Annual Assessment Report

Yingsheng Huang (Supervisor: Yu Jia)

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Institute of High Energy Physics

Outlines

1. Operator Product Expansion for Atomic Wave-functions
2. Meson-meson Scattering in 1+1-d QCD
3. Fragmentation Production of Fully-heavy Tetraquark at LHC

Operator Product Expansion for Atomic Wave-functions

Motivation

☐ Universal behaviors in Coulombic wave-functions, near-the-origin divergence in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} + \cdots \ (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} + \cdots \ (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} + \cdots \ (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} + \cdots \ (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{KG}}(r) \propto \begin{cases} 1 - \frac{r}{a_0} + \frac{1}{2}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \frac{r}{a_0} + \frac{3}{8}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \frac{r}{a_0} + \frac{19}{54}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \frac{r}{a_0} + \frac{11}{32}\frac{r^2}{a_0^2} - Z^2\alpha^2 \log\left(\frac{r}{a_0}\right) + Z^2\alpha^2\left(\frac{r}{a_0}\right)\log\left(\frac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

Motivation

- ☐ Universal behaviors in Coulombic wave-functions, near-the-origin divergence in relativistic wave-functions (i.e. Hydrogen atom, Taylor expanded):

$$R_{n0}^{\mathrm{Schr}}(r) \propto \begin{cases} 1 - \dfrac{r}{a_0} + \frac{1}{2}\dfrac{r^2}{a_0^2} + \cdots \ (n=1) \\ 1 - \dfrac{r}{a_0} + \frac{3}{8}\dfrac{r^2}{a_0^2} + \cdots \ (n=2) \\ 1 - \dfrac{r}{a_0} + \frac{19}{54}\dfrac{r^2}{a_0^2} + \cdots \ (n=3) \\ 1 - \dfrac{r}{a_0} + \frac{11}{32}\dfrac{r^2}{a_0^2} + \cdots \ (n=4) \end{cases},$$

$$R_{n0}^{\mathrm{Dirac}}(r) \propto \begin{cases} 1 - \dfrac{r}{a_0} + \frac{1}{2}\dfrac{r^2}{a_0^2} - \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\dfrac{r}{a_0}\right) + \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\dfrac{r}{a_0}\right) \log\left(\dfrac{r}{a_0}\right) + \cdots (n=1) \\ 1 - \dfrac{r}{a_0} + \frac{3}{8}\dfrac{r^2}{a_0^2} - \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\dfrac{r}{a_0}\right) + \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\dfrac{r}{a_0}\right) \log\left(\dfrac{r}{a_0}\right) + \cdots (n=2) \\ 1 - \dfrac{r}{a_0} + \frac{19}{54}\dfrac{r^2}{a_0^2} - \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\dfrac{r}{a_0}\right) + \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\dfrac{r}{a_0}\right) \log\left(\dfrac{r}{a_0}\right) + \cdots (n=3) \\ 1 - \dfrac{r}{a_0} + \frac{11}{32}\dfrac{r^2}{a_0^2} - \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \log\left(\dfrac{r}{a_0}\right) + \dfrac{\mathbf{1}}{\mathbf{2}} Z^2\alpha^2 \left(\dfrac{r}{a_0}\right) \log\left(\dfrac{r}{a_0}\right) + \cdots (n=4) \end{cases}$$

Attack the problem with OPE & EFT: Construct EFT

- ☐ Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus.

- ☐ Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \mathcal{L}_{\text{Max}} + \mathcal{L}_{\text{NRQED}} + \mathcal{L}_{\text{HNET}} + \delta\mathcal{L}_{\text{contact}} \tag{1}$$

where

$$\begin{aligned}
\mathcal{L}_{\text{Max}} &= -\frac{1}{4} d_\gamma F_{\mu\nu} F^{\mu\nu} + \cdots\,,, \\
\mathcal{L}_{\text{NRQED}} &= \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\boldsymbol{\nabla} \cdot \mathbf{E}]}{8m^2} + \cdots \right\}, \\
\mathcal{L}_{\text{HNET}} &= N^\dagger i D_0 N + \cdots\,, \\
\delta\mathcal{L}_{\text{contact}} &= \frac{c_4}{m^2} \psi^\dagger \psi N^\dagger N + \cdots\,,
\end{aligned}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: Construct EFT

- ☐ Use non-relativistic QED (NRQED) for electron and heavy nucleus effective theory (HNET, similar to HQET) for nucleus, keep only Coulomb potential.

