

Homework: Particle Physics #1

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1. $\gamma^0(\gamma^\mu)^\dagger\gamma^0 = \gamma^\mu$

$$\begin{aligned}\gamma^0\gamma^0\gamma^0 &= \gamma^0, \gamma^0\gamma^{i\dagger}\gamma^0 = 2\delta_i^0 - \gamma^{i\dagger} = -\gamma^{i\dagger} = \gamma^i \\ \implies \gamma^0(\gamma^\mu)^\dagger\gamma^0 &= \gamma^\mu \quad (\text{Used } \gamma^{i\dagger} = \gamma_i = -\gamma^i)\end{aligned}$$

2. From the orthogonal condition, we have the orthogonal condition of spinor ξ : $\xi^\dagger\xi = 1$, then

$$\begin{aligned}\sum_s u_s(p)\bar{u}_s(p) &= \sum_s \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s \end{pmatrix} (\xi^{s\dagger}\sqrt{p\cdot\bar{\sigma}} \quad \xi^{s\dagger}\sqrt{p\cdot\sigma}) \\ &= \sum_s \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s\xi^{s\dagger}\sqrt{p\cdot\bar{\sigma}} & \sqrt{p\cdot\sigma}\xi^s\xi^{s\dagger}\sqrt{p\cdot\sigma} \\ \sqrt{p\cdot\bar{\sigma}}\xi^s\xi^{s\dagger}\sqrt{p\cdot\bar{\sigma}} & \sqrt{p\cdot\bar{\sigma}}\xi^s\xi^{s\dagger}\sqrt{p\cdot\sigma} \end{pmatrix}\end{aligned}$$

(Use $\sum_s \xi^s\xi^{s\dagger} = \mathbb{1}$)

$$\begin{aligned}&= \begin{pmatrix} m & p\cdot\sigma \\ p\cdot\bar{\sigma} & m \end{pmatrix} \\ &= \not{p} + m\end{aligned}$$

Similarly, $\sum_s v_s(p)\bar{v}_s(p) = \not{p} - m$.

3. $m = 0$, $k^\mu = \{k_0, k_x, k_y, k_z\}$, there's two transverse polarization vector, we can choose (note that the orthogonal and completeness conditions are also considered)

$$k\cdot\epsilon = 0 \implies \epsilon^1 = \left(0, \frac{\sqrt{k_y^2+k_z^2}}{\sqrt{k_x^2+k_y^2+k_z^2}}, -\frac{k_xk_y}{\sqrt{k_y^2+k_z^2}\sqrt{k_x^2+k_y^2+k_z^2}}, -\frac{k_xk_z}{\sqrt{k_y^2+k_z^2}\sqrt{k_x^2+k_y^2+k_z^2}}\right), \epsilon^2 = \left(0, 0, \frac{k_z}{\sqrt{k_y^2+k_z^2}}, -\frac{k_y}{\sqrt{k_y^2+k_z^2}}\right)$$

4. Consider a general transformation

$$f(x) \rightarrow f'(x')$$

and define

$$\delta f \equiv f'(x') - f(x) = f'(x + \delta x^\mu) - f(x) \cong f'(x) - f(x) + \delta x^\mu \partial_\mu f + \mathcal{O}(\delta x^2)$$

and also define

$$\delta_0 f = f'(x) - f(x)$$

then

$$\delta f = \delta_0 f + \delta x^\mu \partial_\mu f$$

Now we deal with this problem from the least action principle first

$$\begin{aligned}\delta S &= 0 \\ &= \int d^4x \delta \mathcal{L} + \int \delta(d^4x) \mathcal{L}\end{aligned}$$

And

$$\begin{aligned}
\delta\mathcal{L} &= \delta_0\mathcal{L} + \delta x^\mu \partial_\mu \mathcal{L} \\
&= \frac{\partial\mathcal{L}}{\partial\phi} \delta_0\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta_0(\partial_\mu\phi) + \delta x^\mu \partial_\mu \mathcal{L} \\
&= \delta x^\mu \partial_\mu \mathcal{L} + \left(\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) \delta_0\phi + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta_0\phi\right)
\end{aligned}$$

