1. 纵向极化的矢量玻色子的对产生

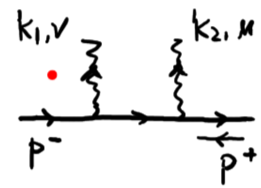
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mu^2 A_\mu A^\mu + \bar{\psi}(i\not\partial - m)\psi - e\bar{\psi}_L \not{A}\psi_L$$

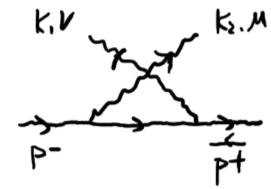
$$-ie\gamma^\mu \frac{1-\gamma_5}{2}$$

① 树图水平上计算 $e^+(p^+)e^-(p^-) \longrightarrow \gamma(k_1,\epsilon_L)\,\gamma(k_2,\epsilon_L)$ 总截面.

$$i\mathcal{M} = (-ie)^2 \frac{1}{4}\epsilon_\mu^*(k_2)\epsilon_\nu^*(k_1)\, i\, \bar{v}(p^+)\left[\gamma^\mu(1-\gamma_5)\frac{\not p^- - \not k_1 + m}{(p^- - k_1)^2 - m^2}\gamma^\nu(1-\gamma_5) + \gamma^\nu(1-\gamma_5)\frac{\not k_1 - \not p^+ + m}{(k_1 - p^+)^2 - m^2}\gamma^\mu(1-\gamma_5)\right]u(p^-)$$

$$= -ie^2\bar{v}_L(p^+)\left[\frac{\not\epsilon_2^*(\not p^- - \not k_1)\not\epsilon_1^*}{(p^- - k_1)^2 - m^2} + \frac{\not\epsilon_1^*(\not k_1 - \not p^+)\not\epsilon_2^*}{(k_1 - p^+)^2 - m^2}\right]u_L(p^-) \quad Ⓐ$$





$$\gamma^\mu(1-\gamma_5)(\not k_2 - \not p^+ + m)\gamma^\nu(1-\gamma_5) = 2\gamma^\mu(\not k_2 - \not p^+)\gamma^\nu$$

$s > m^2, \mu^2$, $\epsilon^\mu(k) \approx \frac{k^\mu}{\mu}$

$$\bar{v}_L(p^+)\not\epsilon_2^*(\not p^- - \not k_1)\not\epsilon_1^* u_L(p^-)$$
$$= \bar{v}_L(p^+)\not\epsilon_2^*\left[2(p^- - k_1)\cdot\epsilon_1^* - \not\epsilon_1^*(\not p^- - \not k_1)\right]u_L(p^-)$$
$$= \bar{v}_L(p^+)\left[\not\epsilon_2^*(2p^-\cdot\epsilon_1^*) + \not\epsilon_1^*\cdot\not k_1\right]u_L(p^-)$$
$$- m\bar{v}_L(p^+)\not\epsilon_2^*\not\epsilon_1^* u_R(p^-)$$

$$\bar{v}_L(p^+)\not\epsilon_1^*(\not k_1 - \not p^+)\not\epsilon_2^* u_L(p^-)$$
$$= \bar{v}_L(p^+)\left[2(k_1 - p^+)\cdot\epsilon_1^* - (\not k_1 - \not p^+)\not\epsilon_1^*\right]\not\epsilon_2^* u_L(p^-)$$
$$= \bar{v}_L(p^+)\left[-2p^+\cdot\epsilon_1^* - \not k_1\cdot\not\epsilon_1^*\right]\not\epsilon_2^* u_L(p^-)$$
$$- m\bar{v}_R(p^+)\not\epsilon_1^*\not\epsilon_2^* u_L(p^-)$$

$$Ⓐ = -ie^2\bar{v}_L(p^+)\left[\frac{\not{\epsilon}_2^{*}\,2(p^-\cdot k_1)}{-2p^-\cdot k_1} - \frac{-2(p^+\cdot k_1)\not{\epsilon}_2^{*}}{-2p^+\cdot k_1}\right]u_L(p^-)$$

