Chiral basis:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1-\gamma_5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\sigma^\mu_{\alpha\dot\beta} = (1, \vec\sigma)_{\alpha\dot\beta}$$

$$(\bar\sigma^\mu)^{\dot\alpha\beta} = (1, -\vec\sigma)^{\dot\alpha\beta} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\beta\alpha}\sigma^\mu_{\alpha\dot\beta}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{\begin{aligned} \varepsilon^{12} &= +1 \\ \varepsilon_{12} &= -1 \end{aligned}}$$

☆ Spinor-helicity

Dixon: 1310.5353

Elvang & Huang: 1308.1697

massless fermion $\quad \not{p}\, u(p) = 0 = \not{p}\, v(p)$

can take $u(p) = v(p)$

$\bar{u}(p)\not{p} = 0 = \bar{v}(p)\not{p}$

$\bar{u}(p) = \bar{v}(p)$

$$\frac{1-\gamma_5}{2} u(p) \equiv \begin{pmatrix} |p]_\alpha \\ 0 \end{pmatrix} \equiv |p]$$

$$\frac{1+\gamma_5}{2} u(p) \equiv \begin{pmatrix} 0 \\ \underset{\sim}{|p\rangle^{\dot{\alpha}}} \end{pmatrix} \equiv |p\rangle$$

$$\bar{u}(p)\frac{1+\gamma_5}{2} \equiv \left(0 \quad \langle p|_{\dot\alpha}\right) \equiv \langle p|$$

$$\bar{u}(p)\frac{1-\gamma_5}{2} \equiv \left([p|^{\alpha} \quad 0\right) \equiv [p|$$

$$\not{p} = p^\mu \gamma_\mu = \begin{pmatrix} 0 & p\cdot\sigma \\ p\cdot\bar\sigma & 0 \end{pmatrix} = \begin{pmatrix} 0 & \underset{\sim}{p_{\alpha\dot\beta}} \\ p^{\dot\alpha\beta} & 0 \end{pmatrix}$$

$$p_\mu \sigma^\mu_{\alpha\dot\beta} \equiv p_{\alpha\dot\beta}\,, \quad p^\mu \bar\sigma_\mu^{\dot\alpha\beta} \equiv \underset{\sim}{p}^{\dot\alpha\beta} = \begin{pmatrix} p^0+p^3 & p^1-ip^2 \\ p^1+ip^2 & p^0-p^3 \end{pmatrix}$$

$$\det(p^{\dot\alpha\beta}) = p^2 = 0$$

Dirac eq. :

$$p_{\alpha\dot{\beta}} |p\rangle^{\dot{\beta}} = 0$$

$$p^{\dot{\alpha}\beta} |p]_{\beta} = 0$$

$$\langle p|_{\dot{\alpha}} \, p^{\dot{\alpha}\beta} = 0$$

$$[p|^{\alpha} \, p_{\alpha\dot{\beta}} = 0$$