Exercises

- Show that
$$M_i^s = M_i^{s'}$$
where s, s' are different schemes.
- Show that
$$\Lambda^s = k\,\Lambda^{s'}$$
Determine k in terms of $\chi_g^{(1)}\,,\ b_0$.
- What is needed to determine $\chi_g^{(1)}$?

Renormalization of composite fields

mixing with operators of same dimension

$$\langle \phi_{\mathrm{R}1}(x_1)\phi_{\mathrm{R}2}(x_2)\phi_{\mathrm{R}3}(x_3)\phi_{\mathrm{R}4}(x_4)\dots\rangle_{\text{path integral average}}$$

is finite for $x_i \neq x_j$ for $i \neq j$ with

dimensional regularisation, MS

$$\phi_{\mathrm{R},i}^{(D)} = \sum_j Z_{ij}(\epsilon, g^2)\, \Phi_j^{(D)}\,, \qquad [\Phi_j^{(D)}] = [\Phi_i^{(D)}] = D$$

e.g. $[S] = [P^{rs}] = 3$

lattice MS

$$\phi_{\mathrm{R},i}^{(D)} = \sum_j Z_{ij}(\ln(a\mu), g^2)\, \Phi_{\mathrm{sub},j}^{(D)}\,, \quad [\Phi_{\mathrm{sub},j}^{(D)}] = [\Phi_i^{(D)}] = D$$

$$\Phi_{\mathrm{sub},j}^{(D)} = \Phi_j^{(D)} + \sum_{n\geq 1} \mathbf{a}^{-\mathbf{n}} \sum_k d_{jk}(g_0)\Phi_k^{(D-n)}$$

Subtraction coefficients d_{jk} can be chosen purely as functions of g_0, not $\ln(a\mu)$ [M. Testa, hep-th/9803147, Sect. 2]

Exercise: Go through the argument in hep-th/9803147. Does it hold beyond PT?