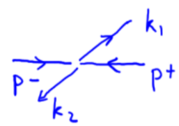
$$+ie^2m\left[\bar{v}_L(p^+)\frac{\not{\epsilon}_2^{*}\not{\epsilon}_1^{*}}{-2p^-\cdot k_1}u_R(p^-) + \bar{v}_R(p^+)\frac{\not{\epsilon}_1^{*}\not{\epsilon}_2^{*}}{-2p^+\cdot k_1}u_L(p^-)\right]$$

$$\approx -i\frac{e^2m}{\mu^2}\left[\bar{v}_L(p^+)\frac{\not{k}_2\not{k}_1}{2p^-\cdot k_1}u_R(p^-) + \bar{v}_R(p^+)\frac{\not{k}_1\not{k}_2}{2p^+\cdot k_1}u_L(p^-)\right] \quad Ⓑ$$

$(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L)$ $\qquad$ $(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L)$

$$\approx -i\frac{e^2m}{\mu^2}\left[\bar{v}_R(k_2)u_L(k_1) + \bar{v}_L(k_1)u_R(k_2)\right]$$

利用旋量的宇称变换性质



$p^-\cdot k_1 = p^+\cdot k_2$

$p^-\cdot k_2 = p^+\cdot k_1$

如果令 $k=(E,\vec{k})$, $\tilde{k}=(E,-\vec{k})$

有: $\gamma^0 u(k) = u(\tilde{k})$

$$\gamma^0 u_L(k) = \gamma^0 P_L u(k) = P_R\gamma^0 u(k) = P_R u(\tilde{k}) = u_R(\tilde{k})$$

$$\gamma^0 u_R(k) = u_L(\tilde{k})$$

在高能极限下 $(E\gg m)$ $\quad v_L(k) = -u_L(k)$, $v_R(k) = -u_R(k)$

$$\bar{u}_R(k)\gamma^0 u_R(k) = 2E,\quad \bar{u}_L(k)\gamma^0 u_L(k) = 2E$$

$$(p^- - k_1)^2 - m^2 = -2p^-\cdot k_1 + \mu^2 \approx -2p^-\cdot k_1$$

$$\not{p}^- u_L(p^-) = \not{p}^- P_L u(p^-) = P_R\not{p}^- u(p^-) = m\,u_R(p^-)$$

$$\bar{v}_L(p^+)\not{p}^+ = \bar{v}(p^+)P_R\not{p}^+ = \bar{v}(p^+)\not{p}^+P_L = -m\bar{v}_R(p^+)$$

$$\not{\epsilon}_2^{*}\not{\epsilon}_1^{*}\not{k}_1 \approx \not{\epsilon}_2^{*}\frac{1}{\mu}k_1^2 \approx 0$$

Peskin & Schroeder, P73, Prob. 3.3
P170, Prob. 5.3

$$u_L(k_1) = \frac{1}{\sqrt{2p^-\cdot k_1}}\not{k}_1 u_R(p^-)$$

$$v_R(k_2) = \frac{1}{\sqrt{2p^+\cdot k_2}}\not{k}_2 v_L(p^+)$$

$$u_R(k_2) = \frac{1}{\sqrt{2p^-\cdot k_2}}\not{k}_2 u_L(p^-)$$

$$v_L(k_1) = \frac{1}{\sqrt{2p^+\cdot k_1}}\not{k}_1 v_R(p^+)$$

从而: $\bar{v}_R(k_2)u_L(k_1) = \bar{v}_R(k_2)\gamma^0 u_R(k_2) = -2E = -\sqrt{s}$

$\bar{v}_L(k_1)u_R(k_2) = \bar{v}_L(k_1)\gamma^0 u_L(k_1) = -2E = -\sqrt{s}$

$$i\mathcal{M}(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L) = i\mathcal{M}(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L) = \frac{ie^2 m}{\mu^2}\sqrt{s}$$

$$\sigma_{tot}(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L) \doteq \sigma_{tot}(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L)$$

$$= \frac{1}{2}\,\frac{1}{2s}\left(\frac{1}{8\pi}\right)\left|\frac{e^2 m}{\mu^2}\sqrt{s}\right|^2 = \frac{1}{32\pi}\,\frac{m^2e^4}{\mu^4}.$$

末态全同粒子　　$4E_{cm}P_{cm} = 2S$　　$\int d\Phi_2 = \int \frac{d\Omega}{16\pi^2}\frac{|\vec{P}|}{E_{cm}} = \frac{1}{8\pi}.$

