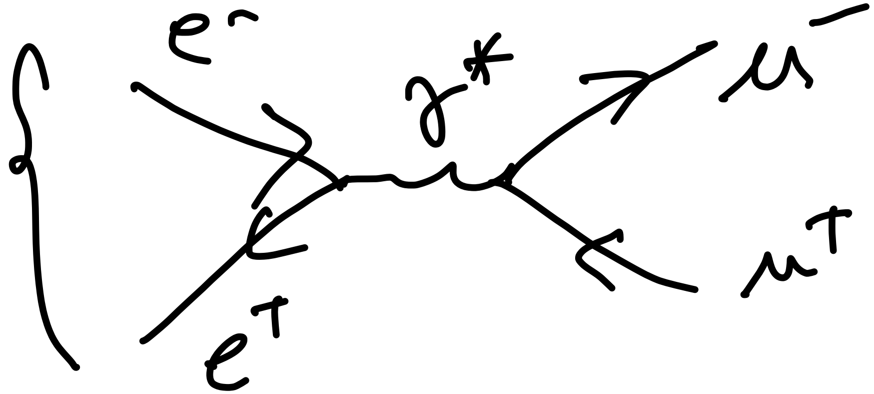
Parton Distribution Functions
&
Collider physics

e^-e^+ collider

1. Leptons $(e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau)$ are elementary particles

Consider $\sqrt{s}$



for $\sqrt{s} \ll M_Z^2$

QED interaction

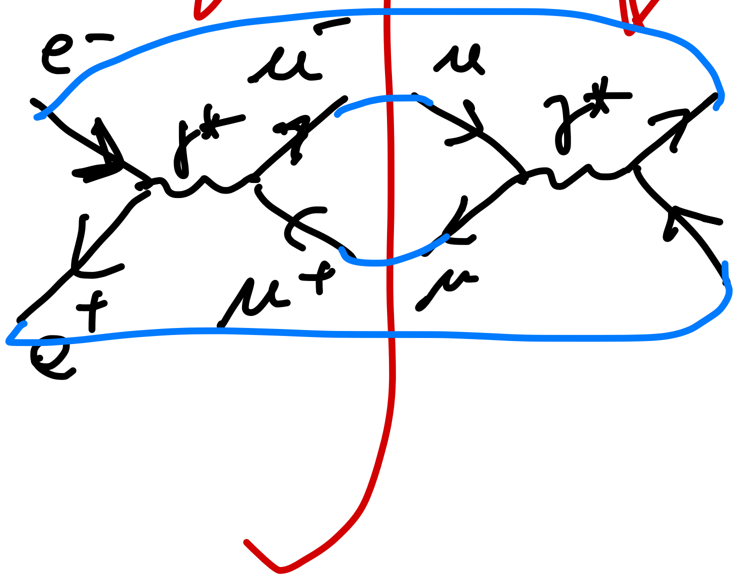
Cross section

$$\sigma \sim \frac{1}{2s} \int d\Phi_2 \, |\overline{\mathcal{M}}|^2$$

final state phase space

$$\mathcal{M} \, \mathcal{M}^*$$

$$\frac{d\sigma}{d\Omega}$$



QED (Quantum Electrodynamics)

① $\mathcal{L} \supset \bar{\psi}(i\not{\partial} - m)\psi$ ⟵ free electron

② To introduce interaction, replace

($U(1)_{em}$ generator gauge sym.)

$\not{\partial} = \partial_\mu \gamma^\mu \longrightarrow \not{D}_\mu = D_\mu \gamma^\mu$

