Two Loop Matching for Quasi PDF

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June 12, 2020

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1 Renormalization

1.1 One loop diagrams

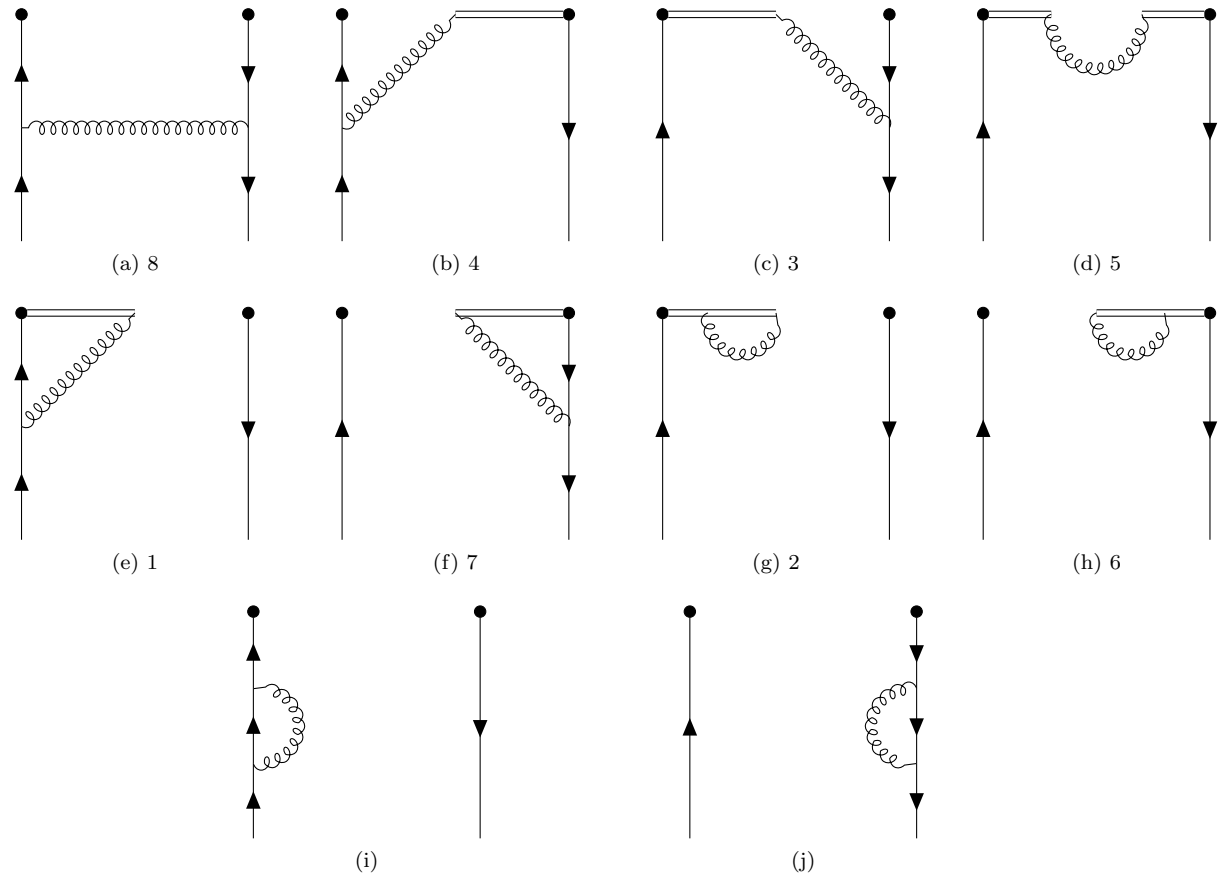
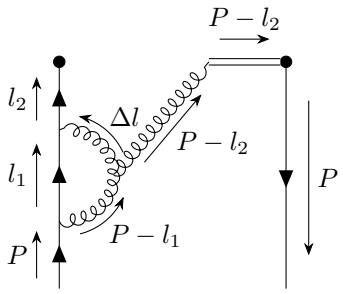


Figure 1: Diagrams of quasi PDF in Feynman gauge.

1.2 Vertex corrections

According to [Ji and Zhang(2015)], the vertex correction diagrams in axial gauge (which corresponds to varieties of diagrams in general covariant gauge) don't have total UV divergence. Rather, they only have subdivergence for sub-diagrams. For example the first column (which involves Figure 3), second row of Table 1 in [Ji and Zhang(2015)] is composed of $\tilde{q}_{11}$ and $\tilde{q}_{12}$, thus we can find some representative diagrams and extract those components ($l \equiv l_1 + l_2, \Delta l \equiv l_1 - l_2$)



$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d} \frac{\mathrm{d}^d l_2}{(2\pi)^d} \frac{1}{[\not{l}_1 - m][\not{l}_2 - m][(P - l_1)^2][(l_1 - l_2)^2][(P - l_2)^2][n \cdot (P - l_2)]} \tag{1}$$

Take the $l_1 \gg l_2$ limit, the integrand becomes

$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[l_1^2\right]\left[\not{l}_2 - m\right]\left[(P-l_2)^2\right]\left[n\cdot(P-l_2)\right]} \tag{2}$$

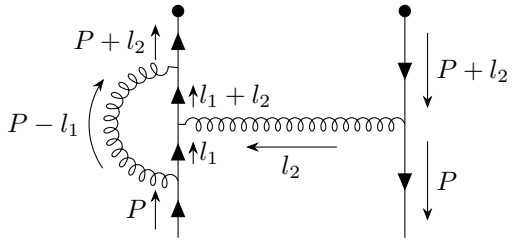
The integral involving l_2 is exactly the integral of $\tilde{q}_{12}$. By adding the gluon self-interacting vertex we can see that the sub-diagram is logarithmic divergent.

Take the $l_2 \gg l_1$ limit, the integrand becomes

$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[l_2^2\right]\left[\not{l}_2 - m\right]\left[(P-l_2)^2\right]\left[n\cdot(P-l_2)\right]} \tag{3}$$

There's another limit where hard loop momentum flows through all paths except the one that's Δl in our current diagram. This configuration gives a finite integral and a power-divergent integral which happens to be a scaleless integral as well. Thus this configuration won't contribute.

What we extracted above is only the $\tilde{q}_{12}$ part, now we will try on the $\tilde{q}_{11}$ part



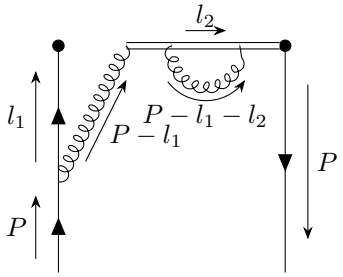
$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d}\frac{\mathrm{d}^d l_2}{(2\pi)^d}\frac{1}{\left[\not{l}_1 - m\right]\left[\not{l}_1 + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[(P-l_1)^2\right]\left[l_2^2\right]} \tag{4}$$

In the $l_1 \gg l_2$ limit we have

$$\frac{1}{\left[\not{l}_1 - m\right]\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[\not{P} + \not{l}_2 - m\right]\left[\not{P} + \not{l}_2 - m\right]\left[l_2^2\right]} \tag{5}$$

and $\tilde{q}_{11}$ is factorized out.

Another example is the sixth row



$$\propto \int \frac{\mathrm{d}^d l_1}{(2\pi)^d}\frac{\mathrm{d}^d l_2}{(2\pi)^d}\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[(P-l_1-l_2)^2\right]\left[n\cdot(P-l_1)\right]\left[n\cdot l_2\right]\left[n\cdot(P-l_1)\right]} \tag{6}$$

Take the $l_2 \gg l_1$ limit, the integrand becomes

$$\frac{1}{\left[\not{l}_1 - m\right]\left[(P-l_1)^2\right]\left[n\cdot(P-l_1)\right]\left[n\cdot(P-l_1)\right]\left[n\cdot l_2\right]\left[(P-l_2)^2\right]} \tag{7}$$

