One Loop Matching for Quasi PDF

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1 Background

The definition of parton distribution function (PDF) is

$$q\left(x, \mu_f\right)=\frac{1}{2} \int \frac{d \eta^-}{2 \pi} e^{-i x P^+ \eta^-}\left\langle P, S\left|\bar{\psi}\left(\eta^-\right) \Gamma \mathcal{W}\left[\eta^- ; 0\right] \psi(0)\right| P, S\right\rangle \tag{1}$$

where with this unpolarized PDF case, $\Gamma=\gamma^+$. $\mathcal{W}$ is the gauge link defined as [Collins(2009)]

$$\mathcal{W}\left[w^-, 0\right]=P\left\{e^{-i g_0 \int_0^{w^-} \mathrm{d} y^- A_{(0) \sigma}^+\left(0, y^-, \mathbf{0}_{\mathrm{T}}\right) t_\sigma}\right\} \tag{2}$$

The definition of quasi PDF is

$$\tilde{q}(x)=\frac{1}{2} \int \frac{\mathrm{d} z}{2 \pi} e^{i x P^z z}\langle P, S|\bar{\psi}(z) \tilde{\Gamma} \tilde{\mathcal{W}}[z, 0] \psi(0)| P, S\rangle \tag{3}$$

where

$$\tilde{\mathcal{W}}[z, 0]=\exp \left[i g \mathcal{P} \int_0^z \mathrm{d} z' n \cdot A^a\left(z'\right) \mathrm{t}^a\right], n=(0,0,0,-1) \tag{4}$$

and $\tilde{\Gamma}=\gamma^z$ in our case.

To make the gauge links equal to unity, we choose light cone gauge for PDF and axial gauge for quasi PDF.

2 Tree Level Matching

In axial gauge, the quasi PDF is

$$\tilde{q}(x)=\frac{1}{4 \pi} \int \mathrm{d} z e^{i x P^z z}\langle P|\bar{\psi}(z) \gamma^z \psi(0)| P\rangle \tag{5}$$

The frame is chosen such that $P^\mu=(P^0, \mathbf{0}, P^z)$. Up to one loop, we can use quark state as the external state to complete the matching process. The quark field ψ reads

$$\psi(x)=\int \frac{\mathrm{d}^3 \vec{k}}{(2 \pi)^3} \frac{1}{2 E_k}\left[u(k) e^{-i k \cdot x} b_k+v(k) e^{i k \cdot x} d_k^{\dagger}\right] \tag{6}$$

Insert it to (5)

$$\tilde{q}^{(0)}(x)=\int \frac{\mathrm{d} z}{4 \pi} e^{i x P^z z}\left\langle 0\right| b_P \int \frac{\mathrm{d}^3 \vec{p}}{(2 \pi)^3} \frac{1}{2 E_p}\left[\bar{u}(p) e^{i p \cdot x} b_p^{\dagger}+\bar{v}(p) e^{-i p \cdot x} d_p\right] \gamma^z \int \frac{\mathrm{d}^3 \vec{k}}{(2 \pi)^3} \frac{1}{2 E_k}\left[u(k) e^{-i k \cdot x} b_k+v(k) e^{i k \cdot x} d_k^{\dagger}\right] b_P^{\dagger}|0\rangle \tag{7}$$

Look at the creation-annihilation operators, we have the following combinations:

$$b_P b_p^\dagger b_k b_P^\dagger,\ b_P d_p b_k b_P^\dagger,\ b_P b_p^\dagger d_k^\dagger b_P^\dagger,\ b_P d_p d_k^\dagger b_P^\dagger \tag{8}$$

Apparently the latter three all go to zero by moving the anti-quark operators to the side:

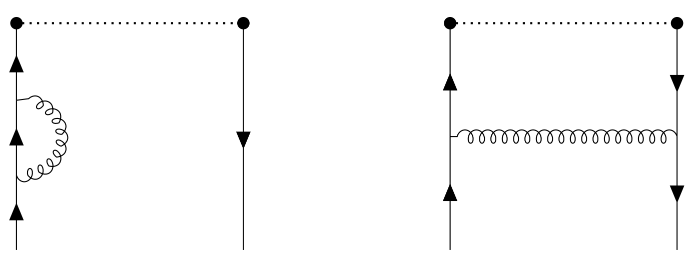
$$\begin{aligned}\tilde{q}^{(0)}(x) &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z} \langle 0| \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2E_p}\bar{u}(p)e^{ip\cdot z} b_P b_p^\dagger \gamma^z \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\frac{1}{2E_k} u(k) e^{-ik\cdot 0} b_k b_P^\dagger |0\rangle \\ &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z} \langle 0| \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{e^{ip\cdot z}}{2E_p}\bar{u}(p)(2\pi)^3 2E_{\mathbf{P}}\delta^{(3)}(\mathbf{p}-\mathbf{P})\gamma^z \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\frac{e^{-ik\cdot 0}}{2E_k} u(k)(2\pi)^3 2E_{\mathbf{P}}\delta^{(3)}(\mathbf{k}-\mathbf{P})|0\rangle \\ &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z + iP\cdot z}\bar{u}(P)\gamma^z u(P)\end{aligned} \tag{9}$$

Using Gordon identity

$$\begin{aligned}\tilde{q}^{(0)}(x) &= \int \frac{\mathrm{d}z}{4\pi} e^{ixP^z z - iP^z z}\bar{u}(P)\frac{P^z}{m}u(P) \\ &= \int \frac{\mathrm{d}z}{2\pi} e^{ixP^z z - iP^z z} P^z \\ &= \delta(1-x)\end{aligned} \tag{10}$$

3 One Loop Quasi PDF

Two diagrams are required with one loop corrections to quasi PDF.



(11)

A Conventions

The quark field ψ reads

$$\psi(x) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\frac{1}{2E_k}\left[u(k)e^{-ik\cdot x}b_k + v(k)e^{ik\cdot x}d_k^\dagger\right] \tag{12}$$

and the projection of single particle state is

$$|p\rangle = b_p^\dagger |0\rangle \tag{13}$$

$$\left\{b_{\mathbf{p}}^r, b_{\mathbf{q}}^{s\dagger}\right\} = (2\pi)^3 2E\delta^{(3)}(\mathbf{p}-\mathbf{q})\delta^{rs} \tag{14}$$

The Dirac spinor is normalized to

$$\bar{u}^s(p)u(p) = 2m\delta^{rs} \tag{15}$$

References

[Collins(2009)] J. Collins, *Foundations of Perturbative QCD* (Cambridge University Press, 2009).