$D_\mu = \partial_\mu + ieQ_e \mathbb{1} A_\mu$

Demand $U(1)_{em}$ Gauge invariance

universal coupling

electric charge of electron ($Q_e = -1$)

gauge boson

③ To make A_μ dynamical,

Introduce field tensor

$$F_{\mu\nu} = \frac{i}{e}[D_\mu, D_\nu] \equiv \frac{i}{e}(D_\mu D_\nu - D_\nu D_\mu)$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu$$

abelian gauge

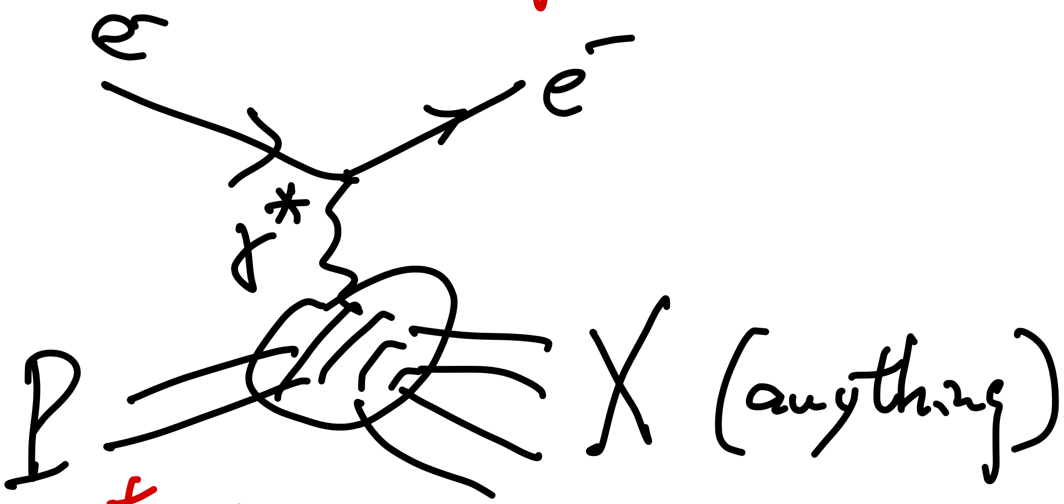
$$\mathcal{L} \supset F_{\mu\nu}F^{\mu\nu}$$

$\Rightarrow$ Complete the QED $\mathcal{L}$.

Electron-Proton Scattering

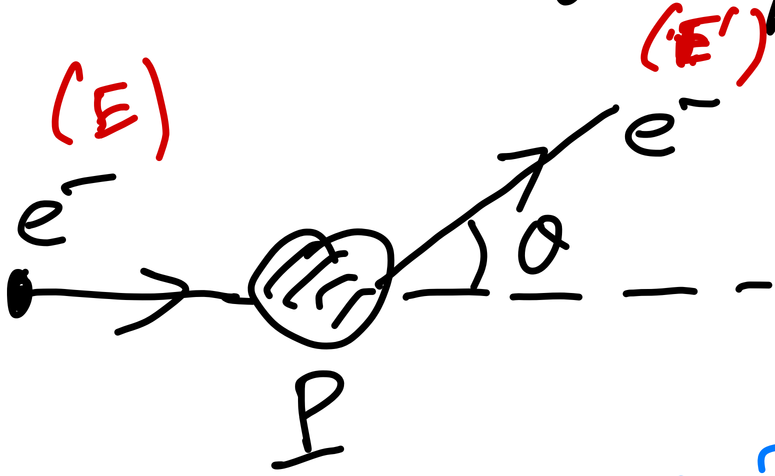
Deep Inelastic Scattering (DIS) process

1.



proton is a composite object, not an elementary particle.