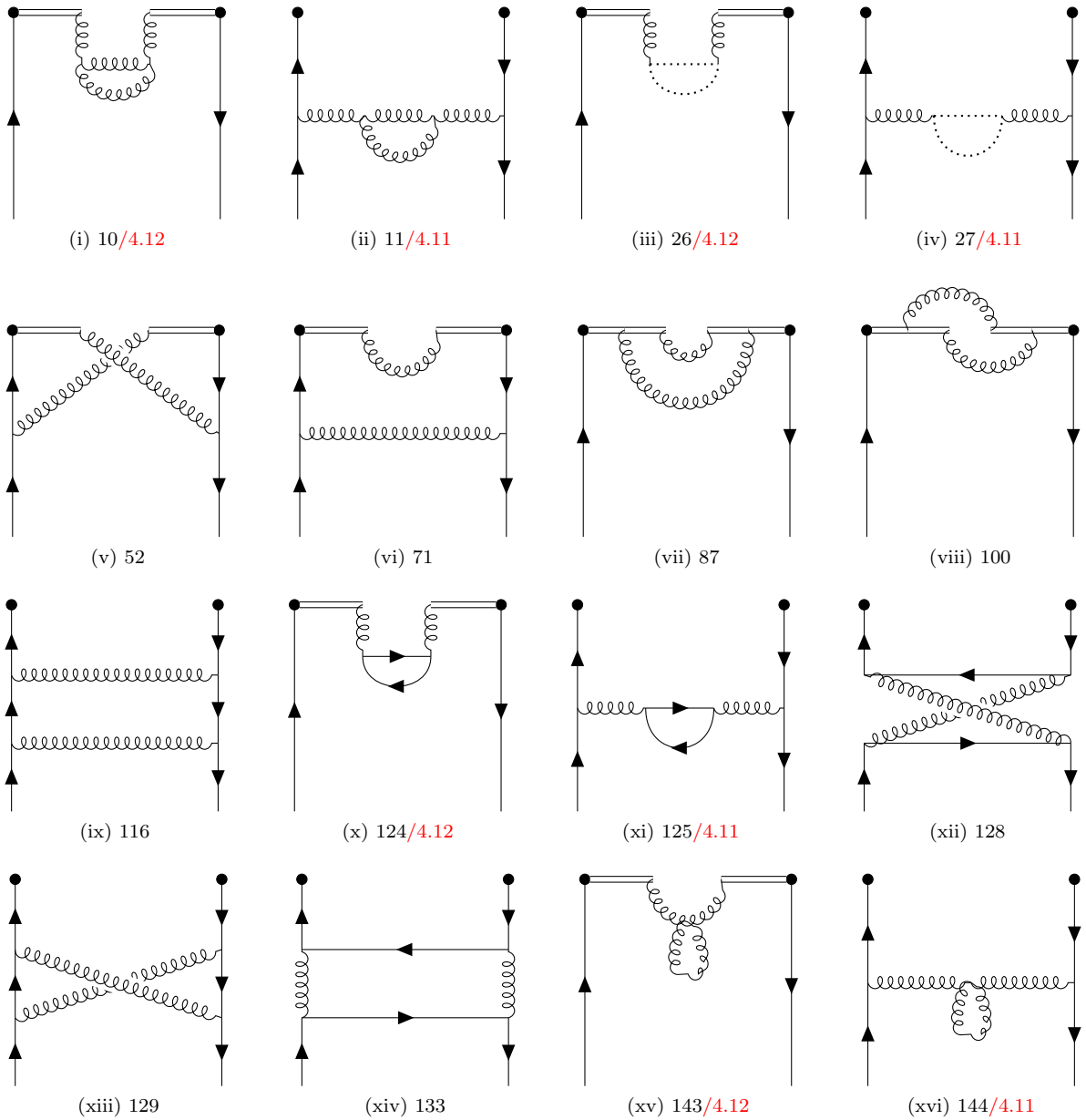
and the integral involving l_2 should give something proportional to $n\cdot(P-l_1)$, thus cancels one eikonal propagator, the remainder is the integral of $\tilde{q}_{12}$.

1.3 Numerical results for one loop diagrams ($z = 1/4$)

2 Real Diagrams

2.1 All diagrams

Figure 2 lists all self-conjugated real diagrams, and Figure 3 lists all non-self-conjugated diagrams, excluding their conjugates.



(i) 10/4.12 (ii) 11/4.11 (iii) 26/4.12 (iv) 27/4.11

(v) 52 (vi) 71 (vii) 87 (viii) 100

(ix) 116 (x) 124/4.12 (xi) 125/4.11 (xii) 128

(xiii) 129 (xiv) 133 (xv) 143/4.12 (xvi) 144/4.11

Figure 2: All self-conjugated diagrams, red n.i marks the diagram number in Ji&Zhang's paper.

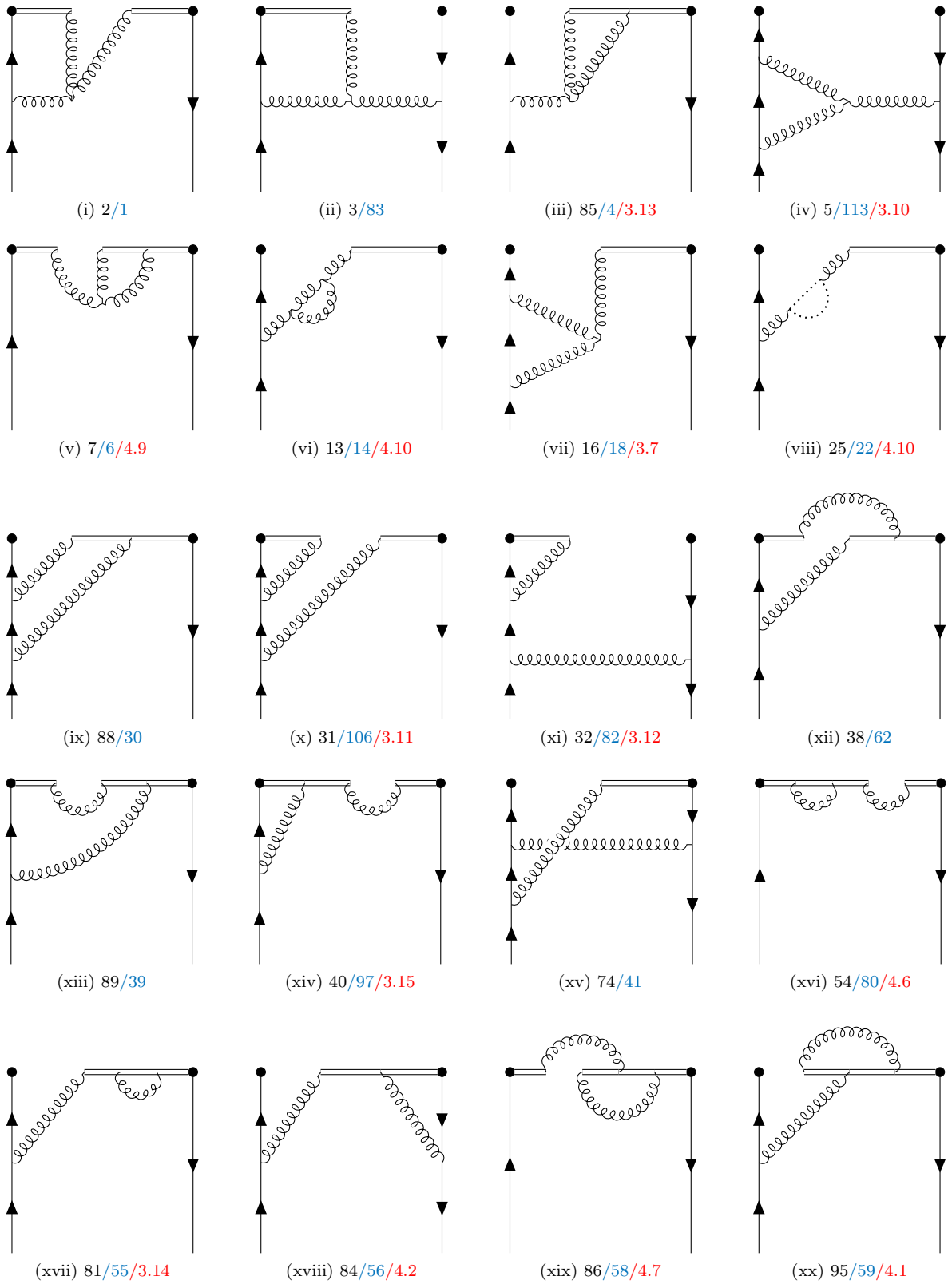


Figure 3: All real diagrams (excluding conjugated diagrams), xxx marks conjugated diagram number, red n.i marks the diagram number in Ji&Zhang's paper.

