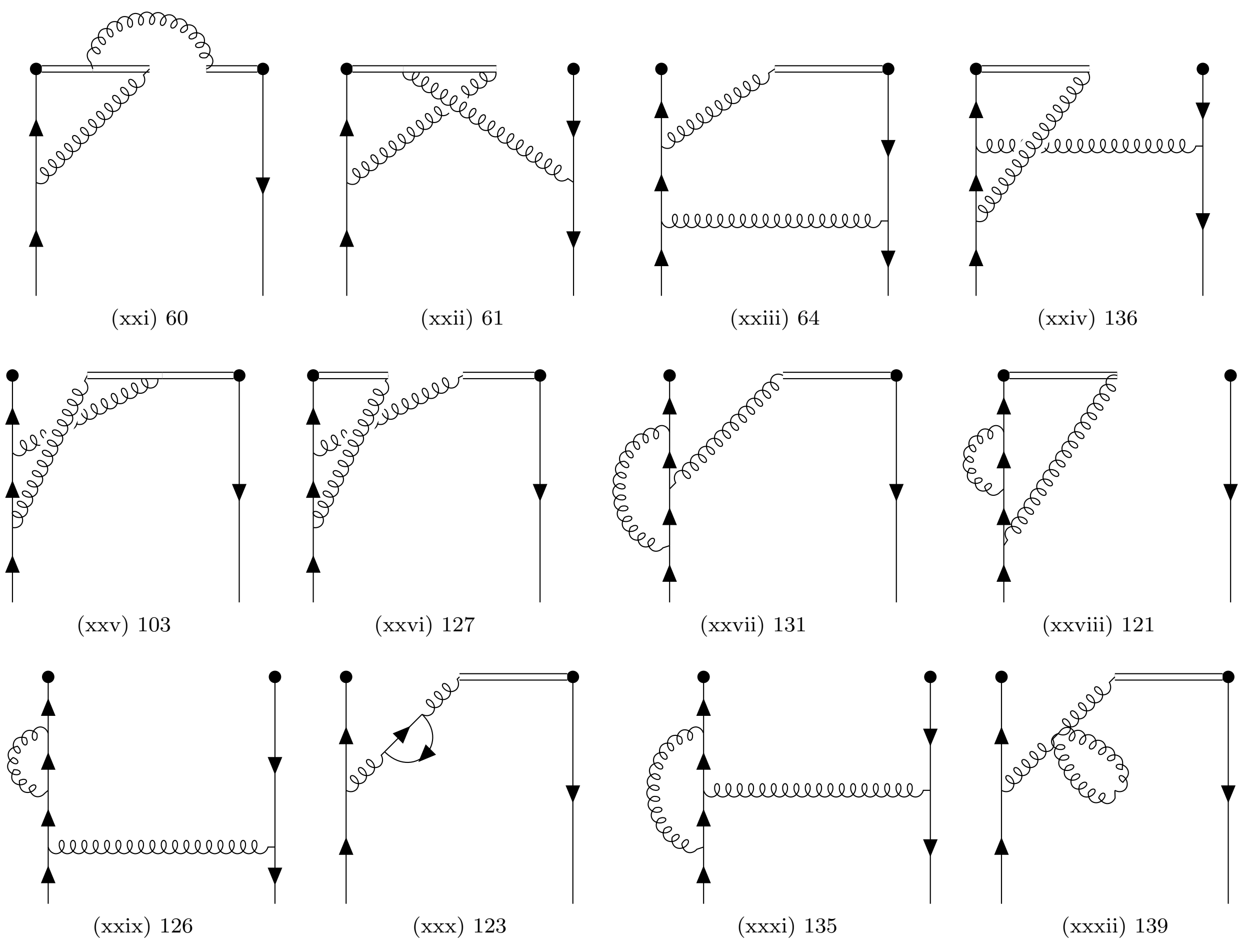


Figure 3: All real diagrams (excluding conjugated and self-conjugated diagrams).

2.2 Amplitude test

First we take diagram 2xv to test if the type of diagrams that is a sub-diagram involving only QCD Feynman rules on top of one loop diagram consist with our manual input.