The other part

$$\delta(d^4x) = \frac{\partial\delta x^\mu}{\partial x^\mu} d^4x = \partial_\mu(\delta x^\mu) \mathcal{L}$$

which can be derived by

$$d^4x' = \left| \frac{\partial x'^\mu}{\partial x^\nu} \right| d^4x = \left| \frac{\partial(x^\mu + \delta x^\mu)}{\partial x^\nu} \right| d^4x = \left(1 + \frac{\partial\delta x^\mu}{\partial x^\mu}\right) d^4x$$

And the whole part of δS becomes (applying Euler-Lagarange equation)

$$\begin{aligned}
\delta S &= \int d^4x \delta x^\mu \partial_\mu \mathcal{L} + \left(\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) \delta_0\phi + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta_0\phi\right) + \partial_\mu(\delta x^\mu) \mathcal{L} \\
&= \int d^4x \partial_\mu(\delta x^\mu \mathcal{L}) + \left(\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right) \delta_0\phi + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta_0\phi\right) \\
&= \int d^4x \partial_\mu(\delta x^\mu \mathcal{L}) + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta_0\phi\right)
\end{aligned}$$

Note that $\delta_0\phi = \delta\phi - \delta x^\mu \partial_\mu \phi$, and

$$\begin{aligned}
\delta S &= \int d^4x \partial_\mu(\delta x^\mu \mathcal{L}) + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta\phi - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta x^\rho \partial_\rho \phi\right) \\
&= \int d^4x \partial_\mu \left\{ \delta x^\mu \mathcal{L} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta\phi - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta x^\rho \partial_\rho \phi \right\} \\
&= \int d^4x \partial_\mu \left\{ (\mathcal{L} g_\rho^\mu - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \partial_\rho \phi) \delta x^\rho + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta\phi \right\}
\end{aligned}$$

5. Given

$$\begin{aligned}
P\psi P^{-1} &= \eta \gamma^0 \psi(t, -\mathbf{x}) \\
P\bar{\psi} P^{-1} &= \eta^* \bar{\psi}(t, -\mathbf{x}) \gamma^0
\end{aligned}$$

The field operators are

$$\begin{aligned}
\psi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s (a_{\mathbf{p}}^s u^s(p) e^{-ip \cdot x} + b_{\mathbf{p}}^{s\dagger} v^s(p) e^{ip \cdot x}) \\
\bar{\psi}(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s (b_{\mathbf{p}}^s \bar{v}^s(p) e^{-ip \cdot x} + a_{\mathbf{p}}^{s\dagger} \bar{u}^s(p) e^{ip \cdot x})
\end{aligned}$$

To satisfy the first two equations when acted on by P operator, the creation & annihilation operators must obey

$$P a_{\mathbf{p}}^s P^{-1} = \eta a_{-\mathbf{p}}^s, P b_{\mathbf{p}}^s P^{-1} = -\eta^* b_{-\mathbf{p}}^s$$

(Note that $u(p^0, \mathbf{p}) = \gamma^0 u(p^0, -\mathbf{p})$, $v(p^0, \mathbf{p}) = -\gamma^0 v(p^0, -\mathbf{p})$)

6. Given

$$\begin{aligned}
C\psi C^{-1} &= -i(\bar{\psi} \gamma^0 \gamma^2)^T = -i\gamma^2 \psi^* \\
C\bar{\psi} C^{-1} &= -i(\gamma^0 \gamma^2 \psi)^T = i\bar{\psi}^* \gamma^2
\end{aligned}$$

Similarly, the creation & annihilation operators must obey

$$C a_{\mathbf{p}}^s C^{-1} = b_{\mathbf{p}}^s, C b_{\mathbf{p}}^s C^{-1} = a_{\mathbf{p}}^s$$

(Note that $u^s(p) = -i\gamma^2 (v^s(p))^*$, $v^s(p) = -i\gamma^2 (u^s(p))^*$)

7. Prove Landau-Yang theorem.

For any vector particles, we can always write the field operator as a single vector field.

