Linear potential $|x|$ – momentum space

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August 30, 2016

Using the parametrised basis mentioned in LiMing's paper (to save time, only 16 bases were used.)

$$\psi_n(p) = \sqrt{\frac{\alpha}{\sqrt{\pi}2^n n!}} H_n(\alpha p) e^{-\frac{1}{2}(\alpha^2)p^2} \tag{1}$$

and the Schrödinger equation in momentum space

$$(E-\mu)\phi(p) = \omega(p)\phi(p) - \frac{\lambda}{\pi}\int_{\infty}^{\infty} \frac{\phi(k)-\phi(p)-(k-p)\phi'(p)}{(p-k)^2} \tag{2}$$

with $\omega(p) = \sqrt{m^2+p^2}$, I have the following results: eigenenergy solved in momentum space

n	0	1	2	3
Schrödinger equation	0.805086	1.84766	2.56684	3.23044
't Hooft equation	0.62674	1.62773	2.32744	2.95498
1st-order perturbation	0.792475	1.81352	2.49903	3.12609
error comparing with 't Hooft eqn	26.44%	11.41%	7.37%	5.79%
momentum representation	0.761624	1.73534	2.35721	2.9225
error comparing with 't Hooft eqn	21.52%	6.61%	1.28%	-1.10%

Table 1: $\mu = 2.5$ with $a = 0.474$

In the process, I find out the best value for parameter $a = 0.474$ (which was determined under 6 bases).

And with larger mass $\mu = 25$, we have

n	0	1	2	3
Schrödinger equation	0.373688	0.857606	1.19142	1.49944
't Hooft equation	0.3541	0.836931	1.17025	1.4773
1st-order perturbation	0.373416	0.85687	1.18996	1.49719
error comparing with 't Hooft eqn	5.45%	2.38%	1.68%	1.34%
momentum representation	0.367673	0.849901	1.18106	1.48622
error comparing with 't Hooft eqn	3.83%	1.55%	0.92%	0.60%

Table 2: $\mu = 25$ with $a = 0.232$

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With $\mu = 250$ and $\alpha = 0.108$

n	0	1	2	3
Schrödinger equation	0.173451	0.398065	0.553009	0.695978
't Hooft equation	0.171459	0.39605	0.550983	0.69393
1st-order perturbation	0.173445	0.39805	0.552978	0.69593
error comparing with 't Hooft eqn	1.16%	0.50%	0.36%	0.29%
momentum representation	0.168524	0.393399	0.548678	0.692045
error comparing with 't Hooft eqn	-1.71%	-0.67%	-0.42%	-0.27%

Table 3: $\mu = 250$ with $a = 0.108$

we might notice that when the energy is low, the result from momentum representation is a little "inferior" to coordinate space with 1st-order perturbation.