

Annual Assessment Report

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- 1 Operator Product Expansion for Atomic Wave-functions
- 2 Meson-meson Scattering in 1+1-d QCD
- 3 Fragmentation Production of Fully-heavy Tetraquark at LHC

Operator Product Expansion for Atomic Wave-functions

- **Universal behaviors** in Coulombic wave-functions, **near-the-origin divergence** in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\text{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2} \frac{r^2}{a_0^2} + \cdots & (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8} \frac{r^2}{a_0^2} + \cdots & (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54} \frac{r^2}{a_0^2} + \cdots & (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32} \frac{r^2}{a_0^2} + \cdots & (n=4) \end{cases},$$

$$R_{n0}^{\text{KG}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2} \frac{r^2}{a_0^2} - Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots & (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8} \frac{r^2}{a_0^2} - Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots & (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54} \frac{r^2}{a_0^2} - Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots & (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32} \frac{r^2}{a_0^2} - Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \cdots & (n=4) \end{cases}$$

Motivation

- **Universal behaviors** in Coulombic wave-functions, **near-the-origin divergence** in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\text{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2} \frac{r^2}{a_0^2} + \dots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8} \frac{r^2}{a_0^2} + \dots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54} \frac{r^2}{a_0^2} + \dots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32} \frac{r^2}{a_0^2} + \dots (n=4) \end{cases},$$

$$R_{n0}^{\text{Dirac}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2} \frac{r^2}{a_0^2} - \frac{1}{2} Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2} Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \dots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8} \frac{r^2}{a_0^2} - \frac{1}{2} Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2} Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \dots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54} \frac{r^2}{a_0^2} - \frac{1}{2} Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2} Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \dots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32} \frac{r^2}{a_0^2} - \frac{1}{2} Z^2 \alpha^2 \log\left(\frac{r}{a_0}\right) + \frac{1}{2} Z^2 \alpha^2 \left(\frac{r}{a_0}\right) \log\left(\frac{r}{a_0}\right) + \dots (n=4) \end{cases}$$

Attack the problem with OPE & EFT: Construct EFT

- Use **non-relativistic QED (NRQED)** for electron and **heavy nucleus effective theory (HNET, similar to HQET)** for nucleus.
- Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \mathcal{L}_{\text{Max}} + \mathcal{L}_{\text{NRQED}} + \mathcal{L}_{\text{HNET}} + \delta\mathcal{L}_{\text{contact}} \quad (1)$$

where

$$\mathcal{L}_{\text{Max}} = -\frac{1}{4}d_\gamma F_{\mu\nu}F^{\mu\nu} + \dots,$$

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\nabla \cdot \mathbf{E}]}{8m^2} + \dots \right\},$$

$$\mathcal{L}_{\text{HNET}} = N^\dagger iD_0 N + \dots,$$

$$\delta\mathcal{L}_{\text{contact}} = \frac{c_4}{m^2} \psi^\dagger \psi N^\dagger N + \dots,$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: Construct EFT

- Use **non-relativistic QED (NRQED)** for electron and **heavy nucleus effective theory (HNET, similar to HQET)** for nucleus, **keep only Coulomb potential**.
- Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \cancel{\mathcal{L}_{\text{Max}}} \mathcal{L}_{\text{Coul}} + \mathcal{L}_{\text{NRQED}} + \mathcal{L}_{\text{HNET}} + \delta\mathcal{L}_{\text{contact}} \quad (1)$$

where

$$\mathcal{L}_{\text{Coul}} = \frac{1}{2} (\nabla A^0)^2,$$

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\nabla \cdot \mathbf{E}]}{8m^2} + \dots \right\},$$

$$\mathcal{L}_{\text{HNET}} = N^\dagger iD_0 N + \dots,$$

$$\delta\mathcal{L}_{\text{contact}} = \frac{c_4}{m^2} \psi^\dagger \psi N^\dagger N + \dots,$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: OPE

- Operator Product Expansion (OPE): The limit when product of local operators at different points approach each other.

