Standard Model Effective Field Theory

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Content

Revisit EFT

Basic idea of effective theory: Low energy approximation of underlying high energy theory.

Examples:

- Astronomy study
- Classical electrodynamics
- Rayleigh scattering
- Hydrogen atom spectra
- ...

Historical review:

- Field-theoretic appearance of current algebra

  From linear σ model to direct derivation
- Wilson's renormalization group
- Applications of EFT in strong interaction

Basic properties and applications of EFT

Power counting:

$$\mathcal{L}_I = \sum_{i=0}^{\infty} \frac{g_i}{\Lambda_i} \hat{O}_i$$

Accuracy increases with higher i.

Renormalizability: Even if the underlying theory is renormalizable, once a finite cutoff is introduced it becomes necessary to introduce every possible interaction, renormalizable or not, to keep physics strictly cutoff independent. From this point of view, it doesnt make much difference whether the underlying theory is renormalizable or not.

Matching.

Famous examples:

- Euler-Heisenberg Lagrangian
- NRQED
- Chiral perturbation theory

Motivation

- SM is not the fundermantal theory.
- LHC: the last piece of standard model has been found, yet no new physics.
- Which directions in the parameter space of deformations of the SM are still unprobed?
- Provide a useful guidance for future experiments.
- Provide insight into some of the many long standing experimental observations that remain unexplained.
- Place constraints on specific UV models.
- Estimate the physics reach (i.e. the largest Λ) of specific UV models.

Standard Model Effective Field Theory

A model independent approach of BSM physics.

Two different ways of BSM: search for new particles, or new interactions of known particles.

SMEFT is of the latter.

Principles of constructing such theory:

- Do not break the framework of SM itself, and respect all axioms of QFT (since it must be a QFT).
- Respect $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. (Unbroken above Higgs mass.)
- Assume that the new interactions decouple from the Standard Model in the limit that the energy scale that characterizes the new interactions goes to infinity.
- Ideally new interaction should be able to calculate any process at any order in both the Standard Model interactions and the new interactions.

Basic idea of SMEFT: Consider Standard Model itself as an effective field theory, and add higher dimensional opeartors to mimic the effect of BSM physics (connect UV models to EW and/or Higgs observables).

The effective Lagrangian is of the form

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{i=0}^{\infty} \frac{c_i}{\Lambda_i^2} \hat{O}_i + \dots$$

- c_i is a dimensionless coefficient. $\hat{O}_i$ is an opeartor of mass dimension six constructed from SM fields. The ellipsis stands for higher mass dimensions.
- Energy scale Λ, interpreted as the scale of new physics and that we assume to be greater than the Higgs mass.
- New interactions are constructed from Standard Model fields and have a coefficient proportional to an inverse power of Λ; thus the Standard Model is recovered in the limit $\Lambda \to \infty$.
- The new interactions are compatible with the calculation of Standard Model radiative corrections. The new interactions may be calculated to any desired order in $1/\Lambda$, with the caveat that the introduction of additional interactions may be necessary at each order in $1/\Lambda$.

Benefit of such theory:

- The possiblity of SM containing non-renormalizable interactions (have coefficients proportional to inverse power of Λ) is therefore supressed below Λ.
- Give guidance of the types of new interactions. (e.g. consider dominant term is of $1/\Lambda$ order, only one possible interaction which is the one gives rise to Majorana masses for neutrinos; but to conserve Baryon and Lepton number B & L, all interactions must proportion to even order of $1/\Lambda$.)
- Easier and, in pratical purposes, usually more accurate than the computation of specific UV models (which may involve loop-order).

An example

Consider a new Z' boson coupled to SM fermions.

At low energy, it's a four fermion interaction.

Each fermion gives mass dimension of 3/2, so it's a dimension-six opeartor.

Similar to the SM Z boson, the vertex is proportional to $\frac{1}{m_{Z'}^2}$, which makes $m_{Z'}$ the energy scale.

$\Longrightarrow$ An effective field theory is not supposed to be valid at any arbitrarily high energy. In this case it's valid below $m_{Z'}$.

Beyond that energy, the new Z' particle must be included explicitly.

Note that in this example, we can also see that measuring the coefficient $\frac{c_i}{\Lambda_i}$ wouldn't give the energy scale Λ, it only gives the overall coupling G_F'.

Construction of dimension-six opeartors

First we study the dimension-five terms (and why only the neutrino mass and mixing part survived).

Applying gauge symmetry of SM, only one possiblity survived.

$$Q_{\nu\nu} = \epsilon_{jk}\epsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n = (\tilde{\varphi}^\dagger l_p)^T C(\tilde{\varphi}^\dagger l_r)$$

where $\tilde{\varphi}^j = \epsilon_{jk}(\varphi^k)^*$, C is the charge conjugate matrix. Clearly it's a dimension-five opeartor, after SSB it provides neutrino Majorana masses and mixing.

For dimension-six opeartors, they can be divide into CP-odd part (all terms containing $\tilde{G}$ and such) and CP-even part. Applying gauge symmetry and EOM, all possible terms are listed in the following tables.

Experiment constrains: B L conservation, $D = 6$ terms relatively small, only CP even part for tree level, etc.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}{}_\mu^I\,\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}{}_\mu^I\,\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Table 3: Four-fermion operators.

How to use SMEFT?

The procedure for any EFT:

1. Matching: matching some UV models onto the EFT at Λ scale
2. Running: running the coupling to weak scale, using RG equation for EFT
3. Mapping: mapping the Wilson coefficients onto observables

Convenient way of matching process: covariant derivative expansion (CDE)

When to choose an opeartor basis?

In matching stage, one should integrate out the massive modes and obtain the effective actions which contains higher-dimension operators.

In the running stage, one can choose the UV-obtained basis or switch to other ones.

After having the parameterized obserables, we can compare it with experimental values, which gives some constrains on parameters.

Summary

- SMEFT is an efficient and systematic way of mimicing the effect of new physics.
- The idea is to take SM as an effective theory where the renormalizable SM might just be the first term of the whole theory.
- The constrains of SMEFT are only some given symmetry requests, Lorentz invariance and the basic property of field theory.
- By integrating out the heavy particles, the effecitve Lagrangian is obtained. With RG equation, we can have EW and Higgs observables at any scale lower than the effecitve limit.

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