Canonical variables

$$A^{A\hat{j}} \qquad \hat{j} = 1, 2$$

$$\pi^{A3} = G^{A30} = \partial^{3} A^{A0}$$

E.L for A^{0}

$$\Rightarrow 0 = -(\partial^{3})^{2} A^{A0} - \partial^{\hat{j}} \pi^{A\hat{j}} + g f^{ABC} \pi^{B\hat{j}} A^{C\hat{j}}$$

$$\Rightarrow A^{A0}\left(\pi^{\hat{j}}, A^{\hat{j}}\right)$$

$$\langle \Omega | T[O_a(x_a) O_b(x_b) \ldots] | \Omega \rangle$$

$$= |N|^2 \int D[A^{Aj}]\, D[\pi^{Aj}]\, \exp\left[i \int d^4x \left(-\pi^{Aj} \dot{A}^{j} - \mathcal{H}(\pi^j, A^j) \right) \right]$$

$$O_a(x_a)\, O_b(x_b) \ldots$$

$$\mathcal{H} = \frac{1}{2} \pi^{Aj} \pi^{Aj} - \pi^{Aj} \left(\partial^j A^{A0} - f^{ABC} A^{B0} A^{Cj} \right)$$

$$+ \frac{1}{4} G^{Aij} G^{Aij} - \frac{1}{2} \partial^3 A^{A0}\, \partial^3 A^{A0}$$

$$J = \int D[A^{Aj}]\, D[A^{A0}]\, D[\pi^{Aj}]$$

$$\exp\left[i \int d^4x \left(-\pi^{Aj} \dot{A}^j - \mathcal{H}' \right) \right]$$

$\mathcal{H}' = \mathcal{H}$ (A^0 free)

$= A^0 (\cdots) A^0 + ( ) A^0$

$+ C.$

$$\int_{-\infty}^{+\infty} dx\, e^{-x^2+2x} = \int dx\, e^{-(x-1)^2+1} = C \cdot e^{1}$$

$$-1^2 + 2 \cdot 1 = 1$$

$$\frac{d}{dx}(-x^2+2x) = -2x+2 = 0 \Rightarrow x = 1$$

$$0 = \frac{dH'}{dA^0} \Rightarrow (\partial^3)^2 A^{A0} + \partial^j \pi^{Aj} - g f^{ABC} \pi^{Bj} A^{Cj} = 0$$

$$H' = \pi^j (\quad) \pi^j + (\quad) \pi^j + C$$

$$\underline{0 = \frac{\partial H'}{\partial \pi^{Aj}}} = \underline{-\pi^{Aj} + G^{Ai0}}$$

$$1 = \int D[A^3]\, \delta(A^{A3})$$

$$\langle \Omega | T[O_a(x_a)\, O_b(x_b) \cdots] | \Omega \rangle$$

$$= |N|^2 \int D[A^{A\mu}]\, \boxed{\delta(A^{A3})} \rightarrow \delta(\varepsilon)$$

$$\exp\left[i \int d^4x \cdot \mathcal{L} \right] O_a(x_a) O_b(x_b) \cdots$$

$$\underline{\delta A^{A3}} \sim \underbrace{f^{ABC} \varepsilon^C A^{B3}}_{\downarrow\, 0} + \underbrace{\partial^3 \varepsilon^A}_{C\text{-}\varepsilon}$$

$$\delta_{\varepsilon}(A^{A3}) \longrightarrow \underline{\delta\left[\boxed{f^A(\varepsilon)} - F^A \right] \det M}$$

$$\boxed{M^{AB} = \left. \frac{\delta f^A(\varepsilon)}{\delta \varepsilon^B} \right|_{\varepsilon=0}}$$

