相互作用量子场

物理学研究对象的惯用技俩：孤立对象，认为它与外界的相互作用可以忽略，研究清楚它的运动律，→ 将外界与它的相互作用作为微扰，研究存在这些相互作用时，对象在其不同状态间的存在规律。

量子场论

(1) 研究量子场在没有相互作用时的运动规律.

—— 自由场

(2) 研究场之间存在相互作用后，场在不同模式之间的转化.

—— 相互作用场

相互作用 { 弱：微扰论.
强：困难.　（说明孤立对象假设不适用）

Level 1. 场在外源下的运动规律

例：van Hove model. (van Hove [1951, 1952])

场 F(x) 的经典运动方程为

$$-\ddot{\varphi}(x) = -(\nabla^2 - m^2)\varphi(x) + \rho(x)$$

→ 外源 $\rho(x)$，与场无关!!!

$$(\Box + m^2)\varphi(x) = -\rho(x)$$

$$\therefore \quad \mathcal{L} = \frac{1}{2}\partial_\mu\varphi \cdot \partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \varphi(x)\rho(x)$$

$$\partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)} - \frac{\partial\mathcal{L}}{\partial\varphi} = (\Box + m^2)\varphi + \rho(x).$$

$$\therefore \quad \pi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \partial_0\varphi$$

$$H = \int d^3\vec{x}\,(\pi\cdot\dot{\varphi} - \mathcal{L})$$

$$= \int d^3\vec{x}\left[\frac{1}{2}\dot{\varphi}\cdot\dot{\varphi} + \frac{1}{2}\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi + \frac{1}{2}m^2\varphi^2 + \varphi(x)\rho(x)\right]$$

在自由标量场的 Hilbert 空间中展开 H,

$$H = \int\frac{d^3p}{(2\pi)^3}E_p\, a_p^\dagger a_p + \int d^3\vec{x}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\left(a_p e^{i\vec{p}\cdot\vec{x}} + a_{-p}^\dagger e^{i\vec{p}\cdot\vec{x}}\right)\rho(\vec{x},t)$$

假定 ρ 不随 t 变化. (Hamilton 量不显含时)

$$H = \int\frac{d^3p}{(2\pi)^3}\left[E_p a_p^\dagger a_p + \frac{\tilde{\rho}(\vec{p})}{\sqrt{2E_p}}(a_p + a_{-p}^\dagger)\right] \quad \text{非对角}$$

$\therefore$ $a_p^\dagger|0\rangle$ 不是 H 本征态, E_p 也不是 H 本征值

$\because$ $\rho(\vec{x})$ 实 $\quad\therefore$ $\tilde{\rho}(-\vec{p}) = \int d^3x\,\rho(\vec{x})e^{-i\vec{p}\cdot\vec{x}} = \left(\int d^3x\,\rho(\vec{x})e^{i\vec{p}\cdot\vec{x}}\right)^* = \tilde{\rho}(\vec{p})^*$

$$\therefore H = \int\frac{d^3p}{(2\pi)^3}\left[E_p a_p^\dagger a_p + \frac{\tilde{\rho}(\vec{p})}{\sqrt{2E_p}}a_p + \frac{\tilde{\rho}(\vec{p})^*}{\sqrt{2E_p}}a_p^\dagger\right]$$

$$= \int\frac{d^3p}{(2\pi)^3}\left[E_p a_p^\dagger a_p + \frac{\tilde{\rho}(\vec{p})}{\sqrt{2E_p}}a_p + \frac{\tilde{\rho}(\vec{p})^*}{\sqrt{2E_p}}a_p^\dagger + \frac{1}{2E_p^2}\tilde{\rho}(\vec{p})^*\tilde{\rho}(\vec{p}) - \frac{1}{2E_p^2}\tilde{\rho}(\vec{p})^*\tilde{\rho}(\vec{p})\right]$$

$$= \int\frac{d^3p}{(2\pi)^3}\left[\left(\sqrt{E_p}a_p + \frac{\tilde{\rho}(\vec{p})^*}{\sqrt{2}E_p}\right)\left(\sqrt{E_p}a_p^\dagger + \frac{\tilde{\rho}(\vec{p})}{\sqrt{2}E_p}\right) - \frac{1}{2E_p^2}\tilde{\rho}(\vec{p})^*\tilde{\rho}(\vec{p})\right]$$

$\therefore$ $\sqrt{E_p}\, a_p^\dagger + \dfrac{\tilde{\rho}(\vec{p})}{\sqrt{2}\, E_p}$ 为H本征态产生算符.