The program gives

$$\Big(\delta_{\mathrm{CI}(9)\,\mathrm{CI}(10)}\,\delta_{\mathrm{CI}(11)\,\mathrm{CI}(12)}\,\delta_{\mathrm{CI}(13)\,\mathrm{CI}(14)}\,g^{\mathrm{LI}(9)\,\mathrm{LI}(10)}\,g^{\mathrm{LI}(11)\,\mathrm{LI}(12)}\,g^{\mathrm{LI}(13)\,\mathrm{LI}(14)}\,g_s^4\,\mathrm{MomC}(-\mathrm{k1})\,\mathrm{n1}^{\mathrm{LI}(9)}\,\mathrm{n2}^{\mathrm{LI}(11)}\,\mathrm{ColorLine}\big(T_{\mathrm{CI}(11)}.T_{\mathrm{CI}(9)},\{p,\,p\}\big)$$
$$\Big(\big(g^{\mathrm{LI}(10)\,\mathrm{LI}(13)}\,g^{\mathrm{LI}(12)\,\mathrm{LI}(14)}-g^{\mathrm{LI}(10)\,\mathrm{LI}(14)}\,g^{\mathrm{LI}(12)\,\mathrm{LI}(13)}\big)\,f_{\mathrm{e\$19}\,\mathrm{CI}(13)\,\mathrm{CI}(14)}\,f_{\mathrm{CI}(10)\,\mathrm{CI}(12)\,\mathrm{e\$19}}+\big(g^{\mathrm{LI}(10)\,\mathrm{LI}(12)}\,g^{\mathrm{LI}(13)\,\mathrm{LI}(14)}-g^{\mathrm{LI}(10)\,\mathrm{LI}(14)}\,g^{\mathrm{LI}(13)\,\mathrm{LI}(12)}\big)$$
$$f_{\mathrm{e\$20}\,\mathrm{CI}(12)\,\mathrm{CI}(14)}\,f_{\mathrm{CI}(10)\,\mathrm{CI}(13)\,\mathrm{e\$20}}+\big(g^{\mathrm{LI}(10)\,\mathrm{LI}(12)}\,g^{\mathrm{LI}(14)\,\mathrm{LI}(13)}-g^{\mathrm{LI}(10)\,\mathrm{LI}(13)}\,g^{\mathrm{LI}(14)\,\mathrm{LI}(12)}\big)\,f_{\mathrm{e\$21}\,\mathrm{CI}(12)\,\mathrm{CI}(13)}\,f_{\mathrm{CI}(10)\,\mathrm{CI}(14)\,\mathrm{e\$21}}\Big)$$
$$\mathrm{SpinLine}(\gamma\cdot n,\,\{p,\,p\})\Big)\Big/\big(2\,\mathrm{k2}^2\,(-p-\mathrm{pe})^2\,(\mathrm{k1}+p+\mathrm{pe})^2\,\mathrm{n1}\cdot(p+\mathrm{pe})\,\mathrm{n2}\cdot(p+\mathrm{pe})\big)$$

which translates to

$$\begin{aligned}&g_s^4\delta(-k_1)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar u(P)\not n u(P)}{2k_2^2(-p-p_e)^2(k_1+p+p_e)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)}\\&\big[\big(g^{l10l13}g^{l12l14}-g^{l10l14}g^{l12l13}\big)f^{e19c13c14}f^{c10c12e19}+\big(g^{l10l12}g^{l13l14}-g^{l10l14}g^{l13l12}\big)f^{e20c12c14}f^{c10c13e20}\\&+\big(g^{l10l12}g^{l14l13}-g^{l10l13}g^{l14l12}\big)f^{e21c12c13}f^{c10c14e21}\big]\big|_{p_e=-xP^z\to-p,p=P,n\to z,k_2=l_2,n_1=n_2=n}\end{aligned}$$

Taking $k_1=p+p_e-l_1$, the first line (that's excluding the four-gluon vertex) becomes

$$g_s^4\delta(-k_1)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar u(P)\not n u(P)}{2k_2^2(-p-p_e)^2(k_1+p+p_e)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)} \tag{8}$$

$$=g_s^4\delta(l_1-p-p_e)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar u(P)\not n u(P)}{2l_2^2(-p-p_e)^2(2p+2p_e-l_1)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)} \tag{9}$$

$$=g_s^4\delta(l_1-p-p_e)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar u(P)\not n u(P)}{2l_2^2l_1^2l_1^2n_1\cdot l_1n_2\cdot l_1} \tag{10}$$

