Exercises for Lattice Flavour Physics

3. Consider semileptonic $B \to \pi \ell \bar{\nu}_\ell$ decays and write the hadronic matrix element in terms of form-factors as follows:

$$\langle \pi^+(p_\pi) \,|\, \bar{u} \gamma^\mu b \,|\bar{B}^0(p_B)\rangle = f^+(q^2)(p_B + p_\pi)^\mu + f^-(q^2)(p_B - p_\pi)^\mu \,,$$

where $q = p_B - p_\pi$. Show that if the masses of the leptons can be neglected then only the form-factor $f^+(q^2)$ contributes to the amplitude for the process $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell$.

4. Using the HQET at leading order, show that f_P, the leptonic decay constant of a heavy pseudoscalar meson P, scales with the mass of the meson (m_P) as

$$f_P \propto \frac{1}{\sqrt{m_P}} \,.$$