Plan for Lectures

- Lecture 1: What is PDF?
- Lecture 2: Large-momentum effective theory and factorization
- Lecture 3: applications: quasi-PDF and spin structure of the proton

Xiangdong Ji, Shanghai Jiao Tong University

Lecture1： What is parton distribution function (PDF)?

outline

- High-energy scattering
- What is parton distribution?
- Momentum dependence and momentum renormalization group
- Gauge invariance and gauge link

High-energy scattering

LHC and high-energy

- LHC is the highest energy collider in the world!
- At 7 TeV (10^{12}eV) ,

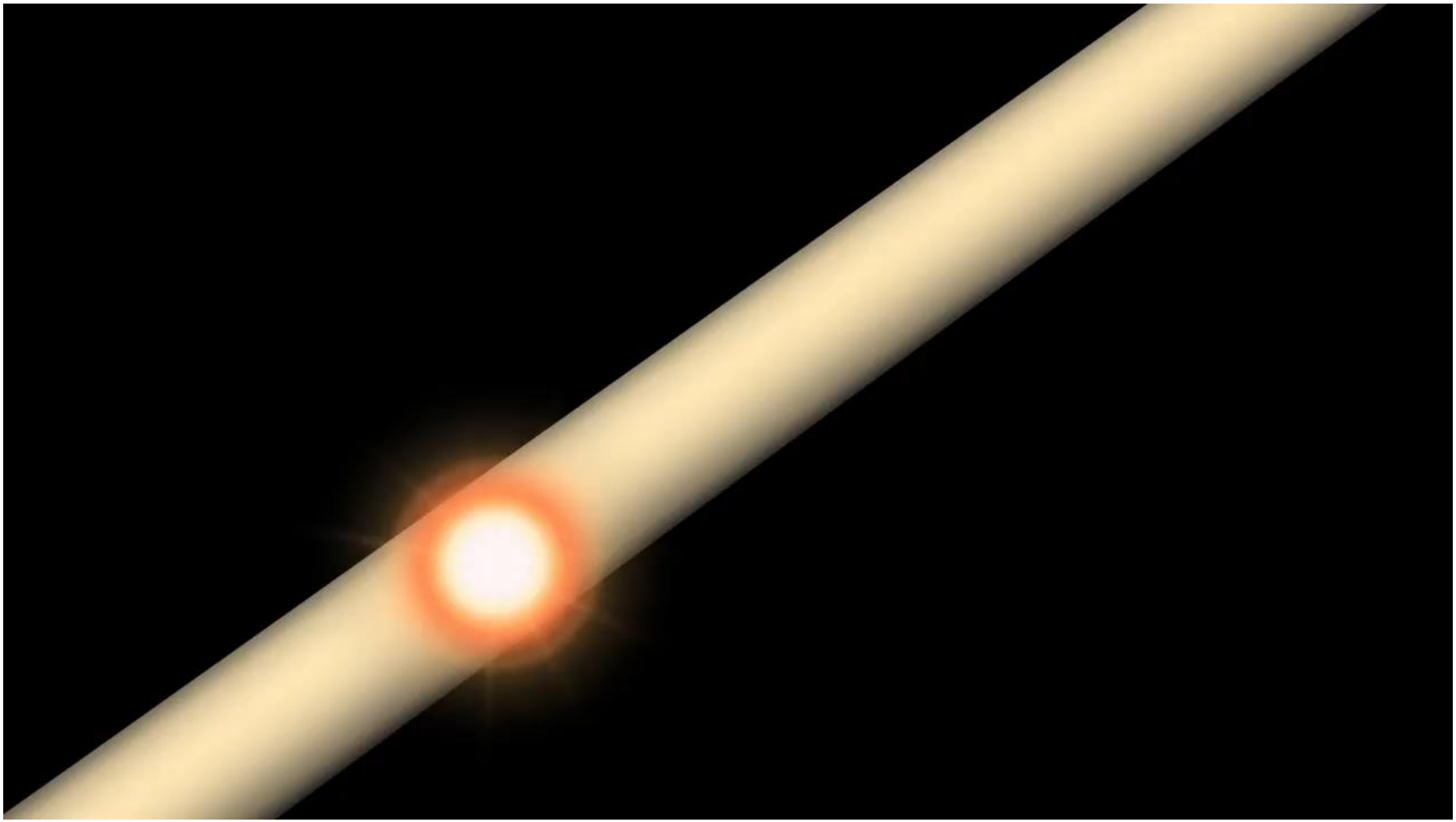
  the proton travels at

  $v=0.999999999c$

  or

  γ**=7463**

Protons in high-energy Collision



How to describe the collisions?

