Homework: Particle Physics #2

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November 16, 2016

1.

Isospin transformation: transform one particle into its conjugated charge state composed of different numbers of u and d quarks. It's built in an abstract space, the group space of SU(2) group. The eigenstates of isospin transformation are u and d and commonly written as $|II_3\rangle$. (In early understanding, the three charge states of π meson are also considered the eigenstates of isospin transformation.) Isospin is conserved in strong interaction.

C transformation (charge conjugation transformation): transform a particle into its antiparticle. The eigenstates of it are neutral particles. C parity is conserved in strong and electromagnetic interactions.

G transformation: change the signs of all internal additive quantum numbers but I_3. The eigenstates are all common mesons. G parity is only conserved in strong interaction.

P transformation: transform the spatial coordinate of particles, or, the parity of particles. The eigenstates are the eigenstates of orbital angular momentum. P parity is conserved in strong and electromagnetic interactions.

CP transformation: transform both the spatial coordinate and the charge state of a particle. The eigenstates are neutral particles (with the states of angular momentum). CP is conserved in strong and electromagnetic interactions.

2. In LAB frame, the reaction is $pp \to \bar{p}X$ where X is of charge +3 and mass m_X. Accually $X = ppp$ so $pp \to \bar{p}ppp$. And for the lowest energy we can assume the products are at rest.

The initial state

$$E^2 = E_{\mathbf{p}} + m_p, \mathbf{p} = \mathbf{p}_p + 0 = \mathbf{p}_p, m = 2m_p, m_p = 1GeV$$

The final state

$$k = k_{\bar{p}} + 3k_p,$$

So

$$p^2 = (E_{\mathbf{p}} + m_p)^2 - \mathbf{p}_p^2 = (p_1 + p_2)^2 = 2m_p^2 + 2E_{\mathbf{p}}m_p$$

$$k^2 = 16m_p^2$$

$$2m_p^2 + 2E_{\mathbf{p}}m_p = 16m_p^2$$

$$E_{\mathbf{p}} = 7m_p$$

The minimum momentum required is

$$\mathbf{p} = \sqrt{E_{\mathbf{p}}^2 - m_p^2} = \sqrt{48}m_p = 4\sqrt{3}m_p \approx 6.928GeV$$

3. For $\phi(1020)$ meson: $I^G(J^{PC}) = 0^-(1^{--})$, $\Gamma = (4.458)MeV$.

1) ${K_L}^0 {K_S}^0$ 33.8%

The partial width of it is $\Gamma = 4.458 \times 33.8\% \approx 1.5MeV$ so it's strong interaction.

2) K^+K^- 49.2%

Same as the first.

3) $\pi^+\pi^-\pi^0$ 15.5%

The isospin of the final state: $\begin{array}{ccc} \pi^+ & \pi^- & \pi^0 \\ |11\rangle & |1-1\rangle & |10\rangle \end{array}$

and all possible I of the final state are: $0, 1,\ I_3 = 0$. From the branching ratio, we know that $\Gamma = 4.458 \times 15.5\% \approx 0.691MeV$ and it's about the order of strong interaction.

4) $\eta\gamma$ 1.3%

γ is involved so it's EM interaction.

5) $\pi^0\gamma$ 1.26×10^{-3}

γ is involved so it's EM interaction.

6) $\mu^+\mu^-$ 2.9×10^{-4}

$\Gamma = 4.458 \times 2.9 \times 10^{-4} \approx 10^3 eV$, so it's EM interaction.

7) $\omega\pi^0$ 5.2×10^{-5}

The isospin of the final state: $\begin{array}{cc} \omega & \pi^0 \\ |00\rangle & |10\rangle \end{array}$. But $\Gamma = 4.458 \times 5.2 \times 10^{-5} \approx 232eV$ so it's EM interaction. G parity is not conserved as well.

8) $\pi^+\pi^-$ 7.3×10^{-5}

The isospin of the final state: $\begin{array}{cc} \pi^+ & \pi^- \\ |11\rangle & |1-1\rangle \end{array}$. But $\Gamma = 4.458 \times 5.2 \times 10^{-5} \approx 10^3 eV$ so it's EM interaction. G parity is not conserved as well.

