Hadron Spectroscopy

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1. Prove Landau-Yang theorem.

   For any vector particles,we can always write the field operator as a single vector field.

   $$A_\mu(x) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\sqrt{2|\mathbf{k}|}}\sum_\lambda (a_{\mathbf{k}}^\lambda \epsilon_\mu^\lambda(k)e^{-ik\cdot x} + a_{\mathbf{k}}^{\lambda\dagger}\epsilon_\mu^{\lambda *}(k)e^{ik\cdot x})$$

   Then the feynman rules can be easily derived. The amplitude of *vector* $\to \gamma\gamma$ is

   $$i\mathcal{M} = \epsilon_1^{*\mu}(p_1)\epsilon_2^{*\nu}(p_2)\epsilon^\alpha(p)\Gamma_{\mu\nu\sigma}$$

   since it must obey Lorentz-invariant

   $$= (\epsilon_1\cdot\epsilon_2)(a_1\epsilon\cdot p_1 + a_2\epsilon\cdot p_2) + a_3(\epsilon_1\cdot\epsilon)(\epsilon_2\cdot p_1) + a_4(\epsilon_2\cdot\epsilon)(\epsilon_1\cdot p_2)$$

   final states symmetry (identical), $a_1 = a_2$, first term vanishes. And $\epsilon_2\cdot p_1 = \epsilon_1\cdot p_2 = 0$

   $$= 0$$

2. $\eta \to \pi\pi$

   For η meson, $I^G J^{PC} = 0^+0^{-+}$, for π meson, $I^G J^{PC} = 1^-0^{-+}$. Charge parity conservation gives the final state angular momentum must be even, so $\pi\pi$ system gives positive parity, parity is not conserved. (For $\pi^0\pi^0$ system, use identical particle instead.)

3. $\eta \to \pi\pi\pi$

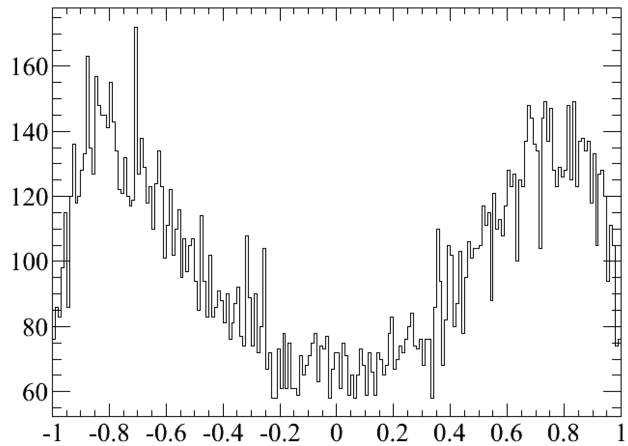
   From previous discussion, we know that this reaction can happen not only under weak interaction for P parity and C parity are conserved. But G parity is not conserved, the final state G parity is negative while the initial state is positive, so it must not be a strong interaction.

4. $\rho \to \pi\pi$

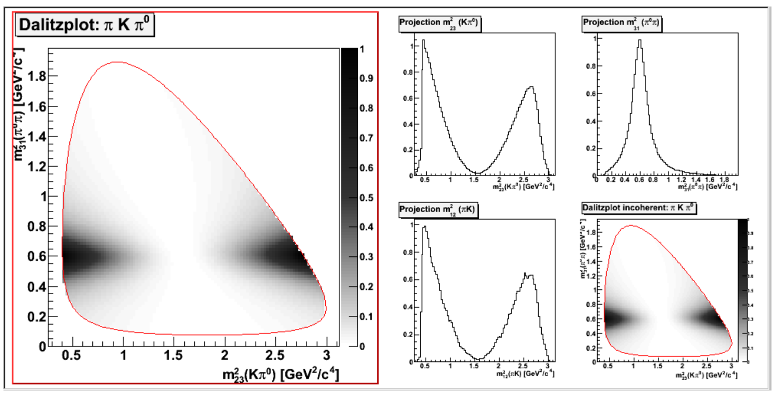
   For ρ meson, $I^G J^{PC} = 1^+1^{--}$. For angular momentum conservation, the final state orbital angular momentum must be $L = 1$ while the spin S is zero. For $\pi^0\pi^0$ scenario, $L + S = even$ is not guaranteed. For $\pi^+\pi^-$ scenario, $CP = (-)^S = +$, no obvious violation, it can happen.

5. $\omega \to \pi^0\pi^0\pi^0$

   For ω meson, $I^G J^{PC} = 0^-1^{--}$. Apparently CP violation.



6. Dalitz plot.



7. $\eta - \eta'$ mixing.

$$\eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

and

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_8 \end{pmatrix}$$

For η and η' to have the same $s\bar{s}$ and $(u\bar{u} + d\bar{d})/\sqrt{2}$ contents, it must have $\tan\theta_P = \frac{1-\sqrt{2}}{1+\sqrt{2}}$, which leads to $\theta_P = 9.7°$.

$\omega - \phi$ mixing.

$$\omega_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$
$$\omega_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

and

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_8 \end{pmatrix}$$

To have

$$\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
$$\phi = s\bar{s}$$

one must make $\frac{1}{\sqrt{3}}\cos\theta + \frac{2}{\sqrt{6}}\sin\theta = 0$ and $-\frac{1}{\sqrt{3}}\sin\theta + \frac{1}{\sqrt{6}}\cos\theta = 0$, which leads to $\tan\theta = \frac{1}{\sqrt{2}}$, $\theta = 35.3°$.