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2|\mathbf{k}|}} \sum_\lambda (a_{\mathbf{k}}^\lambda \epsilon_\mu^\lambda(k) e^{-ik \cdot x} + a_{\mathbf{k}}^{\lambda\dagger} \epsilon_\mu^{\lambda*}(k) e^{ik \cdot x})$$

Then the feynman rules can be easily derived. The amplitude of $vector \rightarrow \gamma\gamma$ is

$$i\mathcal{M} = \epsilon_1^{*\mu}(p_1) \epsilon_2^{*\nu}(p_2) \epsilon^\alpha(p) \Gamma_{\mu\nu\sigma}$$

since it must obey Lorentz-invariant

$$= (\epsilon_1 \cdot \epsilon_2)(a_1 \epsilon \cdot p_1 + a_2 \epsilon \cdot p_2) + a_3(\epsilon_1 \cdot \epsilon)(\epsilon_2 \cdot p_1) + a_4(\epsilon_2 \cdot \epsilon)(\epsilon_1 \cdot p_2)$$

final states symmetry (identical), $a_1 = a_2$, first term vanishes. And $\epsilon_2 \cdot p_1 = \epsilon_1 \cdot p_2 = 0$

$$= 0$$

8. Tensor field.

At lowest order, we can assume there's no propagator.

$$i\mathcal{M} = \varepsilon_{\mu\nu}(p)(i\Gamma^{\mu\nu\rho\sigma}(k_1, k_2))\epsilon_{1\rho}^*(k_1)\epsilon_{2\sigma}^*(k_2)$$

Γ is the effective vertex.

Use Lorenz covariance

$$i\mathcal{M} = \varepsilon \cdot (ak_1 + bk_2)(ck_1 + dk_2)(\epsilon_1 \cdot \epsilon_2) + e(\epsilon_1 \cdot k_2)(\varepsilon \cdot \epsilon_2 k_1) + f(\epsilon_2 \cdot k_1)(\varepsilon \cdot \epsilon_1 k_2) + j\varepsilon_\mu^\mu(\epsilon_1 \cdot \epsilon_2)$$

with $\epsilon_1 \cdot k_2 = \epsilon_2 \cdot k_1 = 0$,

$$i\mathcal{M} = \varepsilon \cdot (ak_1 + bk_2)(ck_1 + dk_2)(\epsilon_1 \cdot \epsilon_2) + j\varepsilon_\mu^\mu(\epsilon_1 \cdot \epsilon_2)$$

9. $n \rightarrow p + e^- + \bar{\nu}_e$.

In centre-of-mass frame, the range is $0 \sim \sqrt{(m_n - m_p)^2 - m_e^2}$.

10. (a). Use natural units:

$$\psi(r) = \frac{g_0}{4\pi r} e^{-rm}$$

and the equation

$$(\nabla^2 - m^2)\psi = 0$$

Obviously if we put ψ in the equation there's a m^2 factor cancelled with the original m^2 , so ψ is a root of the equation.

If closer look is taken, the Laplacian part should be (the coefficients are neglected)

$$\frac{m^2 e^{-mr}}{r} + \frac{2e^{-mr}}{r^3} + \frac{2me^{-mr}}{r^2} + \frac{2\left(-\frac{e^{-mr}}{r^2} - \frac{me^{-mr}}{r}\right)}{r} = \frac{m^2 e^{-mr}}{r}$$

(b). Add the source

$$(\nabla^2 - m^2)\psi = J^\mu$$

11. $\pi^0 \rightarrow \gamma\gamma$.