- Proton is NOT an elementary particle
- It is a bound state of quarks and gluons

✓ How to describe these quarks and gluons in the proton?
✓ How does the collision happens?
✓ What are the results of the collisions?

Feynman parton model

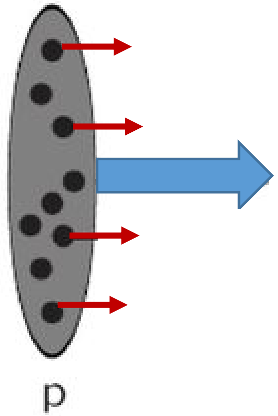
Feynman’s parton model

- When proton travels at v~c, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

Infinite momentum frame

- Proton may be considered as a collection of interaction-free particles called partons
- Partons are quarks and gluons.



Parton distribution functions (PDFs)

- Every parton has k=∞, however,

  $x=k/p$ = finite, $\in [0,1]$

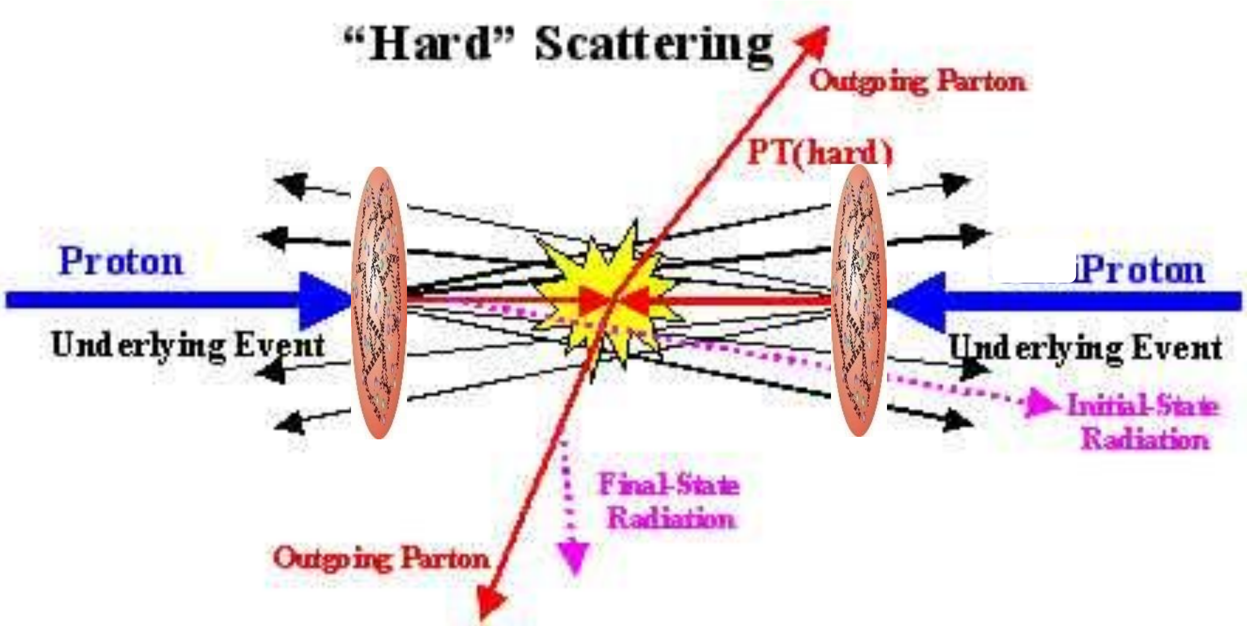
- Parton distribution function

  $f(x)$

is the probability of finding parton in a proton, carrying x fraction of the momentum of the latter.