说明: 在①的计算中，我们也可以不采用 Peskin 书中的旋量的相位处理（那种处理仍然合法，因为从 $u_L(p)\to u_R(k)$ 的定义是合法的，只是我们不能使用在此基础上得出的 $s(k_1,k_2)$ $t(k_1,k_2)$ 之间的关系。这种关系只有在对固定的 p^+ 或 p^- 才有定义）。这时，我们可以直接从Ⓑ出发进行计算。

在Ⓑ中，我们需要对其中的旋量积进行计算：

$$\bar{v}_L(p^+)\frac{\not k_2\not k_1}{2p^-\cdot k_1}u_R(p^-) = \bar{v}_L(p^+)\frac{\not k_2\not k_1}{2p^-\cdot k_1}\gamma^0\gamma^0 u_R(p^-)$$

$$= \bar{v}_L(p^+)\frac{\not k_2\not k_1}{2p^-\cdot k_1}\gamma^0 u_L(p^+) = \mathrm{Tr}\left[u_L(p^+)\bar{v}_L(p^+)\frac{\not k_2\not k_1}{2p^-\cdot k_1}\gamma^0\right]$$

$$\doteq -\mathrm{Tr}\left[u_L(p^+)\bar{u}_L(p^+)\frac{\not k_2\not k_1}{2p^-\cdot k_1}\gamma^0\right] = -\mathrm{Tr}\left[\frac{1-\gamma_5}{2}\not p^+\frac{\not k_2\not k_1}{2p^-\cdot k_1}\gamma^0\right]$$

$$= -\frac{1}{4p^-\cdot k_1}\left(\mathrm{Tr}[\not p^+\not k_2\not k_1\gamma^0] - \mathrm{Tr}[\gamma_5\not p^+\not k_2\not k_1\gamma^0]\right)$$

$$= -\frac{1}{4p^-\cdot k_1}\left(p^+_\alpha k_{2\beta}k_{1\gamma}\mathrm{Tr}[\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^0] - p^+_\alpha k_{2\beta}k_{1\gamma}\mathrm{Tr}[\gamma^5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^0]\right)$$

$$= -\frac{1}{4p^-\cdot k_1}\left(4[p^+\cdot k_2\, k_1^0 - p^+\cdot k_1\, k_2^0 + p^{+0}\, k_1\cdot k_2] - \underbrace{4i\epsilon^{\alpha\beta\gamma 0}p^+_\alpha k_{2\beta}k_{1\gamma}}_{=0}\right)$$

$$= -\frac{E}{p^-\cdot k_1}\left(p^+\cdot k_2 - (p^+ - k_2)\cdot k_1\right) = -\frac{E}{p^-\cdot k_1}\left(p^+\cdot k_2 - (k_1 - p^-)\cdot k_1\right)$$

$$= -\frac{E}{p^-\cdot k_1}\left(p^+\cdot k_2 + p^-\cdot k_1 - k_1^2\right) \approx -2E.$$

同理，有：$\bar{v}_R(p^+)\frac{\not k_1\not k_2}{2p^+\cdot k_1}u_L(p^-) \approx -2E$.

$p^+ = \tilde{p}^-$ （质心系）

$\gamma^0 u_R(p^-) = u_L(\tilde{p}^-) = u_L(p^+)$

在高能极限下，约定 $v_L(p) = -u_L(p)$

$u_L(p)\bar{u}_L(p) = \frac{1-\gamma_5}{2}\not p$

$\mathrm{Tr}\,\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\sigma = 4(g^{\alpha\beta}g^{\rho\sigma} - g^{\alpha\rho}g^{\beta\sigma} + g^{\alpha\sigma}g^{\beta\rho})$

$\mathrm{Tr}\,\gamma^5\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\sigma = 4i\epsilon^{\alpha\beta\rho\sigma}$

在质心系． $k_1 = (E, \vec{k})$

$k_2 = (E, -\vec{k})$

$\epsilon^{\alpha\beta\gamma 0}p^+_\alpha k_{2\beta}k_{1\gamma}$ 反对称

$= -\epsilon^{ijk0}p^+_i k_j k_k = 0$

$(i, j, k = 1, 2, 3)$

$k_1^0 \doteq k_2^0 \doteq p^{+0} = E$, $\quad p^+\cdot k_2 = p^-\cdot k_1$

$k_1^2 = \mu^2 \approx 0 \qquad p^+ - k_1 = k_2 - p^-$

（高能极限）

②. 若考虑模型中加入新的标量粒子，与矢量粒子的耦合如下

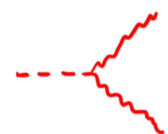
$$\mathcal{L}' = \frac{1}{2}\partial^\mu h \partial_\mu h - \frac{1}{2} m_h^2 h^2 + e^2 v h A_\mu A^\mu - \frac{\lambda_f}{\sqrt{2}}\bar{\psi}\psi h$$

试给出适当的 λ_f，v 的值，使得截面随 s 变化的行为为 $\frac{1}{s}$.

