Canonical variables

A^{Aj} $\qquad j = 1, 2$

$$\Pi^{A3} = G^{A30} = \partial^3 A^{A0}$$

E.L for A^0

$$\Rightarrow 0 = -(\partial^3)^2 A^{A0} - \partial^j \underline{\pi}^{Aj} + g f^{ABC} \underline{\pi}^{Bj} \underline{A}^{Cj}$$

$$\Rightarrow A^{A0}(\pi^j, A^j) \leftarrow$$

$$\langle \Omega | T[O_a(x_a) O_b(x_b) \ldots] | \Omega \rangle$$

$$= \boxed{|N|^2} \int \underline{D[A^{A\hat{j}}] D[\pi^{A\hat{j}}]}$$

$$\exp\left[i \int d^4x \left(-\pi^{Aj} \underset{\frown}{\dot{A}}{}^{\hat{j}} - \mathcal{H}(\underline{\pi}^{\hat{j}}, \underset{\wedge}{A}{}^{\hat{j}}) \right) \right] \swarrow$$

$$O_a(x_a) O_b(x_b) \cdots$$

$$\mathcal{H} = \frac{1}{2} \pi^{A\hat{j}} \pi^{A\hat{j}} - \pi^{A\hat{j}} (\partial^{\hat{j}} A^{A0} - f^{ABC} A^{B0} A^{C\hat{j}})$$

$$+ \frac{1}{4} G^{Aij} G^{Aij} - \frac{1}{2} \partial^3 A^{A0} \partial^3 A^{A0}$$

$$J = \int D[A^{Aj}]\, \underline{D[A^{A0}]}\, \underline{D[\pi^{Aj}]}$$

$$\exp\left[i \int d^4x \,\underline{\left(-\pi^{Aj} \dot{A}^j - H' \right)} \right]$$

$$\underline{H'} = H \,(A^0 \text{ free})$$

$$= A^0 (\cdots) \underline{A}^0 + (\ ) A^0 + C.$$

$$\int_{-\infty}^{+\infty} dx\, e^{-x^2+2x} = \int dx\, e^{-\underline{(x-1)}^2+1} = \boxed{C} \cdot \boxed{\underline{e}^1}$$

$$-1^2 + 2 \cdot 1 = 1$$

$$\frac{d}{dx}(-x^2+2x) = -2x+2 = 0 \Rightarrow x = 1$$

$$0 = \frac{dH'}{dA^0} \Rightarrow (\partial^3)^2 A^{A0} + \partial^j \pi^{Aj} - g f^{ABC} \pi^{Bj} A^{Cj} = 0$$

$$H' = \pi^j (\quad) \pi^j + (\,) \pi^j + C$$

$$0 = \frac{\partial H'}{\underline{\partial \pi^{Aj}}} = \underline{-\pi^{Aj} + G^{Ai0}}$$

$$1 = \int D[A^3]\; \delta(A^{A3})$$

$$\langle \Omega | T[O_a(x_a)\, O_b(x_b) \cdots] | \Omega \rangle$$

$$= |N|^2 \int D[A^{A\mu}]\; \boxed{\underline{\delta(A}^{A3})} \rightarrow \delta(\varepsilon)$$

$$\exp\left[i\int d^4x\cdot\mathcal{L}\right] O_a(x_a)\, O_b(x_b)\cdots$$

$$\underline{\delta A^{A3}} \sim \underset{\overset{\downarrow}{0}}{\underline{f^{ABC}\varepsilon^{C}A^{B3}}} + \underset{C\text{-}\varepsilon}{\underline{\partial^{3}\varepsilon^{A}}}$$

$$\delta\left(A^{A3}\right)_{\varepsilon} \longrightarrow \underline{\delta\left[\boxed{f^{A}(\varepsilon)} - F^{A}\right]\det M}$$

$$\boxed{M^{AB} = \left.\frac{\delta f^{A}(\varepsilon)}{\delta\varepsilon^{B}}\right|_{\varepsilon=0}}$$

$$A^{3} = \boxed{0,1,2}$$

$$\langle\Omega|T(\qquad)|\Omega\rangle$$

$$= |N|^{2}\int \underline{D[A^{A\mu}]}\;\underline{\delta\left[f^{A}(\varepsilon) - F^{A}\right]\det M}$$

$$\exp\left[i \int d^4x \mathcal{L} \right] \cdots$$

$$\underbrace{\int D[F]}\left[\boxed{G[F]\,\delta(f^A - F^A)} \right] = G[f]$$

Choose $G[f] = \exp\left[i \int d^4x \frac{1}{-2\xi} f^A f^A \right]$

$$= |N|^2 \int D[A^{A\mu}]\, \det M \; e^{i\int d^4x\left(\mathcal{L} - \frac{1}{2\xi} f^A f^A\right)} \cdots$$

$$\det M = \int D[\bar{c}^A, c^A]\, e^{\int d^4x\, d^4y\, \underbrace{\bar{c}^A(x)}\, \frac{\delta f^A(x)}{\delta \varepsilon^B(y)}\Big|_{\varepsilon=0} \underbrace{c^B(y)}}$$

$$= |N|^2 \int D[A^{A\mu}]\, D[c^A]\, D[\bar{c}^A]$$

$$\exp\Big[i \int d^4x \Big(\mathcal{L} \underbrace{- \frac{1}{2\xi} f^A f^A}_{\mathcal{L}_{GF}} + \underbrace{\bar{c}^A (-i M^{AB}) c^B}_{\mathcal{L}_{ghost}} \Big) \Big] \ldots$$