- PDF is a bound state property of the proton, and is essential to describe the results of high-energy collisions.

Parton scattering



- Factorization: The scattering cross sections are factorized in terms of parton distribution function (PDFs) and parton scattering.

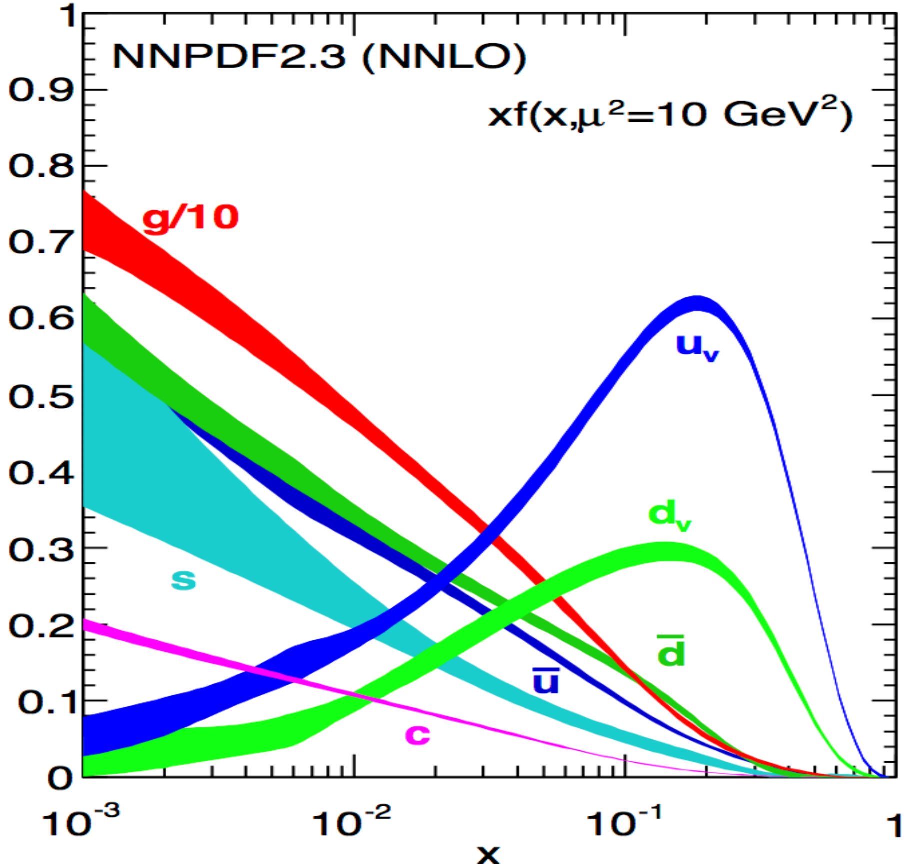
$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

Phenomenological PDFs

- PDFs are non-perturbative properties of the proton structure, and cannot be calculated in pert. theory
- Lacking of a first principle calculation, people resort to phenomenological fits
  - Parametrize the x-dependence
  - Fit the parameters to measured experimental cross sections
  - Resulting PDFs can be used to predict results for new experimental processes.