$A^3 = 0, 1, 2$

$$\langle \Omega | T(\quad) | \Omega \rangle$$

$$= |N|^2 \int \underline{D[A^{A\mu}]}\, \underline{\delta\left[f^A(\varepsilon) - F^A \right] \det M}$$

$$\exp\left[i \int d^4x \, \mathcal{L} \right] \cdots$$

$$\underline{\int D[F] \left[G[F] \, \delta(f^A - F^A) \right] = G[f]}$$

Choose $G[f] = \exp\left[i \int d^4x \frac{1}{-2\xi} f^A f^A \right]$

$$= |N|^2 \int D[A^{A\mu}] \det M \, e^{i\int d^4x \left(\mathcal{L} - \frac{1}{2\xi} f^A f^A\right)} \cdots$$

$$\det M = \int D[\bar{c}^A, c^A] \, e^{\int d^4x \, d^4y \, \bar{c}^A(x) \frac{\delta f^A(x)}{\delta \varepsilon^B(y)}\Big|_{\varepsilon=0} c^B(y)}$$

$$= |N|^2 \int D[A^{A\mu}]\, D[c^A]\, D[\bar{c}^A]$$

$$\exp\Big[i\int d^4x \Big(\mathcal{L} \underbrace{- \frac{1}{2\xi} f^A f^A}_{\mathcal{L}_{GF}} + \underbrace{\bar{c}^A(-iM^{AB})c^B}_{\mathcal{L}_{ghost}}\Big)\Big]$$

Covariant gauge

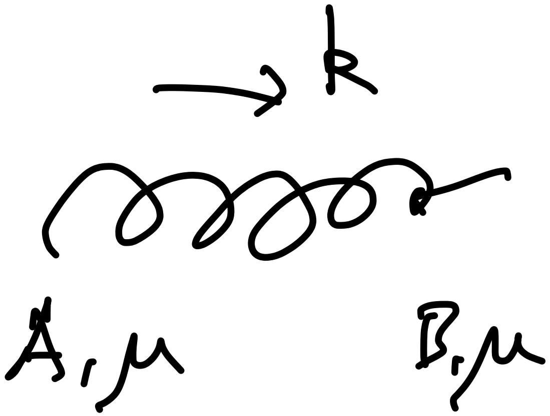
$$f^A = \partial_\mu A^{A\mu}$$

$$\boxed{\text{HW1}}$$

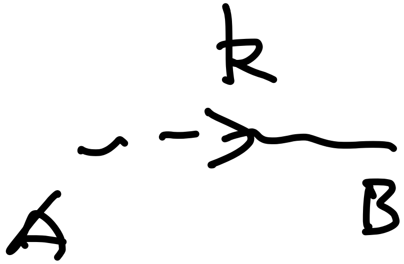
$$\left.\frac{\delta f^A(x)}{\partial \varepsilon^B(y)}\right|_{\varepsilon=0} = \left(\partial^\mu \partial_\mu \delta^{AB} + g f^{ACB} \partial_\mu A^{C\mu}\right)$$

$$\times \frac{f}{g}\, \delta^{(4)}(x-y)$$

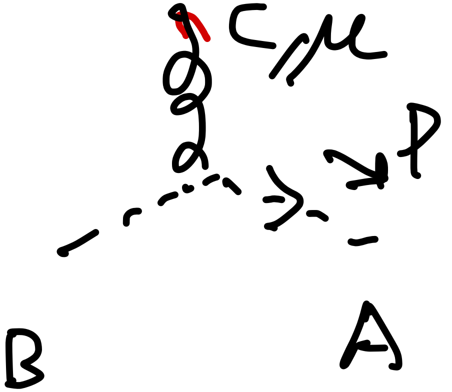
$$\mathcal{L}_{ghost} = (\partial_\mu \bar{c}^A)(\partial^\mu c^A) \underline{- g f^{ABC} (\partial_\mu \bar{c}^A)\, c^B A^{C\mu}}$$



$$\delta^{AB} \frac{i}{k^2+i\epsilon}\left(-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$$



$$\delta^{AB} \frac{i}{k^2+i\epsilon}$$



$g f^{ABC} p^\mu$ $\boxed{\text{HW2}}$

III. Renormalization.

→ **Higher-order calculation.**

UV divergences

part of QFT.

View of EFT.

foms on Low Gnergy

$$\underset{\sim}{M} = M_0 + \frac{\alpha_s}{4\pi} M_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 M_2 + \cdots$$

$\downarrow$ LO $\downarrow$ NLO $\downarrow$ NNLO

loop integral

UV divergences

when $k \to \pm\infty$

Renormalizable:

all divergences can be removed by renormalization of a finite number of couplings in the Lagrangian.