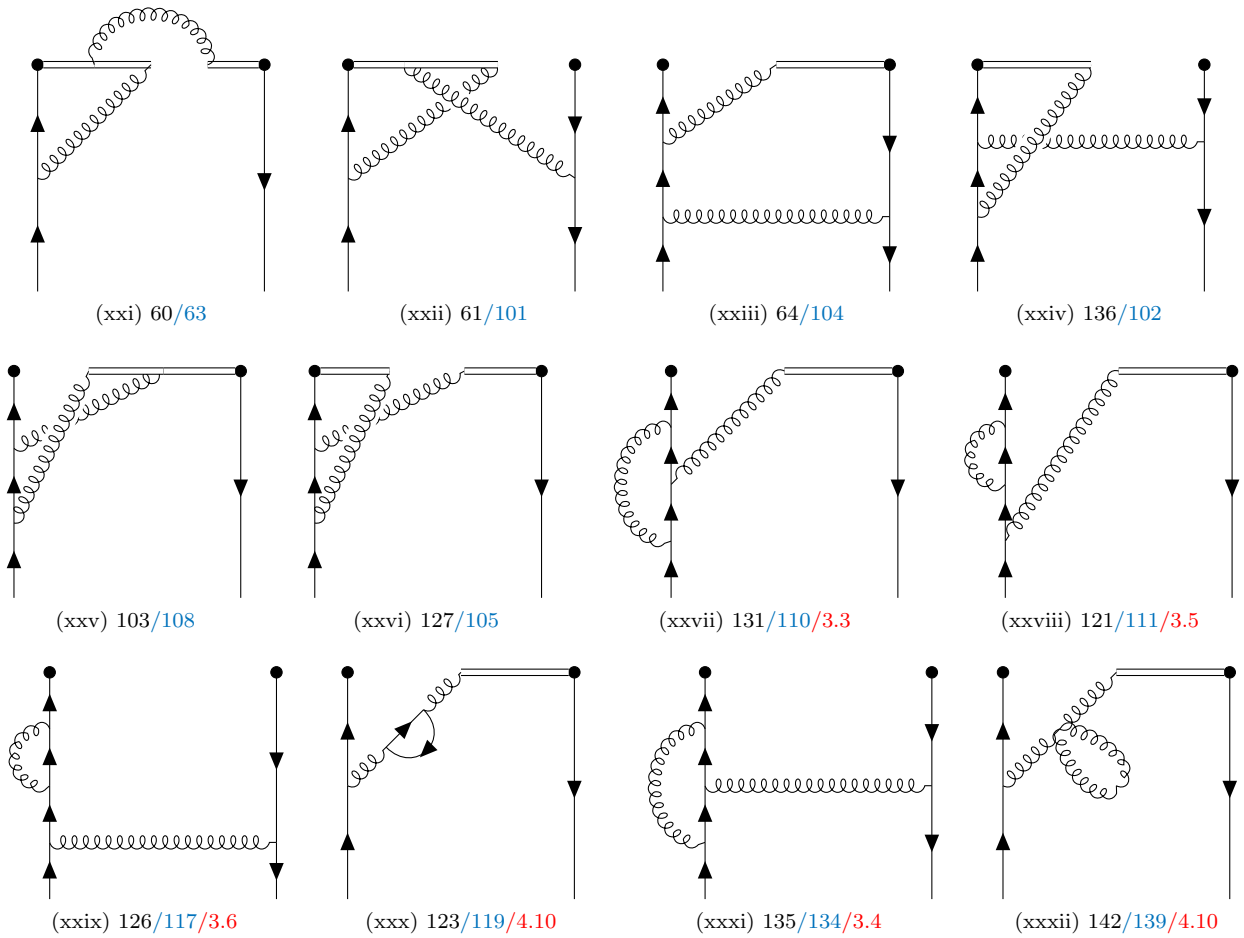


Figure 3: All real diagrams (excluding conjugated and self-conjugated diagrams), xxx marks conjugated diagram number, red n.i marks the diagram number in Ji&Zhang's paper.

2.2 Amplitude test

First we take diagram 2xv to test if the type of diagrams that is a sub-diagram involving only QCD Feynman rules on top of one loop diagram consist with our manual input.

The program gives

$$\begin{aligned}&\left(\delta_{\text{CI(9) CI(10)}}\,\delta_{\text{CI(11) CI(12)}}\,\delta_{\text{CI(13) CI(14)}}\,g^{\text{LI(9) LI(10)}}\,g^{\text{LI(11) LI(12)}}\,g^{\text{LI(13) LI(14)}}\,g_s^4\,\text{MomC(-k1)}\,\text{n1}^{\text{LI(9)}}\,\text{n2}^{\text{LI(11)}}\,\text{ColorLine}\left(T_{\text{CI(11)}}.T_{\text{CI(9)}},\,\{p,\,p\}\right)\right.\\&\quad\left(\left(g^{\text{LI(10) LI(13)}}\,g^{\text{LI(12) LI(14)}}-g^{\text{LI(10) LI(14)}}\,g^{\text{LI(12) LI(13)}}\right)f_{\text{e\$19 CI(13) CI(14)}}\,f_{\text{CI(10) CI(12) e\$19}}+\left(g^{\text{LI(10) LI(12)}}\,g^{\text{LI(13) LI(14)}}-g^{\text{LI(10) LI(14)}}\,g^{\text{LI(13) LI(12)}}\right)\right.\\&\qquad f_{\text{e\$20 CI(12) CI(14)}}\,f_{\text{CI(10) CI(13) e\$20}}+\left(g^{\text{LI(10) LI(12)}}\,g^{\text{LI(14) LI(13)}}-g^{\text{LI(10) LI(13)}}\,g^{\text{LI(14) LI(12)}}\right)f_{\text{e\$21 CI(12) CI(13)}}\,f_{\text{CI(10) CI(14) e\$21}}\Big)\\&\quad\left.\text{SpinLine}(\gamma\cdot n,\,\{p,\,p\})\right)\Big/\left(2\,\text{k2}^2\,(-p-\text{pe})^2\,(\text{k1}+p+\text{pe})^2\,\text{n1}\cdot(p+\text{pe})\,\text{n2}\cdot(p+\text{pe})\right)\end{aligned}$$

which translates to

$$\begin{aligned}&g_s^4\delta(-k_1)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar{u}(P)\not{n}u(P)}{2k_2^2(-p-p_e)^2(k_1+p+p_e)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)}\\&\left[\left(g^{l10l13}g^{l12l14}-g^{l10l14}g^{l12l13}\right)f^{e19c13c14}f^{c10c12e19}+\left(g^{l10l12}g^{l13l14}-g^{l10l14}g^{l13l12}\right)f^{e20c12c14}f^{c10c13e20}\right.\\&\left.+\left(g^{l10l12}g^{l14l13}-g^{l10l13}g^{l14l12}\right)f^{e21c12c13}f^{c10c14e21}\right]\Big|_{p_e=-xP^z\to-p,p=P,n\to z,k_2=l_2,n_1=n_2=n}\end{aligned}$$

Taking $k_1=p+p_e-l_1$, the first line (that's excluding the four-gluon vertex) becomes

$$g_s^4\delta(-k_1)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar{u}(P)\not{n}u(P)}{2k_2^2(-p-p_e)^2(k_1+p+p_e)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)} \tag{8}$$

$$=g_s^4\delta(l_1-p-p_e)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar{u}(P)\not{n}u(P)}{2l_2^2(-p-p_e)^2(2p+2p_e-l_1)^2n_1\cdot(p+p_e)n_2\cdot(p+p_e)} \tag{9}$$

$$=g_s^4\delta(l_1-p-p_e)\delta_{c13c14}g^{l13l14}n_1^{l10}n_1^{l12}t^{c10}t^{c12}\frac{\bar{u}(P)\not{n}u(P)}{2l_2^2l_1^2l_1^2n_1\cdot l_1n_2\cdot l_1} \tag{10}$$

Diagram 2xv gives

$$\begin{aligned}&\frac{-ig_s^4}{2}\bar{u}(P)\not{n}u(P)\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{\mathrm{d}^4l_2}{(2\pi)^4}n_\tau t^i\tilde{D}_G^{\tau\mu,ia}(l_1)\tilde{D}_G^{\sigma\lambda,dj}(l_1)\tilde{D}_G^{\nu\rho,bc}(l_2)n_\lambda t^j\frac{i}{n\cdot l_1+i\epsilon}\frac{i}{-n\cdot l_1+i\epsilon}\delta(l_1^z-(1-x)P^z)\\&\left[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\right]\end{aligned} \tag{11}$$

$$\begin{aligned}&=\frac{\overbrace{(-1)^3i^6}^{1}g_s^4}{2}\bar{u}(P)\not{n}u(P)\int\frac{\mathrm{d}^4l_1}{(2\pi)^4}\frac{\mathrm{d}^4l_2}{(2\pi)^4}\frac{n^\mu n^\sigma g^{\nu\rho}t^i\delta^{ia}t^j\delta^{dj}}{\left[l_1^2\right]^2\left[l_2^2\right]\left[n\cdot l_1\right]^2}\delta(l_1^z-(1-x)P^z)\\&\left[f^{abe}f^{cde}\left(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ace}f^{bde}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho}\right)+f^{ade}f^{bce}\left(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}\right)\right]\end{aligned} \tag{12}$$

Let's compare the color indices:

2.3 Numerical test (massless, ordered as Figure 2 and Figure 3, $z=1/4$)

2.3.1 Self-conjugated

10

```
-((0.20264236728467554288775892641945531671605206 80(3)*10^1)/ep)-0.13142890977(7)*10^2
```

11

```
-((0.3609567167(4))/ep^2)-(0.16328193(1)*10^1)/ep-0.6455896(1)*10^1
```

11 xiong Not handled eqn.