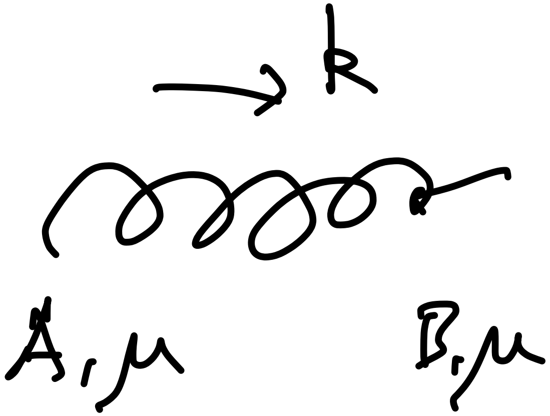
Covariant gauge

$$f^A = \partial_\mu A^{A\mu} \qquad \boxed{HW1}$$

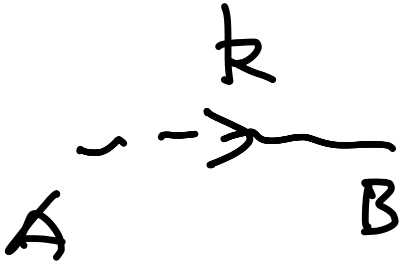
$$\frac{\delta f^A(x)}{\partial \varepsilon^B(y)}\bigg|_{\varepsilon=0} = \left(\partial^\mu \partial_\mu \delta^{AB} + g f^{ACB} \partial_\mu A^{C\mu} \right)$$

$$\times \frac{f}{g}\, \delta^{(4)}(x-y)$$

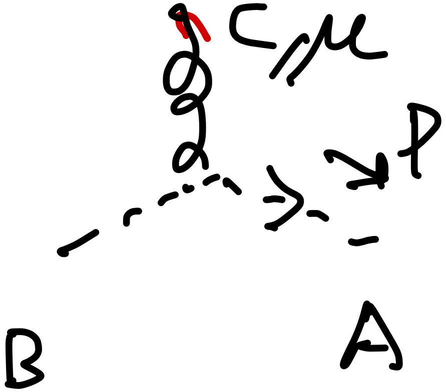
$$\mathcal{L}_{ghost} = (\partial_\mu \bar{c}^A)(\partial^\mu c^A) + \underline{g f^{ABC} (\partial_\mu \bar{c}^A)\, c^B A^{C\mu}}$$



$$\delta^{AB} \frac{i}{k^2 + i\varepsilon} \left(-g_{\mu\nu} + (1-\xi) \frac{k_\mu k_\nu}{k^2} \right)$$



$$\delta^{AB} \frac{i}{k^2 + i\varepsilon}$$



$g f^{ABC} p^\mu$

HW 2

III. Renormalization.

→ Higher-order calculation.

UV divergences

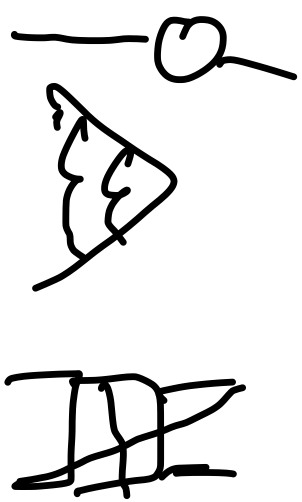
part of QFT.

View of EFT.

foms on Low Energy

$$\underset{\sim}{M} = M_0 + \frac{\alpha_s}{4\pi} M_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 M_2 + \cdots$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \underset{\sim}{\text{LO}} & \underset{\sim}{\text{NLO}} & \text{NNLO} \end{array}$$



loop integral

UV divergences

when $k \to \pm\infty$

Renormalizable:

all divergences can be removed by renormalization of a finite number of couplings in the Lagrangian.

1971. 't Hooft. QCD is renormalizable. $\left(1+\frac{\boxed{\alpha_s}}{4\pi}\frac{1}{\epsilon}\right)\left(1-\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\right)$

In QCD, $\underline{O(m_j, g_s) \text{ finite}}$

$\Rightarrow$ m_j, g_s are divergent.

They cancel exactly the UV divergences in the loops.

$$g_s \bar{\psi} \gamma^\mu T^A \psi A^A$$

$$m \bar{\psi} \psi$$

- Perform renormalization.
- Bare parameter renormalization.

Bare $\mathcal{L}$ and Feynman rules

$O_1 (m_j \cdot g_s)$

Loop integrals are divergent.

"Regularization" $\left\{ \begin{array}{l} \text{cutoff reg.} \\ \text{dimensional reg.} \\ \underline{d = 4 - 2\epsilon} \end{array} \right.$

$$\int_1^\infty dx \, \frac{1}{x} \rightarrow \int_1^\infty dx \cdot \frac{x^{-2\epsilon}}{x} \rightarrow \frac{1}{\epsilon}$$

$\frac{d}{2}$

$\frac{d-2}{2}$

$$\int_1^\infty dx \, \frac{1}{x^2} (\underset{\triangle}{1} - \underset{\triangle}{1} + x) = \int_1^\infty dx \, \frac{1}{x}$$

E^{4-u}

$O_2(m_j, g_s) \qquad O_3(m_j, g_s)$

$$\Rightarrow m_j^R = \underline{m_j} + \boxed{C} + \underline{\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\right)_{\pi\ldots}$$

$$g_s^R = g_s + \boxed{C} + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\right) + \cdots$$

$O_1\left(m_j, g_s, \frac{1}{\epsilon}\right)$

$$m_j^R - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\cdots \qquad g_s^R - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}+\cdots$$

$$= \underline{O_1^R(m_j^R, g_s^R)}$$

$$\frac{\alpha_s}{4\pi}\left(\underset{\underset{\text{loop}}{\uparrow}}{\frac{1}{\epsilon}} - \underset{\underset{m_j^R}{\uparrow}}{\frac{1}{\epsilon}}\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\cdots - \cdots\right)$$