The quantum numbers J^{PC} of π is 0^{-+} with zero spin and orbital angular momentum while for γ it's 1^{--} with spin 1. Assuming we don't know anything about π , the final state angular momentum gives $J(\pi) = J(\gamma\gamma) = L + S$ is even. Since L is odd, the final state system parity is $P = (-)^{L=1} = -$. And $J(\pi) = 0$ (can also be proved by Furry's theorem, $J(\pi)$ can't be 1).

12. $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi$.

Assuming $\gamma - J/\psi$ vertex is $(-iag^{\mu\nu})$

$$i\mathcal{M}^{sr\lambda} = i \frac{ea}{M^2} \bar{v}^s(p_1) \not{\epsilon}^\lambda(p_1 + p_2) u^r(p_2)$$

Use kinematics (centre-of-mass), and choose circular polarization

$$\epsilon^0 = (0, 0, 0, 1), \epsilon^+ = (0, 1, i, 1), \epsilon^- = (0, 1, -i, 1)$$

and only

$$M^{12+} = M^{21-} = \frac{2ae}{M}$$

don't vanish. So J/ψ is circular polarized.

13. $\rho \rightarrow \pi\pi$. ρ is vector particle and π is pseudoscalar particle.

$$i\mathcal{M} = \rho \sim p_1 \sim p_2 = i(ap_1 + bp_2) \cdot \epsilon_\rho(p)$$

So

$$\sum |\mathcal{M}|^2 = \sum [(ap_1 + bp_2) \cdot \epsilon_\rho(p)]^2 = a(p_1 - p_2)^2 \times (I, I_3)$$

14. In $2 \rightarrow 3$ decay, there're total 4 independent 4-momentums, so 5 Lorenz scalars.

For n particles, there're $\frac{n(n-3)}{2}$.

15. 3-particle decay phase space integrals can be derived as follows:

First we know that the gengeral non-relativistic expression for N-body phase space is

$$dn = (2\pi)^3 \prod_{i=1}^N \frac{d^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^N \mathbf{p}_i\right) \quad (1)$$

where \mathbf{p}_a is the momentum of the decaying particle. (This expression can be derived easily from the phase space volume of each particle.) According to Fermi's golden rule (and notice the $(2E_i)^{1/2}$ ratio difference between \mathcal{M}_{fi} and T_{fi}), the decay rate can be written as

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - \sum_{i=1}^N E_i) \delta^3(\mathbf{p}_a - \sum_{i=1}^N \mathbf{p}_i) \prod_{i=1}^N \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \quad (2)$$

So for 3-particle decay the decay rate can be written as

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2 - E_3) \delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3}$$

Now we consider it in the centre-of-mass frame of the decaying particle A, which means $E_a = m_a$ and $\mathbf{p}_a = 0$. Through the integration of delta function and $d^3\mathbf{p}_3$, we have

$$\begin{aligned} \Gamma_{fi} &= \frac{(2\pi)^4}{2m_a} \int |\mathcal{M}_{fi}|^2 \delta(m_a - E_1 - E_2 - E_3) \delta^3(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \\ &= \frac{(2\pi)^4}{2m_a} \int |\mathcal{M}_{fi}|^2 \delta(m_a - E_1 - E_2 - E_3) \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \frac{1}{(2\pi)^3 2E_3} \\ &= \frac{1}{2^4 m_a (2\pi)^5} \int \frac{|\mathcal{M}_{fi}|^2}{E_1 E_2 E_3} \delta(m_a - E_1 - E_2 - E_3) d^3\mathbf{p}_1 d^3\mathbf{p}_2 \\ &= \frac{1}{2^4 m_a (2\pi)^5} \int \frac{|\mathcal{M}_{fi}|^2}{E_1 E_2 E_3} \delta(m_a - E_1 - E_2 - E_3) d^3\mathbf{p}_1 d^3\mathbf{p}_2 \\ &= \frac{1}{2^4 m_a (2\pi)^5} \int \frac{|\mathcal{M}_{fi}|^2 |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 dp_1 d(\cos\theta_1) d\phi_1 dp_2 d(\cos\theta_2) d\phi_2}{E_1 E_2 \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos\theta_2 + m_3^2}} \\ &\quad \delta(m_a - E_1 - E_2 - \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos\theta_2 + m_3^2}) \end{aligned}$$