解答：新的拉氏量中给出新的 Feynman 规则

标量子传播子：$= \frac{i}{p^2-m_h^2+i\epsilon}$

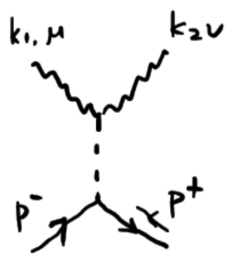
新的相互作用顶点：


$= 2ig_{\mu\nu}e^2 v$


$= -\frac{i\lambda_f}{\sqrt{2}}$

新的作用量计算对过程

$e^+(p^+)e^-(p^-) \to \gamma_L(k_1)\gamma_L(k_2)$ 的贡献



$$i\mathcal{M}' = \left(-i\frac{\lambda_f}{\sqrt{2}}\right)\bar{v}(p^+)u(p^-)\frac{i}{(p^++p^-)^2-m_h^2}(2ie^2v)g_{\mu\nu}\epsilon^{*\mu}(k_1)\epsilon^{*\nu}(k_2)$$

$$\approx i\frac{2\lambda_f e^2 v}{\sqrt{2}}\frac{\bar{v}_L(p^+)u_R(p^-)+\bar{v}_R(p^+)u_L(p^-)}{s-m_h^2}\frac{k_1\cdot k_2}{\mu^2}$$

$$\approx i\frac{2\lambda_f e^2 v}{\sqrt{2}\mu^2}\frac{s}{2}\frac{\bar{v}_L(p^+)u_R(p^-)+\bar{v}_R(p^+)u_L(p^-)}{s-m_h^2}$$

在高能极限下，$s \gg \mu^2, m^2, m_h^2$，

$$i\mathcal{M}' = i\frac{\lambda_f e^2 v}{2\sqrt{2}\mu^2}\left(\bar{v}_L(p^+)u_R(p^-) + \bar{v}_R(p^+)u_L(p^-)\right)$$

同样，$\bar{v}_L(p^+)u_R(p^-) = \bar{v}_R(p^+)u_L(p^-) = -\sqrt{s}$

我们有：$i\mathcal{M}'(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L) = i\mathcal{M}'(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L) = -\frac{i\lambda_f e^2 v}{\sqrt{2}\mu^2}\sqrt{s}$

因此，考虑引入新的标量粒子与矢量场和费米子的耦合，总振幅加上，

$$i\tilde{\mathcal{M}}(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L) = \frac{ie^2}{M^2}\left(m - \frac{\lambda_f v}{\sqrt{2}}\right)\sqrt{s} = i\tilde{\mathcal{M}}(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L)$$

当取参数 $m = \frac{\lambda_f v}{\sqrt{2}}$ 时，这两种贡献相消

$$i\tilde{\mathcal{M}}(e^+_\uparrow e^-_\downarrow \to \gamma_L\gamma_L) = i\tilde{\mathcal{M}}(e^+_\downarrow e^-_\uparrow \to \gamma_L\gamma_L) = 0$$

当矢量粒子和非守恒流耦合时，会导致危险的紫外行为，可能破坏幺正性；可以通过引入新的标量场和特定的耦合方式，来抵消危险的紫外行为。

2. 在正则量子化下推导有质量矢量玻色子的传播子.