- BPHZ scheme

O^R finite

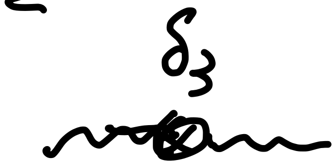
$O^R(m_j^R, g_s^R)$

$$\begin{cases} q_j = Z_{2,j}^{1/2} \, q_{j,R} \\ A^\mu = Z_3^{1/2} \, A_R^\mu \\ c^a = Z_2^{c\,1/2} \, c_R^a \end{cases}$$

$$\mathcal{L}_{QCD} = \underline{\mathcal{L}^{R}_{QCD}} + \mathcal{L}^{C.T.}_{QCD}$$

$$\begin{tikzcd}[column sep=small]
\mathcal{L}^{R}_{QCD} \arrow[ddrr, bend left] & = & \mathcal{L}_{QCD}(m_j \to m_j^{R},\ g_s \to g_s^{R}) \\
& A\,\textcircled{R}\text{: dropped.} & \\
\mathcal{L}^{C.T.}_{QCD} & = & \underline{\mathcal{L}_{QCD} - \mathcal{L}^{R}_{QCD}}
\end{tikzcd}$$

$$= -\frac{1}{4}\delta_3\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)^2$$



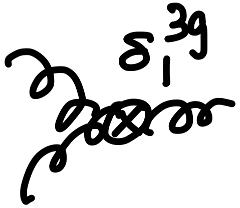
$$\underline{+\sum_j \bar{q}_j\left(i\delta_2^j \not\partial - \delta_m^j\right) q_j}$$

$$-\delta_2^c\, \bar{c}^a \partial^2 c^a$$

$$+\sum_j \underline{g_s^R}\, \delta_1^j A_\mu^a \bar{q}_j \gamma^\mu q_j$$

$$-g_s^R\, \delta_1^{3g} f^{abc}\left(\partial_\mu A_\nu^a\right) A_\mu^b A_\nu^c$$

$$+\frac{1}{4} g_s^{R^2} \delta_1^{4g} (f^{eab} A_\mu^a A_\nu^b)(f^{ecd} A_\mu^c A_\nu^d)$$



$$- g_s^R \delta_1^c f^{abc} \bar{c}^a \partial^\mu A_\mu^b c^c$$

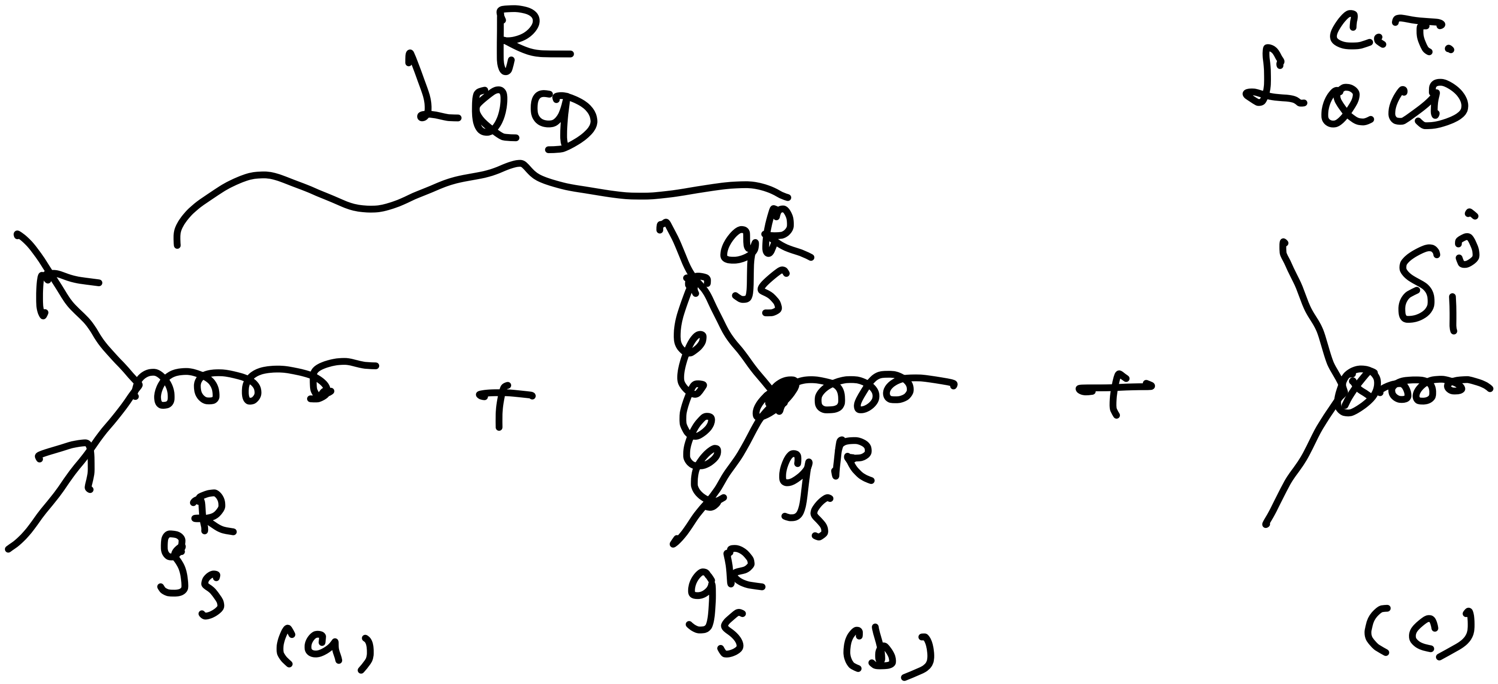
$$\delta_2^j = Z_{2,j} - 1 \qquad \delta_2^c = Z_2^c - 1$$