26

```
-0.4052847345693513
```

26 xiong

```
-0.405285
```

27

```
-((0.1899772193(3)*10^-1)/ep^2)+(0.871746(1)*10^-2)/ep-0.4783185(9)*10^-1
```

27 xiong

52

```
(0.225158185871861714319732140466061463017835631 1(1)*10^-1)/ep^3+(0.302543(3)*10^-2)/ep^2
↪ -(0.947637(3)*10^-1)/ep-0.692705(2)-I
↪ 0.0(1)*10^-5
```

71

```
-((0.40528473456935108577551785283891063343210413 60(3))/ep^2)-(0.19519788014(4)*10^1)/ep
↪ -0.823627359(1)*10^1
```

71 xiong

87

```
-((0.72050619478995748582314284949139668165707401 96(4))/ep)-0.5969939054(1)*10^1
```

87 xiong

100

```
-0.18012654869748937145578571237284917041426850 49(1)
```

116

```
(0.6291855(4)*10^-1)/ep^2-(0.5159034(4))/ep-0.1258567(2)*10^1
```

116 xiong Zero FIRE.py input file. AutoEnd Failed @ ALL possible bisections!

124

```
(0.270189823046234057183678568559273755621402 7573(3))/ep+0.16983474990(6)*10^1
```

124 xiong

125

```
(0.5066059182(3)*10^-1)/ep^2+(0.19628268(1))/ep+0.76838637(9)
```

125 xiong Not handled eqn.

128

```
(0.7100744(2))/ep+0.1478379(1)*10^1
```

128 xiong

129

```
-((0.2725043(8)*10^-1)/ep^2)-(0.1354903(1))/ep+0.265727(5)*10^-1
```

129 xiong

133

```
(0.2880197(3)*10^-1)/ep^2-(0.1100423(3))/ep+0.1811499(1)*10^1
```

133 xiong AutoEnd Failed @ ALL possible bisections!

143

```
0
```

143 xiong

144

```
0
```

2.3.2 Remaining diagrams

2 Not handled eqn.

1 Not handled eqn.

3

83 Not handled eqn.

85 Not handled eqn.

4

5 Zero FIRE.py input file.

113 Zero FIRE.py input file.

7

0

6

0

13

(0.16042520743(9))/ep^2-(0.24060641(3))/ep-0.5034791(3)

14

(0.16042520743(9))/ep^2-(0.24060641(3))/ep-0.5034791(3)

16

(0.72172896(3)*10^-1)/ep^2+(0.13916914(8))/ep+0.10188133(7)*10^1

18

(0.72172896(3)*10^-1)/ep^2+(0.13916914(8))/ep+0.10188133(7)*10^1

25

(0.8443431970(5)*10^-2)/ep^2+(0.99323076(2)*10^-1)/ep+0.32267783(2)

22

(0.8443431970(5)*10^-2)/ep^2+(0.99323076(2)*10^-1)/ep+0.32267783(2)

88

-((0.30021091(3)*10^-1)/ep^3)+(0.11491921(3)+I 0.18862808(2))/ep^2+(0.1510373(2)*10^1+I
↪ 0.13143394(4)*10^1)/ep+I 0.669150(1)*10^1+0.1158914(9)*10^2

30

-((0.300210915(7)*10^-1)/ep^3)+(-0.11023898(3)-I 0.18862808(1))/ep^2+(-0.5735834(7)-I
↪ 0.13143394(4)*10^1)/ep-0.227073(4)*10^1-I 0.6691519(9)*10^1

31 Numbers overflow.

106 Not handled eqn. Failed in GiNaC_Parallel!

3.11

32 Numbers overflow.

82 Not handled eqn.

3.12

38

-((0.450316371743723428639464280932122926035671262 2(3)*10^-1)/ep^2)+(-0.26393587(2)+I
↪ 0.14147106(4))/ep-0.6590036(3)+I 0.6663843(5)

62

```
-((0.450316371743723428639464280932122926035671262 2)*10^-1)/ep^2)+(-0.26393587(2)-I
↪ 0.14147106(1))/ep-0.65900358(7)-I 0.6663842(5)
```

89 (3MIs)

```
(0.18012654869748937145578571237284917041426850 49(1))/ep^2+(0.6273774024(2))/ep
↪ +0.2771696389(5)*10^1
```

39 (2MIs)

```
(0.18012654869748937145578571237284917041426850 49(1))/ep^2+(0.6273774024(2))/ep
↪ +0.2771696389(5)*10^1
```

40

```
-((0.558142 - 0.565884 I)/ep) - (3.52786 - 2.48104 I)
```

97

```
(-0.5581423(1)-I 0.565884242(3))/ep-I 0.24810455(5)*10^1-0.3527861(1)*10^1
```

3.15

```
-(0.360253/ep)
```

74 Zero FIRE.py input file.

41 Failed in GiNaC_Parallel! power::eval(): division by zero.

54

```
-((0.72050619478996(4))/ep)-I 0.226353697(2)*10^1-0.45289267(6)*10^1
```

80

```
-((0.720506194789957(2))/ep)+I 0.2263536968(3)*10^1-0.45289267(5)*10^1
```

4.6

81

```
(0.1801265486974894(3))/ep^2+(0.78974126(8)-I 0.56588424(2))/ep+0.2441972(1)*10^1-I
↪ 0.24810455(7)*10^1
```

55

```
(0.1801265486974 9(3))/ep^2+(0.78974126(9)+I 0.565884242(4))/ep+0.24419718(8)*10^1+I
↪ 0.24810453(5)*10^1
```

3.14

84

```
(0.1395356(1)+I 0.1414710(2))/ep^2+(0.6166541(5)+I 0.3512700(5))/ep+I
↪ 0.1219312(2)*10^1+0.1898527(2)*10^1
```

56

```
(0.1395356(2)-I 0.1414711(2))/ep^2+(0.6166541(7)-I 0.3512700(8))/ep-I
↪ 0.1219311(2)*10^1+0.1898527(2)*10^1
```

4.2

86

```
-((0.900632743487447(2)*10^-1)/ep)+I 0.2829421211(4)-0.5661159(5)
```

58

```
-((0.90063274348745(5)*10^-1)/ep)-I 0.282942121(3)-0.56611583(7)
```

95

```
(0.2251581858718617(4)*10^-1)/ep^2+(0.9871766(7)*10^-1-I
↪ 0.70735530(2)*10^-1)/ep+0.305247(1)-I 0.3101307(3)
```

59

```
(0.22515818587186(3)*10^-1)/ep^2+(0.9871766(1)*10^-1+I
↪ 0.707355303(5)*10^-1)/ep+0.3052465(1)+I 0.31013067(6)
```

60

```
(0.45031637(2)*10^-1)/ep^2+(0.14234890(2)-I 0.70735530(8)*10^-1)/ep+0.2902786(2)-I
↪ 0.3562536(3)
```

63

```
(0.4503163717437(1)*10^-1)/ep^2+(0.14234890(7)+I 0.70735530(2)*10^-1)/ep+0.2902786(8)+I
↪ 0.3562535(2)
```

61

```
-((0.11257909294(3)*10^-1)/ep^3)+(-0.141816(2)*10^-1+I
↪ 0.176839(2)*10^-1)/ep^2+(-0.61242(7)*10^-2+I 0.554395(8)*10^-1)/ep+0.120741(2)+I 0.225341(2)
```

101

```
-((0.11257909293593(2)*10^-1)/ep^3)+(-0.1418158(7)*10^-1-I
↪ 0.176838826(4)*10^-1)/ep^2+(-0.61242(3)*10^-2-I 0.5543947(5)*10^-1)/ep+0.120741(1)-I
↪ 0.2253407(6)
```

64

```
(0.675474558(3)*10^-1)/ep^3+(0.55505288(5)-I 0.42441318(2))/ep^2+(0.24553798(8)*10^1-I
↪ 0.31694700(3)*10^1)/ep+0.5380347(5)*10^1-I 0.17422348(5)*10^2
```

104

```
(0.675474558(2)*10^-1)/ep^3+(0.48750543(5)+I 0.42441318(1))/ep^2+(0.19366015(6)*10^1+I
↪ 0.31694701(3)*10^1)/ep+0.2877105(4)*10^1+I 0.17422349(4)*10^2
```

136

102 Not handled eqn.

103

```
(0.750527286(1)*10^-2)/ep^3+(0.13063137(6)*10^-1-I
↪ 0.1178926(2)*10^-1)/ep^2+(-0.281112(5)*10^-1+I 0.376194(2)*10^-1)/ep+I
↪ 0.243805(2)-0.58512(3)*10^-1
```

