$\bar{c}\gamma^\mu c$ matrix element

Yingsheng Huang

April 12, 2019

1 Kinematics

Quark and antiquark momenta are

$$p_1 = P/2 + p = (E, \mathbf{p}) p_2 = P/2 - p = (E, -\mathbf{p}) \tag{1}$$

where in rest frame

$$P = (2E(p), 0) p = (0, \mathbf{p}) \tag{2}$$

2 State Projection

The bound state is [Weinberg(2015)]

$$|P, E; J, m_j; L; S\rangle = \int \mathrm{d}\Omega_{\mathbf{p_1}} \sum_{s_1 s_2 s_z m_l} Y_l^m(\hat{\mathbf{p_1}}) \langle S s_z | S_1 s_{1z} S_2 s_{2z}\rangle \langle J m_J | S s_z L m_l\rangle |\mathbf{p_1}, s_{1z}\rangle |\mathbf{P} - \mathbf{p_1}, s_{2z}\rangle \tag{3}$$

3 $^3S_1$

Ignore the overall factor:

$$\langle 0 | \bar{c}\gamma^\mu c | ^3S_1\rangle = \int \mathrm{d}\Omega \, \mathrm{tr}[\Pi_1 \gamma^\mu] \propto \sqrt{2}\pi(\frac{m}{3E} + \frac{2}{3})\epsilon^\mu$$

4 $^3D_1$

The matrix element reads:

$$\langle 0 | \bar{c}\gamma^\mu c | ^3D_1\rangle = \int \mathrm{d}\Omega \sum_{\lambda_1 \lambda_2 S_z m} \mathrm{tr}\{\Pi_1 \gamma^\mu\} \langle 1 J_z | 2m; 1 S_z\rangle Y_{2m}(\theta, \phi)$$

while the trace part is the same as $^3S_1$:

$$\mathrm{tr}\{\Pi_1 \gamma^\mu\} = \frac{\sqrt{2} p^\mu (p \cdot \epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \epsilon^{(0)} = (0, 0, 0, 1), \epsilon^{(+)} = \frac{1}{\sqrt{2}}(0, -1, -i, 0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} 0 & \frac{2\sqrt{2\pi}p^2}{3E(m+E)} & -\frac{2i\sqrt{2\pi}p^2}{3E(m+E)} & 0 \\ 0 & 0 & 0 & \frac{4\sqrt{\pi}p^2}{3E(m+E)} \\ 0 & -\frac{2\sqrt{2\pi}p^2}{3E(m+E)} & -\frac{2i\sqrt{2\pi}p^2}{3E(m+E)} & 0 \end{pmatrix}$$

and the decay constant is $\frac{4\sqrt{\pi}p^2}{3E(E+m)}$ where $p = \mathbf{p} = E^2 - m^2$.

References

[Weinberg(2015)] S. Weinberg, *Lectures on Quantum Mechanics* (Cambridge University Pr., 2015).