$$\delta_3 = Z_3 - 1 \qquad \delta_m^j = Z_{2,j} m_j - m_j^R$$

$$\underline{\delta_1^{\bar{j}}} = \frac{\boxed{g_s}}{\underline{g_s^R}^{(\mu)}} \overset{\swarrow}{Z}_{\underline{2},j}\, \underline{Z_3}^{1/2\,\swarrow} - 1 \,, \quad \delta_1^{3g} = \frac{g_s}{g_s^R} Z_3^{3/2} - 1$$

$$\delta_1^{4g} = \frac{g_s^2}{g_s^{R2}} Z_3^2 - 1 \,, \qquad \delta_1^{c} = \frac{g_s}{g_s^R} Z_2^{c} Z_3^{1/2} - 1$$

7.18



$\delta_1^j = \frac{\alpha_s}{4\pi}\delta_1^{j(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\delta_1^{j(2)}$

LO

$$\int d^4k \frac{1}{k^2}\frac{1}{k}\frac{1}{k} = \text{finite}$$

$\rightarrow$ UV $\frac{1}{\epsilon_{UV}}$

$\int d^4p$

$$\overset{4-2\epsilon}{\downarrow}$$

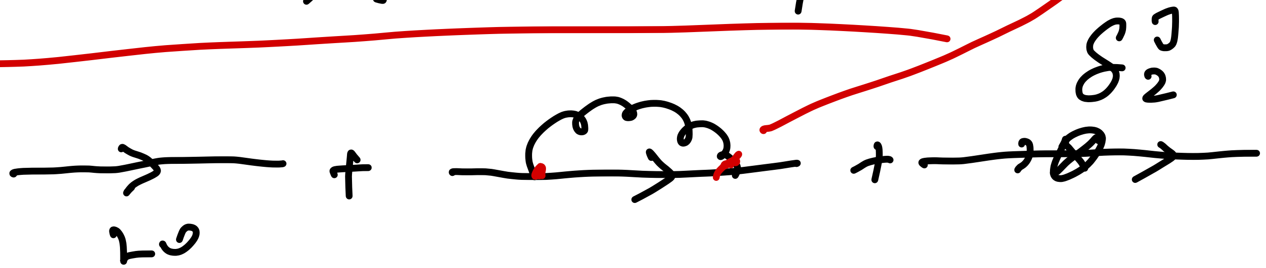
$$= \int d^dp \frac{g_s^{R^2}\underline{\mu^{2\epsilon}}}{(p^2)^2}$$