108

```
(0.7505272862(2)*10^-2)/ep^3+(0.13063137(6)*10^-1+I
↪ 0.1178926(2)*10^-1)/ep^2+(-0.281109(4)*10^-1-I 0.376194(2)*10^-1)/ep-I
↪ 0.243805(3)-0.58513(2)*10^-1
```

127

```
-((0.7505272862(2)*10^-2)/ep^3)+(-0.3668912(2)*10^-1+I
↪ 0.1178926(4)*10^-1)/ep^2+(-0.1256239(4)-I 0.376194(2)*10^-1)/ep-I 0.24381(1)-0.617659(3)
```

105

```
-((0.7505272862(1)*10^-2)/ep^3)+(-0.20284742(6)*10^-1-I
↪ 0.1178926(2)*10^-1)/ep^2+(0.569092(5)*10^-1+I 0.1103759(2))/ep+0.312777(2)+I 0.679341(2)
```

131

```
-((0.21235731(2)*10^-1)/ep^2)-(0.7878081(2)*10^-1)/ep-0.4849994(2)
```

110

```
-((0.21235731(2)*10^-1)/ep^2)-(0.7878081(2)*10^-1)/ep-0.4849994(1)
```

121

```
-((0.4503163717(1)*10^-1)/ep^2)-(0.143889556(4))/ep-0.9841648(2)
```

111

```
-((0.4503163717(1)*10^-1)/ep^2)-(0.143889556(4))/ep-0.9841648(2)
```

126

```
(0.10132118364(2))/ep^2+(0.154882861(5))/ep+0.8068632(2)
```

117

```
(0.10132118364(2))/ep^2+(0.154882861(5))/ep+0.8068632(2)
```

123

```
-((0.2251581859(1)*10^-1)/ep^2)+(0.27844106(4)*10^-1)/ep+0.2197552(4)*10^-1
```

119

```
-((0.2251581859(1)*10^-1)/ep^2)+(0.27844106(4)*10^-1)/ep+0.2197552(4)*10^-1
```

135

```
(0.8824885(6)*10^-2)/ep^2-(0.4259655(6)*10^-1)/ep-0.1357314(2)
```

134

```
(0.8824885(5)*10^-2)/ep^2-(0.4259655(5)*10^-1)/ep-0.1357314(2)
```

3.4

142

```
0
```

139

```
0
```

2.4 Numerical test (ordered as Figure 2 and Figure 3, $z = 1/4$)

2.4.1 Self-conjugated

10

```
-13.142892209573214 - 2.0264236728467586/ep
```

11

```
(-0.721913433451656621537641175369309494168040468603083244938
↪ log(s)-0.214490046(8))/ep+0.721913433451656621537641175369309494168040468603083244938
↪ log^2(s)-0.2836658516(3)*10^1 log(s)-0.327680(1)*10^1
```

11 xiong Not handled eqn.

```
968.417+24.7655/ep
```

26

```
-0.4052847345693513
```

26 xiong

```
-0.405285
```

27

```
(0.95364928(1)*10^-1-0.037995443865876664291454798703647857472224696482761990896
↪ log(s))/ep+0.037995443865876664291454798703647857472224696482761990896 log^2(s)-0.1732949391(5)
↪ log(s)+0.174213(4)
```

27 xiong

```
50.484+1.38493/ep
```

52 Not handled eqn.

```
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
NAN=198 v.s. RUN=99
smin = -1, optimized lambda NOT found!
JPCquasiR: SD.cpp:3732: void HepLib::SD::Integrates(const char*, const char*, int):
↪  Assertion `false' failed.
```

71

```
(0.40528473456935108577551785283891061393612511304014832705485
↪  (-log(s))-0.423786494(1))/ep+0.20264236728467554288775892641945530696806255652007416352742
↪  log^2(s)-0.1528192308(1)*10^1 log(s)-0.49684439(1)*10^1
```

71 xiong

```
(-0.405285 log(s)-0.423787)/ep+0.202642 log^2(s)-1.52819 log(s)-4.96844
```

87

```
-((0.7205061947899575(4))/ep)-0.5969939(1)*10^1
```

87 xiong

```
-5.96994 - 0.720506/ep
```

100

```
-0.18012654869748937145578590196646876085951030436161682934 42
```

116 Not handled eqn.

```
Failed IDs: {12, 13, 20, 21, 22, }
```

116 xiong Zero FIRE.py input file. AutoEnd Failed @ ALL possible bisections!

124

```
(0.270190 CV(1,3))/ep+1.69835 CV(1,3)
```

124 xiong

```
(0.27019 CV(1,3))/ep+1.69835 CV(1,3)
```

125 Not handled eqn.

```
(CV(1,3) (0.101321 log(s)+0.0158833))/ep+0.151982 (-CV(1,3)) log^2(s)-I 0.318310 CV(1,3)
↪  log(s)+0.473347 CV(1,3) log(s)+(0.0(5)*10^-7 CV(1,3))/s+(I 0.3(3)*10^-7 CV(1,3))/s+I
↪  0.48(4)*10^1 CV(1,3)+0.83(4)*10^1 CV(1,3)
```

125 xiong Not handled eqn.

128

```
        -((0.0(5)*10^-7 CV(1,3))/ep^2)+((0.81056947(3) CV(1,3))/s+0.0(4)*10^-7 CV(1,3)
↪  log(s)+0.2026424(3) CV(1,3))/ep+0.5074321(1) CV(1,3) log(s)-(0.810569 CV(1,3)
↪  log(s))/s+(0.45612729(2)*10^1 CV(1,3))/s+0.0(2)*10^-7 (-CV(1,3)) log^2(s)+0.13766891(1)*10^2
↪  CV(1,3)
```

128 xiong

```
        ((0.810569 CV(1,3))/s+0.202642 CV(1,3))/ep+(4.56127 CV(1,3))/s-(0.810569 CV(1,3)
↪  log(s))/s+0.507432 CV(1,3) log(s)+13.7669 CV(1,3)
```

129 Not handled eqn.

```
        ((0.500352 CV(1,3))/s+0.500352 CV(1,3)+(0.0180127 -2.54985*10^-8
↪  I)/s-(9.42776*10^-8+7.15663*10^-8 I))/ep+(2.78522 CV(1,3))/s-(0.500352 CV(1,3)
↪  log(s))/s-0.500352 CV(1,3) log(s)+2.78522 CV(1,3)+(0.526293 -0.0565893 I)/s+(0.0725309
↪  -4.81307*10^-8 I) log(s)+(0.253132 -0.452708 I)+(0.148304 log(s))/s
```

129 xiong

```
((0.810569 CV(1,3))/s+0.202642 CV(1,3))/ep+(4.56127 CV(1,3))/s-(0.810569 CV(1,3) log(s))/s+0.507432
↪  CV(1,3) log(s)+13.7669 CV(1,3)
```

133 Not handled eqn.

```
        AutoEnd Failed @ ALL possible bisections!
        JPCquasiR: SD.cpp:138: std::vector<std::pair<GiNaC::container<std::list>,
        ↪  GiNaC::container<std::list> > >
        ↪  HepLib::SD::AutoEnd(std::pair<GiNaC::container<std::list>, GiNaC::container<std::list>
        ↪  >): Assertion `false' failed.
```

133 xiong AutoEnd Failed @ ALL possible bisections!

143

```
        0
```

143 xiong

```
AutoEnd Failed @ ALL possible bisections!
```

144

```
        0
```

2.4.2 Remaining diagrams

2

1 Not handled eqn.

```
    -((0.455945 log(s)+0.2886892(1)+I 0.143239449(8)*10^1)/ep^2)-(0.227973 (-s) log^2(s)+s
↪ (0.0(1)*10^6-I 0.3(3)*10^6)+(1.21585+s (0.424773500(9)*10^1+I 0.15915495(6)))
↪ log(s)+0.0(1)*10^6-I 0.3(2)*10^6)/(ep s)-(3.64756 log(s))/s^2-(0.0(4)*10^6)/s^2+(I
↪ 1.0(7)*10^6)/s^2-0.0759909 log^3(s)+(0.607927 log^2(s))/s-(22.2444 log(s))/s+(0.0(2)*10^7)/s+(I
↪ 0.2(4)*10^7)/s+I 0.7957747(3)*10^-1 log^2(s)+0.212386750(4)*10^1 log^2(s)-I 0.13271482(2)*10^1
↪ log(s)-0.214233808(5)*10^2 log(s)+0.1(2)*10^7+I 0.3(4)*10^7
```

