Scalar QED

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1 Hydrogen Wavefunction Divergence in Klein-Gordon Equation and Schrödinger Equation

2 Non-relativistic Scalar QED (NRSQED) Matching

2.1 Feynman Rules

2.1.1 Scalar QED (SQED)

Lagrangian

$$\mathcal{L}_{SQED} = |D_\mu \phi|^2 - m^2 |\phi|^2 + \Phi_v^* i v \cdot D \Phi_v \tag{1}$$

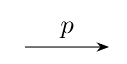
with

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

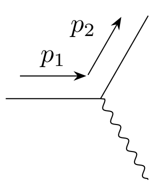
and

$$D_\mu \Phi_v = \partial_\mu \Phi - iZeA_\mu \Phi_v$$

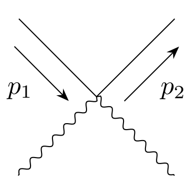
But note that no $\mathbf{A}$ can appear in actual calculation because here only static scalar potential exists. And the Feynman rules



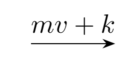
$$= \frac{i}{p^2 - m^2 + i\epsilon}$$



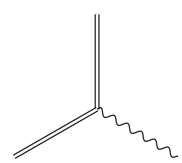
$$= -ie(p_1^\mu + p_2^\mu)$$



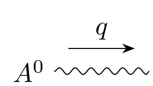
$$= 2ie^2 g^{\mu\nu}$$



$$= \frac{i}{v \cdot k}$$



$$= iZev^\mu$$



$$= \frac{i}{\mathbf{q}^2}$$

2.1.2 NRSQED

Lagrangian

$$\mathcal{L}_{NRSQED} = \varphi^* \left(iD_0 + \frac{\mathbf{D}^2}{2m} \right) \varphi + \delta\mathcal{L} + \Phi_v^* i v \cdot D \Phi_v \tag{2}$$

with the same notation above. Here $\mathbf{D} = \nabla - ie\mathbf{A}$.

Since we need to match it to $\mathcal{O}(v^2)$ order

$$\delta\mathcal{L} = (D_0\varphi)^*(D_0\varphi) = \frac{\dot{\varphi}^*\dot{\varphi}}{2m} + \frac{e^2\varphi^*\varphi A_0^2}{2m} - \frac{ie}{2m}A_0(\varphi^*\dot{\varphi} - \dot{\varphi}^*\varphi) \tag{3}$$

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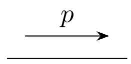
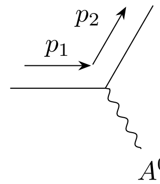
Feynman rules are also the same except for the scalar electron side which becomes



$$= \frac{i}{E - \frac{\mathbf{p}^2}{2m} + i\epsilon}$$



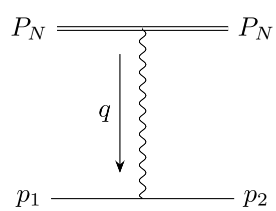
$$= -ie$$

We can ignore all interacting terms involving $\mathbf{A}$.

2.2 LO

2.2.1 SQED

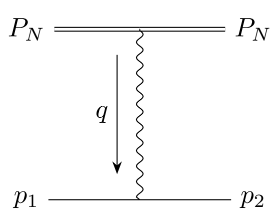
$$i\mathcal{M}^{(0)}_{SQED} =$$



$$= -e^2 v^0 \frac{i(p_1^0 + p_2^0)}{\mathbf{q}^2}$$

2.2.2 NRSQED

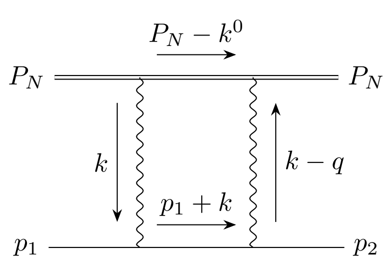
$$i\mathcal{M}^{(0)}_{NRSQED} =$$



$$= -e^2 v^0 \frac{i}{\mathbf{q}^2}$$

2.3 NLO

$$i\mathcal{M}^{(1)}_{SQED} =$$



$$= -e^2 v^0 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{i}{\mathbf{k}^2}$$