Homework: Quantum Field Theory #8

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1. $T\{\phi(x)\phi(y)\}$. (Some definition are the same as P&S so I'll skip them. The result would be $\phi^+|0\rangle=0$ and $\langle 0|\phi^-=0$.)

$$\begin{aligned}
\langle 0|T\{\phi(x)\phi(y)\}|0\rangle &= \langle 0|T\big\{\phi^+(x)\phi^+(y)+\phi^+(x)\phi^-(y)+\phi^-(x)\phi^+(y)+\phi^-(x)\phi^-(y)\big\}|0\rangle \\
&= \langle 0|T\big\{\phi^+(x)\phi^+(y)+\phi^-(y)\phi^+(x)+\phi^-(x)\phi^+(y)+\phi^-(x)\phi^-(y)+[\phi^+(x),\phi^-(y)]\big\}|0\rangle \\
&= \begin{cases}[\phi^+(x),\phi^-(y)], x^0>y^0\\ [\phi^+(y),\phi^-(x)], y^0>x^0\end{cases} \\
&\equiv \wick{\c\phi(x)\c\phi(y)} = D_F(x-y)
\end{aligned}$$

$$T\{\phi(x)\phi(y)\} = N\Big\{\phi(x)\phi(y)+\wick{\c\phi(x)\c\phi(y)}\Big\}$$

2. $T\{\phi(x)\phi(y)\phi(z)\phi(t)\}$.

For the convenience of writing we set $\phi(x_n)$ as ϕ_n.

$$\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle = \langle 0|T\big\{(\phi_1^+\phi_2^+ + \phi_1^-\phi_2^+ + \phi_1^+\phi_2^- + \phi_1^-\phi_2^-)\phi_3\phi_4\big\}|0\rangle$$

we set $x^0>y^0>z^0>t^0$ for now

$$\begin{aligned}
&= \langle 0|(\phi_1^+\phi_2^+ + \phi_1^+\phi_2^-)\phi_3\phi_4|0\rangle \\
&= \langle 0|(\phi_1^+\phi_2^+ + \phi_1^+\phi_2^-)(\phi_3^+\phi_4^+ + \phi_3^-\phi_4^+ + \phi_3^+\phi_4^- + \phi_3^-\phi_4^-)|0\rangle \\
&= \langle 0|(\phi_1^+\phi_2^+ + \phi_1^+\phi_2^-)(\phi_3^+\phi_4^- + \phi_3^-\phi_4^-)|0\rangle \\
&= \langle 0|(\phi_1^+\phi_2^+ + [\phi_1^+,\phi_2^-])([\phi_3^+,\phi_4^-] + \phi_3^-\phi_4^-)|0\rangle \\
&= \langle 0|\phi_1^+\phi_2^+[\phi_3^+,\phi_4^-] + \phi_1^+\phi_2^+\phi_3^-\phi_4^- + [\phi_1^+,\phi_2^-]\phi_3^-\phi_4^-|0\rangle + [\phi_1^+,\phi_2^-][\phi_3^+,\phi_4^-] \\
&= \langle 0|\phi_1^+\phi_2^+\phi_3^-\phi_4^-|0\rangle + [\phi_1^+,\phi_2^-][\phi_3^+,\phi_4^-] \\
&= \langle 0|\phi_1^+([\phi_2^+,\phi_3^-] + \phi_3^-\phi_2^+)\phi_4^-|0\rangle + [\phi_1^+,\phi_2^-][\phi_3^+,\phi_4^-] \\
&= \langle 0|[\phi_1^+,\phi_4^-][\phi_2^+,\phi_3^-] + \phi_4^-\phi_1^+[\phi_2^+,\phi_3^-] + \phi_1^+\phi_3^-\phi_4^-\phi_2^+ + \phi_1^+\phi_3^-[\phi_2^+,\phi_4^-]|0\rangle + [\phi_1^+,\phi_2^-][\phi_3^+,\phi_4^-] \\
&= [\phi_1^+,\phi_4^-][\phi_2^+,\phi_3^-] + [\phi_3^-,\phi_1^+][\phi_2^+,\phi_4^-] + [\phi_1^+,\phi_2^-][\phi_3^+,\phi_4^-]
\end{aligned}$$