LO: (a) + (c) $\Rightarrow$ $\delta_1^{j(0)} = 0$

NLO: (a) + (b) + (c) finite $\Rightarrow$ $\delta_1^{j(1)}$

$\sim g_s^{R^2}\mu^{2\epsilon}\frac{1}{\epsilon}$

$\sim g_s^{R^2}\left(\frac{1}{\epsilon} + \ln\mu^2\right)$

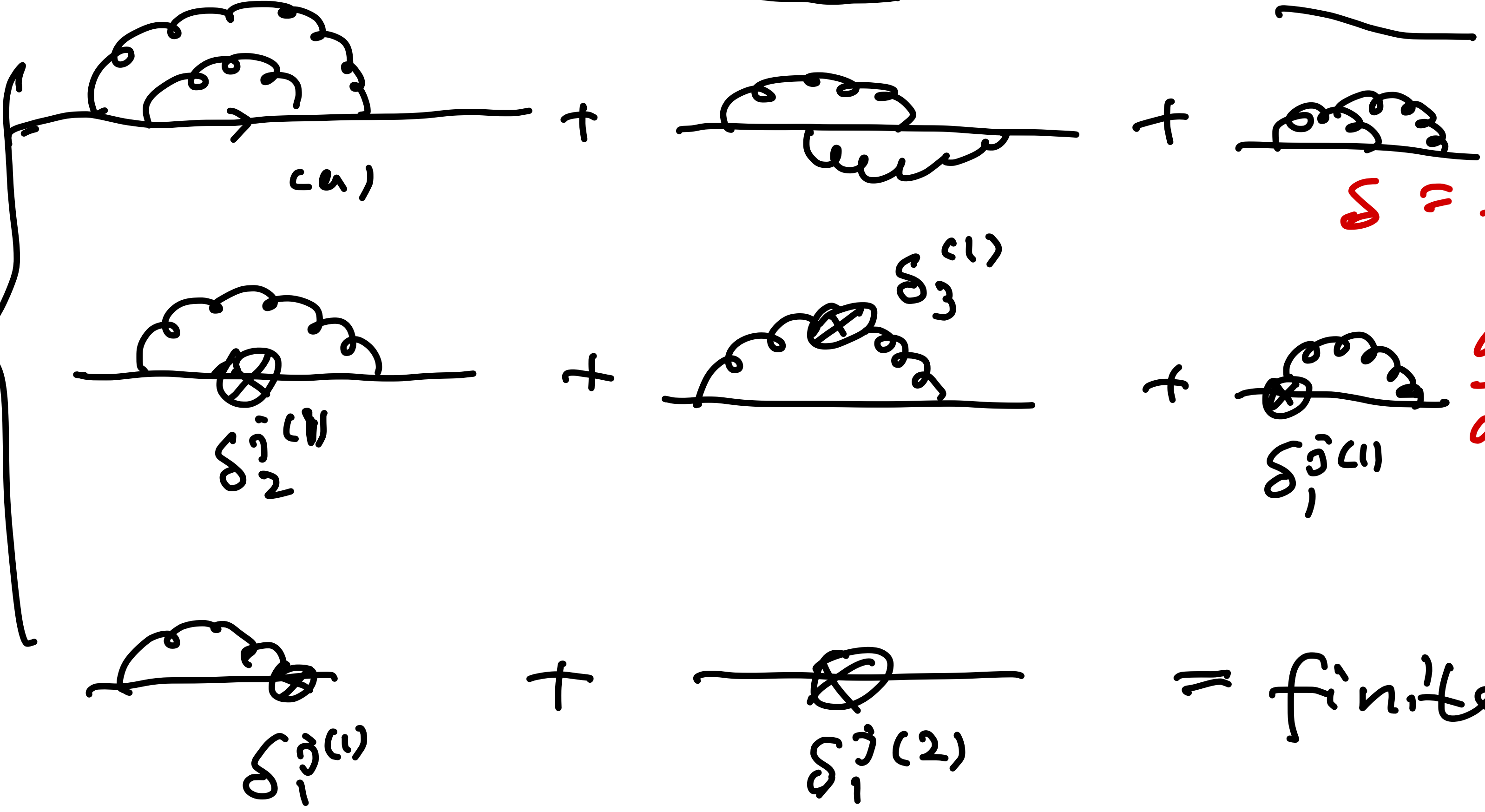


= finite

$\Rightarrow \delta_2^{j(1)}$ $\overbrace{\mathcal{L}^{R}_{QCD}}$ $\overbrace{\mathcal{L}^{C.T.}_{QCD}}$

$$\frac{dg_s^R}{d\ln\mu} = \epsilon g_s$$

1PI



= finite

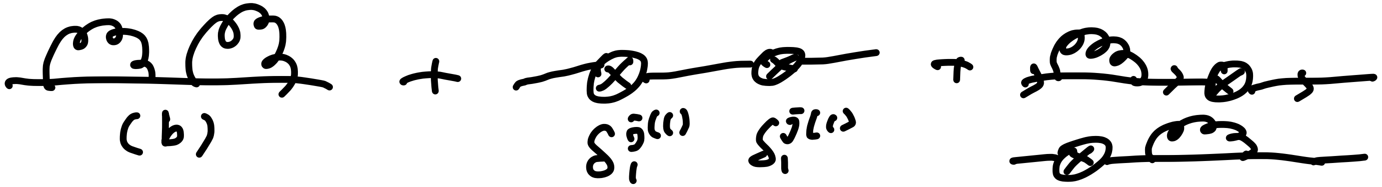
$$\underline{\delta = g_s \frac{1}{\epsilon}}$$

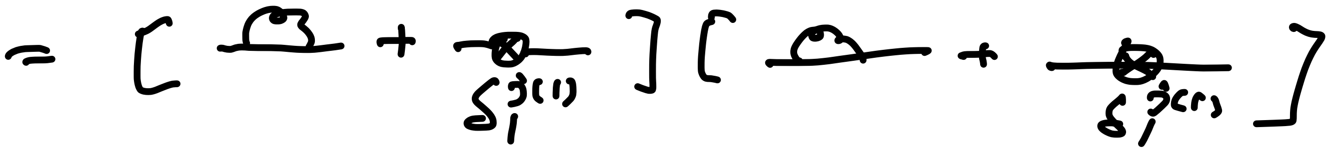
$$\frac{d\delta}{d\ln\mu} = \frac{dg_s}{d\ln\mu}\frac{1}{\epsilon}$$

$$= (\epsilon \cdot g_s)\frac{1}{\epsilon}$$

$$= g_s$$

1PR.

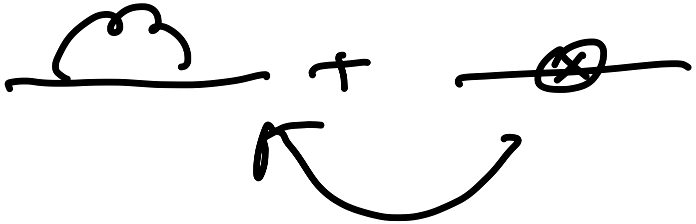




UV finite UV finite

Peskin's Chapter 16

LLY's Lecture



= finite

0, 1, 2

$\left(\frac{1}{\epsilon}+1\right)$ $-\left(\frac{1}{\epsilon}+1\right)$

$-\frac{1}{\epsilon}$

Renormalization scheme.

MS : $-\frac{1}{\epsilon_{UV}}$

$\overline{MS}$: $-\frac{1}{\epsilon_{UV}} + \ln 4\pi + \gamma_E$

on-shell : $-\frac{1}{\epsilon_{UV}} + \frac{1}{\epsilon_{IR}}$

The relation between diff. scheme is universal.

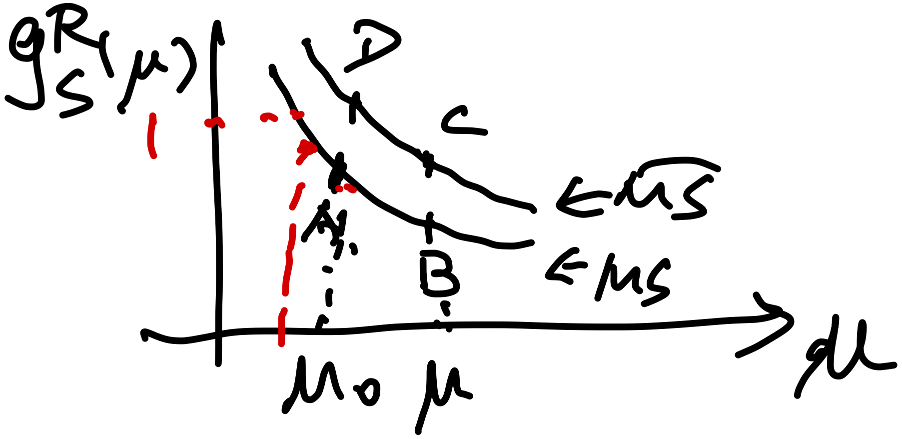
When comparing two observables, one should use the same scheme.