where θ_1 , ϕ_1 and ϕ_2 are independent of the integral and therefore can be integrated first. Note that

$$\frac{d|\mathbf{p}_i|}{dE_i} = \frac{E_i}{|\mathbf{p}_i|}$$

which means

$$d|\mathbf{p}_i| = \frac{E_i}{|\mathbf{p}_i|} dE_i$$

and mark the kernel of δ function as

$$f(\cos \theta_2) \equiv m_a - E_1 - E_2 - \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta_2 + m_3^2}$$

we have

$$f'(\cos \theta_2) = -\frac{2|\mathbf{p}_1||\mathbf{p}_2|}{2\sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta_2 + m_3^2}}$$

and the real root of $f(\cos \theta_2) = 0$ is

$$\cos \theta'_2 = \frac{(m_a - E_1 - E_2)^2 - m_3^2 - |\mathbf{p}_1|^2 - |\mathbf{p}_2|^2}{2|\mathbf{p}_1||\mathbf{p}_2|}$$

So we have

$$\begin{aligned} & \delta(m_a - E_1 - E_2 - \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta_2 + m_3^2}) \\ &= \frac{\delta(\cos \theta_2 - \cos \theta'_2)}{\frac{2|\mathbf{p}_1||\mathbf{p}_2|}{2\sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta'_2 + m_3^2}}} \end{aligned}$$

And the original formula becomes

$$\begin{aligned} \Gamma_{fi} &= \frac{8\pi^2}{2^4 m_a (2\pi)^5} \int \frac{|\mathcal{M}_{fi}|^2 |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 d\mathbf{p}_1 d\mathbf{p}_2 d(\cos \theta_2)}{E_1 E_2 \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta_2 + m_3^2}} \frac{\delta(\cos \theta_2 - \cos \theta'_2)}{\frac{2|\mathbf{p}_1||\mathbf{p}_2|}{2\sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta'_2 + m_3^2}}} \\ &= \frac{1}{2^3 m_a (2\pi)^3} \int \frac{|\mathcal{M}_{fi}|^2 |\mathbf{p}_1|^2 |\mathbf{p}_2|^2 d\mathbf{p}_1 d\mathbf{p}_2}{E_1 E_2 \sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta'_2 + m_3^2}} \frac{2\sqrt{|\mathbf{p}_1|^2 + |\mathbf{p}_2|^2 + 2|\mathbf{p}_1||\mathbf{p}_2| \cos \theta'_2 + m_3^2}}{2|\mathbf{p}_1||\mathbf{p}_2|} \\ &= \frac{1}{2^3 m_a (2\pi)^3} \int \frac{|\mathcal{M}_{fi}|^2 |\mathbf{p}_1||\mathbf{p}_2| d\mathbf{p}_1 d\mathbf{p}_2}{E_1 E_2} \\ &= \frac{1}{2^3 m_a (2\pi)^3} \int \frac{|\mathcal{M}_{fi}|^2 |\mathbf{p}_1||\mathbf{p}_2| \frac{E_1}{|\mathbf{p}_1|} dE_1 \frac{E_2}{|\mathbf{p}_2|} dE_2}{E_1 E_2} \\ &= \frac{1}{8 m_a (2\pi)^3} \int |\mathcal{M}_{fi}|^2 dE_1 dE_2 \end{aligned}$$

which is exactly the form of square Dalitz plot. Transform it a little bit and we have the standard form of Dalitz plot (note that $s_2 = (p_2 + p_3)^2 = (p_a - p_1)^2 \rightarrow ds_2 = -2m_a dE_1$ and similar for s_3)

$$\Gamma_{fi} = \frac{1}{32 m_a (2\pi)^3} \int |\mathcal{M}_{fi}|^2 ds_2 ds_3$$

Now let's review another form of the standard Dalitz form

$$\frac{d\Gamma_{fi}}{ds_2 ds_3} = \frac{1}{32 m_a (2\pi)^3} |\mathcal{M}_{fi}|^2$$

and its physical meaning is obvious: the density of data points on a Dalitz plot is proportional to the decay matrix element.