Diagram 2xv gives

$$\begin{aligned}&\frac{-ig_s^4}{2}\bar u(P)\not n u(P)\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{\mathrm{d}^4l_2}{(2\pi)^4}n_\tau t^i\tilde D_G^{\tau\mu,ia}(l_1)\tilde D_G^{\sigma\lambda,dj}(l_1)\tilde D_G^{\nu\rho,bc}(l_2)n_\lambda t^j\frac{i}{n\cdot l_1+i\epsilon}\frac{i}{-n\cdot l_1+i\epsilon}\delta(l_1^z-(1-x)P^z)\\&\big[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\big]\end{aligned} \tag{11}$$

$$\begin{aligned}=&\frac{\overbrace{(-1)^3i^6}^{1}g_s^4}{2}\bar u(P)\not n u(P)\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{\mathrm{d}^4l_2}{(2\pi)^4}\frac{n^\mu n^\sigma g^{\nu\rho}t^i\delta^{ia}t^j\delta^{dj}}{\left[l_1^2\right]^2\left[l_2^2\right][n\cdot l_1]^2}\delta(l_1^z-(1-x)P^z)\\&\big[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\big]\end{aligned} \tag{12}$$

Let's compare the color indices:

2.3 Numerical test (ordered as Figure 2 and Figure 3, $z=1/4$)

2.3.1 Self-conjugated

10

```
(0.135095 -1.68869/s)/ep-(13.0007 -5.30516 I)/s+(0.991423 -0.424413 I)+(1.68869
↪ log(s))/s-0.135095 log(s)
```

11

```
-((2.71097 -1.0865 I)/s^3)+(13.6036 -5.53435 I)/s^2+(0.351595 -0.203718
↪ I)/s+(-(0.345843/s^3)+1.76164/s^2+0.0648456/s)/ep+(0.345843 log(s))/s^3-(1.76164
↪ log(s))/s^2-(0.0648456 log(s))/s-7.57481*10^-7
```

26

```
(0.991423 -0.424413 I)/s-(0.563259 -0.212207 I)+(0.135095/s-0.0675475)/ep-(0.135095
↪ log(s))/s+0.0675475 log(s)
```

27

```
-((1.24375 -0.509296 I)/s^2)-(0.24875 -0.101859
↪ I)/s+(-(0.162114/s^2)-0.0324228/s)/ep+(0.162114 log(s))/s^2+(0.0324228 log(s))/s-3.8192*10^-7
```

52

71

87

100

```
-0.180127
```

116

125

```
(0.194537 CV(1,3))/(ep s)+(0.885127 CV(1,3))/s-(0.194537 CV(1,3) log(s))/s-4.09664*10^-15
↪ CV(1,3)
```

128

133

143

```
0
```

144

```
0
```

2.3.2 Half of the remainder

2

3

```
-((2.67042 +2.12028 I)/s^2)-((0.260912 -1.01859 I) log(s))/s^2+(14.6734 +1.31932
↪ I)/s-((0.324228 +3.72868*10^-9 I) log^2(s))/s+(0.222907 -5.53015*10^-9 I) log^2(s)+((0.881442
↪ -3.43775 I) log(s))/s+(0.673891 -1.68704 I) log(s)+(13.4354 -26.5232
↪ I)+(-((4.70467*10^-9+1.04672*10^-9 I)/s^2)-(1.49279*10^-8+4.88207*10^-9
↪ I)/s+(-3.02109*10^-8-6.73934*10^-9 I))/ep^2+(1/ep)(-((2.61261*10^-8+1.68009*10^-8
↪ I)/s^2)-((4.70467*10^-9+1.04672*10^-9 I) log(s))/s^2-(1.30934*10^-7+9.62535*10^-8
↪ I)/s-((1.49279*10^-8+4.88207*10^-9 I) log(s))/s-(3.02109*10^-8+6.73934*10^-9 I)
↪ log(s)-(3.63837*10^-7+2.4728*10^-7 I))-((3.64128*10^-9+8.50597*10^-10 I) log^2(s))/s^2
```

