Day 1

A. preparation

Ref. An Introduction to QFT

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Pauli & Dirac matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

products of sigma matrices

$$\sigma^i \sigma^j = \delta^{ij} + i \epsilon^{ijk} \sigma^k$$

Weyl representation

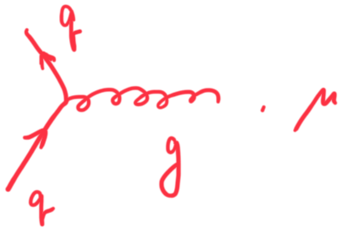
$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & \\ 0 & \end{pmatrix}$$

Anti-commutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \{\gamma^\mu, \gamma^5\} = 0$$

SM Feynman Rules :

$$i g \cdot \gamma^\mu P_L \cdot \delta_{ij} / \sqrt{2}$$



$$i g_s \gamma^\mu T^a_{ij}$$

$$\frac{i}{\not{p} - m + i\epsilon} \delta_{ij}$$

$$\frac{-i g_{\mu\nu}}{p^2 + i\epsilon} \delta_{ab}$$

$$\frac{-i}{p^2 - m_W^2 + i\epsilon} \cdot \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m_W^2} \right)$$

Feynman Gauge & unitary Gauge

Physical constants (PDG 2019):

$m_t = 172.9 \pm 0.4$ GeV

$m_W = 80.379 \pm 0.012$ GeV

m_b, m_ℓ, m_q set to 0, CKM to diagona

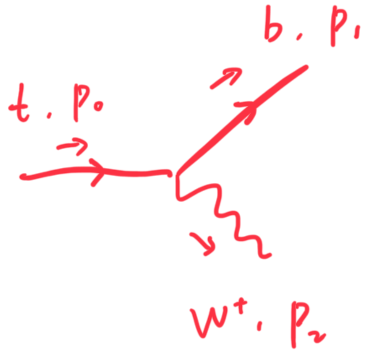
QCD coupling ($\overline{MS}$ scheme)

$$\alpha_s(\mu) = \frac{g_s^2(\mu)}{4\pi}, \quad \alpha_s(M_Z) = 0.118 \pm 0.00$$

EW coupling (G_F scheme)

$$G_F = \frac{\sqrt{2}\, g^2}{8 m_W^2} = 1.16638 \times 10^{-5} \text{ GeV}^{-2}$$

B. Top quark decay at LO (total width)



$$iM = \bar{u}(p_1)\,\gamma^\mu P_L\, u(p_0)\cdot \epsilon^*_\mu(p_2)$$
$$\cdot\, i g/\sqrt{2}$$

for unpolarized top and sum over final state spin

$$\overline{\sum}|M|^2 = \frac{1}{2N_c}\cdot Tr\left[\not p_1\, \gamma^\mu P_L \cdot (\not p_0 + m_t)\cdot \gamma^\nu P_L\right]\cdot N_c$$
$$\cdot \sum_\lambda \epsilon^{*,\lambda}_\mu(p_2)\,\epsilon^\lambda_\nu(p_2)\cdot \frac{g^2}{2}$$

recall spin sum for massive vector boson,

$$\sum_{\lambda=\pm,L} \epsilon^{*,\lambda}_\mu(p_2)\,\epsilon^\lambda_\nu(p_2) = -g_{\mu\nu} + \frac{p_{2\mu}\,p_{2\nu}}{m_W^2}.$$

thus

$$\overline{\sum}|M|^2 = \frac{g^2}{8}\left\{ 2\,Tr[\not p_1 \not p_0] + \frac{1}{m_W^2}\cdot Tr[\not p_1 \not p_2 \not p_0 \not p_2\right.$$

with a bit calculation,

$$\overline{|M|^2} = m_t^2 \cdot (1-x) \cdot \left(1+\frac{1}{2x}\right) \cdot \frac{g^2}{2}, \quad x = \frac{m_W^2}{m_t^2}$$

with the usual formula for decay width,

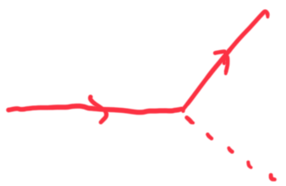
$$\Gamma = \frac{1}{2m_t} \cdot \int d\Pi_2 \, \overline{|M|^2}$$

$$= \frac{1}{2m_t} \cdot m_t^2 (1-x) \cdot \left(1+\frac{1}{2x}\right) \cdot \frac{4 m_W^2 G_F}{\sqrt{2}} \cdot \frac{1}{8\pi} \cdot (1-x)$$

$$= \frac{\sqrt{2}\, G_F m_W^2}{8\pi} \cdot m_t \cdot (1-x)^2 \left(1+\frac{1}{2x}\right)$$

numerically, $\Gamma \approx 1.49$ GeV

note when $m_t \gg m_W$, $\Gamma \propto m_t^3$ due to the longitudinal W / goldstone contribution, e.g.,



$$\overline{|M_L|^2} \approx \frac{1}{2} \mathrm{Tr}[\underbrace{\not{p}_2 \not{p}_1}_{2}] \cdot \left(\frac{\sqrt{2} m_t}{v}\right)^2$$

using $v = \frac{2 m_W}{g}$, $\overline{|M_L|^2} \approx m_t^2 \cdot \frac{1}{2x} \cdot \frac{g^2}{2}$.