Same for other time order, so

$$\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle = \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4} + \wick{\c1\phi_1\c1\phi_3\c2\phi_2\c2\phi_4} + \wick{\c1\phi_1\c1\phi_4\c2\phi_2\c2\phi_3}$$

$$\begin{aligned}
T\{\phi_1\phi_2\phi_3\phi_4\} = N\Big\{&\phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\wick{\c\phi_3\c\phi_4} + \wick{\c\phi_1\c\phi_2}\phi_3\phi_4 + \phi_1\wick{\c\phi_2\phi_3\c\phi_4} + \wick{\c\phi_1\phi_2\c\phi_3}\phi_4 + \phi_1\wick{\c\phi_2\c\phi_3}\phi_4 + \wick{\c\phi_1\phi_2\phi_3\c\phi_4} \\
&+\wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4} + \wick{\c1\phi_1\c1\phi_3\c2\phi_2\c2\phi_4} + \wick{\c1\phi_1\c1\phi_4\c2\phi_2\c2\phi_3}\Big\}
\end{aligned}$$

For more strict prove:

for $x^0>y^0>z^0>t^0$

$$\begin{aligned}
T\{\phi_1\phi_2\phi_3\phi_4\} &= (\phi_1^+\phi_2^+ + \phi_1^-\phi_2^+ + \phi_1^+\phi_2^- + \phi_1^-\phi_2^-)\phi_3\phi_4 \\
&= N\{\phi_1\phi_2\}\phi_3\phi_4 + \wick{\c\phi_1\c\phi_2}\phi_3\phi_4 \\
&= N\{\phi_1\phi_2\}(\phi_3^+\phi_4^+ + \phi_3^-\phi_4^+ + \phi_3^+\phi_4^- + \phi_3^-\phi_4^-) + \wick{\c\phi_1\c\phi_2}N\{\phi_3\phi_4\} + + \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4} \\
&= N\{\phi_1\phi_2\}N\{\phi_3\phi_4\} + N\{\phi_1\phi_2\}\wick{\c\phi_3\c\phi_4} + \wick{\c\phi_1\c\phi_2}N\{\phi_3\phi_4\} + \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4}
\end{aligned}$$

now we look at the first term

$$
\begin{aligned}
N\{\phi_1\phi_2\}N\{\phi_3\phi_4\} &= (\phi_1^+\phi_2^+ + \phi_1^-\phi_2^+ + \phi_2^-\phi_1^+ + \phi_1^-\phi_2^-)(\phi_3^+\phi_4^+ + \phi_3^-\phi_4^+ + \phi_4^-\phi_3^+ + \phi_3^-\phi_4^-) \\
&= \phi_1^+\phi_2^+\phi_3^+\phi_4^+ + \phi_1^+\phi_2^+\phi_3^-\phi_4^+ + \phi_1^+\phi_2^+\phi_4^-\phi_3^+ + \phi_1^+\phi_2^+\phi_3^-\phi_4^- + \phi_1^-\phi_2^+\phi_3^+\phi_4^+ + \phi_1^-\phi_2^+\phi_3^-\phi_4^+ + \phi_1^-\phi_2^+\phi_4^-\phi_3^+ + \phi_1^-\phi_2^+\phi_3^-\phi_4^- \\
&+ \phi_2^-\phi_1^+\phi_3^+\phi_4^+ + \phi_2^-\phi_1^+\phi_3^-\phi_4^+ + \phi_2^-\phi_1^+\phi_4^-\phi_3^+ + \phi_2^-\phi_1^+\phi_3^-\phi_4^- + \phi_1^-\phi_2^-\phi_3^+\phi_4^+ + \phi_1^-\phi_2^-\phi_3^-\phi_4^+ + \phi_1^-\phi_2^-\phi_4^-\phi_3^+ + \phi_1^-\phi_2^-\phi_3^-\phi_4^-
\end{aligned}
$$

note that $N\{\phi_1\phi_2\phi_3\phi_4\} = \phi_1^+\phi_2^+\phi_3^+\phi_4^+ + \phi_4^-\phi_1^+\phi_2^+\phi_3^+ + \phi_1^-\phi_2^+\phi_3^+\phi_4^+ + \phi_2^-\phi_1^+\phi_3^+\phi_4^+ + \phi_3^-\phi_1^+\phi_2^+\phi_4^+ + \phi_1^-\phi_2^-\phi_3^+\phi_4^+ + \phi_1^-\phi_3^-\phi_2^+\phi_4^+ +$ $\phi_1^-\phi_4^-\phi_2^+\phi_3^+ + \phi_2^-\phi_3^-\phi_1^+\phi_4^+ + \phi_2^-\phi_4^-\phi_1^+\phi_3^+ + \phi_3^-\phi_4^-\phi_1^+\phi_2^+ + \phi_1^-\phi_2^-\phi_3^-\phi_4^+ + \phi_1^-\phi_2^-\phi_4^-\phi_3^+ + \phi_1^-\phi_3^-\phi_4^-\phi_2^+ + \phi_2^-\phi_3^-\phi_4^-\phi_1^+ + \phi_1^-\phi_2^-\phi_3^-\phi_4^-$