Phenomenological PDFs

- Use experimental data (50 yrs) to extract PDFs



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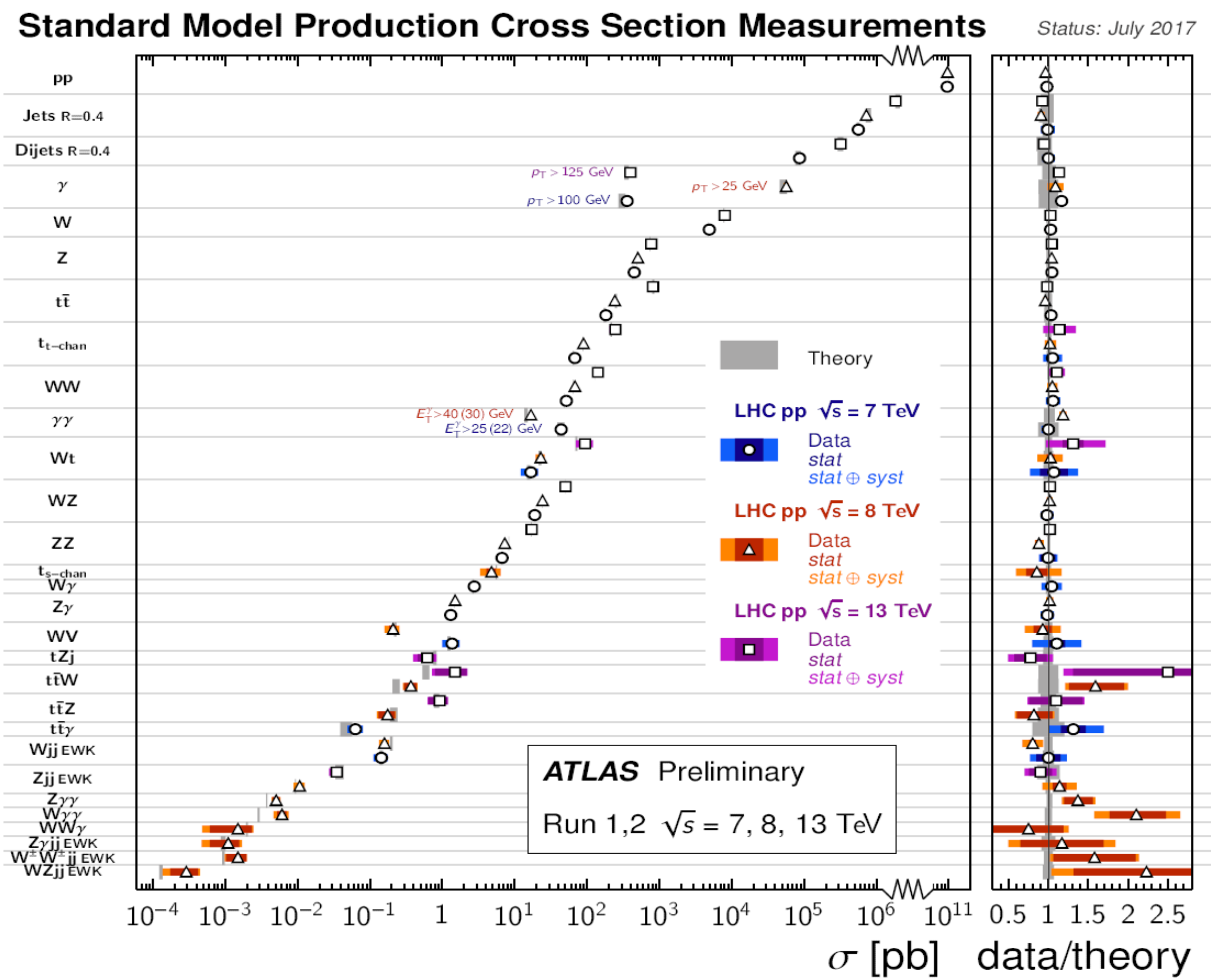
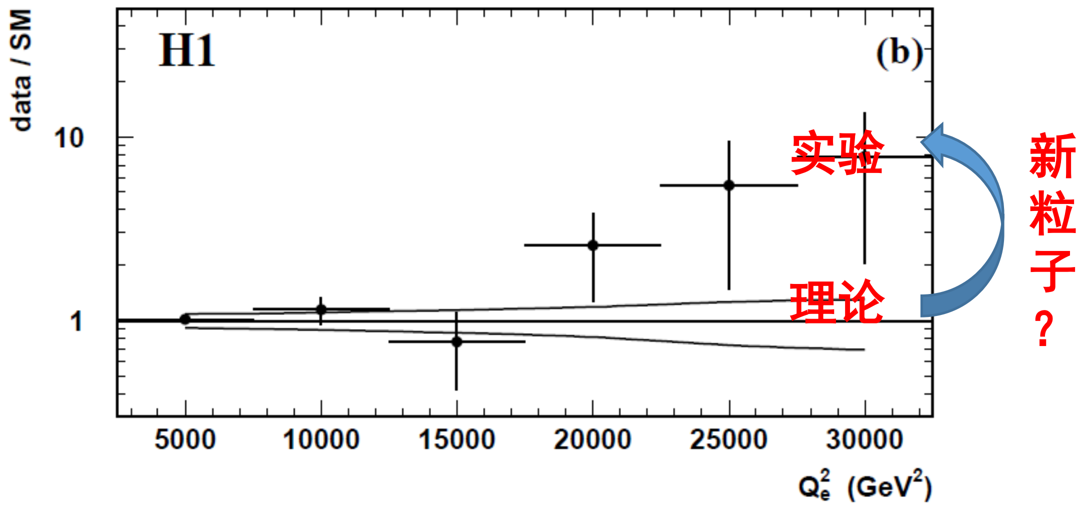