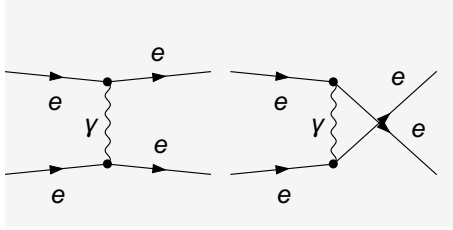
16. $J^{PC}(^{2S+1}L_J)$ of $q\bar{q}$ system. ($P = (-)^{L+1}$, $C = (-)^{L+S}$)

$L \backslash S$	0	1
0	$0^{-+}({}^1S_0)$	$1^{--}({}^3S_1)$
1	$1^{+-}({}^1P_1)$	$0^{++}({}^3P_0)$ $1^{++}({}^3P_1)$ $2^{++}({}^3P_2)$
2	$2^{-+}({}^1D_2)$	$1^{--}({}^3D_1)$ $2^{--}({}^3D_2)$ $3^{--}({}^3D_3)$

17. Moller scattering.

Use FeynCalc.

```
<<FeynCalc `;
topMoeller = CreateTopologies[0, 2 -> 2];
diagsMoeller = InsertFields[topMoeller, {F[2, {1}], F[2, {1}]} -> {F[
2, {1}], F[2, {1}]}], InsertionLevel -> {Classes}, Model -> "SM",
ExcludeParticles -> {S[1], S[2], V[2]}];
Paint[diagsMoeller, ColumnsXRows -> {2, 1}, Numbering -> None,
SheetHeader->None, ImageSize->{512,256}];
```



```
ampMoeller=FCFAConvert[CreateFeynAmp[diagsMoeller, Truncated -> False],
IncomingMomenta->{p1,p2},OutgoingMomenta->{k1,k2},UndoChiralSplittings->True,
ChangeDimension->4,List->False,SMP->True]
```

$$i\mathcal{M} = \frac{\bar{g}^{\text{Lor1Lor2}}(\varphi(\bar{k}1, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor1}}) \cdot (\varphi(\bar{p}1, m_e)) (\varphi(\bar{k}2, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor2}}) \cdot (\varphi(\bar{p}2, m_e))}{(\bar{k}2 - \bar{p}2)^2} \\ - \frac{\bar{g}^{\text{Lor1Lor2}}(\varphi(\bar{k}1, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor2}}) \cdot (\varphi(\bar{p}2, m_e)) (\varphi(\bar{k}2, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor1}}) \cdot (\varphi(\bar{p}1, m_e))}{(\bar{k}1 - \bar{p}2)^2}$$

```
SetMandelstam[s, t, u, p1, p2, -k1, -k2, SMP["m_e"], SMP["m_e"], SMP["m_e"], SMP["m_e"]];
sqAmpMoeller =(ampMoeller (ComplexConjugate[ampMoeller]//FCRenameDummyIndices))//
PropagatorDenominatorExplicit//Contract//FermionSpinSum[#, ExtraFactor -> 1/2^2]&//
ReplaceAll[#, DiracTrace :> Tr] & // Contract//Simplify
```

$$\sum |\mathcal{M}|^2 = \frac{2e^4 (-4m_e^2 (s(t^2 + 3tu + u^2) + t^3 - 2t^2u - 2tu^2 + u^3) + 8m_e^4 (t^2 + tu + u^2) + s^2(t + u)^2 + t^4 + u^4)}{t^2 u^2}$$

