$\bar{c}\gamma^\mu c$ matrix element

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1 $^3S_1$

Ignore the overall factor:

$$\langle 0|\bar{c}\gamma^\mu c|^3S_1\rangle = \int \mathrm{d}\Omega\,\mathrm{tr}[\Pi_1\gamma^\mu] \propto \sqrt{2}\pi(\frac{m}{3E}+\frac{2}{3})\epsilon^\mu$$

2 $^3D_1$

The matrix element reads:

$$\langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle = \int \mathrm{d}\Omega \sum_{\lambda_1\lambda_2 S_z m} \mathrm{tr}\{\Pi_1\gamma^\mu\}\,\langle 1J_z|2m;1S_z\rangle\, Y_{2m}(\theta,\phi)$$

while the trace part is the same as $^3S_1$:

$$\mathrm{tr}\{\Pi_1\gamma^\mu\} = \frac{\sqrt{2}p^\mu(p\cdot\epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = (0,1,-i,0), \epsilon^{(0)} = (0,0,0,-1), \epsilon^{(+)} = (0,1,+i,0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} \langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(-)} \\ \langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(0)} \\ \langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(+)} \end{pmatrix} = \begin{pmatrix} 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(E-m)}{15E} & 0 \\ 0 & 0 & \frac{2i\sqrt{2\pi}(E-m)}{5E} & \frac{8\sqrt{\pi}(E-m)}{15E} \\ 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(m-E)}{15E} & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(E-m)}{15E} & 0 \\ 0 & 0 & \frac{2i\sqrt{2\pi}(E-m)}{5E} & \frac{8\sqrt{\pi}(E-m)}{15E} \\ 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(m-E)}{15E} & 0 \end{pmatrix}$$

Below $\bar{\epsilon 0}, \bar{\epsilon 1}, \bar{\epsilon 2}$ stands for spin state $|1-\rangle, |10\rangle, |1+\rangle$:

$$\langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(-)} = \int \mathrm{d}\Omega \frac{e^{-2i\phi}\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 0}\right)\left(-3\sqrt{2}e^{i\phi}\sin(2\theta)+3\left(-1+e^{2i\phi}\right)\cos(2\theta)+e^{2i\phi}+3\right)}{8\sqrt{\pi}E(E+m)}$$

$$\langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(0)} = \int \mathrm{d}\Omega \frac{-e^{-i\phi}\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 1}\right)\left(6e^{i\phi}\cos^2(\theta)+3\sqrt{2}\left(-1+e^{2i\phi}\right)\sin(\theta)\cos(\theta)-2e^{i\phi}\right)}{4\sqrt{\pi}E(E+m)}$$

$$\langle 0|\bar{c}\gamma^\mu c|^3D_1\rangle^{(+)} = \int \mathrm{d}\Omega \frac{\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 2}\right)\left(3e^{2i\phi}\sin^2(\theta)+3\sqrt{2}e^{i\phi}\sin(\theta)\cos(\theta)+3\cos^2(\theta)-1\right)}{4\sqrt{\pi}E(E+m)}$$

The four component of the matrix element with spin 0:

$$\left\{0,0,-\frac{2i\sqrt{2\pi}p^2}{5E(E+m)},-\frac{8\sqrt{\pi}p^2}{15E(E+m)}\right\}$$

All posible spin-0 polarization vectors that produces result orthigonal to itself:

$$\left\{\begin{pmatrix} 0 \\ 1 \\ -\frac{1+i}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \frac{1-i}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -\frac{1-i}{\sqrt{2}} \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{1+i}{\sqrt{2}} \\ 1 \end{pmatrix}\right\}$$