$\bar{c}\gamma^\mu c$ matrix element

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1 $^3S_1$

Ignore the overall factor:

$$\left\langle 0 \middle| \bar{c}\gamma^\mu c \middle| ^3S_1 \right\rangle = \int \mathrm{d}\Omega \, \mathrm{tr}[\Pi_1 \gamma^\mu] \propto \sqrt{2}\pi(\frac{m}{3E} + \frac{2}{3})\epsilon^\mu$$

2 $^3D_1$

The matrix element reads:

$$\left\langle 0 \middle| \bar{c}\gamma^\mu c \middle| ^3D_1 \right\rangle = \int \mathrm{d}\Omega \sum_{\lambda_1 \lambda_2 S_z m} \mathrm{tr}\{\Pi_1 \gamma^\mu\} \left\langle 1 J_z | 2m; 1 S_z \right\rangle Y_{2m}(\theta, \phi)$$

while the trace part is the same as $^3S_1$:

$$\mathrm{tr}\{\Pi_1 \gamma^\mu\} = \frac{\sqrt{2} p^\mu (p \cdot \epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \epsilon^{(0)} = (0, 0, 0, -1), \epsilon^{(+)} = \frac{1}{\sqrt{2}}(0, -1, -i, 0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} 0 & -\frac{4p^2\sqrt{2\pi}}{15E(m+E)} & \frac{4ip^2\sqrt{2\pi}}{15E(m+E)} & 0 \\ 0 & 0 & 0 & -\frac{4p^2\sqrt{\pi}}{15E(m+E)} \\ 0 & \frac{4p^2\sqrt{2\pi}}{15E(m+E)} & \frac{4ip^2\sqrt{2\pi}}{15E(m+E)} & 0 \end{pmatrix}$$