矢量场的平面波展开:

$$A_\mu(x)=\sum_\lambda\int\frac{d^3k}{(2\pi)^3 2E_k}\left(a_k^{(\lambda)}\epsilon_\mu^{(\lambda)}(k)e^{-ik\cdot x}+a_k^{(\lambda)\dagger}\epsilon_\mu^{(\lambda)*}(k)e^{ik\cdot x}\right)\Big|_{E_k=\sqrt{\mu^2+\vec{k}^2}}$$

$$A_\nu(y)=\sum_{\lambda'}\int\frac{d^3k'}{(2\pi)^3 2E_{k'}}\left(a_{k'}^{(\lambda')}\epsilon^{(\lambda')}(k')e^{-ik'\cdot y}+a_{k'}^{(\lambda')\dagger}\epsilon_\nu^{(\lambda')*}(k')e^{ik'\cdot y}\right)\Big|_{E_{k'}=\sqrt{\mu^2+\vec{k}'^2}}$$

利用产生、湮灭算符的正则对易关系:

$$[a_{\vec{k}}^{(\lambda)},a_{\vec{k}'}^{(\lambda')\dagger}]=(2\pi)^3 2E_{\vec{k}}\delta_{\lambda\lambda'}\delta^3(\vec{k}-\vec{k}').\qquad [a_{\vec{k}}^{(\lambda)},a_{\vec{k}'}^{(\lambda')}]=[a_{\vec{k}}^{(\lambda)\dagger},a_{\vec{k}'}^{(\lambda')\dagger}]=0$$

$$a_{\vec{k}}^{(\lambda)\dagger}|0\rangle=|\vec{k},\lambda\rangle,\quad \langle 0|a_{\vec{k}}^{(\lambda)}=\langle\vec{k},\lambda\rangle,\quad a_{\vec{k}}^{(\lambda)}|0\rangle=0,\ \langle 0|a_{\vec{k}}^{(\lambda)\dagger}=0$$

$$\langle\vec{k},\lambda|\vec{k}'\lambda'\rangle=(2\pi)^3 2E_{\vec{k}}\delta^3(\vec{k}-\vec{k}')\delta_{\lambda\lambda'}$$

$$\langle 0|A_\mu(x)A_\nu(y)|0\rangle=\left\langle 0\right|\sum_{\lambda\lambda'}\int\frac{d^3k\,d^3k'}{(2\pi)^6 2E_k 2E_{k'}}\left(a_k^{(\lambda)}\epsilon_\mu^{(\lambda)}(k)\,a_{k'}^{(\lambda')\dagger}\epsilon_\nu^{(\lambda')*}(k')e^{-ik\cdot x+ik'\cdot y}\right)\left|0\right\rangle$$

$$=\sum_\lambda\int\frac{d^3k}{(2\pi)^3 2E_k}\epsilon_\mu^{(\lambda)}(k)\epsilon_\nu^{(\lambda)*}(k)e^{-ik(x-y)}=\int\frac{d^3k}{(2\pi)^3 2E_k}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{\mu^2}\right)e^{-ik\cdot(x-y)}$$

同理，我们有： $\langle 0|A_\nu(y)A_\mu(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3 2E_k}(-g_{\mu\nu}+\frac{k_\mu k_\nu}{\mu^2})e^{ik\cdot(x-y)}$

利用 $\theta(x)$ 的积分表示： $\theta(x^0-y^0) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} ds \frac{e^{-is(x_0-y_0)}}{s+i\epsilon}$

$$\langle 0|TA_\mu(x)A_\nu(y)|0\rangle = \theta(x^0-y^0)\langle 0|A_\mu(x)A_\nu(y)|0\rangle + \theta(y^0-x^0)\langle 0|A_\nu(y)A_\mu(x)|0\rangle$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_k}(-g_{\mu\nu}+\frac{k_\mu k_\nu}{\mu^2})\left[\theta(x^0-y^0)e^{-ik(x-y)} + \theta(y^0-x^0)e^{ik(x-y)}\right]$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_k}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{\mu^2}\right)\frac{i}{2\pi}\int_{-\infty}^{+\infty} ds \left[\frac{e^{-i(k^0+s)(x^0-y^0)}}{s+i\epsilon}e^{i\vec{k}\cdot(\vec{x}-\vec{y})} + \frac{e^{i(k^0+s)(x^0-y^0)}}{s+i\epsilon}e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}\right]_{k^0=E_k}$$

$$= \int \frac{d^3k}{(2\pi)^3 2k^0}\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{\mu^2}\right)\frac{i}{2\pi}\int_{-\infty}^{+\infty} dq^0 \left(\frac{e^{-iq\cdot(x-y)}}{q^0-k^0+i\epsilon} + \frac{e^{iq\cdot(x-y)}}{q^0-k^0+i\epsilon}\right)$$