归一化条件 $|d|^2 [\sqrt{E_q}\, a_q + \dfrac{\tilde{\rho}(\vec{q})^*}{\sqrt{2} E_q}, \sqrt{E_p}\, a_p^\dagger + \dfrac{\tilde{\rho}(\vec{p})}{\sqrt{2} E_p}] = (2\pi)^3 \delta^3(\vec{p}-\vec{q})$

$$\Rightarrow \quad c_p^\dagger = a_p^\dagger + \frac{\tilde{\rho}(\vec{p})}{\sqrt{2}\, E_p^{3/2}}$$

$$\therefore \quad H = \int \frac{d^3\vec{p}}{(2\pi)^3} E_p c_p^\dagger c_p - \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{2E_p^2} \tilde{\rho}(\vec{p}) \tilde{\rho}(\vec{p})^*$$

$$= \int \frac{d^3\vec{p}}{(2\pi)^3} E_p c_p^\dagger c_p - \frac{1}{2} \int d^3\vec{x}\, d^3\vec{y}\, \rho(\vec{x}) V(\vec{x}-\vec{y}) \rho(\vec{y})$$

$$V(\vec{r}) = -\frac{e^{-m|r|}}{4\pi r^2}$$

以此 $c_p, c_p^\dagger$ 构造Fock空间表示.

小结: 场在不含时外源下的演化, 现实例子: 平均场模型,
核力的Yukawa模型, 外场中的声子运动, …

Level 2: 外源本身亦变化, 且被场影响, $\rightarrow$ 外源也是场.
场与场的相互作用.

例, $\lambda-\varphi^4$ 模型.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 \qquad \lambda \in \mathbb{R}^+ \;,\; \lambda \ll 1.$$

$\lambda \ll 1$ 保证相互作用为"小扰动"(?)

回顾 non-SR QM. 谐振子 $H_0 = \frac{1}{2}\left(-\frac{d^2}{dx^2} + x^2\right)$

若有 $\propto x^4$ 的相互作用. $H = H_0 + H_1 = \frac{1}{2}\left(-\frac{d^2}{dx^2} + x^2\right) + \frac{\lambda}{4!}x^4$

演化 $\rightarrow$ 含时微扰论 $\rightarrow$ 相互作用表象.

任意算符的期望. $\langle \psi^S(t) | \mathcal{O}^S | \psi^S(t) \rangle$

$\langle \psi^S(t) | \mathcal{O}^S | \psi^S(t) \rangle = \langle \psi^S(0) | e^{iHt} \mathcal{O}^S e^{-iHt} | \psi^S(0) \rangle$

$|\psi^H\rangle \equiv |\psi^S(0)\rangle \qquad \mathcal{O}^H(t) = e^{iHt}\mathcal{O}^S e^{-iHt}$ Heisenberg picture

$|\psi^S(t)\rangle = e^{-iHt}|\psi^H\rangle \qquad \mathcal{O}^S \equiv \mathcal{O}^H(0)$ Schrödinger picture

时间演化算符 $U(t, t_0) = \exp[-iH^S(t-t_0)] \quad U(t) \equiv U(t,0)$

$\therefore \mathcal{O}^H(t) = U(t)^\dagger \mathcal{O}^S U(t)$

$\because H^H(t) = U(t)^\dagger H^S U(t) = H^S \equiv H$

Question: if $H = H_0 + H_I$, what will happen?