$$T\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\mathcal{O}}(x^{\mu})[\mathcal{O}(0)]_R \quad (2)$$

Correct OPE relation in coordinate space

$$\psi(\mathbf{r})N(\mathbf{0}) = (1 - mZ\alpha|\mathbf{r}|) [\psi N](\mathbf{0}) + (1 - mZ\alpha|\mathbf{r}|/2)\mathbf{r} \cdot [\nabla\psi N](\mathbf{0}) + \dots$$

Correct OPE relation in momentum space

$$\begin{aligned} \tilde{\psi}(\mathbf{q})N(\mathbf{0}) &\equiv \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \psi(\mathbf{r})N(\mathbf{0}) \\ &= \frac{8\pi Z\alpha m}{\mathbf{q}^4} [\psi N](\mathbf{0}) - \frac{16i\pi Z\alpha m}{\mathbf{q}^6} \mathbf{q} \cdot [\nabla\psi N](\mathbf{0}) + \dots \end{aligned}$$

Reproduce wave functions near the origin

- With operator definition of the wave-functions

$$\Psi_{nlm}(\mathbf{r}) = \langle 0 | \psi(\mathbf{r}) N(\mathbf{0}) | nlm \rangle \quad (3)$$

Wave-function origin (Schrödinger equation)

$$R_{n0}(r) = R_{n0}(0) \left[1 - \frac{r}{a_0} + \mathcal{O}(r/a_0)^2 \right] \quad (4)$$

- Add **relativistic corrections** in OPE with higher order Lagrangian to account for the logarithms in **relativistic wave-functions (Klein-Gordon, Dirac)**.
- Use renormalization group equation to reproduce all leading logarithms.

Meson-meson Scattering in 1+1-d

QCD

't Hooft equation

- Large-N Expansion
- In 1+1-d, **ONLY PLANAR DIAGRAM!!!**

Steps:

1. Obtain mesons' 't Hooft wave-functions with 't Hooft equation (Fig 1).
2. Obtain effective meson-meson vertex function with Bethe-Salpeter equation (Fig 2).
3. Calculate meson-meson scattering amplitude with said vertex functions and wave-functions.

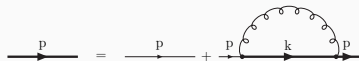


Figure 1: The Dyson-Schwinger equation for the quark self-energy.

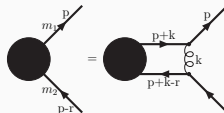


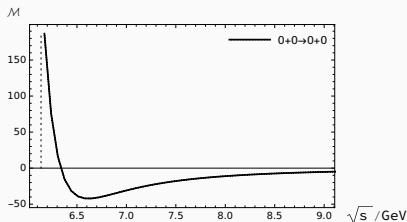
Figure 2: The Bethe-Salpeter equation for the $q\bar{q}$ bound state.

't Hooft equation

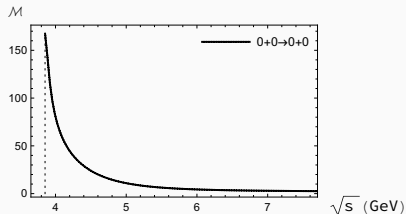
$$\mu^2 \varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x} \right) \varphi(x) - P \int_0^1 dy \frac{\varphi(y)}{(x-y)^2}. \quad (5)$$

μ is the mass of the meson, α_i is rescaled quark mass, P marks principle value.

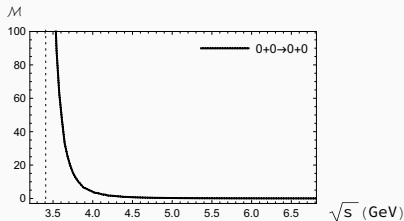
Results (NO INDICATION OF TETRAQUARK!!!)



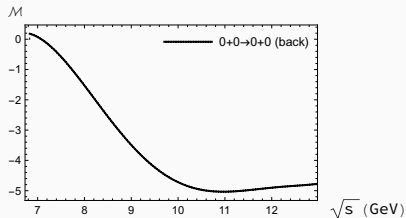
(a) Amplitudes for the contact term in $A(c\bar{c}) + B(c\bar{c}) \rightarrow C(c\bar{c}) + D(c\bar{c})$.