$$
\begin{aligned}
&= N\{\phi_1\phi_2\phi_3\phi_4\} + \wick{\phi_1^+\c\phi_2\c\phi_3\phi_4^+} + \wick{\c\phi_1\phi_2^+\c\phi_3\phi_4^+} + \wick{\phi_1^+\c\phi_2\c\phi_4\phi_3^+} + \wick{\c\phi_1\phi_2^+\c\phi_4\phi_3^+} + \wick{\phi_1^+\c\phi_2\c\phi_3\phi_4^-} + \wick{\c\phi_1\phi_2^+\c\phi_3\phi_4^-} \\
&+ \wick{\phi_3^-\phi_1^+\c\phi_2\c\phi_4} + \wick{\phi_3^-\c\phi_1\phi_2^+\c\phi_4} + \wick{\phi_1^-\c\phi_2\c\phi_3\phi_4^+} + \wick{\phi_1^-\c\phi_2\phi_3^+\c\phi_4} + \wick{\phi_1^-\c\phi_2\c\phi_3\phi_4^-} + \wick{\phi_1^-\c\phi_2\phi_3^-\c\phi_4} + \wick{\c\phi_1\phi_2^-\c\phi_3\phi_4^+} + \wick{\c\phi_1\phi_2^-\phi_3^+\c\phi_4} \\
&+ \wick{\phi_2^-\c\phi_1\c\phi_3\phi_4^-} + \wick{\phi_2^-\c\phi_1\phi_3^-\c\phi_4} \\
&= N\{\phi_1\phi_2\phi_3\phi_4\} + N\{\phi_1\phi_4\}\wick{\c\phi_2\c\phi_3} + \wick{\c1\phi_1\c2\phi_2\c2\phi_3\c1\phi_4} + N\{\phi_2\phi_4\}\wick{\c\phi_1\c\phi_3} + \wick{\c1\phi_1\c2\phi_2\c1\phi_3\c2\phi_4} + N\{\phi_1\phi_3\}\wick{\c\phi_2\c\phi_4} + N\{\phi_2\phi_3\}\wick{\c\phi_1\c\phi_4}
\end{aligned}
$$

and similar for the rest time orderings.

Or:

$$
\begin{aligned}
T\{\phi_1\phi_2\phi_3\phi_4\} &= (\phi_1^+\phi_2^+ + \phi_1^-\phi_2^+ + \phi_1^+\phi_2^- + \phi_1^-\phi_2^-)(\phi_3^+\phi_4^+ + \phi_3^-\phi_4^+ + \phi_3^+\phi_4^- + \phi_3^-\phi_4^-) \\
&= (1^+2^+3^+4^+ + 1^+2^+3^-4^+ + 1^+2^+3^+4^- + 1^+2^+3^-4^-) + (1^-2^+3^+4^+ + 1^-2^+3^-4^+ + 1^-2^+3^+4^- + 1^-2^+3^-4^-) \\
&+ (1^+2^-3^+4^+ + 1^+2^-3^-4^+ + 1^+2^-3^+4^- + 1^+2^-3^-4^-) + (1^-2^-3^+4^+ + 1^-2^-3^-4^+ + 1^-2^-3^+4^- + 1^-2^-3^-4^-) \\
&= N\{1234\} + (\wick{1^+\c2\c34^+} + \wick{\c12^+\c34^+}) + (\wick{1^+2^+\c3\c4} + \wick{1^+\c23^+\c4} + \wick{\c12^+3^+\c4}) + (\wick{1^+\c2\c34^-} + \wick{\c12^+\c34^-} + \wick{3^-\c21^+\c4} + \wick{\c13^-2^+\c4}) \\
&+ (\wick{1^-\c2\c34^+}) + (\wick{1^-2^+\c3\c4} + \wick{1^-\c23^+\c4}) + (\wick{1^-\c2\c34^-} + \wick{1^-\c23^-\c4}) + (\wick{\c1\c23^+4^+}) + (\wick{\c1\c23^-4^+} + \wick{\c12^-\c34^+}) \\
&+ (\wick{\c1\c23^+4^-} + \wick{2^-1^+\c3\c4} + \wick{\c12^-3^+\c4}) + (\wick{\c1\c23^-4^-} + \wick{\c12^-\c34^-} + \wick{\c12^-3^-\c4}) + (\wick{1^-2^-\c3\c4}) \\
&= N\{1234\} + (\wick{1^+\c2\c34^+} + \wick{1^+\c2\c34^-} + \wick{1^-\c2\c34^+} + \wick{1^-\c2\c34^-}) + (\wick{\c12^+3^+\c4} + \wick{\c13^-2^+\c4} + \wick{\c12^-3^+\c4} + \wick{\c12^-3^-\c4}) \\
&+ (\wick{\c12^+\c34^+} + \wick{\c12^+\c34^-} + \wick{\c12^-\c34^+} + \wick{\c12^-\c34^-}) + (\wick{1^+\c23^+\c4} + \wick{3^-\c21^+\c4} + \wick{1^-\c23^+\c4} + \wick{1^-\c23^-\c4}) \\
&+ (\wick{1^+2^+\c3\c4} + \wick{1^-2^+\c3\c4} + \wick{2^-1^+\c3\c4} + \wick{1^-2^-\c3\c4}) + (\wick{\c1\c23^+4^+} + \wick{\c1\c23^-4^+} + \wick{\c1\c23^+4^-} + \wick{\c1\c23^-4^-}) \\
&= N\{\phi_1\phi_2\phi_3\phi_4\} + N\{\phi_1\phi_4\}\wick{\c\phi_2\c\phi_3} + \wick{\c1\phi_1\c2\phi_2\c2\phi_3\c1\phi_4} + N\{\phi_2\phi_4\}\wick{\c\phi_1\c\phi_3} + \wick{\c1\phi_1\c2\phi_2\c1\phi_3\c2\phi_4} + N\{\phi_1\phi_3\}\wick{\c\phi_2\c\phi_4} + N\{\phi_2\phi_3\}\wick{\c\phi_1\c\phi_4} \\
&+ N\{\phi_1\phi_2\}\wick{\c\phi_3\c\phi_4} + \wick{\c\phi_1\c\phi_2}N\{\phi_3\phi_4\} + \wick{\c1\phi_1\c1\phi_2\c2\phi_3\c2\phi_4}
\end{aligned}
$$

3. $T\{\psi(x)\bar{\psi}(y)\}$.

Set

$$
\psi^+(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s u^s(p)a_{\mathbf{p}}^s e^{-ip\cdot x}
$$

$$
\psi^-(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s v^s(p)b_{\mathbf{p}}^{s\dagger} e^{ip\cdot x}
$$

$$
\bar{\psi}^+(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \bar{v}^s(p)b_{\mathbf{p}}^s e^{-ip\cdot x}
$$

$$
\bar{\psi}^-(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s \bar{u}^s(p)a_{\mathbf{p}}^{s\dagger} e^{ip\cdot x}
$$

and

$$
\psi^+(x)\left|0\right\rangle = \bar{\psi}^+(x)\left|0\right\rangle = 0
$$

$$
\left\langle 0\right|\psi^-(x) = \left\langle 0\right|\bar{\psi}^-(x) = 0
$$

Assuming $x^0 > y^0$ (we ignore the variables since we can distinguish them by the conjugation)

$$T\{\psi(x)\bar{\psi}(y)\} = \psi^+\bar{\psi}^+ + \psi^+\bar{\psi}^- + \psi^-\bar{\psi}^+ + \psi^-\bar{\psi}^- = N\{\psi\bar{\psi}\} + \left[\{\psi^+, \bar{\psi}^-\} \equiv \overbracket{\psi\bar{\psi}} = S_F(x-y)\right]$$

and for $x^0 < y^0$

$$T\{\psi(x)\bar{\psi}(y)\} = -(\bar{\psi}^+\psi^+ + \bar{\psi}^+\psi^- + \bar{\psi}^-\psi^+ + \bar{\psi}^-\psi^-) = N\{\psi\bar{\psi}\} + \left[-\{\bar{\psi}^+, \psi^-\} \equiv \overbracket{\psi\bar{\psi}}\right]$$

so

$$T\{\psi(x)\bar{\psi}(y)\} = N\{\psi\bar{\psi}\} + \overbracket{\psi\bar{\psi}}$$

4. $T\{A_\mu(x)A_\nu(y)\}$.