$H = H_0 + H_I$

相互作用的存在，使得 $|\psi^S(t)\rangle$ 的演化偏离自由演化 $|\psi_0^S(t)\rangle$

$|\psi_0^S(t)\rangle = U_0(t)|\psi^S(0)\rangle = \exp(-iH_0t)|\psi^S(0)\rangle$

$|\psi^S(t)\rangle = U(t)|\psi^S(0)\rangle = \exp(-i(H_0+H_I)t)|\psi^S(0)\rangle$

$\therefore\ U_0(t)^\dagger|\psi^S(t)\rangle = U_0(t)^\dagger U(t)|\psi^S(0)\rangle$

when $H_I \to 0$, $U_0(t)^\dagger U(t)|\psi^S(0)\rangle \to |\psi^S(0)\rangle$

相当于态没有变化.

$\Rightarrow$ 态的演化被相互作用决定.

$|\psi^I(t)\rangle \equiv U_0(t)^\dagger|\psi^S(t)\rangle = U_0(t)^\dagger U(t)|\psi^S(0)\rangle$

$$\therefore\ \frac{d}{dt}|\psi^I(t)\rangle = \left(\frac{dU_0(t)^\dagger}{dt}\cdot U(t) + U_0(t)^\dagger\frac{dU(t)}{dt}\right)|\psi^S(0)\rangle$$

$$= iH_0\,U_0(t)^\dagger U(t) - U_0(t)^\dagger(iH)U(t)|\psi^S(0)\rangle$$

$$= iH_0|\psi^I(t)\rangle - iU_0(t)^\dagger H\,U_0(t)\cdot U_0(t)^\dagger U(t)|\psi^S(0)\rangle$$

$$= i(H_0 - U_0(t)^\dagger H\,U_0(t))|\psi^I(t)\rangle$$

$$= i\,U_0(t)^\dagger(H_0 - H)U_0(t)|\psi^I(t)\rangle$$

$$= -i\,U_0(t)^\dagger H_I\,U_0(t)|\psi^I(t)\rangle$$

如果定义 $H_I^I(t) = U_0(t)^\dagger H_I\,U_0(t)$ （演化为自由演化）

则 $\begin{cases} i\frac{d}{dt}|\psi^I(t)\rangle = H_I^I(t)|\psi^I(t)\rangle \\ i\frac{d}{dt}H^I(t) = [H^I(t), H_0] \end{cases}$

$|\psi^I(t)\rangle = U_0(t)^\dagger U(t)|\psi^S(0)\rangle = U_0(t)^\dagger U(t)|\psi^I(0)\rangle$

其演化（时间平移生成元）算符为 $U^I(t) = U_0(t)^\dagger U(t)$

$$U^I(t) = U_0(t)^\dagger U(t) = e^{iH_0 t} \cdot e^{-i(H_0+H_I^S)t}$$

! $[H_0, H_I] \neq 0$ $\therefore$ $e^{iH_0 t} e^{-i(H_0+H_I^S)t} \neq e^{-iH_I^S t}$.

$\therefore$ $U^I(t)$ 应通过求解 $i\frac{d}{dt}U^I(t) = H_I^I(t)\,U^I(t)$ 得到

小结：以上的讨论适用于一般的量子理论，量子场论也是量子理论，只是 H 的形式由场表示，

Comment：注意，相互作用表象在 $H_I^S = 0$ 时回到 Heisenberg 表象，其算符演化（即使 $H_I^S \neq 0$）与 Heisenberg 表象相同，
$\therefore$ 其成立的前提，是加入相互作用后，理论的解的 Hilbert 空间（对易关系的表示）与自由理论么正等价。
在场论中，即自由场必须穷尽所有可能的单粒子态
也就是说，束缚态需要以自由场的形式放在自由 Hamilton 中。

