$\bar{c}\gamma^\mu c$ matrix element

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1 $^3S_1$

Ignore the overall factor:

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3S_1\right\rangle = \int \mathrm{d}\Omega\, \mathrm{tr}[\Pi_1\gamma^\mu] \propto \sqrt{2}\pi(\frac{m}{3E}+\frac{2}{3})\epsilon^\mu$$

2 $^3D_1$

The matrix element reads:

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle = \int \mathrm{d}\Omega \sum_{\lambda_1\lambda_2 S_z m} \mathrm{tr}\{\Pi_1\gamma^\mu\} \left\langle 1J_z\middle|2m;1S_z\right\rangle Y_{2m}(\theta,\phi)$$

while the trace part is the same as $^3S_1$:

$$\mathrm{tr}\{\Pi_1\gamma^\mu\} = \frac{\sqrt{2}p^\mu(p\cdot\epsilon)}{E(E+m)} + \epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)} = \frac{1}{\sqrt{2}}(0,1,-i,0), \epsilon^{(0)} = (0,0,0,1), \epsilon^{(+)} = \frac{1}{\sqrt{2}}(0,-1,-i,0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} 0 & \frac{2\sqrt{2\pi}p^2}{3E(m+E)} & -\frac{2i\sqrt{2\pi}p^2}{3E(m+E)} & 0 \\ 0 & 0 & 0 & \frac{4\sqrt{\pi}p^2}{3E(m+E)} \\ 0 & -\frac{2\sqrt{2\pi}p^2}{3E(m+E)} & -\frac{2i\sqrt{2\pi}p^2}{3E(m+E)} & 0 \end{pmatrix}$$

and the decay constant is $\frac{4\sqrt{\pi}p^2}{3E(E+m)}$ where $p = \mathbf{p} = E^2 - m^2$.