

# $\bar{c}\gamma^\mu c$ matrix element

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## 1 $^3S_1$

Ignore the overall factor:

$$\langle 0 | \bar{c}\gamma^\mu c | ^3S_1 \rangle = \int d\Omega \text{tr}[\Pi_1 \gamma^\mu] \propto \sqrt{2}\pi \left( \frac{m}{3E} + \frac{2}{3} \right) \epsilon^\mu$$

## 2 $^3D_1$

The matrix element reads:

$$\langle 0 | \bar{c}\gamma^\mu c | ^3D_1 \rangle = \int d\Omega \sum_{\lambda_1 \lambda_2 S_z m} \text{tr}\{\Pi_1 \gamma^\mu\} \langle 1J_z | 2m; 1S_z \rangle Y_{2m}(\theta, \phi)$$

while the trace part is the same as  $^3S_1$ :

$$\text{tr}\{\Pi_1 \gamma^\mu\} = \frac{\sqrt{2}p^\mu (p \cdot \epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \epsilon^{(0)} = (0, 0, 0, 1), \epsilon^{(+)} = \frac{1}{\sqrt{2}}(0, -1, -i, 0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} 0 & \frac{2\sqrt{2}\pi p^2}{3E(m+E)} & -\frac{2i\sqrt{2}\pi p^2}{3E(m+E)} & 0 \\ 0 & 0 & 0 & \frac{4\sqrt{\pi}p^2}{3E(m+E)} \\ 0 & -\frac{2\sqrt{2}\pi p^2}{3E(m+E)} & -\frac{2i\sqrt{2}\pi p^2}{3E(m+E)} & 0 \end{pmatrix}$$

and the decay constant is  $\frac{4\sqrt{\pi}p^2}{3E(E+m)}$  where  $p = \mathbf{p} = E^2 - m^2$ .