1971. 't Hooft. QCD is renormalizable. $\left(1+\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\right)\left(1-\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\right)$

In QCD. $\underline{O(m_j, g_s) \text{ finite}}$

$\Rightarrow$ m_j, g_s are divergent.

They cancel exactly the UV divergences in the loops.

$$g_s \bar{\psi} \gamma^\mu T^A \psi A^A$$

$$m \bar{\psi} \psi$$

- Perform renormalization.
- Bare parameter renormalization.

Bare $\mathcal{L}$ and Feynman rules

$O_1 (m_j \cdot g_s)$

Loop integrals are divergent.

"Regularization" $\begin{cases} \text{cutoff reg.} \\ \text{dimensional reg.} \end{cases}$

$\underline{d = 4 - 2\epsilon}$

$$\int_1^\infty dx \frac{1}{x} \to \int_1^\infty dx \cdot \frac{x^{-2\epsilon}}{x} \to \frac{1}{\epsilon}$$

$\frac{d}{2}$

$\frac{d-2}{2}$

$$\int_1^\infty dx \frac{1}{x^2} (1 - 1 + x) = \int_1^\infty dx \frac{1}{x}$$

$$E^{4-n}$$

$$O_2(m_j, g_s) \qquad O_3(m_j, g_s)$$

$$\Rightarrow \quad m_j^R = m_j + C + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\right)_{+\cdots}$$

$$g_s^R = g_s + C + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\right) + \cdots$$

$$O_1\left(m_j, g_s, \frac{1}{\epsilon}\right)$$

$$m_j^R - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\cdots \qquad g_s^R - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \cdots$$

$$= O_1^R(m_j^R, g_s^R)$$

$$\frac{\alpha_s}{4\pi}\left(\frac{1}{\underset{\uparrow \text{loop}}{\epsilon}} - \frac{1}{\underset{\uparrow m_j^R}{\epsilon}}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2(\cdots - \cdots)$$

- BPHZ scheme

$$O^R \quad \text{finite}$$

$$O^R(m_j^R, g_s^R)$$

$$q_j = Z_{2,j}^{1/2}\, q_{j,R}$$

$$A^\mu = Z_3^{1/2}\, A_R^\mu$$

$$c^a = Z_2^{c\,1/2}\, c_R^a$$

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{R} + \mathcal{L}_{QCD}^{C.T.}$$

$$\mathcal{L}_{QCD}^{R} = \mathcal{L}_{QCD}\left(m_j \to m_j^R, \; g_s \to g_s^R\right)$$

A (R): dropped.

$$\mathcal{L}_{QCD}^{C.T.} = \underline{\mathcal{L}_{QCD} - \mathcal{L}_{QCD}^{R}}$$

$$= -\frac{1}{4} \delta_3 (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2$$

$$+ \sum_j \bar{q}_j (i \delta_2^j \not{\partial} - \delta_m^j) q_j$$

$$- \delta_2^c \, \bar{c}^a \partial^2 c^a$$

$$+ \sum_j \underline{g_s^R} \, \delta_1^j A^a_\mu \bar{q}_j \gamma^\mu q_j$$

$$- g_s^R \, \delta_1^{3g} f^{abc} (\partial_\mu A^a_\nu) A^b_\mu A^c_\nu$$

$$+\frac{1}{4} g_s^{R\,2} \delta_1^{4g} \left(f^{eab} A_\mu^a A_\nu^b\right) \left(f^{ecd} A_\mu^c A_\nu^d\right)$$

$$-g_s^R \delta_1^c f^{abc} \bar{c}^a \partial^\mu A_\mu^b c^c$$

$$\delta_2^j = Z_{2,j} - 1 \qquad \delta_2^c = Z_2^c - 1$$

$$\delta_3 = Z_3 - 1 \qquad \delta_m^j = Z_{2,j} m_j - m_j^R$$

$$\delta_1^{\tilde{g}} = \frac{g_s}{g_s^R} Z_{2,\tilde{g}} Z_3^{1/2} - 1, \quad \delta_1^{3g} = \frac{g_s}{g_s^R} Z_3^{3/2} - 1$$

$$\delta_1^{4g} = \frac{g_s^2}{g_s^{R^2}} Z_3^2 - 1, \quad \delta_1^{c} = \frac{g_s}{g_s^R} Z_2^c Z_3^{1/2} - 1$$