$\boxed{HW1}$

$$\beta(g_s) \equiv \frac{d\, g_s^R}{d\ln\mu} = g_s^R \frac{d}{d\ln\mu}\left[-\ln(1+\delta_1^i) + \ln(1+\delta_2^i) + \frac{1}{2}\ln(1+\delta_3)\right]$$

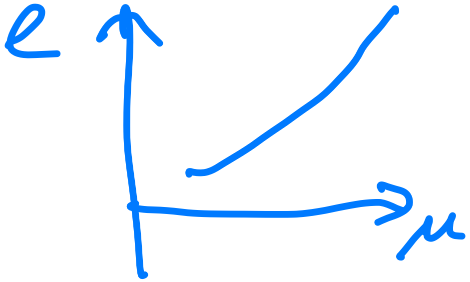
$$\simeq -\frac{g_s^{R3}}{(4\pi)^2} \times \underline{\left(\overset{\downarrow}{\underset{\smile}{11}} - \frac{2}{3}\overset{\overset{\text{QED like}}{\downarrow}}{n_f}\right)} = \beta_0 \quad \text{for QCD}$$

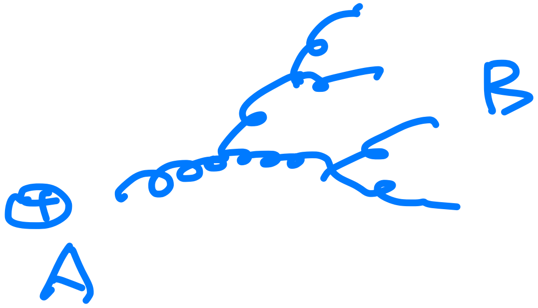
$C_A = 3, \; C_F = \frac{4}{3}$

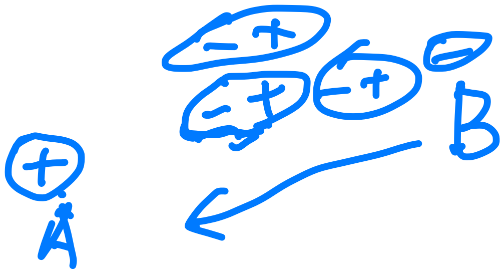


$n_f = 6$

$11 - \frac{2}{3} \times 6 = 7$







$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$\Rightarrow \alpha_s(\mu) = \frac{\alpha_s(\mu_0) \leftarrow 0.1}{1 + \underbrace{\frac{\alpha_s}{4\pi} \beta_0 \ln \frac{\mu^2}{\mu_0^2}}_{0}}$$

< 1

$\sim \alpha_s$, 0.1

$\mu \to \infty$, $\alpha_s(\mu) \to 0$

$\mu \to 0$, $\alpha_s(\mu) \to \infty$

Λ_{QCD}

$$1+\frac{\alpha_s}{4\pi}\beta_0 \ln\frac{\Lambda^2_{QCD}}{\underline{\mu_0}} = \underline{0,\ \boxed{0.1}}$$

$\Rightarrow \Lambda_{QCD}.\ \sim (200\ \text{MeV} \sim 400\ \text{MeV})$

pQCD. scale > 2 GeV

$$\int_1^{\infty} dk\,\frac{1}{k} = \int_1^{\boxed{a}} dk\,\frac{1}{k} + \int_a^{\infty} dk\,\frac{1}{k}$$

$$= \underline{\ln \boxed{a}} + \int_a^{\infty} dk\,\frac{1}{k}$$

$\nearrow$

$1+\epsilon$

$$\int_1^\infty \overset{d}{d}\overset{k}{k}\,\frac{\boxed{k^{-\epsilon}}\,\mu^\epsilon}{k} = \frac{1}{\epsilon}\boxed{\mu}^\epsilon = \overset{\swarrow}{\frac{1}{\epsilon}} + \boxed{\ln\mu}$$

IV QCD at hadron colliders

$$A(p_A) + B(p_B) \to X(p_1) + X(p_2) + \cdots$$

$$\underline{d\sigma} = \frac{1}{2s}\, d\Phi_n \,|\underline{M}(p_A + p_B \to p_1 + p_2 + \cdots)|^2$$

Scattering Amp. Lorentz inv.

$\downarrow$

$\sqrt{s}$ c.of.m. | energy, lorentz boost invariant along the beam line.