Recall actually width of unstable particles are with defined as imaginary part of 1PI two-point corr

It's with optical theorem.

$$\mathrm{Im}$$  $$= \sum_X \Big|$$  $$X \Big|^2 \cdot \frac{1}{2}$$

(unitary condition $|1+i\hat{T}|^2 = 1 \Rightarrow -i(\hat{T}-\hat{T}^\dagger) = \hat{T}^\dagger\hat{T}$

That implies for scalar $\Gamma = -\dfrac{\mathrm{Im}(M(p^2))}{m}$.

For the case of fermion, define

$$-i\Sigma(\not{p}) =$$  1PI

$$= -i\left(\Sigma_+(\not{p})\, P_R + \Sigma_-(\not{p})\, P_L\right)$$

then

$$\Gamma = -\,\mathrm{Im}\left(\Sigma_+(\not{p}) + \Sigma_-(\not{p})\right)\Big|_{\not{p}=m},$$

arxiv: 0801.0669

alternative way,

in dimensional regularizatio $d = 4-2\epsilon$

$$-i\Sigma(\not{p}) = \mu^{2\epsilon} \cdot \int \frac{d^d k}{(2\pi)^d} \cdot (-g^2/2) \cdot \frac{1}{(k^2-m_W^2)((k-p$$

$$\cdot \left\{ \gamma^\mu (\not p - \not k) \gamma_\mu - \frac{1}{m_W^2} \cdot \not k (\not p - \not k) \not k \right\} \cdot P_L$$

the Dirac algebra,

$$\doteq -2(1-\epsilon)(\not p - \not k) - \frac{2p\cdot k}{m_W^2}\not k + \frac{k^2}{m_W^2}(\not k + \not p)$$

let

$$\frac{1}{i\pi^2 \cdot r_\Gamma}\cdot(\pi\mu^2)^\epsilon \cdot \int d^dk \cdot \frac{1}{(k^2-m_W^2)((k-p)^2)} \equiv B_0(p^2, m_W^2, 0)$$

$$\frac{1}{i\pi^2 \cdot r_\Gamma}\cdot(\pi\mu^2)^\epsilon \cdot \int d^dk \cdot \frac{k^\mu}{(k^2-m_W^2)((k-p)^2)} \equiv B_1(p^2, m_W^2, 0)\cdot p$$

(using Passarino - Veltman notation that will be introduced later, $r_\Gamma = \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}$)

for the second term,

$$-\frac{2p\cdot k \not k}{m_W^2}\cdot\frac{1}{(k^2-m_W^2)((k-p)^2)} \doteq -\left(1+\frac{p^2}{m_W^2}\right)\cdot\frac{\not k}{(k^2-m_W^2)((k-p)^2)}$$

$$\frac{k^2}{m_W^2}\cdot(\not k+\not p)\cdot\frac{1}{\quad} \doteq \frac{\not k+\not p}{\quad}$$

$m_W \quad \cdots \quad (k^2-m_W^2)((k-p)^2)$

$$\left[\text{e.g.}, \quad \int d^dk \cdot \frac{1}{(k-p)^2} = 0, \quad \int d^dk \, \frac{k^\mu}{k^2-m_W^2} = 0 \right]$$

thus

$$-i\Sigma(p) = i \cdot \frac{(4\pi)^\epsilon}{16\pi^2} \cdot \gamma_\mu \cdot \left(-\frac{g^2}{2}\right) \cdot \Big\{ (2\epsilon-1) B_0(p^2, m_W^2, 0)$$

$$+ \left(2-2\epsilon-\frac{p^2}{m_W^2}\right) \cdot B_1(p^2, m_W^2, 0) \cdot \not{p} \Big\} P_L ,$$

using

$$B_1(p^2, m_W^2, 0) = \frac{1}{2p^2} \cdot A_0(m_W^2) - \frac{1}{2}\left(1+\frac{m_W^2}{p^2}\right) B_0(p^2, m_W^2, \ldots$$

also

$$A_0(m_W^2) = m_W^2 \cdot \left(\frac{\mu^2}{m_W^2 - i\epsilon}\right)^\epsilon \left(\frac{1}{\epsilon} + 1 \right\} + O(\epsilon)$$

$$B_0(p^2, m_W^2, 0) = \left(\frac{\mu^2}{m_W^2}\right)^\epsilon \left\{ \frac{1}{\epsilon} + 2 + \frac{m_W^2 - p^2}{p^2} \ln\left(\frac{m_W^2 - p^2 - i\epsilon}{m_W^2}\right)\right.$$

one find the imaginary part,

$$\operatorname{Im} A_0 = 0,$$

$$\operatorname{Im} B_0(p^2, m_W^2, 0) = \frac{p^2 - m_W^2}{p^2} \pi \cdot \theta(p^2 - m_W^2)$$

finally.

$$\operatorname{Im} \Sigma_-(\not{p}) = \frac{1}{16\pi^2} \cdot \frac{g^2}{2} \cdot \not{p} \cdot \left(\frac{m_W^2}{p^2} - \frac{p^2}{2m_W^2} - \frac{1}{2} \right) \cdot$$

$$\left(1 - \frac{m_W^2}{p^2}\right) \pi \cdot \theta(p^2 - m_W^2)$$

So

$$\Gamma = -\operatorname{Im} \Sigma_-(p) \Big|_{\not{p} = m_t} = \frac{\sqrt{2}\, G_F m_W^2}{8\pi} \cdot m_t (1-x)^2 \cdot \left(1 + \frac{1}{2x}\right)$$

Proof of optical theory at Feynman diagram level $\longrightarrow$ Cutkosky rules !!