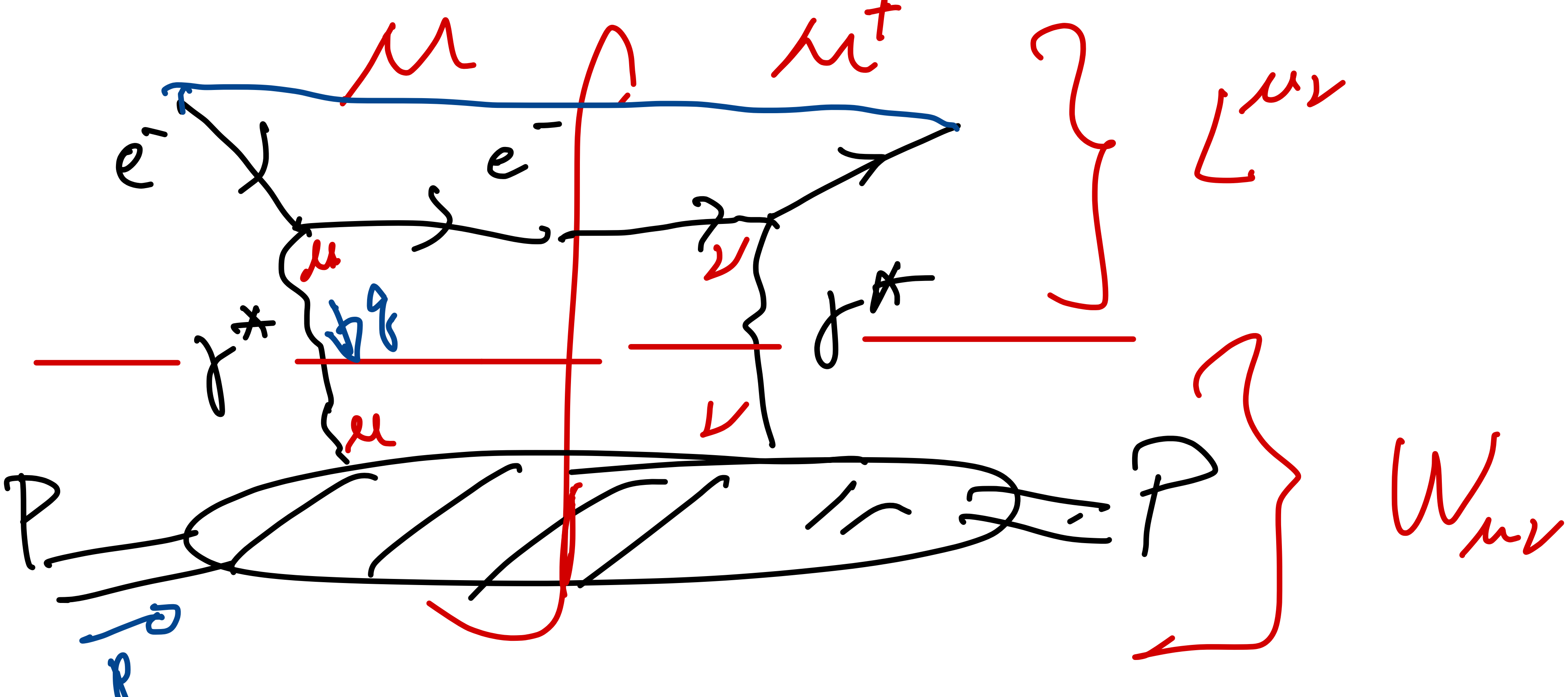
Fixed-target exp.



inclusive measurement

Three kinematic variables $\Rightarrow$ measure only lepton kinematics

(E, E', θ)



$$\sigma \sim L^{\mu\nu} W_{\mu\nu}$$

Quark Model

$Q_u = \frac{2}{3}$

$P = (\underline{u}\,\underline{u}\,\underline{d})$

$Q_d = -\frac{1}{3}$

$n = (\underline{d}\,\underline{d}\,\underline{u})$

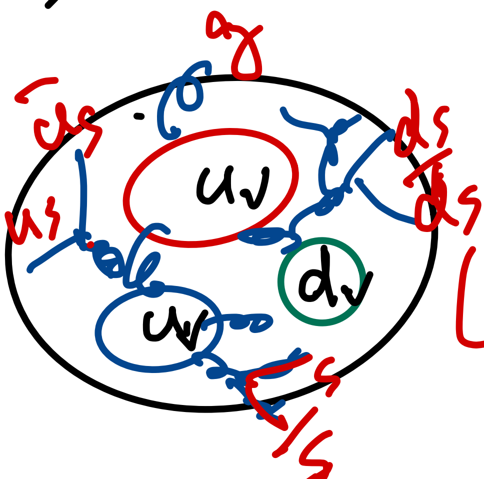
They are constituent quarks for static proton, $m_{\underline{u}} \simeq m_{\underline{d}} \simeq \frac{1}{3} m_{proton}$

$\simeq 300$ MeV

$m_{proton} \sim 1$ GeV

However, in QCD (Quantum Chromodynamics) $\mathcal{L}$

P



Quarks have colors (3) and (8) gluons bound them together

$SU(3)_{color}$

① $\mathcal{L}_{QCD} \supset \bar{\psi}(i\not{\partial} - m_q)\psi$

quarks are not "free" particles

(Except ~~top quark~~)

Top quark is a bare, free, quark.