And the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{cm}^2}$$

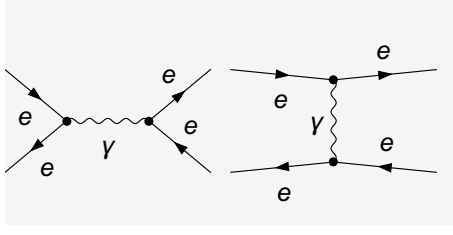
18. Bhabha scattering. Similarly, use FeynCalc.

```
<<FeynCalc `;
topBhabha = CreateTopologies[0, 2 -> 2];
diagsBhabha = InsertFields[topBhabha, {F[2, {1}], -F[2, {1}]} ->
```

```

{F[ 2, {1}], -F[2, {1}]], InsertionLevel -> {Classes},
Model -> "SM", ExcludeParticles -> {S[1], S[2], V[2]}};
Paint[diagsBhabha, ColumnsXRows -> {2, 1}, Numbering -> None, SheetHeader->None,
ImageSize -> {512, 256}];

```



```

ampBhabha=FCFAConvert[CreateFeynAmp[diagsBhabha, Truncated -> False],
IncomingMomenta->{p1, p2}, OutgoingMomenta->{k1, k2}, UndoChiralSplittings->True, ChangeDim
ension->4, List->False, SMP->True]

```

$$i\mathcal{M} = \frac{\bar{g}^{\text{Lor1Lor2}}(\varphi(\bar{k}1, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor2}}) \cdot (\varphi(-\bar{k}2, m_e)) (\varphi(-\bar{p}2, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor1}}) \cdot (\varphi(\bar{p}1, m_e))}{(\bar{k}1 + \bar{k}2)^2} \\ - \frac{\bar{g}^{\text{Lor1Lor2}}(\varphi(\bar{k}1, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor1}}) \cdot (\varphi(\bar{p}1, m_e)) (\varphi(-\bar{p}2, m_e)) \cdot (ie\bar{\gamma}^{\text{Lor2}}) \cdot (\varphi(-\bar{k}2, m_e))}{(\bar{k}2 - \bar{p}2)^2}$$

```

SetMandelstam[s, t, u, p1, p2, -k1, -k2, SMP["m_e"], SMP["m_e"], SMP["m_e"], SMP["m_e"]];
sqAmpBhabha =
(ampBhabha (ComplexConjugate[ampBhabha]//FCRenameDummyIndices))
//PropagatorDenominatorExplicit//
Contract//FermionSpinSum[#, ExtraFactor -> 1/2^2] & // ReplaceAll [# ,
DiracTrace :> Tr]& //Contract//Simplify
masslessSqAmpBhabha = (sqAmpBhabha /. {SMP["m_e"] -> 0})//Simplify

```

$$\sum |\mathcal{M}|^2 = \frac{2e^4 (s^4 + s^2 u^2 + 2stu^2 + t^4 + t^2 u^2)}{s^2 t^2}$$

And the differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{cm}^2}$$

19. Mott scattering. (Assuming the scalar-photon vertex is $ig(k_1 - k_2)^\mu$, ignore electron mass.)

$$i\mathcal{M} = p_1 \quad p_2 = ige\bar{u}(p_2)\gamma^\mu u(p_1) \frac{1}{(p_2 - p_1)^2 + i\epsilon} (k_1 - k_2)_\mu$$

$$\frac{1}{4} \sum |\mathcal{M}|^2 = \frac{g^2 e^2}{(p_2 - p_1)^4} [2(k_1 - k_2) \cdot p_1 (k_1 - k_2) \cdot p_2 - (k_1 - k_2) \cdot (k_1 - k_2) p_1 \cdot p_2]$$