Same as above, define

$$A_\mu^+(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2|\mathbf{k}|}}\sum_\lambda a_\mathbf{k}^\lambda \epsilon_\mu^\lambda(k)e^{-ik\cdot x}$$

$$A_\mu^-(x) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2|\mathbf{k}|}}\sum_\lambda a_\mathbf{k}^{\lambda\dagger} \epsilon_\mu^{\lambda *}(k)e^{ik\cdot x}$$

As spin-1 field ($x^0 > y^0$)

$$T\{A_\mu(x)A_\nu(y)\} = A_\mu^+A_\nu^+ + A_\mu^+A_\nu^- + A_\mu^-A_\nu^+ + A_\mu^-A_\nu^- = N\{A_\mu(x)A_\nu(y)\} + \overbracket{A_\mu(x)A_\nu(y)}$$

where

$$\overbracket{A_\mu(x)A_\nu(y)} \equiv \begin{cases} [A_\mu^+, A_\nu^-], x^0 > y^0 \\ [A_\nu^+, A_\mu^-], y^0 > x^0 \end{cases}$$

5. Repeat $S = Te^{\cdots\int\cdots}$.

First in interaction picture, the S matrix

$$S_{fi} = \langle f|\psi(\infty)\rangle = \langle f|U(\infty, -\infty)|i\rangle = \langle f|S_I|i\rangle$$

and the Schrödinger equation in interaction picture

$$i\frac{\mathrm{d}}{\mathrm{d}t}U(t_f, t_i) = H_I(t_f)U(t_f, t_i)$$

so

$$\begin{aligned}|\psi(t)\rangle_I &= |i\rangle + (-i)\int_{-\infty}^t \mathrm{d}t_1 H_I(t_1)\,|\psi(t_1)\rangle_I \\ &= |i\rangle + (-i)\int_{-\infty}^t \mathrm{d}t_1 H_I(t_1)(|i\rangle + (-i)\int_{-\infty}^{t_1}\mathrm{d}t_2 H_I(t_2)\,|\psi(t_2)\rangle_I)\end{aligned}$$

and such on and on till

$$S = \sum_{n=0}^\infty (-i)^n \int_{-\infty}^t \mathrm{d}t_1 \int_{-\infty}^{t_1}\mathrm{d}t_2 \cdots \int_{-\infty}^{t_{n-1}} \mathrm{d}t_n H_I(t_1)H_I(t_2)\cdots H_I(t_n)$$

Now a little tweak on the integral variables:

$$\begin{aligned}&\int_{t_0}^t \mathrm{d}t_1 \int_{t_0}^{t_1}\mathrm{d}t_2 H_I(t_1)H_I(t_2), \quad t_1 > t_2 \\ =&\int_{t_0}^t \mathrm{d}t_2 \int_{t_0}^{t_2}\mathrm{d}t_1 H_I(t_2)H_I(t_1), \quad t_2 > t_1 \\ =&\frac{1}{2}\int_{t_0}^t \mathrm{d}t_1 \int_{t_0}^{t}\mathrm{d}t_2 T\{H_I(t_1)H_I(t_2)\}\end{aligned}$$

so

$$
\begin{aligned}
S &= \sum_{n=0}^{\infty} \frac{(-i)^n}{2^n} \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t} \mathrm{d}t_2 \cdots \int_{-\infty}^{t} \mathrm{d}t_n T\{H_I(t_1)H_I(t_2)\cdots H_I(t_n)\}|_{t=\infty} \\
&= Te^{-i\int_{-\infty}^{\infty} \mathrm{d}t H_I(t)} \\
&= Te^{i\int_{-\infty}^{\infty} \mathrm{d}^4x \mathcal{L}_I(x)}
\end{aligned}
$$

6. $\mathcal{L} = \mathcal{L}_{KG} - \frac{g}{3!}\phi^3$.

(i) Write down the T matrix of $\langle p|S|p\rangle$ to g^2 order and draw corresponding Feynman diagram.