Current quark mass

$m_u \simeq 2.2\ \text{MeV}$

$m_d \simeq 4.7\ \text{MeV}$

$m_s \simeq 96\ \text{MeV}$

② To Introduce interaction, replace

$$\partial_\mu \longrightarrow (D_\mu)_{ij} = \delta_{ij}\partial_i - ig \sum_a (T^a)_{ij} A^a_\mu$$

$a = 1, 2, \cdots 8$

universal coupling

$SU(3)_c$ generators

$\left(T^a \equiv \frac{\lambda^a}{2} \right)$

gauge bosons (gluons)

$i, j = 1, 2, 3$

$a = 1, 2, 3 \cdots 8$

③ Make A_μ^a dynamical

Introduce field tensor

$$\mathbb{F}_{\mu\nu} \equiv T^a F^a_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

$$\mathbb{A}_\mu \equiv T^a A^a_\mu$$

$$= \partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu - \underbrace{ig[\mathbb{A}_\mu, \mathbb{A}_\nu]}_{\text{non-abelian gauge group}},$$

$SU(3)_c$

$$T^a \equiv \frac{\lambda^a}{2}$$

then,

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$[T^a, T^b] \equiv T^a T^b - T^b T^a = i f_{abc} T^c$$

$SU(3)_c$ structure constant

$$\mathcal{L} \supset \mathrm{Tr}(\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu})$$

But, there is no free (u, d, s, c, b) quarks.

(Top Quark is a bare, free, quark)

Thus, they have to form

Hadrons $\begin{cases} (q\bar{q}) & \text{Mesons} \quad \pi, K, \dots \\ (qqq) & \text{Baryons} \quad p, n, \dots \end{cases}$