(b) Amplitudes for the contact term in $A(c\bar{s}) + B(c\bar{s}) \rightarrow C(c\bar{s}) + D(c\bar{s})$.



(c) Amplitudes for the contact term in $A(c\bar{u}) + B(c\bar{d}) \rightarrow C(c\bar{u}) + D(c\bar{d})$.

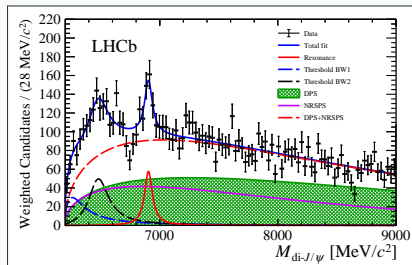


(d) Amplitudes for the contact term in $A(c\bar{d}) + B(b\bar{s}) \rightarrow C(b\bar{d}) + D(c\bar{s})$ with particle B moving backwards. No near-threshold enhancement.

Fragmentation Production of Fully-heavy Tetraquark at LHC

Factorization theorem for $T_{4c/b}$ production

- LHCb discovered a narrow structure near 6.9 GeV in the di- J/ψ invariant mass spectrum ($> 5\sigma$): $X(6900)$.
- Strong candidate for fully-charmed tetraquark.



- QCD factorization theorem for fully-heavy tetraquark ($T_{4c/b}$) exclusive production at high- p_T

$$\begin{aligned}
 d\sigma \left(pp \rightarrow T_{4c/b}(p_T) + X \right) &= \sum_i \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz f_{a/p}(x_a, \mu) f_{b/p}(x_b, \mu) \\
 &\quad \times d\hat{\sigma}(ab \rightarrow i(p_T/z) + X, \mu) D_{i \rightarrow T_{4c/b}}(z, \mu) + \mathcal{O}(1/p_T).
 \end{aligned}
 \tag{6}$$

- Dominate partonic channel is $gg \rightarrow gg$, rather than $gg \rightarrow q\bar{q}$.

Fragmentation Function

Collins-Soper definition of fragmentation function:

$$D_{g \rightarrow T_{4c}}(z, \mu) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ (N_c^2 - 1) (d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+ x^-} \\ \times \sum_X \langle 0 | G_c^{+\mu}(0) \mathcal{E}^\dagger(0, 0, \mathbf{0}_\perp)_{cb} | T_{4c}(P) + X \rangle \langle T_{4c}(P) + X | \mathcal{E}(0, x^-, \mathbf{0}_\perp)_{ba} G_a^{+\nu}(0, x^-, \mathbf{0}_\perp) | 0 \rangle$$

□ NRQCD factorization:

$$D_{g \rightarrow H}(z) = \sum_n d_n(z) \langle 0 | \mathcal{O}_n^H | 0 \rangle$$

- Perturbative matching to determine short distance coefficients.
- Use wave-function origin (S-wave) from potential models to determine long range matrix elements in order to yield a phenomenological result.
- More details in Jia-Yue Zhang's talk this afternoon.

