#### RI/MOM Scheme and Quasi PDF

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#### Outlines

#### RI/MOM Scheme

RI/MOM Scheme[Martinelli et al., 1995]

## Momentum Subtraction Schemes (MOM)

Recall procedure in continuum perturbation theory:

- example: renormalisation of the pseudoscalar density  $P^a(x) = \overline{\psi}(x)\gamma_5\frac{1}{2}\tau^a\psi(x)$ :
- Correlation functions in momentum space with external quark states:

$$\left\langle \widetilde{\psi}(p)\overline{\psi}(q) \right\rangle = (2\pi)^4 \delta(p+q)S(p)$$
 quark propagator  $\left\langle \widetilde{\psi}(p)\widetilde{P}^a(q)\widetilde{\overline{\psi}}(p') \right\rangle = (2\pi)^4 \delta(p+q+p')S(p)\Gamma_P^a(p,q)S(p+q),$ 

At tree-level:

$$\Gamma_P^a(p,q)|_{\mathrm{tree}} = \gamma_5 \frac{1}{2} \tau^a,$$
  
 $\Rightarrow \frac{1}{4} \sum_{b=1}^3 \operatorname{tr} \left\{ \gamma_5 \tau^b \Gamma_P^a(p,q)|_{\mathrm{tree}} \right\} = 1$ 

Renormalised fields:

$$\psi_{\mathrm{R}} = Z_{\psi} \psi, \qquad \overline{\psi}_{\mathrm{R}} = Z_{\psi} \overline{\psi}, \qquad P_{\mathrm{R}}^{\mathsf{a}} = Z_{\mathrm{P}} P^{\mathsf{a}}$$

⇒ renormalised vertex function:

$$\Gamma_{P,\mathbf{R}}^{a}(p,q) = Z_{\mathbf{P}}Z_{\psi}^{-2}\Gamma_{P}^{a}(p,q)$$

typical MOM renormalisation condition (quark masses set to zero):

$$\Gamma^a_{P,\mathbf{R}}(p,0)|_{\mu^2=p^2} = \gamma_5 \frac{1}{2} \tau^a \qquad \Rightarrow \qquad Z_{\mathbf{P}} Z_{\psi}^{-2}$$

equivalently using "projector":

$$rac{1}{4}\sum_{b=1}^3 \operatorname{tr}\left\{\gamma_5 au^b\, \mathsf{\Gamma}^{m{a}}_{P,\mathrm{R}}(m{
ho},0)|_{\mu^2=m{
ho}^2}
ight\}=1$$

• Determine  $Z_{\psi}$  either from propagator or use MOM scheme for vertex function of a conserved current

$$\Gamma_{V,R}(p,q) = Z_{\psi}^{-2} \Gamma_{V}(p,q)$$

### Summary: MOM schemes in the continuum

- Renormalisation condions are imposed on vertex functions in the gauge fixed theory with external quark, gluon or ghost lines
- The vertex functions are taken in momentum space.
- A particular momentum configuration is chosen, such that the vertex function becomes a function of a single momentum p; quark masses are set to zero
- MOM condition: a renormalised vertex function at subtraction scale  $\mu^2=p^2$  equals its tree-level expression
- Can also be used to define a renormalised gauge coupling: take vertex function of either the 3-gluon vertex, the quark-gluon vertex or the ghost-gluon vertex.
- Renormalisation constants depend on the chosen gauge! Need wave function renormalisation for quark, gluon and ghost fields.

# RI/MOM Schemes (RI = Regularisation Independent; MOM = Momentum Subtraction)

[Martinelli et al '95]: mimick the procedure in perturbation theory:

• choose Landau gauge

$$\partial_{\mu}A_{\mu}=0$$

can be implemented on the lattice by a minimisation procedure

- RI/MOM schemes are very popular: many major collaborations use it because
  - it is straightforward to implement on the lattice; many improvements over the years regarding algorithmic questions
  - it can be used on the very same gauge configurations which are produced for hadronic physics
- Regularisation Independence (RI) means: correlation functions of a renormalised operator do not depend on the regularisation used (up to cutoff effects).



#### RI/MOM schemes, discussion

 Suppose we have calculated a renormalised hadronic matrix element of the multiplicatively renormalisable operator O

$$\mathcal{M}_O(\mu) = \lim_{a \to 0} \langle h | O_{\mathbf{R}}(\mu) | h' \rangle$$

• Provided  $\mu$  is in the perturbative regime, one may evaluate the MOM scheme in continuum perturbation theory and evolve to a different scale:

$$\mathcal{M}_{O}(\mu') = U(\mu', \mu) \mathcal{M}_{O}(\mu),$$

$$U(\mu', \mu) = \exp \left\{ \int_{\bar{g}(\mu)}^{\bar{g}(\mu')} \frac{\gamma_{O}(g)}{\beta(g)} dg \right\}$$

 N.B. Continuum perturbation theory is available to 3-loops in some cases!



## RI/MOM schemes, what could go wrong?

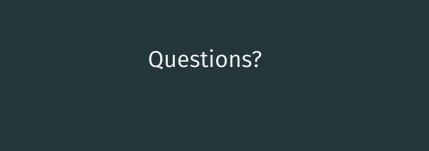
ullet The scale  $\mu$  could be too low; need to hope for a "window"

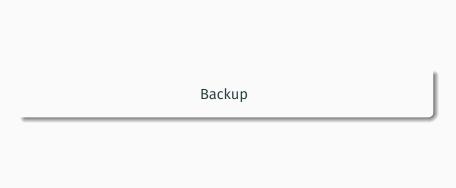
$$\Lambda_{\rm QCD} \ll \mu \ll a^{-1}$$

In practice scales are often too low: non-perturbative effects (e.g. pion poles, condensates) are then eliminated by fitting to expected functional form (from OPE in fixed gauge);

- ⇒ errors are difficult to quantify!
  - Gribov copies: the (Landau) gauge condition does not have a unique solution on the full gauge orbit
  - Perturbative calculations are made using
    - infinite volume
    - vanishing quark masses
- ⇒ inconvenient for numerical simulations especially in full QCD.
  - Wilson quarks: a priori cutoff effects are O(a) even in on-shell O(a) improved theory.







References \_\_\_\_\_



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