Research Summary & Plans

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Outlines

1. Operator Product Expansion for Atomic Wave Functions

2. Meson-meson scattering in 1+1 dimensional QCD

3. NRQCD Factorization for Fully-heavy Tetraquark Production

Operator Product Expansion for Atomic Wave Functions

Motivation

- Logarithmic divergence appearing near the origin of Hydrogen wave functions given by Dirac equation
- Mentioned in textbooks, i.e.:

The Dirac wave functions with $j=\frac{1}{2}$ ($l=0$ or 1), unlike the other Dirac functions and all Schrödinger wave functions, are *singular* at the origin for all principal quantum numbers n. If $Z\alpha \approx Z/137$ is small, however, this singularity is a very weak one. Consider, for instance, the states with $j=\frac{1}{2}$ and $l=0$ (for any value of n). For small distances, $\varrho \ll 1$, the Schrödinger function $R(\varrho)$ is approximately equal to a constant $R(0)$, but the Dirac function $g(\varrho)$ is given by

$$g(\varrho) \sim R(0)\,\varrho^{\gamma-1} \sim R(0) \exp\left[\frac{1}{2}(Z\alpha)^2 \log \frac{1}{\varrho}\right].$$

Thus $g(\varrho)$ is infinite at the origin but, at finite distances ϱ larger than $\exp(-1/Z^2\alpha^2)$, $(g-R)/R$ is still only of order $\frac{1}{2}(Z\alpha)^2 \log \varrho$. Only for exceedingly small distances, ϱ of the order of $\exp[-2(137/Z)^2]$, does $(g-R)/R$ become of order unity or greater. For all but very large Z, this distance is well inside the nucleus. For the $j=\frac{1}{2}$, $l=1$ states, the singular term is smaller by a factor of order $(Z\alpha)^2$ than for the $l=0$ states. Since $R(\varrho)$ is proportional to ϱ for small ϱ if $l=1$, in this case $(g-R)/R$ is of order $(Z\alpha)^2/\varrho$.

Figure 1: QM by Bethe & Salpeter

Motivation

☐ Universal behaviors in Coulombic wave functions, near-the-origin divergence in relativistic wave functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} + \cdots \ (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} + \cdots \ (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} + \cdots \ (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} + \cdots \ (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{KG}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

Motivation

☐ Universal behaviors in Coulombic wave functions, near-the-origin divergence in relativistic wave functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} + \cdots \; (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} + \cdots \; (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} + \cdots \; (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} + \cdots \; (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{Dirac}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} - \frac{1}{2}Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2}Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} - \frac{1}{2}Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2}Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} - \frac{1}{2}Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2}Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} - \frac{1}{2}Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2}Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

Question:

- ☐ What caused the state-independent logarithmic divergences?
- ☐ Hint:
  - Logarithms from the wave functions appear at order-α^2,
  - anomalous dimensions of NRQCD also appear at order-α_s^2,
  - Bethe-Salpeter wave function is defined as $\langle 0|\psi|\Psi\rangle$ in QFT, state-independency implies operator properties
- ☐ Treat the nucleus as an infinitely heavy field, similar to HQET
- ☐ Natural assumption: the QFT correspondence of Dirac equation is QED
- ☐ UV behavior of Dirac wave functions + operator behavior $\Rightarrow$Operator product expansion (OPE)

Failed Attempt: OPE & QED

☐ QED + heavy nucleus effective theory (HNET):

$$\mathcal{L}_{\mathrm{UV}} = \bar{\Psi}(i\not{D} - m)\Psi + N^{\dagger} i D_0 N - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{1}$$

Ψ: QED electron field

N: nucleus field

F: photon field, only considering Coulomb potential

☐ Dirac wave function:

$$\Psi_{njm}(\mathbf{r}) \equiv \langle 0|\Psi(\mathbf{r}) N(0)|njm\rangle, \tag{2}$$

Failed Attempt: OPE & QED

- Operator Product Expansion (OPE): The limit when product of local operators at different points approach each other.

$$T\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\mathcal{O}}(x^{\mu})[\mathcal{O}(0)]_R \tag{3}$$

- Expand $\Psi(\mathbf{r})N(0)$ with OPE:

OPE relation in coordinate space (QED)

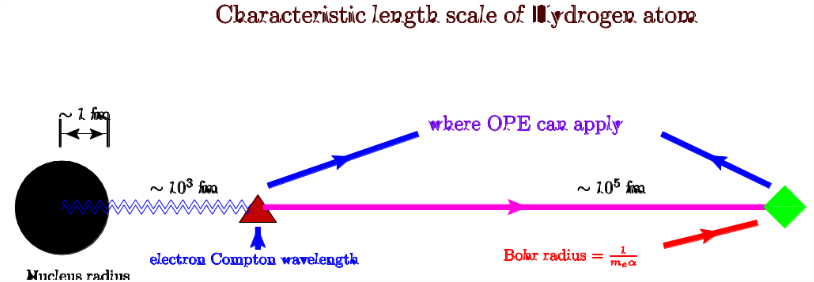
$$\Psi(\mathbf{r})N(\mathbf{0}) = (1 + \frac{Z\alpha}{\pi}\ln r)\,[\Psi N](\mathbf{0}) + \cdots$$

- Logarithms at order-α!

- Why?

  The UV behavior of QED does not reflect the UV behavior of Dirac wave function.

The scales of an atom



- ☐ electron Compton wavelength is the IR scale of QED
- ☐ Desired OPE should probe the UV limit of an EFT whose effectiveness stays below m_e.

Attack the problem with OPE & EFT: Construct EFT

- Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus.

- Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \mathcal{L}_{\mathrm{Max}} + \mathcal{L}_{\mathrm{NRQED}} + \mathcal{L}_{\mathrm{HNET}} + \delta\mathcal{L}_{\mathrm{contact}} \tag{4}$$

where

$$\begin{aligned}
\mathcal{L}_{\mathrm{Max}} &= -\frac{1}{4} d_\gamma F_{\mu\nu} F^{\mu\nu} + \cdots\,,\,, \\
\mathcal{L}_{\mathrm{NRQED}} &= \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\boldsymbol{\nabla} \cdot \mathbf{E}]}{8m^2} + \cdots \right\}, \\
\mathcal{L}_{\mathrm{HNET}} &= N^\dagger i D_0 N + \cdots\,, \\
\delta\mathcal{L}_{\mathrm{contact}} &= \frac{c_4}{m^2} \psi^\dagger \psi N^\dagger N + \cdots\,,
\end{aligned}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: Construct EFT

- ☐ Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus, keep only Coulomb potential.

- ☐ Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \cancel{\mathcal{L}_{\rm Max}}\ \mathcal{L}_{\rm Coul} + \mathcal{L}_{\rm NRQED} + \mathcal{L}_{\rm HNET} + \delta\mathcal{L}_{\rm contact} \tag{4}$$

where

$$\mathcal{L}_{\rm Coul} = \frac{1}{2}\left(\nabla A^0\right)^2,$$

$$\mathcal{L}_{\rm NRQED} = \psi^\dagger\left\{iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e\frac{[\boldsymbol{\nabla}\cdot\mathbf{E}]}{8m^2} + \cdots\right\},$$

$$\mathcal{L}_{\rm HNET} = N^\dagger iD_0 N + \cdots\,,$$

$$\xcancel{\delta\mathcal{L}_{\rm contact} = \frac{c_4}{m^2}\psi^\dagger\psi N^\dagger N + \cdots\,,}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: Renormalization of the local operator

- Use 4-point Green function as testing ground.

- The renormalization constant of $[\psi N](0)$:

$$[\psi N]_R = Z_{\mathcal{S}} \psi N \tag{5}$$

- The total divergence coming from the local operator (MS scheme)

$$Z_{\mathcal{S}} = 1 - \frac{Z^2\alpha^2}{4\epsilon} + \cdots \,. \tag{6}$$

- The anomalous dimension of the operator ψN then reads

$$\gamma_{\mathcal{S}} \equiv \frac{d \ln Z_{\mathcal{S}}}{d \ln \mu} = \frac{Z^2\alpha^2}{2} . \tag{7}$$

