Note on Braaten's Paper

Yingsheng Huang

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1 Intro

Hamiltonian [1]:

$$\mathcal{H}=\sum_\sigma \frac{1}{2m}\nabla\psi_\sigma^\dagger\cdot\nabla\psi_\sigma^{(\Lambda)}+\frac{g(\Lambda)}{m}\psi_1^\dagger\psi_2^\dagger\psi_3\psi_4^{(\Lambda)}+\mathcal{V} \tag{1}$$

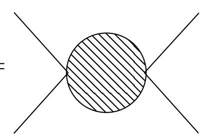
where the renormalized coupling

$$g(\Lambda)=\frac{4\pi a}{1-2a\Lambda/\pi} \tag{2}$$

2 Amplitude

Consider:

$$i\mathcal{A}=\langle 34|\psi^\dagger\psi|12\rangle= \tag{3}$$



Define $P=p_1+p_2=(E,\mathbf{0})$, and $E=p^2/m$. The integral equation is

$$i\mathcal{A}=-\frac{ig(\Lambda)}{m}\left(1+i\mathcal{A}\int\left[\mathrm{d}^4k\right]\frac{i}{k^0-\frac{\mathbf{k}^2}{2m}+i\epsilon}\frac{i}{k^0-p^0-\frac{|\mathbf{k}-\mathbf{p}|^2}{2m}+i\epsilon}\right) \tag{4}$$

The integral gives (redefine $\epsilon\to 2m\epsilon$)

$$\mathcal{I}=\frac{im\left(-\Lambda+\sqrt{-mE-i\epsilon}\tan^{-1}\left(\frac{\Lambda}{\sqrt{-mE-i\epsilon}}\right)\right)}{2\pi^2} \tag{5}$$

and

$$i\mathcal{A}=\frac{-1}{\mathcal{I}+\frac{m}{ig(\Lambda)}}=\frac{-1}{\frac{im\sqrt{-mE-i\epsilon}\tan^{-1}\left(\frac{\Lambda}{\sqrt{-mE-i\epsilon}}\right)}{2\pi^2}-\frac{im}{4\pi a}} \tag{6}$$

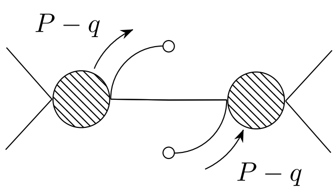
$$\overset{\Lambda\to\infty}{=\!=}\frac{4i\pi/m}{-1/a+\sqrt{-mE-i\epsilon}} \tag{7}$$

Question: Why only infinite bubbles in s-channel are considered? What about other channels?

3 OPE

3.1 l.h.s.

Take what we got in the last section as a new non-perturbative vertex, we only need to deal with tree diagram this way. First we have Figure 2(a) in Braaten's paper:



$$= \langle 34|\psi^\dagger\left(-\frac{\mathbf{r}}{2}\right)\psi\left(\frac{\mathbf{r}}{2}\right)|12\rangle \tag{8}$$

$$= \mathcal{A}^2\int\frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{i}{q^0-\frac{\mathbf{q}^2}{2m}+i\epsilon}\frac{i}{q^0-\frac{E-\mathbf{q}^2}{2m}+i\epsilon}\frac{i}{E-q^0-\frac{\mathbf{q}^2}{2m}+i\epsilon}e^{i\mathbf{q}\cdot\mathbf{r}} \tag{9}$$

$$= -\mathcal{A}^2\int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\frac{m^2 e^{i\mathbf{q}\cdot\mathbf{r}}}{(\mathbf{q}^2-p^2-i\epsilon)^2} \tag{10}$$

$$= -\frac{im^2\mathcal{A}^2e^{ipr}}{8\pi p} \tag{11}$$

References

[1] Eric Braaten and Lucas Platter. Exact relations for a strongly interacting fermi gas from the operator product expansion. *Phys. Rev. Lett.*, 100(20), may 2008.