

# Hadron Spectroscopy

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1. Prove Landau-Yang theorem.

For any vector particles, we can always write the field operator as a single vector field.

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2|\mathbf{k}|}} \sum_\lambda (a_{\mathbf{k}}^\lambda \epsilon_\mu^\lambda(k) e^{-ik \cdot x} + a_{\mathbf{k}}^{\lambda\dagger} \epsilon_\mu^{\lambda*}(k) e^{ik \cdot x})$$

Then the feynnman rules can be easily derived. The amplitude of *vector*  $\rightarrow \gamma\gamma$  is

$$i\mathcal{M} = \epsilon_1^{*\mu}(p_1) \epsilon_2^{*\nu}(p_2) \epsilon^\alpha(p) \Gamma_{\mu\nu\sigma}$$

since it must obey Lorentz-invariant

$$= (\epsilon_1 \cdot \epsilon_2)(a_1 \epsilon \cdot p_1 + a_2 \epsilon \cdot p_2) + a_3(\epsilon_1 \cdot \epsilon)(\epsilon_2 \cdot p_1) + a_4(\epsilon_2 \cdot \epsilon)(\epsilon_1 \cdot p_2)$$

final states symmetry (identical),  $a_1 = a_2$ , first term vanishes. And  $\epsilon_2 \cdot p_1 = \epsilon_1 \cdot p_2 = 0$

$$= 0$$

2.  $\eta \rightarrow \pi\pi$

For  $\eta$  meson,  $I^G J^{PC} = 0^+ 0^{-+}$ , for  $\pi$  meson,  $I^G J^{PC} = 1^- 0^{-+}$ . Charge parity conservation gives the final state angular momentum must be even, so  $\pi\pi$  system gives positive parity, parity is not conserved. (For  $\pi^0\pi^0$  system, use identical particle instead.)

3.  $\eta \rightarrow \pi\pi\pi$

From previous discussion, we know that this reaction can happen not only under weak interaction for  $P$  parity and  $C$  parity are conserved. But  $G$  parity is not conserved, the final state  $G$  parity is negative while the initial state is positive, so it must not be a strong interaction.

4.  $\rho \rightarrow \pi\pi$

For  $\rho$  meson,  $I^G J^{PC} = 1^+ 1^{--}$ . For angular momentum conservation, the final state orbital angular momentum must be  $L = 1$  while the spin  $S$  is zero. For  $\pi^0\pi^0$  scenario,  $L + S = \text{even}$  is not guaranteed. For  $\pi^+\pi^-$  scenario,  $CP = (-)^S = +$ , no obvious violation, it can happen.

5.  $\omega \rightarrow \pi^0\pi^0\pi^0$

For  $\omega$  meson,  $I^G J^{PC} = 0^- 1^{--}$ . Apparently CP violation.