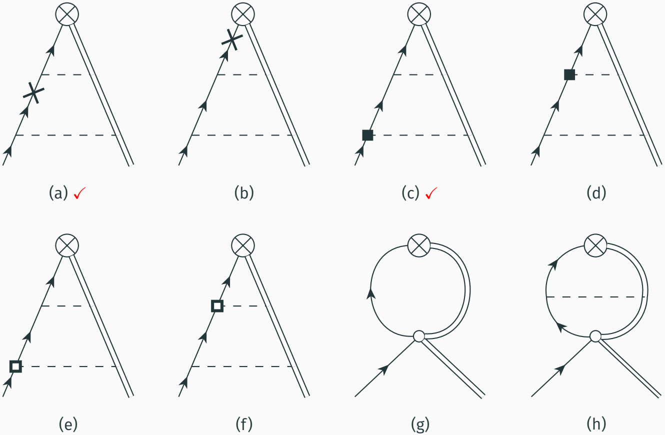


Figure 2: Representative diagrams for local operator renormalization. The cap represents the insertion of the operator ψN, cross stands for the $\mathbf{p}^4$ relativistic correction, solid square for the Darwin vertex, while empty square for spin-orbital vertex, which making vanishing contribution in this case. The empty circle represents the contact interaction. The last two diagrams are beyond the prescribed accuracy of $\mathcal{O}(Z^2\alpha^2)$.

Attack the problem with OPE & EFT: Wilson coefficients

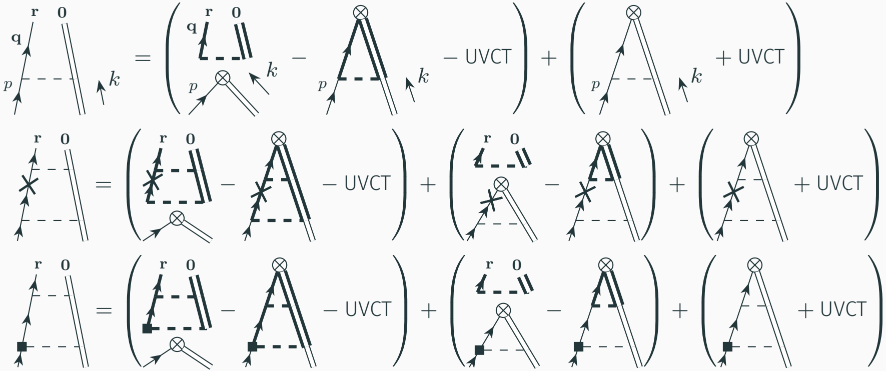


Figure 3: Illustration of the OPE structure of the four-point Green functions through order $Z^2\alpha^2$. The first line is for the Wilson coefficient $\mathcal{C}^{(1)}(r)$, the two bottom lines for the Wilson coefficient $\mathcal{C}^{(2)}(r)$. The thick line indicates the corresponding loop momentum to be "hard" ($\sim \mathbf{q}$).

Correct OPE relation in coordinate space

$$\psi(\mathbf{r})N(\mathbf{0}) = \left[1 - mZ\alpha r - \frac{Z^2\alpha^2}{2}\left(\ln\mu r + \text{const}\right) + \mathcal{O}(Z^3\alpha^3)\right][\psi N](\mathbf{0}) + \cdots$$

Correct OPE relation in momentum space

$$\begin{aligned}\widetilde{\psi}(\mathbf{q})N(\mathbf{0}) &\equiv \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\psi(\mathbf{r})N(\mathbf{0}) \\ &= \left[\frac{8\pi mZ\alpha + \mathcal{O}(Z^3\alpha^3)}{|\mathbf{q}|^4} - \frac{\pi^2 Z^2\alpha^2 + \mathcal{O}(Z^4\alpha^4)}{|\mathbf{q}|^3}\right][\psi N](\mathbf{0}) + \cdots .\end{aligned}$$

- ☐ $\log r$ behavior is the same with the Dirac wave function.

Attack the problem with OPE & EFT: Resumming logarithms with RGE

- ☐ The l.h.s. of the OPE relation is scale independent.