85

5

6

13

16

```
-((0.211474 +1.27324 I)/s^2)+(2.28897 -1.08225 I)/s-(4.7286 -0.668451
↪ I)+(-(6.7167*10^-16/s^2)+0.486342/s-0.729513)/ep-(3.00378*10^-16 log(s))/s^2-(0.486342
↪ log(s))/s+0.729513 log(s)
```

25

88

31

```
-((0.162114 +1.91875*10^-8 I)/ep^3)+(-0.324228 log(s)-(92.0766 +2.65448 I))/ep^2+1/ep
↪ (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) log(s)-0.0405285 log^2(s)-(1.58061
↪ log(s))/s-(1.91238*10^10+1.98727*10^10 I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46
↪ I)/s-(16.9633 +0.31831 I) log^2(s)-(4.74183 log(s))/s^2+0.148604 log^3(s)+(0.790305
↪ log^2(s))/s+(81.2751 log(s))/s+(4.05197*10^10-6.80923*10^10 I)
↪ log(s)+(6.59315*10^13-1.55809*10^14 I)
```

32

```
-((0.162114 +1.91875*10^-8 I)/ep^3)+(-0.324228 log(s)-(92.0766 +2.65448 I))/ep^2+1/ep
↪ (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) log(s)-0.0405285 log^2(s)-(1.58061
↪ log(s))/s-(1.91238*10^10+1.98727*10^10 I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46
↪ I)/s-(16.9633 +0.31831 I) log^2(s)-(4.74183 log(s))/s^2+0.148604 log^3(s)+(0.790305
↪ log^2(s))/s+(81.2751 log(s))/s+(4.05197*10^10-6.80923*10^10 I)
↪ log(s)+(6.59315*10^13-1.55809*10^14 I)
```

38

89

40

74

54

81

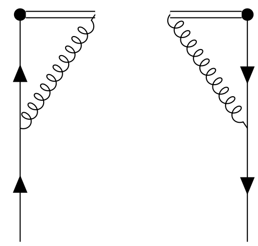
```
((0.180127 -9.13154*10^-9 I) log(s)-(0.231946 +7.6754*10^-8 I))/ep+(1.02169 -0.565884 I)
↪ log(s)-(0.815813 -0.728676 I)-(5.2095*10^-10+1.21468*10^-8 I)/ep^2+(-0.0900633-4.02323*10^-9 I)
↪ log^2(s)
```

84

```
((0.226837 -4.76795*10^-7 I)+(2.53148*10^-36+9.52697*10^-70 I)/s+(-0.0900633-5.77113*10^-8
↪ I) log(s))/ep+(0.27463 +0.141471 I) log^2(s)-(1.20417 +0.36434 I) log(s)+(1.63449 +0.234572
↪ I)-(6.78712*10^-8+5.87408*10^-8 I)/ep^2-(7.40521*10^-8+3.24692*10^-8 I)/s
```

3 Virtual Diagrams (Excluding Gauge Link Self-Energy Diagrams)

3.1 All diagrams



(i) 8

Figure 4: All self-conjugated virtual diagrams (actually there's only one).