Use kinematics:

$$\begin{aligned} \frac{1}{4} \sum |\mathcal{M}|^2 &= \frac{g^2 e^2}{(p_2 - p_1)^4} [2(k_1 - k_2) \cdot p_1 (k_1 - k_2) \cdot (k_1 - k_2 + p_1)] - \frac{g^2 e^2}{(p_2 - p_1)^2} [p_1 \cdot (k_1 + p_1 - k_2)] \\ &= \frac{2g^2 e^2}{(p_2 - p_1)^4} [p_2 \cdot p_1]^2 + \frac{g^2 e^2}{(p_2 - p_1)^2} [p_2 \cdot p_1] \\ &= \frac{2g^2 e^2}{(p_2 - p_1)^2} [p_2 \cdot p_1] \\ &= 4g^2 e^2 \end{aligned}$$

The cross section is

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{2k_1^0 2p_1^0 |v_{k1} - vp_1|} \frac{1}{16\pi} \frac{2|p_1|}{E_{cm}} |\mathcal{M}|^2$$

20. Form factor.

$$F^E(q^2) = \frac{1}{e} \int d^3r \rho(r) e^{iq \cdot r} = \frac{2\pi}{e} \int dr d\theta \rho(r) e^{iqr \cos\theta} r^2 \sin\theta$$

$$F^E(q^2) = 1 - \frac{|q|^2}{6} \langle r^2 \rangle$$

if no angular part, $\langle r^2 \rangle \equiv \frac{1}{e} \int d^3r \rho(r) r^2 = \frac{4\pi}{e} \int dr \rho(r) r^4$.

$\rho(r) = \delta(r)$:

$$F^E(q^2) = 1$$

$\rho(r) = \frac{\alpha^2}{4\pi} \frac{e^{-\alpha r}}{r}$:

$$F^E(q^2) = \frac{\alpha^2}{\alpha^2 + q^2} = 1 - \frac{|q|^2}{\alpha^2}$$

$\rho(r) = \frac{m^3}{8\pi} e^{-mr}$:

$$F^E(q^2) = \frac{m}{(m^2 + q^2)^2} = 1 - \frac{2|q|^2}{m^2}$$

21. CP eigenvalues of $K^0 \rightarrow \pi\pi$ & $K^0 \rightarrow \pi\pi\pi$ ($CP = (-)^{S-2s}$ for neutral system).

For 2π system obviously they're all positive.

For 3π system, consider $\pi^0\pi^0\pi^0$, $CP = (+)(-)^L(-) = (-)^{L+1}$ where L is the orbital angular momentum between a $\pi^0\pi^0$ system and the other π^0 . Also consider $\pi^+\pi^-\pi^0$, $CP = (-)^{L+1}$ where L is the orbital angular momentum between a $\pi^+\pi^-$ system and π^0 .

22. Meson mass.

From Gell-Mann-Okubo formula

$$M(I, Y) = A + CY + B(I(I+1) - \frac{1}{4}Y^2)$$

Take $J^P = 0^-$, we have

$$M(K) = M(\frac{1}{2}, 1) = A + \frac{1}{2}B$$

$$M(\pi) = M(1, 0) = A + 2B$$

$$M(\bar{K}) = M(\frac{1}{2}, -1) = A + \frac{1}{2}B$$

$$M(\eta) = M(0, 0) = A$$

so

$$4m_K^2 = 3m_{\eta_s}^2 + m_\pi^2$$

Experiments give a deviation of 6%.

23. $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow J/\psi \rightarrow \mu^+\mu^-$.

Check Peskin 5.1, the unpolarized cross section is

$$\sigma = \frac{4\pi\alpha^2}{3E_{cm}^2} \sqrt{1 - \frac{m_\mu^2}{E^2}} (1 + \frac{1}{2} \frac{m_\mu^2}{E^2})$$

and for J/ψ production we need to work with the propagator:

add resonance structure to the propagator, it becomes (in Feynman gauge)

$$\frac{-g^{\mu\nu}}{p^2 - (m_{J/\psi} + i\frac{\Gamma}{2})^2}$$

24. Use CP invariance (if not violated) to determine P parity of strange particles via weak interaction.