- ☐ We can write down the renormalization group equation of the Wilson coefficient:

$$\mu \frac{\partial \mathcal{C}(r,\mu)}{\partial \mu} + \gamma_{\mathcal{S}} \mathcal{C}(r,\mu) = 0, \tag{8}$$

- ☐ Dimensional analysis leads to

$$r \frac{\partial \mathcal{C}}{\partial r} + \gamma_{\mathcal{S}} \mathcal{C} = 0. \tag{9}$$

- ☐ We then recovers the leading logs:

$$\mathcal{C}(r,\mu) = \mathcal{C}(r_0,\mu) \left(\frac{r}{r_0}\right)^{-\frac{Z^2\alpha^2}{2}}. \tag{10}$$

Recover the Dirac wave function

☐ Solution to Dirac equation expressed by Pauli spinor:

$$\Psi_{n\frac{1}{2}m}(\mathbf{r}) = \begin{pmatrix} F_n(r)\sqrt{\frac{1}{4\pi}}\,\xi_m \\ G_n(r)\sqrt{\frac{3}{4\pi}}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\,\xi_m \end{pmatrix}, \tag{11}$$

☐ We only consider the upper component, whose asymptotic behavior is

$$F_n(r) \approx R_n^{\mathrm{Sch}}(0)\left(\frac{2r}{na_0}\right)^{-\frac{Z^2\alpha^2}{2}}, \tag{12}$$

☐ Set $r_0 = \frac{na_0}{2}$ for the nS hydrogen state, and $\mu_0 = 1/r_0$. The boundary condition is $\mathcal{C}(r = r_0; \mu = \mu_0) = 1$.

$$\langle 0|[\psi N]_R(0;\mu_0)|nS_{1/2}, m\rangle \approx \frac{1}{\sqrt{4\pi}}\,R_{n0}^{\mathrm{Sch}}(0)\,\xi_m, \tag{13}$$

☐ We reproduced the asymptotic form of the wave function in (12).

Summary

- ☐ We attempted to understand the divergence of Dirac hydrogen wave function near the origin with OPE & NREFT.
- ☐ OPE + QED won't work.
- ☐ NREFT (NRQED + HNET) shows anomalous dimension of the local operators at order-α^2.
- ☐ $\log r$ behavior is reproduced in the Wilson coefficient of the OPE.
- ☐ Resummed leading logs with RGE to recover the asymptotic behavior of the wave function with exponents.

Meson-meson scattering in 1+1 dimensional QCD

- ☐ Large-N Expansion
- ☐ In 1+1-d, ONLY PLANAR DIAGRAM!!!

Steps:

1. Obtain mesons' 't Hooft wave-functions with 't Hooft equation (Fig 4).
2. Obtain effective meson-meson vertex function with Bethe-Salpeter equation (Fig 5).
3. Calculate meson-meson scattering amplitude with said vertex functions and wave-functions.

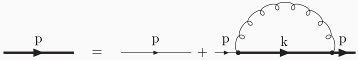


Figure 4: The Dyson-Schwinger equation for the quark self-energy.

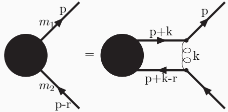


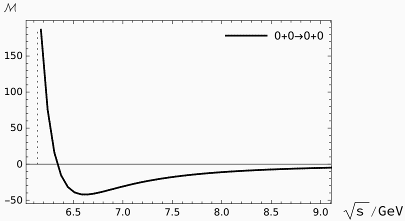
Figure 5: The Bethe-Salpeter equation for the $q\bar{q}$ bound state.

't Hooft equation

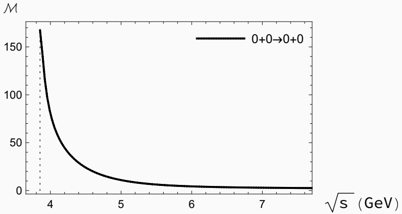
$$\mu^2 \varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right)\varphi(x) - P\int_0^1 dy \frac{\varphi(y)}{(x-y)^2}. \tag{14}$$

μ is the mass of the meson, α_i is rescaled quark mass, P marks principle value.

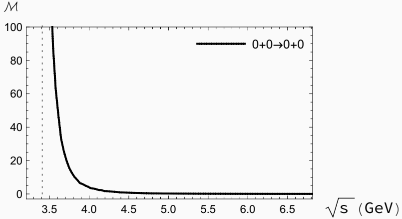
Results (No indication of tetraquark!!!)