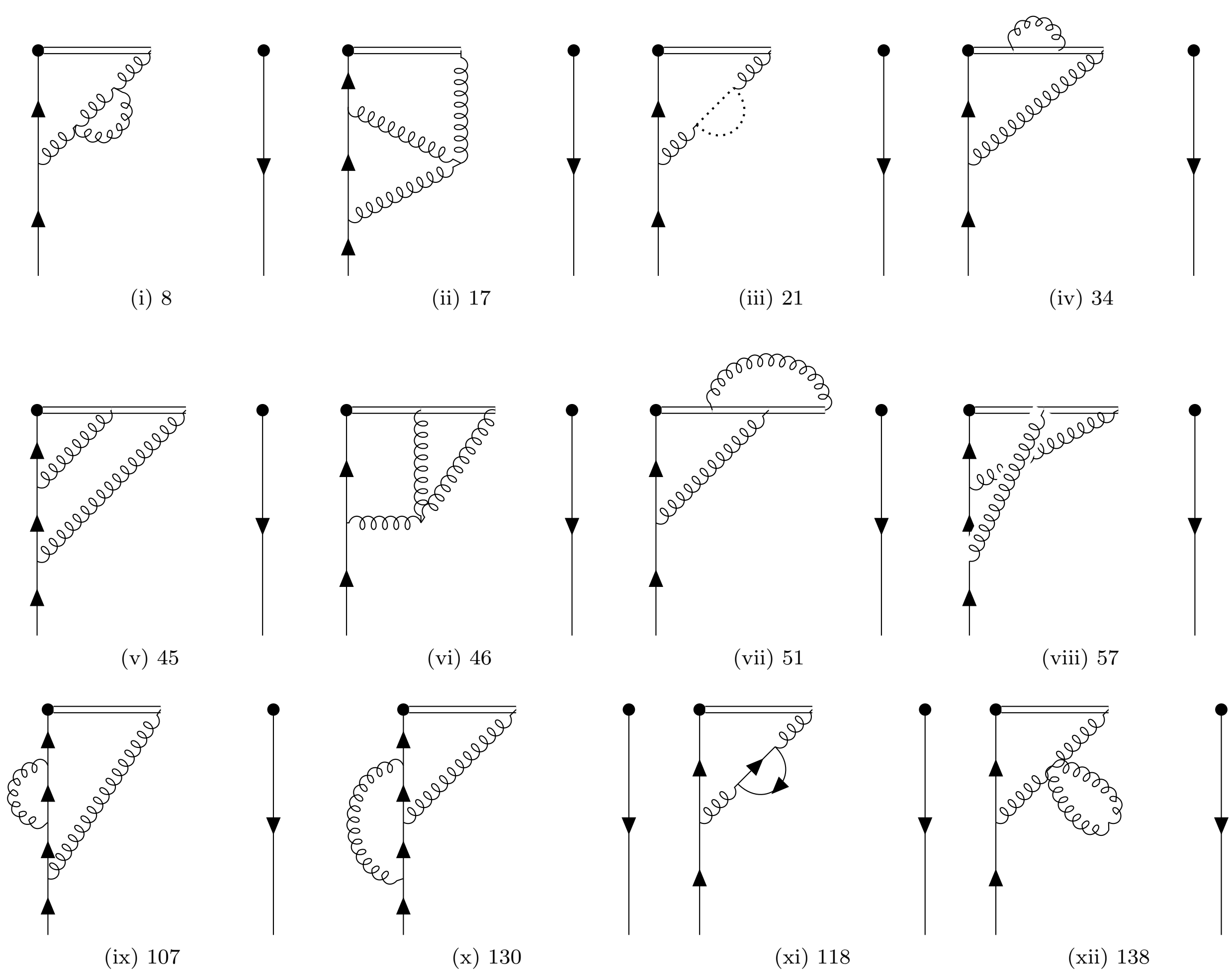
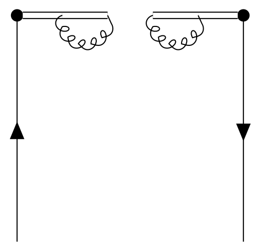


Figure 5: All virtual diagrams (excluding conjugated and self-conjugated diagrams).

4 Gauge Link Self-Energy Diagrams

4.1 All diagrams



(i) 8

Figure 6: All self-conjugated gauge link self-energy diagrams (actually there's only one).