Standard Model Production Cross Section Measurements
Status: July 2017
pp
Jets R=0.4
Dijets R=0.4
γ
W
Z
tt̄
t_{t-chan}
WW
γγ
Wt
WZ
ZZ
t_{s-chan}
Wγ
Zγ
WV
tZj
tt̄W
tt̄Z
tt̄γ
Wjj EWK
Zjj EWK
Zγγ
Wγγ
WWγ
Zγjj EWK
W±W±jj EWK
WZjj EWK
p_T > 125 GeV
p_T > 100 GeV
p_T > 25 GeV
E_T^γ > 40 (30) GeV
E_T^γ > 25 (22) GeV
Theory
LHC pp √s = 7 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 8 TeV
Data
stat
stat ⊕ syst
LHC pp √s = 13 TeV
Data
stat
stat ⊕ syst
ATLAS Preliminary
Run 1,2 √s = 7, 8, 13 TeV
10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6 10^11
0.5 1 1.5 2 2.5
σ [pb]
data/theory

Problem with experimental PDFs

- Large uncertainty in certain regions (large x)
- Less known about polarized PDFs

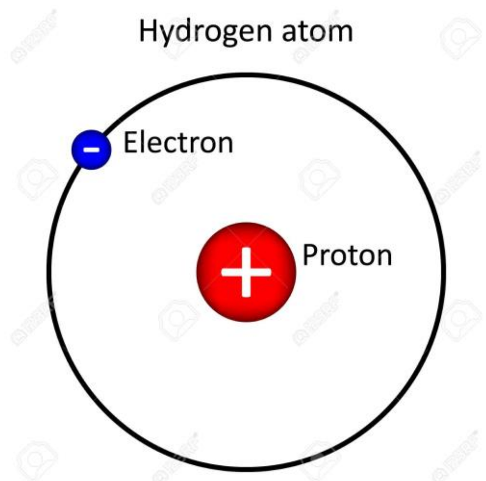


1997年德国HERA对撞机上疑似发现了一类新粒子leptoquark，后来证明是因为部分子分布函数不精确而导致的错误！

What is pdf?

What are parton densities?

- Can be understood simply from a H-atom



$$\psi_{100} = \frac{1}{\sqrt{\pi r_0^3}} e^{-\frac{r}{r_0}}$$

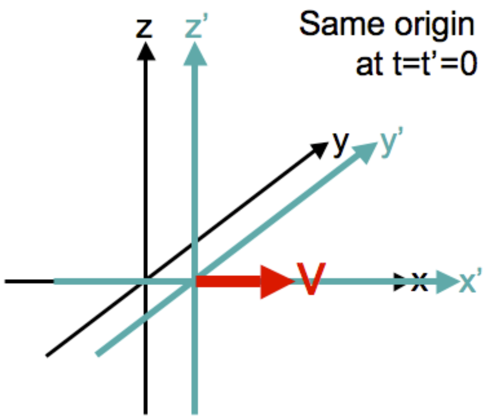
- Momentum density is just $n(k) = |\psi(k)|^2$

Center-of-Mass and Internal Motions : non-relativistic case

- In non-relativistic systems, the COM motion is decoupled from the internal motion in the sense that the internal dynamics is independent of the COM momentum.

  $$H = H_{com} + H_{int}$$

- H_{int} is independent of P (COM momentum) and R (COM position)
- Wave function of the H-atom is independent of its speed.