- ☐ Lagrangian for non-relativistic atoms:

$$\mathcal{L} = \cancel{\mathcal{L}_{\text{Max}}}\ \mathcal{L}_{\text{Coul}} + \mathcal{L}_{\text{NRQED}} + \mathcal{L}_{\text{HNET}} + \delta\mathcal{L}_{\text{contact}} \tag{1}$$

where

$$\mathcal{L}_{\text{Coul}} = \frac{1}{2}\left(\nabla A^0\right)^2,$$

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{D}^4}{8m^3} + c_D e \frac{[\boldsymbol{\nabla}\cdot\mathbf{E}]}{8m^2} + \cdots \right\},$$

$$\mathcal{L}_{\text{HNET}} = N^\dagger i D_0 N + \cdots,$$

$$\xcancel{\delta\mathcal{L}_{\text{contact}} = \frac{c_4}{m^2}\psi^\dagger\psi N^\dagger N + \cdots,}$$

where $D^\mu = \partial^\mu + ieA^\mu$.

Attack the problem with OPE & EFT: OPE

- Operator Product Expansion (OPE): The limit when product of local operators at different points approach each other.

$$T\phi(x)\phi(0) \sim \sum_{\mathcal{O}} C_{\mathcal{O}}(x^\mu)[\mathcal{O}(0)]_R \tag{2}$$

Correct OPE relation in coordinate space

$$\psi(\mathbf{r})N(\mathbf{0}) = (1 - mZ\alpha|\mathbf{r}|)\,[\psi N](\mathbf{0}) + (1 - mZ\alpha|\mathbf{r}|/2)\mathbf{r}\cdot[\nabla\psi N](\mathbf{0}) + \cdots$$

Correct OPE relation in momentum space

$$\begin{aligned}\widetilde{\psi}(\mathbf{q})N(\mathbf{0}) &\equiv \int d^3\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\psi(\mathbf{r})N(\mathbf{0}) \\ &= \frac{8\pi Z\alpha m}{\mathbf{q}^4}[\psi N](\mathbf{0}) - \frac{16i\pi Z\alpha m}{\mathbf{q}^6}\boldsymbol{q}\cdot[\nabla\psi N](\mathbf{0}) + \cdots .\end{aligned}$$

Reproduce Wave-function origin

☐ With operator definition of the wave-functions

$$\Psi_{nlm}(\mathbf{r}) = \langle 0 \,|\psi(\mathbf{r}) N(\mathbf{0})|\, nlm \rangle \tag{3}$$

Wave-function origin (Schrödinger equation)

$$R_{n0}(r) = R_{n0}(0)\left[1 - \frac{r}{a_0} + \mathcal{O}\left(r/a_0\right)^2\right] \tag{4}$$

☐ Add relativistic corrections in OPE with higher order Lagrangian to account for the logarithms in relativistic wave-functions (Klein-Gordon, Dirac).

☐ Use renormalization group equation to reproduce all leading logarithms.

Meson-meson Scattering in 1+1-d QCD

’t Hooft equation

- ☐ Large-N Expansion
- ☐ In 1+1-d, ONLY PLANAR DIAGRAM!!!

Steps:

1. Obtain mesons’ ’t Hooft wave-functions with ’t Hooft equation (Fig 1).
2. Obtain effective meson-meson vertex function with Bethe-Salpeter equation (Fig 2).
3. Calculate meson-meson scattering amplitude with said vertex functions and wave-functions.

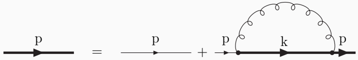


Figure 1: The Dyson-Schwinger equation for the quark self-energy.

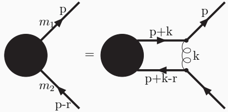


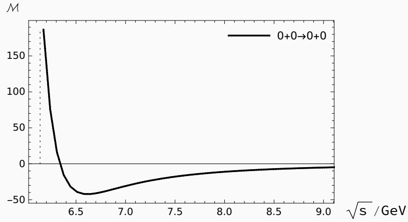
Figure 2: The Bethe-Salpeter equation for the $q\bar{q}$ bound state.

’t Hooft equation

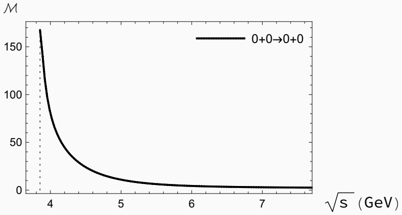
$$\mu^2\varphi(x) = \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right)\varphi(x) - P\int_0^1 dy\,\frac{\varphi(y)}{(x-y)^2}. \tag{5}$$

μ is the mass of the meson, α_i is rescaled quark mass, P marks principle value.

