Meson-meson scattering in 1+1 Dimension

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Content

1+1-d QCD and 't Hooft model

Lagrangian:

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu\ i}^{\ \ \ j} G^{\mu\nu\ \ i}_{\ \ \ j} + \bar{q}^{ai}(i\gamma^{\mu} D_{\mu} - m_a) q_i^a, \tag{1}$$

where

$$\begin{aligned} G_{\mu\nu\ i}^{\ \ \ j} &= \partial_\mu A_i^{\ j}{}_{\nu} - \partial_\nu A_i^{\ j}{}_{\mu} + ig[A_\mu, A_\nu]_i^{\ j}, \\ D_\mu q_i^a &= \partial_\mu q_i^a + ig A_i^{\ j}{}_{\mu} q_j^a, \\ i, j &= 1, 2, ..., N_c, \qquad a = 1, 2, ..., N_f. \end{aligned} \tag{2}$$

Choose light-cone gauge condition

$$A_- = A^+ = 0, \tag{3}$$

where $A_- = \frac{1}{\sqrt{2}}(A^0 + A^1) = \frac{1}{\sqrt{2}}(A_0 - A_1)$.

Using Dyson-Schwinger equation and Bethe-Salpeter equation in large N_c limit we obtain the famous 't Hooft equation

$$\mu^2 \varphi(x) = (\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x})\varphi(x) - P\int_0^1 dy \frac{\varphi(y)}{(x-y)^2}. \tag{4}$$

Meson-meson scattering amplitude (Gou-ying Chen and Rui Yu)

The Bethe-Salpeter equation for the quark-antiquark scattering amplitude can be written as

$$\mathcal{T}(p,p';r) = -\frac{ig^2}{(p_- - p'_-)^2} + i4N_c g^2 \int \frac{d^2k}{(2\pi)^2}\frac{1}{(k_- - p_-)^2}\tilde{S}(k)\tilde{S}(k-r)\mathcal{T}(k,p';r), \quad (5)$$

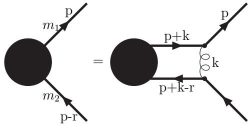

Figure: The Bethe-Salpeter equation of the bound state. Arrow lines are dressed quark propagators.

where $\tilde{S}(p)\gamma_+ = S(p)$. This equation has been solved (Callan, Coote and Gross, 1975) and the result is

$$\begin{aligned}\mathcal{T}(x,x';r) &= -\frac{ig^2}{r_-^2(x-x')^2} + \sum_n \frac{i}{r^2 - r_n^2}\left\{\varphi_n(x)\frac{g^2}{|r_-|}\sqrt{\frac{N_c}{\pi}}\left[\theta(x(1-x))\frac{2|r_-|}{\lambda} + \frac{\alpha_1}{x} + \frac{\alpha_2}{1-x} - \mu_n^2\right]\right\} \\ &\quad \times\left\{\varphi_n^*(x')\frac{g^2}{|r_-|}\sqrt{\frac{N_c}{\pi}}\left[\theta(x'(1-x'))\frac{2|r_-|}{\lambda} + \frac{\alpha_1}{x'} + \frac{\alpha_2}{1-x'} - \mu_n^2\right]\right\}, \quad (6)\end{aligned}$$

For process $A(q^a\bar{q}^b) + B(q^c\bar{q}^a) \to C(q^a\bar{q}^b) + D(q^c\bar{q}^a)$ (where a, b, c are different flavor indexes), the amplitude reads

$$i\mathcal{M} = (1+\mathcal{C})i\mathcal{M}_0,$$
$$i\mathcal{M}_0 = \theta(\omega_2-\omega_1)i4g^2\omega_1\int_0^1 dx\int_0^1 dy\,\frac{1}{(y\omega_1-\omega_2-x)^2}\varphi_A\Big(\frac{\omega_2-\omega_1+x}{\omega_2-\omega_1+1}\Big)\varphi_B(y)\varphi_C(x)\varphi_D\Big(\frac{y\omega_1}{\omega_2}\Big),$$

where

$$\omega_1 = \frac{r_{B-}}{r_{C-}}, \quad \omega_2 = \frac{r_{D-}}{r_{C-}}. \tag{7}$$

Here and in the following, we define the operation $(A \leftrightarrow C,\ \ B \leftrightarrow D,\ \ \omega_1 \to \frac{\omega_2}{1+\omega_2-\omega_1},\ \ \omega_2 \to \frac{\omega_1}{1+\omega_2-\omega_1})$ as $\mathcal{C}$. One can find that the final expression is infra-red safe, thus we postpone $\lambda \to 0$ in our final expression.

$A(q^a\bar{q}^b) + B(q^b\bar{q}^a) \to C(q^a\bar{q}^b) + D(q^b\bar{q}^a)$ reads

$$i\mathcal{M} = (1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0. \tag{8}$$

where the operation $\mathcal{P}$ is defined as $\mathcal{P} = (A \leftrightarrow B,\ \ C \leftrightarrow D,\ \ \omega_1 \to \frac{1+\omega_2-\omega_1}{\omega_2},\ \ \omega_2 \to \frac{1}{\omega_2})$.

$A(q^a\bar{q}^a) + B(q^a\bar{q}^a) \to C(q^a\bar{q}^a) + D(q^a\bar{q}^a)$ reads

$$i\mathcal{M} = (1+\mathcal{R})(1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0 + (1+\mathcal{R})i\mathcal{M}_1, \tag{9}$$

where

$$\begin{aligned} & i\mathcal{M}_1 \\ &= -(1+\mathcal{Q})\theta(1-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1\omega_2}{[(y-1)\omega_1+(1-x)\omega_2]^2}\varphi_A(\frac{x\omega_2}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(y\omega_1)\varphi_D(x) \\ &-(1+\mathcal{C})\theta(\omega_2-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1}{(y\omega_1-x)^2}\varphi_A(\frac{x+\omega_2-\omega_1}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(x)\varphi_D(\frac{(y-1)\omega_1+\omega_2}{\omega_2}) \\ &-(1+\mathcal{Q}+\mathcal{P}+\mathcal{C})\theta(\omega_2-\omega_1)\theta(\omega_1-1)i\frac{4\pi}{N_c}\int_0^1 dx\left[2r_{C+}r_{C-}+2r_{D+}r_{C-}+\frac{M_a^2}{x-\omega_1}+\frac{M_a^2}{x-1}\right. \\ &\left.-\frac{M_a^2}{x-\omega_1+\omega_2}-\frac{M_a^2}{x}\right]\times\varphi_A(\frac{x-\omega_1+\omega_2}{1+\omega_2-\omega_1})\varphi_B(x/\omega_1)\varphi_C(x)\varphi_D(\frac{x-\omega_1+\omega_2}{\omega_2}), \end{aligned}$$

and

$$\begin{aligned} \mathcal{R} &= (C\leftrightarrow D,\ \ \omega_1\to\frac{\omega_1}{\omega_2},\ \ \omega_2\to 1/\omega_2), \\ \mathcal{Q} &= (B\leftrightarrow C,\ \ A\leftrightarrow D,\ \ \omega_1\to 1/\omega_1,\ \ \omega_2\to\frac{1+\omega_2-\omega_1}{\omega_1}). \end{aligned} \tag{10}$$

Numerical Calculation (Yingsheng Huang)

Conclusion