The T matrix is

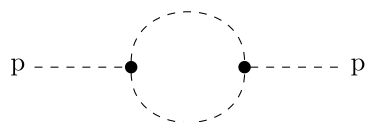
$$
\langle p|iT|p\rangle = \langle p|T\{\frac{1}{2!}(\frac{-ig}{3!})^2 \int \mathrm{d}^4x\phi\phi\phi \int \mathrm{d}^4y\phi\phi\phi\}|p\rangle
$$

so with a few contractions

$$
\langle p|iT|p\rangle = (-ig)^2 \langle p| \int \mathrm{d}^4x\phi\phi\phi \int \mathrm{d}^4y\phi\phi\phi |p\rangle \times (\text{Symmetry factor})
$$

where (Symmetry factor) = 2.

The corresponding feynman diagram



(ii) Write down some things for $\langle p_1p_2|S|p_Ap_B\rangle$. Calculate $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$ and σ_{tot}.

The T matrix is

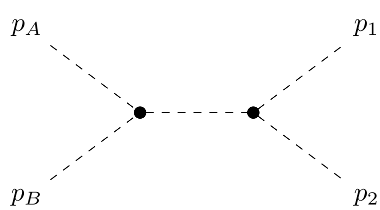
$$
\langle p_1p_2|iT|p_Ap_B\rangle = \langle p_1p_2|T\{\frac{1}{2!}(\frac{-ig}{3!})^2 \int \mathrm{d}^4x\phi\phi\phi \int \mathrm{d}^4y\phi\phi\phi\}|p_Ap_B\rangle
$$

so with a few contractions

$$
\langle p_1p_2|iT|p_Ap_B\rangle = (-ig)^2 \langle p_1p_2| \int \mathrm{d}^4x\phi\phi\phi \int \mathrm{d}^4y\phi\phi\phi |p_Ap_B\rangle \times (\text{Symmetry factor})
$$

where (Symmetry factor) = 1 (stands for all tree level process).

The corresponding feynman diagram



The cross section:

$$
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}|_{CM} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{CM}^2}
$$

and

$$
\begin{aligned}
\langle p_1p_2|iT|p_Ap_B\rangle &= (-ig)^2 \int \mathrm{d}^4x \int \mathrm{d}^4y \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{ip_1\cdot x} e^{ip_2\cdot x} e^{-ip_A\cdot y} e^{-ip_B\cdot y} \frac{i}{p^2-m^2} e^{-ip\cdot(x-y)} \\
&= (-ig)^2 \int \mathrm{d}^4p (2\pi)^4 \delta^4(p_1+p_2-p)\delta^4(p-p_A-p_B) \frac{i}{p^2-m^2} \\
&= (-ig)^2 (2\pi)^4 \delta^4(p_1+p_2-p_A-p_B) \frac{i}{(p_1+p_2)^2-m^2}
\end{aligned}
$$

so (of course you can always write it down directly from the feynman diagram)

$$i\mathcal{M} = -g^2 \frac{i}{(p_1+p_2)^2 - m^2} = i\frac{-g^2}{(p_1+p_2)^2 - m^2}$$

Thus we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}|_{CM} = \frac{g^4}{64\pi^2 E_{CM}^2[(p_1+p_2)^2 - m^2]^2}\frac{1}{2} = \frac{g^4}{2m^4}\frac{1}{64\pi^2 E_{CM}^2}$$

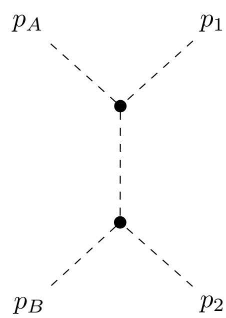
and the total cross section

$$\sigma|_{CM} = \frac{g^4}{2m^4}\frac{1}{16\pi E_{CM}^2}$$

There's also t-channel diagram:

$$\langle p_1 p_2 | iT | p_A p_B\rangle = (-ig)^2 \langle p_1 p_2 | \int \mathrm{d}^4 x \phi\phi\phi \int \mathrm{d}^4 y \phi\phi\phi \, | p_A p_B \rangle$$

and the feynman diagram for t-channel



and u-channel and they should be the same for the two identical out-state particles.

The scattering matrix and cross section should differ with only s, t and u.

$$i\mathcal{M}_t = i\frac{-g^2}{t - m^2}$$

and

$$\frac{\mathrm{d}\sigma_t}{\mathrm{d}\Omega}|_{CM} = \frac{g^4}{64\pi^2 E_{CM}^2[t - m^2]^2}\frac{1}{2}$$

$$\sigma|_{CM} = \frac{1}{m^4 - 4k^2p^2}\frac{1}{16\pi E_{CM}^2}$$

where p and k are the momentum values of p_A and p_1.