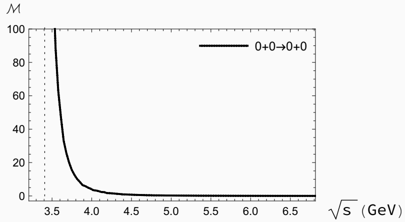
Results (No indication of tetraquark!!!)



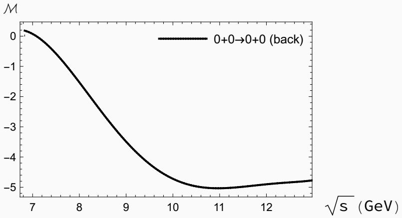
(a) Amplitudes for the contact term in $A(c\bar{c}) + B(c\bar{c}) \to C(c\bar{c}) + D(c\bar{c})$.



(b) Amplitudes for the contact term in $A(c\bar{s}) + B(c\bar{s}) \to C(c\bar{s}) + D(c\bar{s})$.



(c) Amplitudes for the contact term in $A(c\bar{u}) + B(c\bar{d}) \to C(c\bar{u}) + D(c\bar{d})$.

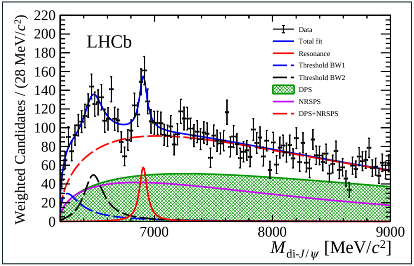


(d) Amplitudes for the contact term in $A(c\bar{d}) + B(b\bar{s}) \to C(b\bar{d}) + D(c\bar{s})$ with particle B moving backwards. No near-threshold enhancement.

Fragmentation Production of Fully-heavy Tetraquark at LHC

Factorization theorem for $T_{4c/b}$ production

- LHCb discovered a narrow structure near 6.9 GeV in the di-J/ψ invariant mass spectrum ($> 5\sigma$): $X(6900)$.
- Strong candidate for fully-charmed tetraquark.



- QCD factorization theorem for fully-heavy tetraquark ($T_{4c/b}$) exclusive production at high-p_T

$$
\begin{aligned}
\mathrm{d}\sigma\left(pp \to T_{4c/b}\left(p_{\mathrm{T}}\right)+X\right) = \sum_i \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int_0^1 \mathrm{d}z \; f_{a/p}(x_a,\mu) f_{b/p}(x_b,\mu) \\
\times \mathrm{d}\hat{\sigma}(ab \to i(p_T/z)+X,\mu) D_{i\to T_{4c/b}}(z,\mu) + \mathcal{O}(1/p_T).
\end{aligned}
\tag{6}
$$

- Dominate partonic channel is $gg \to gg$, rather than $gg \to q\bar{q}$.

Fragmentation Function

Collins-Soper definition of fragmentation function:

$$D_{g\to T_{4c}}(z,\mu)=\frac{-g_{\mu\nu}z^{d-3}}{2\pi k^{+}\left(N_c^2-1\right)(d-2)}\int_{-\infty}^{+\infty}dx^{-}\,e^{-ik^{+}x^{-}}$$

$$\times\sum_{X}\left\langle 0\left|G_c^{+\mu}(0)\mathcal{E}^{\dagger}\left(0,0,\mathbf{0}_{\perp}\right)_{cb}\left|T_{4c}(P)+X\right\rangle\left\langle T_{4c}(P)+X\right|\mathcal{E}\left(0,x^{-},\mathbf{0}_{\perp}\right)_{ba}G_a^{+\nu}\left(0,x^{-},\mathbf{0}_{\perp}\right)\right|0\right\rangle$$

- ☐ NRQCD factorization:

$$D_{g\to H}(z)=\sum_{n}d_n(z)\left\langle 0\left|\mathcal{O}_n^H\right|0\right\rangle$$

- ☐ Perturbative matching to determine short distance coefficients.
- ☐ Use wave-function origin (S-wave) from potential models to determine long range matrix elements in order to yield a phenomenological result.
- ☐ More details in Jia-Yue Zhang's talk this afternoon.