3

```
    -((2.67042 +2.12028 I)/s^2)-((0.260912 -1.01859 I) log(s))/s^2+(14.6734 +1.31932
↪ I)/s-((0.324228 +3.72868*10^-9 I) log^2(s))/s+(0.222907 -5.53015*10^-9 I) log^2(s)+((0.881442
↪ -3.43775 I) log(s))/s+(0.673891 -1.68704 I) log(s)+(13.4354 -26.5232
↪ I)+(-((4.70467*10^-9+1.04672*10^-9 I)/s^2)-(1.49279*10^-8+4.88207*10^-9
↪ I)/s+(-3.02109*10^-8-6.73934*10^-9 I))/ep^2+(1/ep)(-((2.61261*10^-8+1.68009*10^-8
↪ I)/s^2)-((4.70467*10^-9+1.04672*10^-9 I) log(s))/s^2-(1.30934*10^-7+9.62535*10^-8
↪ I)/s-((1.49279*10^-8+4.88207*10^-9 I) log(s))/s-(3.02109*10^-8+6.73934*10^-9 I)
↪ log(s)-(3.63837*10^-7+2.4728*10^-7 I))-((3.64128*10^-9+8.50597*10^-10 I) log^2(s))/s^2
```

83 Not handled eqn.

85 Not handled eqn.

4

```
    -((2.42703 -0.625114 I)/s)+((0.0810569 -3.18897*10^-9 I) log^2(s))/s-(0.0405285
↪ +2.8617*10^-9 I) log^2(s)+((0.0994902 +1.01859 I) log(s))/s+(0.0313118 -0.509296 I)
↪ log(s)+(1.1822 +0.196737 I)+((-5.16401*10^-10-3.70072*10^-9 I)-(5.50793*10^-10+3.94743*10^-9
↪ I)/s)/ep^2+(-((2.78596*10^-8+7.69087*10^-8 I)/s)-((5.50793*10^-10+3.94743*10^-9 I)
↪ log(s))/s-(5.16401*10^-10+3.70072*10^-9 I) log(s)-(3.69889*10^-8+6.98559*10^-8 I))/ep
```

5

```
    (-((0.34195899479288997862309318833283073871522184913))/s)-0.12918450914(7)*10^1)/ep^2+(1/ep)(-((0.1
↪ log^2(s)+(0.34195899479288997862309318833283074349804313682(7)
↪ log(s))/s-(0.8382561761(1)*10^1)/s+0.12918450914(6)*10^1
↪ log(s)-0.1470414820(6)*10^2)-(0.30776309531360098076078386949954766506204314813(3)*10^1)/s^3-(0.3540645512
↪ log(s))/s^2+0.11398633159762999287436439611094360171680492877(1)
↪ log^3(s)-(0.51293849218933496793463978249924613414931972596(7)
↪ log^2(s))/s-(0.132919978(2)*10^3)/s-0.19124846658(3)*10^1 log^2(s)+(0.104343157298(2)*10^2
↪ log(s))/s+0.2985081056(3)*10^2 log(s)-0.17571608(1)*10^3
```

113

```
    (-((0.34195899479288997862309318833283073734308474213(3))/s)-0.14184965710(7)*10^1)/ep^2+(1/ep)(-((0.1
↪ log^2(s)+(0.34195899479288997862309318833283073307983918224(6)
↪ log(s))/s-(0.99213772376(4)*10^1)/s+0.14184965710(6)*10^1
↪ log(s)-0.1937716819(6)*10^2)-(0.30776309531360098076078386949954766367304963583(3)*10^1)/s^3-(0.4002290155
↪ log(s))/s^2+0.11398633159762999287436439611094359096909904074(1)
↪ log^3(s)-(0.51293849218933496793463978249924612373111577138(6)
↪ log^2(s))/s-(0.166808716(2)*10^3)/s-0.24887488978(3)*10^1 log^2(s)+(0.119731312064(2)*10^2
↪ log(s))/s+0.3760146151(3)*10^2 log(s)-0.22781445(1)*10^3
```

7

```
    0
```

6

0

4.9

0

13 Zero FIRE.py input file.

14

(0.108076/s^2-0.770041/s)/ep+1.09363/s^2-6.67758/s-1.08404*10^-6

16

-((0.211474 +1.27324 I)/s^2)+(2.28897 -1.08225 I)/s-(4.7286 -0.668451
↪ I)+(-(6.7167*10^-16/s^2)+0.486342/s-0.729513)/ep-(3.00378*10^-16 log(s))/s^2-(0.486342
↪ log(s))/s+0.729513 log(s)

18 Zero FIRE.py input file.

25 Zero FIRE.py input file.

22

(0.0810569/s-0.054038/s^2)/ep-0.411721/s^2+0.617579/s

88 Zero FIRE.py input file.

30

-((0.394011 +1.25922*10^-8 I)/s)+(0.782248 -2.46415*10^-8 I)+(0.168118 log(s))/s-0.354249
↪ log(s)

31 Numbers overflow.

-((0.162114 +1.91875*10^-8 I)/ep^3)+(-0.324228 log(s)-(92.0766 +2.65448 I))/ep^2+1/ep
↪ (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) log(s)-0.0405285 log^2(s)-(1.58061
↪ log(s))/s-(1.91238*10^10+1.98727*10^10 I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46
↪ I)/s-(16.9633 +0.31831 I) log^2(s)-(4.74183 log(s))/s^2+0.148604 log^3(s)+(0.790305
↪ log^2(s))/s+(81.2751 log(s))/s+(4.05197*10^10-6.80923*10^10 I)
↪ log(s)+(6.59315*10^13-1.55809*10^14 I)

106 Not handled eqn. Failed in GiNaC_Parallel!

3.11

32 Numbers overflow.

-((0.162114 +1.91875*10^-8 I)/ep^3)+(-0.324228 log(s)-(92.0766 +2.65448 I))/ep^2+1/ep
↪ (-((158.541 +3.48593 I)/s)+(32.6052 +0.63662 I) log(s)-0.0405285 log^2(s)-(1.58061
↪ log(s))/s-(1.91238*10^10+1.98727*10^10 I))-(475.619 +10.4578 I)/s^2-(12502.7 +1351.46
↪ I)/s-(16.9633 +0.31831 I) log^2(s)-(4.74183 log(s))/s^2+0.148604 log^3(s)+(0.790305
↪ log^2(s))/s+(81.2751 log(s))/s+(4.05197*10^10-6.80923*10^10 I)
↪ log(s)+(6.59315*10^13-1.55809*10^14 I)

82 Not handled eqn.

3.12

31+32

38 Zero FIRE.py input file.

62 Zero FIRE.py input file.

89 Divergent integrand.

39 Divergent integrand.

40

```
-((0.3602531(6))/ep)+0.1978893(4) (-log(s))+I 0.5658842(7) log(s)-I
↪ 0.72868(2)-0.170150(1)*10^1
```

97

```
-((0.3602531(4))/ep)+0.1978892(3) (-log(s))-I 0.5658841(5) log(s)+I
↪ 0.72868(1)-0.170150(1)*10^1
```

3.15

```
-(0.360253/ep)
```

74 Zero FIRE.py input file.

41 Failed in GiNaC_Parallel! power::eval(): division by zero.

54

```
-((0.72050619478996(4))/ep)-I 0.226353697(2)*10^1-0.45289267(6)*10^1
```

80

```
-((0.720506194789957(2))/ep)+I 0.2263536968(3)*10^1-0.4528927(6)*10^1
```

4.6

```
-2*(0.720506/ep)
```

81

```
((0.180127 -9.13154*10^-9 I) log(s)-(0.231946 +7.6754*10^-8 I))/ep+(1.02169 -0.565884 I)
↪ log(s)-(0.815813 -0.728676 I)-(5.2095*10^-10+1.21468*10^-8 I)/ep^2+(-0.0900633-4.02323*10^-9 I)
↪ log^2(s)
```

55

```
((0.288202 -2.40402*10^-8 I)/s-(0.432304 +3.26984*10^-8 I))/ep+(1.81157 +0.905414
↪ I)/s-(2.71736 +1.35812 I)
```

3.14

```
2*(0.180127 log(s)-0.231946)/ep
```

84

```
(-0.900633(1)*10^-1 log(s)-0.154217(3)-I 0.0(4)*10^-5)/ep+I 0.14147106(8)
↪ log^2(s)+0.27463050(8) log^2(s)-I 0.364339(2) log(s)-0.823115(2) log(s)+I
↪ 0.2156(8)-0.11134(8)*10^1
```

56

```
(-0.1125791(1) log(s)+I 0.0(3)*10^-5-0.80192(3)*10^-1)/ep-I 0.1414711(1)
↪ log^2(s)+0.2858884(1) log^2(s)+I 0.364340(2) log(s)-0.975562(2) log(s)-0.66278(3)-I 0.0(5)*10^10
```