Pions are the lightest hadrons, with $m_{\pi^0} \simeq 135\,\text{MeV}$

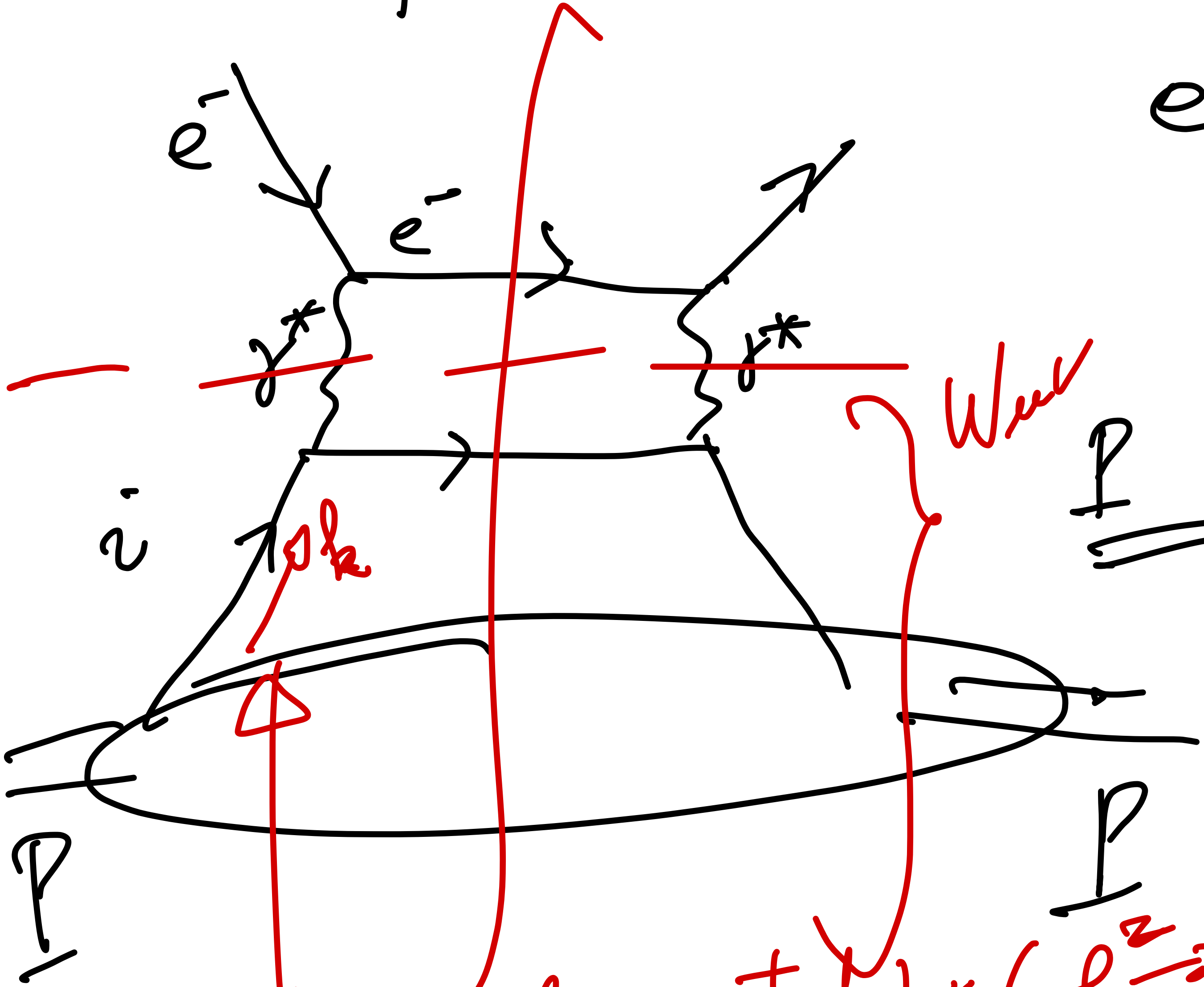