$U^I(t)$ 的求解，Dyson 序列

$\because$ $H^I(t)$ 含时，$\therefore$ $U^I(t) \neq e^{-iH_I^I(t)}$

迭代求解。

(1) 0阶：$H_I^I(t)=0$ $\therefore i\frac{d}{dt}U^I(t)^{(0)}=0$

$$U^I(t)^{(0)}=\mathbb{1}$$

(2) 1阶：$i\frac{d}{dt}U^I(t)^{(1)}=H_I^I(t)U^I(t)^{(0)}=H_I^I(t)$

$$\therefore U^I(t)^{(1)}=C+(-i)\int_0^t H_I^I(\tau)d\tau$$

$\because U^I(0)^{(1)}=\mathbb{1}$ $\therefore C=\mathbb{1}$

$$\therefore U^I(t)^{(1)}=\mathbb{1}+(-i)\int_0^t H_I^I(\tau)d\tau.$$

(3) 2阶：$i\frac{d}{dt}U^I(t)^{(2)}=H_I^I(t)U^I(t)^{(1)}$

$$\therefore U^I(t)^{(2)}=\mathbb{1}+(-i)\int_0^t d\tau_2\, H_I^I(\tau_2)U^I(\tau_2)^{(1)}$$

$$=\mathbb{1}+(-i)\int_0^t d\tau_2\, H_I^I(\tau_2)+(-i)^2\int_0^t d\tau_2\int_0^{\tau_2}d\tau_1\, H_I^I(\tau_2)H_I^I(\tau_1)$$

如此往复

$$U^I(t)^{(n)}=\mathbb{1}+(-i)\int_0^t d\tau_n H_I^I(\tau_n)+(-i)^2\int_0^t d\tau_n\int_0^{\tau_n}d\tau_{n-1}\, H_I^I(\tau_n)H_I^I(\tau_{n-1})$$
$$+\cdots+(-i)^n\int_0^t d\tau_n\cdots\int_0^{\tau_2}d\tau_1\prod_{j=1}^{n}H_I^I(\tau_j)$$

$n\to\infty$ 时，此序列称为 Dyson 序列.

$$i\frac{d}{dt}U^I(t)^{(\infty)}=H_I^I(t)+H_I^I(t)\cdot(-i)\int_0^t d\tau_{n-1}\, H_I^I(\tau_{n-1})+\cdots$$
$$=H_I^I(t)U^I(t)^{(\infty)}$$

满足方程.

思考：上述验证有何问题？

Time order:

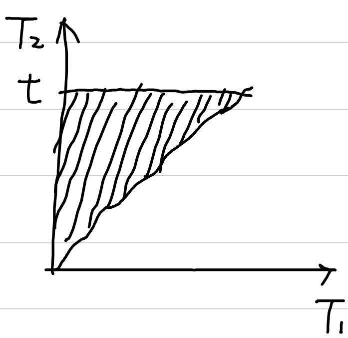
根据迭代. $t \geq \tau_n \geq \tau_{n-1} \geq \cdots \geq \tau_1 \geq 0$

∴ $H_I(\tau_n)H_I(\tau_{n-1})\cdots H_I(\tau_1)$ 为"编时乘积"(time order product)

定义编时乘积

$$T\{H_I(t_1)H_I(t_2)\} = \begin{cases} H_I(t_1)H_I(t_2) & t_1 \geq t_2 \\ H_I(t_2)H_I(t_1) & t_2 > t_1 \end{cases}$$

对于 $\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\, H_I(\tau_2)H_I(\tau_1) \qquad \tau_2 > \tau_1$



考虑 $\int_0^t d\tau_2 \int_{\tau_2}^t d\tau_1\, H_I(\tau_1)H_I(\tau_2) \qquad \tau_1 > \tau_2$

积分变换 $\tau_1 \to t_2 \quad \tau_2 \to t_1 \qquad t_2 > t_1$

$$\Rightarrow \int_0^t d\tau_2 \int_{\tau_2}^t d\tau_1\, H_I(\tau_1)H_I(\tau_2)$$

$$= \int_0^t dt_1 \int_{t_1}^t dt_2\, H_I(t_2)H_I(t_1)$$

$$= \int_0^t dt_2 \int_0^{t_2} dt_1\, H_I(t_2)H_I(t_1) \qquad t_2 > t_1$$

∴ $\tau_1 > \tau_2$ 时. $\int_0^t d\tau_2 \int_{\tau_2}^t d\tau_1\, H_I(\tau_1)H_I(\tau_2) = \int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\, H_I(\tau_2)H_I(\tau_1)$