$q^0 = k^0+s$

$s = q^0 - s$

分三种情形讨论:

① $\mu,\nu=i,\ (i=1,2,3)$ 对第二项做变量替换 $q^0\to -q^0$, $\vec{k}\to-\vec{k}$, 第二项不变号

$$\langle 0|TA_i(x)A_j(y)|0\rangle=\frac{i}{2\pi}\int dq^0\int\frac{d^3\vec{k}}{(2\pi)^3 2k^0}\left(-g_{ij}+\frac{k_ik_j}{\mu^2}\right)e^{-iq(x-y)}\left(\frac{1}{q^0-k^0+i\epsilon}+\frac{1}{-q^0-k^0+i\epsilon}\right)$$

$d^4q=dq^0d^3\vec{k}$

$$=i\int\frac{d^4q}{(2\pi)^4}\frac{1}{2k^0}\left(-g_{ij}+\frac{k_ik_j}{\mu^2}\right)e^{-iq(x-y)}\frac{-2k^0}{-q^2+m^2-i\epsilon}$$

$$=\int\frac{d^4q}{(2\pi)^4}\left(-g_{ij}+\frac{k_ik_j}{\mu^2}\right)\frac{i}{q^2-\mu^2+i\epsilon}e^{-iq(x-y)}$$

$-q^{02}+(k^0-i\epsilon)^2$

$=-q^{02}+k^{02}-i\epsilon$

$=-q^{02}+\vec{k}^2+\mu^2-i\epsilon$

$=-q^2+\mu^2-i\epsilon$

② $\mu=i,\ \nu=0$ 对第二项做变量替换, $q^0\to-q^0$, $\vec{k}\to-\vec{k}$, 第二项变号 ($k_i\to-k_i$

$$\langle 0|TA_i(x)A_0(y)|0\rangle=\frac{i}{2\pi}\int\frac{d^3\vec{k}}{(2\pi)^3 2k_0}\frac{k_ik_0}{\mu^2}\int dq\, e^{-iq\cdot(x-y)}\left(\frac{1}{q^0-k^0+i\epsilon}-\frac{1}{-q^0-k^0+i\epsilon}\right)$$

$$=i\int\frac{d^4q}{(2\pi)^4}\frac{k_i}{2\mu^2}e^{-iq\cdot(x-y)}\frac{-2q^0}{-q^2+\mu^2-i\epsilon}$$

$$=\int\frac{d^4q}{(2\pi)^4}\frac{k_iq^0}{\mu^2}\frac{i}{q^2-\mu^2+i\epsilon}e^{-iq\cdot(x-y)}$$

$\nu=i,\ \mu=0$ 同理.

③ $\mu=\nu=0$

$$\langle 0|TA_0(x)A_0(y)|0\rangle = \frac{i}{2\pi}\int\frac{d^4q}{(2\pi)^3 2k_0}\left(-g_{00}+\frac{k_0k_0}{\mu^2}\right)e^{-iq(x-y)}\frac{2k^0}{q^2-\mu^2+i\epsilon}$$

$$= i\int\frac{d^4q}{(2\pi)^4}\left(-g_{00}+\frac{q_0q_0}{\mu^2}\right)e^{-iq(x-y)}\frac{1}{q^2-\mu^2+i\epsilon}$$

$$+ i\int\frac{d^4q}{(2\pi)^4}\left(\frac{k_0k_0}{\mu^2}-\frac{q_0q_0}{\mu^2}\right)e^{-iq(x-y)}\frac{1}{q^2-\mu^2+i\epsilon}$$

$$= \int\frac{d^4q}{(2\pi)^4}\left(-g_{00}+\frac{q_0q_0}{\mu^2}\right)\frac{i}{q^2-\mu^2+i\epsilon}e^{-iq(x-y)}$$

$k_0^2-q_0^2=m^2-q^2$

$$- i\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)} \leftarrow i\delta^4(x-y)$$

thus:

$$\langle 0|TA_\mu(x)A_\nu(y)|0\rangle = \int\frac{d^4q}{(2\pi)^4}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{\mu^2}\right)\frac{i}{q^2-\mu^2+i\epsilon}e^{-ik(x-y)} - \frac{i}{\mu^2}\delta_\mu^0\delta_\nu^0\delta^4(x-y)$$