$\boxed{HW2}$

n-body phase space.   Lorentz invariant

$$d\Phi_n = \left(\prod_{f=1}^{n} \frac{d^3 p_f}{2E_f (2\pi)^3} \right) (2\pi)^4 \delta^{(4)}\left(p_A + p_B - \sum_f p_f \right)$$

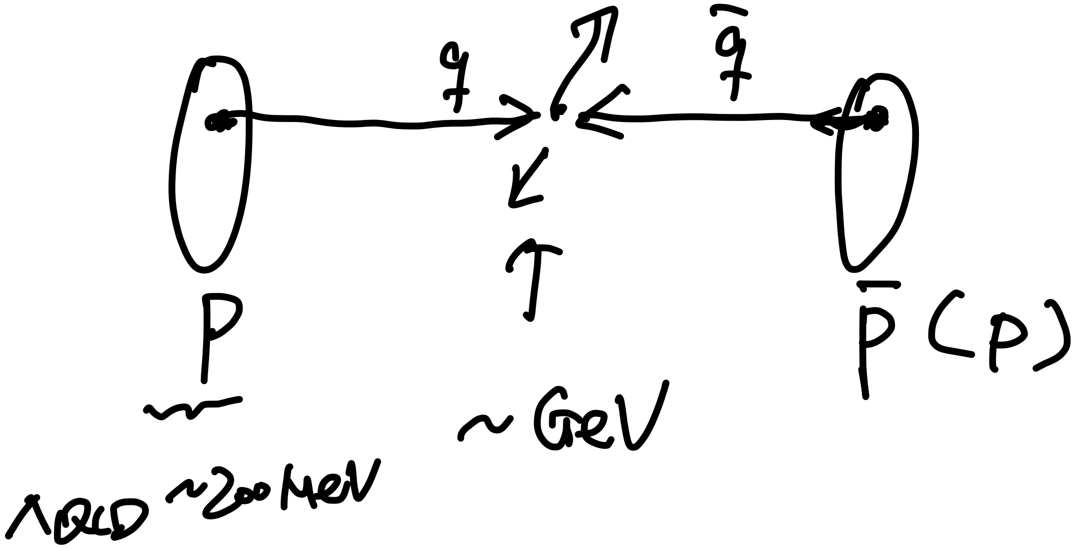
$$(2\pi)^4 \delta^{(4)}\left(p_A + p_B - \sum_f p_f \right) M\left(p_A + p_B \to p_1 \, p_2 \cdots \right)$$

$$= \lim_{t_0 \to \infty(1-i\epsilon)} {}_0\langle \underbrace{p_1 p_2 \cdots}_{+\infty} | \underbrace{T \exp\left[-i \int_{-t_0}^{t_0} dt \, H_I(t) \right]}_{} | \underbrace{p_A p_B}_{-\infty} \rangle_0 \quad C.A.$$

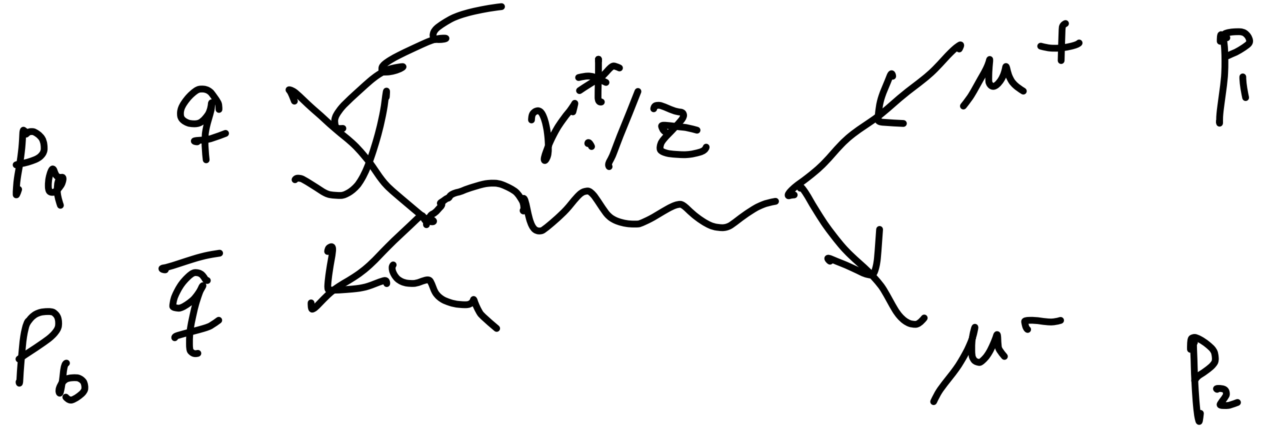
$$+ \int_{-t_0}^{t_0} dt_1 \int_{-t_0}^{t_1} dt_2 \, H_I(t_1) \, \underline{H_I(t_2)}$$

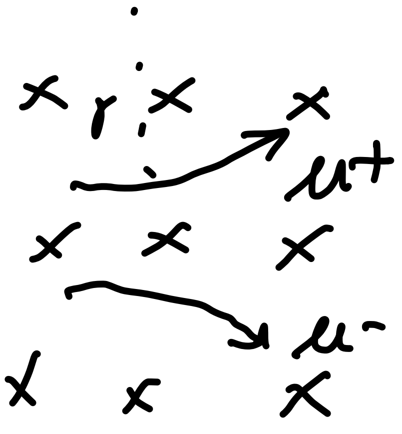
$$1 - i\int_{-t_0}^{t_0} dt \; H_I(t)$$

IV. 1. factorization



$$r \propto \frac{v}{B}$$





$$Q^2 = (P_1 + P_2)^2$$

$$y = \frac{1}{2} \ln \frac{(P_1 + P_2) \cdot P_A}{(P_1 + P_2) \cdot P_B}$$

$$\boxed{\underline{\frac{d\sigma}{dQ^2 dy}}} = \underline{\sum_{a,b} \int_{\underline{x_A}}^{1} d\xi_A \int_{\underline{x_B}}^{1} d\xi_B} \, \underline{f_{a/p}}(\xi_A, \mu)$$

$$\underline{f_{b/\bar{p}}(\xi_B, \mu)} \times \underline{H_{ab}\left(\boxed{\frac{\xi_A}{x_A}}, \frac{\xi_B}{x_B}, Q, \mu, \alpha_s \right)}$$

$$+ O\left(\frac{\Lambda_{QCD}}{Q^2} \right)$$

$$x_A = e^{y} \sqrt{\frac{Q^2}{s}}$$

$$P_A \cdot \xi_A = P_a$$

$$x_B = e^{-y}\sqrt{\frac{Q^2}{s}}$$

$$x_A x_B = \frac{Q^2}{s}$$

$$\underline{Q^2} = \underline{x_a x_b \cdot s}$$

$$f_{q/p}(\xi,\mu) = \frac{1}{4\pi}\int dx^- e^{-i\xi p^+ x^-} \langle p|\underline{\bar{\psi}(0, x^-, 0_\perp)\gamma^+}$$