∴ $\int_0^t d\tau_2 \int_0^{\tau_2} d\tau_1\, H_I(\tau_2)H_I(\tau_1) = \frac{1}{2!}\int_0^t d\tau_1 d\tau_2\, T\{H_I(\tau_1)H_I(\tau_2)\}$

类似地, 第 n 项可以写为 $\frac{1}{n!}\int_0^t d\tau_1 \cdots d\tau_n\, T\{H_I(\tau_1)\cdots H_I(\tau_n)\}$

$$\therefore U^I(t) = \mathbb{1} + (-i)\int_0^t d\tau_1 H_I(\tau_1) + \cdots + \frac{(-i)^n}{n!}\int_0^t d\tau_1 \cdots d\tau_n T\{H_I(\tau_1)\cdots H_I(\tau_n)\}$$
$$+ \cdots$$

$$\equiv T\left\{\exp\left[-i\int_0^t d\tau H_I(\tau)\right]\right\}$$

至此，我们就得到了算符的演化规律（相互作用表象下，场算符的演化与自由场 Heisenberg 表象相同）和态的演化规律

$$U^I(t, t_0) = T\left\{\exp\left[-i\int_{t_0}^t d\tau H_I(\tau)\right]\right\}$$

对于场论，最重要的量为 n 点关联函数，可以证明，如果任意 n 点的真空关联函数都已给定，则场论已经被完全确定。

考虑两点真空关联函数 $\langle\Omega|\phi(x)\phi(y)|\Omega\rangle$

在相互作用表象中，$\phi(x) = \phi_0^H(x)$

$|0_f\rangle$（自由真空态）的演化？

考虑 H（不是 H_0）的完备归一本征态集 $|n\rangle$，根据定义

$$U^H(T)|0_f\rangle = e^{-iHT}|0_f\rangle = \sum_n e^{-iE_nT}|n\rangle\langle n|0_f\rangle$$

设 $\langle 0|0_f\rangle \neq 0$

$$U^H(T)|0_f\rangle = e^{-iE_0T}\left(|0\rangle\cdot\langle 0|0_f\rangle + \sum_{n\neq 0} e^{-i(E_n-E_0)T}|n\rangle\langle n|0_f\rangle\right)$$

真空唯一，$E_n - E_0 > 0$，

$T \to +\infty(1-i\varepsilon)$ （虚时间 $\Leftrightarrow$ 非绝对 0 度）

$$\Rightarrow \lim_{T\to+\infty(1-i\varepsilon)} e^{-iHT}|0_f\rangle = e^{-iE_0T}|0\rangle\langle 0|0_f\rangle$$

$$\therefore |0\rangle = \frac{1}{\langle 0|0_f\rangle}\cdot\lim_{T\to+\infty(1-i\varepsilon)} e^{iE_0T}e^{-iHT}|0_f\rangle$$

$|0_f\rangle$ 为自由场 Heisenberg 表象下的真空，

而 $e^{iH_0T}|0_f\rangle = e^{i0\times T}|0_f\rangle = |0_f\rangle$ （选 $H_0|0\rangle$ 为能量零点）

$$\therefore |0\rangle = \frac{1}{\langle 0|0_f\rangle}\lim_{T\to+\infty(1-i\varepsilon)} e^{iE_0T}\cdot e^{-iHT}\cdot e^{iH_0T}|0_f\rangle$$

$$e^{-iHT}e^{iH_0T} = \left(e^{iH_0(-T)}e^{-iH(-T)}\right)^\dagger$$
$$= U(-T,0)^\dagger \overset{?}{=} U(0,-T)$$

$$i\frac{\partial}{\partial t}U(t,t_0) = H_I(t)U(t,t_0)$$

$$\therefore\ iU^\dagger(t,t_0)\frac{\partial}{\partial t}U(t,t_0) = U^\dagger(t,t_0)H_I(t)U(t,t_0)$$

$$-i\frac{\partial}{\partial t}U^\dagger(t,t_0) = U^\dagger(t,t_0)H_I^\dagger(t)$$