$m_{\pi^\pm} \simeq 140\,\text{MeV}$

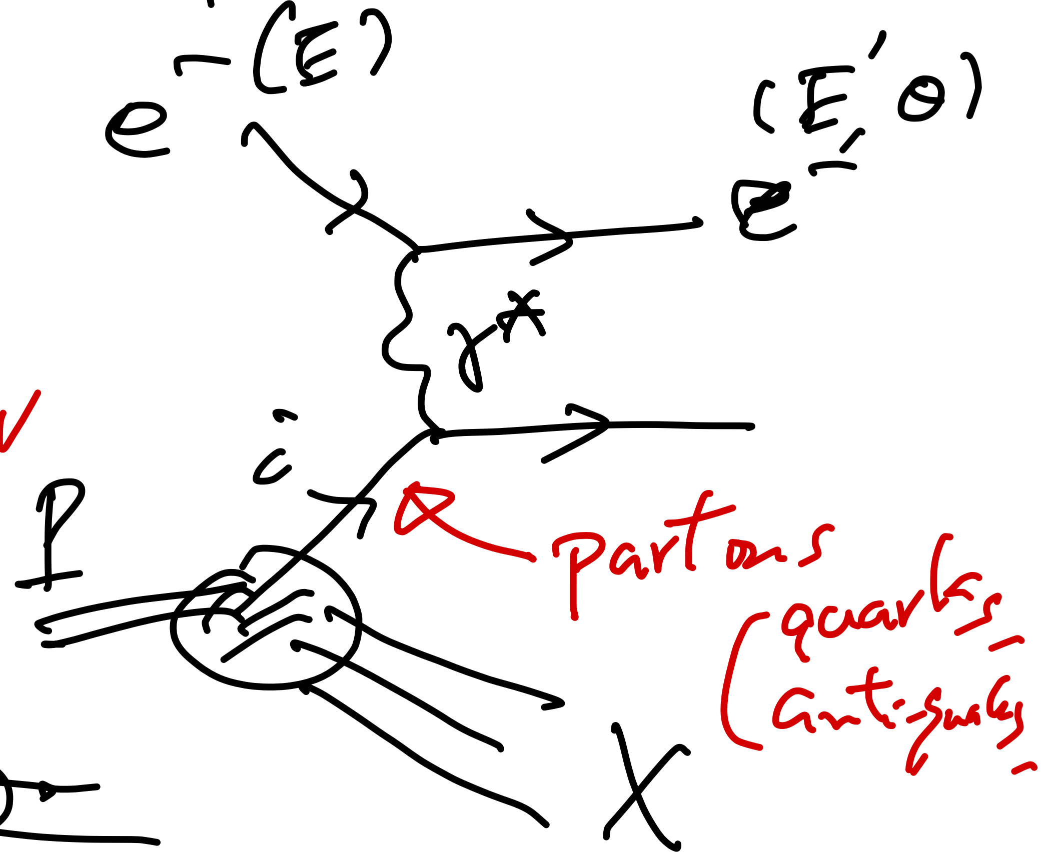
$\Rightarrow$ QCD has both non-perturbative & perturbative aspects.

