Notes:Scattering Theory

hys

In the previous study,scattering theory was not been introduced in detail.The textbook just briefly shows that the whole wavefunction,the formal solution of the Schrodinger equation can be described as following:

$$\psi(r,\theta) \approx A\left\{e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right\}, \tag{1}$$

and thus led to the scattering amplitude $f(\theta)$.In the interest of understanding the Lippmann-Schwinger equation,I choose to start from the very beginning of scattering theory.

1 Scattering as a time-dependent perturbation

1.1

Given the Hamiltonian H's form:

$$H = H_0 + V(\boldsymbol{r}),$$

where

$$H_0 = \frac{\boldsymbol{p^2}}{2m},$$

if we consider $V(\boldsymbol{r})$ as a perturbation,and we already know that H_0's eigenvectors is the plane-wave solution,set as $|\boldsymbol{k}\rangle$,the eigenvalue can be easily written as

$$E_{\boldsymbol{k}} = \frac{\hbar^2\boldsymbol{k}^2}{2m}.$$

Assuming $V(\boldsymbol{r})$ is time-independent(from the incoming particle's view,the potential of the scatterer $V(\boldsymbol{r})$ is only visible during the short period of the interaction,so it's logical to consider $V(\boldsymbol{r})$ as a time-independent value),using the time-dependent perturbation theory,the transition amplitude therefore is:

$$\langle n| U_I(t, t0) |i\rangle . \tag{2}$$