Galilei transformation

Center-of-Mass and Internal Motions : Relativistic case

- In relativistic theory, the internal dynamics DOES depend on the total momentum of the system.
- The internal wave function of a system is frame-dependent.
- Wave functions in the different frame is related by Lorentz boost

$|p\rangle = U(\Lambda(p))\,|p=0\rangle$, Λ is related to the boost K_i

- Bound state properties do depend on the COM momentum p.

Momentum distribution of constituents

- Consider the momentum distribution of the constituent

$$n(k) = \langle p|a_{\mathrm{k}}^{+} a_k|p\rangle$$

In relativistic bound state, this becomes a COM momentum-dependent quantity,

$$n(k) \to n(k,p) \text{ or } n_p(k)$$

How to compute the momentum dependence?

Computing the momentum dependence

- computing the momentum dependence of an observable O(p) is in principle possible through commutation relation,

$$[O, K_i] = ...$$

However, in relativistic theories, the boost operator K is highly non-trivial, it is interaction-dependent, just like the Hamiltonian.

- Thus, computing the p-dependence of an observable is just as difficult as studying the dynamical evolution.

Feynman density

- Consider the H-atom moves with a speed v in z-direction.
- The wave function shall be the $\psi_v(\vec{k})$
- As v-> c, the momentum of H-atom is $P_H \to \infty$

  and

  $k_z \to xP_H \to \infty$ (0<x<1, not the case if v≠c)
- Feynman distribution

$$f(x, k_\perp) = |\psi_{v=c}(x, k_\perp)|^2$$

a momentum density as the system travel at c

Large momentum limit

- Feynman parton distribution corresponds to the large momentum limit

  $$p \to \infty, \quad or \quad v \to c$$

  Is this limit smooth?

  NO

- There are large logarithms lnP associated with the limit, as one can compute from pert. theory.
- Thus there is a UV divergence in the limit, which can be regularized.

Asymptotic freedom (AF) and large momentum case

- QCD is an asymptotic-free theory. As such, once there is a large scale in the problem, such a scale dependence can be studied in pert. theory.

- One can establish a momentum renormalization group eq.

  $$dO(p,\mu)/dlnp \ = \gamma_o(p,\ \mu)O(p,\mu)$$

  γ is a perturbative expansion

  in the strong interaction coupling constant.

Fixed point, parton physics, and critical point

- The RG equation has a fixed point at P=∞,

$$\gamma_o(p=\infty)=0$$

- This is the infinite momentum limit at which the partons were first introduced. Thus the parton physics corresponds to frame-dependent physical obervables at the fixed point of the frame transformations.

- P=∞ is like the critical point in phase transition diagram.

Taking $P=\infty$ limit

Formally taking the limit

- One can ignore the infinity and taking $p=\infty$ in Feynman diagrams.
- In this case, one gets the so-called light-cone or light-front limit (Weinberg, Drell)
- Parton distributions become light-cone correlations.

Parton distribution (Schrodinger rep)

- Can be formulated in as the matrix elements of the boost-invariant light-front correlations .

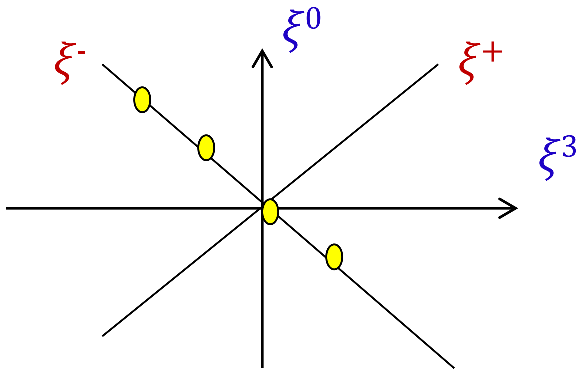
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle \ ,$$