而 $H_I(t) = U_0^\dagger(t)H_I U_0(t)$

$\therefore H_I(t)^\dagger = U_0^\dagger(t)H_I^\dagger U_0(t) = H_I(t)$

$$\therefore -i\left[\frac{\partial}{\partial t}U^\dagger(t,t_0)\right]U(t,t_0) = U^\dagger(t,t_0)H_I(t)U(t,t_0)$$

$$\Rightarrow \frac{\partial}{\partial t}\left(U^\dagger(t,t_0)U(t,t_0)\right) = 0 \Rightarrow U^\dagger(t,t_0)U(t,t_0)\equiv \mathbb{1}.$$

$$\therefore\ |0\rangle = \frac{1}{\langle 0|0_f\rangle}\lim_{T\to+\infty(1-i\varepsilon)} e^{iE_0T}\,U(0,-T)|0_f\rangle$$

or，一般地. 时间原点为 t_0 时.

$$|0\rangle = \frac{1}{\langle 0|0_f\rangle}\lim_{T\to+\infty(1-i\varepsilon)} e^{iE_0(T+t_0)}\,U(t_0,-T)|0_f\rangle$$

类似地 $\langle 0_f|\,U^H(-T) = \sum_n \langle 0_f|n\rangle\langle n|\,e^{iHT}$

$$= \langle 0_f|0\rangle e^{-iE_0T}\langle 0| + \sum_{n\neq 0}\langle 0_f|n\rangle\langle n|\,e^{-i(E_n-E_0)T}$$

$$T\to+\infty(1-i\varepsilon) \to \langle 0_f|0\rangle e^{-iE_0T}\cdot\langle 0|$$

$$\therefore\quad \langle 0_f|\,e^{iHT} = \langle 0_f|0\rangle e^{-iE_0T}\cdot\langle 0|.$$

$$\therefore\quad \langle 0| \to \frac{1}{\langle 0_f|0\rangle} e^{iE_0T}\langle 0_f|\,\underline{e^{iH_0T}\cdot e^{iHT}}$$

$$= \frac{1}{\langle 0_f|0\rangle} e^{iE_0T}\langle 0_f|\,U(T,0)$$

$$\text{or}\quad \langle 0| = \frac{1}{\langle 0_f|0\rangle}\lim_{T\to+\infty(1-i\varepsilon)} e^{iE_0(T-t_0)}\langle 0_f|\,U(T,t_0)$$

$$\therefore\ \langle 0|\,\phi(x)\,\phi(y)|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)}\frac{1}{|\langle 0_f|0\rangle|^2}\cdot e^{2iE_0T}$$

$$\langle 0_f|U(T,t_0)\,\phi^H(x)\,\phi^H(y)\,U(t_0,-T)|0_f\rangle$$

但是. $\phi^I(x) = \phi^H_0(x)$ $\qquad \phi^H_0(x) = U_0^\dagger\phi^S U_0$

$$\phi^H(x) = U^{H\dagger}\phi^S U^H$$

$$\therefore \phi^H = U_H^\dagger U_0 \phi^I U_0^\dagger U_H = U^\dagger \phi^I U.$$

$$\therefore \langle 0|\phi(x)\phi(y)|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)} \frac{e^{2iE_0T}}{|\langle 0|0_f\rangle|^2}$$

$$\times \langle 0_f|U(T,t_0)U^\dagger(x^0,t_0)\phi^I(x)U(x^0,t_0)\cdot U^\dagger(y^0,t_0)\phi^I(y)$$

$$U(y^0,t_0)U(t_0,-T)|0_f\rangle$$

$$= \lim_{T\to+\infty(1-i\varepsilon)} \frac{e^{2iE_0T}}{|\langle 0|0_f\rangle|^2} \langle 0_f|U(T,x^0)\phi^I(x)U(x^0,y^0)\phi^I(y)$$

$$\cdot U(y^0,-T)|0_f\rangle$$

而 $1 = \langle 0|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)} \frac{e^{2iE_0T}}{|\langle 0|0_f\rangle|^2} \cdot \langle 0_f|U(T,-T)|0_f\rangle$

$$\therefore \langle 0|\phi(x)\phi(y)|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)} \frac{\langle 0_f|U(T,x^0)\phi^I(x)U(x^0,y^0)\phi^I(y)U(y^0,-T)|0_f\rangle}{\langle 0_f|U(T,-T)|0_f\rangle}$$