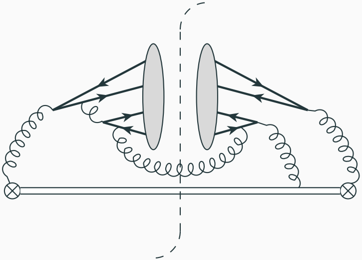
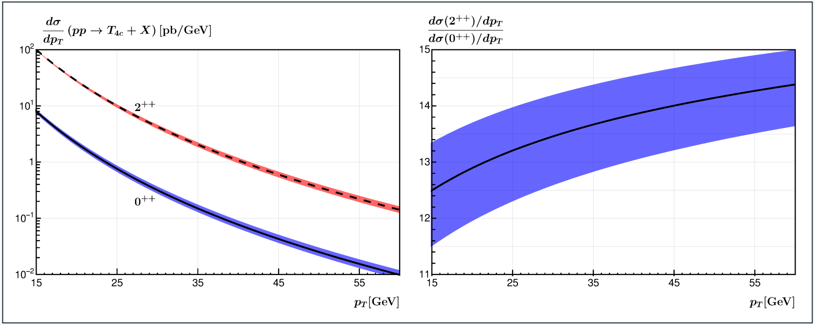
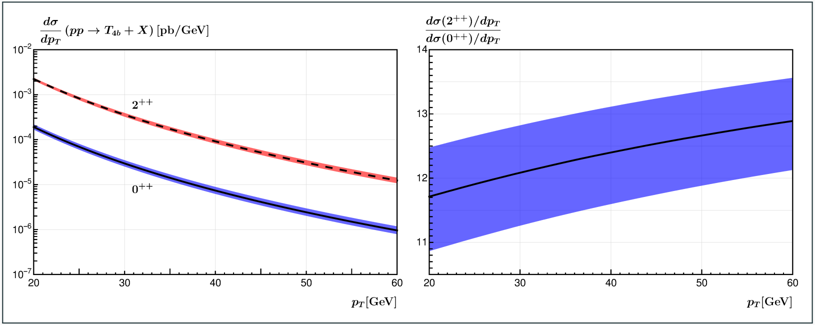


Figure 3: A representative Feynman diagram for the fragmentation function of gluon into T_{4c}. The grey blob indicates the C-even tetraquark. Horizontal double line denotes the eikonal line.

Phenomenology for $T_{4c/b}$ production at LHC





Phenomenology for $T_{4c/b}$ production at LHC

- ☐ 2^{++} cross section is about 10 times larger than 0^{++}.
- ☐ We obtain the yields of the accumulated event number for T_{4c} at HL-LHC are a hundred million for 0^{++} and 8 hundreds million for 2^{++} (with integrated luminosity 3000 fb^{-1}).
- ☐ The prediction for T_{4b} is highly suppressed, mainly due to the relative larger bottom mass suppression.
- ☐ The total cross section we obtained is unreliable mainly due to the fact that fragmentation only works at high-p_T, and our integration is done within approximately $15 \leq p_T \leq 60\mathrm{GeV}$.

Publications

Publications?

- Huang, Y., Jia, Y., & Yu, R. (2018a). Deciphering the coalescence behavior of Coulomb-Schrödinger atomic wave functions from operator product expansion.arXiv 1809.09023 **(rejected by PRL, waiting for resubmission)**
- Huang, Y., Jia, Y., & Yu, R. (2018b). Near-the-origin divergence of Klein-Gordon wave functions for hydrogen-like atoms and operator product expansion.arXiv 1812.11957 **(Submitted to PRD, referee comments received)**
- Huang, Y., Jia, Y., & Yu, R. (2019). Near-the-origin divergence of Dirac wave functions of hydrogen and operator product expansion.arXiv 1901.04971 **(rejected by PRL, waiting for appeal)**
- Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391 **(Submitted to PRD and received positive response.)**
- Feng, F., Huang, Y., Jia, Y., Sang, W.-L., Xiong, X., & Zhang, J.-Y. (2020). Fragmentation production of fully-charmed tetraquarks at lhc.arXiv 2009.08450 **(To be submitted to PRL)**

Questions?

Backup

References

Chen, G.-Y., Huang, Y., Jia, Y., & Rui, Y. (2019). Meson-meson scattering in two-dimensional qcd.arXiv 1904.13391.

Feng, F., Huang, Y., Jia, Y., Sang, W.-L., Xiong, X., & Zhang, J.-Y. (2020). Fragmentation production of fully-charmed tetraquarks at lhc.arXiv 2009.08450.

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