where the light-front coordinates,

$$\xi^\pm = \frac{\xi^0 \pm \xi^3}{\sqrt{2}}$$

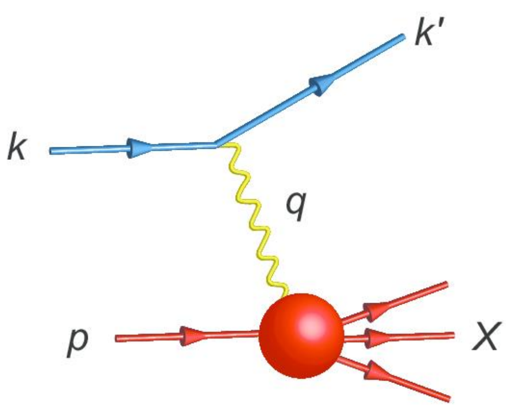
Partons as light-front correlations

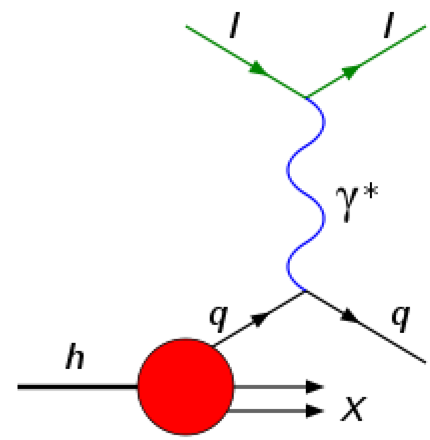
- Quark and gluon fields are distributed along the light-cone ξ^- direction



- Parton physics involves time-dependent dynamics.
- This is very general, parton physics = "light cone physics" of bound states.

Where does the gauge-dependence come from? Deep-inelastic scattering

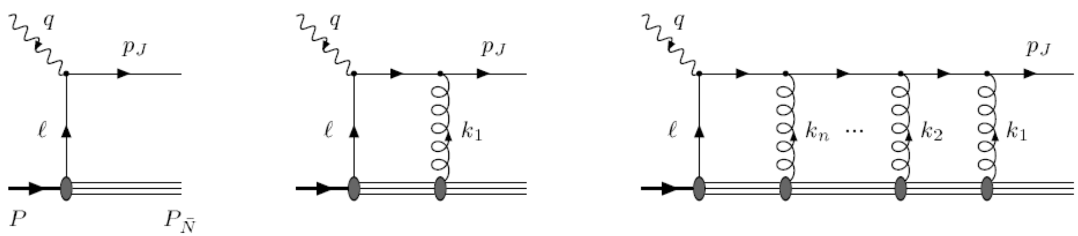




Quarks entering into a scattering do not know about gauge-invariance.

Final state interactions

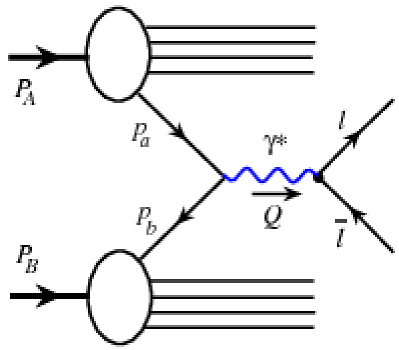
- Gauge invariance is a result of summing many Feynman diagrams.



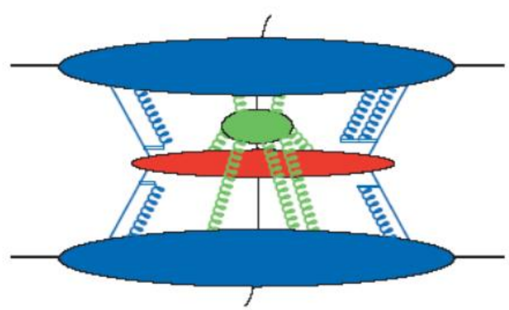
- The outgoing quarks scatter successively in the background fields of the nucleon.

Drell-yan process

- Simple process

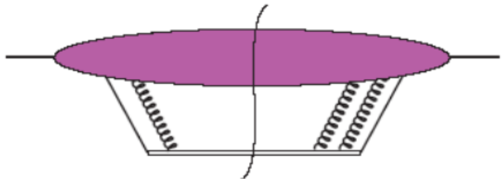


- Considering higher-order processes



Factorization of Drell-Yan process

- Parton distribution in the Drell-Yan process.



which contains the gauge-link needed for gauge symmetry.

- This gauge link comes from initial state interactions.
- A general proof does not exist for other processes.