编时乘积

$$\therefore \langle 0|T\{\phi(x)\phi(y)\}|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)} \frac{\langle 0_f|T\{\phi^I(x)\phi^I(y)\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0_f\rangle}{\langle 0_f|T\{\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0_f\rangle}$$

一般地，对于场 $\phi_1(x_1), \phi_2(x_2) \cdots \phi_n(x_n)$ 构成的多项式 $P[\phi]$.

$$\langle 0|T\{P[\phi]\}|0\rangle = \lim_{T\to+\infty(1-i\varepsilon)} \frac{\langle 0_f|T\{P[\phi]\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0_f\rangle}{\langle 0_f|T\{\exp[-i\int_{-T}^{T}dt\,H_I(t)]\}|0_f\rangle}$$

Gell-Mann Low formula

$|\psi^S(t)\rangle$, $|\psi^I(t)\rangle$, $|\psi^H(t)\rangle$:

$|\psi^H(t)\rangle \equiv |\psi^H(0)\rangle = |\psi^H\rangle$ 不演化.

$|\psi^S(t)\rangle = e^{-iH(t-t_0)}|\psi^S(t_0)\rangle \qquad i\frac{\partial}{\partial t}|\psi^S(t)\rangle = H|\psi^S(t)\rangle$

$t=0$ 时 $|\psi^S(0)\rangle = |\psi^H\rangle$，则 $|\psi^S(t)\rangle = e^{-iHt}|\psi^H\rangle$

$|\psi^I(t)\rangle = U_0(t)^\dagger U(t)|\psi^I(0)\rangle = e^{iH_0t}e^{-iHt}|\psi^I(0)\rangle$

$t=0$ 时 $|\psi^I(0)\rangle = |\psi^S(0)\rangle = |\psi^H\rangle$

$\Rightarrow |\psi^I(t)\rangle = e^{iH_0t}e^{-iHt}|\psi^H\rangle$

$= e^{iH_0t}e^{-iHt}\cdot e^{iHt}|\psi^S(t)\rangle = e^{iH_0t}|\psi^S(t)\rangle$

$i\frac{\partial}{\partial t}|\psi^I(t)\rangle = H_I(t)|\psi^I(t)\rangle$

$H_I(t) \equiv e^{iH_0t}H_I e^{-iH_0t}$

$\varphi^S(x)$, $\varphi^I(x)$, $\varphi^H(x)$

$\varphi^S(x) \equiv \varphi^S(0,\vec{x})$ 不演化.

$\varphi^H(x) = ?$ 考虑 $\langle O(x)\rangle \equiv \langle\psi_1|\varphi(x)|\psi_2\rangle$

$\because \langle\psi_1^S(t)|\varphi^S(0,\vec{x})|\psi_2^S(t)\rangle = \langle O(x)\rangle = \langle\psi_1^H|\varphi^H(x)|\psi_2^H\rangle$

$\therefore \langle\psi_1^H|e^{iHt}\varphi^S(0,\vec{x})e^{-iHt}|\psi_2^H\rangle = \langle\psi_1^H|\varphi^H(x)|\psi_2^H\rangle$

由 ψ_1^H, ψ_2^H 的完备性 $\Rightarrow \varphi^H(x) = e^{iHt}\varphi^S(0,\vec{x})e^{-iHt}$

同理 $\Rightarrow \varphi^I(x) = e^{iH_0t}\varphi^S(0,\vec{x})e^{-iH_0t}$

$$\therefore \quad \varphi^I(x) = \underbrace{e^{iH_0t}\, e^{-iHt}}\, \varphi^H(x)\, e^{iHt}\, e^{-iH_0t}$$