$\uparrow$ characterized by Λ_{QCD} $(\hbar c \simeq 197\,\text{MeV.fm})$

$\Rightarrow$ (non-pert. scale $\sim 200\,\text{MeV}$)

DIS process in "Naive" Parton Model

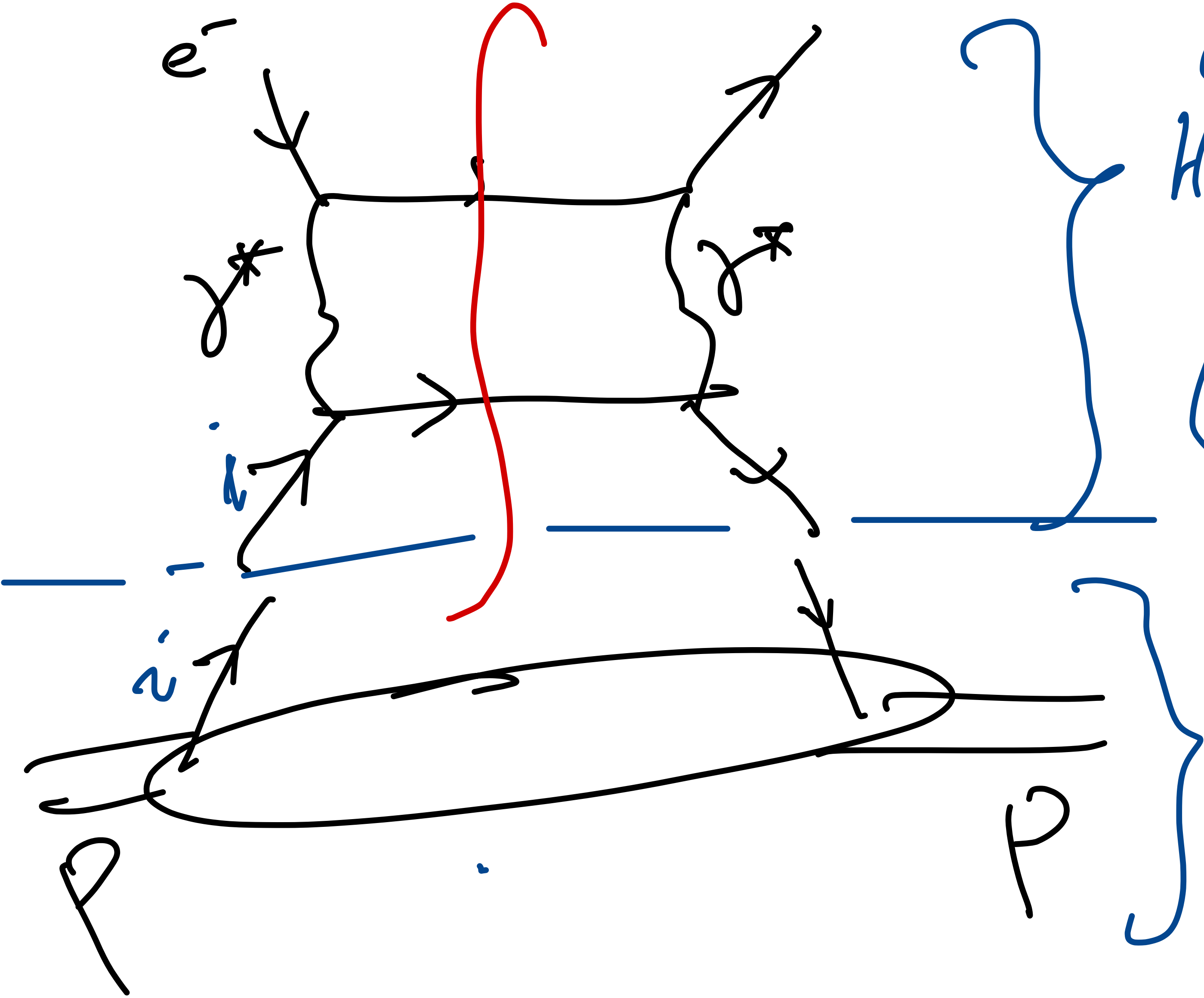




dominated by ($k^2 \to 0$) contribution, for the propagator $\frac{1}{k^2}$

$\Longrightarrow$ "Pinch" approximation

(massless quarks, $m_q^2 \ll Q^2$)



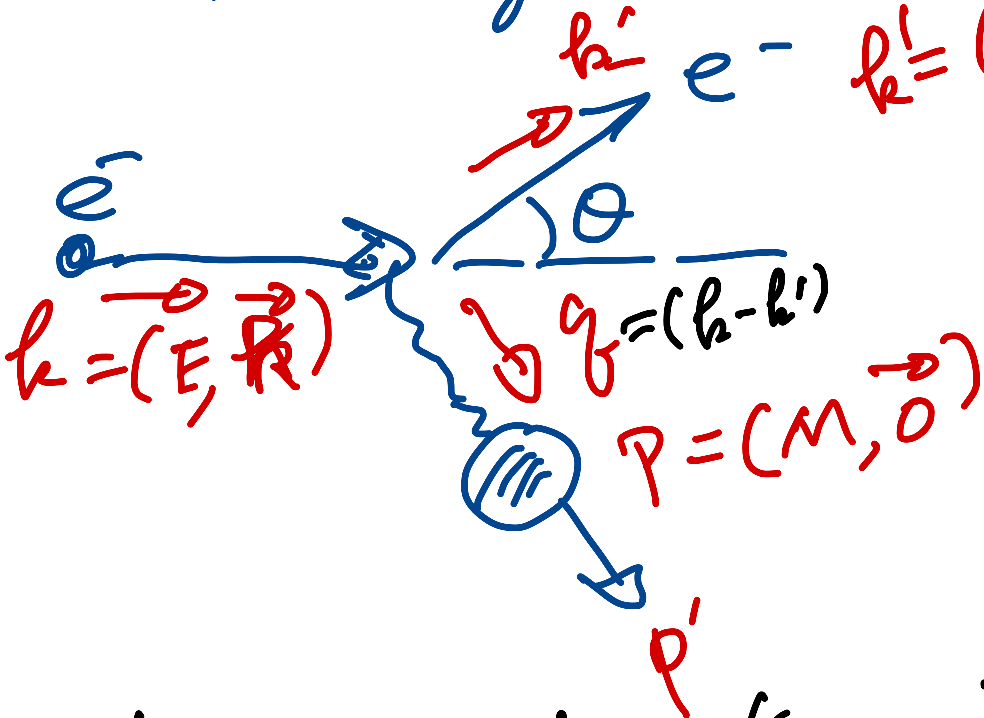
(short distant)
Hard part
Constituent cross section
(Wilson coefficient function)

Soft part
(Long distance part)
Parton distribution function
(parton density)

SLAC-MIT DIS exp. (1967–)

