Ising Model & Monte Carlo method

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February 20, 2017

Ising Model:

- Hamiltonian of the system:

$$H(\sigma) \xlongequal{} -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - h\sum_j \sigma_j$$
$$\xlongequal{h=0} -J\sum_{\langle ij\rangle}\sigma_i\sigma_j$$

  h is the external magnetic field (for simplicity we now consider $h = 0$), and $J > 0$ which means it's ferromagnetic. (And it's reasonable to consider the lowest energy state is when the spins are all +1.)

- Total Energy at configuration $\{\sigma_i\}$:

$$E_{\{\sigma_i\}} = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - h\sum_j \sigma_j$$

- Spin state σ_i is differed by

$$\sigma_i = \begin{cases} +1 \\ -1 \end{cases}$$

- Configuration probability:

$$P_\beta(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z_\beta}$$

  where $\beta = (k_B T)^{-1}$ and Z_β is the partition function.

- Partition function:

$$Z_\beta = \sum_\sigma e^{-\beta H(\sigma)}$$

Mento Carlo Method (Metropolis Method):

- Given L^d lattice point. (For instance $d = 1$, which means 1-d Ising model.)

  (1)Give a initial state, and calculate its energy. Calculate the magnetic dipole and get the phase transition point.

  [1.] 随机生成态 s1 随机生成态 s2 计算 s1 与 s2 的权重 p1，p2（$p = exp[-H(s)]$）如果 $p2 > p1$，s2 为第二取样点如果 p2<=p1, 生成随机数 r，如果 $r <= \frac{p2}{p1}$,s2 为第二取样点，否则 s1 为第二个取样点。