4.2

```
2*(0.115973 -0.0900633 log(s))/ep
```

86

```
-((0.900632743487447(2)*10^-1)/ep)+I 0.2829421211(4)-0.566116(6)
```

58

```
-((0.9006327434874(2)*10^-1)/ep)-I 0.282942121(3)-0.56611583(7)
```

4.7

```
-2*(0.0900633/ep)
```

95

```
(0.225158185872(7)*10^-1 log(s)-0.289932(5)*10^-1)/ep+0.1277109(3)
↪ log(s)-0.112579092936(3)*10^-1 log^2(s)-I 0.707355(5)*10^-1 log(s)-0.10198(1)+I 0.9108(1)*10^-1
```

59

```
(0.225158185871861714319732140466061 33(7)*10^-1 log(s)-0.289932(5)*10^-1)/ep+0.1277109(2)
↪ log(s)-0.112579092935930857159866070233030 66(3)*10^-1 log^2(s)+I 0.707355(2)*10^-1
↪ log(s)-0.101977(9)-I 0.9108(1)*10^-1
```

4.1

```
2*(0.0289932 -0.0225158 log(s))/ep
```

60 Zero FIRE.py input file.

63 Zero FIRE.py input file.

61

```
((0.0249708 +0.0565884 I)/s-(0.0409356 +0.113177 I)-(0.0180127 log(s))/s+0.0360253
↪ log(s))/ep+(0.121805 +0.302986 I)/s-((0.0819104 +0.0282942 I) log(s))/s+(0.123545 +0.12025 I)
↪ log(s)-(0.132015 +0.412085 I)+((-1.70997*10^-8-7.51261*10^-9 I)-(3.80204*10^-10+2.72486*10^-9
↪ I)/s)/ep^2-1.51762*10^-7/s^2+(0.00900633 log^2(s))/s-0.0180127 log^2(s)
```

101

```
((0. -5.44316*10^-9 I) log(s)-(5.06726*10^-8+1.09474*10^-7 I))/ep^2+((0.0301033 -0.0353678
↪ I) log(s)-(0.0200969 -0.045542 I)-(1.32299*10^-8+4.49679*10^-9 I)/s+(-0.0112579-3.05202*10^-9 I)
↪ log^2(s))/ep+(0.0112579 -9.07187*10^-10 I) log^3(s)-(0.0593367 -0.0353677 I) log^2(s)+(0.0923378
↪ -0.190433 I) log(s)-(0.0423955 -0.117755 I)-(1.47432*10^-10+8.25276*10^-9
↪ I)/ep^3-(4.6093*10^-7+4.84678*10^-8 I)/s
```

64 Zero FIRE.py input file.

104 Zero FIRE.py input file.

136

```
(-((0.00499433 +0.0113178 I)/s)+(0.000695404 +0.00565863 I)+(0.00360253 log(s))/s-0.00180128
↪ log(s))/ep+(0.232843 -0.0984924 I)/s+((0.0363448 +0.0226354 I) log(s))/s+(0.0153548
↪ -1.05329*10^-7 I) log(s)-(0.653916 -3.42529 I)+((-1.16122*10^-8-2.31214*10^-9
↪ I)-(4.75779*10^-9+2.54986*10^-9 I)/s)/ep^2-2.41702*10^-7/s^2-(0.0054038 log^2(s))/s+0.00270189
↪ log^2(s)
```

102 Not handled eqn.

103 Zero FIRE.py input file.

108 Zero FIRE.py input file.

127 Zero FIRE.py input file.

105 Zero FIRE.py input file.

131

```
(0.027019 -0.0180127/s)/ep+0.0890396/s+(0.0180127 log(s))/s-0.027019 log(s)+0.197988
```

110 Zero FIRE.py input file.

121 Zero FIRE.py input file.

111 Zero FIRE.py input file.

126

```
-(0.478416/(ep s))-3.57259/s+(0.478416 log(s))/s
```

117

```
(0.337737 log(s)+(12.5141 +0.065117 I))/ep+(4.9247 +0.043133 I)/s-(7.6631 +0.248286 I)
↪ log(s)+(960.52 -2.012 I)-0.371511 log^2(s)+(0.0506606 log(s))/s
```

123 Zero FIRE.py input file.

119

```
((0.108076 CV(1,3))/s-(0.0720506 CV(1,3))/s^2)/ep-(0.621011 CV(1,3))/s^2-(0.221294
↪ CV(1,3))/s+0.51236 CV(1,3) log(s)+0.289007 CV(1,3)
```

135

```
    -((0.75052728623953904773244046822021(1)*10^-2)/ep^2)+(-((0.22164008921761387503348633257712
↪ 791698673512155(5)*10^-1)/s)+0.32835568772979833338294270484634 2(2)*10^-1
↪ log(s)+0.1195769047(6))/ep-(0.66492026765284162510045897731383754797319145 60(6)*10^-1)/s^2-(0.4
↪ 6125458376(8))/s-0.40311198621749568716728944276108 3(2)*10^-1
↪ log^2(s)+(0.22164008921761387503348632577127917203974334809(4)*10^-1
↪ log(s))/s-0.275899853(5)*10^-1 log(s)+0.935583(2)
```

134

```
    (0.93815910779942380966555(8)*10^-3)/ep^2+(-((0.1583143494(1)*10^-1)/s)+0.24392136802785019
↪ 0513043(1)*10^-1
↪ log(s)+0.1710538662(7))/ep-(0.47494304832345830364318498379559824599397842 69(4)*10^-1)/s^2-(0.3
↪ 21325679(2))/s-0.33978624644103458001486 5(1)*10^-1 log^2(s)+(0.15831434944(1)*10^-1
↪ log(s))/s-0.917320947(5)*10^-1 log(s)+0.1254593(2)*10^1
```

142

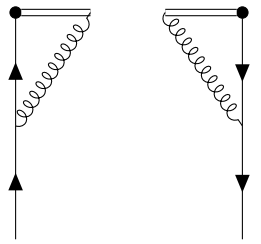
```
    0
```

139

```
    0
```

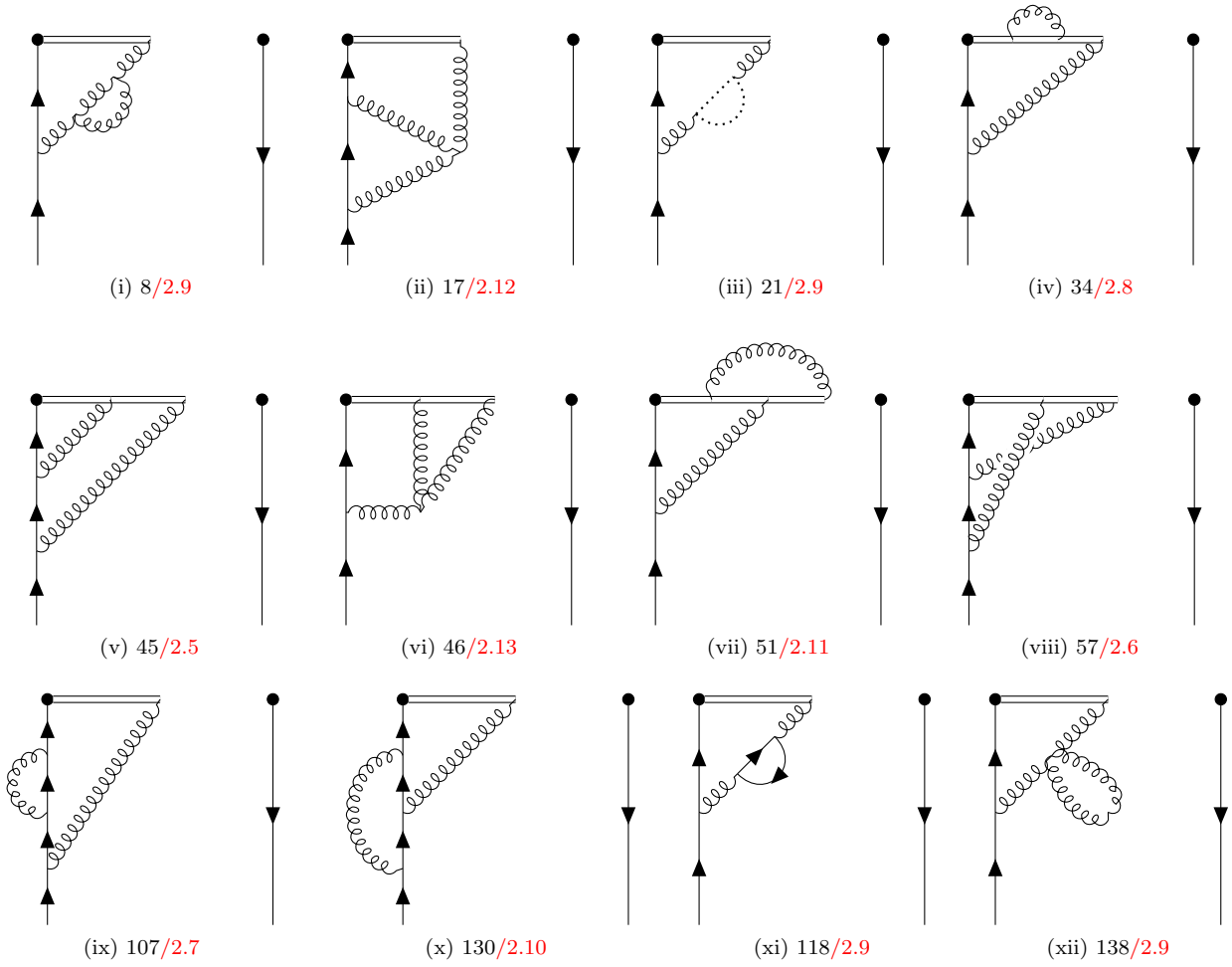
3 Virtual Diagrams (Excluding Gauge Link Self-Energy Diagrams)

3.1 All diagrams



(i) 75

Figure 4: All self-conjugated virtual diagrams (actually there's only one).