4. Neutral system composed of two π.

$\pi^+\pi^-$: C number is $(-)^{L+S}$, G number is $C(-)^I$ and P number is $(-)^L$. For general identical particle, $L + S + I - 2i = even$, and here $I = 0, 1, 2,\ i = 1,\ S = 0$.

We have $\begin{cases} I = 0, 2, L + S = even, C = +, P = +, G = + \\ I = 1, L + S = odd, C = -, P = -, G = + \end{cases}$.

$\pi^0\pi^0$: Similarly, for π^0 we have $C = +$, $P = -$, $G = -$. And for this system we have $L + S = even$, $S = 0$, so $C = +$, $P = - - (-)^L = +$, $G = +$.

5. Neutral system composed of K^+K^- $(S = 0)$.

We have $\begin{cases} I = 1, C = (-)^L, P = (-)^L, G = (-)^{L+1} \\ I = 0, C = (-)^L, P = (-)^L, G = (-)^L \end{cases}$

6. It's hard to identify K^0 and $\bar{K}^0$ meson via decay type. It's because their differences are only in strangeness and I_3. And in weak interaction these are not conserved. In the process decaying to π mesons, if we view it in the eigenstates of CP transformation, we'll find one eigenstate have much longer lifetime than the other.

PS: $\begin{cases} K^0: \ S=1, I_3=\frac{1}{2} \\ \bar{K}^0: \ S=-1, I_3=-\frac{1}{2} \end{cases}$, and the eigenstates of CP are $\begin{cases} K_L = \frac{1}{\sqrt{2}}(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle) \\ K_S = \frac{1}{\sqrt{2}}(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle) \end{cases}$

7.

1) $\pi^- p \to K^+ \Sigma^-$ Strangeness is conserved. So it's strong interaction.

2) $pp \to n\pi^+\Sigma^-$ Charge not conserved. It can't happen.

3) $\pi^0 \to e^+e^-e^+e^-$ The final states involves leptons, so it's EM interaction. G is not conserved.

4) $\rho^0 \to \eta\pi^0$ If it can happen, CP must be conserved. So $(+) = (-)(-)(-)^L \Longrightarrow L = even$. And the angular momentum must be conserved, so $1 = 0 + 0 + L \Longrightarrow L = odd$. So it can't happen.

5) $J/\Psi \to \pi^+\pi^-$ Charm's conserved. CP is conserved. If C is conserved, $L + S = odd$. If P is conserved, $L = odd$. And we know $L + S + I - 2i = even$. If G is conserved, $L + S + I = odd$, so G isn't conserved, it can't be strong interaction. It's EM or weak interaction.

6) $J/\Psi \to \pi\rho$ Charm's conserved. C is conserved. If P is conserved, $L = odd$. The angular momentum $1 \leq 0 + 1 + L$. G is conserved. It's strong interaction.

7) $p\bar{n} \to K^+K^0\pi^0$ Strangeness's not conserved. Baryon number is conserved. It can happen via weak interaction.

8) $K^+ \to \pi^+\pi^0$ Isospin and strangeness are not conserved. It's weak interaction.

9) $\rho \to \pi^0\pi^0$ C parity not conserved. CP is conserved. We know that $L + S = even$, but for conserved angular momentum $L = 1$. so it can't happen.

10) $K^0 \to \pi^+\pi^-\pi^0$ Strangeness is not conserved. K^0 is the linear combination of K^L and K_S, so its decay products should contain both two pion system and three pion system. It can happen via weak decay in the form of K_L.

11) $\eta \to \pi\pi$ CP is not conserved. It can't happen.

12) $\eta \to \pi^+\pi^-\pi^0$ CP is conserved. G is not conserved (for n pion system $G = (-1)^n$ but $G(\eta) = -$). It's EM or weak interaction.

13) $\phi \to K^S K^S$ $C(K_S) = -, C(K_L) = +$, $L + S = even, S = 0$, $P(K_S K_S) = (-)^L = +$, $CP(\phi) = +$, $CP(K_S K_S) = -$, so it can't happen.

14) $\eta \to \pi^+\pi^-\gamma$ Photon is involved, C and P are conserved, it's EM interaction.

15) $\omega \to \pi^+\pi^-$ G is not conserved. C and P are conserved. It's EM or weak interaction.