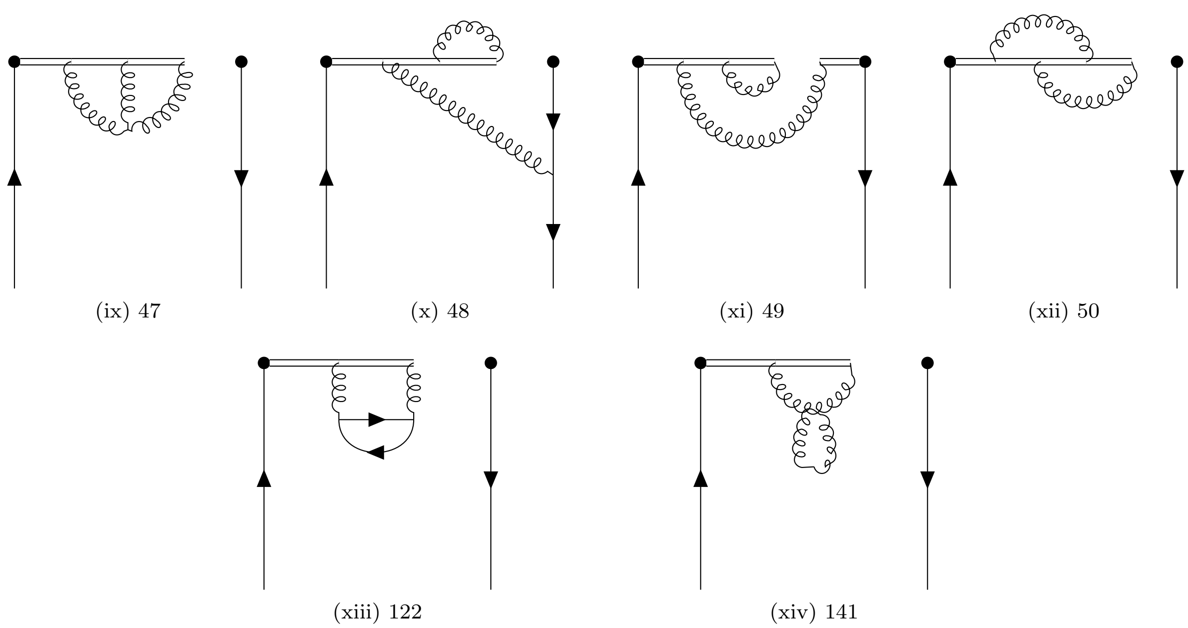
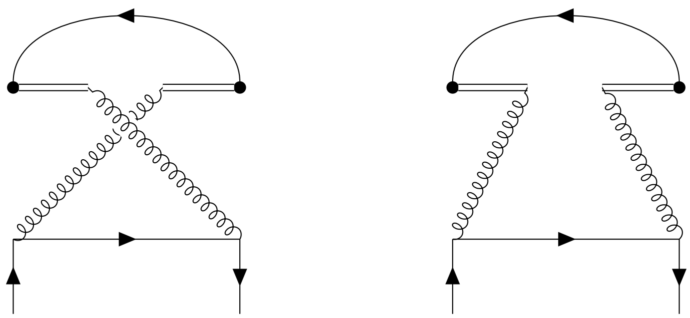


Figure 7: All gauge link self-energy diagrams (excluding conjugated and self-conjugated diagrams).

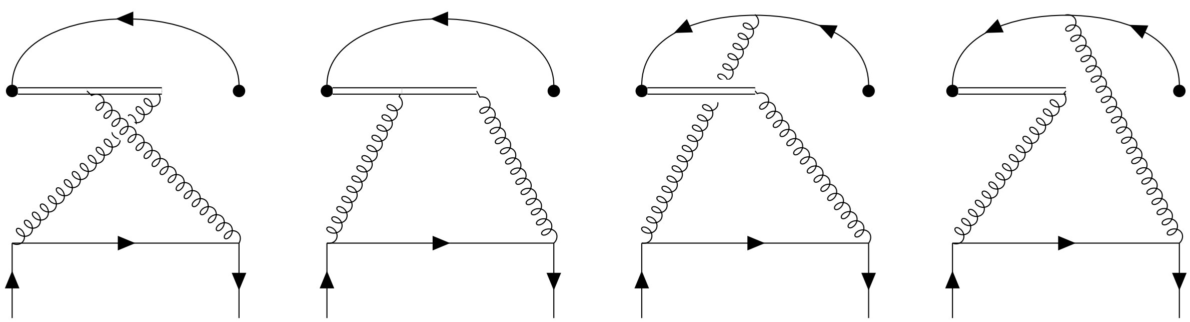
5 Diagrams with Direct Contracting $\bar{\psi}(z)\psi(0)$

5.1 All diagrams



(a) 53 (b) 76

Figure 8: All self-conjugated quark contraction diagrams



(a) 28 (b) 29 (c) 42 (d) 132

Figure 9: All quark contraction diagrams (excluding conjugated and self-conjugated diagrams).

6 HQET Correspondence

References

[Ji and Zhang(2015)] X. Ji and J.-H. Zhang, Phys. Rev. **D92**, 034006 (2015), arXiv:1505.07699 [hep-ph] .