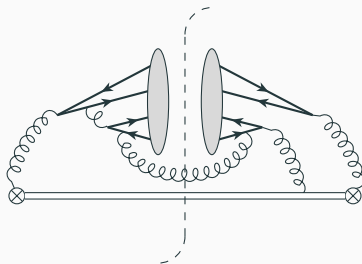
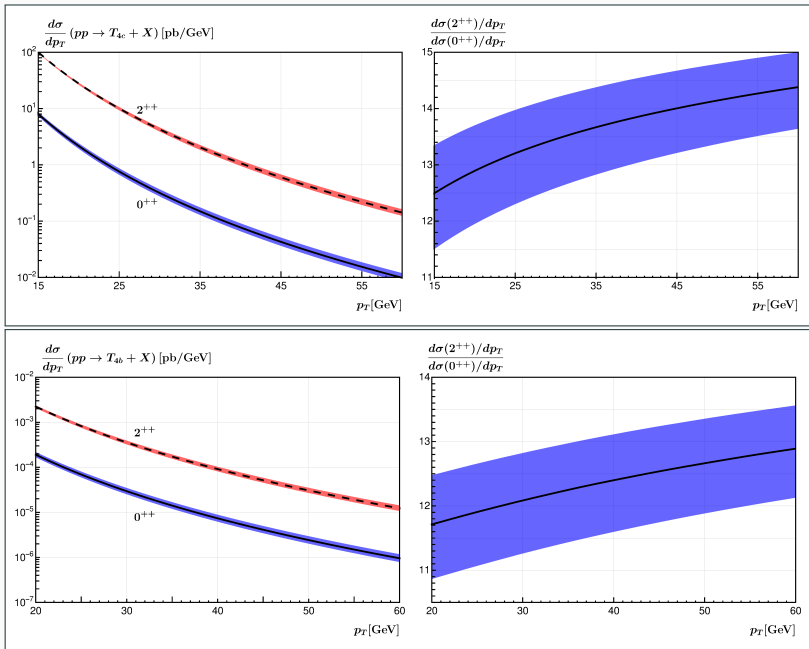


Figure 3: A representative Feynman diagram for the fragmentation function of gluon into T_{4c} . The grey blob indicates the C -even tetraquark. Horizontal double line denotes the eikonal line.

Phenomenology for $T_{4c/b}$ production at LHC



- 2^{++} cross section is about 10 times larger than 0^{++} .
- We obtain the yields of the accumulated event number for T_{4c} at HL-LHC are a hundred million for 0^{++} and 8 hundreds million for 2^{++} (with integrated luminosity 3000 fb^{-1}).
- The prediction for T_{4b} is highly suppressed, mainly due to the relative larger bottom mass suppression.
- The total cross section we obtained is unreliable mainly due to the fact that fragmentation only works at high- p_T , and our integration is done within approximately $15 \leq p_T \leq 60 \text{ GeV}$.

Publications






Publications?

- ▶ Huang, Y., Jia, Y., & Yu, R. (2018a). Deciphering the coalescence behavior of Coulomb-Schrödinger atomic wave functions from operator product expansion.arXiv 1809.09023 (**rejected by PRL, waiting for resubmission**)
- ▶ Huang, Y., Jia, Y., & Yu, R. (2018b). Near-the-origin divergence of Klein-Gordon wave functions for hydrogen-like atoms and operator product expansion.arXiv 1812.11957 (**Submitted to PRD, referee comments received**)
- ▶ Huang, Y., Jia, Y., & Yu, R. (2019). Near-the-origin divergence of Dirac wave functions of hydrogen and operator product expansion.arXiv 1901.04971 (**rejected by PRL, waiting for appeal**)
- ▶ Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391 (**Submitted to PRD and received positive response.)**
- ▶ Feng, F., Huang, Y., Jia, Y., Sang, W.-L., Xiong, X., & Zhang, J.-Y. (2020). Fragmentation production of fully-charmed tetraquarks at lhc.arXiv 2009.08450 (**To be submitted to PRL**)

Questions?

Backup

References

-  Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391.
-  Feng, F., Huang, Y., Jia, Y., Sang, W.-L., Xiong, X., & Zhang, J.-Y. (2020). Fragmentation production of fully-charmed tetraquarks at lhc.arXiv 2009.08450.
-  Huang, Y., Jia, Y., & Yu, R. (2018a). Deciphering the coalescence behavior of Coulomb-Schrödinger atomic wave functions from operator product expansion.arXiv 1809.09023.
-  Huang, Y., Jia, Y., & Yu, R. (2018b). Near-the-origin divergence of Klein-Gordon wave functions for hydrogen-like atoms and operator product expansion.arXiv 1812.11957.
-  Huang, Y., Jia, Y., & Yu, R. (2019). Near-the-origin divergence of Dirac wave functions of hydrogen and operator product expansion.arXiv 1901.04971.