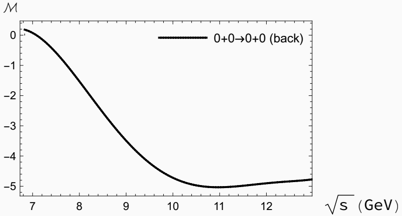
(a) Amplitudes for the contact term in $A(c\bar{c}) + B(c\bar{c}) \to C(c\bar{c}) + D(c\bar{c})$.



(b) Amplitudes for the contact term in $A(c\bar{s}) + B(c\bar{s}) \to C(c\bar{s}) + D(c\bar{s})$.



(c) Amplitudes for the contact term in $A(c\bar{u}) + B(c\bar{d}) \to C(c\bar{u}) + D(c\bar{d})$.

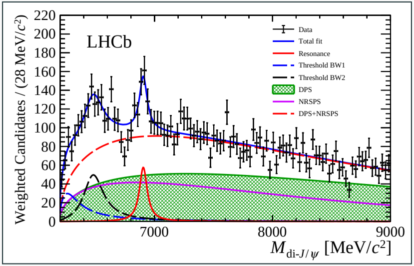


(d) Amplitudes for the contact term in $A(c\bar{d}) + B(b\bar{s}) \to C(b\bar{d}) + D(c\bar{s})$ with particle B moving backwards. No near-threshold enhancement.

NRQCD Factorization for Fully-heavy Tetraquark Production

Factorization theorem for $T_{4c/b}$ production

- LHCb discovered a narrow structure near 6.9 GeV in the di-J/ψ invariant mass spectrum ($> 5\sigma$): $X(6900)$.
- Strong candidate for fully-charmed tetraquark.



- QCD factorization theorem for fully-heavy tetraquark ($T_{4c/b}$) exclusive production at high-p_T

$$
\mathrm{d}\sigma\left(pp \to T_{4c/b}(p_\mathrm{T}) + X\right) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z \; f_{a/p}(x_a,\mu) f_{b/p}(x_b,\mu)
$$

$$
\times \mathrm{d}\hat{\sigma}(ab \to i(p_T/z) + X, \mu) D_{i \to T_{4c/b}}(z,\mu) + \mathcal{O}(1/p_T). \tag{15}
$$

- Dominate partonic channel is $gg \to gg$, rather than $gg \to q\bar{q}$.

Fragmentation Function

Collins-Soper definition of fragmentation function:

$$D_{g\to T_{4c}}(z,\mu) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ \left(N_c^2-1\right)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-}$$

$$\times \sum_X \left\langle 0 \left| G_c^{+\mu}(0)\mathcal{E}^\dagger\left(0,0,\mathbf{0}_\perp\right)_{cb} \right| T_{4c}(P)+X\right\rangle \left\langle T_{4c}(P)+X \right| \mathcal{E}\left(0,x^-,\mathbf{0}_\perp\right)_{ba} G_a^{+\nu}\left(0,x^-,\mathbf{0}_\perp\right)\Big| 0 \Big\rangle$$

- ☐ c/b quarks are heavy enough such that Fock states with light quarks or gluons are suppressed
- ☐ Similar to quarkonium cases
- ☐ NRQCD factorization for $T_{4c/b}$ production:

$$\begin{aligned} D_{g\to T_{4c}}\left(z,\mu_\Lambda\right) = &\frac{d_{3,3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\langle 0|\mathscr{O}_{3,3}^{(J)}|0\rangle + \frac{d_{6,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\langle 0|\mathscr{O}_{6,6}^{(J)}|0\rangle \\ &+ \frac{d_{3,6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9} 2\mathrm{Re}\left[\langle 0|\mathscr{O}_{3,6}^{(J)}|0\rangle\right] + \cdots, \end{aligned} \tag{16}$$

