

# $\bar{c}\gamma^\mu c$ matrix element

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## 1 Kinematics and Conventions

Quark and antiquark momenta are

$$p_1 = P/2 + p = (E, \mathbf{p}) \quad (1)$$

$$p_2 = P/2 - p = (E, -\mathbf{p}) \quad (2)$$

where in rest frame

$$P = (2E(p), 0) \quad (3)$$

$$p = (0, \mathbf{p}) \quad (4)$$

Dirac spinors are normalized as following

$$u(\mathbf{p}) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \xi \\ \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{E+m}\xi \end{pmatrix} \quad (5)$$

$$v(\mathbf{p}) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \frac{-\mathbf{p}\cdot\boldsymbol{\sigma}}{E+m}\eta \\ \eta \end{pmatrix} \quad (6)$$

where

$$\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (7)$$

$$\eta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad (8)$$

## 2 State Projection

The bound state is [Weinberg(2015)]

$$|P, E; J, m_J; L; S\rangle = \int d\Omega_{\mathbf{p}_1} \sum_{s_1 s_2 s_z m_l} Y_l^m(\hat{\mathbf{p}}_1) \langle S_1 \lambda_1 S_2 \lambda_2 | S s_z \rangle \langle S s_z L m_l | J m_J \rangle |\mathbf{p}_1, \lambda_1\rangle |\mathbf{P} - \mathbf{p}_1, \lambda_2\rangle \quad (9)$$

## 3 Bilinears [Bodwin and Petrelli(2002)]

$$\Pi_0(P, p) \equiv - \sum_{\lambda_1, \lambda_2} u(\mathbf{p}, \lambda_1) \bar{v}(-\mathbf{p}, \lambda_2) \left\langle \frac{1}{2} \lambda_1 \frac{1}{2} \lambda_2 \left| 00 \right. \right\rangle \quad (10)$$

$$= \frac{1}{2\sqrt{2}E(E+m)} \left( \frac{1}{2} \not{p} + m + \not{p} \right) \frac{\not{P} + 2E}{4E} \gamma_5 \left( \frac{1}{2} \not{p} - m - \not{p} \right) \quad (11)$$

$$\Pi_1(P, p) \equiv \sum_{\lambda_1, \lambda_2} u(\mathbf{p}, \lambda_1) \bar{v}(-\mathbf{p}, \lambda_2) \left\langle \frac{1}{2} \lambda_1 \frac{1}{2} \lambda_2 \left| 1\epsilon \right. \right\rangle \quad (12)$$

$$= \frac{-1}{2\sqrt{2}E(E+m)} \left( \frac{1}{2} \not{p} + m + \not{p} \right) \frac{\not{P} + 2E}{4E} \not{\epsilon} \left( \frac{1}{2} \not{p} - m - \not{p} \right) \quad (13)$$

## 4 $^3S_1$

Average over  $\mathbf{p}$  with Bodwin's convention (extra  $1/(4\pi)$ ):

$$\langle 0 | \bar{c} \gamma^\mu c | ^3S_1 \rangle^{(0)} = \frac{1}{4\pi} \int d\Omega \text{tr}[\Pi_1 \gamma^\mu] = \sqrt{2} \left( \frac{m}{3E} + \frac{2}{3} \right) \epsilon^\mu$$

## 5 $^3D_1$

The matrix element reads:

$$\langle 0 | \bar{c} \gamma^\mu c | ^3D_1 \rangle^{(0)} = \int d\Omega \sum_{\lambda_1 \lambda_2 s_z m_l} \text{tr}\{\Pi_1 \gamma^\mu\} \langle 2m_l; 1s_z | 1J_z \rangle Y_{2m_l}(\theta, \phi)$$

while the trace part is the same as  $^3S_1$ :

$$\text{tr}\{\Pi_1 \gamma^\mu\} = \frac{\sqrt{2} p^\mu (p \cdot \epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \epsilon^{(0)} = (0, 0, 0, 1), \epsilon^{(+)} = \frac{1}{\sqrt{2}}(0, -1, -i, 0)$$

Result (the first row and the last are orthogonal):

$$\left( \begin{array}{c|ccc} \epsilon^{(-)} & 0 & \frac{2\sqrt{2}\pi p^2}{3E(m+E)} & -\frac{2i\sqrt{2}\pi p^2}{3E(m+E)} & 0 \\ \hline \epsilon^{(0)} & 0 & 0 & 0 & \frac{4\sqrt{\pi} p^2}{3E(m+E)} \\ \hline \epsilon^{(+)} & 0 & -\frac{2\sqrt{2}\pi p^2}{3E(m+E)} & -\frac{2i\sqrt{2}\pi p^2}{3E(m+E)} & 0 \end{array} \right)$$

and the decay constant is  $\frac{4\sqrt{\pi} p^2}{3E(E+m)}$  where  $p^2 = \mathbf{p}^2 = E^2 - m^2$ .

## 6 NRQCD $^3D_1$

$$\langle 0 | \frac{1}{2m^2} \chi^\dagger D^{\{i} D^{j\}} \sigma^j \psi | Q\bar{Q} [^3D_1(\epsilon)] \rangle^{(0)} = \int d\Omega \sum_{\lambda_1 \lambda_2 s_z m_l} \frac{-p^{\{i} p^{j\}}}{2m^2} \eta_{\lambda_2}^\dagger \sigma^j \xi_{\lambda_1} \langle 2m_l; 1s_z | 1J_z \rangle \left\langle \frac{1}{2} \lambda_1 \frac{1}{2} \lambda_2 \middle| 1s_z \right\rangle Y_{2m_l}(\theta, \phi) \quad (14)$$

$$= \sqrt{\pi} \frac{4p^2}{3m^2} \epsilon^i \quad (15)$$

if  $\partial^{\{i} \partial^{j\}} = 2\partial^i \partial^j$ .

## References

[Weinberg(2015)] S. Weinberg, *Lectures on Quantum Mechanics* (Cambridge University Pr., 2015).

[Bodwin and Petrelli(2002)] G. T. Bodwin and A. Petrelli, *Phys. Rev. D* **66**, 094011 (2002), [Erratum: *Phys. Rev. D* 87, no.3, 039902(2013)], [arXiv:hep-ph/0205210 \[hep-ph\]](#) .