Fixed-target DIS

$$\left(Q \gg m_e,\ m_q,\quad Q \gg m_p \right)$$



In the Lab frame

$$p = (M, \vec{0})$$

There are three kinematic variables (E, E', θ)

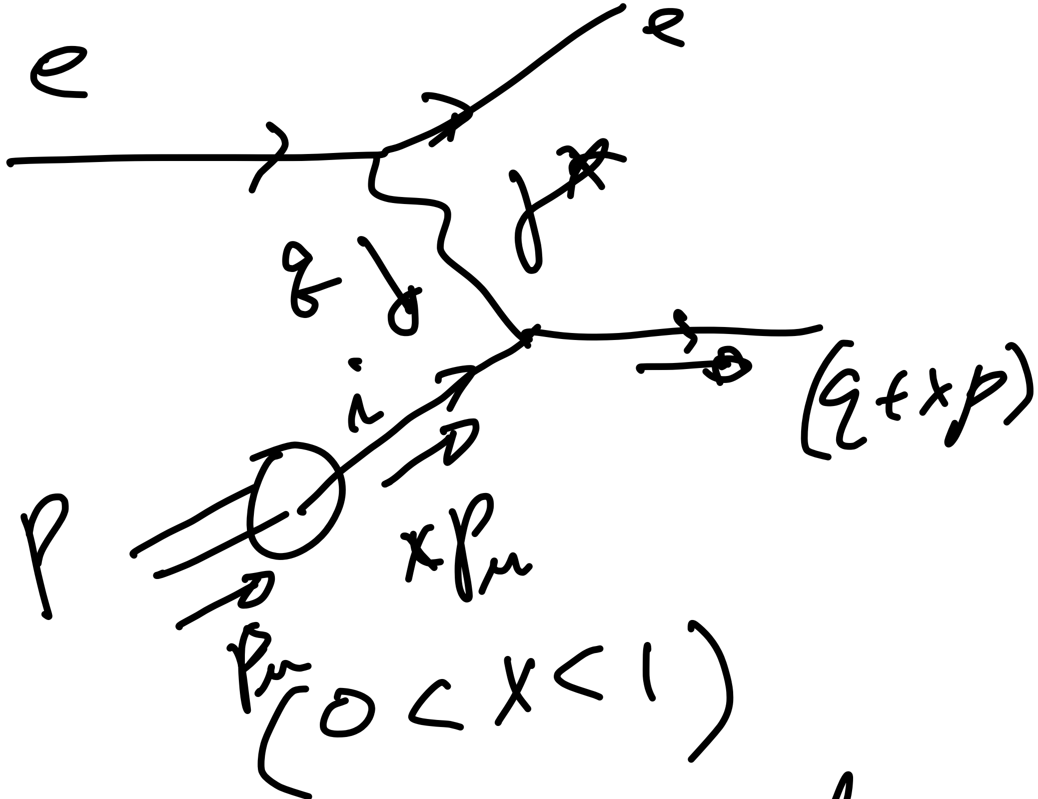
Defin $\quad Q^2 \equiv -q^2 = -(k-k')^2 \qquad (k^2 = k'^2 = 0)$

$$= 2k\cdot k'$$

$$= 2(EE' - \vec{k}\cdot\vec{k}') = 2EE'(1-\cos\theta)$$

$$= 4EE'\sin^2\frac{\theta}{2}$$

Define



(xp) = fraction of momentum of proton carried by the parton i

$(q+xp)^2 = 0$ for massless quark

$= q^2 + 2q\cdot xp + \cancel{x^2p^2} \rightarrow 0$

$\Rightarrow x = \dfrac{-q^2}{2p\cdot q} = \dfrac{Q^2}{2p\cdot q}$

Thus

$$\frac{d\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{xQ^4}\left\{(1-y)F_2(x,Q^2) + (xy^2)F_1(x,Q^2)\right\}$$

where $F_2(x,Q^2) = \sum_i x\, e_i^2 f_i(x)$

$$= 2xF_1(x,Q^2)$$

in Naive parton Model.

$$\left(\text{where } y \equiv \frac{E-E'}{E}\right)$$