(i) 8/2.9 (ii) 17/2.12 (iii) 21/2.9 (iv) 34/2.8

(v) 45/2.5 (vi) 46/2.13 (vii) 51/2.11 (viii) 57/2.6

(ix) 107/2.7 (x) 130/2.10 (xi) 118/2.9 (xii) 138/2.9

Figure 5: All virtual diagrams (excluding conjugated and self-conjugated diagrams), red n.i marks the diagram number in Ji&Zhang's paper.

3.2 Numerical test (ordered as Figure 4 and Figure 3, $z = 1/4$)

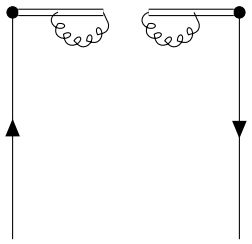
3.2.1 Self-conjugated

75

3.2.2 Remaining diagrams (exclude conjugated ones)

4 Gauge Link Self-Energy Diagrams

4.1 All diagrams



(i) 8

Figure 6: All self-conjugated gauge link self-energy diagrams (actually there's only one).

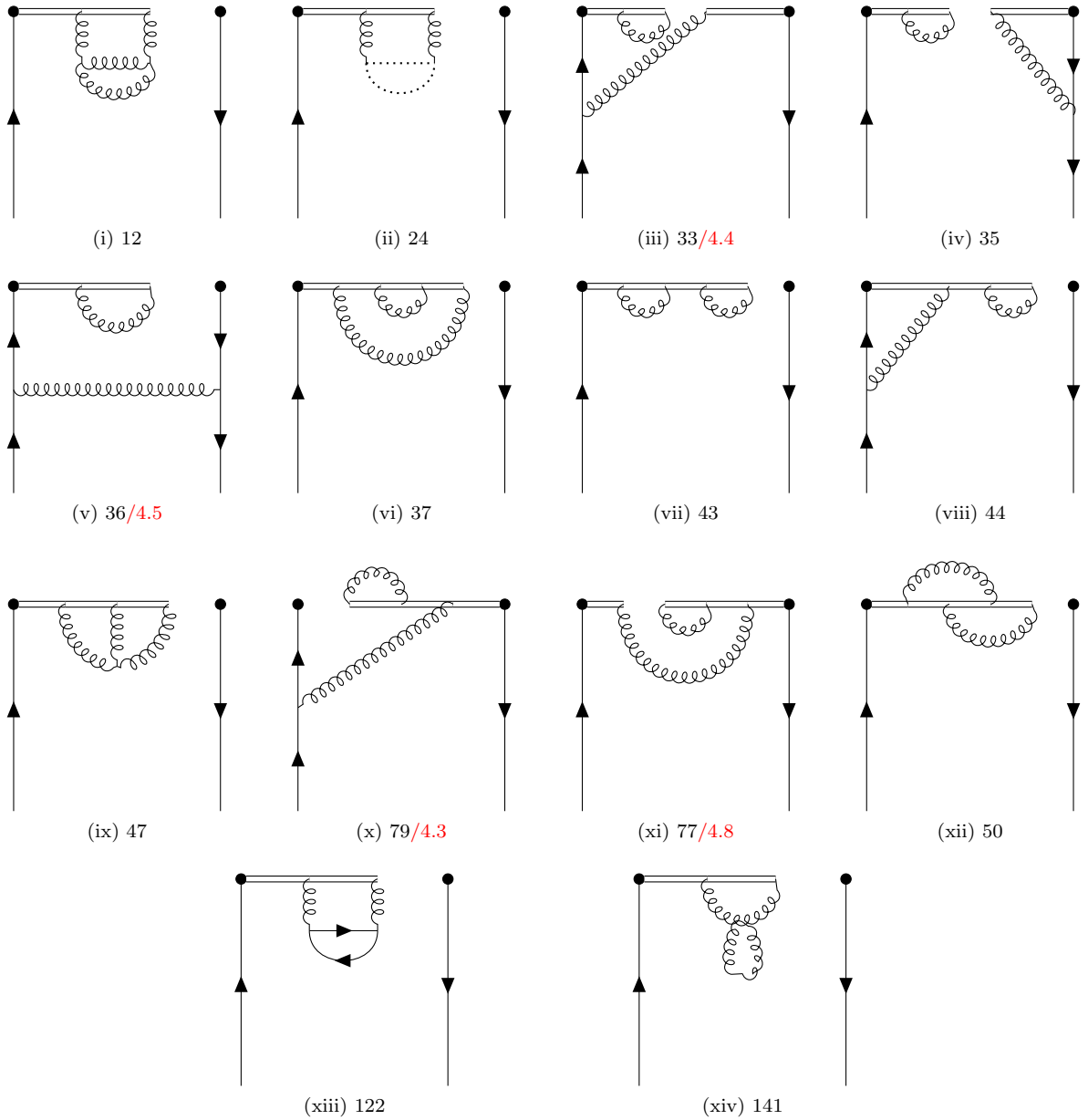
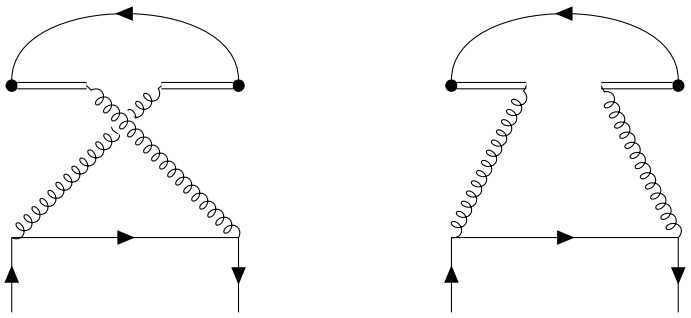


Figure 7: All gauge link self-energy diagrams (excluding conjugated and self-conjugated diagrams), red n.i marks the diagram number in Ji&Zhang's paper.

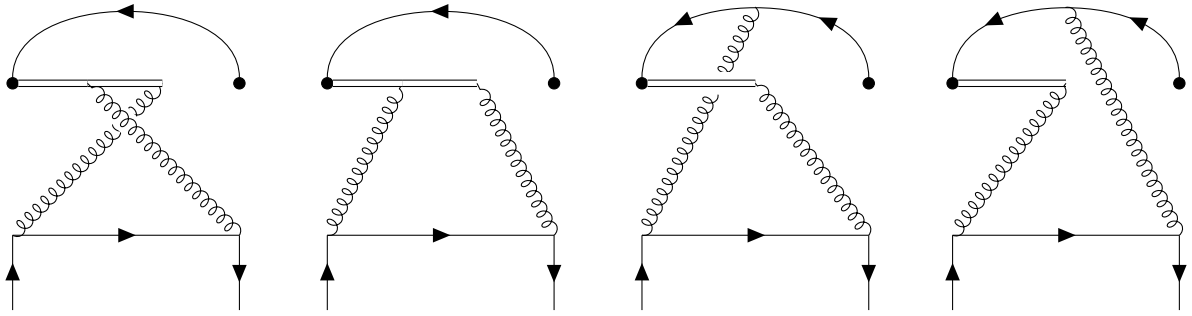
5 Diagrams with Direct Contracting $\bar{\psi}(z)\psi(0)$

5.1 All diagrams



(a) 53 (b) 76

Figure 8: All self-conjugated quark contraction diagrams



(a) 28 (b) 29 (c) 42 (d) 132

Figure 9: All quark contraction diagrams (excluding conjugated and self-conjugated diagrams).

6 HQET Correspondence

References

[Ji and Zhang(2015)] X. Ji and J.-H. Zhang, Phys. Rev. **D92**, 034006 (2015), arXiv:1505.07699 [hep-ph] .