Collins-Soper definition of fragmentation function:

$$D_{g\to T_{4c}}(z,\mu)=\frac{-g_{\mu\nu}z^{d-3}}{2\pi k^{+}\left(N_c^2-1\right)(d-2)}\int_{-\infty}^{+\infty}dx^{-}e^{-ik^{+}x^{-}}$$

$$\times\sum_{X}\left\langle 0\left|G_c^{+\mu}(0)\mathcal{E}^{\dagger}\left(0,0,\mathbf{0}_{\perp}\right)_{cb}\left|T_{4c}(P)+X\right\rangle\left\langle T_{4c}(P)+X\right|\mathcal{E}\left(0,x^{-},\mathbf{0}_{\perp}\right)_{ba}G_a^{+\nu}\left(0,x^{-},\mathbf{0}_{\perp}\right)\right|0\right\rangle$$

- ☐ c/b quarks are heavy enough such that Fock states with light quarks or gluons are suppressed
- ☐ Similar to quarkonium cases
- ☐ Vacuum saturation approximation to suppress extra intermediate states

$$\mathscr{O}_{3,3}^{(J)}=\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_{X}\left|T_{4c}^{J}+X\right\rangle\left\langle T_{4c}^{J}+X\right|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)\dagger}$$

$$\mathscr{O}_{6,6}^{(J)}=\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)}\sum_{X}\left|T_{4c}^{J}+X\right\rangle\left\langle T_{4c}^{J}+X\right|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}$$

$$\mathscr{O}_{3,6}^{(J)}=\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_{X}\left|T_{4c}^{J}+X\right\rangle\left\langle T_{4c}^{J}+X\right|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}$$

☐ Local tetraquark operators:

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}}, \tag{16a}$$

$$\mathcal{O}^{\alpha\beta;(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\,\Gamma^{\alpha\beta;mn}\,\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}}, \tag{16b}$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}, \tag{16c}$$

The rank-4 Lorentz tensor is given by $\Gamma^{\alpha\beta;mn} \equiv \frac{1}{2}[g^{\alpha m}g^{\beta n} + g^{\alpha n}g^{\beta m} - \frac{1}{2}g^{\alpha\beta}g^{mn}]$, and the rank-4 color tensors read

$$\mathcal{C}^{ab;cd}_{\mathbf{3}\otimes\bar{\mathbf{3}}} \equiv \frac{1}{(\sqrt{2})^2}\epsilon^{abm}\epsilon^{cdn}\frac{\delta^{mn}}{\sqrt{N_c}} = \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}) \tag{17a}$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{6}}\otimes\mathbf{6}} \equiv \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}). \tag{17b}$$

- NRQCD factorization:

$$D_{g\to H}(z) = \sum_n d_n(z) \left\langle 0 \left| \mathcal{O}_n^H \right| 0 \right\rangle$$

- Perturbative matching to determine short distance coefficients.
- Use wave-function origin (S-wave) from potential models to determine long range matrix elements in order to yield a phenomenological result.
- More details in Jia-Yue Zhang's talk this afternoon.

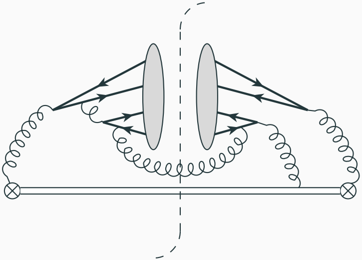
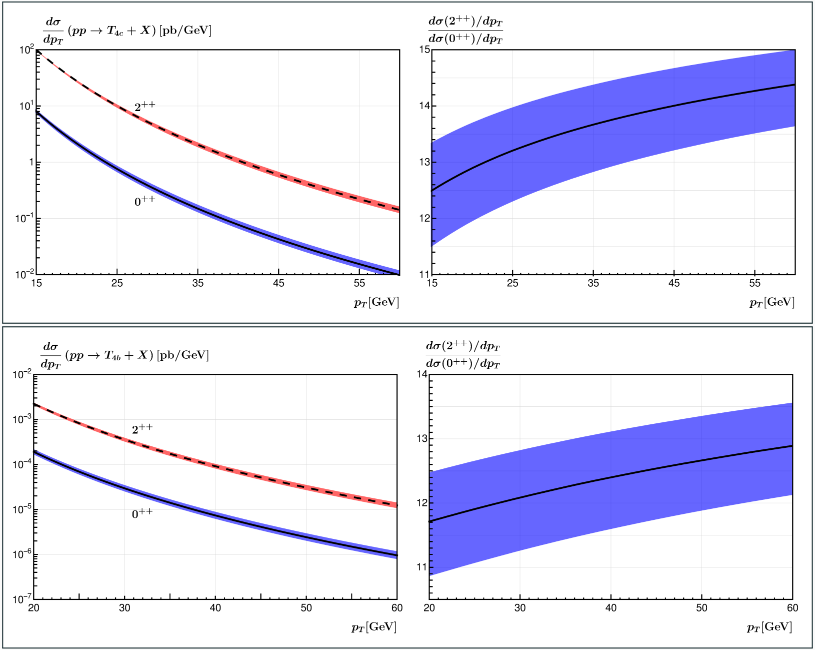


