$\bar{c}\gamma^\mu c$ matrix element

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1 $^3S_1$

Ignore the overall factor:

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3S_1\right\rangle = \int \mathrm{d}\Omega\,\mathrm{tr}[\Pi_1\gamma^\mu] \propto \sqrt{2}\pi(\frac{m}{3E}+\frac{2}{3})\epsilon^\mu$$

2 $^3D_1$

The matrix element reads:

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle = \int \mathrm{d}\Omega \sum_{\lambda_1\lambda_2 S_z m} \mathrm{tr}\{\Pi_1\gamma^\mu\} \left\langle 1J_z\middle|2m;1S_z\right\rangle Y_{2m}(\theta,\phi)$$

while the trace part is the same as $^3S_1$:

$$\mathrm{tr}\{\Pi_1\gamma^\mu\} = \frac{\sqrt{2}p^\mu(p\cdot\epsilon)}{E(E+m)}+\epsilon^\mu$$

Chosen polarization vectors:

$$\epsilon^{(-)}=(0,1,-i,0),\epsilon^{(0)}=(0,0,0,-1),\epsilon^{(+)}=(0,1,+i,0)$$

Result (the first row and the last are orthogonal):

$$\begin{pmatrix} \left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(-)} \\ \left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(0)} \\ \left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(+)} \end{pmatrix} = \begin{pmatrix} 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(E-m)}{15E} & 0 \\ 0 & 0 & \frac{2i\sqrt{2\pi}(E-m)}{5E} & \frac{8\sqrt{\pi}(E-m)}{15E} \\ 0 & \frac{2\sqrt{\pi}(m-E)}{15E} & \frac{2i\sqrt{\pi}(m-E)}{15E} & 0 \end{pmatrix}$$

Below $\bar{\epsilon 0},\bar{\epsilon 1},\bar{\epsilon 2}$ stands for spin state $\left|1-\right\rangle,\left|10\right\rangle,\left|1+\right\rangle$:

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(-)} = \int \mathrm{d}\Omega\frac{e^{-2i\phi}\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 0}\right)\left(-3\sqrt{2}e^{i\phi}\sin(2\theta)+3\left(-1+e^{2i\phi}\right)\cos(2\theta)+e^{2i\phi}+3\right)}{8\sqrt{\pi}E(E+m)}$$

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(0)} = \int \mathrm{d}\Omega\frac{-e^{-i\phi}\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 1}\right)\left(6e^{i\phi}\cos^2(\theta)+3\sqrt{2}\left(-1+e^{2i\phi}\right)\sin(\theta)\cos(\theta)-2e^{i\phi}\right)}{4\sqrt{\pi}E(E+m)}$$

$$\left\langle 0\middle|\bar{c}\gamma^\mu c\middle|^3D_1\right\rangle^{(+)} = \int \mathrm{d}\Omega\frac{\overline{p}^\mu\left(\overline{p}\cdot\overline{\epsilon 2}\right)\left(3e^{2i\phi}\sin^2(\theta)+3\sqrt{2}e^{i\phi}\sin(\theta)\cos(\theta)+3\cos^2(\theta)-1\right)}{4\sqrt{\pi}E(E+m)}$$

The four component of the matrix element with spin 0:

$$\left\{0,0,-\frac{2i\sqrt{2\pi}p^2}{5E(E+m)},-\frac{8\sqrt{\pi}p^2}{15E(E+m)}\right\}$$