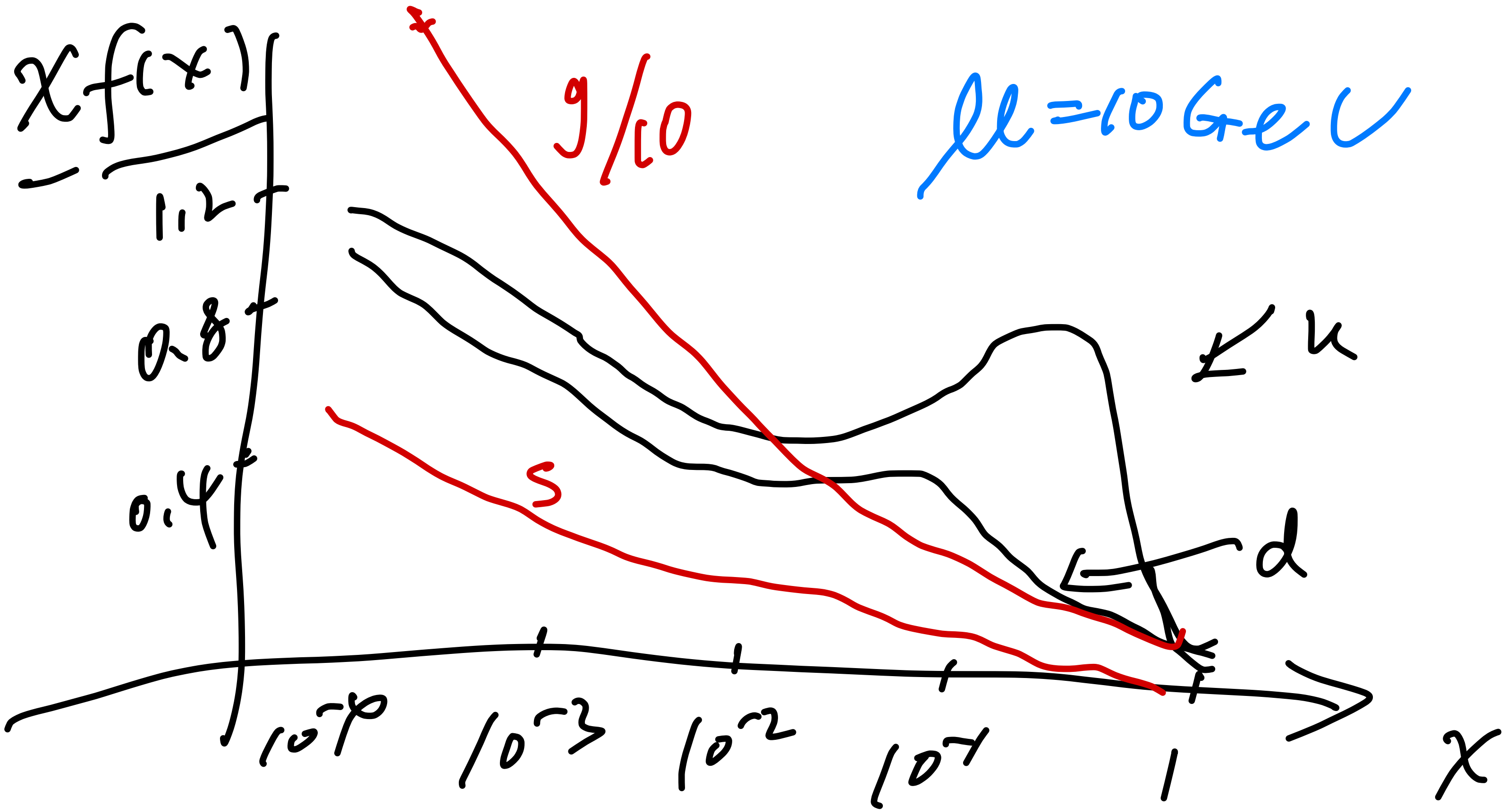
$$\underleftarrow{G\,\psi(0, 0, 0_\perp)|p\rangle}$$

$$\frac{\psi^\dagger \psi}{|\psi|^2}$$

$$G = P\exp\left[ig\int_0^{x^-} dy^- A_c^+(0, y^-, 0_\perp)\, t_c\right]$$

$$x^{\pm} = (x^0 \pm x^3)/\sqrt{2}$$

$$p^{\pm} = (p^0 \pm p^3)/\sqrt{2}$$



globle fitting

IR safety

0. insensitive to the collinear and soft emission

$|M|^2 \cdot F_J$

① $F_J^{n+1}(p_1, \cdots, p_j = \lambda q, \cdots)$ $\lambda \to 0$

$\to F_J^{n}(p_1 \cdots, \cdots)$

② $F_J^{n+1}(\cdots p_i, p_j \cdots)$

$p_i = z \cdot p$

$p_j = (1-z) \cdot p$

$\to \quad F_J^n(\cdots p, \cdots\cdots)$

③ $F_J^n(p_1, \cdots, p_n) \to 0 \quad$ if $\quad p_i \cdot p_j \to 0$