Figure 6: A representative Feynman diagram for the fragmentation function of gluon into T_{4c}. The grey blob indicates the C-even tetraquark. Horizontal double line denotes the eikonal line.

Phenomenology for $T_{4c/b}$ production at LHC



Phenomenology for $T_{4c/b}$ production at LHC

- ☐ 2^{++} cross section is about 10 times larger than 0^{++}.
- ☐ We obtain the yields of the accumulated event number for T_{4c} at HL-LHC are a hundred million for 0^{++} and 8 hundreds million for 2^{++} (with integrated luminosity 3000 fb^{-1}).
- ☐ The prediction for T_{4b} is highly suppressed, mainly due to the relative larger bottom mass suppression.
- ☐ The total cross section we obtained is unreliable mainly due to the fact that fragmentation only works at high-p_T, and our integration is done within approximately $15 \leq p_T \leq 60\mathrm{GeV}$.

Summary & Outlook

Summary:

- ☐ Production mechanism for fully-heavy tetraquark $T_{4c/b}$ hasn't been well discussed on a QCD basis
- ☐ We propose a framework for $T_{4c/b}$ production based on NRQCD factorization
- ☐ We adopt fragmentation mechanism for $T_{4c/b}$ production @LHC
- ☐ Calculate FF from QCD-based Collins-Soper definition, FF is factorized into SDCs and NRQCD LDMEs
- ☐ Defined LO NRQCD local operators and derived LO SDCs for $D_{g\to T_{4c}}$
- ☐ We use phenomenological wave functions at origin to obtain predictions of cross sections

Outlook:

- ☐ Production at other experiments
- ☐ P-wave $T_{4c/b}$

Research Plans

☐ New projects: Anything QCD or EFT related

☐ Old projects:

- Three body OPE to the 1st order
- Fully reconstruct Schroedinger wave function with OPE (renormalization problem for QM)
- P-wave tetraquark fragmentation function
- Top loop Coulomb resummation

Questions?

Backup

bottom versus charm

- ☐ Charm:

$$\alpha_s(4m_c) = 0.22,\ m_c = 1.5\text{GeV},\ \mathrm{R_{D_c}}(0) = 0.523\ \text{GeV}^{3/2},\ \mathrm{R_{T_c}}(0) = 2.902\ \text{GeV}^{3/2}$$

- ☐ Bottom:

$$\alpha_s(4m_b) = 0.17,\ m_b = 4.8\text{GeV},\ \mathrm{R_{D_b}}(0) = 0.703\ \text{GeV}^{3/2},\ \mathrm{R_{T_b}}(0) = 5.579\ \text{GeV}^{3/2}$$

- ☐ The ratio is

$$\begin{aligned}
&\left(\frac{\alpha_s(4m_c)}{\alpha_s(4m_b)}\right)^4 \left(\frac{m_c}{m_b}\right)^{-9} \left(\frac{R_{T_c}(0)}{R_{T_b}(0)}\right)^2 \left(\frac{R_{D_c}(0)}{R_{D_b}(0)}\right)^4 \\
=&\left(\frac{0.22}{0.17}\right)^4 \left(\frac{1.5}{4.8}\right)^{-9} \left(\frac{2.902}{5.579}\right)^2 \left(\frac{0.523}{0.703}\right)^4 \\
\approx&10^4
\end{aligned}$$