RI/MOM Scheme and Quasi PDF

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Outlines

RI/MOM Scheme

RI/MOM Scheme[Martinelli et al., 1995]

Momentum Subtraction Schemes (MOM)

Recall procedure in continuum perturbation theory:

- example: renormalisation of the pseudoscalar density $P^a(x) = \overline{\psi}(x)\gamma_5 \frac{1}{2}\tau^a \psi(x)$:
- Correlation functions in momentum space with external quark states:

$$
\begin{aligned}
\left\langle \tilde{\psi}(p)\tilde{\overline{\psi}}(q) \right\rangle &= (2\pi)^4 \delta(p+q) S(p) \qquad \text{quark propagator} \\
\left\langle \tilde{\psi}(p)\tilde{P}^a(q)\tilde{\overline{\psi}}(p') \right\rangle &= (2\pi)^4 \delta(p+q+p') S(p) \Gamma_P^a(p,q) S(p+q)\,,
\end{aligned}
$$

- At tree-level:

$$
\begin{aligned}
\Gamma_P^a(p,q)|_{\mathrm{tree}} &= \gamma_5 \tfrac{1}{2}\tau^a, \\
\Rightarrow \quad \tfrac{1}{4}\sum_{b=1}^{3} \mathrm{tr}\left\{ \gamma_5 \tau^b \Gamma_P^a(p,q)|_{\mathrm{tree}} \right\} &= 1
\end{aligned}
$$

- Renormalised fields:

$$\psi_{\mathrm{R}} = Z_\psi \psi, \qquad \overline{\psi}_{\mathrm{R}} = Z_\psi \overline{\psi}, \qquad P^a_{\mathrm{R}} = Z_{\mathrm{P}} P^a$$

$\Rightarrow$ renormalised vertex function:

$$\Gamma^a_{P,\mathrm{R}}(p,q) = Z_{\mathrm{P}} Z_\psi^{-2} \Gamma^a_P(p,q)$$

- typical MOM renormalisation condition (quark masses set to zero):

$$\Gamma^a_{P,\mathrm{R}}(p,0)|_{\mu^2=p^2} = \gamma_5 \tfrac{1}{2}\tau^a \qquad \Rightarrow \qquad Z_{\mathrm{P}} Z_\psi^{-2}$$

- equivalently using "projector":

$$\tfrac{1}{4}\sum_{b=1}^{3} \mathrm{tr}\,\left\{\gamma_5 \tau^b\, \Gamma^a_{P,\mathrm{R}}(p,0)|_{\mu^2=p^2}\right\} = 1$$

- Determine Z_ψ either from propagator or use MOM scheme for vertex function of a conserved current

$$\Gamma_{V,\mathrm{R}}(p,q) = Z_\psi^{-2} \Gamma_V(p,q)$$

Summary: MOM schemes in the continuum

- Renormalisation condions are imposed on vertex functions in the gauge fixed theory with external quark, gluon or ghost lines
- The vertex functions are taken in momentum space.
- A particular momentum configuration is chosen, such that the vertex function becomes a function of a single momentum p; quark masses are set to zero
- MOM condition: a renormalised vertex function at subtraction scale $\mu^2 = p^2$ equals its tree-level expression
- Can also be used to define a renormalised gauge coupling: take vertex function of either the 3-gluon vertex, the quark-gluon vertex or the ghost-gluon vertex.
- Renormalisation constants depend on the chosen gauge! Need wave function renormalisation for quark, gluon and ghost fields.

RI/MOM Schemes (RI = Regularisation Independent; MOM = Momentum Subtraction)

[Martinelli et al '95]: mimick the procedure in perturbation theory:

- choose Landau gauge

$$\partial_\mu A_\mu = 0$$

  can be implemented on the lattice by a minimisation procedure
- RI/MOM schemes are very popular: many major collaborations use it because
  - it is straightforward to implement on the lattice; many improvements over the years regarding algorithmic questions
  - it can be used on the very same gauge configurations which are produced for hadronic physics
- Regularisation Independence (RI) means: correlation functions of a renormalised operator do not depend on the regularisation used (up to cutoff effects).

RI/MOM schemes, discussion

- Suppose we have calculated a renormalised hadronic matrix element of the multiplicatively renormalisable operator O

$$\mathcal{M}_O(\mu) = \lim_{a\to 0} \langle h|O_{\rm R}(\mu)|h'\rangle$$

- Provided μ is in the perturbative regime, one may evaluate the MOM scheme in continuum perturbation theory and evolve to a different scale:

$$\begin{aligned}
\mathcal{M}_O(\mu') &= U(\mu',\mu)\mathcal{M}_O(\mu), \\
U(\mu',\mu) &= \exp\left\{\int_{\bar{g}(\mu)}^{\bar{g}(\mu')} \frac{\gamma_O(g)}{\beta(g)} \mathrm{d}g\right\}
\end{aligned}$$

- N.B. Continuum perturbation theory is available to 3-loops in some cases!

RI/MOM schemes, what could go wrong?

- The scale μ could be too low; need to hope for a "window"

$$\Lambda_{\mathrm{QCD}} \ll \mu \ll a^{-1}$$

In practice scales are often too low: non-perturbative effects (e.g. pion poles, condensates) are then eliminated by fitting to expected functional form (from OPE in fixed gauge);

$\Rightarrow$ errors are difficult to quantify!

- Gribov copies: the (Landau) gauge condition does not have a unique solution on the full gauge orbit
- Perturbative calculations are made using
  - infinite volume
  - vanishing quark masses

$\Rightarrow$ inconvenient for numerical simulations especially in full QCD.

- Wilson quarks: a priori cutoff effects are $\mathrm{O}(a)$ even in on-shell $\mathrm{O}(a)$ improved theory.

END

Questions?

Backup

References

Martinelli, G., Pittori, C., Sachrajda, C. T., Testa, M., and Vladikas, A. (1995). A General method for nonperturbative renormalization of lattice operators. *Nucl. Phys.*, B445:81–108.